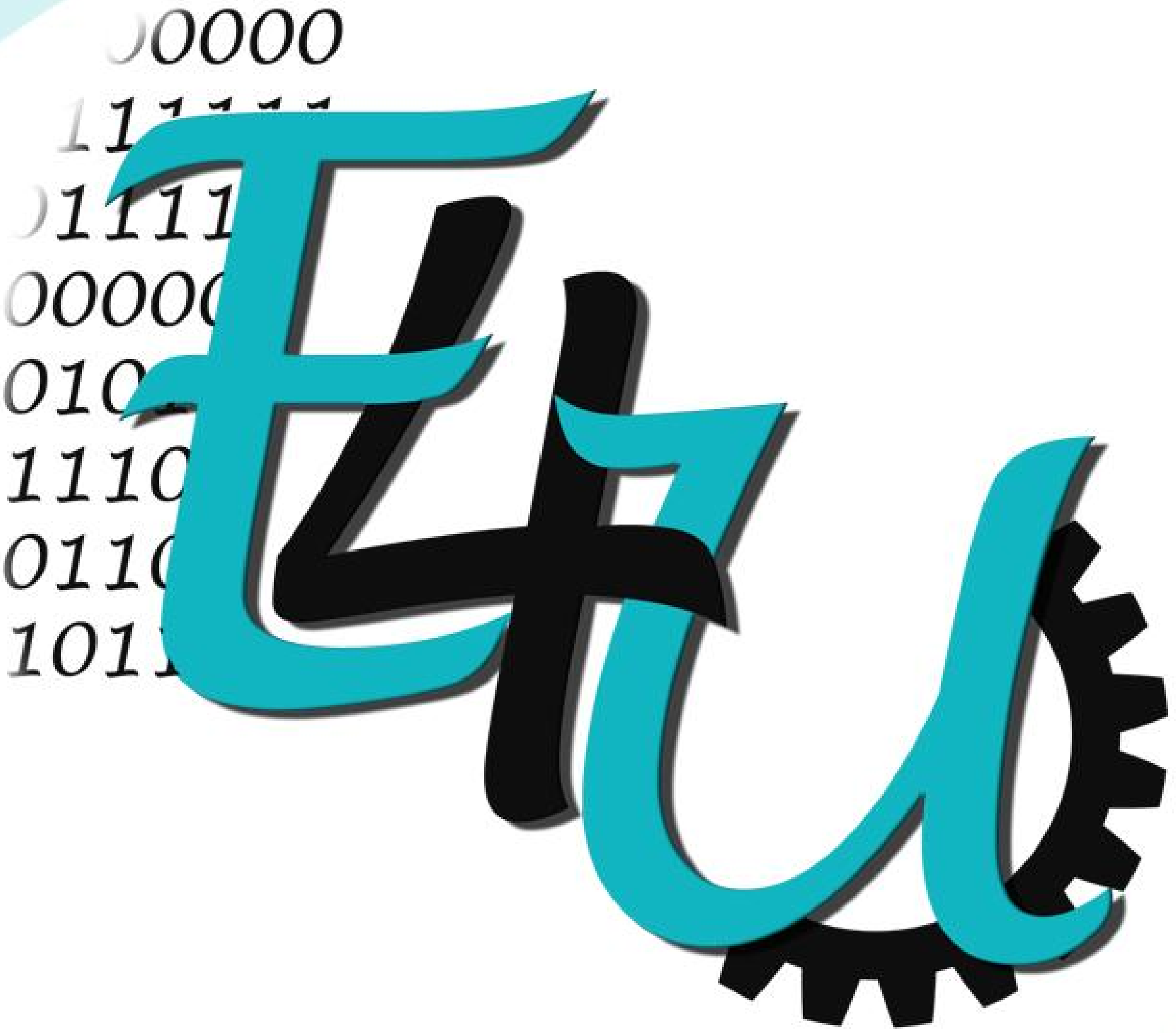

# Engineers For You

# لجنة كلية الهندسة

**الاتجاه الاسلامي – الجامعة الهاشمية**

INSTRUCTOR'S SOLUTIONS MANUAL

FOR

SERWAY AND JEWETT'S

# PHYSICS

## FOR SCIENTISTS AND ENGINEERS

SIXTH EDITION

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# 1

# Physics and Measurement

## CHAPTER OUTLINE

1.1 Standards of Length, Mass, and Time
1.2 Matter and Model-Building
1.3 Density and Atomic Mass
1.4 Dimensional Analysis
1.5 Conversion of Units
1.6 Estimates and Order-of-Magnitude Calculations
1.7 Significant Figures

## ANSWERS TO QUESTIONS

**Q1.1** Atomic clocks are based on electromagnetic waves which atoms emit. Also, pulsars are highly regular astronomical clocks.

**Q1.2** Density varies with temperature and pressure. It would be necessary to measure both mass and volume very accurately in order to use the density of water as a standard.

**Q1.3** People have different size hands. Defining the unit precisely would be cumbersome.

**Q1.4** (a) 0.3 millimeters (b) 50 microseconds (c) 7.2 kilograms

**Q1.5** (b) and (d). You cannot add or subtract quantities of different dimension.

**Q1.6** A dimensionally correct equation need not be true. Example: 1 chimpanzee = 2 chimpanzee is dimensionally correct. If an equation is not dimensionally correct, it cannot be correct.

**Q1.7** If I were a runner, I might walk or run $10^1$ miles per day. Since I am a college professor, I walk about $10^0$ miles per day. I drive about 40 miles per day on workdays and up to 200 miles per day on vacation.

**Q1.8** On February 7, 2001, I am 55 years and 39 days old.

$$55\text{ yr}\left(\frac{365.25\text{ d}}{1\text{ yr}}\right)+39\text{ d}=20\,128\text{ d}\left(\frac{86\,400\text{ s}}{1\text{ d}}\right)=1.74\times10^{9}\text{ s}\sim10^{9}\text{ s}.$$

Many college students are just approaching 1 Gs.

**Q1.9** Zero digits. An order-of-magnitude calculation is accurate only within a factor of 10.

**Q1.10** The mass of the forty-six chapter textbook is on the order of $10^0$ kg .

**Q1.11** With one datum known to one significant digit, we have 80 million yr + 24 yr = 80 million yr.

## SOLUTIONS TO PROBLEMS

### Section 1.1 **Standards of Length, Mass, and Time**

No problems in this section

### Section 1.2 **Matter and Model-Building**

**P1.1** From the figure, we may see that the spacing between diagonal planes is half the distance between diagonally adjacent atoms on a flat plane. This diagonal distance may be obtained from the Pythagorean theorem, $L_{\text{diag}} = \sqrt{L^2 + L^2}$ . Thus, since the atoms are separated by a distance $L = 0.200\ \text{nm}$, the diagonal planes are separated by $\frac{1}{2}\sqrt{L^2 + L^2} = \boxed{0.141\ \text{nm}}$ .

### Section 1.3 **Density and Atomic Mass**

***P1.2** Modeling the Earth as a sphere, we find its volume as $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(6.37 \times 10^6\ \text{m}\right)^3 = 1.08 \times 10^{21}\ \text{m}^3$. Its density is then $\rho = \frac{m}{V} = \frac{5.98 \times 10^{24}\ \text{kg}}{1.08 \times 10^{21}\ \text{m}^3} = \boxed{5.52 \times 10^3\ \text{kg/m}^3}$ . This value is intermediate between the tabulated densities of aluminum and iron. Typical rocks have densities around 2 000 to 3 000 $\text{kg/m}^3$ . The average density of the Earth is significantly higher, so higher-density material must be down below the surface.

**P1.3** With $V = (\text{base area})(\text{height})$ $V = \left(\pi r^2\right)h$ and $\rho = \frac{m}{V}$, we have

$$\rho = \frac{m}{\pi r^2 h} = \frac{1\ \text{kg}}{\pi (19.5\ \text{mm})^2 (39.0\ \text{mm})}\left(\frac{10^9\ \text{mm}^3}{1\ \text{m}^3}\right)$$

$$\rho = \boxed{2.15 \times 10^4\ \text{kg/m}^3}.$$

***P1.4** Let *V* represent the volume of the model, the same in $\rho = \frac{m}{V}$ for both. Then $\rho_{\text{iron}} = 9.35\ \text{kg}/V$ and $\rho_{\text{gold}} = \frac{m_{\text{gold}}}{V}$. Next, $\frac{\rho_{\text{gold}}}{\rho_{\text{iron}}} = \frac{m_{\text{gold}}}{9.35\ \text{kg}}$ and $m_{\text{gold}} = 9.35\ \text{kg}\left(\frac{19.3 \times 10^3\ \text{kg/m}^3}{7.86 \times 10^3\ \text{kg/m}^3}\right) = \boxed{23.0\ \text{kg}}$.

**P1.5** $V = V_o - V_i = \frac{4}{3}\pi\left(r_2^3 - r_1^3\right)$

$\rho = \frac{m}{V}$, so $m = \rho V = \rho\left(\frac{4}{3}\pi\right)\left(r_2^3 - r_1^3\right) = \boxed{\frac{4\pi\rho\left(r_2^3 - r_1^3\right)}{3}}$.

**P1.6** For either sphere the volume is $V = \frac{4}{3}\pi r^3$ and the mass is $m = \rho V = \rho\frac{4}{3}\pi r^3$. We divide this equation for the larger sphere by the same equation for the smaller:

$$\frac{m_\ell}{m_s} = \frac{\rho 4\pi r_\ell^3 3}{\rho 4\pi r_s^3 3} = \frac{r_\ell^3}{r_s^3} = 5.$$

Then $r_\ell = r_s\sqrt[3]{5} = 4.50\text{ cm}(1.71) = \boxed{7.69\text{ cm}}$.

**P1.7** Use $1\text{ u} = 1.66 \times 10^{-24}\text{ g}$.

(a) For He, $m_0 = 4.00\text{ u}\left(\frac{1.66 \times 10^{-24}\text{ g}}{1\text{ u}}\right) = \boxed{6.64 \times 10^{-24}\text{ g}}$.

(b) For Fe, $m_0 = 55.9\text{ u}\left(\frac{1.66 \times 10^{-24}\text{ g}}{1\text{ u}}\right) = \boxed{9.29 \times 10^{-23}\text{ g}}$.

(c) For Pb, $m_0 = 207\text{ u}\left(\frac{1.66 \times 10^{-24}\text{ g}}{1\text{ u}}\right) = \boxed{3.44 \times 10^{-22}\text{ g}}$.

***P1.8** (a) The mass of any sample is the number of atoms in the sample times the mass $m_0$ of one atom: $m = Nm_0$. The first assertion is that the mass of one aluminum atom is

$$m_0 = 27.0\text{ u} = 27.0\text{ u} \times 1.66 \times 10^{-27}\text{ kg/1 u} = 4.48 \times 10^{-26}\text{ kg}.$$

Then the mass of $6.02 \times 10^{23}$ atoms is

$$m = Nm_0 = 6.02 \times 10^{23} \times 4.48 \times 10^{-26}\text{ kg} = 0.027\,0\text{ kg} = 27.0\text{ g}.$$

Thus the first assertion implies the second. Reasoning in reverse, the second assertion can be written $m = Nm_0$.

$$0.027\,0\text{ kg} = 6.02 \times 10^{23} m_0,\text{ so } m_0 = \frac{0.027\text{ kg}}{6.02 \times 10^{23}} = 4.48 \times 10^{-26}\text{ kg},$$

in agreement with the first assertion.

(b) The general equation $m = Nm_0$ applied to one mole of any substance gives $M\text{ g} = NM\text{ u}$, where $M$ is the numerical value of the atomic mass. It divides out exactly for all substances, giving $1.000\,000\,0 \times 10^{-3}\text{ kg} = N1.660\,540\,2 \times 10^{-27}\text{ kg}$. With eight-digit data, we can be quite sure of the result to seven digits. For one mole the number of atoms is

$$N = \left(\frac{1}{1.660\,540\,2}\right)10^{-3+27} = \boxed{6.022\,137 \times 10^{23}}.$$

(c) The atomic mass of hydrogen is 1.008 0 u and that of oxygen is 15.999 u. The mass of one molecule of $H_2O$ is $2(1.008\,0) + 15.999\text{ u} = 18.0\text{ u}$. Then the molar mass is $\boxed{18.0\text{ g}}$.

(d) For $CO_2$ we have $12.011\text{ g} + 2(15.999\text{ g}) = \boxed{44.0\text{ g}}$ as the mass of one mole.

**P1.9** Mass of gold abraded: $|\Delta m| = 3.80\text{ g} - 3.35\text{ g} = 0.45\text{ g} = (0.45\text{ g})\left(\frac{1\text{ kg}}{10^3\text{ g}}\right) = 4.5\times10^{-4}\text{ kg}$.

Each atom has mass $m_0 = 197\text{ u} = 197\text{ u}\left(\frac{1.66\times10^{-27}\text{ kg}}{1\text{ u}}\right) = 3.27\times10^{-25}\text{ kg}$.

Now, $|\Delta m| = |\Delta N|m_0$, and the number of atoms missing is

$$|\Delta N| = \frac{|\Delta m|}{m_0} = \frac{4.5\times10^{-4}\text{ kg}}{3.27\times10^{-25}\text{ kg}} = 1.38\times10^{21}\text{ atoms}.$$

The rate of loss is

$$\frac{|\Delta N|}{\Delta t} = \frac{1.38\times10^{21}\text{ atoms}}{50\text{ yr}}\left(\frac{1\text{ yr}}{365.25\text{ d}}\right)\left(\frac{1\text{ d}}{24\text{ h}}\right)\left(\frac{1\text{ h}}{60\text{ min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)$$

$$\frac{|\Delta N|}{\Delta t} = \boxed{8.72\times10^{11}\text{ atoms/s}}.$$

**P1.10** (a) $m = \rho L^3 = (7.86\text{ g/cm}^3)(5.00\times10^{-6}\text{ cm})^3 = \boxed{9.83\times10^{-16}\text{ g}} = 9.83\times10^{-19}\text{ kg}$

(b) $N = \frac{m}{m_0} = \frac{9.83\times10^{-19}\text{ kg}}{55.9\text{ u}(1.66\times10^{-27}\text{ kg/1 u})} = \boxed{1.06\times10^7\text{ atoms}}$

**P1.11** (a) The cross-sectional area is

$$A = 2(0.150\text{ m})(0.010\text{ m}) + (0.340\text{ m})(0.010\text{ m}) = 6.40\times10^{-3}\text{ m}^2.$$

The volume of the beam is

$$V = AL = (6.40\times10^{-3}\text{ m}^2)(1.50\text{ m}) = 9.60\times10^{-3}\text{ m}^3.$$

Thus, its mass is

$$m = \rho V = (7.56\times10^3\text{ kg/m}^3)(9.60\times10^{-3}\text{ m}^3) = \boxed{72.6\text{ kg}}.$$

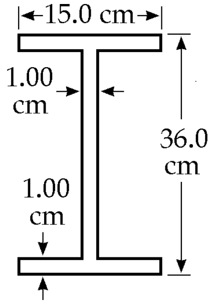

**FIG. P1.11**

(b) The mass of one typical atom is $m_0 = (55.9\text{ u})\left(\frac{1.66\times10^{-27}\text{ kg}}{1\text{ u}}\right) = 9.28\times10^{-26}\text{ kg}$. Now $m = Nm_0$ and the number of atoms is $N = \frac{m}{m_0} = \frac{72.6\text{ kg}}{9.28\times10^{-26}\text{ kg}} = \boxed{7.82\times10^{26}\text{ atoms}}$.

**P1.12** (a) The mass of one molecule is $m_0 = 18.0\ \text{u}\left(\frac{1.66\times10^{-27}\ \text{kg}}{1\ \text{u}}\right) = 2.99\times10^{-26}\ \text{kg}$. The number of molecules in the pail is

$$N_{\text{pail}} = \frac{m}{m_0} = \frac{1.20\ \text{kg}}{2.99\times10^{-26}\ \text{kg}} = \boxed{4.02\times10^{25}\ \text{molecules}}.$$

(b) Suppose that enough time has elapsed for thorough mixing of the hydrosphere.

$$N_{\text{both}} = N_{\text{pail}}\left(\frac{m_{\text{pail}}}{M_{\text{total}}}\right) = (4.02\times10^{25}\ \text{molecules})\left(\frac{1.20\ \text{kg}}{1.32\times10^{21}\ \text{kg}}\right),$$

or

$$N_{\text{both}} = \boxed{3.65\times10^{4}\ \text{molecules}}.$$

## Section 1.4 **Dimensional Analysis**

**P1.13** The term $x$ has dimensions of L, $a$ has dimensions of $\text{LT}^{-2}$, and $t$ has dimensions of T. Therefore, the equation $x = ka^m t^n$ has dimensions of

$$\text{L} = \left(\text{LT}^{-2}\right)^m (\text{T})^n \text{ or } \text{L}^1\text{T}^0 = \text{L}^m\text{T}^{n-2m}.$$

The powers of L and T must be the same on each side of the equation. Therefore,

$$\text{L}^1 = \text{L}^m \text{ and } \boxed{m = 1}.$$

Likewise, equating terms in T, we see that $n - 2m$ must equal 0. Thus, $\boxed{n = 2}$. The value of $k$, a dimensionless constant, $\boxed{\text{cannot be obtained by dimensional analysis}}$.

***P1.14** (a) Circumference has dimensions of L.

(b) Volume has dimensions of $\text{L}^3$.

(c) Area has dimensions of $\text{L}^2$.

Expression (i) has dimension $\text{L}\left(\text{L}^2\right)^{1/2} = \text{L}^2$, so this must be area (c).
Expression (ii) has dimension L, so it is (a).
Expression (iii) has dimension $\text{L}\left(\text{L}^2\right) = \text{L}^3$, so it is (b). Thus, $\boxed{\text{(a)} = \text{ii};\quad \text{(b)} = \text{iii},\ \text{(c)} = \text{i}}$.

**P1.15** (a) $\boxed{\text{This is incorrect}}$ since the units of $[ax]$ are $\mathrm{m^2/s^2}$, while the units of $[v]$ are m/s.

(b) $\boxed{\text{This is correct}}$ since the units of $[y]$ are m, and $\cos(kx)$ is dimensionless if $[k]$ is in $\mathrm{m^{-1}}$.

***P1.16** (a) $a \propto \frac{\sum F}{m}$ or $a = k\frac{\sum F}{m}$ represents the proportionality of acceleration to resultant force and the inverse proportionality of acceleration to mass. If $k$ has no dimensions, we have

$$[a]=[k]\frac{[F]}{[m]},\ \frac{\mathrm{L}}{\mathrm{T}^2}=1\frac{[\mathrm{F}]}{\mathrm{M}},\ [\mathrm{F}]=\boxed{\frac{\mathrm{M}\cdot\mathrm{L}}{\mathrm{T}^2}}.$$

(b) In units, $\frac{\mathrm{M}\cdot\mathrm{L}}{\mathrm{T}^2}=\frac{\mathrm{kg}\cdot\mathrm{m}}{\mathrm{s}^2}$, so $\boxed{1\ \text{newton}=1\ \mathrm{kg\cdot m/s^2}}$.

**P1.17** Inserting the proper units for everything except $G$,

$$\left[\frac{\mathrm{kg\ m}}{\mathrm{s}^2}\right]=\frac{G[\mathrm{kg}]^2}{[\mathrm{m}]^2}.$$

Multiply both sides by $[\mathrm{m}]^2$ and divide by $[\mathrm{kg}]^2$; the units of $G$ are $\boxed{\frac{\mathrm{m}^3}{\mathrm{kg}\cdot\mathrm{s}^2}}$.

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## Section 1.5 **Conversion of Units**

***P1.18** Each of the four walls has area $(8.00\ \mathrm{ft})(12.0\ \mathrm{ft})=96.0\ \mathrm{ft}^2$. Together, they have area $4\left(96.0\ \mathrm{ft}^2\right)\left(\frac{1\ \mathrm{m}}{3.28\ \mathrm{ft}}\right)^2=\boxed{35.7\ \mathrm{m}^2}$.

**P1.19** Apply the following conversion factors:

$$1\ \mathrm{in}=2.54\ \mathrm{cm},\ 1\ \mathrm{d}=86\,400\ \mathrm{s},\ 100\ \mathrm{cm}=1\ \mathrm{m},\ \text{and}\ 10^9\ \mathrm{nm}=1\ \mathrm{m}$$

$$\left(\frac{1}{32}\ \mathrm{in/day}\right)\frac{(2.54\ \mathrm{cm/in})\left(10^{-2}\ \mathrm{m/cm}\right)\left(10^9\ \mathrm{nm/m}\right)}{86\,400\ \mathrm{s/day}}=\boxed{9.19\ \mathrm{nm/s}}.$$

This means the proteins are assembled at a rate of many layers of atoms each second!

***P1.20** $8.50\ \mathrm{in}^3=8.50\ \mathrm{in}^3\left(\frac{0.025\,4\ \mathrm{m}}{1\ \mathrm{in}}\right)^3=\boxed{1.39\times10^{-4}\ \mathrm{m}^3}$

**P1.21** *Conceptualize*: We must calculate the area and convert units. Since a meter is about 3 feet, we should expect the area to be about $A \approx (30\text{ m})(50\text{ m}) = 1\,500\text{ m}^2$.

*Categorize*: We model the lot as a perfect rectangle to use Area = Length × Width. Use the conversion: $1\text{ m} = 3.281\text{ ft}$.

*Analyze*: $A = LW = (100\text{ ft})\left(\dfrac{1\text{ m}}{3.281\text{ ft}}\right)(150\text{ ft})\left(\dfrac{1\text{ m}}{3.281\text{ ft}}\right) = 1\,390\text{ m}^2 = \boxed{1.39\times 10^3\text{ m}^2}$.

*Finalize*: Our calculated result agrees reasonably well with our initial estimate and has the proper units of $\text{m}^2$. Unit conversion is a common technique that is applied to many problems.

**P1.22** (a) $V = (40.0\text{ m})(20.0\text{ m})(12.0\text{ m}) = 9.60\times 10^3\text{ m}^3$
$V = 9.60\times 10^3\text{ m}^3(3.28\text{ ft}/1\text{ m})^3 = \boxed{3.39\times 10^5\text{ ft}^3}$

(b) The mass of the air is

$$m = \rho_{\text{air}}V = \left(1.20\text{ kg/m}^3\right)\left(9.60\times 10^3\text{ m}^3\right) = 1.15\times 10^4\text{ kg}.$$

The student must look up weight in the index to find

$$F_g = mg = \left(1.15\times 10^4\text{ kg}\right)\left(9.80\text{ m/s}^2\right) = 1.13\times 10^5\text{ N}.$$

Converting to pounds,

$$F_g = \left(1.13\times 10^5\text{ N}\right)\left(1\text{ lb}/4.45\text{ N}\right) = \boxed{2.54\times 10^4\text{ lb}}.$$

**P1.23** (a) Seven minutes is 420 seconds, so the rate is

$$r = \frac{30.0\text{ gal}}{420\text{ s}} = \boxed{7.14\times 10^{-2}\text{ gal/s}}.$$

(b) Converting gallons first to liters, then to $\text{m}^3$,

$$r = \left(7.14\times 10^{-2}\text{ gal/s}\right)\left(\frac{3.786\text{ L}}{1\text{ gal}}\right)\left(\frac{10^{-3}\text{ m}^3}{1\text{ L}}\right)$$
$$r = \boxed{2.70\times 10^{-4}\text{ m}^3/\text{s}}.$$

(c) At that rate, to fill a 1-$\text{m}^3$ tank would take

$$t = \left(\frac{1\text{ m}^3}{2.70\times 10^{-4}\text{ m}^3/\text{s}}\right)\left(\frac{1\text{ h}}{3\,600}\right) = \boxed{1.03\text{ h}}.$$

***P1.24** (a) Length of Mammoth Cave $= 348\ \text{mi}\left(\dfrac{1.609\ \text{km}}{1\ \text{mi}}\right) = \boxed{560\ \text{km} = 5.60\times 10^{5}\ \text{m} = 5.60\times 10^{7}\ \text{cm}}$.

(b) Height of Ribbon Falls $= 1\,612\ \text{ft}\left(\dfrac{0.304\,8\ \text{m}}{1\ \text{ft}}\right) = \boxed{491\ \text{m} = 0.491\ \text{km} = 4.91\times 10^{4}\ \text{cm}}$.

(c) Height of Denali $= 20\,320\ \text{ft}\left(\dfrac{0.304\,8\ \text{m}}{1\ \text{ft}}\right) = \boxed{6.19\ \text{km} = 6.19\times 10^{3}\ \text{m} = 6.19\times 10^{5}\ \text{cm}}$.

(d) Depth of King's Canyon $= 8\,200\ \text{ft}\left(\dfrac{0.304\,8\ \text{m}}{1\ \text{ft}}\right) = \boxed{2.50\ \text{km} = 2.50\times 10^{3}\ \text{m} = 2.50\times 10^{5}\ \text{cm}}$.

**P1.25** From Table 1.5, the density of lead is $1.13\times 10^{4}\ \text{kg/m}^{3}$, so we should expect our calculated value to be close to this number. This density value tells us that lead is about 11 times denser than water, which agrees with our experience that lead sinks.

Density is defined as mass per volume, in $\rho = \dfrac{m}{V}$. We must convert to SI units in the calculation.

$$\rho = \frac{23.94\ \text{g}}{2.10\ \text{cm}^{3}}\left(\frac{1\ \text{kg}}{1\,000\ \text{g}}\right)\left(\frac{100\ \text{cm}}{1\ \text{m}}\right)^{3} = \boxed{1.14\times 10^{4}\ \text{kg/m}^{3}}$$

At one step in the calculation, we note that *one million* cubic centimeters make one cubic meter. Our result is indeed close to the expected value. Since the last reported significant digit is not certain, the difference in the two values is probably due to measurement uncertainty and should not be a concern. One important common-sense check on density values is that objects which sink in water must have a density greater than $1\ \text{g/cm}^{3}$, and objects that float must be less dense than water.

**P1.26** It is often useful to remember that the 1 600-m race at track and field events is approximately 1 mile in length. To be precise, there are 1 609 meters in a mile. Thus, 1 acre is equal in area to

$$(1\ \text{acre})\left(\frac{1\ \text{mi}^{2}}{640\ \text{acres}}\right)\left(\frac{1\,609\ \text{m}}{\text{mi}}\right)^{2} = \boxed{4.05\times 10^{3}\ \text{m}^{2}}.$$

***P1.27** The weight flow rate is $1\,200\ \dfrac{\text{ton}}{\text{h}}\left(\dfrac{2\,000\ \text{lb}}{\text{ton}}\right)\left(\dfrac{1\ \text{h}}{60\ \text{min}}\right)\left(\dfrac{1\ \text{min}}{60\ \text{s}}\right) = \boxed{667\ \text{lb/s}}$.

**P1.28** $1\ \text{mi} = 1\,609\ \text{m} = 1.609\ \text{km}$; thus, to go from mph to km/h, multiply by 1.609.

(a) $1\ \text{mi/h} = \boxed{1.609\ \text{km/h}}$

(b) $55\ \text{mi/h} = \boxed{88.5\ \text{km/h}}$

(c) $65\ \text{mi/h} = 104.6\ \text{km/h}$. Thus, $\Delta v = \boxed{16.1\ \text{km/h}}$.

**P1.29** (a) $$\left(\frac{6\times10^{12}\ \$}{1\,000\ \$/\text{s}}\right)\left(\frac{1\ \text{h}}{3\,600\ \text{s}}\right)\left(\frac{1\ \text{day}}{24\ \text{h}}\right)\left(\frac{1\ \text{yr}}{365\ \text{days}}\right)=\boxed{190\ \text{years}}$$

(b) The circumference of the Earth at the equator is $2\pi\left(6.378\times10^3\ \text{m}\right)=4.01\times10^7\ \text{m}$. The length of one dollar bill is 0.155 m so that the length of 6 trillion bills is $9.30\times10^{11}\ \text{m}$. Thus, the 6 trillion dollars would encircle the Earth

$$\frac{9.30\times10^{11}\ \text{m}}{4.01\times0^7\ \text{m}}=\boxed{2.32\times10^4\ \text{times}}.$$

**P1.30** $$N_{\text{atoms}}=\frac{m_{\text{Sun}}}{m_{\text{atom}}}=\frac{1.99\times10^{30}\ \text{kg}}{1.67\times10^{-27}\ \text{kg}}=\boxed{1.19\times10^{57}\ \text{atoms}}$$

**P1.31** $$V=At\ \text{so}\ t=\frac{V}{A}=\frac{3.78\times10^{-3}\ \text{m}^3}{25.0\ \text{m}^2}=\boxed{1.51\times10^{-4}\ \text{m (or 151}\ \mu\text{m)}}$$

**P1.32** $$V=\frac{1}{3}Bh=\frac{\left[(13.0\ \text{acres})\left(43\,560\ \text{ft}^2/\text{acre}\right)\right]}{3}(481\ \text{ft})$$
$$=9.08\times10^7\ \text{ft}^3,$$

or

$$V=\left(9.08\times10^7\ \text{ft}^3\right)\left(\frac{2.83\times10^{-2}\ \text{m}^3}{1\ \text{ft}^3}\right)$$
$$=\boxed{2.57\times10^6\ \text{m}^3}$$

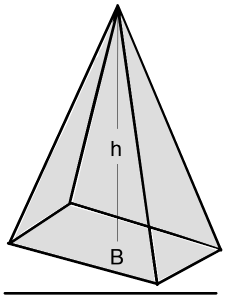

FIG. P1.32

**P1.33** $$F_g=(2.50\ \text{tons/block})\left(2.00\times10^6\ \text{blocks}\right)(2\,000\ \text{lb/ton})=\boxed{1.00\times10^{10}\ \text{lbs}}$$

***P1.34** The area covered by water is

$$A_w=0.70A_{\text{Earth}}=(0.70)\left(4\pi R_{\text{Earth}}^2\right)=(0.70)(4\pi)\left(6.37\times10^6\ \text{m}\right)^2=3.6\times10^{14}\ \text{m}^2.$$

The average depth of the water is

$$d=(2.3\ \text{miles})(1\,609\ \text{m}/1\ \text{mile})=3.7\times10^3\ \text{m}.$$

The volume of the water is

$$V=A_wd=\left(3.6\times10^{14}\ \text{m}^2\right)\left(3.7\times10^3\ \text{m}\right)=1.3\times10^{18}\ \text{m}^3$$

and the mass is

$$m=\rho V=\left(1\,000\ \text{kg/m}^3\right)\left(1.3\times10^{18}\ \text{m}^3\right)=\boxed{1.3\times10^{21}\ \text{kg}}.$$

**P1.35** (a) $$d_{\text{nucleus, scale}}=d_{\text{nucleus, real}}\left(\frac{d_{\text{atom, scale}}}{d_{\text{atom, real}}}\right)=\left(2.40\times10^{-15}\text{ m}\right)\left(\frac{300\text{ ft}}{1.06\times10^{-10}\text{ m}}\right)=6.79\times10^{-3}\text{ ft, or}$$

$$d_{\text{nucleus, scale}}=\left(6.79\times10^{-3}\text{ ft}\right)\left(304.8\text{ mm/1 ft}\right)=\boxed{2.07\text{ mm}}$$

(b) $$\frac{V_{\text{atom}}}{V_{\text{nucleus}}}=\frac{\frac{4\pi r_{\text{atom}}^3}{3}}{\frac{4\pi r_{\text{nucleus}}^3}{3}}=\left(\frac{r_{\text{atom}}}{r_{\text{nucleus}}}\right)^3=\left(\frac{d_{\text{atom}}}{d_{\text{nucleus}}}\right)^3=\left(\frac{1.06\times10^{-10}\text{ m}}{2.40\times10^{-15}\text{ m}}\right)^3$$

$$=\boxed{8.62\times10^{13}\text{ times as large}}$$

***P1.36** $$\begin{matrix}\text{scale distance}\\\text{between}\end{matrix}=\begin{pmatrix}\text{real}\\\text{distance}\end{pmatrix}\begin{pmatrix}\text{scale}\\\text{factor}\end{pmatrix}=\left(4.0\times10^{13}\text{ km}\right)\left(\frac{7.0\times10^{-3}\text{ m}}{1.4\times10^{9}\text{ m}}\right)=\boxed{200\text{ km}}$$

**P1.37** The scale factor used in the "dinner plate" model is

$$S=\frac{0.25\text{ m}}{1.0\times10^{5}\text{ lightyears}}=2.5\times10^{-6}\text{ m/lightyears}.$$

The distance to Andromeda in the scale model will be

$$D_{\text{scale}}=D_{\text{actual}}S=\left(2.0\times10^{6}\text{ lightyears}\right)\left(2.5\times10^{-6}\text{ m/lightyears}\right)=\boxed{5.0\text{ m}}.$$

**P1.38** (a) $$\frac{A_{\text{Earth}}}{A_{\text{Moon}}}=\frac{4\pi r_{\text{Earth}}^2}{4\pi r_{\text{Moon}}^2}=\left(\frac{r_{\text{Earth}}}{r_{\text{Moon}}}\right)^2=\left(\frac{\left(6.37\times10^{6}\text{ m}\right)\left(100\text{ cm/m}\right)}{1.74\times10^{8}\text{ cm}}\right)^2=\boxed{13.4}$$

(b) $$\frac{V_{\text{Earth}}}{V_{\text{Moon}}}=\frac{\frac{4\pi r_{\text{Earth}}^3}{3}}{\frac{4\pi r_{\text{Moon}}^3}{3}}=\left(\frac{r_{\text{Earth}}}{r_{\text{Moon}}}\right)^3=\left(\frac{\left(6.37\times10^{6}\text{ m}\right)\left(100\text{ cm/m}\right)}{1.74\times10^{8}\text{ cm}}\right)^3=\boxed{49.1}$$

**P1.39** To balance, $m_{\text{Fe}}=m_{\text{Al}}$ or $\rho_{\text{Fe}}V_{\text{Fe}}=\rho_{\text{Al}}V_{\text{Al}}$

$$\rho_{\text{Fe}}\left(\frac{4}{3}\right)\pi r_{\text{Fe}}{}^3=\rho_{\text{Al}}\left(\frac{4}{3}\right)\pi r_{\text{Al}}{}^3$$

$$r_{\text{Al}}=r_{\text{Fe}}\left(\frac{\rho_{\text{Fe}}}{\rho_{\text{Al}}}\right)^{1/3}=(2.00\text{ cm})\left(\frac{7.86}{2.70}\right)^{1/3}=\boxed{2.86\text{ cm}}.$$

**P1.40** The mass of each sphere is

$$m_{\text{Al}} = \rho_{\text{Al}} V_{\text{Al}} = \frac{4\pi \rho_{\text{Al}} r_{\text{Al}}^3}{3}$$

and

$$m_{\text{Fe}} = \rho_{\text{Fe}} V_{\text{Fe}} = \frac{4\pi \rho_{\text{Fe}} r_{\text{Fe}}^3}{3}.$$

Setting these masses equal,

$$\frac{4\pi \rho_{\text{Al}} r_{\text{Al}}^3}{3} = \frac{4\pi \rho_{\text{Fe}} r_{\text{Fe}}^3}{3} \text{ and } \boxed{r_{\text{Al}} = r_{\text{Fe}} \sqrt[3]{\frac{\rho_{\text{Fe}}}{\rho_{\text{Al}}}}}.$$

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## Section 1.6 **Estimates and Order-of-Magnitude Calculations**

**P1.41** Model the room as a rectangular solid with dimensions 4 m by 4 m by 3 m, and each ping-pong ball as a sphere of diameter 0.038 m. The volume of the room is $4 \times 4 \times 3 = 48 \text{ m}^3$, while the volume of one ball is

$$\frac{4\pi}{3}\left(\frac{0.038 \text{ m}}{2}\right)^3 = 2.87 \times 10^{-5} \text{ m}^3.$$

Therefore, one can fit about $\frac{48}{2.87 \times 10^{-5}} \sim \boxed{10^6}$ ping-pong balls in the room.

As an aside, the actual number is smaller than this because there will be a lot of space in the room that cannot be covered by balls. In fact, even in the best arrangement, the so-called "best packing fraction" is $\frac{1}{6}\pi\sqrt{2} = 0.74$ so that at least 26% of the space will be empty. Therefore, the above estimate reduces to $1.67 \times 10^6 \times 0.740 \sim 10^6$.

**P1.42** A reasonable guess for the diameter of a tire might be 2.5 ft, with a circumference of about 8 ft. Thus, the tire would make $(50\,000 \text{ mi})(5\,280 \text{ ft/mi})(1 \text{ rev}/8 \text{ ft}) = 3 \times 10^7 \text{ rev} \sim \boxed{10^7 \text{ rev}}$.

**P1.43** In order to reasonably carry on photosynthesis, we might expect a blade of grass to require at least $\frac{1}{16} \text{ in}^2 = 43 \times 10^{-5} \text{ ft}^2$. Since $1 \text{ acre} = 43\,560 \text{ ft}^2$, the number of blades of grass to be expected on a quarter-acre plot of land is about

$$n = \frac{\text{total area}}{\text{area per blade}} = \frac{(0.25 \text{ acre})\left(43\,560 \text{ ft}^2/\text{acre}\right)}{43 \times 10^{-5} \text{ ft}^2/\text{blade}} = 2.5 \times 10^7 \text{ blades} \sim \boxed{10^7 \text{ blades}}.$$

**P1.44** A typical raindrop is spherical and might have a radius of about 0.1 inch. Its volume is then approximately $4\times10^{-3}\ \text{in}^3$. Since $1\ \text{acre} = 43\,560\ \text{ft}^2$, the volume of water required to cover it to a depth of 1 inch is

$$(1\ \text{acre})(1\ \text{inch}) = (1\ \text{acre}\cdot\text{in})\left(\frac{43\,560\ \text{ft}^2}{1\ \text{acre}}\right)\left(\frac{144\ \text{in}^2}{1\ \text{ft}^2}\right)\approx 6.3\times10^6\ \text{in}^3.$$

The number of raindrops required is

$$n=\frac{\text{volume of water required}}{\text{volume of a single drop}}=\frac{6.3\times10^6\ \text{in}^3}{4\times10^{-3}\ \text{in}^3}=1.6\times10^9\sim\boxed{10^9}\,.$$

***P1.45** Assume the tub measures 1.3 m by 0.5 m by 0.3 m. One-half of its volume is then

$$V=(0.5)(1.3\ \text{m})(0.5\ \text{m})(0.3\ \text{m})=0.10\ \text{m}^3\,.$$

The mass of this volume of water is

$$m_{\text{water}}=\rho_{\text{water}}V=\left(1\,000\ \text{kg/m}^3\right)\left(0.10\ \text{m}^3\right)=100\ \text{kg}\boxed{\sim10^2\ \text{kg}}\,.$$

Pennies are now mostly zinc, but consider copper pennies filling 50% of the volume of the tub. The mass of copper required is

$$m_{\text{copper}}=\rho_{\text{copper}}V=\left(8\,920\ \text{kg/m}^3\right)\left(0.10\ \text{m}^3\right)=892\ \text{kg}\boxed{\sim10^3\ \text{kg}}\,.$$

**P1.46** The typical person probably drinks 2 to 3 soft drinks daily. Perhaps half of these were in aluminum cans. Thus, we will estimate 1 aluminum can disposal per person per day. In the U.S. there are ~250 million people, and 365 days in a year, so

$$\left(250\times10^6\ \text{cans/day}\right)(365\ \text{days/year})\cong\boxed{10^{11}\ \text{cans}}$$

are thrown away or recycled each year. Guessing that each can weighs around $1/10$ of an ounce, we estimate this represents

$$\left(10^{11}\ \text{cans}\right)(0.1\ \text{oz/can})(1\ \text{lb/16 oz})(1\ \text{ton/2\,000 lb})\approx3.1\times10^5\ \text{tons/year}.\ \boxed{\sim10^5\ \text{tons}}$$

**P1.47** Assume: Total population $=10^7$; one out of every 100 people has a piano; one tuner can serve about 1 000 pianos (about 4 per day for 250 weekdays, assuming each piano is tuned once per year). Therefore,

$$\#\ \text{tuners}\sim\left(\frac{1\ \text{tuner}}{1\,000\ \text{pianos}}\right)\left(\frac{1\ \text{piano}}{100\ \text{people}}\right)(10^7\ \text{people})=\boxed{100}\,.$$

## Section 1.7 **Significant Figures**

***P1.48** METHOD ONE

We treat the best value with its uncertainty as a binomial $(21.3 \pm 0.2)\text{ cm }(9.8 \pm 0.1)\text{ cm}$,

$$A = \left[21.3(9.8) \pm 21.3(0.1) \pm 0.2(9.8) \pm (0.2)(0.1)\right]\text{ cm}^2 .$$

The first term gives the best value of the area. The cross terms add together to give the uncertainty and the fourth term is negligible.

$$A = \boxed{209\text{ cm}^2 \pm 4\text{ cm}^2} .$$

METHOD TWO

We add the fractional uncertainties in the data.

$$A = (21.3\text{ cm})(9.8\text{ cm}) \pm \left(\frac{0.2}{21.3} + \frac{0.1}{9.8}\right) = 209\text{ cm}^2 \pm 2\% = 209\text{ cm}^2 \pm 4\text{ cm}^2$$

**P1.49** (a)

$$\begin{aligned}\pi r^2 &= \pi(10.5\text{ m} \pm 0.2\text{ m})^2\\ &= \pi\left[(10.5\text{ m})^2 \pm 2(10.5\text{ m})(0.2\text{ m}) + (0.2\text{ m})^2\right]\\ &= \boxed{346\text{ m}^2 \pm 13\text{ m}^2}\end{aligned}$$

(b) $2\pi r = 2\pi(10.5\text{ m} \pm 0.2\text{ m}) = \boxed{66.0\text{ m} \pm 1.3\text{ m}}$

**P1.50** (a) $\boxed{3}$ (b) $\boxed{4}$ (c) $\boxed{3}$ (d) $\boxed{2}$

**P1.51** $r = (6.50 \pm 0.20)\text{ cm} = (6.50 \pm 0.20) \times 10^{-2}\text{ m}$

$m = (1.85 \pm 0.02)\text{ kg}$

$$\rho = \frac{m}{\left(\frac{4}{3}\right)\pi r^3}$$

also,

$$\frac{\delta\rho}{\rho} = \frac{\delta m}{m} + \frac{3\delta r}{r} .$$

In other words, the percentages of uncertainty are cumulative. Therefore,

$$\frac{\delta\rho}{\rho} = \frac{0.02}{1.85} + \frac{3(0.20)}{6.50} = 0.103 ,$$

$$\rho = \frac{1.85}{\left(\frac{4}{3}\right)\pi\left(6.5 \times 10^{-2}\text{ m}\right)^3} = \boxed{1.61 \times 10^3\text{ kg/m}^3}$$

and

$$\rho \pm \delta\rho = \boxed{(1.61 \pm 0.17) \times 10^3\text{ kg/m}^3} = (1.6 \pm 0.2) \times 10^3\text{ kg/m}^3 .$$

**P1.52** (a)

$$\begin{array}{r} 756.?? \\ 37.2? \\ 0.83 \\ \underline{+\ 2.5?} \\ 796.\cancel{5}\cancel{3} \end{array} = \boxed{797}$$

(b)

$$0.003\,2(2\text{ s.f.})\times 356.3(4\text{ s.f.}) = 1.140\,16 = (2\text{ s.f.})\ \boxed{1.1}$$

(c)

$$5.620(4\text{ s.f.})\times \pi(>4\text{ s.f.}) = 17.656 = (4\text{ s.f.})\ \boxed{17.66}$$

***P1.53** We work to nine significant digits:

$$1\text{ yr} = 1\text{ yr}\left(\frac{365.242\,199\text{ d}}{1\text{ yr}}\right)\left(\frac{24\text{ h}}{1\text{ d}}\right)\left(\frac{60\text{ min}}{1\text{ h}}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right) = \boxed{31\,556\,926.0\text{ s}}.$$

**P1.54** The distance around is $38.44\text{ m} + 19.5\text{ m} + 38.44\text{ m} + 19.5\text{ m} = 115.88\text{ m}$, but this answer must be rounded to 115.9 m because the distance 19.5 m carries information to only one place past the decimal. $\boxed{115.9\text{ m}}$

**P1.55**

$$V = 2V_1 + 2V_2 = 2(V_1 + V_2)$$

$$V_1 = (17.0\text{ m} + 1.0\text{ m} + 1.0\text{ m})(1.0\text{ m})(0.09\text{ m}) = 1.70\text{ m}^3$$

$$V_2 = (10.0\text{ m})(1.0\text{ m})(0.090\text{ m}) = 0.900\text{ m}^3$$

$$V = 2\left(1.70\text{ m}^3 + 0.900\text{ m}^3\right) = \boxed{5.2\text{ m}^3}$$

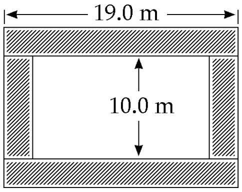


**FIG. P1.55**

$$\left.\begin{array}{l} \dfrac{\delta \ell_1}{\ell_1} = \dfrac{0.12\text{ m}}{19.0\text{ m}} = 0.0063 \\ \dfrac{\delta w_1}{w_1} = \dfrac{0.01\text{ m}}{1.0\text{ m}} = 0.010 \\ \dfrac{\delta t_1}{t_1} = \dfrac{0.1\text{ cm}}{9.0\text{ cm}} = 0.011 \end{array}\right\} \frac{\delta V}{V} = 0.006 + 0.010 + 0.011 = 0.027 = \boxed{3\%}$$

## Additional Problems

**P1.56** It is desired to find the distance $x$ such that

$$\frac{x}{100\text{ m}} = \frac{1\,000\text{ m}}{x}$$

(i.e., such that $x$ is the same multiple of 100 m as the multiple that 1 000 m is of $x$). Thus, it is seen that

$$x^2 = (100\text{ m})(1\,000\text{ m}) = 1.00\times 10^5\text{ m}^2$$

and therefore

$$x = \sqrt{1.00\times 10^5\text{ m}^2} = \boxed{316\text{ m}}.$$

***P1.57** Consider one cubic meter of gold. Its mass from Table 1.5 is 19 300 kg. One atom of gold has mass

$$m_0 = (197\ \text{u})\left(\frac{1.66\times10^{-27}\ \text{kg}}{1\ \text{u}}\right) = 3.27\times10^{-25}\ \text{kg}\,.$$

So, the number of atoms in the cube is

$$N = \frac{19\,300\ \text{kg}}{3.27\times10^{-25}\ \text{kg}} = 5.90\times10^{28}\,.$$

The imagined cubical volume of each atom is

$$d^3 = \frac{1\ \text{m}^3}{5.90\times10^{28}} = 1.69\times10^{-29}\ \text{m}^3\,.$$

So

$$d = \boxed{2.57\times10^{-10}\ \text{m}}\,.$$

**P1.58** $$A_{\text{total}} = (N)\left(A_{\text{drop}}\right) = \left(\frac{V_{\text{total}}}{V_{\text{drop}}}\right)\left(A_{\text{drop}}\right) = \left(\frac{V_{\text{total}}}{\frac{4\pi r^3}{3}}\right)\left(4\pi r^2\right)$$

$$A_{\text{total}} = \left(\frac{3V_{\text{total}}}{r}\right) = 3\left(\frac{30.0\times10^{-6}\ \text{m}^3}{2.00\times10^{-5}\ \text{m}}\right) = \boxed{4.50\ \text{m}^2}$$

**P1.59** One month is

$$1\ \text{mo} = (30\ \text{day})(24\ \text{h/day})(3\,600\ \text{s/h}) = 2.592\times10^6\ \text{s}\,.$$

Applying units to the equation,

$$V = \left(1.50\ \text{Mft}^3/\text{mo}\right)t + \left(0.008\,00\ \text{Mft}^3/\text{mo}^2\right)t^2\,.$$

Since $1\ \text{Mft}^3 = 10^6\ \text{ft}^3$,

$$V = \left(1.50\times10^6\ \text{ft}^3/\text{mo}\right)t + \left(0.008\,00\times10^6\ \text{ft}^3/\text{mo}^2\right)t^2\,.$$

Converting months to seconds,

$$V = \frac{1.50\times10^6\ \text{ft}^3/\text{mo}}{2.592\times10^6\ \text{s/mo}}t + \frac{0.008\,00\times10^6\ \text{ft}^3/\text{mo}^2}{\left(2.592\times10^6\ \text{s/mo}\right)^2}t^2\,.$$

Thus, $\boxed{V\,[\text{ft}^3] = \left(0.579\ \text{ft}^3/\text{s}\right)t + \left(1.19\times10^{-9}\ \text{ft}^3/\text{s}^2\right)t^2}$.

**P1.60**

| $\alpha'$(deg) | $\alpha$(rad) | $\tan(\alpha)$ | $\sin(\alpha)$ | difference |
|---|---|---|---|---|
| 15.0 | 0.262 | 0.268 | 0.259 | 3.47% |
| 20.0 | 0.349 | 0.364 | 0.342 | 6.43% |
| 25.0 | 0.436 | 0.466 | 0.423 | 10.2% |
| 24.0 | 0.419 | 0.445 | 0.407 | 9.34% |
| 24.4 | 0.426 | 0.454 | 0.413 | 9.81% |
| 24.5 | 0.428 | 0.456 | 0.415 | 9.87% |
| 24.6 | 0.429 | 0.458 | 0.416 | 9.98% |
| 24.7 | 0.431 | 0.460 | 0.418 | 10.1% |

$\boxed{24.6^\circ}$

**P1.61**

$$2\pi r = 15.0\text{ m}$$

$$r = 2.39\text{ m}$$

$$\frac{h}{r} = \tan 55.0^\circ$$

$$h = (2.39\text{ m})\tan(55.0^\circ) = \boxed{3.41\text{ m}}$$

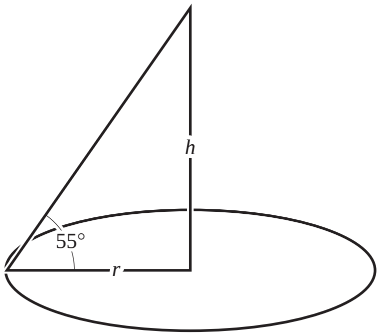


**FIG. P1.61**

***P1.62** Let *d* represent the diameter of the coin and *h* its thickness. The mass of the gold is

$$m = \rho V = \rho A t = \rho\left(\frac{2\pi d^2}{4} + \pi d h\right)t$$

where *t* is the thickness of the plating.

$$m = 19.3\left[2\pi\frac{(2.41)^2}{4} + \pi(2.41)(0.178)\right]\left(0.18\times10^{-4}\right)$$

$$= 0.003\,64\text{ grams}$$

$$\text{cost} = 0.003\,64\text{ grams}\times\$10/\text{gram} = \$0.036\,4 = \boxed{3.64\text{ cents}}$$

This is negligible compared to $4.98.

**P1.63** The actual number of seconds in a year is

$$(86\,400\text{ s/day})(365.25\text{ day/yr}) = 31\,557\,600\text{ s/yr}.$$

The percent error in the approximation is

$$\frac{\left|\left(\pi\times10^7\text{ s/yr}\right) - (31\,557\,600\text{ s/yr})\right|}{31\,557\,600\text{ s/yr}}\times100\% = \boxed{0.449\%}.$$

**P1.64** (a) $[V]=\mathrm{L}^3,\ [A]=\mathrm{L}^2,\ [h]=\mathrm{L}$

$$[V]=[A][h]$$

$\mathrm{L}^3=\mathrm{L}^2\mathrm{L}=\mathrm{L}^3$. Thus, the equation is dimensionally correct.

(b) $V_{\text{cylinder}}=\pi R^2h=\left(\pi R^2\right)h=Ah$, where $\boxed{A=\pi R^2}$

$V_{\text{rectangular object}}=\ell wh=(\ell w)h=Ah$, where $\boxed{A=\ell w}$

**P1.65** (a) The speed of rise may be found from

$$v=\frac{(\text{Vol rate of flow})}{(\text{Area: }\frac{\pi D^2}{4})}=\frac{16.5\ \text{cm}^3/\text{s}}{\frac{\pi(6.30\ \text{cm})^2}{4}}=\boxed{0.529\ \text{cm/s}}.$$

(b) Likewise, at a 1.35 cm diameter,

$$v=\frac{16.5\ \text{cm}^3/\text{s}}{\frac{\pi(1.35\ \text{cm})^2}{4}}=\boxed{11.5\ \text{cm/s}}.$$

**P1.66** (a) 1 cubic meter of water has a mass

$$m=\rho V=\left(1.00\times10^{-3}\ \text{kg/cm}^3\right)\left(1.00\ \text{m}^3\right)\left(10^2\ \text{cm/m}\right)^3=\boxed{1\,000\ \text{kg}}$$

(b) As a rough calculation, we treat each item as if it were 100% water.

cell: $m=\rho V=\rho\left(\frac{4}{3}\pi R^3\right)=\rho\left(\frac{1}{6}\pi D^3\right)=\left(1\,000\ \text{kg/m}^3\right)\left(\frac{1}{6}\pi\right)\left(1.0\times10^{-6}\ \text{m}\right)^3$

$=\boxed{5.2\times10^{-16}\ \text{kg}}$

kidney: $m=\rho V=\rho\left(\frac{4}{3}\pi R^3\right)=\left(1.00\times10^{-3}\ \text{kg/cm}^3\right)\left(\frac{4}{3}\pi\right)(4.0\ \text{cm})^3$

$=\boxed{0.27\ \text{kg}}$

fly: $m=\rho\left(\frac{\pi}{4}D^2h\right)=\left(1\times10^{-3}\ \text{kg/cm}^3\right)\left(\frac{\pi}{4}\right)(2.0\ \text{mm})^2(4.0\ \text{mm})\left(10^{-1}\ \text{cm/mm}\right)^3$

$=\boxed{1.3\times10^{-5}\ \text{kg}}$

**P1.67** $V_{20\text{ mpg}}=\dfrac{(10^8\ \text{cars})(10^4\ \text{mi/yr})}{20\ \text{mi/gal}}=5.0\times10^{10}\ \text{gal/yr}$

$V_{25\text{ mpg}}=\dfrac{(10^8\ \text{cars})(10^4\ \text{mi/yr})}{25\ \text{mi/gal}}=4.0\times10^{10}\ \text{gal/yr}$

Fuel saved $=V_{25\text{ mpg}}-V_{20\text{ mpg}}=\boxed{1.0\times10^{10}\ \text{gal/yr}}$

**P1.68** $$v=\left(5.00\,\frac{\text{furlongs}}{\text{fortnight}}\right)\left(\frac{220\text{ yd}}{1\text{ furlong}}\right)\left(\frac{0.914\,4\text{ m}}{1\text{ yd}}\right)\left(\frac{1\text{ fortnight}}{14\text{ days}}\right)\left(\frac{1\text{ day}}{24\text{ hrs}}\right)\left(\frac{1\text{ hr}}{3\,600\text{ s}}\right)=\boxed{8.32\times10^{-4}\text{ m/s}}$$

This speed is almost 1 mm/s; so we might guess the creature was a snail, or perhaps a sloth.

**P1.69** The volume of the galaxy is

$$\pi r^2 t=\pi\left(10^{21}\text{ m}\right)^2\left(10^{19}\text{ m}\right)\sim10^{61}\text{ m}^3.$$

If the distance between stars is $4\times10^{16}$ m, then there is one star in a volume on the order of

$$\left(4\times10^{16}\text{ m}\right)^3\sim10^{50}\text{ m}^3.$$

The number of stars is about $\dfrac{10^{61}\text{ m}^3}{10^{50}\text{ m}^3/\text{star}}\sim\boxed{10^{11}\text{ stars}}$.

**P1.70** The density of each material is $\rho=\dfrac{m}{V}=\dfrac{m}{\pi r^2h}=\dfrac{4m}{\pi D^2h}$.

Al: $\rho=\dfrac{4(51.5\text{ g})}{\pi(2.52\text{ cm})^2(3.75\text{ cm})}=\boxed{2.75\,\dfrac{\text{g}}{\text{cm}^3}}$ The tabulated value $\left(2.70\,\dfrac{\text{g}}{\text{cm}^3}\right)$ is $\boxed{2\%}$ smaller.

Cu: $\rho=\dfrac{4(56.3\text{ g})}{\pi(1.23\text{ cm})^2(5.06\text{ cm})}=\boxed{9.36\,\dfrac{\text{g}}{\text{cm}^3}}$ The tabulated value $\left(8.92\,\dfrac{\text{g}}{\text{cm}^3}\right)$ is $\boxed{5\%}$ smaller.

Brass: $\rho=\dfrac{4(94.4\text{ g})}{\pi(1.54\text{ cm})^2(5.69\text{ cm})}=\boxed{8.91\,\dfrac{\text{g}}{\text{cm}^3}}$

Sn: $\rho=\dfrac{4(69.1\text{ g})}{\pi(1.75\text{ cm})^2(3.74\text{ cm})}=\boxed{7.68\,\dfrac{\text{g}}{\text{cm}^3}}$

Fe: $\rho=\dfrac{4(216.1\text{ g})}{\pi(1.89\text{ cm})^2(9.77\text{ cm})}=\boxed{7.88\,\dfrac{\text{g}}{\text{cm}^3}}$ The tabulated value $\left(7.86\,\dfrac{\text{g}}{\text{cm}^3}\right)$ is $\boxed{0.3\%}$ smaller.

**P1.71** (a) $(3\,600\text{ s/hr})(24\text{ hr/day})(365.25\text{ days/yr})=\boxed{3.16\times10^{7}\text{ s/yr}}$

(b) $$V_{\text{mm}}=\frac{4}{3}\pi r^3=\frac{4}{3}\pi\left(5.00\times10^{-7}\text{ m}\right)^3=5.24\times10^{-19}\text{ m}^3$$

$$\frac{V_{\text{cube}}}{V_{\text{mm}}}=\frac{1\text{ m}^3}{5.24\times10^{-19}\text{ m}^3}=1.91\times10^{18}\text{ micrometeorites}$$

This would take $\dfrac{1.91\times10^{18}\text{ micrometeorites}}{3.16\times10^{7}\text{ micrometeorites/yr}}=\boxed{6.05\times10^{10}\text{ yr}}$.

## ANSWERS TO EVEN PROBLEMS

| | |
|---|---|
| **P1.2** | $5.52 \times 10^3 \text{ kg/m}^3$, between the densities of aluminum and iron, and greater than the densities of surface rocks. |
| **P1.4** | 23.0 kg |
| **P1.6** | 7.69 cm |
| **P1.8** | (a) and (b) see the solution, $N_A = 6.022\,137 \times 10^{23}$; (c) 18.0 g;<br>(d) 44.0 g |
| **P1.10** | (a) $9.83 \times 10^{-16}$ g; (b) $1.06 \times 10^7$ atoms |
| **P1.12** | (a) $4.02 \times 10^{25}$ molecules;<br>(b) $3.65 \times 10^4$ molecules |
| **P1.14** | (a) ii; (b) iii; (c) i |
| **P1.16** | (a) $\dfrac{\text{M}\cdot\text{L}}{\text{T}^2}$; (b) $1 \text{ newton} = 1 \text{ kg}\cdot\text{m/s}^2$ |
| **P1.18** | $35.7 \text{ m}^2$ |
| **P1.20** | $1.39 \times 10^{-4} \text{ m}^3$ |
| **P1.22** | (a) $3.39 \times 10^5 \text{ ft}^3$; (b) $2.54 \times 10^4$ lb |
| **P1.24** | (a) $560 \text{ km} = 5.60 \times 10^5 \text{ m} = 5.60 \times 10^7 \text{ cm}$;<br>(b) $491 \text{ m} = 0.491 \text{ km} = 4.91 \times 10^4 \text{ cm}$;<br>(c) $6.19 \text{ km} = 6.19 \times 10^3 \text{ m} = 6.19 \times 10^5 \text{ cm}$;<br>(d) $2.50 \text{ km} = 2.50 \times 10^3 \text{ m} = 2.50 \times 10^5 \text{ cm}$ |
| **P1.26** | $4.05 \times 10^3 \text{ m}^2$ |
| **P1.28** | (a) $1 \text{ mi/h} = 1.609 \text{ km/h}$; (b) $88.5 \text{ km/h}$;<br>(c) $16.1 \text{ km/h}$ |
| **P1.30** | $1.19 \times 10^{57}$ atoms |
| **P1.32** | $2.57 \times 10^6 \text{ m}^3$ |
| **P1.34** | $1.3 \times 10^{21}$ kg |
| **P1.36** | 200 km |
| **P1.38** | (a) 13.4; (b) 49.1 |
| **P1.40** | $r_{\text{Al}} = r_{\text{Fe}}\left(\dfrac{\rho_{\text{Fe}}}{\rho_{\text{Al}}}\right)^{1/3}$ |
| **P1.42** | $\sim 10^7$ rev |
| **P1.44** | $\sim 10^9$ raindrops |
| **P1.46** | $\sim 10^{11}$ cans; $\sim 10^5$ tons |
| **P1.48** | $(209 \pm 4) \text{ cm}^2$ |
| **P1.50** | (a) 3; (b) 4; (c) 3; (d) 2 |
| **P1.52** | (a) 797; (b) 1.1; (c) 17.66 |
| **P1.54** | 115.9 m |
| **P1.56** | 316 m |
| **P1.58** | $4.50 \text{ m}^2$ |
| **P1.60** | see the solution; 24.6° |
| **P1.62** | 3.64 cents; no |
| **P1.64** | see the solution |
| **P1.66** | (a) 1 000 kg; (b) $5.2 \times 10^{-16}$ kg; 0.27 kg;<br>$1.3 \times 10^{-5}$ kg |
| **P1.68** | $8.32 \times 10^{-4} \text{ m/s}$; a snail |
| **P1.70** | see the solution |

# 2

# Motion in One Dimension

**CHAPTER OUTLINE**

2.1 Position, Velocity, and Speed
2.2 Instantaneous Velocity and Speed
2.3 Acceleration
2.4 Motion Diagrams
2.5 One-Dimensional Motion with Constant Acceleration
2.6 Freely Falling Objects
2.7 Kinematic Equations Derived from Calculus

## ANSWERS TO QUESTIONS

**Q2.1** If I count 5.0 s between lightning and thunder, the sound has traveled $(331\ \text{m/s})(5.0\ \text{s}) = 1.7\ \text{km}$. The transit time for the light is smaller by

$$\frac{3.00 \times 10^{8}\ \text{m/s}}{331\ \text{m/s}} = 9.06 \times 10^{5}\ \text{times},$$

so it is negligible in comparison.

**Q2.2** Yes. Yes, if the particle winds up in the $+x$ region at the end.

**Q2.3** Zero.

**Q2.4** Yes. Yes.

**Q2.5** No. Consider a sprinter running a straight-line race. His average velocity would simply be the length of the race divided by the time it took for him to complete the race. If he stops along the way to tie his shoe, then his instantaneous velocity at that point would be zero.

**Q2.6** We assume the object moves along a straight line. If its average velocity is zero, then the displacement must be zero over the time interval, according to Equation 2.2. The object might be stationary throughout the interval. If it is moving to the right at first, it must later move to the left to return to its starting point. Its velocity must be zero as it turns around. The graph of the motion shown to the right represents such motion, as the initial and final positions are the same. In an $x$ vs. $t$ graph, the instantaneous velocity at any time $t$ is the slope of the curve at that point. At $t_0$ in the graph, the slope of the curve is zero, and thus the instantaneous velocity at that time is also zero.

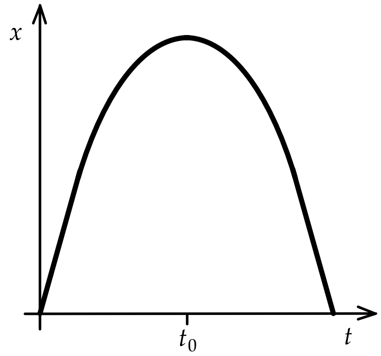


**FIG. Q2.6**

**Q2.7** Yes. If the velocity of the particle is nonzero, the particle is in motion. If the acceleration is zero, the velocity of the particle is unchanging, or is a constant.

**Q2.8** Yes. If you drop a doughnut from rest $(v = 0)$, then its acceleration is not zero. A common misconception is that immediately after the doughnut is released, both the velocity and acceleration are zero. If the acceleration were zero, then the velocity would not change, leaving the doughnut floating at rest in mid-air.

**Q2.9** No: Car A might have greater acceleration than B, but they might both have zero acceleration, or otherwise equal accelerations; or the driver of B might have tramped hard on the gas pedal in the recent past.

**Q2.10** Yes. Consider throwing a ball straight up. As the ball goes up, its velocity is upward $(v > 0)$, and its acceleration is directed down $(a < 0)$. A graph of $v$ vs. $t$ for this situation would look like the figure to the right. The acceleration is the slope of a $v$ vs. $t$ graph, and is always negative in this case, even when the velocity is positive.

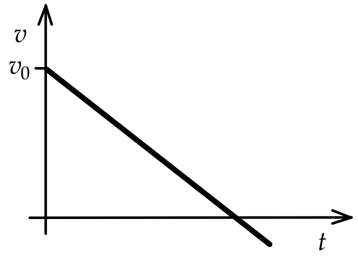


**FIG. Q2.10**

**Q2.11** (a) Accelerating East (b) Braking East (c) Cruising East

(d) Braking West (e) Accelerating West (f) Cruising West

(g) Stopped but starting to move East

(h) Stopped but starting to move West

**Q2.12** No. Constant acceleration only. Yes. Zero is a constant.

**Q2.13** The position does depend on the origin of the coordinate system. Assume that the cliff is 20 m tall, and that the stone reaches a maximum height of 10 m above the top of the cliff. If the origin is taken as the top of the cliff, then the maximum height reached by the stone would be 10 m. If the origin is taken as the bottom of the cliff, then the maximum height would be 30 m.

The velocity is independent of the origin. Since the *change* in position is used to calculate the instantaneous velocity in Equation 2.5, the choice of origin is arbitrary.

**Q2.14** Once the objects leave the hand, both are in free fall, and both experience the same downward acceleration equal to the free-fall acceleration, $-g$.

**Q2.15** They are the same. After the first ball reaches its apex and falls back downward past the student, it will have a downward velocity equal to $v_i$. This velocity is the same as the velocity of the second ball, so after they fall through equal heights their impact speeds will also be the same.

**Q2.16** With $h = \frac{1}{2}gt^2$,

(a) $0.5h = \frac{1}{2}g(0.707t)^2$. The time is later than $0.5t$.

(b) The distance fallen is $0.25h = \frac{1}{2}g(0.5t)^2$. The elevation is $0.75h$, greater than $0.5h$.

**Q2.17** Above. Your ball has zero initial speed and smaller average speed during the time of flight to the passing point.

## SOLUTIONS TO PROBLEMS

### Section 2.1 **Position, Velocity, and Speed**

**P2.1** (a) $\bar{v} = \boxed{2.30\ \text{m/s}}$

(b) $$v = \frac{\Delta x}{\Delta t} = \frac{57.5\ \text{m} - 9.20\ \text{m}}{3.00\ \text{s}} = \boxed{16.1\ \text{m/s}}$$

(c) $$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{57.5\ \text{m} - 0\ \text{m}}{5.00\ \text{s}} = \boxed{11.5\ \text{m/s}}$$

***P2.2** (a) $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{20\ \text{ft}}{1\ \text{yr}}\left(\frac{1\ \text{m}}{3.281\ \text{ft}}\right)\left(\frac{1\ \text{yr}}{3.156 \times 10^{7}\ \text{s}}\right) = \boxed{2 \times 10^{-7}\ \text{m/s}}$ or in particularly windy times

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{100\ \text{ft}}{1\ \text{yr}}\left(\frac{1\ \text{m}}{3.281\ \text{ft}}\right)\left(\frac{1\ \text{yr}}{3.156 \times 10^{7}\ \text{s}}\right) = \boxed{1 \times 10^{-6}\ \text{m/s}}.$$

(b) The time required must have been

$$\Delta t = \frac{\Delta x}{\bar{v}} = \frac{3\,000\ \text{mi}}{10\ \text{mm/yr}}\left(\frac{1\,609\ \text{m}}{1\ \text{mi}}\right)\left(\frac{10^{3}\ \text{mm}}{1\ \text{m}}\right) = \boxed{5 \times 10^{8}\ \text{yr}}.$$

**P2.3** (a) $$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{10\ \text{m}}{2\ \text{s}} = \boxed{5\ \text{m/s}}$$

(b) $$\bar{v} = \frac{5\ \text{m}}{4\ \text{s}} = \boxed{1.2\ \text{m/s}}$$

(c) $$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{5\ \text{m} - 10\ \text{m}}{4\ \text{s} - 2\ \text{s}} = \boxed{-2.5\ \text{m/s}}$$

(d) $$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{-5\ \text{m} - 5\ \text{m}}{7\ \text{s} - 4\ \text{s}} = \boxed{-3.3\ \text{m/s}}$$

(e) $$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{0 - 0}{8 - 0} = \boxed{0\ \text{m/s}}$$

**P2.4** $x = 10t^2$: For

$$\begin{array}{lclll} t(\text{s}) & = & 2.0 & 2.1 & 3.0 \\ x(\text{m}) & = & 40 & 44.1 & 90 \end{array}$$

(a) $$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{50\ \text{m}}{1.0\ \text{s}} = \boxed{50.0\ \text{m/s}}$$

(b) $$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{4.1\ \text{m}}{0.1\ \text{s}} = \boxed{41.0\ \text{m/s}}$$

**P2.5** (a) Let $d$ represent the distance between A and B. Let $t_1$ be the time for which the walker has the higher speed in $5.00\text{ m/s} = \dfrac{d}{t_1}$. Let $t_2$ represent the longer time for the return trip in $-3.00\text{ m/s} = -\dfrac{d}{t_2}$. Then the times are $t_1 = \dfrac{d}{(5.00\text{ m/s})}$ and $t_2 = \dfrac{d}{(3.00\text{ m/s})}$. The average speed is:

$$\bar{v} = \frac{\text{Total distance}}{\text{Total time}} = \frac{d+d}{\frac{d}{(5.00\text{ m/s})} + \frac{d}{(3.00\text{ m/s})}} = \frac{2d}{\frac{(8.00\text{ m/s})d}{(15.0\text{ m}^2/\text{s}^2)}}$$

$$\bar{v} = \frac{2(15.0\text{ m}^2/\text{s}^2)}{8.00\text{ m/s}} = \boxed{3.75\text{ m/s}}$$

(b) She starts and finishes at the same point A. With total displacement = 0, average velocity $= \boxed{0}$.

---

Section 2.2 **Instantaneous Velocity and Speed**

**P2.6** (a) At any time, $t$, the position is given by $x = (3.00\text{ m/s}^2)t^2$.
Thus, at $t_i = 3.00\text{ s}$: $x_i = (3.00\text{ m/s}^2)(3.00\text{ s})^2 = \boxed{27.0\text{ m}}$.

(b) At $t_f = 3.00\text{ s} + \Delta t$: $x_f = (3.00\text{ m/s}^2)(3.00\text{ s} + \Delta t)^2$, or

$$x_f = \boxed{27.0\text{ m} + (18.0\text{ m/s})\Delta t + (3.00\text{ m/s}^2)(\Delta t)^2}.$$

(c) The instantaneous velocity at $t = 3.00\text{ s}$ is:

$$v = \lim_{\Delta t \to 0}\left(\frac{x_f - x_i}{\Delta t}\right) = \lim_{\Delta t \to 0}\left(18.0\text{ m/s} + (3.00\text{ m/s}^2)\Delta t\right) = \boxed{18.0\text{ m/s}}.$$

**P2.7** (a) at $t_i = 1.5\text{ s}$, $x_i = 8.0\text{ m}$ (Point A)
at $t_f = 4.0\text{ s}$, $x_f = 2.0\text{ m}$ (Point B)

$$\bar{v} = \frac{x_f - x_i}{t_f - t_i} = \frac{(2.0 - 8.0)\text{ m}}{(4 - 1.5)\text{ s}} = -\frac{6.0\text{ m}}{2.5\text{ s}} = \boxed{-2.4\text{ m/s}}$$

(b) The slope of the tangent line is found from points $C$ and $D$. $(t_C = 1.0\text{ s},\ x_C = 9.5\text{ m})$ and $(t_D = 3.5\text{ s},\ x_D = 0)$,

$$v \cong \boxed{-3.8\text{ m/s}}.$$

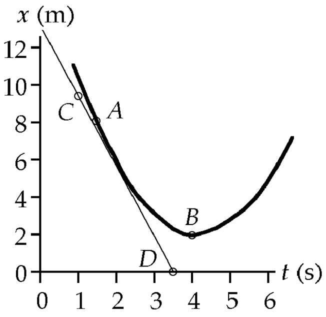
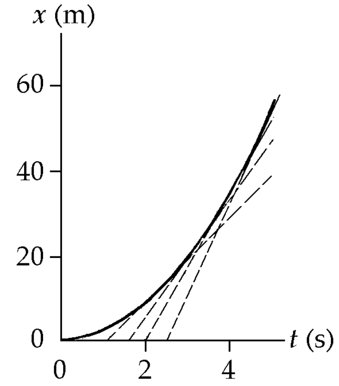
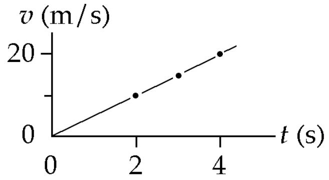


**FIG. P2.7**

(c) The velocity is zero when $x$ is a minimum. This is at $t \cong \boxed{4\text{ s}}$.

**P2.8** (a)





(b) At $t = 5.0$ s, the slope is $v \cong \dfrac{58\text{ m}}{2.5\text{ s}} \cong \boxed{23\text{ m/s}}$.

At $t = 4.0$ s, the slope is $v \cong \dfrac{54\text{ m}}{3\text{ s}} \cong \boxed{18\text{ m/s}}$.

At $t = 3.0$ s, the slope is $v \cong \dfrac{49\text{ m}}{3.4\text{ s}} \cong \boxed{14\text{ m/s}}$.

At $t = 2.0$ s, the slope is $v \cong \dfrac{36\text{ m}}{4.0\text{ s}} \cong \boxed{9.0\text{ m/s}}$.

(c) $\bar{a} = \dfrac{\Delta v}{\Delta t} \cong \dfrac{23\text{ m/s}}{5.0\text{ s}} \cong \boxed{4.6\text{ m/s}^2}$

(d) Initial velocity of the car was $\boxed{\text{zero}}$.

**P2.9** (a) $v = \dfrac{(5-0)\text{ m}}{(1-0)\text{ s}} = \boxed{5\text{ m/s}}$

(b) $v = \dfrac{(5-10)\text{ m}}{(4-2)\text{ s}} = \boxed{-2.5\text{ m/s}}$

(c) $v = \dfrac{(5\text{ m}-5\text{ m})}{(5\text{ s}-4\text{ s})} = \boxed{0}$

(d) $v = \dfrac{0-(-5\text{ m})}{(8\text{ s}-7\text{ s})} = \boxed{+5\text{ m/s}}$

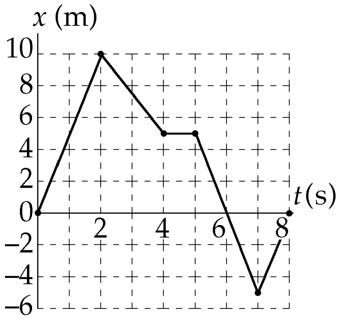


**FIG. P2.9**

***P2.10** Once it resumes the race, the hare will run for a time of

$$t = \frac{x_f - x_i}{v_x} = \frac{1\,000\text{ m} - 800\text{ m}}{8\text{ m/s}} = 25\text{ s}.$$

In this time, the tortoise can crawl a distance

$$x_f - x_i = (0.2\text{ m/s})(25\text{ s}) = \boxed{5.00\text{ m}}.$$

## Section 2.3 **Acceleration**

**P2.11** Choose the positive direction to be the outward direction, perpendicular to the wall.

$$v_f = v_i + at:\ a = \frac{\Delta v}{\Delta t} = \frac{22.0\ \text{m/s} - (-25.0\ \text{m/s})}{3.50 \times 10^{-3}\ \text{s}} = \boxed{1.34 \times 10^4\ \text{m/s}^2}.$$

**P2.12** (a) Acceleration is constant over the first ten seconds, so at the end,

$$v_f = v_i + at = 0 + \left(2.00\ \text{m/s}^2\right)(10.0\ \text{s}) = \boxed{20.0\ \text{m/s}}.$$

Then $a = 0$ so $v$ is constant from $t = 10.0$ s to $t = 15.0$ s. And over the last five seconds the velocity changes to

$$v_f = v_i + at = 20.0\ \text{m/s} + \left(3.00\ \text{m/s}^2\right)(5.00\ \text{s}) = \boxed{5.00\ \text{m/s}}.$$

(b) In the first ten seconds,

$$x_f = x_i + v_i t + \frac{1}{2}at^2 = 0 + 0 + \frac{1}{2}\left(2.00\ \text{m/s}^2\right)(10.0\ \text{s})^2 = 100\ \text{m}.$$

Over the next five seconds the position changes to

$$x_f = x_i + v_i t + \frac{1}{2}at^2 = 100\ \text{m} + (20.0\ \text{m/s})(5.00\ \text{s}) + 0 = 200\ \text{m}.$$

And at $t = 20.0$ s,

$$x_f = x_i + v_i t + \frac{1}{2}at^2 = 200\ \text{m} + (20.0\ \text{m/s})(5.00\ \text{s}) + \frac{1}{2}\left(-3.00\ \text{m/s}^2\right)(5.00\ \text{s})^2 = \boxed{262\ \text{m}}.$$

***P2.13** (a) The average speed during a time interval $\Delta t$ is $\bar{v} = \frac{\text{distance traveled}}{\Delta t}$. During the first quarter mile segment, Secretariat's average speed was

$$\bar{v}_1 = \frac{0.250\ \text{mi}}{25.2\ \text{s}} = \frac{1\,320\ \text{ft}}{25.2\ \text{s}} = \boxed{52.4\ \text{ft/s}}\quad (35.6\ \text{mi/h}).$$

During the second quarter mile segment,

$$\bar{v}_2 = \frac{1\,320\ \text{ft}}{24.0\ \text{s}} = \boxed{55.0\ \text{ft/s}}\quad (37.4\ \text{mi/h}).$$

For the third quarter mile of the race,

$$\bar{v}_3 = \frac{1\,320\ \text{ft}}{23.8\ \text{s}} = \boxed{55.5\ \text{ft/s}}\quad (37.7\ \text{mi/h}),$$

and during the final quarter mile,

$$\bar{v}_4 = \frac{1\,320\ \text{ft}}{23.0\ \text{s}} = \boxed{57.4\ \text{ft/s}}\quad (39.0\ \text{mi/h}).$$

*continued on next page*

(b) Assuming that $v_f = \overline{v}_4$ and recognizing that $v_i = 0$, the average acceleration during the race was

$$\overline{a} = \frac{v_f - v_i}{\text{total elapsed time}} = \frac{57.4\ \text{ft/s} - 0}{(25.2 + 24.0 + 23.8 + 23.0)\ \text{s}} = \boxed{0.598\ \text{ft/s}^2}.$$

**P2.14** (a) Acceleration is the slope of the graph of $v$ vs $t$.

For $0 < t < 5.00$ s, $a = 0$.

For $15.0\ \text{s} < t < 20.0$ s, $a = 0$.

For $5.0\ \text{s} < t < 15.0$ s, $a = \dfrac{v_f - v_i}{t_f - t_i}$.

$$a = \frac{8.00 - (-8.00)}{15.0 - 5.00} = 1.60\ \text{m/s}^2$$

We can plot $a(t)$ as shown.

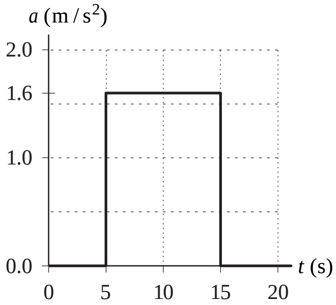


**FIG. P2.14**

(b) $a = \dfrac{v_f - v_i}{t_f - t_i}$

(i) For $5.00\ \text{s} < t < 15.0\ \text{s}$, $t_i = 5.00$ s, $v_i = -8.00$ m/s,

$$t_f = 15.0\ \text{s}$$
$$v_f = 8.00\ \text{m/s}$$
$$a = \frac{v_f - v_i}{t_f - t_i} = \frac{8.00 - (-8.00)}{15.0 - 5.00} = \boxed{1.60\ \text{m/s}^2}.$$

(ii) $t_i = 0$, $v_i = -8.00$ m/s, $t_f = 20.0$ s, $v_f = 8.00$ m/s

$$a = \frac{v_f - v_i}{t_f - t_i} = \frac{8.00 - (-8.00)}{20.0 - 0} = \boxed{0.800\ \text{m/s}^2}$$

**P2.15** $x = 2.00 + 3.00t - t^2$, $v = \dfrac{dx}{dt} = 3.00 - 2.00t$, $a = \dfrac{dv}{dt} = -2.00$

At $t = 3.00$ s:

(a) $x = (2.00 + 9.00 - 9.00)\ \text{m} = \boxed{2.00\ \text{m}}$

(b) $v = (3.00 - 6.00)\ \text{m/s} = \boxed{-3.00\ \text{m/s}}$

(c) $a = \boxed{-2.00\ \text{m/s}^2}$

**P2.16** (a) At $t = 2.00\ \text{s}$, $x = \left[3.00(2.00)^2 - 2.00(2.00) + 3.00\right]\ \text{m} = 11.0\ \text{m}$.

At $t = 3.00\ \text{s}$, $x = \left[3.00(9.00)^2 - 2.00(3.00) + 3.00\right]\ \text{m} = 24.0\ \text{m}$

so

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{24.0\ \text{m} - 11.0\ \text{m}}{3.00\ \text{s} - 2.00\ \text{s}} = \boxed{13.0\ \text{m/s}}.$$

(b) At all times the instantaneous velocity is

$$v = \frac{d}{dt}\left(3.00t^2 - 2.00t + 3.00\right) = (6.00t - 2.00)\ \text{m/s}$$

At $t = 2.00\ \text{s}$, $v = \left[6.00(2.00) - 2.00\right]\ \text{m/s} = \boxed{10.0\ \text{m/s}}$.
At $t = 3.00\ \text{s}$, $v = \left[6.00(3.00) - 2.00\right]\ \text{m/s} = \boxed{16.0\ \text{m/s}}$.

(c) $\bar{a} = \dfrac{\Delta v}{\Delta t} = \dfrac{16.0\ \text{m/s} - 10.0\ \text{m/s}}{3.00\ \text{s} - 2.00\ \text{s}} = \boxed{6.00\ \text{m/s}^2}$

(d) At all times $a = \dfrac{d}{dt}(6.00 - 2.00) = \boxed{6.00\ \text{m/s}^2}$. (This includes both $t = 2.00\ \text{s}$ and $t = 3.00\ \text{s}$).

**P2.17** (a) $a = \dfrac{\Delta v}{\Delta t} = \dfrac{8.00\ \text{m/s}}{6.00\ \text{s}} = \boxed{1.3\ \text{m/s}^2}$

(b) Maximum positive acceleration is at $t = 3\ \text{s}$, and is approximately $\boxed{2\ \text{m/s}^2}$.

(c) $a = 0$, at $\boxed{t = 6\ \text{s}}$, and also for $\boxed{t > 10\ \text{s}}$.

(d) Maximum negative acceleration is at $t = 8\ \text{s}$, and is approximately $\boxed{-1.5\ \text{m/s}^2}$.

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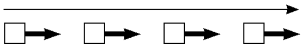
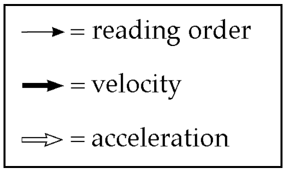
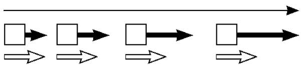
## Section 2.4 **Motion Diagrams**

**P2.18** (a)

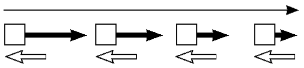




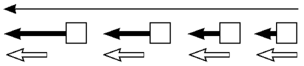
(b)



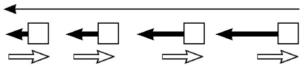
(c)



(d)



(e)



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(f) One way of phrasing the answer: The spacing of the successive positions would change with less regularity.
Another way: The object would move with some combination of the kinds of motion shown in (a) through (e). Within one drawing, the accelerations vectors would vary in magnitude and direction.

## Section 2.5 **One-Dimensional Motion with Constant Acceleration**

**P2.19** From $v_f^2 = v_i^2 + 2ax$, we have $\left(10.97\times10^3\ \text{m/s}\right)^2 = 0 + 2a(220\ \text{m})$, so that $\boxed{a = 2.74\times10^5\ \text{m/s}^2}$ which is $\boxed{a = 2.79\times10^4 \text{ times } g}$.

**P2.20** (a) $x_f - x_i = \frac{1}{2}\left(v_i + v_f\right)t$ becomes $40\ \text{m} = \frac{1}{2}(v_i + 2.80\ \text{m/s})(8.50\ \text{s})$ which yields $v_i = \boxed{6.61\ \text{m/s}}$.

(b) $a = \dfrac{v_f - v_i}{t} = \dfrac{2.80\ \text{m/s} - 6.61\ \text{m/s}}{8.50\ \text{s}} = \boxed{-0.448\ \text{m/s}^2}$

**P2.21** Given $v_i = 12.0\ \text{cm/s}$ when $x_i = 3.00\ \text{cm}(t = 0)$, and at $t = 2.00\ \text{s}$, $x_f = -5.00\ \text{cm}$,

$$x_f - x_i = v_i t + \frac{1}{2}at^2:\ -5.00 - 3.00 = 12.0(2.00) + \frac{1}{2}a(2.00)^2$$
$$-8.00 = 24.0 + 2a \qquad a = -\frac{32.0}{2} = \boxed{-16.0\ \text{cm/s}^2}.$$

***P2.22** (a) Let *i* be the state of moving at $60\ \text{mi/h}$ and *f* be at rest

$$v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$$
$$0 = \left(60\ \text{mi/h}\right)^2 + 2a_x(121\ \text{ft} - 0)\left(\frac{1\ \text{mi}}{5\,280\ \text{ft}}\right)$$
$$a_x = \frac{-3\,600\ \text{mi}}{242\ \text{h}^2}\left(\frac{5\,280\ \text{ft}}{1\ \text{mi}}\right)\left(\frac{1\ \text{h}}{3\,600\ \text{s}}\right) = \boxed{-21.8\ \text{mi/h}\cdot\text{s}}$$
$$= -21.8\ \text{mi/h}\cdot\text{s}\left(\frac{1\,609\ \text{m}}{1\ \text{mi}}\right)\left(\frac{1\ \text{h}}{3\,600\ \text{s}}\right) = \boxed{-9.75\ \text{m/s}^2}.$$

(b) Similarly,

$$0 = \left(80\ \text{mi/h}\right)^2 + 2a_x(211\ \text{ft} - 0)$$
$$a_x = -\frac{6\,400(5\,280)}{422(3\,600)}\ \text{mi/h}\cdot\text{s} = \boxed{-22.2\ \text{mi/h}\cdot\text{s} = -9.94\ \text{m/s}^2}.$$

(c) Let *i* be moving at $80\ \text{mi/h}$ and *f* be moving at $60\ \text{mi/h}$.

$$v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$$
$$\left(60\ \text{mi/h}\right)^2 = \left(80\ \text{mi/h}\right)^2 + 2a_x(211\ \text{ft} - 121\ \text{ft})$$
$$a_x = -\frac{2\,800(5\,280)}{2(90)(3\,600)}\ \text{mi/h}\cdot\text{s} = \boxed{-22.8\ \text{mi/h}\cdot\text{s} = -10.2\ \text{m/s}^2}.$$

***P2.23** (a) Choose the initial point where the pilot reduces the throttle and the final point where the boat passes the buoy:

$$x_i = 0,\ x_f = 100\text{ m},\ v_{xi} = 30\text{ m/s},\ v_{xf} = ?,\ a_x = -3.5\text{ m/s}^2,\ t = ?$$

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2:$$

$$100\text{ m} = 0 + (30\text{ m/s})t + \frac{1}{2}\left(-3.5\text{ m/s}^2\right)t^2$$

$$\left(1.75\text{ m/s}^2\right)t^2 - (30\text{ m/s})t + 100\text{ m} = 0.$$

We use the quadratic formula:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{30\text{ m/s} \pm \sqrt{900\text{ m}^2/\text{s}^2 - 4\left(1.75\text{ m/s}^2\right)(100\text{ m})}}{2\left(1.75\text{ m/s}^2\right)} = \frac{30\text{ m/s} \pm 14.1\text{ m/s}}{3.5\text{ m/s}^2} = 12.6\text{ s or } \boxed{4.53\text{ s}}.$$

The smaller value is the physical answer. If the boat kept moving with the same acceleration, it would stop and move backward, then gain speed, and pass the buoy again at 12.6 s.

(b) $v_{xf} = v_{xi} + a_x t = 30\text{ m/s} - \left(3.5\text{ m/s}^2\right)4.53\text{ s} = \boxed{14.1\text{ m/s}}$

**P2.24** (a) Total displacement = area under the $(v,\ t)$ curve from $t = 0$ to 50 s.

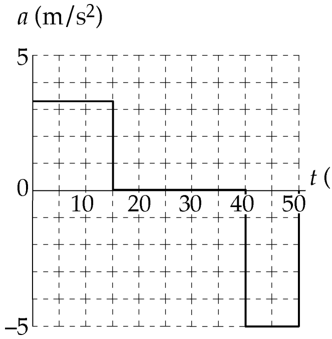


**FIG. P2.24**

$$\Delta x = \frac{1}{2}(50\text{ m/s})(15\text{ s}) + (50\text{ m/s})(40 - 15)\text{ s}$$

$$+\frac{1}{2}(50\text{ m/s})(10\text{ s})$$

$$\Delta x = \boxed{1\,875\text{ m}}$$

(b) From $t = 10\text{ s}$ to $t = 40\text{ s}$, displacement is

$$\Delta x = \frac{1}{2}(50\text{ m/s} + 33\text{ m/s})(5\text{ s}) + (50\text{ m/s})(25\text{ s}) = \boxed{1\,457\text{ m}}.$$

(c) $0 \le t \le 15\text{ s}$: $a_1 = \dfrac{\Delta v}{\Delta t} = \dfrac{(50 - 0)\text{ m/s}}{15\text{ s} - 0} = \boxed{3.3\text{ m/s}^2}$

$15\text{ s} < t < 40\text{ s}$: $\boxed{a_2 = 0}$

$40\text{ s} \le t \le 50\text{ s}$: $a_3 = \dfrac{\Delta v}{\Delta t} = \dfrac{(0 - 50)\text{ m/s}}{50\text{ s} - 40\text{ s}} = \boxed{-5.0\text{ m/s}^2}$

*continued on next page*

(d) (i) $x_1 = 0 + \frac{1}{2}a_1 t^2 = \frac{1}{2}\left(3.3\ \text{m/s}^2\right)t^2$ or $\boxed{x_1 = \left(1.67\ \text{m/s}^2\right)t^2}$

(ii) $x_2 = \frac{1}{2}(15\ \text{s})[50\ \text{m/s} - 0] + (50\ \text{m/s})(t - 15\ \text{s})$ or $\boxed{x_2 = (50\ \text{m/s})t - 375\ \text{m}}$

(iii) For $40\ \text{s} \le t \le 50\ \text{s}$,

$$x_3 = \begin{pmatrix}\text{area under } v \text{ vs } t \\ \text{from } t = 0 \text{ to } 40\ \text{s}\end{pmatrix} + \frac{1}{2}a_3(t - 40\ \text{s})^2 + (50\ \text{m/s})(t - 40\ \text{s})$$

or

$$x_3 = 375\ \text{m} + 1\,250\ \text{m} + \frac{1}{2}\left(-5.0\ \text{m/s}^2\right)(t - 40\ \text{s})^2 + (50\ \text{m/s})(t - 40\ \text{s})$$

which reduces to

$$\boxed{x_3 = (250\ \text{m/s})t - \left(2.5\ \text{m/s}^2\right)t^2 - 4\,375\ \text{m}}.$$

(e) $\bar{v} = \frac{\text{total displacement}}{\text{total elapsed time}} = \frac{1\,875\ \text{m}}{50\ \text{s}} = \boxed{37.5\ \text{m/s}}$

**P2.25** (a) Compare the position equation $x = 2.00 + 3.00t - 4.00t^2$ to the general form

$$x_f = x_i + v_i t + \frac{1}{2}at^2$$

to recognize that $x_i = 2.00\ \text{m}$, $v_i = 3.00\ \text{m/s}$, and $a = -8.00\ \text{m/s}^2$. The velocity equation, $v_f = v_i + at$, is then

$$v_f = 3.00\ \text{m/s} - \left(8.00\ \text{m/s}^2\right)t.$$

The particle changes direction when $v_f = 0$, which occurs at $t = \frac{3}{8}\ \text{s}$. The position at this time is:

$$x = 2.00\ \text{m} + (3.00\ \text{m/s})\left(\frac{3}{8}\ \text{s}\right) - \left(4.00\ \text{m/s}^2\right)\left(\frac{3}{8}\ \text{s}\right)^2 = \boxed{2.56\ \text{m}}.$$

(b) From $x_f = x_i + v_i t + \frac{1}{2}at^2$, observe that when $x_f = x_i$, the time is given by $t = -\frac{2v_i}{a}$. Thus, when the particle returns to its initial position, the time is

$$t = \frac{-2(3.00\ \text{m/s})}{-8.00\ \text{m/s}^2} = \frac{3}{4}\ \text{s}$$

and the velocity is $v_f = 3.00\ \text{m/s} - \left(8.00\ \text{m/s}^2\right)\left(\frac{3}{4}\ \text{s}\right) = \boxed{-3.00\ \text{m/s}}$.

***P2.26** The time for the Ford to slow down we find from

$$x_f = x_i + \frac{1}{2}\left(v_{xi} + v_{xf}\right)t$$

$$t = \frac{2\Delta x}{v_{xi} + v_{xf}} = \frac{2(250\text{ m})}{71.5\text{ m/s} + 0} = 6.99\text{ s}.$$

Its time to speed up is similarly

$$t = \frac{2(350\text{ m})}{0 + 71.5\text{ m/s}} = 9.79\text{ s}.$$

The whole time it is moving at less than maximum speed is $6.99\text{ s} + 5.00\text{ s} + 9.79\text{ s} = 21.8\text{ s}$. The Mercedes travels

$$x_f = x_i + \frac{1}{2}\left(v_{xi} + v_{xf}\right)t = 0 + \frac{1}{2}(71.5 + 71.5)(\text{m/s})(21.8\text{ s})$$
$$= 1\,558\text{ m}$$

while the Ford travels $250 + 350\text{ m} = 600\text{ m}$, to fall behind by $1\,558\text{ m} - 600\text{ m} = \boxed{958\text{ m}}$.

**P2.27** (a) $v_i = 100\text{ m/s}$, $a = -5.00\text{ m/s}^2$, $v_f = v_i + at$ so $0 = 100 - 5t$, $v_f^2 = v_i^2 + 2a\left(x_f - x_i\right)$ so $0 = (100)^2 - 2(5.00)\left(x_f - 0\right)$. Thus $x_f = 1\,000\text{ m}$ and $t = \boxed{20.0\text{ s}}$.

(b) At this acceleration the plane would overshoot the runway: $\boxed{\text{No}}$.

**P2.28** (a) Take $t_i = 0$ at the bottom of the hill where $x_i = 0$, $v_i = 30.0\text{ m/s}$, $a = -2.00\text{ m/s}^2$. Use these values in the general equation

$$x_f = x_i + v_i t + \frac{1}{2}at^2$$

to find

$$x_f = 0 + (30.0t\text{ m/s}) + \frac{1}{2}(-2.00\text{ m/s}^2)t^2$$

when $t$ is in seconds

$$\boxed{x_f = \left(30.0t - t^2\right)\text{ m}}.$$

To find an equation for the velocity, use $v_f = v_i + at = 30.0\text{ m/s} + \left(-2.00\text{ m/s}^2\right)t$,

$$\boxed{v_f = (30.0 - 2.00t)\text{ m/s}}.$$

(b) The distance of travel $x_f$ becomes a maximum, $x_{\max}$, when $v_f = 0$ (turning point in the motion). Use the expressions found in part (a) for $v_f$ to find the value of $t$ when $x_f$ has its maximum value:

From $v_f = (3.00 - 2.00t)\text{ m/s}$, $v_f = 0$ when $t = 15.0\text{ s}$. Then

$$x_{\max} = \left(30.0t - t^2\right)\text{ m} = (30.0)(15.0) - (15.0)^2 = \boxed{225\text{ m}}.$$

**P2.29** In the simultaneous equations:

$$\left\{\begin{array}{c} v_{xf}=v_{xi}+a_x t \\ x_f-x_i=\dfrac{1}{2}\left(v_{xi}+v_{xf}\right)t \end{array}\right\} \text{ we have } \left\{\begin{array}{c} v_{xf}=v_{xi}-\left(5.60\ \text{m/s}^2\right)(4.20\ \text{s}) \\ 62.4\ \text{m}=\dfrac{1}{2}\left(v_{xi}+v_{xf}\right)(4.20\ \text{s}) \end{array}\right\}.$$

So substituting for $v_{xi}$ gives $62.4\ \text{m}=\dfrac{1}{2}\left[v_{xf}+\left(56.0\ \text{m/s}^2\right)(4.20\ \text{s})+v_{xf}\right](4.20\ \text{s})$

$$14.9\ \text{m/s}=v_{xf}+\frac{1}{2}\left(5.60\ \text{m/s}^2\right)(4.20\ \text{s}).$$

Thus

$$v_{xf}=\boxed{3.10\ \text{m/s}}.$$

**P2.30** Take any two of the standard four equations, such as $\left\{\begin{array}{c} v_{xf}=v_{xi}+a_x t \\ x_f-x_i=\dfrac{1}{2}\left(v_{xi}+v_{xf}\right)t \end{array}\right\}$. Solve one for $v_{xi}$, and substitute into the other: $v_{xi}=v_{xf}-a_x t$

$$x_f-x_i=\frac{1}{2}\left(v_{xf}-a_x t+v_{xf}\right)t.$$

Thus

$$\boxed{x_f-x_i=v_{xf}t-\frac{1}{2}a_x t^2}.$$

Back in problem 29, $62.4\ \text{m}=v_{xf}(4.20\ \text{s})-\dfrac{1}{2}\left(-5.60\ \text{m/s}^2\right)(4.20\ \text{s})^2$

$$v_{xf}=\frac{62.4\ \text{m}-49.4\ \text{m}}{4.20\ \text{s}}=\boxed{3.10\ \text{m/s}}.$$

**P2.31** (a) $a=\dfrac{v_f-v_i}{t}=\dfrac{632\left(\frac{5\,280}{3\,600}\right)}{1.40}=\boxed{-662\ \text{ft/s}^2}=-202\ \text{m/s}^2$

(b) $x_f=v_i t+\dfrac{1}{2}at^2=(632)\left(\dfrac{5\,280}{3\,600}\right)(1.40)-\dfrac{1}{2}(662)(1.40)^2=\boxed{649\ \text{ft}}=\boxed{198\ \text{m}}$

**P2.32** (a) The time it takes the truck to reach $20.0\ \text{m/s}$ is found from $v_f = v_i + at$. Solving for $t$ yields

$$t = \frac{v_f - v_i}{a} = \frac{20.0\ \text{m/s} - 0\ \text{m/s}}{2.00\ \text{m/s}^2} = 10.0\ \text{s}.$$

The total time is thus

$$10.0\ \text{s} + 20.0\ \text{s} + 5.00\ \text{s} = \boxed{35.0\ \text{s}}.$$

(b) The average velocity is the total distance traveled divided by the total time taken. The distance traveled during the first 10.0 s is

$$x_1 = \bar{v}t = \left(\frac{0 + 20.0}{2}\right)(10.0) = 100\ \text{m}.$$

With $a$ being 0 for this interval, the distance traveled during the next 20.0 s is

$$x_2 = v_i t + \frac{1}{2}at^2 = (20.0)(20.0) + 0 = 400\ \text{m}.$$

The distance traveled in the last 5.00 s is

$$x_3 = \bar{v}t = \left(\frac{20.0 + 0}{2}\right)(5.00) = 50.0\ \text{m}.$$

The total distance $x = x_1 + x_2 + x_3 = 100 + 400 + 50 = 550\ \text{m}$, and the average velocity is given by $\bar{v} = \frac{x}{t} = \frac{550}{35.0} = \boxed{15.7\ \text{m/s}}$.

**P2.33** We have $v_i = 2.00 \times 10^4\ \text{m/s}$, $v_f = 6.00 \times 10^6\ \text{m/s}$, $x_f - x_i = 1.50 \times 10^{-2}\ \text{m}$.

(a) $$x_f - x_i = \frac{1}{2}(v_i + v_f)t:\ t = \frac{2(x_f - x_i)}{v_i + v_f} = \frac{2(1.50 \times 10^{-2}\ \text{m})}{2.00 \times 10^4\ \text{m/s} + 6.00 \times 10^6\ \text{m/s}} = \boxed{4.98 \times 10^{-9}\ \text{s}}$$

(b) $v_f^2 = v_i^2 + 2a_x(x_f - x_i)$:

$$a_x = \frac{v_f^2 - v_i^2}{2(x_f - x_i)} = \frac{(6.00 \times 10^6\ \text{m/s})^2 - (2.00 \times 10^4\ \text{m/s})^2}{2(1.50 \times 10^{-2}\ \text{m})} = \boxed{1.20 \times 10^{15}\ \text{m/s}^2}$$

***P2.34** (a) $v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$: $\left[0.01\left(3\times10^8\ \text{m/s}\right)\right]^2 = 0 + 2a_x(40\ \text{m})$

$$a_x = \frac{\left(3\times10^6\ \text{m/s}\right)^2}{80\ \text{m}} = \boxed{1.12\times10^{11}\ \text{m/s}^2}$$

(b) We must find separately the time $t_1$ for speeding up and the time $t_2$ for coasting:

$$x_f - x_i = \frac{1}{2}\left(v_{xf} + v_{xi}\right)t_1:\quad 40\ \text{m} = \frac{1}{2}\left(3\times10^6\ \text{m/s} + 0\right)t_1$$

$$t_1 = 2.67\times10^{-5}\ \text{s}$$

$$x_f - x_i = \frac{1}{2}\left(v_{xf} + v_{xi}\right)t_2:\quad 60\ \text{m} = \frac{1}{2}\left(3\times10^6\ \text{m/s} + 3\times10^6\ \text{m/s}\right)t_2$$

$$t_2 = 2.00\times10^{-5}\ \text{s}$$

total time $= \boxed{4.67\times10^{-5}\ \text{s}}$.

***P2.35** (a) Along the time axis of the graph shown, let $i = 0$ and $f = t_m$. Then $v_{xf} = v_{xi} + a_x t$ gives $v_c = 0 + a_m t_m$

$$\boxed{a_m = \frac{v_c}{t_m}}.$$

(b) The displacement between 0 and $t_m$ is

$$x_f - x_i = v_{xi}t + \frac{1}{2}a_x t^2 = 0 + \frac{1}{2}\frac{v_c}{t_m}t_m^2 = \frac{1}{2}v_c t_m.$$

The displacement between $t_m$ and $t_0$ is

$$x_f - x_i = v_{xi}t + \frac{1}{2}a_x t^2 = v_c(t_0 - t_m) + 0.$$

The total displacement is

$$\Delta x = \frac{1}{2}v_c t_m + v_c t_0 - v_c t_m = \boxed{v_c\left(t_0 - \frac{1}{2}t_m\right)}.$$

(c) For constant $v_c$ and $t_0$, $\Delta x$ is minimized by maximizing $t_m$ to $t_m = t_0$. Then $\Delta x_{\min} = v_c\left(t_0 - \frac{1}{2}t_0\right) = \boxed{\frac{v_c t_0}{2}}$.

(e) This is realized by having the servo motor on all the time.

(d) We maximize $\Delta x$ by letting $t_m$ approach zero. In the limit $\Delta x = v_c(t_0 - 0) = \boxed{v_c t_0}$.

(e) This cannot be attained because the acceleration must be finite.

***P2.36** Let the glider enter the photogate with velocity $v_i$ and move with constant acceleration $a$. For its motion from entry to exit,

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$$\ell = 0 + v_i\Delta t_d + \frac{1}{2}a\Delta t_d^2 = v_d\Delta t_d$$

$$v_d = v_i + \frac{1}{2}a\Delta t_d$$

(a) The speed halfway through the photogate in space is given by

$$v_{hs}^2 = v_i^2 + 2a\left(\frac{\ell}{2}\right) = v_i^2 + av_d\Delta t_d .$$

$v_{hs} = \sqrt{v_i^2 + av_d\Delta t_d}$ and this is not equal to $v_d$ unless $a = 0$.

(b) The speed halfway through the photogate in time is given by $v_{ht} = v_i + a\left(\frac{\Delta t_d}{2}\right)$ and this is equal to $v_d$ as determined above.

**P2.37** (a) Take initial and final points at top and bottom of the incline. If the ball starts from rest,

$$v_i = 0,\ a = 0.500\ \text{m/s}^2,\ x_f - x_i = 9.00\ \text{m}.$$

Then

$$v_f^2 = v_i^2 + 2a(x_f - x_i) = 0^2 + 2(0.500\ \text{m/s}^2)(9.00\ \text{m})$$

$$v_f = \boxed{3.00\ \text{m/s}}.$$

(b) $x_f - x_i = v_i t + \frac{1}{2}at^2$

$$9.00 = 0 + \frac{1}{2}(0.500\ \text{m/s}^2)t^2$$

$$t = \boxed{6.00\ \text{s}}$$

(c) Take initial and final points at the bottom of the planes and the top of the second plane, respectively:

$$v_i = 3.00\ \text{m/s},\ v_f = 0,\ x_f - x_i = 15.00\ \text{m}.$$

$v_f^2 = v_i^2 + 2a(x_f - x_i)$ gives

$$a = \frac{v_f^2 - v_i^2}{2(x_f - x_i)} = \frac{\left[0 - (3.00\ \text{m/s})^2\right]}{2(15.0\ \text{m})} = \boxed{-0.300\ \text{m/s}^2}.$$

(d) Take the initial point at the bottom of the planes and the final point 8.00 m along the second: $v_i = 3.00\ \text{m/s}$, $x_f - x_i = 8.00\ \text{m}$, $a = -0.300\ \text{m/s}^2$

$$v_f^2 = v_i^2 + 2a(x_f - x_i) = (3.00\ \text{m/s})^2 + 2(-0.300\ \text{m/s}^2)(8.00\ \text{m}) = 4.20\ \text{m}^2/\text{s}^2$$

$$v_f = \boxed{2.05\ \text{m/s}}.$$

**P2.38** Take the original point to be when Sue notices the van. Choose the origin of the $x$-axis at Sue's car. For her we have $x_{is}=0$, $v_{is}=30.0\text{ m/s}$, $a_s=-2.00\text{ m/s}^2$ so her position is given by

$$x_s(t)=x_{is}+v_{is}t+\frac{1}{2}a_st^2=(30.0\text{ m/s})t+\frac{1}{2}\left(-2.00\text{ m/s}^2\right)t^2.$$

For the van, $x_{iv}=155\text{ m}$, $v_{iv}=5.00\text{ m/s}$, $a_v=0$ and

$$x_v(t)=x_{iv}+v_{iv}t+\frac{1}{2}a_vt^2=155+(5.00\text{ m/s})t+0.$$

To test for a collision, we look for an instant $t_c$ when both are at the same place:

$$30.0t_c-t_c^2=155+5.00t_c$$
$$0=t_c^2-25.0t_c+155.$$

From the quadratic formula

$$t_c=\frac{25.0\pm\sqrt{(25.0)^2-4(155)}}{2}=13.6\text{ s or }\boxed{11.4\text{ s}}.$$

The smaller value is the collision time. (The larger value tells when the van would pull ahead again if the vehicles could move through each other). The wreck happens at position

$$155\text{ m}+(5.00\text{ m/s})(11.4\text{ s})=\boxed{212\text{ m}}.$$

***P2.39** As in the algebraic solution to Example 2.8, we let $t$ represent the time the trooper has been moving. We graph

$$x_{\text{car}}=45+45t$$

and

$$x_{\text{trooper}}=1.5t^2.$$

They intersect at

$$t=\boxed{31\text{ s}}.$$

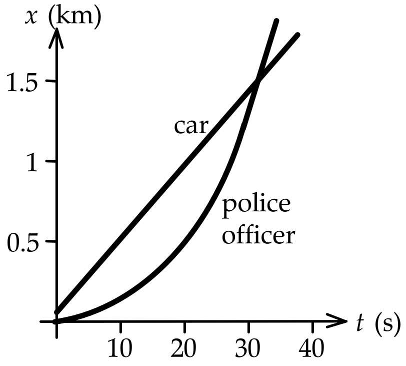


**FIG. P2.39**

## Section 2.6 **Freely Falling Objects**

**P2.40** Choose the origin $(y=0,\ t=0)$ at the starting point of the ball and take upward as positive. Then $y_i=0$, $v_i=0$, and $a=-g=-9.80\ \text{m/s}^2$. The position and the velocity at time $t$ become:

$$y_f-y_i=v_it+\frac{1}{2}at^2:\ y_f=-\frac{1}{2}gt^2=-\frac{1}{2}\left(9.80\ \text{m/s}^2\right)t^2$$

and

$$v_f=v_i+at:\ v_f=-gt=-\left(9.80\ \text{m/s}^2\right)t.$$

(a) at $t=1.00\ \text{s}$: $y_f=-\frac{1}{2}\left(9.80\ \text{m/s}^2\right)(1.00\ \text{s})^2=\boxed{-4.90\ \text{m}}$

at $t=2.00\ \text{s}$: $y_f=-\frac{1}{2}\left(9.80\ \text{m/s}^2\right)(2.00\ \text{s})^2=\boxed{-19.6\ \text{m}}$

at $t=3.00\ \text{s}$: $y_f=-\frac{1}{2}\left(9.80\ \text{m/s}^2\right)(3.00\ \text{s})^2=\boxed{-44.1\ \text{m}}$

(b) at $t=1.00\ \text{s}$: $v_f=-\left(9.80\ \text{m/s}^2\right)(1.00\ \text{s})=\boxed{-9.80\ \text{m/s}}$

at $t=2.00\ \text{s}$: $v_f=-\left(9.80\ \text{m/s}^2\right)(2.00\ \text{s})=\boxed{-19.6\ \text{m/s}}$

at $t=3.00\ \text{s}$: $v_f=-\left(9.80\ \text{m/s}^2\right)(3.00\ \text{s})=\boxed{-29.4\ \text{m/s}}$

**P2.41** Assume that air resistance may be neglected. Then, the acceleration at all times during the flight is that due to gravity, $a=-g=-9.80\ \text{m/s}^2$. During the flight, Goff went 1 mile (1 609 m) up and then 1 mile back down. Determine his speed just after launch by considering his upward flight:

$$v_f^2=v_i^2+2a\left(y_f-y_i\right):\quad 0=v_i^2-2\left(9.80\ \text{m/s}^2\right)(1\,609\ \text{m})$$
$$v_i=178\ \text{m/s}.$$

His time in the air may be found by considering his motion from just after launch to just before impact:

$$y_f-y_i=v_it+\frac{1}{2}at^2:\ 0=(178\ \text{m/s})t-\frac{1}{2}\left(-9.80\ \text{m/s}^2\right)t^2.$$

The root $t=0$ describes launch; the other root, $t=36.2\ \text{s}$, describes his flight time. His rate of pay may then be found from

$$\text{pay rate}=\frac{\$1.00}{36.2\ \text{s}}=(0.027\,6\ \$/\text{s})(3\,600\ \text{s/h})=\boxed{\$99.3/\text{h}}.$$

We have assumed that the workman's flight time, "a mile", and "a dollar", were measured to three-digit precision. We have interpreted "up in the sky" as referring to the free fall time, not to the launch and landing times. Both the takeoff and landing times must be several seconds away from the job, in order for Goff to survive to resume work.

**P2.42** We have $y_f = -\frac{1}{2}gt^2 + v_i t + y_i$

$$0 = -\left(4.90\ \mathrm{m/s^2}\right)t^2 - (8.00\ \mathrm{m/s})t + 30.0\ \mathrm{m}.$$

Solving for $t$,

$$t = \frac{8.00 \pm \sqrt{64.0 + 588}}{-9.80}.$$

Using only the positive value for $t$, we find that $t = \boxed{1.79\ \mathrm{s}}$.

**P2.43** (a) $y_f - y_i = v_i t + \frac{1}{2}at^2$: $4.00 = (1.50)v_i - (4.90)(1.50)^2$ and $v_i = \boxed{10.0\ \mathrm{m/s\ upward}}$.

(b) $v_f = v_i + at = 10.0 - (9.80)(1.50) = -4.68\ \mathrm{m/s}$

$$v_f = \boxed{4.68\ \mathrm{m/s\ downward}}$$

**P2.44** The bill starts from rest $v_i = 0$ and falls with a downward acceleration of $9.80\ \mathrm{m/s^2}$ (due to gravity). Thus, in 0.20 s it will fall a distance of

$$\Delta y = v_i t - \frac{1}{2}gt^2 = 0 - \left(4.90\ \mathrm{m/s^2}\right)(0.20\ \mathrm{s})^2 = -0.20\ \mathrm{m}.$$

This distance is about twice the distance between the center of the bill and its top edge ($\cong 8$ cm). $\boxed{\text{Thus, David will be unsuccessful}}$.

***P2.45** (a) From $\Delta y = v_i t + \frac{1}{2}at^2$ with $v_i = 0$, we have

$$t = \sqrt{\frac{2(\Delta y)}{a}} = \sqrt{\frac{2(-23\ \mathrm{m})}{-9.80\ \mathrm{m/s^2}}} = \boxed{2.17\ \mathrm{s}}.$$

(b) The final velocity is $v_f = 0 + \left(-9.80\ \mathrm{m/s^2}\right)(2.17\ \mathrm{s}) = \boxed{-21.2\ \mathrm{m/s}}$.

(c) The time take for the sound of the impact to reach the spectator is

$$t_{\text{sound}} = \frac{\Delta y}{v_{\text{sound}}} = \frac{23\ \mathrm{m}}{340\ \mathrm{m/s}} = 6.76 \times 10^{-2}\ \mathrm{s},$$

so the total elapsed time is $t_{\text{total}} = 2.17\ \mathrm{s} + 6.76 \times 10^{-2}\ \mathrm{s} \approx \boxed{2.23\ \mathrm{s}}$.

**P2.46** At any time $t$, the position of the ball released from rest is given by $y_1 = h - \frac{1}{2}gt^2$. At time $t$, the position of the ball thrown vertically upward is described by $y_2 = v_i t - \frac{1}{2}gt^2$. The time at which the first ball has a position of $y_1 = \frac{h}{2}$ is found from the first equation as $\frac{h}{2} = h - \frac{1}{2}gt^2$, which yields $t = \sqrt{\frac{h}{g}}$. To require that the second ball have a position of $y_2 = \frac{h}{2}$ at this time, use the second equation to obtain $\frac{h}{2} = v_i\sqrt{\frac{h}{g}} - \frac{1}{2}g\left(\frac{h}{g}\right)$. This gives the required initial upward velocity of the second ball as $\boxed{v_i = \sqrt{gh}}$.

**P2.47** (a) $v_f = v_i - gt$: $v_f = 0$ when $t = 3.00\text{ s}$, $g = 9.80\text{ m/s}^2$. Therefore,

$$v_i = gt = (9.80\text{ m/s}^2)(3.00\text{ s}) = \boxed{29.4\text{ m/s}}.$$

(b) $y_f - y_i = \frac{1}{2}(v_f + v_i)t$

$$y_f - y_i = \frac{1}{2}(29.4\text{ m/s})(3.00\text{ s}) = \boxed{44.1\text{ m}}$$

***P2.48** (a) Consider the upward flight of the arrow.

$$v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$$

$$0 = (100\text{ m/s})^2 + 2(-9.8\text{ m/s}^2)\Delta y$$

$$\Delta y = \frac{10\,000\text{ m}^2/\text{s}^2}{19.6\text{ m/s}^2} = \boxed{510\text{ m}}$$

(b) Consider the whole flight of the arrow.

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$0 = 0 + (100\text{ m/s})t + \frac{1}{2}(-9.8\text{ m/s}^2)t^2$$

The root $t = 0$ refers to the starting point. The time of flight is given by

$$t = \frac{100\text{ m/s}}{4.9\text{ m/s}^2} = \boxed{20.4\text{ s}}.$$

**P2.49** Time to fall 3.00 m is found from Eq. 2.12 with $v_i = 0$, $3.00\text{ m} = \frac{1}{2}(9.80\text{ m/s}^2)t^2$, $t = 0.782\text{ s}$.

(a) With the horse galloping at 10.0 m/s, the horizontal distance is $vt = \boxed{7.82\text{ m}}$.

(b) $t = \boxed{0.782\text{ s}}$

**P2.50** Take downward as the positive $y$ direction.

(a) While the woman was in free fall,

$$\Delta y = 144\text{ ft},\ v_i = 0,\text{ and } a = g = 32.0\ \text{ft/s}^2.$$

Thus, $\Delta y = v_i t + \frac{1}{2}at^2 \rightarrow 144\text{ ft} = 0 + \left(16.0\ \text{ft/s}^2\right)t^2$ giving $t_{\text{fall}} = 3.00\text{ s}$. Her velocity just before impact is:

$$v_f = v_i + gt = 0 + \left(32.0\ \text{ft/s}^2\right)(3.00\text{ s}) = \boxed{96.0\ \text{ft/s}}.$$

(b) While crushing the box, $v_i = 96.0\ \text{ft/s}$, $v_f = 0$, and $\Delta y = 18.0\text{ in.} = 1.50\text{ ft}$. Therefore,

$$a = \frac{v_f^2 - v_i^2}{2(\Delta y)} = \frac{0 - (96.0\ \text{ft/s})^2}{2(1.50\text{ ft})} = -3.07\times10^3\ \text{ft/s}^2,\text{ or } \boxed{a = 3.07\times10^3\ \text{ft/s}^2\text{ upward}}.$$

(c) Time to crush box: $\Delta t = \frac{\Delta y}{\bar{v}} = \frac{\Delta y}{\frac{v_f + v_i}{2}} = \frac{2(1.50\text{ ft})}{0 + 96.0\ \text{ft/s}}$ or $\boxed{\Delta t = 3.13\times10^{-2}\text{ s}}$.

**P2.51** $y = 3.00t^3$: At $t = 2.00\text{ s}$, $y = 3.00(2.00)^3 = 24.0\text{ m}$ and

$$v_y = \frac{dy}{dt} = 9.00t^2 = 36.0\ \text{m/s}\uparrow.$$

If the helicopter releases a small mailbag at this time, the equation of motion of the mailbag is

$$y_b = y_{bi} + v_i t - \frac{1}{2}gt^2 = 24.0 + 36.0t - \frac{1}{2}(9.80)t^2.$$

Setting $y_b = 0$,

$$0 = 24.0 + 36.0t - 4.90t^2.$$

Solving for $t$, (only positive values of $t$ count), $\boxed{t = 7.96\text{ s}}$.

***P2.52** Consider the last 30 m of fall. We find its speed 30 m above the ground:

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$0 = 30\text{ m} + v_{yi}(1.5\text{ s}) + \frac{1}{2}\left(-9.8\ \text{m/s}^2\right)(1.5\text{ s})^2$$

$$v_{yi} = \frac{-30\text{ m} + 11.0\text{ m}}{1.5\text{ s}} = -12.6\ \text{m/s}.$$

Now consider the portion of its fall above the 30 m point. We assume it starts from rest

$$v_{yf}^2 = v_{yi}^2 + 2a_y\left(y_f - y_i\right)$$

$$(-12.6\ \text{m/s})^2 = 0 + 2\left(-9.8\ \text{m/s}^2\right)\Delta y$$

$$\Delta y = \frac{160\ \text{m}^2/\text{s}^2}{-19.6\ \text{m/s}^2} = -8.16\text{ m}.$$

Its original height was then $30\text{ m} + |-8.16\text{ m}| = \boxed{38.2\text{ m}}$.

## Section 2.7 **Kinematic Equations Derived from Calculus**

**P2.53** (a) $J = \dfrac{da}{dt} = \text{constant}$

$$da = J\,dt$$

$$a = J\int dt = Jt + c_1$$

but $a = a_i$ when $t = 0$ so $c_1 = a_i$. Therefore, $\boxed{a = Jt + a_i}$

$$\begin{aligned} a &= \frac{dv}{dt} \\ dv &= a\,dt \\ v &= \int a\,dt = \int (Jt + a_i)\,dt = \frac{1}{2}Jt^2 + a_i t + c_2 \end{aligned}$$

but $v = v_i$ when $t = 0$, so $c_2 = v_i$ and $\boxed{v = \frac{1}{2}Jt^2 + a_i t + v_i}$

$$\begin{aligned} v &= \frac{dx}{dt} \\ dx &= v\,dt \\ x &= \int v\,dt = \int \left(\frac{1}{2}Jt^2 + a_i t + v_i\right)dt \\ x &= \frac{1}{6}Jt^3 + \frac{1}{2}a_i t^2 + v_i t + c_3 \\ x &= x_i \end{aligned}$$

when $t = 0$, so $c_3 = x_i$. Therefore, $\boxed{x = \frac{1}{6}Jt^3 + \frac{1}{2}a_i t^2 + v_i t + x_i}$.

(b)

$$\begin{aligned} a^2 &= (Jt + a_i)^2 = J^2t^2 + a_i^2 + 2Ja_i t \\ a^2 &= a_i^2 + \left(J^2t^2 + 2Ja_i t\right) \\ a^2 &= a_i^2 + 2J\left(\frac{1}{2}Jt^2 + a_i t\right) \end{aligned}$$

Recall the expression for $v$: $v = \frac{1}{2}Jt^2 + a_i t + v_i$. So $(v - v_i) = \frac{1}{2}Jt^2 + a_i t$. Therefore,

$$\boxed{a^2 = a_i^2 + 2J(v - v_i)}.$$

**P2.54** (a) See the graphs at the right.

Choose $x=0$ at $t=0$.

At $t=3\text{ s}$, $x=\frac{1}{2}(8\text{ m/s})(3\text{ s})=12\text{ m}$.

At $t=5\text{ s}$, $x=12\text{ m}+(8\text{ m/s})(2\text{ s})=28\text{ m}$.

At $t=7\text{ s}$, $x=28\text{ m}+\frac{1}{2}(8\text{ m/s})(2\text{ s})=36\text{ m}$.

(b) For $0<t<3\text{ s}$, $a=\frac{8\text{ m/s}}{3\text{ s}}=2.67\text{ m/s}^2$.

For $3<t<5\text{ s}$, $a=0$.

(c) For $5\text{ s}<t<9\text{ s}$, $a=-\frac{16\text{ m/s}}{4\text{ s}}=\boxed{-4\text{ m/s}^2}$.

(d) At $t=6\text{ s}$, $x=28\text{ m}+(6\text{ m/s})(1\text{ s})=\boxed{34\text{ m}}$.

(e) At $t=9\text{ s}$, $x=36\text{ m}+\frac{1}{2}(-8\text{ m/s})(2\text{ s})=\boxed{28\text{ m}}$.

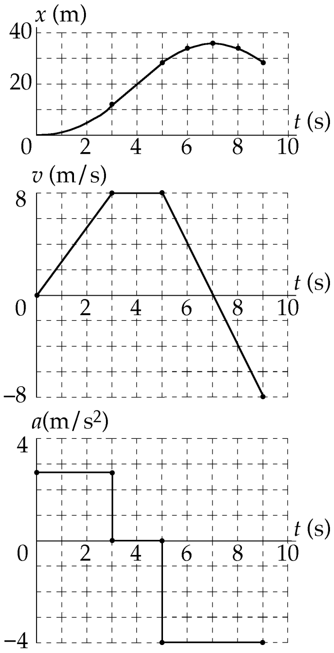


FIG. P2.54

**P2.55** (a) $a=\frac{dv}{dt}=\frac{d}{dt}\left[-5.00\times10^7t^2+3.00\times10^5t\right]$

$$\boxed{a=-\left(10.0\times10^7\text{ m/s}^3\right)t+3.00\times10^5\text{ m/s}^2}$$

Take $x_i=0$ at $t=0$. Then $v=\frac{dx}{dt}$

$$x-0=\int_0^t vdt=\int_0^t\left(-5.00\times10^7t^2+3.00\times10^5t\right)dt$$

$$x=-5.00\times10^7\frac{t^3}{3}+3.00\times10^5\frac{t^2}{2}$$

$$\boxed{x=-\left(1.67\times10^7\text{ m/s}^3\right)t^3+\left(1.50\times10^5\text{ m/s}^2\right)t^2}.$$

(b) The bullet escapes when $a=0$, at $-\left(10.0\times10^7\text{ m/s}^3\right)t+3.00\times10^5\text{ m/s}^2=0$

$$t=\frac{3.00\times10^5\text{ s}}{10.0\times10^7}=\boxed{3.00\times10^{-3}\text{ s}}.$$

(c) New $v=\left(-5.00\times10^7\right)\left(3.00\times10^{-3}\right)^2+\left(3.00\times10^5\right)\left(3.00\times10^{-3}\right)$

$$v=-450\text{ m/s}+900\text{ m/s}=\boxed{450\text{ m/s}}.$$

(d) $x=-\left(1.67\times10^7\right)\left(3.00\times10^{-3}\right)^3+\left(1.50\times10^5\right)\left(3.00\times10^{-3}\right)^2$

$x=-0.450\text{ m}+1.35\text{ m}=\boxed{0.900\text{ m}}$

**P2.56** $a=\frac{dv}{dt}=-3.00v^2$, $v_i=1.50\ \text{m/s}$

Solving for $v$, $\frac{dv}{dt}=-3.00v^2$

$$\int_{v=v_i}^{v} v^{-2}dv=-3.00\int_{t=0}^{t} dt$$

$$-\frac{1}{v}+\frac{1}{v_i}=-3.00t \text{ or } 3.00t=\frac{1}{v}-\frac{1}{v_i}.$$

When $v=\frac{v_i}{2}$, $t=\frac{1}{3.00v_i}=\boxed{0.222\ \text{s}}$.

---

## Additional Problems

***P2.57** The distance the car travels at constant velocity, $v_0$, during the reaction time is $(\Delta x)_1=v_0\Delta t_r$. The time for the car to come to rest, from initial velocity $v_0$, after the brakes are applied is

$$t_2=\frac{v_f-v_i}{a}=\frac{0-v_0}{a}=-\frac{v_0}{a}$$

and the distance traveled during this braking period is

$$(\Delta x)_2=\bar{v}t_2=\left(\frac{v_f+v_i}{2}\right)t_2=\left(\frac{0+v_0}{2}\right)\left(-\frac{v_0}{a}\right)=-\frac{v_0^2}{2a}.$$

Thus, the total distance traveled before coming to a stop is

$$s_{\text{stop}}=(\Delta x)_1+(\Delta x)_2=\boxed{v_0\Delta t_r-\frac{v_0^2}{2a}}.$$

***P2.58** (a) If a car is a distance $s_{\text{stop}}=v_0\Delta t_r-\frac{v_0^2}{2a}$ (See the solution to Problem 2.57) from the intersection of length $s_i$ when the light turns yellow, the distance the car must travel before the light turns red is

$$\Delta x=s_{\text{stop}}+s_i=v_0\Delta t_r-\frac{v_0^2}{2a}+s_i.$$

Assume the driver does not accelerate in an attempt to "beat the light" (an extremely dangerous practice!). The time the light should remain yellow is then the time required for the car to travel distance $\Delta x$ at constant velocity $v_0$. This is

$$\Delta t_{\text{light}}=\frac{\Delta x}{v_0}=\frac{v_0\Delta t_r-\frac{v_0^2}{2a}+s_i}{v_0}=\boxed{\Delta t_r-\frac{v_0}{2a}+\frac{s_i}{v_0}}.$$

(b) With $s_i=16\ \text{m}$, $v=60\ \text{km/h}$, $a=-2.0\ \text{m/s}^2$, and $\Delta t_r=1.1\ \text{s}$,

$$\Delta t_{\text{light}}=1.1\ \text{s}-\frac{60\ \text{km/h}}{2\left(-2.0\ \text{m/s}^2\right)}\left(\frac{0.278\ \text{m/s}}{1\ \text{km/h}}\right)+\frac{16\ \text{m}}{60\ \text{km/h}}\left(\frac{1\ \text{km/h}}{0.278\ \text{m/s}}\right)=\boxed{6.23\ \text{s}}.$$

***P2.59** (a) As we see from the graph, from about –50 s to 50 s Acela is cruising at a constant positive velocity in the $+x$ direction. From 50 s to 200 s, Acela accelerates in the $+x$ direction reaching a top speed of about 170 mi/h. Around 200 s, the engineer applies the brakes, and the train, still traveling in the $+x$ direction, slows down and then stops at 350 s. Just after 350 s, Acela reverses direction ($v$ becomes negative) and steadily gains speed in the $-x$ direction.

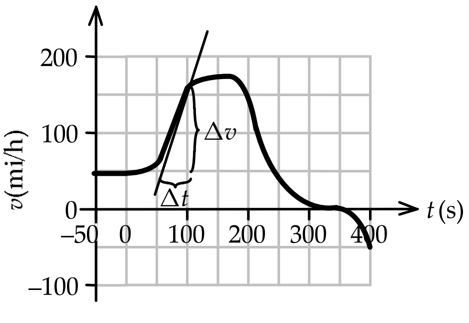


**FIG. P2.59(a)**

(b) The peak acceleration between 45 and 170 mi/h is given by the slope of the steepest tangent to the $v$ versus $t$ curve in this interval. From the tangent line shown, we find

$$a = \text{slope} = \frac{\Delta v}{\Delta t} = \frac{(155 - 45)\ \text{mi/h}}{(100 - 50)\ \text{s}} = \boxed{2.2\ (\text{mi/h})/\text{s}} = 0.98\ \text{m/s}^2.$$

(c) Let us use the fact that the area under the $v$ versus $t$ curve equals the displacement. The train's displacement between 0 and 200 s is equal to the area of the gray shaded region, which we have approximated with a series of triangles and rectangles.

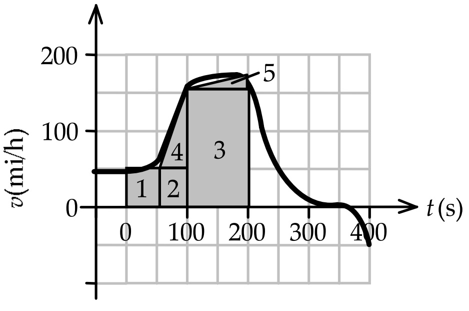


**FIG. P2.59(c)**

$$\begin{aligned}\Delta x_{0\to 200\ \text{s}} &= \text{area}_1 + \text{area}_2 + \text{area}_3 + \text{area}_4 + \text{area}_5\\ &\approx (50\ \text{mi/h})(50\ \text{s}) + (50\ \text{mi/h})(50\ \text{s})\\ &+ (160\ \text{mi/h})(100\ \text{s})\\ &+ \frac{1}{2}(50\ \text{s})(100\ \text{mi/h})\\ &+ \frac{1}{2}(100\ \text{s})(170\ \text{mi/h} - 160\ \text{mi/h})\\ &= 24\,000(\text{mi/h})(\text{s})\end{aligned}$$

Now, at the end of our calculation, we can find the displacement in miles by converting hours to seconds. As $1\ \text{h} = 3\,600\ \text{s}$,

$$\Delta x_{0\to 200\ \text{s}} \approx \left(\frac{24\,000\ \text{mi}}{3\,600\ \text{s}}\right)(\text{s}) = \boxed{6.7\ \text{mi}}.$$

***P2.60** Average speed of every point on the train as the first car passes Liz:

$$\frac{\Delta x}{\Delta t}=\frac{8.60\ \text{m}}{1.50\ \text{s}}=5.73\ \text{m/s}.$$

The train has this as its instantaneous speed halfway through the 1.50 s time. Similarly, halfway through the next 1.10 s, the speed of the train is $\frac{8.60\ \text{m}}{1.10\ \text{s}}=7.82\ \text{m/s}$. The time required for the speed to change from 5.73 m/s to 7.82 m/s is

$$\frac{1}{2}(1.50\ \text{s})+\frac{1}{2}(1.10\ \text{s})=1.30\ \text{s}$$

so the acceleration is: $a_x=\frac{\Delta v_x}{\Delta t}=\frac{7.82\ \text{m/s}-5.73\ \text{m/s}}{1.30\ \text{s}}=\boxed{1.60\ \text{m/s}^2}$.

**P2.61** The rate of hair growth is a velocity and the rate of its increase is an acceleration. Then $v_{xi}=1.04\ \text{mm/d}$ and $a_x=0.132\left(\frac{\text{mm/d}}{\text{w}}\right)$. The increase in the length of the hair (i.e., displacement) during a time of $t=5.00\ \text{w}=35.0\ \text{d}$ is

$$\Delta x=v_{xi}t+\frac{1}{2}a_xt^2$$

$$\Delta x=(1.04\ \text{mm/d})(35.0\ \text{d})+\frac{1}{2}(0.132\ \text{mm/d}\cdot\text{w})(35.0\ \text{d})(5.00\ \text{w})$$

or $\boxed{\Delta x=48.0\ \text{mm}}$.

**P2.62** Let point 0 be at ground level and point 1 be at the end of the engine burn. Let point 2 be the highest point the rocket reaches and point 3 be just before impact. The data in the table are found for each phase of the rocket's motion.

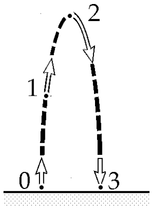


**FIG. P2.62**

(0 to 1) $v_f^2-(80.0)^2=2(4.00)(1\,000)$ so $v_f=120\ \text{m/s}$

$120=80.0+(4.00)t$ giving $t=10.0\ \text{s}$

(1 to 2) $0-(120)^2=2(-9.80)(x_f-x_i)$ giving $x_f-x_i=735\ \text{m}$

$0-120=-9.80t$ giving $t=12.2\ \text{s}$

This is the time of maximum height of the rocket.

(2 to 3) $v_f^2-0=2(-9.80)(-1\,735)$

$v_f=-184=(-9.80)t$ giving $t=18.8\ \text{s}$

(a) $t_{\text{total}}=10+12.2+18.8=\boxed{41.0\ \text{s}}$

(b) $(x_f-x_i)_{\text{total}}=\boxed{1.73\ \text{km}}$

*continued on next page*

(c) $v_{\text{final}} = \boxed{-184\ \text{m/s}}$

| | | $t$ | $x$ | $v$ | $a$ |
|---|---|---|---|---|---|
| 0 | Launch | 0.0 | 0 | 80 | +4.00 |
| #1 | End Thrust | 10.0 | 1 000 | 120 | +4.00 |
| #2 | Rise Upwards | 22.2 | 1 735 | 0 | –9.80 |
| #3 | Fall to Earth | 41.0 | 0 | –184 | –9.80 |

**P2.63** Distance traveled by motorist $= (15.0\ \text{m/s})t$

Distance traveled by policeman $= \dfrac{1}{2}\left(2.00\ \text{m/s}^2\right)t^2$

(a) intercept occurs when $15.0t = t^2$, or $t = \boxed{15.0\ \text{s}}$

(b) $v(\text{officer}) = \left(2.00\ \text{m/s}^2\right)t = \boxed{30.0\ \text{m/s}}$

(c) $x(\text{officer}) = \dfrac{1}{2}\left(2.00\ \text{m/s}^2\right)t^2 = \boxed{225\ \text{m}}$

**P2.64** Area $A_1$ is a rectangle. Thus, $A_1 = hw = v_{xi}t$.

Area $A_2$ is triangular. Therefore $A_2 = \dfrac{1}{2}bh = \dfrac{1}{2}t\left(v_x - v_{xi}\right)$.

The total area under the curve is

$$A = A_1 + A_2 = v_{xi}t + \frac{\left(v_x - v_{xi}\right)t}{2}$$

and since $v_x - v_{xi} = a_x t$

$$\boxed{A = v_{xi}t + \frac{1}{2}a_x t^2}.$$

The displacement given by the equation is: $x = v_{xi}t + \dfrac{1}{2}a_x t^2$, the same result as above for the total area.

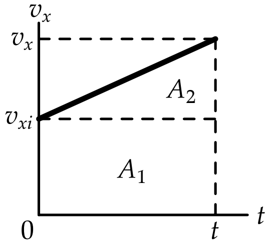


FIG. P2.64

**P2.65** (a) Let $x$ be the distance traveled at acceleration $a$ until maximum speed $v$ is reached. If this is achieved in time $t_1$ we can use the following three equations:

$$x = \frac{1}{2}(v + v_i)t_1,\ 100 - x = v(10.2 - t_1) \text{ and } v = v_i + at_1.$$

The first two give

$$100 = \left(10.2 - \frac{1}{2}t_1\right)v = \left(10.2 - \frac{1}{2}t_1\right)at_1$$

$$a = \frac{200}{(20.4 - t_1)t_1}.$$

$$\text{For Maggie: } a = \frac{200}{(18.4)(2.00)} = \boxed{5.43\ \text{m/s}^2}$$

$$\text{For Judy: } a = \frac{200}{(17.4)(3.00)} = \boxed{3.83\ \text{m/s}^2}$$

(b) $v = a_1 t$

$$\text{Maggie: } v = (5.43)(2.00) = \boxed{10.9\ \text{m/s}}$$

$$\text{Judy: } v = (3.83)(3.00) = \boxed{11.5\ \text{m/s}}$$

(c) At the six-second mark

$$x = \frac{1}{2}at_1^2 + v(6.00 - t_1)$$

$$\text{Maggie: } x = \frac{1}{2}(5.43)(2.00)^2 + (10.9)(4.00) = 54.3\ \text{m}$$

$$\text{Judy: } x = \frac{1}{2}(3.83)(3.00)^2 + (11.5)(3.00) = 51.7\ \text{m}$$

Maggie is ahead by $\boxed{2.62\ \text{m}}$.

**P2.66** $a_1 = 0.100\ \text{m/s}^2$

$a_2 = -0.500\ \text{m/s}^2$

$x = 1\,000\ \text{m} = \frac{1}{2}a_1t_1^2 + v_1t_2 + \frac{1}{2}a_2t_2^2$

$t = t_1 + t_2$ and $v_1 = a_1t_1 = -a_2t_2$

$$1\,000 = \frac{1}{2}a_1t_1^2 + a_1t_1\left(-\frac{a_1t_1}{a_2}\right) + \frac{1}{2}a_2\left(\frac{a_1t_1}{a_2}\right)^2$$

$$1\,000 = \frac{1}{2}a_1\left(1 - \frac{a_1}{a_2}\right)t_1^2$$

$$t_1 = \sqrt{\frac{20\,000}{1.20}} = \boxed{129\ \text{s}}$$

$$t_2 = \frac{a_1t_1}{-a_2} = \frac{12.9}{0.500} \approx 26\ \text{s}$$

$$\text{Total time} = t = \boxed{155\ \text{s}}$$

**P2.67** Let the ball fall 1.50 m. It strikes at speed given by

$$v_{xf}^2 = v_{xi}^2 + 2a(x_f - x_i):$$

$$v_{xf}^2 = 0 + 2(-9.80\ \text{m/s}^2)(-1.50\ \text{m})$$

$$v_{xf} = -5.42\ \text{m/s}$$

and its stopping is described by

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$

$$0 = (-5.42\ \text{m/s})^2 + 2a_x(-10^{-2}\ \text{m})$$

$$a_x = \frac{-29.4\ \text{m}^2/\text{s}^2}{-2.00 \times 10^{-2}\ \text{m}} = +1.47 \times 10^3\ \text{m/s}^2.$$

Its maximum acceleration will be larger than the average acceleration we estimate by imagining constant acceleration, but will still be of order of magnitude $\boxed{\sim 10^3\ \text{m/s}^2}$.

***P2.68** (a) $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$. We assume the package starts from rest.

$$-145\ \text{m} = 0 + 0 + \frac{1}{2}(-9.80\ \text{m/s}^2)t^2$$

$$t = \sqrt{\frac{2(-145\ \text{m})}{-9.80\ \text{m/s}^2}} = \boxed{5.44\ \text{s}}$$

(b) $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 = 0 + 0 + \frac{1}{2}(-9.80\ \text{m/s}^2)(5.18\ \text{s})^2 = -131\ \text{m}$

$$\text{distance fallen} = |x_f| = \boxed{131\ \text{m}}$$

(c) $\text{speed} = |v_{xf}| = |v_{xi} + a_x t| = \left|0 + (-9.8\ \text{m/s}^2)5.18\ \text{s}\right| = \boxed{50.8\ \text{m/s}}$

(d) The remaining distance is

$$145\ \text{m} - 131.5\ \text{m} = 13.5\ \text{m}.$$

During deceleration,

$$v_{xi} = -50.8\ \text{m/s},\ v_{xf} = 0,\ x_f - x_i = -13.5\ \text{m}$$

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i):$$

$$0 = (-50.8\ \text{m/s})^2 + 2a_x(-13.5\ \text{m})$$

$$a_x = \frac{-2\,580\ \text{m}^2/\text{s}^2}{2(-13.5\ \text{m})} = +95.3\ \text{m/s}^2 = \boxed{95.3\ \text{m/s}^2\ \text{upward}}.$$

**P2.69** (a) $$y_f = v_{i1}t + \frac{1}{2}at^2 = 50.0 = 2.00t + \frac{1}{2}(9.80)t^2,$$

$$4.90t^2 + 2.00t - 50.0 = 0$$

$$t = \frac{-2.00 + \sqrt{2.00^2 - 4(4.90)(-50.0)}}{2(4.90)}$$

Only the positive root is physically meaningful:

$t = \boxed{3.00\text{ s}}$ after the first stone is thrown.

(b) $y_f = v_{i2}t + \frac{1}{2}at^2$ and $t = 3.00 - 1.00 = 2.00$ s

substitute $50.0 = v_{i2}(2.00) + \frac{1}{2}(9.80)(2.00)^2$:

$v_{i2} = \boxed{15.3\text{ m/s}}$ downward

(c) $v_{1f} = v_{i1} + at = 2.00 + (9.80)(3.00) = \boxed{31.4\text{ m/s}}$ downward

$v_{2f} = v_{i2} + at = 15.3 + (9.80)(2.00) = \boxed{34.8\text{ m/s}}$ downward

**P2.70** (a) $d = \frac{1}{2}(9.80)t_1^2$ $\qquad d = 336t_2$

$t_1 + t_2 = 2.40$ $\qquad 336t_2 = 4.90(2.40 - t_2)^2$

$4.90t_2^2 - 359.5t_2 + 28.22 = 0$ $\qquad t_2 = \frac{359.5 \pm \sqrt{359.5^2 - 4(4.90)(28.22)}}{9.80}$

$t_2 = \frac{359.5 \pm 358.75}{9.80} = 0.076\,5\text{ s}$ so $d = 336t_2 = \boxed{26.4\text{ m}}$

(b) Ignoring the sound travel time, $d = \frac{1}{2}(9.80)(2.40)^2 = 28.2\text{ m}$, an error of $\boxed{6.82\%}$.

**P2.71** (a) In walking a distance $\Delta x$, in a time $\Delta t$, the length of rope $\ell$ is only increased by $\Delta x \sin\theta$.

$\therefore$ The pack lifts at a rate $\frac{\Delta x}{\Delta t}\sin\theta$.

$$v = \frac{\Delta x}{\Delta t}\sin\theta = v_{\text{boy}}\frac{x}{\ell} = \boxed{v_{\text{boy}}\frac{x}{\sqrt{x^2+h^2}}}$$

(b) $$a = \frac{dv}{dt} = \frac{v_{\text{boy}}}{\ell}\frac{dx}{dt} + v_{\text{boy}}x\frac{d}{dt}\left(\frac{1}{\ell}\right)$$

$$a = v_{\text{boy}}\frac{v_{\text{boy}}}{\ell} - \frac{v_{\text{boy}}x}{\ell^2}\frac{d\ell}{dt}, \text{ but } \frac{d\ell}{dt} = v = v_{\text{boy}}\frac{x}{\ell}$$

$$\therefore a = \frac{v_{\text{boy}}^2}{\ell}\left(1 - \frac{x^2}{\ell^2}\right) = \frac{v_{\text{boy}}^2}{\ell}\frac{h^2}{\ell^2} = \boxed{\frac{h^2 v_{\text{boy}}^2}{\left(x^2+h^2\right)^{3/2}}}$$

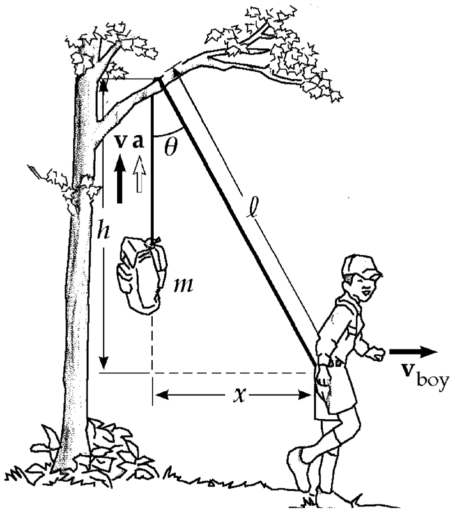


**FIG. P2.71**

(c) $\frac{v_{\text{boy}}^2}{h}$, 0

(d) $v_{\text{boy}}$, 0

**P2.72** $h = 6.00\text{ m},\ v_{\text{boy}} = 2.00\text{ m/s}\ \ v = \dfrac{\Delta x}{\Delta t}\sin\theta = v_{\text{boy}}\dfrac{x}{\ell} = \dfrac{v_{\text{boy}}x}{\left(x^2+h^2\right)^{1/2}}.$

However, $x = v_{\text{boy}}t : \therefore v = \dfrac{v_{\text{boy}}^2 t}{\left(v_{\text{boy}}^2 t^2 + h^2\right)^{1/2}} = \dfrac{4t}{\left(4t^2+36\right)^{1/2}}.$

(a)

| $t$(s) | $v$(m/s) |
|---|---|
| 0 | 0 |
| 0.5 | 0.32 |
| 1 | 0.63 |
| 1.5 | 0.89 |
| 2 | 1.11 |
| 2.5 | 1.28 |
| 3 | 1.41 |
| 3.5 | 1.52 |
| 4 | 1.60 |
| 4.5 | 1.66 |
| 5 | 1.71 |

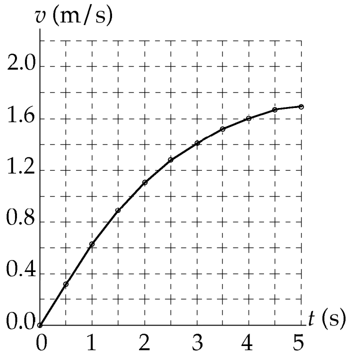


**FIG. P2.72(a)**

(b) From problem 2.71 above, $a = \dfrac{h^2 v_{\text{boy}}^2}{\left(x^2+h^2\right)^{3/2}} = \dfrac{h^2 v_{\text{boy}}^2}{\left(v_{\text{boy}}^2 t^2 + h^2\right)^{3/2}} = \dfrac{144}{\left(4t^2+36\right)^{3/2}}.$

| $t$(s) | $a\left(\text{m/s}^2\right)$ |
|---|---|
| 0 | 0.67 |
| 0.5 | 0.64 |
| 1 | 0.57 |
| 1.5 | 0.48 |
| 2 | 0.38 |
| 2.5 | 0.30 |
| 3. | 0.24 |
| 3.5 | 0.18 |
| 4. | 0.14 |
| 4.5 | 0.11 |
| 5 | 0.09 |

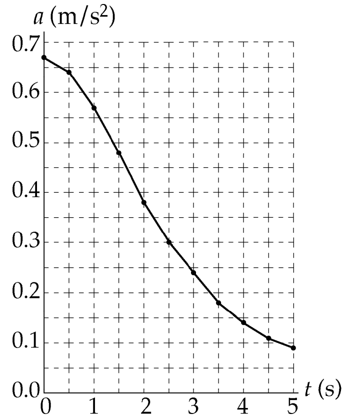


**FIG. P2.72(b)**

**P2.73** (a) We require $x_s = x_k$ when $t_s = t_k + 1.00$

$$x_s = \frac{1}{2}\left(3.50\text{ m/s}^2\right)\left(t_k+1.00\right)^2 = \frac{1}{2}\left(4.90\text{ m/s}^2\right)\left(t_k\right)^2 = x_k$$

$$t_k + 1.00 = 1.183t_k$$

$$t_k = \boxed{5.46\text{ s}}.$$

(b) $x_k = \dfrac{1}{2}\left(4.90\text{ m/s}^2\right)(5.46\text{ s})^2 = \boxed{73.0\text{ m}}$

(c) $v_k = \left(4.90\text{ m/s}^2\right)(5.46\text{ s}) = \boxed{26.7\text{ m/s}}$

$v_s = \left(3.50\text{ m/s}^2\right)(6.46\text{ s}) = \boxed{22.6\text{ m/s}}$

**P2.74**

| Time $t$ (s) | Height $h$ (m) | $\Delta h$ (m) | $\Delta t$ (s) | $\bar{v}$ (m/s) | midpt time $t$ (s) |
|---|---|---|---|---|---|
| 0.00 | 5.00 | | | | |
| | | 0.75 | 0.25 | 3.00 | 0.13 |
| 0.25 | 5.75 | | | | |
| | | 0.65 | 0.25 | 2.60 | 0.38 |
| 0.50 | 6.40 | | | | |
| | | 0.54 | 0.25 | 2.16 | 0.63 |
| 0.75 | 6.94 | | | | |
| | | 0.44 | 0.25 | 1.76 | 0.88 |
| 1.00 | 7.38 | | | | |
| | | 0.34 | 0.25 | 1.36 | 1.13 |
| 1.25 | 7.72 | | | | |
| | | 0.24 | 0.25 | 0.96 | 1.38 |
| 1.50 | 7.96 | | | | |
| | | 0.14 | 0.25 | 0.56 | 1.63 |
| 1.75 | 8.10 | | | | |
| | | 0.03 | 0.25 | 0.12 | 1.88 |
| 2.00 | 8.13 | | | | |
| | | –0.06 | 0.25 | –0.24 | 2.13 |
| 2.25 | 8.07 | | | | |
| | | –0.17 | 0.25 | –0.68 | 2.38 |
| 2.50 | 7.90 | | | | |
| | | –0.28 | 0.25 | –1.12 | 2.63 |
| 2.75 | 7.62 | | | | |
| | | –0.37 | 0.25 | –1.48 | 2.88 |
| 3.00 | 7.25 | | | | |
| | | –0.48 | 0.25 | –1.92 | 3.13 |
| 3.25 | 6.77 | | | | |
| | | –0.57 | 0.25 | –2.28 | 3.38 |
| 3.50 | 6.20 | | | | |
| | | –0.68 | 0.25 | –2.72 | 3.63 |
| 3.75 | 5.52 | | | | |
| | | –0.79 | 0.25 | –3.16 | 3.88 |
| 4.00 | 4.73 | | | | |
| | | –0.88 | 0.25 | –3.52 | 4.13 |
| 4.25 | 3.85 | | | | |
| | | –0.99 | 0.25 | –3.96 | 4.38 |
| 4.50 | 2.86 | | | | |
| | | –1.09 | 0.25 | –4.36 | 4.63 |
| 4.75 | 1.77 | | | | |
| | | –1.19 | 0.25 | –4.76 | 4.88 |
| 5.00 | 0.58 | | | | |

**TABLE P2.74**

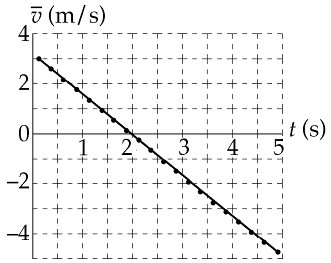


**FIG. P2.74**

acceleration = slope of line is constant.

$$\bar{a} = -1.63\ \text{m/s}^2 = \boxed{1.63\ \text{m/s}^2\ \text{downward}}$$

**P2.75** The distance $x$ and $y$ are always related by $x^2 + y^2 = L^2$. Differentiating this equation with respect to time, we have

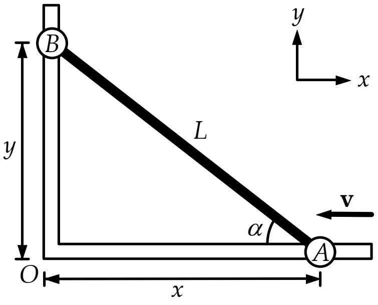


FIG. P2.75

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

Now $\frac{dy}{dt}$ is $v_B$, the unknown velocity of $B$; and $\frac{dx}{dt} = -v$.
From the equation resulting from differentiation, we have

$$\frac{dy}{dt} = -\frac{x}{y}\left(\frac{dx}{dt}\right) = -\frac{x}{y}(-v).$$

But $\frac{y}{x} = \tan\alpha$ so $v_B = \left(\frac{1}{\tan\alpha}\right)v$. When $\alpha = 60.0°$, $v_B = \frac{v}{\tan 60.0°} = \frac{v\sqrt{3}}{3} = \boxed{0.577v}$.

## ANSWERS TO EVEN PROBLEMS

| | |
|---|---|
| **P2.2** | (a) $2\times10^{-7}$ m/s; $1\times10^{-6}$ m/s;<br>(b) $5\times10^{8}$ yr |
| **P2.4** | (a) 50.0 m/s; (b) 41.0 m/s |
| **P2.6** | (a) 27.0 m;<br>(b) $27.0\text{ m} + (18.0\text{ m/s})\Delta t + (3.00\text{ m/s}^2)(\Delta t)^2$;<br>(c) 18.0 m/s |
| **P2.8** | (a), (b), (c) see the solution; 4.6 $\text{m/s}^2$; (d) 0 |
| **P2.10** | 5.00 m |
| **P2.12** | (a) 20.0 m/s; 5.00 m/s; (b) 262 m |
| **P2.14** | (a) see the solution;<br>(b) 1.60 $\text{m/s}^2$; 0.800 $\text{m/s}^2$ |
| **P2.16** | (a) 13.0 m/s; (b) 10.0 m/s; 16.0 m/s;<br>(c) 6.00 $\text{m/s}^2$; (d) 6.00 $\text{m/s}^2$ |
| **P2.18** | see the solution |
| **P2.20** | (a) 6.61 m/s; (b) –0.448 $\text{m/s}^2$ |
| **P2.22** | (a) $-21.8\text{ mi/h}\cdot\text{s} = -9.75\text{ m/s}^2$;<br>(b) $-22.2\text{ mi/h}\cdot\text{s} = -9.94\text{ m/s}^2$;<br>(c) $-22.8\text{ mi/h}\cdot\text{s} = -10.2\text{ m/s}^2$ |
| **P2.24** | (a) 1.88 km; (b) 1.46 km;<br>(c) see the solution;<br>(d) (i) $x_1 = (1.67\text{ m/s}^2)t^2$;<br>(ii) $x_2 = (50\text{ m/s})t - 375\text{ m}$;<br>(iii) $x_3 = (250\text{ m/s})t - (2.5\text{ m/s}^2)t^2 - 4\,375\text{ m}$;<br>(e) 37.5 m/s |
| **P2.26** | 958 m |
| **P2.28** | (a) $x_f = (30.0t - t^2)$ m; $v_f = (30.0 - 2t)$ m/s;<br>(b) 225 m |
| **P2.30** | $x_f - x_i = v_{xf}t - \frac{1}{2}a_x t^2$; 3.10 m/s |
| **P2.32** | (a) 35.0 s; (b) 15.7 m/s |
| **P2.34** | (a) $1.12\times10^{11}\text{ m/s}^2$; (b) $4.67\times10^{-5}$ s |
| **P2.36** | (a) False unless the acceleration is zero; see the solution; (b) True |
| **P2.38** | Yes; 212 m; 11.4 s |
| **P2.40** | (a) –4.90 m; –19.6 m; –44.1 m;<br>(b) –9.80 m/s; –19.6 m/s; –29.4 m/s |
| **P2.42** | 1.79 s |

| | |
|---|---|
| **P2.44** | No; see the solution |
| **P2.46** | The second ball is thrown at speed $v_i = \sqrt{gh}$ |
| **P2.48** | (a) 510 m; (b) 20.4 s |
| **P2.50** | (a) $96.0\ \text{ft/s}$;<br>(b) $a = 3.07 \times 10^3\ \text{ft/s}^2$ upward;<br>(c) $\Delta t = 3.13 \times 10^{-2}\ \text{s}$ |
| **P2.52** | 38.2 m |
| **P2.54** | (a) and (b) see the solution; (c) $-4\ \text{m/s}^2$;<br>(d) 34 m; (e) 28 m |
| **P2.56** | 0.222 s |
| **P2.58** | (a) see the solution; (b) 6.23 s |
| **P2.60** | $1.60\ \text{m/s}^2$ |
| **P2.62** | (a) 41.0 s; (b) 1.73 km; (c) $-184\ \text{m/s}$ |
| **P2.64** | $v_{xi}t + \frac{1}{2}a_x t^2$; displacements agree |
| **P2.66** | 155 s; 129 s |
| **P2.68** | (a) 5.44 s; (b) 131 m; (c) $50.8\ \text{m/s}$;<br>(d) $95.3\ \text{m/s}^2$ upward |
| **P2.70** | (a) 26.4 m; (b) 6.82% |
| **P2.72** | see the solution |
| **P2.74** | see the solution; $a_x = -1.63\ \text{m/s}^2$ |

# 3

# Vectors

**CHAPTER OUTLINE**

3.1 Coordinate Systems
3.2 Vector and Scalar Quantities
3.3 Some Properties of Vectors
3.4 Components of a Vector and Unit Vectors

## ANSWERS TO QUESTIONS

**Q3.1** No. The sum of two vectors can only be zero if they are in opposite directions and have the same magnitude. If you walk 10 meters north and then 6 meters south, you won't end up where you started.

**Q3.2** No, the magnitude of the displacement is always less than or equal to the distance traveled. If two displacements in the same direction are added, then the magnitude of their sum will be equal to the distance traveled. Two vectors in any other orientation will give a displacement less than the distance traveled. If you first walk 3 meters east, and then 4 meters south, you will have walked a total distance of 7 meters, but you will only be 5 meters from your starting point.

**Q3.3** The largest possible magnitude of $\mathbf{R}=\mathbf{A}+\mathbf{B}$ is 7 units, found when **A** and **B** point in the same direction. The smallest magnitude of $\mathbf{R}=\mathbf{A}+\mathbf{B}$ is 3 units, found when **A** and **B** have opposite directions.

**Q3.4** Only force and velocity are vectors. None of the other quantities requires a direction to be described.

**Q3.5** If the direction-angle of **A** is between 180 degrees and 270 degrees, its components are both negative. If a vector is in the second quadrant or the fourth quadrant, its components have opposite signs.

**Q3.6** The book's displacement is zero, as it ends up at the point from which it started. The distance traveled is 6.0 meters.

**Q3.7** 85 miles. The magnitude of the displacement is the distance from the starting point, the 260-mile mark, to the ending point, the 175-mile mark.

**Q3.8** Vectors **A** and **B** are perpendicular to each other.

**Q3.9** No, the magnitude of a vector is always positive. A minus sign in a vector only indicates direction, not magnitude.

**Q3.10** Any vector that points along a line at 45° to the $x$ and $y$ axes has components equal in magnitude.

**Q3.11** $A_x = B_x$ and $A_y = B_y$.

**Q3.12** Addition of a vector to a scalar is not defined. Think of apples and oranges.

**Q3.13** One difficulty arises in determining the individual components. The relationships between a vector and its components such as $A_x = A\cos\theta$, are based on right-triangle trigonometry. Another problem would be in determining the magnitude or the direction of a vector from its components. Again, $A = \sqrt{A_x^2 + A_y^2}$ only holds true if the two component vectors, $\mathbf{A}_x$ and $\mathbf{A}_y$, are perpendicular.

**Q3.14** If the direction of a vector is specified by giving the angle of the vector measured clockwise from the positive $y$-axis, then the $x$-component of the vector is equal to the sine of the angle multiplied by the magnitude of the vector.

## SOLUTIONS TO PROBLEMS

### Section 3.1 **Coordinate Systems**

**P3.1**

$$x = r\cos\theta = (5.50\text{ m})\cos 240° = (5.50\text{ m})(-0.5) = \boxed{-2.75\text{ m}}$$

$$y = r\sin\theta = (5.50\text{ m})\sin 240° = (5.50\text{ m})(-0.866) = \boxed{-4.76\text{ m}}$$

**P3.2** (a) $x = r\cos\theta$ and $y = r\sin\theta$, therefore
$x_1 = (2.50\text{ m})\cos 30.0°$, $y_1 = (2.50\text{ m})\sin 30.0°$, and

$$(x_1, y_1) = \boxed{(2.17,\ 1.25)\text{ m}}$$

$x_2 = (3.80\text{ m})\cos 120°$, $y_2 = (3.80\text{ m})\sin 120°$, and

$$(x_2, y_2) = \boxed{(-1.90,\ 3.29)\text{ m}}.$$

(b)

$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{16.6 + 4.16} = \boxed{4.55\text{ m}}$$

**P3.3** The $x$ distance out to the fly is 2.00 m and the $y$ distance up to the fly is 1.00 m.

(a) We can use the Pythagorean theorem to find the distance from the origin to the fly.

$$\text{distance} = \sqrt{x^2 + y^2} = \sqrt{(2.00\text{ m})^2 + (1.00\text{ m})^2} = \sqrt{5.00\text{ m}^2} = \boxed{2.24\text{ m}}$$

(b)

$$\theta = \tan^{-1}\left(\frac{1}{2}\right) = 26.6°;\ \mathbf{r} = \boxed{2.24\text{ m},\ 26.6°}$$

**P3.4** (a) $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(2.00-[-3.00])^2+(-4.00-3.00)^2}$

$d=\sqrt{25.0+49.0}=\boxed{8.60\text{ m}}$

(b) $r_1=\sqrt{(2.00)^2+(-4.00)^2}=\sqrt{20.0}=\boxed{4.47\text{ m}}$

$\theta_1=\tan^{-1}\left(-\frac{4.00}{2.00}\right)=\boxed{-63.4°}$

$r_2=\sqrt{(-3.00)^2+(3.00)^2}=\sqrt{18.0}=\boxed{4.24\text{ m}}$

$\theta_2=\boxed{135°}$ measured from the $+x$ axis.

**P3.5** We have $2.00=r\cos 30.0°$

$$r=\frac{2.00}{\cos 30.0°}=\boxed{2.31}$$

and $y=r\sin 30.0°=2.31\sin 30.0°=\boxed{1.15}$.

**P3.6** We have $r=\sqrt{x^2+y^2}$ and $\theta=\tan^{-1}\left(\frac{y}{x}\right)$.

(a) The radius for this new point is

$$\sqrt{(-x)^2+y^2}=\sqrt{x^2+y^2}=\boxed{r}$$

and its angle is

$$\tan^{-1}\left(\frac{y}{-x}\right)=\boxed{180°-\theta}.$$

(b) $\sqrt{(-2x)^2+(-2y)^2}=\boxed{2r}$. This point is in the third quadrant if $(x, y)$ is in the first quadrant or in the fourth quadrant if $(x, y)$ is in the second quadrant. It is at an angle of $\boxed{180°+\theta}$.

(c) $\sqrt{(3x)^2+(-3y)^2}=\boxed{3r}$. This point is in the fourth quadrant if $(x, y)$ is in the first quadrant or in the third quadrant if $(x, y)$ is in the second quadrant. It is at an angle of $\boxed{-\theta}$.

## Section 3.2 Vector and Scalar Quantities

## Section 3.3 Some Properties of Vectors

**P3.7**

$$\tan 35.0° = \frac{x}{100\text{ m}}$$

$$x = (100\text{ m})\tan 35.0° = \boxed{70.0\text{ m}}$$

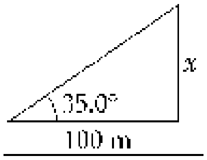


FIG. P3.7

**P3.8**

$$R = \boxed{14\text{ km}}$$

$$\theta = \boxed{65°\text{ N of E}}$$

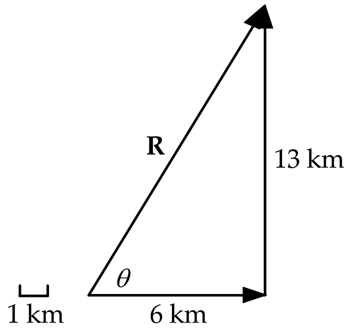


FIG. P3.8

**P3.9**

$$-\mathbf{R} = \boxed{310\text{ km at }57°\text{ S of W}}$$

(Scale: 1 unit = 20 km)

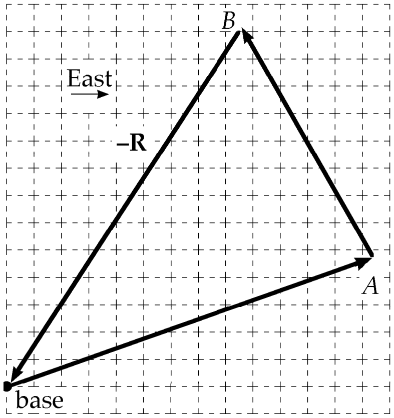


FIG. P3.9

**P3.10** (a) Using graphical methods, place the tail of vector **B** at the head of vector **A**. The new vector $\mathbf{A}+\mathbf{B}$ has a magnitude of $\boxed{6.1\text{ at }112°}$ from the $x$-axis.

(b) The vector difference $\mathbf{A}-\mathbf{B}$ is found by placing the negative of vector **B** at the head of vector **A**. The resultant vector $\mathbf{A}-\mathbf{B}$ has magnitude $\boxed{14.8}$ units at an $\boxed{\text{angle of }22°}$ from the + $x$-axis.

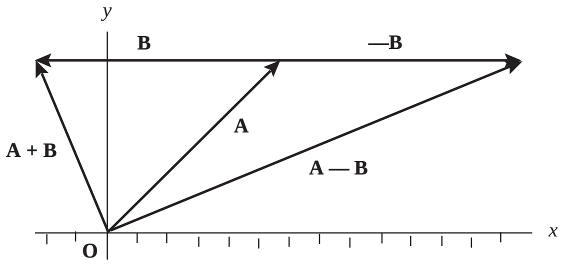


FIG. P3.10

**P3.11** (a) $|\mathbf{d}| = |-10.0\hat{\mathbf{i}}| = \boxed{10.0\text{ m}}$ since the displacement is in a straight line from point A to point B.

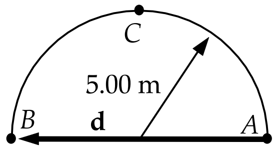


FIG. P3.11

(b) The actual distance skated is not equal to the straight-line displacement. The distance follows the curved path of the semi-circle (*ACB*).

$$s = \frac{1}{2}(2\pi r) = 5\pi = \boxed{15.7\text{ m}}$$

(c) If the circle is complete, **d** begins and ends at point **A**. Hence, $|\mathbf{d}| = \boxed{0}$.

**P3.12** Find the resultant $\mathbf{F}_1 + \mathbf{F}_2$ graphically by placing the tail of $\mathbf{F}_2$ at the head of $\mathbf{F}_1$. The resultant force vector $\mathbf{F}_1 + \mathbf{F}_2$ is of magnitude $\boxed{9.5\text{ N}}$ and at an angle of $\boxed{57° \text{ above the } x\text{-axis}}$.

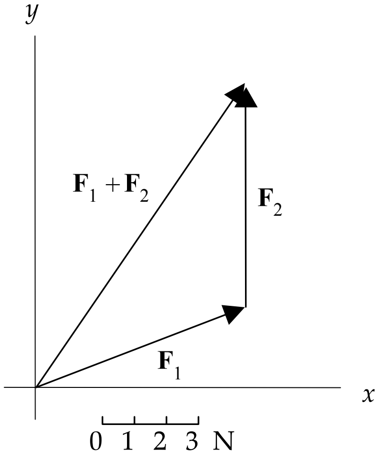


FIG. P3.12

**P3.13** (a) The large majority of people are standing or sitting at this hour. Their instantaneous foot-to-head vectors have upward vertical components on the order of 1 m and randomly oriented horizontal components. The citywide sum will be $\boxed{\sim 10^5 \text{ m upward}}$.

(b) Most people are lying in bed early Saturday morning. We suppose their beds are oriented north, south, east, west quite at random. Then the horizontal component of their total vector height is very nearly zero. If their compressed pillows give their height vectors vertical components averaging 3 cm, and if one-tenth of one percent of the population are on-duty nurses or police officers, we estimate the total vector height as $\sim 10^5(0.03\text{ m}) + 10^2(1\text{ m})$ $\boxed{\sim 10^3 \text{ m upward}}$.

**P3.14** Your sketch should be drawn to scale, and should look somewhat like that pictured to the right. The angle from the westward direction, $\theta$, can be measured to be $\boxed{4^\circ \text{ N of W}}$, and the distance $R$ from the sketch can be converted according to the scale to be $\boxed{7.9 \text{ m}}$.

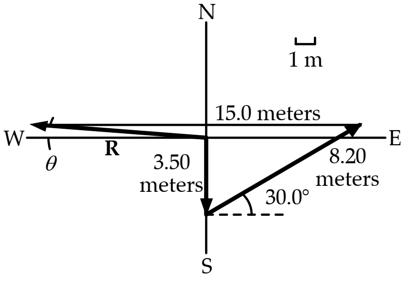


FIG. P3.14

**P3.15** To find these vector expressions graphically, we draw each set of vectors. Measurements of the results are taken using a ruler and protractor. (Scale: 1 unit = 0.5 m)

(a) $\mathbf{A}+\mathbf{B} = 5.2$ m at 60°

(b) $\mathbf{A}-\mathbf{B} = 3.0$ m at 330°

(c) $\mathbf{B}-\mathbf{A} = 3.0$ m at 150°

(d) $\mathbf{A}-2\mathbf{B} = 5.2$ m at 300°.

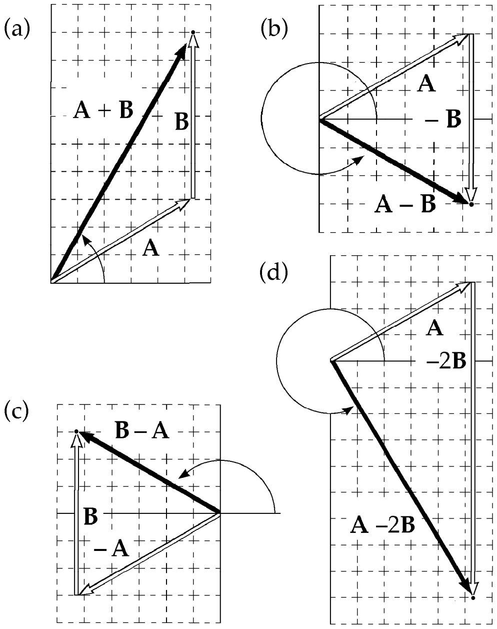


FIG. P3.15

***P3.16** The three diagrams shown below represent the graphical solutions for the three vector sums: $\mathbf{R}_1 = \mathbf{A}+\mathbf{B}+\mathbf{C}$, $\mathbf{R}_2 = \mathbf{B}+\mathbf{C}+\mathbf{A}$, and $\mathbf{R}_3 = \mathbf{C}+\mathbf{B}+\mathbf{A}$. You should observe that $\mathbf{R}_1 = \mathbf{R}_2 = \mathbf{R}_3$, illustrating that the sum of a set of vectors is not affected by the order in which the vectors are added.

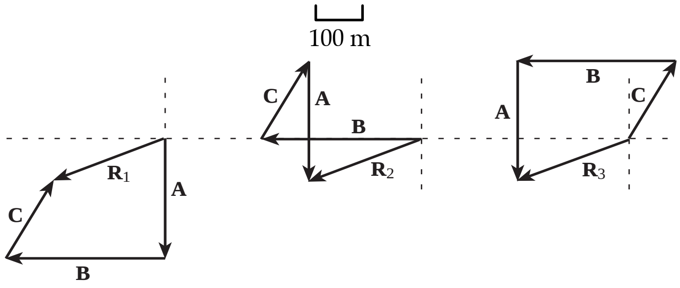
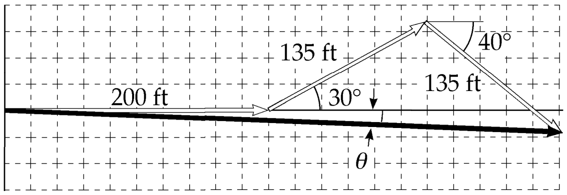


FIG. P3.16

**P3.17** The scale drawing for the graphical solution should be similar to the figure to the right. The magnitude and direction of the final displacement from the starting point are obtained by measuring $d$ and $\theta$ on the drawing and applying the scale factor used in making the drawing. The results should be

$$\boxed{d = 420 \text{ ft and } \theta = -3°}$$



(Scale: 1 unit = 20 ft)

**FIG. P3.17**

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Section 3.4 **Components of a Vector and Unit Vectors**

**P3.18** Coordinates of the super-hero are:

$$x = (100 \text{ m})\cos(-30.0°) = \boxed{86.6 \text{ m}}$$

$$y = (100 \text{ m})\sin(-30.0°) = \boxed{-50.0 \text{ m}}$$

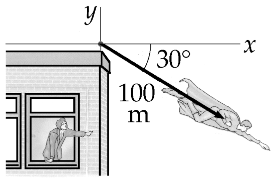


**FIG. P3.18**

**P3.19** $A_x = -25.0$

$A_y = 40.0$

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(-25.0)^2 + (40.0)^2} = \boxed{47.2 \text{ units}}$$

We observe that

$$\tan\phi = \frac{|A_y|}{|A_x|}.$$

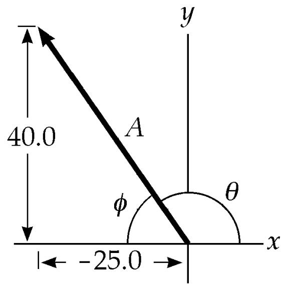


**FIG. P3.19**

So

$$\phi = \tan^{-1}\left(\frac{A_y}{|A_x|}\right) = \tan\frac{40.0}{25.0} = \tan^{-1}(1.60) = 58.0°.$$

The diagram shows that the angle from the $+x$ axis can be found by subtracting from 180°:

$$\theta = 180° - 58° = \boxed{122°}.$$

**P3.20** The person would have to walk $3.10\sin(25.0°) = \boxed{1.31 \text{ km north}}$, and

$$3.10\cos(25.0°) = \boxed{2.81 \text{ km east}}.$$

**P3.21** $x = r\cos\theta$ and $y = r\sin\theta$, therefore:

(a) $x = 12.8\cos 150°$, $y = 12.8\sin 150°$, and $(x,\ y) = \left(-11.1\hat{\mathbf{i}} + 6.40\hat{\mathbf{j}}\right)$ m

(b) $x = 3.30\cos 60.0°$, $y = 3.30\sin 60.0°$, and $(x,\ y) = \left(1.65\hat{\mathbf{i}} + 2.86\hat{\mathbf{j}}\right)$ cm

(c) $x = 22.0\cos 215°$, $y = 22.0\sin 215°$, and $(x,\ y) = \left(-18.0\hat{\mathbf{i}} - 12.6\hat{\mathbf{j}}\right)$ in

**P3.22** $x = d\cos\theta = (50.0\text{ m})\cos(120) = -25.0\text{ m}$
$y = d\sin\theta = (50.0\text{ m})\sin(120) = 43.3\text{ m}$
$\mathbf{d} = \boxed{(-25.0\text{ m})\hat{\mathbf{i}} + (43.3\text{ m})\hat{\mathbf{j}}}$

***P3.23** (a) Her net $x$ (east-west) displacement is $-3.00 + 0 + 6.00 = +3.00$ blocks, while her net $y$ (north-south) displacement is $0 + 4.00 + 0 = +4.00$ blocks. The magnitude of the resultant displacement is

$$R = \sqrt{(x_{\text{net}})^2 + (y_{\text{net}})^2} = \sqrt{(3.00)^2 + (4.00)^2} = 5.00\text{ blocks}$$

and the angle the resultant makes with the $x$-axis (eastward direction) is

$$\theta = \tan^{-1}\left(\frac{4.00}{3.00}\right) = \tan^{-1}(1.33) = 53.1°.$$

The resultant displacement is then $\boxed{5.00\text{ blocks at }53.1°\text{ N of E}}$.

(b) The total distance traveled is $3.00 + 4.00 + 6.00 = \boxed{13.0\text{ blocks}}$.

***P3.24** Let $\hat{\mathbf{i}} = \text{east}$ and $\hat{\mathbf{j}} = \text{north}$. The unicyclist's displacement is, in meters

$$280\hat{\mathbf{j}} + 220\hat{\mathbf{i}} + 360\hat{\mathbf{j}} - 300\hat{\mathbf{i}} - 120\hat{\mathbf{j}} + 60\hat{\mathbf{i}} - 40\hat{\mathbf{j}} - 90\hat{\mathbf{i}} + 70\hat{\mathbf{j}}.$$

$$\begin{aligned}\mathbf{R} &= -110\hat{\mathbf{i}} + 550\hat{\mathbf{j}} \\ &= \sqrt{(110\text{ m})^2 + (550\text{ m})^2}\text{ at }\tan^{-1}\frac{110\text{ m}}{550\text{ m}}\text{ west of north} \\ &= 561\text{ m at }11.3°\text{ west of north}.\end{aligned}$$

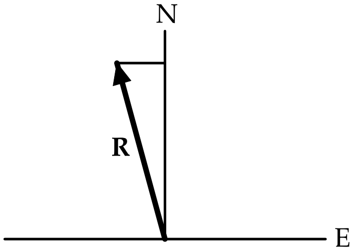


**FIG. P3.24**

The crow's velocity is

$$\begin{aligned}\mathbf{v} &= \frac{\Delta\mathbf{x}}{\Delta t} = \frac{561\text{ m at }11.3°\text{ W of N}}{40\text{ s}} \\ &= \boxed{14.0\text{ m/s at }11.3°\text{ west of north}}.\end{aligned}$$

**P3.25** $+x$ East, $+y$ North

$$\sum x = 250 + 125\cos 30° = 358 \text{ m}$$
$$\sum y = 75 + 125\sin 30° – 150 = –12.5 \text{ m}$$
$$d = \sqrt{\left(\sum x\right)^2 + \left(\sum y\right)^2} = \sqrt{(358)^2 + (–12.5)^2} = 358 \text{ m}$$
$$\tan\theta = \frac{\left(\sum y\right)}{\left(\sum x\right)} = -\frac{12.5}{358} = –0.0349$$
$$\theta = –2.00°$$
$$\boxed{\mathbf{d} = 358 \text{ m at } 2.00° \text{ S of E}}$$

**P3.26** The east and north components of the displacement from Dallas (D) to Chicago (C) are the sums of the east and north components of the displacements from Dallas to Atlanta (A) and from Atlanta to Chicago. In equation form:

$$d_{\text{DC east}} = d_{\text{DA east}} + d_{\text{AC east}} = 730\cos 5.00° – 560\sin 21.0° = 527 \text{ miles}.$$
$$d_{\text{DC north}} = d_{\text{DA north}} + d_{\text{AC north}} = 730\sin 5.00° + 560\cos 21.0° = 586 \text{ miles}.$$

By the Pythagorean theorem, $d = \sqrt{(d_{\text{DC east}})^2 + (d_{\text{DC north}})^2} = 788 \text{ mi}$.

$$\text{Then } \tan\theta = \frac{d_{\text{DC north}}}{d_{\text{DC east}}} = 1.11 \text{ and } \theta = 48.0°.$$

Thus, Chicago is $\boxed{788 \text{ miles at } 48.0° \text{ northeast of Dallas}}$.

**P3.27** (a) See figure to the right.

(b) $\mathbf{C} = \mathbf{A} + \mathbf{B} = 2.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}} + 3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}} = \boxed{5.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}}}$

$\mathbf{C} = \sqrt{25.0 + 16.0} \text{ at } \tan^{-1}\left(\frac{4}{5}\right) = \boxed{6.40 \text{ at } 38.7°}$

$\mathbf{D} = \mathbf{A} - \mathbf{B} = 2.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}} - 3.00\hat{\mathbf{i}} + 2.00\hat{\mathbf{j}} = \boxed{-1.00\hat{\mathbf{i}} + 8.00\hat{\mathbf{j}}}$

$\mathbf{D} = \sqrt{(–1.00)^2 + (8.00)^2} \text{ at } \tan^{-1}\left(\frac{8.00}{–1.00}\right)$

$\mathbf{D} = 8.06 \text{ at } (180° - 82.9°) = \boxed{8.06 \text{ at } 97.2°}$

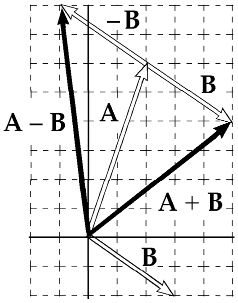


**FIG. P3.27**

**P3.28** $d = \sqrt{(x_1 + x_2 + x_3)^2 + (y_1 + y_2 + y_3)^2}$

$= \sqrt{(3.00 - 5.00 + 6.00)^2 + (2.00 + 3.00 + 1.00)^2} = \sqrt{52.0} = \boxed{7.21 \text{ m}}$

$\theta = \tan^{-1}\left(\frac{6.00}{4.00}\right) = \boxed{56.3°}$

**P3.29** We have $\mathbf{B} = \mathbf{R} - \mathbf{A}$:

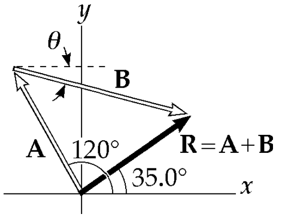


**FIG. P3.29**

$$A_x = 150\cos 120° = -75.0 \text{ cm}$$
$$A_y = 150\sin 120° = 130 \text{ cm}$$
$$R_x = 140\cos 35.0° = 115 \text{ cm}$$
$$R_y = 140\sin 35.0° = 80.3 \text{ cm}$$

Therefore,

$$\mathbf{B} = [115 - (-75)]\hat{\mathbf{i}} + [80.3 - 130]\hat{\mathbf{j}} = (190\hat{\mathbf{i}} - 49.7\hat{\mathbf{j}}) \text{ cm}$$
$$|\mathbf{B}| = \sqrt{190^2 + 49.7^2} = \boxed{196 \text{ cm}}$$
$$\theta = \tan^{-1}\left(-\frac{49.7}{190}\right) = \boxed{-14.7°}.$$

**P3.30** $\mathbf{A} = -8.70\hat{\mathbf{i}} + 15.0\hat{\mathbf{j}}$ and $\mathbf{B} = 13.2\hat{\mathbf{i}} - 6.60\hat{\mathbf{j}}$

$\mathbf{A} - \mathbf{B} + 3\mathbf{C} = 0$:

$$3\mathbf{C} = \mathbf{B} - \mathbf{A} = 21.9\hat{\mathbf{i}} - 21.6\hat{\mathbf{j}}$$
$$\mathbf{C} = 7.30\hat{\mathbf{i}} - 7.20\hat{\mathbf{j}}$$

or

$$C_x = \boxed{7.30 \text{ cm}};\ C_y = \boxed{-7.20 \text{ cm}}$$

**P3.31** (a) $(\mathbf{A} + \mathbf{B}) = (3\hat{\mathbf{i}} - 2\hat{\mathbf{j}}) + (-\hat{\mathbf{i}} - 4\hat{\mathbf{j}}) = \boxed{2\hat{\mathbf{i}} - 6\hat{\mathbf{j}}}$

(b) $(\mathbf{A} - \mathbf{B}) = (3\hat{\mathbf{i}} - 2\hat{\mathbf{j}}) - (-\hat{\mathbf{i}} - 4\hat{\mathbf{j}}) = \boxed{4\hat{\mathbf{i}} + 2\hat{\mathbf{j}}}$

(c) $|\mathbf{A} + \mathbf{B}| = \sqrt{2^2 + 6^2} = \boxed{6.32}$

(d) $|\mathbf{A} - \mathbf{B}| = \sqrt{4^2 + 2^2} = \boxed{4.47}$

(e) $\theta_{|\mathbf{A}+\mathbf{B}|} = \tan^{-1}\left(-\frac{6}{2}\right) = -71.6° = \boxed{288°}$

$\theta_{|\mathbf{A}-\mathbf{B}|} = \tan^{-1}\left(\frac{2}{4}\right) = \boxed{26.6°}$

**P3.32** (a) $\mathbf{D} = \mathbf{A} + \mathbf{B} + \mathbf{C} = 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$

$|\mathbf{D}| = \sqrt{2^2 + 4^2} = \boxed{4.47 \text{ m at } \theta = 63.4°}$

(b) $\mathbf{E} = -\mathbf{A} - \mathbf{B} + \mathbf{C} = -6\hat{\mathbf{i}} + 6\hat{\mathbf{j}}$

$|\mathbf{E}| = \sqrt{6^2 + 6^2} = \boxed{8.49 \text{ m at } \theta = 135°}$

**P3.33** $d_1 = \left(-3.50\hat{\mathbf{j}}\right)\text{ m}$

$d_2 = 8.20\cos 45.0°\,\hat{\mathbf{i}} + 8.20\sin 45.0°\,\hat{\mathbf{j}} = \left(5.80\hat{\mathbf{i}} + 5.80\hat{\mathbf{j}}\right)\text{ m}$

$d_3 = \left(-15.0\hat{\mathbf{i}}\right)\text{ m}$

$\mathbf{R} = d_1 + d_2 + d_3 = (-15.0 + 5.80)\hat{\mathbf{i}} + (5.80 - 3.50)\hat{\mathbf{j}} = \boxed{\left(-9.20\hat{\mathbf{i}} + 2.30\hat{\mathbf{j}}\right)\text{ m}}$

(or 9.20 m west and 2.30 m north)

The magnitude of the resultant displacement is

$$|\mathbf{R}| = \sqrt{R_x^2 + R_y^2} = \sqrt{(-9.20)^2 + (2.30)^2} = \boxed{9.48\text{ m}}.$$

The direction is $\theta = \arctan\left(\dfrac{2.30}{-9.20}\right) = \boxed{166°}$.

**P3.34** Refer to the sketch

$$\mathbf{R} = \mathbf{A} + \mathbf{B} + \mathbf{C} = -10.0\hat{\mathbf{i}} - 15.0\hat{\mathbf{j}} + 50.0\hat{\mathbf{i}}$$

$$= 40.0\hat{\mathbf{i}} - 15.0\hat{\mathbf{j}}$$

$$|\mathbf{R}| = \left[(40.0)^2 + (-15.0)^2\right]^{1/2} = \boxed{42.7\text{ yards}}$$

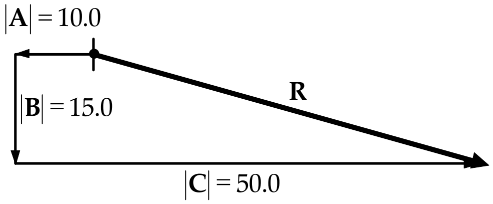


**FIG. P3.34**

**P3.35** (a) $\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2$

$\mathbf{F} = 120\cos(60.0°)\hat{\mathbf{i}} + 120\sin(60.0°)\hat{\mathbf{j}} - 80.0\cos(75.0°)\hat{\mathbf{i}} + 80.0\sin(75.0°)\hat{\mathbf{j}}$

$\mathbf{F} = 60.0\hat{\mathbf{i}} + 104\hat{\mathbf{j}} - 20.7\hat{\mathbf{i}} + 77.3\hat{\mathbf{j}} = \left(39.3\hat{\mathbf{i}} + 181\hat{\mathbf{j}}\right)\text{ N}$

$|\mathbf{F}| = \sqrt{39.3^2 + 181^2} = \boxed{185\text{ N}}$

$\theta = \tan^{-1}\left(\dfrac{181}{39.3}\right) = \boxed{77.8°}$

(b) $\mathbf{F}_3 = -\mathbf{F} = \boxed{\left(-39.3\hat{\mathbf{i}} - 181\hat{\mathbf{j}}\right)\text{ N}}$

**P3.36**

| East | West |
|---|---|
| $x$ | $y$ |
| 0 m | 4.00 m |
| 1.41 | 1.41 |
| –0.500 | –0.866 |
| +0.914 | 4.55 |

$|\mathbf{R}| = \sqrt{|x|^2 + |y|^2} = \boxed{4.64\text{ m at }78.6°\text{ N of E}}$

**P3.37** $\mathbf{A} = 3.00\text{ m},\ \theta_A = 30.0°$ $\mathbf{B} = 3.00\text{ m},\ \theta_B = 90.0°$

$A_x = A\cos\theta_A = 3.00\cos 30.0° = 2.60\text{ m}$ $A_y = A\sin\theta_A = 3.00\sin 30.0° = 1.50\text{ m}$

$$\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} = \left(2.60\hat{\mathbf{i}} + 1.50\hat{\mathbf{j}}\right)\text{ m}$$

$B_x = 0,\ B_y = 3.00\text{ m}$ so $\mathbf{B} = 3.00\hat{\mathbf{j}}\text{ m}$

$$\mathbf{A} + \mathbf{B} = \left(2.60\hat{\mathbf{i}} + 1.50\hat{\mathbf{j}}\right) + 3.00\hat{\mathbf{j}} = \boxed{\left(2.60\hat{\mathbf{i}} + 4.50\hat{\mathbf{j}}\right)\text{ m}}$$

**P3.38** Let the positive $x$-direction be eastward, the positive $y$-direction be vertically upward, and the positive $z$-direction be southward. The total displacement is then

$$\mathbf{d} = \left(4.80\hat{\mathbf{i}} + 4.80\hat{\mathbf{j}}\right)\text{ cm} + \left(3.70\hat{\mathbf{j}} - 3.70\hat{\mathbf{k}}\right)\text{ cm} = \left(4.80\hat{\mathbf{i}} + 8.50\hat{\mathbf{j}} - 3.70\hat{\mathbf{k}}\right)\text{ cm}.$$

(a) The magnitude is $d = \sqrt{(4.80)^2 + (8.50)^2 + (-3.70)^2}\text{ cm} = \boxed{10.4\text{ cm}}$.

(b) Its angle with the $y$-axis follows from $\cos\theta = \dfrac{8.50}{10.4}$, giving $\boxed{\theta = 35.5°}$.

**P3.39** $\mathbf{B} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}} = 4.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}} + 3.00\hat{\mathbf{k}}$

$$|\mathbf{B}| = \sqrt{4.00^2 + 6.00^2 + 3.00^2} = \boxed{7.81}$$

$$\alpha = \cos^{-1}\left(\frac{4.00}{7.81}\right) = \boxed{59.2°}$$

$$\beta = \cos^{-1}\left(\frac{6.00}{7.81}\right) = \boxed{39.8°}$$

$$\gamma = \cos^{-1}\left(\frac{3.00}{7.81}\right) = \boxed{67.4°}$$

**P3.40** The $y$ coordinate of the airplane is constant and equal to $7.60 \times 10^3$ m whereas the $x$ coordinate is given by $x = v_i t$ where $v_i$ is the constant speed in the horizontal direction.

At $t = 30.0$ s we have $x = 8.04 \times 10^3$, so $v_i = 268$ m/s. The position vector as a function of time is

$$\mathbf{P} = (268\text{ m/s})t\hat{\mathbf{i}} + \left(7.60 \times 10^3\text{ m}\right)\hat{\mathbf{j}}.$$

At $t = 45.0$ s, $\mathbf{P} = \left[1.21 \times 10^4\hat{\mathbf{i}} + 7.60 \times 10^3\hat{\mathbf{j}}\right]$ m. The magnitude is

$$\mathbf{P} = \sqrt{\left(1.21 \times 10^4\right)^2 + \left(7.60 \times 10^3\right)^2}\text{ m} = \boxed{1.43 \times 10^4\text{ m}}$$

and the direction is

$$\theta = \arctan\left(\frac{7.60 \times 10^3}{1.21 \times 10^4}\right) = \boxed{32.2°\text{ above the horizontal}}.$$

**P3.41** (a) $\mathbf{A} = \boxed{8.00\hat{\mathbf{i}} + 12.0\hat{\mathbf{j}} - 4.00\hat{\mathbf{k}}}$

(b) $\mathbf{B} = \dfrac{\mathbf{A}}{4} = \boxed{2.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}} - 1.00\hat{\mathbf{k}}}$

(c) $\mathbf{C} = -3\mathbf{A} = \boxed{-24.0\hat{\mathbf{i}} - 36.0\hat{\mathbf{j}} + 12.0\hat{\mathbf{k}}}$

**P3.42** $\mathbf{R} = 75.0\cos 240°\,\hat{\mathbf{i}} + 75.0\sin 240°\,\hat{\mathbf{j}} + 125\cos 135°\,\hat{\mathbf{i}} + 125\sin 135°\,\hat{\mathbf{j}} + 100\cos 160°\,\hat{\mathbf{i}} + 100\sin 160°\,\hat{\mathbf{j}}$

$\mathbf{R} = -37.5\hat{\mathbf{i}} - 65.0\hat{\mathbf{j}} - 88.4\hat{\mathbf{i}} + 88.4\hat{\mathbf{j}} - 94.0\hat{\mathbf{i}} + 34.2\hat{\mathbf{j}}$

$\mathbf{R} = \boxed{-220\hat{\mathbf{i}} + 57.6\hat{\mathbf{j}}}$

$\mathbf{R} = \sqrt{(-220)^2 + 57.6^2}$ at $\arctan\left(\dfrac{57.6}{220}\right)$ above the $-x$-axis

$\mathbf{R} = \boxed{227 \text{ paces at } 165°}$

**P3.43** (a) $\mathbf{C} = \mathbf{A} + \mathbf{B} = \boxed{\left(5.00\hat{\mathbf{i}} - 1.00\hat{\mathbf{j}} - 3.00\hat{\mathbf{k}}\right) \text{ m}}$

$|\mathbf{C}| = \sqrt{(5.00)^2 + (1.00)^2 + (3.00)^2} \text{ m} = \boxed{5.92 \text{ m}}$

(b) $\mathbf{D} = 2\mathbf{A} - \mathbf{B} = \boxed{\left(4.00\hat{\mathbf{i}} - 11.0\hat{\mathbf{j}} + 15.0\hat{\mathbf{k}}\right) \text{ m}}$

$|\mathbf{D}| = \sqrt{(4.00)^2 + (11.0)^2 + (15.0)^2} \text{ m} = \boxed{19.0 \text{ m}}$

**P3.44** The position vector from radar station to ship is

$$\mathbf{S} = \left(17.3\sin 136°\,\hat{\mathbf{i}} + 17.3\cos 136°\,\hat{\mathbf{j}}\right) \text{ km} = \left(12.0\hat{\mathbf{i}} - 12.4\hat{\mathbf{j}}\right) \text{ km}.$$

From station to plane, the position vector is

$$\mathbf{P} = \left(19.6\sin 153°\,\hat{\mathbf{i}} + 19.6\cos 153°\,\hat{\mathbf{j}} + 2.20\hat{\mathbf{k}}\right) \text{ km},$$

or

$$\mathbf{P} = \left(8.90\hat{\mathbf{i}} - 17.5\hat{\mathbf{j}} + 2.20\hat{\mathbf{k}}\right) \text{ km}.$$

(a) To fly to the ship, the plane must undergo displacement

$$\mathbf{D} = \mathbf{S} - \mathbf{P} = \boxed{\left(3.12\hat{\mathbf{i}} + 5.02\hat{\mathbf{j}} - 2.20\hat{\mathbf{k}}\right) \text{ km}}.$$

(b) The distance the plane must travel is

$$D = |\mathbf{D}| = \sqrt{(3.12)^2 + (5.02)^2 + (2.20)^2} \text{ km} = \boxed{6.31 \text{ km}}.$$

**P3.45** The hurricane's first displacement is $\left(\frac{41.0\text{ km}}{\text{h}}\right)(3.00\text{ h})$ at 60.0° N of W, and its second displacement is $\left(\frac{25.0\text{ km}}{\text{h}}\right)(1.50\text{ h})$ due North. With $\hat{\mathbf{i}}$ representing east and $\hat{\mathbf{j}}$ representing north, its total displacement is:

$$\left(41.0\frac{\text{km}}{\text{h}}\cos 60.0^\circ\right)(3.00\text{ h})\left(-\hat{\mathbf{i}}\right)+\left(41.0\frac{\text{km}}{\text{h}}\sin 60.0^\circ\right)(3.00\text{ h})\hat{\mathbf{j}}+\left(25.0\frac{\text{km}}{\text{h}}\right)(1.50\text{ h})\hat{\mathbf{j}}=61.5\text{ km}\left(-\hat{\mathbf{i}}\right)+144\text{ km}\,\hat{\mathbf{j}}$$

with magnitude $\sqrt{(61.5\text{ km})^2+(144\text{ km})^2}=\boxed{157\text{ km}}$.

**P3.46** (a) $\mathbf{E}=(17.0\text{ cm})\cos 27.0^\circ\,\hat{\mathbf{i}}+(17.0\text{ cm})\sin 27.0^\circ\,\hat{\mathbf{j}}$

$\mathbf{E}=\boxed{\left(15.1\hat{\mathbf{i}}+7.72\hat{\mathbf{j}}\right)\text{ cm}}$

(b) $\mathbf{F}=-(17.0\text{ cm})\sin 27.0^\circ\,\hat{\mathbf{i}}+(17.0\text{ cm})\cos 27.0^\circ\,\hat{\mathbf{j}}$

$\mathbf{F}=\boxed{\left(-7.72\hat{\mathbf{i}}+15.1\hat{\mathbf{j}}\right)\text{ cm}}$

(c) $\mathbf{G}=+(17.0\text{ cm})\sin 27.0^\circ\,\hat{\mathbf{i}}+(17.0\text{ cm})\cos 27.0^\circ\,\hat{\mathbf{j}}$

$\mathbf{G}=\boxed{\left(+7.72\hat{\mathbf{i}}+15.1\hat{\mathbf{j}}\right)\text{ cm}}$

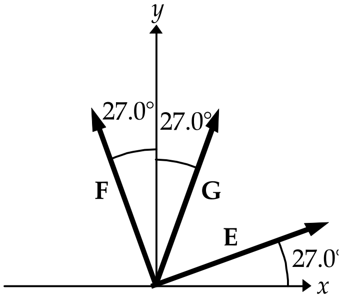


FIG. P3.46

**P3.47** $A_x=-3.00$, $A_y=2.00$

(a) $\mathbf{A}=A_x\hat{\mathbf{i}}+A_y\hat{\mathbf{j}}=\boxed{-3.00\hat{\mathbf{i}}+2.00\hat{\mathbf{j}}}$

(b) $|\mathbf{A}|=\sqrt{A_x^2+A_y^2}=\sqrt{(-3.00)^2+(2.00)^2}=\boxed{3.61}$

$\tan\theta=\frac{A_y}{A_x}=\frac{2.00}{(-3.00)}=-0.667$, $\tan^{-1}(-0.667)=-33.7^\circ$

$\theta$ is in the 2nd quadrant, so $\theta=180^\circ+(-33.7^\circ)=\boxed{146^\circ}$.

(c) $\mathbf{R}_x=0$, $\mathbf{R}_y=-4.00$, $\mathbf{R}=\mathbf{A}+\mathbf{B}$ thus $\mathbf{B}=\mathbf{R}-\mathbf{A}$ and

$B_x=R_x-A_x=0-(-3.00)=3.00$, $B_y=R_y-A_y=-4.00-2.00=-6.00$.

Therefore, $\mathbf{B}=\boxed{3.00\hat{\mathbf{i}}-6.00\hat{\mathbf{j}}}$.

**P3.48** Let $+x =$ East, $+y =$ North,

| $x$ | $y$ |
|---|---|
| 300 | 0 |
| –175 | 303 |
| 0 | 150 |
| 125 | 453 |

(a) $\theta = \tan^{-1}\dfrac{y}{x} = \boxed{74.6° \text{ N of E}}$

(b) $|\mathbf{R}| = \sqrt{x^2 + y^2} = \boxed{470 \text{ km}}$

**P3.49** (a)

$$R_x = 40.0\cos 45.0° + 30.0\cos 45.0° = 49.5$$

$$R_y = 40.0\sin 45.0° - 30.0\sin 45.0° + 20.0 = 27.1$$

$$\mathbf{R} = \boxed{49.5\hat{\mathbf{i}} + 27.1\hat{\mathbf{j}}}$$

(b)

$$|\mathbf{R}| = \sqrt{(49.5)^2 + (27.1)^2} = \boxed{56.4}$$

$$\theta = \tan^{-1}\left(\frac{27.1}{49.5}\right) = \boxed{28.7°}$$

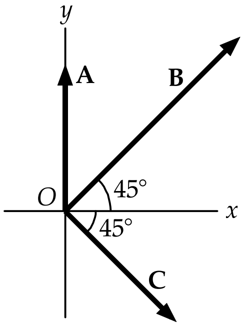


**FIG. P3.49**

**P3.50** Taking components along $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$, we get two equations:

$$6.00a - 8.00b + 26.0 = 0$$

and

$$-8.00a + 3.00b + 19.0 = 0.$$

Solving simultaneously,

$$\boxed{a = 5.00,\ b = 7.00}.$$

Therefore,

$$5.00\mathbf{A} + 7.00\mathbf{B} + \mathbf{C} = 0.$$

## Additional Problems

**P3.51** Let $\theta$ represent the angle between the directions of **A** and **B**. Since **A** and **B** have the same magnitudes, **A**, **B**, and $\mathbf{R} = \mathbf{A} + \mathbf{B}$ form an isosceles triangle in which the angles are $180° - \theta$, $\dfrac{\theta}{2}$, and $\dfrac{\theta}{2}$. The magnitude of **R** is then $R = 2A\cos\left(\dfrac{\theta}{2}\right)$. [*Hint:* apply the law of cosines to the isosceles triangle and use the fact that $B = A$.]

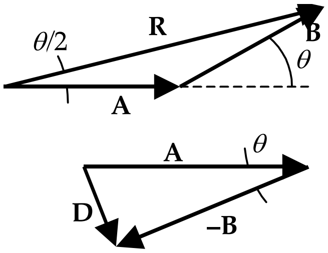


FIG. P3.51

Again, **A**, –**B**, and $\mathbf{D} = \mathbf{A} - \mathbf{B}$ form an isosceles triangle with apex angle $\theta$. Applying the law of cosines and the identity

$$(1 - \cos\theta) = 2\sin^2\left(\frac{\theta}{2}\right)$$

gives the magnitude of **D** as $D = 2A\sin\left(\dfrac{\theta}{2}\right)$.

The problem requires that $R = 100D$.

Thus, $2A\cos\left(\dfrac{\theta}{2}\right) = 200A\sin\left(\dfrac{\theta}{2}\right)$. This gives $\tan\left(\dfrac{\theta}{2}\right) = 0.010$ and $\boxed{\theta = 1.15°}$.

**P3.52** Let $\theta$ represent the angle between the directions of **A** and **B**. Since **A** and **B** have the same magnitudes, **A**, **B**, and $\mathbf{R} = \mathbf{A} + \mathbf{B}$ form an isosceles triangle in which the angles are $180° - \theta$, $\dfrac{\theta}{2}$, and $\dfrac{\theta}{2}$. The magnitude of **R** is then $R = 2A\cos\left(\dfrac{\theta}{2}\right)$. [*Hint*: apply the law of cosines to the isosceles triangle and use the fact that $B = A$. ]

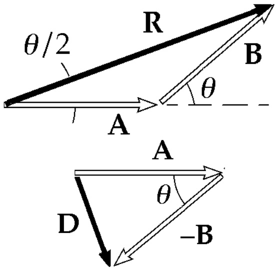


FIG. P3.52

Again, **A**, –**B**, and $\mathbf{D} = \mathbf{A} - \mathbf{B}$ form an isosceles triangle with apex angle $\theta$. Applying the law of cosines and the identity

$$(1 - \cos\theta) = 2\sin^2\left(\frac{\theta}{2}\right)$$

gives the magnitude of **D** as $D = 2A\sin\left(\dfrac{\theta}{2}\right)$.

The problem requires that $R = nD$ or $\cos\left(\dfrac{\theta}{2}\right) = n\sin\left(\dfrac{\theta}{2}\right)$ giving

$$\boxed{\theta = 2\tan^{-1}\left(\frac{1}{n}\right)}.$$

**P3.53** (a) $R_x = \boxed{2.00}$, $R_y = \boxed{1.00}$, $R_z = \boxed{3.00}$

(b) $$|\mathbf{R}| = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{4.00 + 1.00 + 9.00} = \sqrt{14.0} = \boxed{3.74}$$

(c) $$\cos\theta_x = \frac{R_x}{|\mathbf{R}|} \Rightarrow \theta_x = \cos^{-1}\left(\frac{R_x}{|\mathbf{R}|}\right) = \boxed{57.7^\circ \text{ from } +x}$$
$$\cos\theta_y = \frac{R_y}{|\mathbf{R}|} \Rightarrow \theta_y = \cos^{-1}\left(\frac{R_y}{|\mathbf{R}|}\right) = \boxed{74.5^\circ \text{ from } +y}$$
$$\cos\theta_z = \frac{R_z}{|\mathbf{R}|} \Rightarrow \theta_z = \cos^{-1}\left(\frac{R_z}{|\mathbf{R}|}\right) = \boxed{36.7^\circ \text{ from } +z}$$

***P3.54** Take the $x$-axis along the tail section of the snake. The displacement from tail to head is

$$240 \text{ m}\hat{\mathbf{i}} + (420 - 240) \text{ m}\cos(180° - 105°)\hat{\mathbf{i}} - 180 \text{ m}\sin 75°\,\hat{\mathbf{j}} = 287 \text{ m}\hat{\mathbf{i}} - 174 \text{ m}\hat{\mathbf{j}}.$$

Its magnitude is $\sqrt{(287)^2 + (174)^2}\text{ m} = 335 \text{ m}$. From $v = \dfrac{\text{distance}}{\Delta t}$, the time for each child's run is

$$\text{Inge: } \Delta t = \frac{\text{distance}}{v} = \frac{335 \text{ m(h)(1 km)(3 600 s)}}{\text{(12 km)(1 000 m)(1 h)}} = 101 \text{ s}$$
$$\text{Olaf: } \Delta t = \frac{420 \text{ m}\cdot\text{s}}{3.33 \text{ m}} = 126 \text{ s}.$$

Inge wins by $126 - 101 = \boxed{25.4 \text{ s}}$.

***P3.55** The position vector from the ground under the controller of the first airplane is

$$\mathbf{r}_1 = (19.2 \text{ km})(\cos 25°)\hat{\mathbf{i}} + (19.2 \text{ km})(\sin 25°)\hat{\mathbf{j}} + (0.8 \text{ km})\hat{\mathbf{k}}$$
$$= \left(17.4\hat{\mathbf{i}} + 8.11\hat{\mathbf{j}} + 0.8\hat{\mathbf{k}}\right) \text{ km}.$$

The second is at

$$\mathbf{r}_2 = (17.6 \text{ km})(\cos 20°)\hat{\mathbf{i}} + (17.6 \text{ km})(\sin 20°)\hat{\mathbf{j}} + (1.1 \text{ km})\hat{\mathbf{k}}$$
$$= \left(16.5\hat{\mathbf{i}} + 6.02\hat{\mathbf{j}} + 1.1\hat{\mathbf{k}}\right) \text{ km}.$$

Now the displacement from the first plane to the second is

$$\mathbf{r}_2 - \mathbf{r}_1 = \left(-0.863\hat{\mathbf{i}} - 2.09\hat{\mathbf{j}} + 0.3\hat{\mathbf{k}}\right) \text{ km}$$

with magnitude

$$\sqrt{(0.863)^2 + (2.09)^2 + (0.3)^2} = \boxed{2.29 \text{ km}}.$$

***P3.56** Let $A$ represent the distance from island 2 to island 3. The displacement is $\mathbf{A} = A$ at $159°$. Represent the displacement from 3 to 1 as $\mathbf{B} = B$ at $298°$. We have 4.76 km at $37° \ +\mathbf{A}+\mathbf{B}=0$.

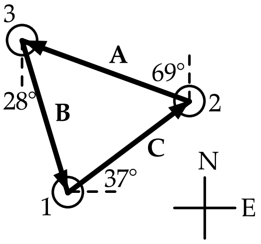


**FIG. P3.56**

For $x$-components

$$(4.76\text{ km})\cos 37° + A\cos 159° + B\cos 298° = 0$$
$$3.80\text{ km} - 0.934A + 0.469B = 0$$
$$B = -8.10\text{ km} + 1.99A$$

For $y$-components,

$$(4.76\text{ km})\sin 37° + A\sin 159° + B\sin 298° = 0$$
$$2.86\text{ km} + 0.358A - 0.883B = 0$$

(a) We solve by eliminating $B$ by substitution:

$$2.86\text{ km} + 0.358A - 0.883(-8.10\text{ km} + 1.99A) = 0$$
$$2.86\text{ km} + 0.358A + 7.15\text{ km} - 1.76A = 0$$
$$10.0\text{ km} = 1.40A$$
$$A = \boxed{7.17\text{ km}}$$

(b) $B = -8.10\text{ km} + 1.99(7.17\text{ km}) = \boxed{6.15\text{ km}}$

***P3.57** (a) We first express the corner's position vectors as sets of components

$$\mathbf{A} = (10\text{ m})\cos 50°\,\hat{\mathbf{i}} + (10\text{ m})\sin 50°\,\hat{\mathbf{j}} = 6.43\text{ m}\hat{\mathbf{i}} + 7.66\text{ m}\hat{\mathbf{j}}$$
$$\mathbf{B} = (12\text{ m})\cos 30°\,\hat{\mathbf{i}} + (12\text{ m})\sin 30°\,\hat{\mathbf{j}} = 10.4\text{ m}\hat{\mathbf{i}} + 6.00\text{ m}\hat{\mathbf{j}}.$$

The horizontal width of the rectangle is

$$10.4\text{ m} - 6.43\text{ m} = 3.96\text{ m}.$$

Its vertical height is

$$7.66\text{ m} - 6.00\text{ m} = 1.66\text{ m}.$$

Its perimeter is

$$2(3.96 + 1.66)\text{ m} = \boxed{11.2\text{ m}}.$$

(b) The position vector of the distant corner is $B_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} = 10.4\text{ m}\hat{\mathbf{i}} + 7.66\text{ m}\hat{\mathbf{j}} = \sqrt{10.4^2 + 7.66^2}$ m at $\tan^{-1}\dfrac{7.66\text{ m}}{10.4\text{ m}} = \boxed{12.9\text{ m at }36.4°}$.

**P3.58** Choose the $+x$-axis in the direction of the first force. The total force, in newtons, is then

$$12.0\hat{\mathbf{i}} + 31.0\hat{\mathbf{j}} - 8.40\hat{\mathbf{i}} - 24.0\hat{\mathbf{j}} = \boxed{\left(3.60\hat{\mathbf{i}}\right) + \left(7.00\hat{\mathbf{j}}\right)\text{ N}}.$$

The magnitude of the total force is

$$\sqrt{(3.60)^2 + (7.00)^2}\text{ N} = \boxed{7.87\text{ N}}$$

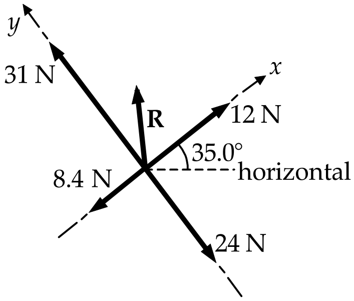


FIG. P3.58

and the angle it makes with our $+x$-axis is given by $\tan\theta = \frac{(7.00)}{(3.60)}$, $\theta = 62.8°$. Thus, its angle counterclockwise from the horizontal is $35.0° + 62.8° = \boxed{97.8°}$.

**P3.59**

$\mathbf{d}_1 = 100\hat{\mathbf{i}}$

$\mathbf{d}_2 = -300\hat{\mathbf{j}}$

$\mathbf{d}_3 = -150\cos(30.0°)\hat{\mathbf{i}} - 150\sin(30.0°)\hat{\mathbf{j}} = -130\hat{\mathbf{i}} - 75.0\hat{\mathbf{j}}$

$\mathbf{d}_4 = -200\cos(60.0°)\hat{\mathbf{i}} + 200\sin(60.0°)\hat{\mathbf{j}} = -100\hat{\mathbf{i}} + 173\hat{\mathbf{j}}$

$\mathbf{R} = \mathbf{d}_1 + \mathbf{d}_2 + \mathbf{d}_3 + \mathbf{d}_4 = \boxed{\left(-130\hat{\mathbf{i}} - 202\hat{\mathbf{j}}\right)\text{ m}}$

$|\mathbf{R}| = \sqrt{(-130)^2 + (-202)^2} = \boxed{240\text{ m}}$

$\phi = \tan^{-1}\left(\frac{202}{130}\right) = 57.2°$

$\theta = 180 + \phi = \boxed{237°}$

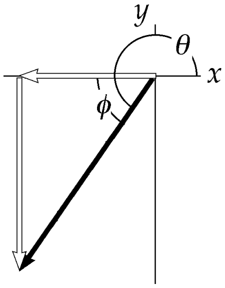


FIG. P3.59

**P3.60**

$$\frac{d\mathbf{r}}{dt} = \frac{d\left(4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2t\hat{\mathbf{j}}\right)}{dt} = 0 + 0 - 2\hat{\mathbf{j}} = \boxed{-(2.00\text{ m/s})\hat{\mathbf{j}}}$$

The position vector at $t = 0$ is $4\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$. At $t = 1$ s, the position is $4\hat{\mathbf{i}} + 1\hat{\mathbf{j}}$, and so on. The object is moving straight downward at 2 m/s, so

$$\frac{d\mathbf{r}}{dt}\text{ represents }\boxed{\text{its velocity vector}}.$$

**P3.61**

$\mathbf{v} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} = (300 + 100\cos 30.0°)\hat{\mathbf{i}} + (100\sin 30.0°)\hat{\mathbf{j}}$

$\mathbf{v} = \left(387\hat{\mathbf{i}} + 50.0\hat{\mathbf{j}}\right)\text{ mi/h}$

$|\mathbf{v}| = \boxed{390\text{ mi/h at }7.37°\text{ N of E}}$

**P3.62** (a) You start at point $A$: $\mathbf{r}_1 = \mathbf{r}_A = \left(30.0\hat{\mathbf{i}} - 20.0\hat{\mathbf{j}}\right)$ m.

The displacement to $B$ is

$$\mathbf{r}_B - \mathbf{r}_A = 60.0\hat{\mathbf{i}} + 80.0\hat{\mathbf{j}} - 30.0\hat{\mathbf{i}} + 20.0\hat{\mathbf{j}} = 30.0\hat{\mathbf{i}} + 100\hat{\mathbf{j}}.$$

You cover half of this, $\left(15.0\hat{\mathbf{i}} + 50.0\hat{\mathbf{j}}\right)$ to move to $\mathbf{r}_2 = 30.0\hat{\mathbf{i}} - 20.0\hat{\mathbf{j}} + 15.0\hat{\mathbf{i}} + 50.0\hat{\mathbf{j}} = 45.0\hat{\mathbf{i}} + 30.0\hat{\mathbf{j}}$.

Now the displacement from your current position to $C$ is

$$\mathbf{r}_C - \mathbf{r}_2 = -10.0\hat{\mathbf{i}} - 10.0\hat{\mathbf{j}} - 45.0\hat{\mathbf{i}} - 30.0\hat{\mathbf{j}} = -55.0\hat{\mathbf{i}} - 40.0\hat{\mathbf{j}}.$$

You cover one-third, moving to

$$\mathbf{r}_3 = \mathbf{r}_2 + \Delta\mathbf{r}_{23} = 45.0\hat{\mathbf{i}} + 30.0\hat{\mathbf{j}} + \frac{1}{3}\left(-55.0\hat{\mathbf{i}} - 40.0\hat{\mathbf{j}}\right) = 26.7\hat{\mathbf{i}} + 16.7\hat{\mathbf{j}}.$$

The displacement from where you are to $D$ is

$$\mathbf{r}_D - \mathbf{r}_3 = 40.0\hat{\mathbf{i}} - 30.0\hat{\mathbf{j}} - 26.7\hat{\mathbf{i}} - 16.7\hat{\mathbf{j}} = 13.3\hat{\mathbf{i}} - 46.7\hat{\mathbf{j}}.$$

You traverse one-quarter of it, moving to

$$\mathbf{r}_4 = \mathbf{r}_3 + \frac{1}{4}(\mathbf{r}_D - \mathbf{r}_3) = 26.7\hat{\mathbf{i}} + 16.7\hat{\mathbf{j}} + \frac{1}{4}\left(13.3\hat{\mathbf{i}} - 46.7\hat{\mathbf{j}}\right) = 30.0\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}}.$$

The displacement from your new location to **E** is

$$\mathbf{r}_E - \mathbf{r}_4 = -70.0\hat{\mathbf{i}} + 60.0\hat{\mathbf{j}} - 30.0\hat{\mathbf{i}} - 5.00\hat{\mathbf{j}} = -100\hat{\mathbf{i}} + 55.0\hat{\mathbf{j}}$$

of which you cover one-fifth the distance, $-20.0\hat{\mathbf{i}} + 11.0\hat{\mathbf{j}}$, moving to

$$\mathbf{r}_4 + \Delta\mathbf{r}_{45} = 30.0\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}} - 20.0\hat{\mathbf{i}} + 11.0\hat{\mathbf{j}} = 10.0\hat{\mathbf{i}} + 16.0\hat{\mathbf{j}}.$$

The treasure is at $\boxed{(10.0\text{ m, }16.0\text{ m})}$.

(b) Following the directions brings you to the average position of the trees. The steps we took numerically in part (a) bring you to

$$\mathbf{r}_A + \frac{1}{2}(\mathbf{r}_B - \mathbf{r}_A) = \left(\frac{\mathbf{r}_A + \mathbf{r}_B}{2}\right)$$

then to $\dfrac{(\mathbf{r}_A + \mathbf{r}_B)}{2} + \dfrac{\mathbf{r}_C - \frac{(\mathbf{r}_A + \mathbf{r}_B)}{2}}{3} = \dfrac{\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C}{3}$

then to $\dfrac{(\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C)}{3} + \dfrac{\mathbf{r}_D - \frac{(\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C)}{3}}{4} = \dfrac{\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C + \mathbf{r}_D}{4}$

and last to $\dfrac{(\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C + \mathbf{r}_D)}{4} + \dfrac{\mathbf{r}_E - \frac{(\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C + \mathbf{r}_D)}{4}}{5} = \dfrac{\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C + \mathbf{r}_D + \mathbf{r}_E}{5}$.

This center of mass of the tree distribution is the same location whatever order we take the trees in.

***P3.63** (a) Let $T$ represent the force exerted by each child. The $x$-component of the resultant force is

$$T\cos 0 + T\cos 120° + T\cos 240° = T(1) + T(-0.5) + T(-0.5) = 0.$$

The $y$-component is

$$T\sin 0 + T\sin 120 + T\sin 240 = 0 + 0.866T - 0.866T = 0.$$

Thus,

$$\sum \mathbf{F} = 0.$$

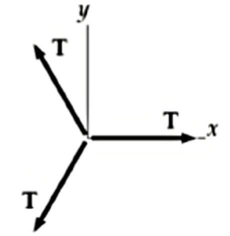


FIG. P3.63

(b) If the total force is not zero, it must point in some direction. When each child moves one space clockwise, the total must turn clockwise by that angle, $\frac{360°}{N}$. Since each child exerts the same force, the new situation is identical to the old and the net force on the tire must still point in the original direction. The contradiction indicates that we were wrong in supposing that the total force is not zero. The total force *must* be zero.

**P3.64** (a) From the picture, $\mathbf{R}_1 = a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$ and $|\mathbf{R}_1| = \sqrt{a^2 + b^2}$.

(b) $\mathbf{R}_2 = a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + c\hat{\mathbf{k}}$; its magnitude is

$$\sqrt{|\mathbf{R}_1|^2 + c^2} = \sqrt{a^2 + b^2 + c^2}.$$

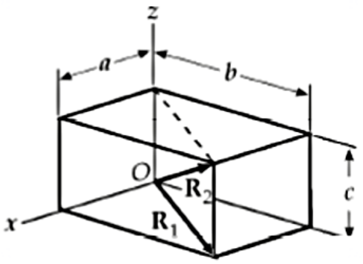


FIG. P3.64

**P3.65** Since

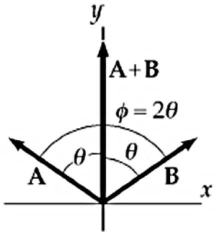


**FIG. P3.65**

$$\mathbf{A}+\mathbf{B}=6.00\hat{\mathbf{j}},$$

we have

$$\left(A_x+B_x\right)\hat{\mathbf{i}}+\left(A_y+B_y\right)\hat{\mathbf{j}}=0\hat{\mathbf{i}}+6.00\hat{\mathbf{j}}$$

giving

$$A_x+B_x=0 \text{ or } A_x=-B_x \qquad [1]$$

and

$$A_y+B_y=6.00. \qquad [2]$$

Since both vectors have a magnitude of 5.00, we also have

$$A_x^2+A_y^2=B_x^2+B_y^2=5.00^2.$$

From $A_x=-B_x$, it is seen that

$$A_x^2=B_x^2.$$

Therefore, $A_x^2+A_y^2=B_x^2+B_y^2$ gives

$$A_y^2=B_y^2.$$

Then, $A_y=B_y$ and Eq. [2] gives

$$A_y=B_y=3.00.$$

Defining $\theta$ as the angle between either **A** or **B** and the $y$ axis, it is seen that

$$\cos\theta=\frac{A_y}{A}=\frac{B_y}{B}=\frac{3.00}{5.00}=0.600 \text{ and } \theta=53.1°.$$

The angle between **A** and **B** is then $\boxed{\phi=2\theta=106°}$.

***P3.66** Let $\theta$ represent the angle the $x$-axis makes with the horizontal. Since angles are equal if their sides are perpendicular right side to right side and left side to left side, $\theta$ is also the angle between the weight and our $y$ axis. The $x$-components of the forces must add to zero:

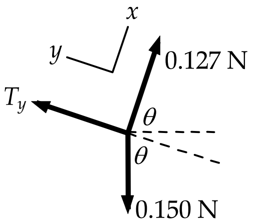


**FIG. P3.66**

$$-0.150\text{ N}\sin\theta+0.127\text{ N}=0.$$

(b) $\theta=\boxed{57.9^\circ}$

(a) The $y$-components for the forces must add to zero:

$$+T_y-(0.150\text{ N})\cos 57.9^\circ=0,\ T_y=\boxed{0.079\,8\text{ N}}.$$

(c) The angle between the $y$ axis and the horizontal is $90.0^\circ-57.9^\circ=\boxed{32.1^\circ}$.

**P3.67** The displacement of point $P$ is invariant under rotation of the coordinates. Therefore, $r=r'$ and $r^2=(r')^2$ or, $x^2+y^2=(x')^2+(y')^2$. Also, from the figure, $\beta=\theta-\alpha$

$$\therefore \tan^{-1}\left(\frac{y'}{x'}\right)=\tan^{-1}\left(\frac{y}{x}\right)-\alpha$$

$$\frac{y'}{x'}=\frac{\left(\frac{y}{x}\right)-\tan\alpha}{1+\left(\frac{y}{x}\right)\tan\alpha}$$

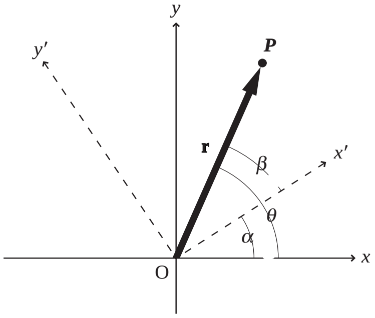


**FIG. P3.67**

Which we simplify by multiplying top and bottom by $x\cos\alpha$. Then,

$$x'=x\cos\alpha+y\sin\alpha,\ y'=-x\sin\alpha+y\cos\alpha.$$

## ANSWERS TO EVEN PROBLEMS

| | |
|---|---|
| **P3.2** | (a) (2.17 m, 1.25 m); (–1.90 m, 3.29 m);<br>(b) 4.55 m |
| **P3.4** | (a) 8.60 m;<br>(b) 4.47 m at –63.4°; 4.24 m at 135° |
| **P3.6** | (a) $r$ at $180^\circ-\theta$; (b) $2r$ at $180^\circ+\theta$; (c) $3r$ at $-\theta$ |
| **P3.8** | 14 km at 65° north of east |
| **P3.10** | (a) 6.1 at 112°; (b) 14.8 at 22° |
| **P3.12** | 9.5 N at 57° |
| **P3.14** | 7.9 m at 4° north of west |
| **P3.16** | see the solution |
| **P3.18** | 86.6 m and –50.0 m |
| **P3.20** | 1.31 km north; 2.81 km east |
| **P3.22** | $-25.0\text{ m}\,\hat{\mathbf{i}}+43.3\text{ m}\,\hat{\mathbf{j}}$ |
| **P3.24** | 14.0 m/s at 11.3° west of north |
| **P3.26** | 788 mi at 48.0° north of east |
| **P3.28** | 7.21 m at 56.3° |
| **P3.30** | $\mathbf{C}=7.30\text{ cm}\,\hat{\mathbf{i}}-7.20\text{ cm}\,\hat{\mathbf{j}}$ |

| | |
|---|---|
| **P3.32** | (a) 4.47 m at 63.4°; (b) 8.49 m at 135° |
| **P3.34** | 42.7 yards |
| **P3.36** | 4.64 m at 78.6° |
| **P3.38** | (a) 10.4 cm; (b) 35.5° |
| **P3.40** | $1.43 \times 10^4$ m at 32.2° above the horizontal |
| **P3.42** | $-220\hat{\mathbf{i}} + 57.6\hat{\mathbf{j}} = 227$ paces at 165° |
| **P3.44** | (a) $\left(3.12\hat{\mathbf{i}} + 5.02\hat{\mathbf{j}} - 2.20\hat{\mathbf{k}}\right)$ km; (b) 6.31 km |
| **P3.46** | (a) $\left(15.1\hat{\mathbf{i}} + 7.72\hat{\mathbf{j}}\right)$ cm;<br>(b) $\left(-7.72\hat{\mathbf{i}} + 15.1\hat{\mathbf{j}}\right)$ cm;<br>(c) $\left(+7.72\hat{\mathbf{i}} + 15.1\hat{\mathbf{j}}\right)$ cm |
| **P3.48** | (a) 74.6° north of east; (b) 470 km |
| **P3.50** | $a = 5.00$, $b = 7.00$ |
| **P3.52** | $2\tan^{-1}\left(\dfrac{1}{n}\right)$ |
| **P3.54** | 25.4 s |
| **P3.56** | (a) 7.17 km; (b) 6.15 km |
| **P3.58** | 7.87 N at 97.8° counterclockwise from a horizontal line to the right |
| **P3.60** | $(-2.00\ \text{m/s})\hat{\mathbf{j}}$; its velocity vector |
| **P3.62** | (a) (10.0 m, 16.0 m); (b) see the solution |
| **P3.64** | (a) $\mathbf{R}_1 = a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$; $\lvert\mathbf{R}_1\rvert = \sqrt{a^2 + b^2}$;<br>(b) $\mathbf{R}_2 = a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + c\hat{\mathbf{k}}$; $\lvert\mathbf{R}_2\rvert = \sqrt{a^2 + b^2 + c^2}$ |
| **P3.66** | (a) 0.079 8N; (b) 57.9°; (c) 32.1° |

# 4

# Motion in Two Dimensions

## CHAPTER OUTLINE

4.1 The Position, Velocity, and Acceleration Vectors
4.2 Two-Dimensional Motion with Constant Acceleration
4.3 Projectile Motion
4.4 Uniform Circular Motion
4.5 Tangential and Radial Acceleration
4.6 Relative Velocity and Relative Acceleration

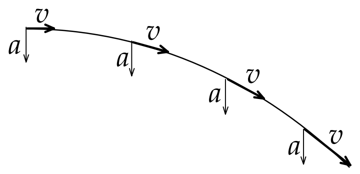
## ANSWERS TO QUESTIONS

**Q4.1** Yes. An object moving in uniform circular motion moves at a constant speed, but changes its direction of motion. An object cannot accelerate if its velocity is constant.

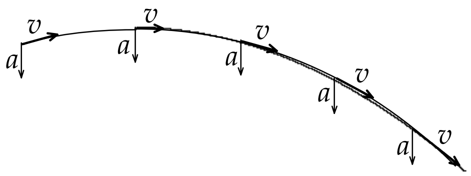
**Q4.2** No, you cannot determine the instantaneous velocity. Yes, you can determine the average velocity. The points could be widely separated. In this case, you can only determine the average velocity, which is

$$\bar{\mathbf{v}} = \frac{\Delta \mathbf{x}}{\Delta t}.$$

**Q4.3** (a)



(b)



**Q4.4** (a) $10\hat{\mathbf{i}}$ m/s (b) $-9.80\hat{\mathbf{j}}$ $\text{m/s}^2$

**Q4.5** The easiest way to approach this problem is to determine acceleration first, velocity second and finally position.

Vertical: In free flight, $a_y = -g$. At the top of a projectile's trajectory, $v_y = 0$. Using this, the maximum height can be found using $v_{fy}^2 = v_{iy}^2 + 2a_y\left(y_f - y_i\right)$.

Horizontal: $a_x = 0$, so $v_x$ is always the same. To find the horizontal position at maximum height, one needs the flight time, *t*. Using the vertical information found previously, the flight time can be found using $v_{fy} = v_{iy} + a_y t$. The horizontal position is $x_f = v_{ix} t$.

If air resistance is taken into account, then the acceleration in both the *x* and *y*-directions would have an additional term due to the drag.

**Q4.6** A parabola.

**Q4.7** The balls will be closest together as the second ball is thrown. Yes, the first ball will always be moving faster, since its flight time is larger, and thus the vertical component of the velocity is larger. The time interval will be one second. No, since the vertical component of the motion determines the flight time.

**Q4.8** The ball will have the greater speed. Both the rock and the ball will have the same vertical component of the velocity, but the ball will have the additional horizontal component.

**Q4.9** (a) yes (b) no (c) no (d) yes (e) no

**Q4.10** Straight up. Throwing the ball any other direction than straight up will give a nonzero speed at the top of the trajectory.

**Q4.11** No. The projectile with the larger vertical component of the initial velocity will be in the air longer.

**Q4.12** The projectile is in free fall. Its vertical component of acceleration is the downward acceleration of gravity. Its horizontal component of acceleration is zero.

**Q4.13** (a) no (b) yes (c) yes (d) no

**Q4.14** 60°. The projection angle appears in the expression for horizontal range in the function $\sin 2\theta$. This function is the same for 30° and 60°.

**Q4.15** The optimal angle would be less than 45°. The longer the projectile is in the air, the more that air resistance will change the components of the velocity. Since the vertical component of the motion determines the flight time, an angle less than 45° would increase range.

**Q4.16** The projectile on the moon would have both the larger range and the greater altitude. *Apollo* astronauts performed the experiment with golf balls.

**Q4.17** Gravity only changes the vertical component of motion. Since both the coin and the ball are falling from the same height with the same vertical component of the initial velocity, they must hit the floor at the same time.

**Q4.18** (a) no (b) yes

In the second case, the particle is continuously changing the direction of its velocity vector.

**Q4.19** The racing car rounds the turn at a constant *speed* of 90 miles per hour.

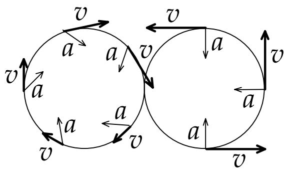
**Q4.20** The acceleration cannot be zero because the pendulum does not remain at rest at the end of the arc.

**Q4.21** (a) The velocity is not constant because the object is constantly changing the direction of its motion.

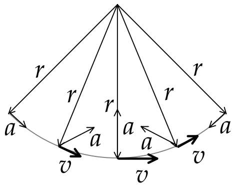
(b) The acceleration is not constant because the acceleration always points towards the center of the circle. The magnitude of the acceleration is constant, but not the direction.

**Q4.22** (a) straight ahead (b) in a circle or straight ahead

**Q4.23**



**Q4.24**



**Q4.25** The unit vectors $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\theta}}$ are in different directions at different points in the $xy$ plane. At a location along the $x$-axis, for example, $\hat{\mathbf{r}} = \hat{\mathbf{i}}$ and $\hat{\boldsymbol{\theta}} = \hat{\mathbf{j}}$, but at a point on the $y$-axis, $\hat{\mathbf{r}} = \hat{\mathbf{j}}$ and $\hat{\boldsymbol{\theta}} = -\hat{\mathbf{i}}$. The unit vector $\hat{\mathbf{i}}$ is equal everywhere, and $\hat{\mathbf{j}}$ is also uniform.

**Q4.26** The wrench will hit at the base of the mast. If air resistance is a factor, it will hit slightly leeward of the base of the mast, displaced in the direction in which air is moving relative to the deck. If the boat is scudding before the wind, for example, the wrench's impact point can be in front of the mast.

**Q4.27** (a) The ball would move straight up and down as observed by the passenger. The ball would move in a parabolic trajectory as seen by the ground observer.

(b) Both the passenger and the ground observer would see the ball move in a parabolic trajectory, although the two observed paths would not be the same.

**Q4.28** (a) $g$ downward (b) $g$ downward

The horizontal component of the motion does not affect the vertical acceleration.

## SOLUTIONS TO PROBLEMS

### Section 4.1 **The Position, Velocity, and Acceleration Vectors**

**P4.1**

| $x(m)$ | $y(m)$ |
|---|---|
| 0 | –3 600 |
| –3 000 | 0 |
| –1 270 | 1 270 |
| –4 270 m | –2 330 m |

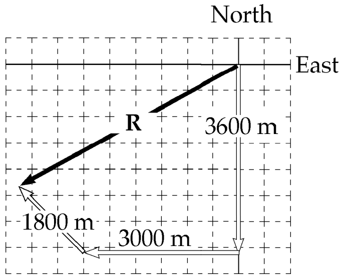


**FIG. P4.1**

(a) Net displacement $= \sqrt{x^2 + y^2}$

$$= \boxed{4.87 \text{ km at } 28.6° \text{ S of W}}$$

(b) $$\text{Average speed} = \frac{(20.0 \text{ m/s})(180 \text{ s}) + (25.0 \text{ m/s})(120 \text{ s}) + (30.0 \text{ m/s})(60.0 \text{ s})}{180 \text{ s} + 120 \text{ s} + 60.0 \text{ s}} = \boxed{23.3 \text{ m/s}}$$

(c) $$\text{Average velocity} = \frac{4.87 \times 10^3 \text{ m}}{360 \text{ s}} = \boxed{13.5 \text{ m/s along } \mathbf{R}}$$

**P4.2** (a) $\mathbf{r}=\boxed{18.0t\hat{\mathbf{i}}+\left(4.00t-4.90t^2\right)\hat{\mathbf{j}}}$

(b) $\mathbf{v}=\boxed{(18.0\ \text{m/s})\hat{\mathbf{i}}+\left[4.00\ \text{m/s}-\left(9.80\ \text{m/s}^2\right)t\right]\hat{\mathbf{j}}}$

(c) $\mathbf{a}=\boxed{\left(-9.80\ \text{m/s}^2\right)\hat{\mathbf{j}}}$

(d) $\mathbf{r}(3.00\ \text{s})=\boxed{(54.0\ \text{m})\hat{\mathbf{i}}-(32.1\ \text{m})\hat{\mathbf{j}}}$

(e) $\mathbf{v}(3.00\ \text{s})=\boxed{(18.0\ \text{m/s})\hat{\mathbf{i}}-(25.4\ \text{m/s})\hat{\mathbf{j}}}$

(f) $\mathbf{a}(3.00\ \text{s})=\boxed{\left(-9.80\ \text{m/s}^2\right)\hat{\mathbf{j}}}$

***P4.3** The sun projects onto the ground the $x$-component of her velocity:

$$5.00\ \text{m/s}\ \cos(-60.0°)=\boxed{2.50\ \text{m/s}}.$$

**P4.4** (a) From $x=-5.00\sin\omega t$, the $x$-component of velocity is

$$v_x=\frac{dx}{dt}=\left(\frac{d}{dt}\right)(-5.00\sin\omega t)=-5.00\omega\cos\omega t$$

and $a_x=\dfrac{dv_x}{dt}=+5.00\omega^2\sin\omega t$

similarly, $v_y=\left(\dfrac{d}{dt}\right)(4.00-5.00\cos\omega t)=0+5.00\omega\sin\omega t$

and $a_y=\left(\dfrac{d}{dt}\right)(5.00\omega\sin\omega t)=5.00\omega^2\cos\omega t$.

At $t=0$, $\mathbf{v}=-5.00\omega\cos 0\hat{\mathbf{i}}+5.00\omega\sin 0\hat{\mathbf{j}}=\boxed{\left(5.00\omega\hat{\mathbf{i}}+0\hat{\mathbf{j}}\right)\ \text{m/s}}$

and $\mathbf{a}=5.00\omega^2\sin 0\hat{\mathbf{i}}+5.00\omega^2\cos 0\hat{\mathbf{j}}=\boxed{\left(0\hat{\mathbf{i}}+5.00\omega^2\hat{\mathbf{j}}\right)\ \text{m/s}^2}$.

(b) $\mathbf{r}=x\hat{\mathbf{i}}+y\hat{\mathbf{j}}=\boxed{(4.00\ \text{m})\hat{\mathbf{j}}+(5.00\ \text{m})\left(-\sin\omega t\,\hat{\mathbf{i}}-\cos\omega t\,\hat{\mathbf{j}}\right)}$

$\mathbf{v}=\boxed{(5.00\ \text{m})\omega\left[-\cos\omega t\,\hat{\mathbf{i}}+\sin\omega t\,\hat{\mathbf{j}}\right]}$

$\mathbf{a}=\boxed{(5.00\ \text{m})\omega^2\left[\sin\omega t\,\hat{\mathbf{i}}+\cos\omega t\,\hat{\mathbf{j}}\right]}$

(c) The object moves in $\boxed{\text{a circle of radius 5.00 m centered at }(0,\ 4.00\text{ m})}$.

## Section 4.2 **Two-Dimensional Motion with Constant Acceleration**

**P4.5** (a) $\mathbf{v}_f=\mathbf{v}_i+\mathbf{a}t$

$$\mathbf{a}=\frac{\mathbf{v}_f-\mathbf{v}_i}{t}=\frac{\left(9.00\hat{\mathbf{i}}+7.00\hat{\mathbf{j}}\right)-\left(3.00\hat{\mathbf{i}}-2.00\hat{\mathbf{j}}\right)}{3.00}=\boxed{\left(2.00\hat{\mathbf{i}}+3.00\hat{\mathbf{j}}\right)\text{ m/s}^2}$$

(b) $\mathbf{r}_f=\mathbf{r}_i+\mathbf{v}_it+\frac{1}{2}\mathbf{a}t^2=\left(3.00\hat{\mathbf{i}}-2.00\hat{\mathbf{j}}\right)t+\frac{1}{2}\left(2.00\hat{\mathbf{i}}+3.00\hat{\mathbf{j}}\right)t^2$

$\boxed{x=\left(3.00t+t^2\right)\text{ m}}$ and $\boxed{y=\left(1.50t^2-2.00t\right)\text{ m}}$

**P4.6** (a) $\mathbf{v}=\frac{d\mathbf{r}}{dt}=\left(\frac{d}{dt}\right)\left(3.00\hat{\mathbf{i}}-6.00t^2\hat{\mathbf{j}}\right)=\boxed{-12.0t\,\hat{\mathbf{j}}\text{ m/s}}$

$\mathbf{a}=\frac{d\mathbf{v}}{dt}=\left(\frac{d}{dt}\right)\left(-12.0t\,\hat{\mathbf{j}}\right)=\boxed{-12.0\hat{\mathbf{j}}\text{ m/s}^2}$

(b) $\boxed{\mathbf{r}=\left(3.00\hat{\mathbf{i}}-6.00\hat{\mathbf{j}}\right)\text{ m; }\mathbf{v}=-12.0\hat{\mathbf{j}}\text{ m/s}}$

**P4.7** $\mathbf{v}_i=\left(4.00\hat{\mathbf{i}}+1.00\hat{\mathbf{j}}\right)$ m/s and $\mathbf{v}(20.0)=\left(20.0\hat{\mathbf{i}}-5.00\hat{\mathbf{j}}\right)$ m/s

(a) $a_x=\frac{\Delta v_x}{\Delta t}=\frac{20.0-4.00}{20.0}\text{ m/s}^2=\boxed{0.800\text{ m/s}^2}$

$a_y=\frac{\Delta v_y}{\Delta t}=\frac{-5.00-1.00}{20.0}\text{ m/s}^2=\boxed{-0.300\text{ m/s}^2}$

(b) $\theta=\tan^{-1}\left(\frac{-0.300}{0.800}\right)=-20.6°=\boxed{339°\text{ from }+x\text{ axis}}$

(c) At $t=25.0$ s

$$x_f=x_i+v_{xi}t+\frac{1}{2}a_xt^2=10.0+4.00(25.0)+\frac{1}{2}(0.800)(25.0)^2=\boxed{360\text{ m}}$$
$$y_f=y_i+v_{yi}t+\frac{1}{2}a_yt^2=-4.00+1.00(25.0)+\frac{1}{2}(-0.300)(25.0)^2=\boxed{-72.7\text{ m}}$$
$$v_{xf}=v_{xi}+a_xt=4+0.8(25)=24\text{ m/s}$$
$$v_{yf}=v_{yi}+a_yt=1-0.3(25)=-6.5\text{ m/s}$$
$$\theta=\tan^{-1}\left(\frac{v_y}{v_x}\right)=\tan^{-1}\left(\frac{-6.50}{24.0}\right)=\boxed{-15.2°}$$

**P4.8** $\mathbf{a}=3.00\hat{\mathbf{j}}\ \text{m/s}^2$ ; $\mathbf{v}_i=5.00\hat{\mathbf{i}}\ \text{m/s}$ ; $\mathbf{r}_i=0\hat{\mathbf{i}}+0\hat{\mathbf{j}}$

(a)
$$\mathbf{r}_f=\mathbf{r}_i+\mathbf{v}_i t+\frac{1}{2}\mathbf{a}t^2=\boxed{\left[5.00t\hat{\mathbf{i}}+\frac{1}{2}3.00t^2\hat{\mathbf{j}}\right]\ \text{m}}$$
$$\mathbf{v}_f=\mathbf{v}_i+\mathbf{a}t=\boxed{\left(5.00\hat{\mathbf{i}}+3.00t\hat{\mathbf{j}}\right)\ \text{m/s}}$$

(b)
$$t=2.00\ \text{s},\ \mathbf{r}_f=5.00(2.00)\,\hat{\mathbf{i}}+\frac{1}{2}(3.00)(2.00)^2\,\hat{\mathbf{j}}=\left(10.0\hat{\mathbf{i}}+6.00\hat{\mathbf{j}}\right)\ \text{m}$$
so $x_f=\boxed{10.0\ \text{m}}$, $y_f=\boxed{6.00\ \text{m}}$

$$\mathbf{v}_f=5.00\hat{\mathbf{i}}+3.00(2.00)\,\hat{\mathbf{j}}=\left(5.00\hat{\mathbf{i}}+6.00\hat{\mathbf{j}}\right)\ \text{m/s}$$
$$v_f=\left|\mathbf{v}_f\right|=\sqrt{v_{xf}^2+v_{yf}^2}=\sqrt{(5.00)^2+(6.00)^2}=\boxed{7.81\ \text{m/s}}$$

***P4.9** (a) For the $x$-component of the motion we have $x_f=x_i+v_{xi}t+\frac{1}{2}a_x t^2$.

$$0.01\ \text{m}=0+\left(1.80\times10^7\ \text{m/s}\right)t+\frac{1}{2}\left(8\times10^{14}\ \text{m/s}^2\right)t^2$$
$$\left(4\times10^{14}\ \text{m/s}^2\right)t^2+\left(1.80\times10^7\ \text{m/s}\right)t-10^{-2}\ \text{m}=0$$
$$t=\frac{-1.80\times10^7\ \text{m/s}\pm\sqrt{\left(1.8\times10^7\ \text{m/s}\right)^2-4\left(4\times10^{14}\ \text{m/s}^2\right)\left(-10^{-2}\text{m}\right)}}{2\left(4\times10^{14}\ \text{m/s}^2\right)}$$
$$=\frac{-1.8\times10^7\pm1.84\times10^7\ \text{m/s}}{8\times10^{14}\ \text{m/s}^2}$$

We choose the + sign to represent the physical situation

$$t=\frac{4.39\times10^5\ \text{m/s}}{8\times10^{14}\ \text{m/s}^2}=5.49\times10^{-10}\ \text{s}.$$

Here

$$y_f=y_i+v_{yi}t+\frac{1}{2}a_y t^2=0+0+\frac{1}{2}\left(1.6\times10^{15}\ \text{m/s}^2\right)\left(5.49\times10^{-10}\ \text{s}\right)^2=2.41\times10^{-4}\ \text{m}.$$

So, $\boxed{\mathbf{r}_f=\left(10.0\ \hat{\mathbf{i}}+0.241\ \hat{\mathbf{j}}\right)\ \text{mm}}$.

(b)
$$\mathbf{v}_f=\mathbf{v}_i+\mathbf{a}t=1.80\times10^7\ \text{m/s}\,\hat{\mathbf{i}}+\left(8\times10^{14}\ \text{m/s}^2\ \hat{\mathbf{i}}+1.6\times10^{15}\ \text{m/s}^2\ \hat{\mathbf{j}}\right)\left(5.49\times10^{-10}\ \text{s}\right)$$
$$=\left(1.80\times10^7\ \text{m/s}\right)\hat{\mathbf{i}}+\left(4.39\times10^5\ \text{m/s}\right)\hat{\mathbf{i}}+\left(8.78\times10^5\ \text{m/s}\right)\hat{\mathbf{j}}$$
$$=\boxed{\left(1.84\times10^7\ \text{m/s}\right)\hat{\mathbf{i}}+\left(8.78\times10^5\ \text{m/s}\right)\hat{\mathbf{j}}}$$

(c)
$$\left|\mathbf{v}_f\right|=\sqrt{\left(1.84\times10^7\ \text{m/s}\right)^2+\left(8.78\times10^5\ \text{m/s}\right)^2}=\boxed{1.85\times10^7\ \text{m/s}}$$

(d)
$$\theta=\tan^{-1}\left(\frac{v_y}{v_x}\right)=\tan^{-1}\left(\frac{8.78\times10^5}{1.84\times10^7}\right)=\boxed{2.73^\circ}$$

## Section 4.3 **Projectile Motion**

**P4.10**

$$x = v_{xi}t = v_i\cos\theta_i t$$

$$x = (300\ \text{m/s})(\cos 55.0°)(42.0\ \text{s})$$

$$x = \boxed{7.23\times 10^3\ \text{m}}$$

$$y = v_{yi}t - \frac{1}{2}gt^2 = v_i\sin\theta_i t - \frac{1}{2}gt^2$$

$$y = (300\ \text{m/s})(\sin 55.0°)(42.0\ \text{s}) - \frac{1}{2}\left(9.80\ \text{m/s}^2\right)(42.0\ \text{s})^2 = \boxed{1.68\times 10^3\ \text{m}}$$

**P4.11** (a) The mug leaves the counter horizontally with a velocity $v_{xi}$ (say). If time $t$ elapses before it hits the ground, then since there is no horizontal acceleration, $x_f = v_{xi}t$, i.e.,

$$t = \frac{x_f}{v_{xi}} = \frac{(1.40\ \text{m})}{v_{xi}}$$

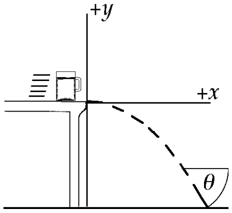


**FIG. P4.11**

In the same time it falls a distance of 0.860 m with acceleration downward of $9.80\ \text{m/s}^2$. Then

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 :\ 0 = 0.860\ \text{m} + \frac{1}{2}\left(-9.80\ \text{m/s}^2\right)\left(\frac{1.40\ \text{m}}{v_{xi}}\right)^2 .$$

Thus,

$$v_{xi} = \sqrt{\frac{\left(4.90\ \text{m/s}^2\right)\left(1.96\ \text{m}^2\right)}{0.860\ \text{m}}} = \boxed{3.34\ \text{m/s}} .$$

(b) The vertical velocity component with which it hits the floor is

$$v_{yf} = v_{yi} + a_y t = 0 + \left(-9.80\ \text{m/s}^2\right)\left(\frac{1.40\ \text{m}}{3.34\ \text{m/s}}\right) = -4.11\ \text{m/s}.$$

Hence, the angle $\theta$ at which the mug strikes the floor is given by

$$\theta = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right) = \tan^{-1}\left(\frac{-4.11}{3.34}\right) = \boxed{-50.9°} .$$

**P4.12** The mug is a projectile from just after leaving the counter until just before it reaches the floor. Taking the origin at the point where the mug leaves the bar, the coordinates of the mug at any time are

$$x_f = v_{xi}t + \frac{1}{2}a_x t^2 = v_{xi}t + 0 \text{ and } y_f = v_{yi}t + \frac{1}{2}a_y t^2 = 0 - \frac{1}{2}gt^2.$$

When the mug reaches the floor, $y_f = -h$ so

$$-h = -\frac{1}{2}gt^2$$

which gives the time of impact as

$$t = \sqrt{\frac{2h}{g}}.$$

(a) Since $x_f = d$ when the mug reaches the floor, $x_f = v_{xi}t$ becomes $d = v_{xi}\sqrt{\frac{2h}{g}}$ giving the initial velocity as

$$\boxed{v_{xi} = d\sqrt{\frac{g}{2h}}}.$$

(b) Just before impact, the $x$-component of velocity is still

$$v_{xf} = v_{xi}$$

while the $y$-component is

$$v_{yf} = v_{yi} + a_y t = 0 - g\sqrt{\frac{2h}{g}}.$$

Then the direction of motion just before impact is below the horizontal at an angle of

$$\theta = \tan^{-1}\left(\frac{|v_{yf}|}{v_{xf}}\right) = \tan^{-1}\left(\frac{g\sqrt{\frac{2h}{g}}}{d\sqrt{\frac{g}{2h}}}\right) = \boxed{\tan^{-1}\left(\frac{2h}{d}\right)}.$$

**P4.13** (a) The time of flight of the first snowball is the nonzero root of $y_f = y_i + v_{yi}t_1 + \frac{1}{2}a_y t_1^2$

$$0 = 0 + (25.0\ \text{m/s})(\sin 70.0°)t_1 - \frac{1}{2}(9.80\ \text{m/s}^2)t_1^2$$

$$t_1 = \frac{2(25.0\ \text{m/s})\sin 70.0°}{9.80\ \text{m/s}^2} = 4.79\ \text{s}.$$

The distance to your target is

$$x_f - x_i = v_{xi}t_1 = (25.0\ \text{m/s})\cos 70.0°(4.79\ \text{s}) = 41.0\ \text{m}.$$

Now the second snowball we describe by

$$y_f = y_i + v_{yi}t_2 + \frac{1}{2}a_y {t_2}^2$$

$$0 = (25.0\ \text{m/s})\sin\theta_2 t_2 - (4.90\ \text{m/s}^2)t_2^2$$

$$t_2 = (5.10\ \text{s})\sin\theta_2$$
$$x_f - x_i = v_{xi}\, t_2$$

$$41.0\ \text{m} = (25.0\ \text{m/s})\cos\theta_2(5.10\ \text{s})\sin\theta_2 = (128\ \text{m})\sin\theta_2\cos\theta_2$$

$$0.321 = \sin\theta_2\cos\theta_2$$

Using $\sin 2\theta = 2\sin\theta\cos\theta$ we can solve $0.321 = \frac{1}{2}\sin 2\theta_2$

$$2\theta_2 = \sin^{-1} 0.643 \text{ and } \theta_2 = \boxed{20.0°}.$$

(b) The second snowball is in the air for time $t_2 = (5.10\ \text{s})\sin\theta_2 = (5.10\ \text{s})\sin 20° = 1.75\ \text{s}$, so you throw it after the first by

$$t_1 - t_2 = 4.79\ \text{s} - 1.75\ \text{s} = \boxed{3.05\ \text{s}}.$$

**P4.14** From Equation 4.14 with $R = 15.0\ \text{m}$, $v_i = 3.00\ \text{m/s}$, $\theta_{\max} = 45.0°$

$$\therefore g = \frac{v_i^2}{R} = \frac{9.00}{15.0} = \boxed{0.600\ \text{m/s}^2}$$

**P4.15** $h=\dfrac{v_i^2\sin^2\theta_i}{2g}$; $R=\dfrac{v_i^2(\sin 2\theta_i)}{g}$; $3h=R$,

so $\dfrac{3v_i^2\sin^2\theta_i}{2g}=\dfrac{v_i^2(\sin 2\theta_i)}{g}$

or $\dfrac{2}{3}=\dfrac{\sin^2\theta_i}{\sin 2\theta_i}=\dfrac{\tan\theta_i}{2}$

thus $\theta_i=\tan^{-1}\left(\dfrac{4}{3}\right)=\boxed{53.1°}$.

***P4.16** (a) To identify the maximum height we let *i* be the launch point and *f* be the highest point:

$$\begin{aligned} v_{yf}^2 &= v_{yi}^2+2a_y(y_f-y_i)\\ 0 &= v_i^2\sin^2\theta_i+2(-g)(y_{\max}-0)\\ y_{\max} &= \frac{v_i^2\sin^2\theta_i}{2g}. \end{aligned}$$

To identify the range we let *i* be the launch and *f* be the impact point; where *t* is not zero:

$$\begin{aligned} y_f &= y_i+v_{yi}t+\frac{1}{2}a_yt^2\\ 0 &= 0+v_i\sin\theta_i t+\frac{1}{2}(-g)t^2\\ t &= \frac{2v_i\sin\theta_i}{g}\\ x_f &= x_i+v_{xi}t+\frac{1}{2}a_xt^2\\ d &= 0+v_i\cos\theta_i\frac{2v_i\sin\theta_i}{g}+0. \end{aligned}$$

For this rock, $d=y_{\max}$

$$\begin{aligned} \frac{v_i^2\sin^2\theta_i}{2g} &= \frac{2v_i^2\sin\theta_i\cos\theta_i}{g}\\ \frac{\sin\theta_i}{\cos\theta_i} &= \tan\theta_i=4\\ \theta_i &= \boxed{76.0°} \end{aligned}$$

(b) Since $g$ divides out, the answer is $\boxed{\text{the same}}$ on every planet.

(c) The maximum range is attained for $\theta_i=45°$:

$$\frac{d_{\max}}{d}=\frac{v_i\cos 45°2v_i\sin 45°g}{gv_i\cos 76°2v_i\sin 76°}=2.125\,.$$

So $d_{\max}=\boxed{\dfrac{17d}{8}}$.

**P4.17** (a) $x_f = v_{xi}\,t = 8.00\cos 20.0°(3.00) = \boxed{22.6\text{ m}}$

(b) Taking $y$ positive downwards,

$$y_f = v_{yi}t + \frac{1}{2}g\,t^2$$

$$y_f = 8.00\sin 20.0°(3.00) + \frac{1}{2}(9.80)(3.00)^2 = \boxed{52.3\text{ m}}.$$

(c) $10.0 = 8.00(\sin 20.0°)t + \frac{1}{2}(9.80)t^2$

$$4.90t^2 + 2.74t - 10.0 = 0$$

$$t = \frac{-2.74 \pm \sqrt{(2.74)^2 + 196}}{9.80} = \boxed{1.18\text{ s}}$$

***P4.18** We interpret the problem to mean that the displacement from fish to bug is

$$2.00\text{ m at }30° = (2.00\text{ m})\cos 30°\,\hat{\mathbf{i}} + (2.00\text{ m})\sin 30°\,\hat{\mathbf{j}} = (1.73\text{ m})\hat{\mathbf{i}} + (1.00\text{ m})\hat{\mathbf{j}}.$$

If the water should drop 0.03 m during its flight, then the fish must aim at a point 0.03 m above the bug. The initial velocity of the water then is directed through the point with displacement

$$(1.73\text{ m})\hat{\mathbf{i}} + (1.03\text{ m})\hat{\mathbf{j}} = 2.015\text{ m at }30.7°.$$

For the time of flight of a water drop we have

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$1.73\text{ m} = 0 + (v_i\cos 30.7°)t + 0$ so

$$t = \frac{1.73\text{ m}}{v_i\cos 30.7°}.$$

The vertical motion is described by

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2.$$

The "drop on its path" is

$$-3.00\text{ cm} = \frac{1}{2}\left(-9.80\text{ m/s}^2\right)\left(\frac{1.73\text{ m}}{v_i\cos 30.7°}\right)^2.$$

Thus,

$$v_i = \frac{1.73\text{ m}}{\cos 30.7°}\sqrt{\frac{9.80\text{ m/s}^2}{2\times 0.03\text{ m}}} = 2.015\text{ m}\left(12.8\text{ s}^{-1}\right) = \boxed{25.8\text{ m/s}}.$$

**P4.19** (a) We use the trajectory equation:

$$y_f = x_f \tan\theta_i - \frac{gx_f^2}{2v_i^2 \cos^2\theta_i}.$$

With

$$x_f = 36.0 \text{ m},\ v_i = 20.0 \text{ m/s, and } \theta = 53.0°$$

we find

$$y_f = (36.0 \text{ m})\tan 53.0° - \frac{\left(9.80 \text{ m/s}^2\right)(36.0 \text{ m})^2}{2(20.0 \text{ m/s})^2 \cos^2(53.0°)} = 3.94 \text{ m}.$$

The ball clears the bar by

$$(3.94 - 3.05) \text{ m} = \boxed{0.889 \text{ m}}.$$

(b) The time the ball takes to reach the maximum height is

$$t_1 = \frac{v_i \sin\theta_i}{g} = \frac{(20.0 \text{ m/s})(\sin 53.0°)}{9.80 \text{ m/s}^2} = 1.63 \text{ s}.$$

The time to travel 36.0 m horizontally is $t_2 = \frac{x_f}{v_{ix}}$

$$t_2 = \frac{36.0 \text{ m}}{(20.0 \text{ m/s})(\cos 53.0°)} = 2.99 \text{ s}.$$

Since $t_2 > t_1$ $\boxed{\text{the ball clears the goal on its way down}}$.

**P4.20** The horizontal component of displacement is $x_f = v_{xi}t = (v_i \cos\theta_i)t$. Therefore, the time required to reach the building a distance $d$ away is $t = \frac{d}{v_i \cos\theta_i}$. At this time, the altitude of the water is

$$y_f = v_{yi}t + \frac{1}{2}a_y t^2 = v_i \sin\theta_i \left(\frac{d}{v_i \cos\theta_i}\right) - \frac{g}{2}\left(\frac{d}{v_i \cos\theta_i}\right)^2.$$

Therefore the water strikes the building at a height $h$ above ground level of

$$h = y_f = \boxed{d \tan\theta_i - \frac{gd^2}{2v_i^2 \cos^2\theta_i}}.$$

***P4.21** (a) For the horizontal motion, we have

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$
$$24\text{ m} = 0 + v_i(\cos 53°)(2.2\text{ s}) + 0$$
$$v_i = \boxed{18.1\text{ m/s}}.$$

(b) As it passes over the wall, the ball is above the street by $y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$

$$y_f = 0 + (18.1\text{ m/s})(\sin 53°)(2.2\text{ s}) + \frac{1}{2}\left(-9.8\text{ m/s}^2\right)(2.2\text{ s})^2 = 8.13\text{ m}.$$

So it clears the parapet by $8.13\text{ m} - 7\text{ m} = \boxed{1.13\text{ m}}$.

(c) Note that the highest point of the ball's trajectory is not directly above the wall. For the whole flight, we have from the trajectory equation

$$y_f = (\tan\theta_i)x_f - \left(\frac{g}{2v_i^2\cos^2\theta_i}\right)x_f^2$$

or

$$6\text{ m} = (\tan 53°)x_f - \left(\frac{9.8\text{ m/s}^2}{2(18.1\text{ m/s})^2\cos^2 53°}\right)x_f^2.$$

Solving,

$$\left(0.041\,2\text{ m}^{-1}\right)x_f^2 - 1.33x_f + 6\text{ m} = 0$$

and

$$x_f = \frac{1.33 \pm \sqrt{1.33^2 - 4(0.0412)(6)}}{2\left(0.0412\text{ m}^{-1}\right)}.$$

This yields two results:

$$x_f = 26.8\text{ m or }5.44\text{ m}$$

The ball passes twice through the level of the roof.

It hits the roof at distance from the wall

$$26.8\text{ m} - 24\text{ m} = \boxed{2.79\text{ m}}.$$

***P4.22** When the bomb has fallen a vertical distance 2.15 km, it has traveled a horizontal distance $x_f$ given by

$$x_f=\sqrt{(3.25\text{ km})^2-(2.15\text{ km})^2}=2.437\text{ km}$$

$$y_f=x_f\tan\theta-\frac{gx_f^2}{2v_i^2\cos^2\theta_i}$$

$$-2\,150\text{ m}=(2\,437\text{ m})\tan\theta_i-\frac{(9.8\text{ m/s}^2)(2\,437\text{ m})^2}{2(280\text{ m/s})^2\cos^2\theta_i}$$

$$\therefore -2\,150\text{ m}=(2\,437\text{ m})\tan\theta_i-(371.19\text{ m})(1+\tan^2\theta_i)$$

$$\therefore \tan^2\theta-6.565\tan\theta_i-4.792=0$$

$$\therefore \tan\theta_i=\frac{1}{2}\left(6.565\pm\sqrt{(6.565)^2-4(1)(-4.792)}\right)=3.283\pm3.945.$$

Select the negative solution, since $\theta_i$ is below the horizontal.

$$\therefore \tan\theta_i=-0.662,\ \boxed{\theta_i=-33.5°}$$

**P4.23** The horizontal kick gives zero vertical velocity to the rock. Then its time of flight follows from

$$y_f=y_i+v_{yi}t+\frac{1}{2}a_yt^2$$

$$-40.0\text{ m}=0+0+\frac{1}{2}(-9.80\text{ m/s}^2)t^2$$

$$t=2.86\text{ s}.$$

The extra time $3.00\text{ s}-2.86\text{ s}=0.143\text{ s}$ is the time required for the sound she hears to travel straight back to the player.

It covers distance

$$(343\text{ m/s})0.143\text{ s}=49.0\text{ m}=\sqrt{x^2+(40.0\text{ m})^2}$$

where $x$ represents the horizontal distance the rock travels.

$$x=28.3\text{ m}=v_{xi}t+0t^2$$

$$\therefore v_{xi}=\frac{28.3\text{ m}}{2.86\text{ s}}=\boxed{9.91\text{ m/s}}$$

**P4.24** From the instant he leaves the floor until just before he lands, the basketball star is a projectile. His vertical velocity and vertical displacement are related by the equation $v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$. Applying this to the upward part of his flight gives $0 = v_{yi}^2 + 2(-9.80\ \text{m/s}^2)(1.85 - 1.02)\ \text{m}$. From this, $v_{yi} = 4.03\ \text{m/s}$. [Note that this is the answer to part (c) of this problem.]

For the downward part of the flight, the equation gives $v_{yf}^2 = 0 + 2(-9.80\ \text{m/s}^2)(0.900 - 1.85)\ \text{m}$. Thus the vertical velocity just before he lands is

$$v_{yf} = -4.32\ \text{m/s}.$$

(a) His hang time may then be found from $v_{yf} = v_{yi} + a_y t$:

$$-4.32\ \text{m/s} = 4.03\ \text{m/s} + (-9.80\ \text{m/s}^2)t$$

or $t = \boxed{0.852\ \text{s}}$.

(b) Looking at the total horizontal displacement during the leap, $x = v_{xi}t$ becomes

$$2.80\ \text{m} = v_{xi}(0.852\ \text{s})$$

which yields $v_{xi} = \boxed{3.29\ \text{m/s}}$.

(c) $v_{yi} = \boxed{4.03\ \text{m/s}}$. See above for proof.

(d) The takeoff angle is: $\theta = \tan^{-1}\left(\dfrac{v_{yi}}{v_{xi}}\right) = \tan^{-1}\left(\dfrac{4.03\ \text{m/s}}{3.29\ \text{m/s}}\right) = \boxed{50.8°}$.

(e) Similarly for the deer, the upward part of the flight gives $v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$:

$$0 = v_{yi}^2 + 2(-9.80\ \text{m/s}^2)(2.50 - 1.20)\ \text{m}$$

so $v_{yi} = 5.04\ \text{m/s}$.

For the downward part, $v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$ yields $v_{yf}^2 = 0 + 2(-9.80\ \text{m/s}^2)(0.700 - 2.50)\ \text{m}$ and $v_{yf} = -5.94\ \text{m/s}$.

The hang time is then found as $v_{yf} = v_{yi} + a_y t$: $-5.94\ \text{m/s} = 5.04\ \text{m/s} + (-9.80\ \text{m/s}^2)t$ and

$$\boxed{t = 1.12\ \text{s}}.$$

***P4.25** The arrow's flight time to the collision point is

$$t=\frac{x_f-x_i}{v_{xi}}=\frac{150\text{ m}}{(45\text{ m/s})\cos 50°}=5.19\text{ s}.$$

The arrow's altitude at the collision is

$$\begin{aligned}y_f&=y_i+v_{yi}t+\frac{1}{2}a_yt^2\\&=0+(45\text{ m/s})(\sin 50°)5.19\text{ s}+\frac{1}{2}\left(-9.8\text{ m/s}^2\right)(5.19\text{ s})^2=47.0\text{ m}.\end{aligned}$$

(a) The required launch speed for the apple is given by

$$\begin{aligned}v_{yf}^2&=v_{yi}^2+2a_y(y_f-y_i)\\0&=v_{yi}^2+2\left(-9.8\text{ m/s}^2\right)(47\text{ m}-0)\\v_{yi}&=\boxed{30.3\text{ m/s}}.\end{aligned}$$

(b) The time of flight of the apple is given by

$$\begin{aligned}v_{yf}&=v_{yi}+a_yt\\0&=30.3\text{ m/s}-9.8\text{ m/s}^2\,t\\t&=3.10\text{ s}.\end{aligned}$$

So the apple should be launched after the arrow by $5.19\text{ s}-3.10\text{ s}=\boxed{2.09\text{ s}}$.

***P4.26** For the smallest impact angle

$$\theta=\tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right),$$

we want to minimize $v_{yf}$ and maximize $v_{xf}=v_{xi}$. The final $y$-component of velocity is related to $v_{yi}$ by $v_{yf}^2=v_{yi}^2+2gh$, so we want to minimize $v_{yi}$ and maximize $v_{xi}$. Both are accomplished by making the initial velocity horizontal. Then $v_{xi}=v$, $v_{yi}=0$, and $v_{yf}=\sqrt{2gh}$. At last, the impact angle is

$$\theta=\tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right)=\boxed{\tan^{-1}\left(\frac{\sqrt{2gh}}{v}\right)}.$$

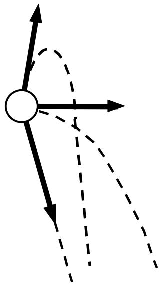


**FIG. P4.26**

## Section 4.4 Uniform Circular Motion

**P4.27** $a_c = \frac{v^2}{r} = \frac{(20.0\ \text{m/s})^2}{1.06\ \text{m}} = \boxed{377\ \text{m/s}^2}$

The mass is unnecessary information.

**P4.28** $a = \frac{v^2}{R},\ T = 24\ \text{h}(3\,600\ \text{s/h}) = 86\,400\ \text{s}$

$$v = \frac{2\pi R}{T} = \frac{2\pi(6.37\times10^6\ \text{m})}{86\,400\ \text{s}} = 463\ \text{m/s}$$

$$a = \frac{(463\ \text{m/s})^2}{6.37\times10^6\ \text{m}} = \boxed{0.033\,7\ \text{m/s}^2\ \text{directed toward the center of Earth}}.$$

**P4.29** $r = 0.500\ \text{m};$

$$v_t = \frac{2\pi r}{T} = \frac{2\pi(0.500\ \text{m})}{\frac{60.0\ \text{s}}{200\ \text{rev}}} = 10.47\ \text{m/s} = \boxed{10.5\ \text{m/s}}$$

$$a = \frac{v^2}{R} = \frac{(10.47)^2}{0.5} = \boxed{219\ \text{m/s}^2\ \text{inward}}$$

**P4.30** $a_c = \frac{v^2}{r}$

$$v = \sqrt{a_c r} = \sqrt{3(9.8\ \text{m/s}^2)(9.45\ \text{m})} = 16.7\ \text{m/s}$$

Each revolution carries the astronaut over a distance of $2\pi r = 2\pi(9.45\ \text{m}) = 59.4\ \text{m}$. Then the rotation rate is

$$16.7\ \text{m/s}\left(\frac{1\ \text{rev}}{59.4\ \text{m}}\right) = \boxed{0.281\ \text{rev/s}}.$$

**P4.31** (a) $v = r\omega$

At $8.00\ \text{rev/s}$, $v = (0.600\ \text{m})(8.00\ \text{rev/s})(2\pi\ \text{rad/rev}) = 30.2\ \text{m/s} = 9.60\pi\ \text{m/s}$.

At $6.00\ \text{rev/s}$, $v = (0.900\ \text{m})(6.00\ \text{rev/s})(2\pi\ \text{rad/rev}) = 33.9\ \text{m/s} = 10.8\pi\ \text{m/s}$.

$\boxed{6.00\ \text{rev/s}}$ gives the larger linear speed.

(b) $\text{Acceleration} = \frac{v^2}{r} = \frac{(9.60\pi\ \text{m/s})^2}{0.600\ \text{m}} = \boxed{1.52\times10^3\ \text{m/s}^2}$.

(c) At $6.00\ \text{rev/s}$, acceleration $= \frac{(10.8\pi\ \text{m/s})^2}{0.900\ \text{m}} = \boxed{1.28\times10^3\ \text{m/s}^2}$.

**P4.32** The satellite is in free fall. Its acceleration is due to gravity and is by effect a centripetal acceleration.

$$a_c = g$$

so

$$\frac{v^2}{r} = g\,.$$

Solving for the velocity, $v = \sqrt{rg} = \sqrt{(6{,}400 + 600)\left(10^3 \text{ m}\right)\left(8.21 \text{ m/s}^2\right)} = \boxed{7.58 \times 10^3 \text{ m/s}}$

$$v = \frac{2\pi r}{T}$$

and

$$T = \frac{2\pi r}{v} = \frac{2\pi\left(7{,}000 \times 10^3 \text{ m}\right)}{7.58 \times 10^3 \text{ m/s}} = \boxed{5.80 \times 10^3 \text{ s}}$$

$$T = 5.80 \times 10^3 \text{ s}\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 96.7 \text{ min}.$$

## Section 4.5 **Tangential and Radial Acceleration**

**P4.33** We assume the train is still slowing down at the instant in question.

$$a_c = \frac{v^2}{r} = 1.29 \text{ m/s}^2$$

$$a_t = \frac{\Delta v}{\Delta t} = \frac{(-40.0 \text{ km/h})\left(10^3 \text{ m/km}\right)\left(\frac{1 \text{ h}}{3\,600 \text{ s}}\right)}{15.0 \text{ s}} = -0.741 \text{ m/s}^2$$

$$a = \sqrt{a_c^2 + a_t^2} = \sqrt{\left(1.29 \text{ m/s}^2\right)^2 + \left(-0.741 \text{ m/s}^2\right)^2}$$

at an angle of $\tan^{-1}\left(\frac{|a_t|}{a_c}\right) = \tan^{-1}\left(\frac{0.741}{1.29}\right)$

$$\mathbf{a} = \boxed{1.48 \text{ m/s}^2 \text{ inward and } 29.9^\circ \text{ backward}}$$

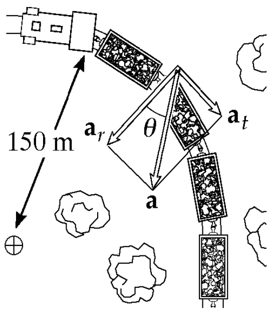


**FIG. P4.33**

**P4.34** (a) $a_t = \boxed{0.600 \text{ m/s}^2}$

(b) $a_r = \frac{v^2}{r} = \frac{(4.00 \text{ m/s})^2}{20.0 \text{ m}} = \boxed{0.800 \text{ m/s}^2}$

(c) $a = \sqrt{a_t^2 + a_r^2} = \boxed{1.00 \text{ m/s}^2}$

$\theta = \tan^{-1}\frac{a_r}{a_t} = \boxed{53.1^\circ \text{ inward from path}}$

**P4.35** $r = 2.50\text{ m},\ a = 15.0\ \text{m/s}^2$

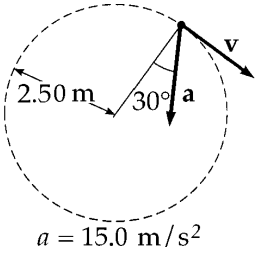


FIG. P4.35

(a) $a_c = a\cos 30.0° = \left(15.0\ \text{m/s}^2\right)(\cos 30°) = \boxed{13.0\ \text{m/s}^2}$

(b) $a_c = \dfrac{v^2}{r}$

so $v^2 = ra_c = 2.50\ \text{m}\left(13.0\ \text{m/s}^2\right) = 32.5\ \text{m}^2/\text{s}^2$

$v = \sqrt{32.5}\ \text{m/s} = \boxed{5.70\ \text{m/s}}$

(c) $a^2 = a_t^2 + a_r^2$

so $a_t = \sqrt{a^2 - a_r^2} = \sqrt{\left(15.0\ \text{m/s}^2\right)^2 - \left(13.0\,\text{m/s}^2\right)} = \boxed{7.50\ \text{m/s}^2}$

**P4.36** (a) See figure to the right.

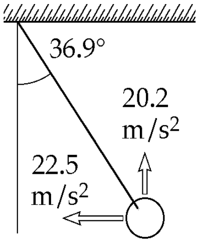


FIG. P4.36

(b) The components of the 20.2 and the $22.5\ \text{m/s}^2$ along the rope together constitute the centripetal acceleration:

$$a_c = \left(22.5\ \text{m/s}^2\right)\cos(90.0°\text{–}36.9°) + \left(20.2\ \text{m/s}^2\right)\cos 36.9° = \boxed{29.7\ \text{m/s}^2}$$

(c) $a_c = \dfrac{v^2}{r}$ so $v = \sqrt{a_c r} = \sqrt{29.7\ \text{m/s}^2\,(1.50\ \text{m})} = 6.67\ \text{m/s}$ tangent to circle

$\mathbf{v} = \boxed{6.67\ \text{m/s at } 36.9° \text{ above the horizontal}}$

***P4.37** Let *i* be the starting point and *f* be one revolution later. The curvilinear motion with constant tangential acceleration is described by

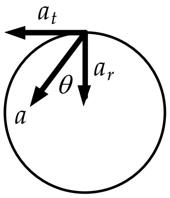


FIG. P4.37

$$\Delta x = v_{xi}t + \frac{1}{2}a_x t^2$$

$$2\pi r = 0 + \frac{1}{2}a_t t^2$$

$$a_t = \frac{4\pi r}{t^2}$$

and $v_{xf} = v_{xi} + a_x t$, $v_f = 0 + a_t t = \dfrac{4\pi r}{t}$. The magnitude of the radial acceleration is $a_r = \dfrac{v_f^2}{r} = \dfrac{16\pi^2 r^2}{t^2 r}$.

Then $\tan\theta = \dfrac{a_t}{a_r} = \dfrac{4\pi r t^2}{t^2 16\pi^2 r} = \dfrac{1}{4\pi}$ $\qquad \theta = \boxed{4.55°}$.

## Section 4.6 **Relative Velocity and Relative Acceleration**

**P4.38** (a)

$$\mathbf{v}_{\mathrm{H}} = 0 + \mathbf{a}_{\mathrm{H}}t = \left(3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}\right) \text{ m/s}^2 (5.00 \text{ s})$$

$$\mathbf{v}_{\mathrm{H}} = \left(15.0\hat{\mathbf{i}} - 10.0\hat{\mathbf{j}}\right) \text{ m/s}$$

$$\mathbf{v}_{\mathrm{J}} = 0 + \mathbf{a}_{j}t = \left(1.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}}\right) \text{ m/s}^2 (5.00 \text{ s})$$

$$\mathbf{v}_{\mathrm{J}} = \left(5.00\hat{\mathbf{i}} + 15.0\hat{\mathbf{j}}\right) \text{ m/s}$$

$$\mathbf{v}_{\mathrm{HJ}} = v_{\mathrm{H}} - v_{\mathrm{J}} = \left(15.0\hat{\mathbf{i}} - 10.0\hat{\mathbf{j}} - 5.00\hat{\mathbf{i}} - 15.0\hat{\mathbf{j}}\right) \text{ m/s}$$

$$\mathbf{v}_{\mathrm{HJ}} = \left(10.0\hat{\mathbf{i}} - 25.0\hat{\mathbf{j}}\right) \text{ m/s}$$

$$\left|\mathbf{v}_{\mathrm{HJ}}\right| = \sqrt{(10.0)^2 + (25.0)^2} \text{ m/s} = \boxed{26.9 \text{ m/s}}$$

(b)

$$\mathbf{r}_{\mathrm{H}} = 0 + 0 + \frac{1}{2}\mathbf{a}_{\mathrm{H}}t^2 = \frac{1}{2}\left(3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}\right) \text{ m/s}^2 (5.00 \text{ s})^2$$

$$\mathbf{r}_{\mathrm{H}} = \left(37.5\hat{\mathbf{i}} - 25.0\hat{\mathbf{j}}\right) \text{ m}$$

$$\mathbf{r}_{\mathrm{J}} = \frac{1}{2}\left(1.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}}\right) \text{ m/s}^2 (5.00\text{s})^2 = \left(12.5\hat{\mathbf{i}} + 37.5\hat{\mathbf{j}}\right) \text{ m}$$

$$\mathbf{r}_{\mathrm{HJ}} = \mathbf{r}_{\mathrm{H}} - \mathbf{r}_{\mathrm{J}} = \left(37.5\hat{\mathbf{i}} - 25.0\hat{\mathbf{j}} - 12.5\hat{\mathbf{i}} - 37.5\hat{\mathbf{j}}\right) \text{ m}$$

$$\mathbf{r}_{\mathrm{HJ}} = \left(25.0\hat{\mathbf{i}} - 62.5\hat{\mathbf{j}}\right) \text{ m}$$

$$\left|\mathbf{r}_{\mathrm{HJ}}\right| = \sqrt{(25.0)^2 + (62.5)^2} \text{ m} = \boxed{67.3 \text{ m}}$$

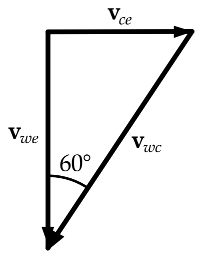
(c)

$$\mathbf{a}_{\mathrm{HJ}} = \mathbf{a}_{\mathrm{H}} - \mathbf{a}_{\mathrm{J}} = \left(3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}} - 1.00\hat{\mathbf{i}} - 3.00\hat{\mathbf{j}}\right) \text{ m/s}^2$$

$$\mathbf{a}_{\mathrm{HJ}} = \boxed{\left(2.00\hat{\mathbf{i}} - 5.00\hat{\mathbf{j}}\right) \text{ m/s}^2}$$

***P4.39** $\mathbf{v}_{ce} =$ the velocity of the car relative to the earth.
$\mathbf{v}_{wc} =$ the velocity of the water relative to the car.
$\mathbf{v}_{we} =$ the velocity of the water relative to the earth.

These velocities are related as shown in the diagram at the right.



$$\mathbf{v}_{we} = \mathbf{v}_{ce} + \mathbf{v}_{wc}$$

FIG. P4.39

(a) Since $\mathbf{v}_{we}$ is vertical, $v_{wc}\sin 60.0° = v_{ce} = 50.0$ km/h or
$\mathbf{v}_{wc} = \boxed{57.7 \text{ km/h at } 60.0° \text{ west of vertical}}$.

(b) Since $\mathbf{v}_{ce}$ has zero vertical component,

$$v_{we} = v_{wc}\cos 60.0° = (57.7 \text{ km/h})\cos 60.0° = \boxed{28.9 \text{ km/h downward}}.$$

**P4.40** The bumpers are initially $100\text{ m} = 0.100\text{ km}$ apart. After time $t$ the bumper of the leading car travels $40.0\,t$, while the bumper of the chasing car travels $60.0t$.

Since the cars are side by side at time $t$, we have

$$0.100 + 40.0t = 60.0t\,,$$

yielding

$$t = 5.00 \times 10^{-3}\text{ h} = \boxed{18.0\text{ s}}\,.$$

**P4.41** Total time in still water $t = \frac{d}{v} = \frac{2\,000}{1.20} = \boxed{1.67 \times 10^{3}\text{ s}}$.

Total time = time upstream plus time downstream:

$$t_{\text{up}} = \frac{1\,000}{(1.20 - 0.500)} = 1.43 \times 10^{3}\text{ s}$$

$$t_{\text{down}} = \frac{1\,000}{1.20 + 0.500} = 588\text{ s}\,.$$

Therefore, $t_{\text{total}} = 1.43 \times 10^{3} + 588 = \boxed{2.02 \times 10^{3}\text{ s}}$.

**P4.42** $v = \sqrt{150^2 + 30.0^2} = \boxed{153\text{ km/h}}$

$\theta = \tan^{-1}\left(\frac{30.0}{150}\right) = \boxed{11.3°\text{ north of west}}$

**P4.43** For Alan, his speed downstream is $c + v$, while his speed upstream is $c - v$.

Therefore, the total time for Alan is

$$t_1 = \frac{L}{c+v} + \frac{L}{c-v} = \boxed{\frac{\frac{2L}{c}}{1 - \frac{v^2}{c^2}}}\,.$$

For Beth, her cross-stream speed (both ways) is

$$\sqrt{c^2 - v^2}\,.$$

Thus, the total time for Beth is $t_2 = \frac{2L}{\sqrt{c^2 - v^2}} = \boxed{\frac{\frac{2L}{c}}{\sqrt{1 - \frac{v^2}{c^2}}}}$.

Since $1 - \frac{v^2}{c^2} < 1$, $t_1 > t_2$, or Beth, who swims cross-stream, returns first.

**P4.44** (a) To an observer at rest in the train car, the bolt accelerates downward and toward the rear of the train.

$$a=\sqrt{(2.50\ \text{m/s})^2+(9.80\ \text{m/s})^2}=\boxed{10.1\ \text{m/s}^2}$$

$$\tan\theta=\frac{2.50\ \text{m/s}^2}{9.80\ \text{m/s}^2}=0.255$$

$$\theta=\boxed{14.3°\ \text{to the south from the vertical}}$$

(b) $a=\boxed{9.80\ \text{m/s}^2\ \text{vertically downward}}$

**P4.45** Identify the student as the $S'$ observer and the professor as the $S$ observer. For the initial motion in $S'$, we have

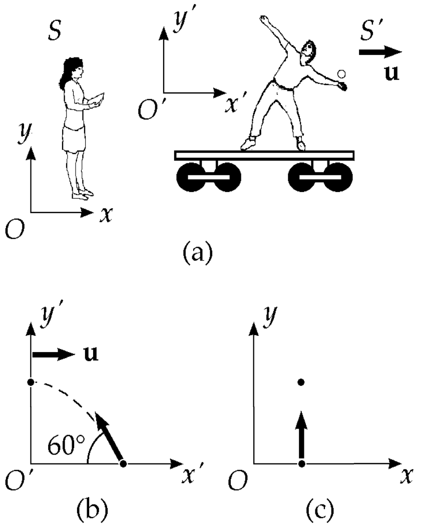
$$\frac{v'_y}{v'_x}=\tan 60.0°=\sqrt{3}.$$

Let $u$ represent the speed of $S'$ relative to $S$. Then because there is no $x$-motion in $S$, we can write $v_x=v'_x+u=0$ so that $v'_x=-u=-10.0\ \text{m/s}$. Hence the ball is thrown backwards in $S'$. Then,

$$v_y=v'_y=\sqrt{3}\left|v'_x\right|=10.0\sqrt{3}\ \text{m/s}.$$

Using $v_y^2=2gh$ we find

$$h=\frac{\left(10.0\sqrt{3}\ \text{m/s}\right)^2}{2\left(9.80\ \text{m/s}^2\right)}=\boxed{15.3\ \text{m}}.$$



(a)

(b) (c)

**FIG. P4.45**

The motion of the ball as seen by the student in $S'$ is shown in diagram (b). The view of the professor in $S$ is shown in diagram (c).

***P4.46** Choose the $x$-axis along the 20-km distance. The $y$-components of the displacements of the ship and the speedboat must agree:

$$(26\ \text{km/h})t\sin(40°-15°)=(50\ \text{km/h})t\sin\alpha$$

$$\alpha=\sin^{-1}\frac{11.0}{50}=12.7°.$$

The speedboat should head

$$15°+12.7°=\boxed{27.7°\ \text{east of north}}.$$

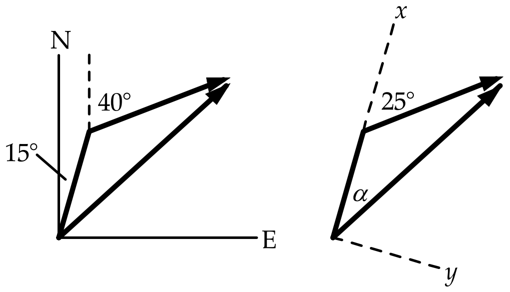


**FIG. P4.46**

## Additional Problems

***P4.47** (a) The speed at the top is $v_x = v_i \cos\theta_i = (143\ \text{m/s})\cos 45° = \boxed{101\ \text{m/s}}$.

(b) In free fall the plane reaches altitude given by

$$\begin{aligned} v_{yf}^2 &= v_{yi}^2 + 2a_y\left(y_f - y_i\right) \\ 0 &= \left(143\ \text{m/s}\sin 45°\right)^2 + 2\left(-9.8\ \text{m/s}^2\right)\left(y_f - 31\,000\ \text{ft}\right) \\ y_f &= 31\,000\ \text{ft} + 522\ \text{m}\left(\frac{3.28\ \text{ft}}{1\ \text{m}}\right) = \boxed{3.27\times10^3\ \text{ft}}. \end{aligned}$$

(c) For the whole free fall motion $v_{yf} = v_{yi} + a_y t$

$$\begin{aligned} -101\ \text{m/s} &= +101\ \text{m/s} - \left(9.8\ \text{m/s}^2\right)t \\ t &= \boxed{20.6\ \text{s}} \end{aligned}$$

(d) $a_c = \dfrac{v^2}{r}$

$$v = \sqrt{a_c r} = \sqrt{0.8\left(9.8\ \text{m/s}^2\right)4{,}130\ \text{m}} = \boxed{180\ \text{m/s}}$$

**P4.48** At any time $t$, the two drops have identical $y$-coordinates. The distance between the two drops is then just twice the magnitude of the horizontal displacement either drop has undergone. Therefore,

$$d = 2\left|x(t)\right| = 2\left(v_{xi}t\right) = 2\left(v_i\cos\theta_i\right)t = \boxed{2v_i t\cos\theta_i}.$$

**P4.49** After the string breaks the ball is a projectile, and reaches the ground at time $t$: $y_f = v_{yi}t + \frac{1}{2}a_y t^2$

$$-1.20\ \text{m} = 0 + \frac{1}{2}\left(-9.80\ \text{m/s}^2\right)t^2$$

so $t = 0.495\ \text{s}$.

Its constant horizontal speed is $v_x = \dfrac{x}{t} = \dfrac{2.00\ \text{m}}{0.495\ \text{s}} = 4.04\ \text{m/s}$

so before the string breaks $a_c = \dfrac{v_x^2}{r} = \dfrac{(4.04\ \text{m/s})^2}{0.300\ \text{m}} = \boxed{54.4\ \text{m/s}^2}$.

**P4.50** (a) $y_f = (\tan\theta_i)(x_f) - \dfrac{g}{2v_i^2\cos^2\theta_i}x_f^2$

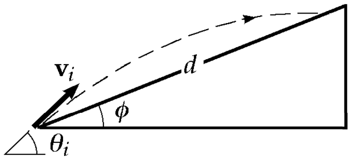


**FIG. P4.50**

Setting $x_f = d\cos\phi$, and $y_f = d\sin\phi$, we have

$$d\sin\phi = (\tan\theta_i)(d\cos\phi) - \frac{g}{2v_i^2\cos^2\theta_i}(d\cos\phi)^2.$$

Solving for $d$ yields, $d = \dfrac{2v_i^2\cos\theta_i[\sin\theta_i\cos\phi - \sin\phi\cos\theta_i]}{g\cos^2\phi}$

or $d = \boxed{\dfrac{2v_i^2\cos\theta_i\sin(\theta_i - \phi)}{g\cos^2\phi}}$.

(b) Setting $\dfrac{dd}{d\theta_i} = 0$ leads to $\boxed{\theta_i = 45° + \dfrac{\phi}{2}}$ and $\boxed{d_{\max} = \dfrac{v_i^2(1-\sin\phi)}{g\cos^2\phi}}$.

**P4.51** Refer to the sketch:

(b) $\Delta x = v_{xi}t$; substitution yields $130 = (v_i\cos 35.0°)t$.

$\Delta y = v_{yi}t + \frac{1}{2}at^2$; substitution yields

$$20.0 = (v_i\sin 35.0°)t + \frac{1}{2}(-9.80)t^2.$$

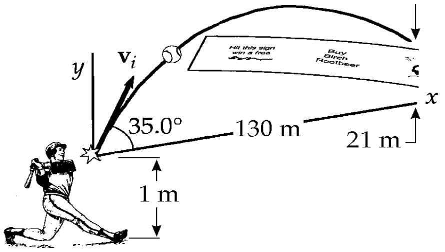


**FIG. P4.51**

Solving the above gives $t = \boxed{3.81 \text{ s}}$.

(a) $v_i = \boxed{41.7 \text{ m/s}}$

(c) $v_{yf} = v_i\sin\theta_i - gt$, $v_x = v_i\cos\theta_i$

At $t = 3.81$ s, $v_{yf} = 41.7\sin 35.0° - (9.80)(3.81) = \boxed{-13.4 \text{ m/s}}$

$$v_x = (41.7\cos 35.0°) = \boxed{34.1 \text{ m/s}}$$

$$v_f = \sqrt{v_x^2 + v_{yf}^2} = \boxed{36.7 \text{ m/s}}.$$

**P4.52** (a) The moon's gravitational acceleration is the probe's centripetal acceleration: (For the moon's radius, see end papers of text.)

$$a = \frac{v^2}{r}$$

$$\frac{1}{6}\left(9.80\ \text{m/s}^2\right) = \frac{v^2}{1.74 \times 10^6\ \text{m}}$$

$$v = \sqrt{2.84 \times 10^6\ \text{m}^2/\text{s}^2} = \boxed{1.69\ \text{km/s}}$$

(b)

$$v = \frac{2\pi r}{T}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi(1.74 \times 10^6\ \text{m})}{1.69 \times 10^3\ \text{m/s}} = 6.47 \times 10^3\ \text{s} = \boxed{1.80\ \text{h}}$$

**P4.53** (a)

$$a_c = \frac{v^2}{r} = \frac{(5.00\ \text{m/s})^2}{1.00\ \text{m}} = \boxed{25.0\ \text{m/s}^2}$$

$$a_t = g = \boxed{9.80\ \text{m/s}^2}$$

(b) See figure to the right.

(c)

$$a = \sqrt{a_c^2 + a_t^2} = \sqrt{\left(25.0\ \text{m/s}^2\right)^2 + \left(9.80\ \text{m/s}^2\right)^2} = \boxed{26.8\ \text{m/s}^2}$$

$$\phi = \tan^{-1}\left(\frac{a_t}{a_c}\right) = \tan^{-1}\frac{9.80\ \text{m/s}^2}{25.0\ \text{m/s}^2} = \boxed{21.4°}$$

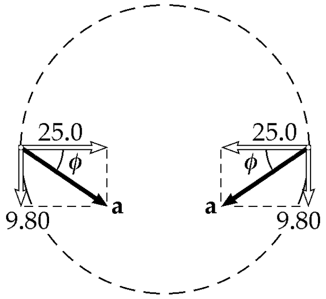


**FIG. P4.53**

**P4.54** $x_f = v_{ix}t = v_i t\cos 40.0°$

Thus, when $x_f = 10.0\ \text{m}$, $t = \dfrac{10.0\ \text{m}}{v_i \cos 40.0°}$.

At this time, $y_f$ should be $3.05\ \text{m} - 2.00\ \text{m} = 1.05\ \text{m}$.

Thus, $1.05\ \text{m} = \dfrac{(v_i \sin 40.0°)10.0\ \text{m}}{v_i \cos 40.0°} + \dfrac{1}{2}\left(-9.80\ \text{m/s}^2\right)\left[\dfrac{10.0\ \text{m}}{v_i \cos 40.0°}\right]^2$.

From this, $v_i = \boxed{10.7\ \text{m/s}}$.

**P4.55** The special conditions allowing use of the horizontal range equation applies.

For the ball thrown at 45°,

$$D=R_{45}=\frac{v_i^2\sin 90}{g}.$$

For the bouncing ball,

$$D=R_1+R_2=\frac{v_i^2\sin 2\theta}{g}+\frac{\left(\frac{v_i}{2}\right)^2\sin 2\theta}{g}$$

where $\theta$ is the angle it makes with the ground when thrown and when bouncing.

(a) We require:

$$\frac{v_i^2}{g}=\frac{v_i^2\sin 2\theta}{g}+\frac{v_i^2\sin 2\theta}{4g}$$

$$\sin 2\theta=\frac{4}{5}$$

$$\boxed{\theta=26.6°}$$

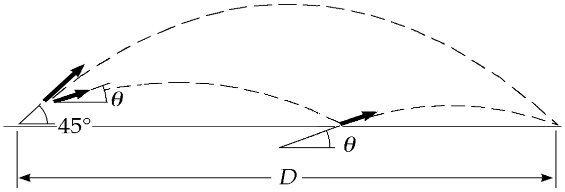


**FIG. P4.55**

(b) The time for any symmetric parabolic flight is given by

$$y_f=v_{yi}t-\frac{1}{2}gt^2$$

$$0=v_i\sin\theta_i t-\frac{1}{2}gt^2.$$

If $t=0$ is the time the ball is thrown, then $t=\frac{2v_i\sin\theta_i}{g}$ is the time at landing.

So for the ball thrown at 45.0°

$$t_{45}=\frac{2v_i\sin 45.0°}{g}.$$

For the bouncing ball,

$$t=t_1+t_2=\frac{2v_i\sin 26.6°}{g}+\frac{2\left(\frac{v_i}{2}\right)\sin 26.6°}{g}=\frac{3v_i\sin 26.6°}{g}.$$

The ratio of this time to that for no bounce is

$$\frac{\frac{3v_i\sin 26.6°}{g}}{\frac{2v_i\sin 45.0°}{g}}=\frac{1.34}{1.41}=\boxed{0.949}.$$

**P4.56** Using the range equation (Equation 4.14)

$$R = \frac{v_i^2 \sin(2\theta_i)}{g}$$

the maximum range occurs when $\theta_i = 45°$, and has a value $R = \frac{v_i^2}{g}$. Given $R$, this yields $v_i = \sqrt{gR}$.

If the boy uses the same speed to throw the ball vertically upward, then

$$v_y = \sqrt{gR} - gt \text{ and } y = \sqrt{gR}\,t - \frac{gt^2}{2}$$

at any time, $t$.

At the maximum height, $v_y = 0$, giving $t = \sqrt{\frac{R}{g}}$, and so the maximum height reached is

$$y_{\max} = \sqrt{gR}\sqrt{\frac{R}{g}} - \frac{g}{2}\left(\sqrt{\frac{R}{g}}\right)^2 = R - \frac{R}{2} = \boxed{\frac{R}{2}}.$$

**P4.57** Choose upward as the positive $y$-direction and leftward as the positive $x$-direction. The vertical height of the stone when released from $A$ or $B$ is

$$y_i = (1.50 + 1.20\sin 30.0°)\text{ m} = 2.10\text{ m}$$

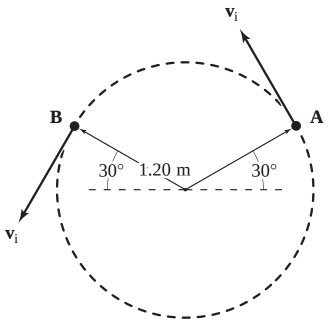


**FIG. P4.57**

(a) The equations of motion after release at $A$ are

$$\begin{aligned} v_y &= v_i \sin 60.0° - gt = (1.30 - 9.80t)\text{ m/s} \\ v_x &= v_i \cos 60.0° = 0.750\text{ m/s} \\ y &= \left(2.10 + 1.30t - 4.90t^2\right)\text{ m} \\ \Delta x_A &= (0.750t)\text{ m} \end{aligned}$$

When $y = 0$, $t = \frac{-1.30 \pm \sqrt{(1.30)^2 + 41.2}}{-9.80} = 0.800\text{ s}$. Then, $\Delta x_A = (0.750)(0.800)\text{ m} = \boxed{0.600\text{ m}}$.

(b) The equations of motion after release at point $B$ are

$$\begin{aligned} v_y &= v_i(-\sin 60.0°) - gt = (-1.30 - 9.80t)\text{ m/s} \\ v_x &= v_i \cos 60.0 = 0.750\text{ m/s} \\ y_i &= \left(2.10 - 1.30t - 4.90t^2\right)\text{ m}. \end{aligned}$$

When $y = 0$, $t = \frac{+1.30 \pm \sqrt{(-1.30)^2 + 41.2}}{-9.80} = 0.536\text{ s}$. Then, $\Delta x_B = (0.750)(0.536)\text{ m} = \boxed{0.402\text{ m}}$.

(c) $a_r = \frac{v^2}{r} = \frac{(1.50\text{ m/s})^2}{1.20\text{ m}} = \boxed{1.87\text{ m/s}^2\text{ toward the center}}$

(d) After release, $\mathbf{a} = -g\,\hat{\mathbf{j}} = \boxed{9.80\text{ m/s}^2\text{ downward}}$

**P4.58** The football travels a horizontal distance

$$R = \frac{v_i^2 \sin(2\theta_i)}{g} = \frac{(20.0)^2 \sin(60.0°)}{9.80} = 35.3 \text{ m}.$$

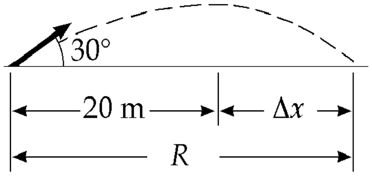


**FIG. P4.58**

Time of flight of ball is

$$t = \frac{2v_i \sin\theta_i}{g} = \frac{2(20.0)\sin 30.0°}{9.80} = 2.04 \text{ s}.$$

The receiver is $\Delta x$ away from where the ball lands and $\Delta x = 35.3 - 20.0 = 15.3$ m. To cover this distance in 2.04 s, he travels with a velocity

$$v = \frac{15.3}{2.04} = \boxed{7.50 \text{ m/s in the direction the ball was thrown}}.$$

**P4.59** (a) $\Delta y = -\frac{1}{2}gt^2$; $\Delta x = v_i t$

Combine the equations eliminating *t*:

$$\Delta y = -\frac{1}{2}g\left(\frac{\Delta x}{v_i}\right)^2.$$

From this, $(\Delta x)^2 = \left(\frac{-2\Delta y}{g}\right)v_i^2$

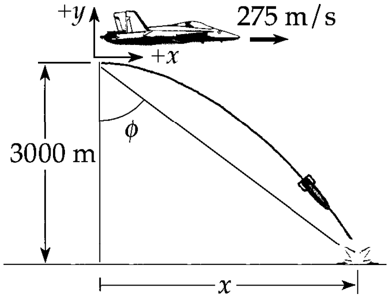


**FIG. P4.59**

thus $\Delta x = v_i\sqrt{\frac{-2\Delta y}{g}} = 275\sqrt{\frac{-2(-300)}{9.80}} = 6.80 \times 10^3 = \boxed{6.80 \text{ km}}$.

(b) The plane has the same velocity as the bomb in the *x* direction. Therefore, the plane will be $\boxed{\text{3 000 m directly above the bomb}}$ when it hits the ground.

(c) When $\phi$ is measured from the vertical, $\tan\phi = \frac{\Delta x}{\Delta y}$

therefore, $\phi = \tan^{-1}\left(\frac{\Delta x}{\Delta y}\right) = \tan^{-1}\left(\frac{6\,800}{3\,000}\right) = \boxed{66.2°}$.

***P4.60** (a) We use the approximation mentioned in the problem. The time to travel 200 m horizontally is $t=\frac{\Delta x}{v_x}=\frac{200\text{ m}}{1{,}000\text{ m/s}}=0.200\text{ s}$. The bullet falls by

$$\Delta y=v_{yi}t+\frac{1}{2}a_y t^2=0+\frac{1}{2}\left(-9.8\text{ m/s}^2\right)(0.2\text{ s})^2=\boxed{-0.196\text{ m}}.$$

(b) The telescope axis must point below the barrel axis by $\theta=\tan^{-1}\frac{0.196\text{ m}}{200\text{ m}}=\boxed{0.0561°}$.

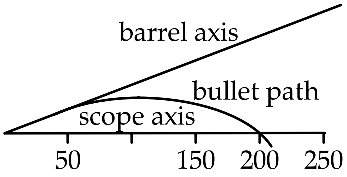


FIG. P4.60(b)

(c) $t=\frac{50.0\text{ m}}{1\,000\text{ m/s}}=0.050\,0\text{ s}$. The bullet falls by only

$$\Delta y=\frac{1}{2}\left(-9.8\text{ m/s}^2\right)(0.05\text{ s})^2=-0.0122\text{ m}.$$

At range $50\text{ m}=\frac{1}{4}(200\text{ m})$, the scope axis points to a location $\frac{1}{4}(19.6\text{ cm})=4.90\text{ cm}$ above the barrel axis, so the sharpshooter must $\boxed{\text{aim low}}$ by $4.90\text{ cm}-1.22\text{ cm}=\boxed{3.68\text{ cm}}$.

(d) $t=\frac{150\text{ m}}{1\,000\text{ m/s}}=0.150\text{ s}$

$\Delta y=\frac{1}{2}\left(-9.8\text{ m/s}^2\right)(0.15\text{ s})^2=0.110\text{ m}$

$\boxed{\text{Aim low}}$ by $\frac{150}{200}(19.6\text{ cm})-11.0\text{ cm}=\boxed{3.68\text{ cm}}$.

(e) $t=\frac{250\text{ m}}{1\,000\text{ m/s}}=0.250\text{ s}$

$\Delta y=\frac{1}{2}\left(-9.8\text{ m/s}^2\right)(0.25\text{ s})^2=0.306\text{ m}$

$\boxed{\text{Aim high}}$ by $30.6\text{ cm}-\frac{250}{200}(19.6\text{ cm})=\boxed{6.12\text{ cm}}$.

(f), (g) Many marksmen have a hard time believing it, but they should aim low in both cases. As in case (a) above, the time of flight is very nearly 0.200 s and the bullet falls below the barrel axis by 19.6 cm on its way. The 0.0561° angle would cut off a 19.6-cm distance on a vertical wall at a horizontal distance of 200 m, but on a vertical wall up at 30° it cuts off distance $h$ as shown, where $\cos 30°=19.6\text{ cm}/h$, $h=22.6\text{ cm}$. The marksman must $\boxed{\text{aim low}}$ by $22.6\text{ cm}-19.6\text{ cm}=3.03\text{ cm}$. The answer can be obtained by considering limiting cases. Suppose the target is nearly straight above or below you. Then gravity will not cause deviation of the path of the bullet, and one must aim low as in part (c) to cancel out the sighting-in of the telescope.

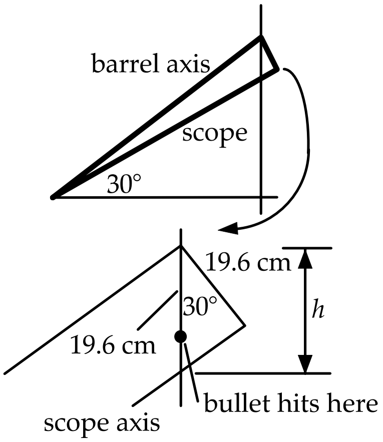


FIG. P4.60(f–g)

**P4.61** (a) From Part (c), the raptor dives for $6.34 - 2.00 = 4.34$ s undergoing displacement 197 m downward and $(10.0)(4.34) = 43.4$ m forward.

$$v = \frac{\Delta d}{\Delta t} = \frac{\sqrt{(197)^2 + (43.4)^2}}{4.34} = \boxed{46.5 \text{ m/s}}$$

(b) $\alpha = \tan^{-1}\left(\frac{-197}{43.4}\right) = \boxed{-77.6°}$

(c) $197 = \frac{1}{2}gt^2$, $\boxed{t = 6.34 \text{ s}}$

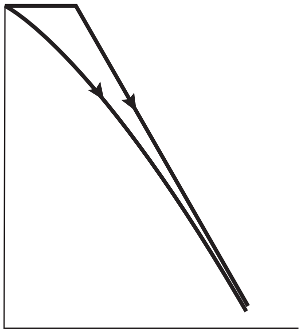


FIG. P4.61

**P4.62** Measure heights above the level ground. The elevation $y_b$ of the ball follows

$$y_b = R + 0 - \frac{1}{2}gt^2$$

with $x = v_i t$ so $y_b = R - \frac{gx^2}{2v_i^2}$.

(a) The elevation $y_r$ of points on the rock is described by

$$y_r^2 + x^2 = R^2.$$

We will have $y_b = y_r$ at $x = 0$, but for all other $x$ we require the ball to be above the rock surface as in $y_b > y_r$. Then $y_b^2 + x^2 > R^2$

$$\left(R - \frac{gx^2}{2v_i^2}\right)^2 + x^2 > R^2$$

$$R^2 - \frac{gx^2R}{v_i^2} + \frac{g^2x^4}{4v_i^4} + x^2 > R^2$$

$$\frac{g^2x^4}{4v_i^4} + x^2 > \frac{gx^2R}{v_i^2}.$$

If this inequality is satisfied for $x$ approaching zero, it will be true for all $x$. If the ball's parabolic trajectory has large enough radius of curvature at the start, the ball will clear the whole rock: $1 > \frac{gR}{v_i^2}$

$$\boxed{v_i > \sqrt{gR}}.$$

(b) With $v_i = \sqrt{gR}$ and $y_b = 0$, we have $0 = R - \frac{gx^2}{2gR}$

or $x = R\sqrt{2}$.

The distance from the rock's base is

$$x - R = \boxed{\left(\sqrt{2} - 1\right)R}.$$

**P4.63** (a) While on the incline

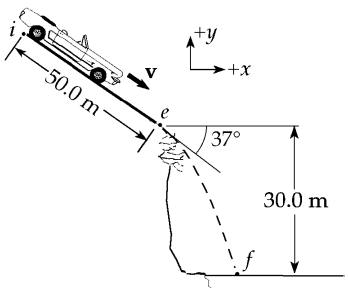


FIG. P4.63

$$v_f^2 - v_i^2 = 2a\Delta x$$
$$v_f - v_i = at$$
$$v_f^2 - 0 = 2(4.00)(50.0)$$
$$20.0 - 0 = 4.00t$$
$$v_f = \boxed{20.0 \text{ m/s}}$$
$$t = \boxed{5.00 \text{ s}}$$

(b) Initial free-flight conditions give us

$$v_{xi} = 20.0\cos 37.0° = 16.0 \text{ m/s}$$

and

$$v_{yi} = -20.0\sin 37.0° = -12.0 \text{ m/s}$$
$$v_{xf} = v_{xi} \text{ since } a_x = 0$$
$$v_{yf} = -\sqrt{2a_y \Delta y + v_{yi}{}^2} = -\sqrt{2(-9.80)(-30.0) + (-12.0)^2} = -27.1 \text{ m/s}$$
$$v_f = \sqrt{v_{xf}{}^2 + v_{yf}{}^2} = \sqrt{(16.0)^2 + (-27.1)^2} = \boxed{31.5 \text{ m/s at } 59.4° \text{ below the horizontal}}$$

(c) $t_1 = 5 \text{ s};\ t_2 = \dfrac{v_{yf} - v_{yi}}{a_y} = \dfrac{-27.1 + 12.0}{-9.80} = 1.53 \text{ s}$

$$t = t_1 + t_2 = \boxed{6.53 \text{ s}}$$

(d) $\Delta x = v_{xi} t_1 = 16.0(1.53) = \boxed{24.5 \text{ m}}$

**P4.64** Equation of bank: $y^2 = 16x$ (1)

Equations of motion: $x = v_i t$ (2)

$y = -\dfrac{1}{2} g t^2$ (3)

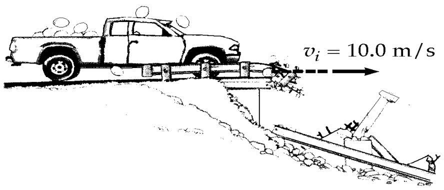


FIG. P4.64

Substitute for $t$ from (2) into (3) $y = -\dfrac{1}{2} g\left(\dfrac{x^2}{v_i^2}\right)$. Equate $y$ from the bank equation to $y$ from the equations of motion:

$$16x = \left[-\frac{1}{2} g\left(\frac{x^2}{v_i^2}\right)\right]^2 \Rightarrow \frac{g^2 x^4}{4v_i^4} - 16x = x\left(\frac{g^2 x^3}{4v_i^4} - 16\right) = 0.$$

From this, $x = 0$ or $x^3 = \dfrac{64 v_i^4}{g^2}$ and $x = 4\left(\dfrac{10^4}{9.80^2}\right)^{1/3} = \boxed{18.8 \text{ m}}$. Also,

$$y = -\frac{1}{2} g\left(\frac{x^2}{v_i^2}\right) = -\frac{1}{2}\frac{(9.80)(18.8)^2}{(10.0)^2} = \boxed{-17.3 \text{ m}}.$$

**P4.65** (a) Coyote: $\Delta x = \frac{1}{2}at^2$; $70.0 = \frac{1}{2}(15.0)t^2$

Roadrunner: $\Delta x = v_i t$; $70.0 = v_i t$

Solving the above, we get

$$v_i = \boxed{22.9\ \text{m/s}}\ \text{and}\ t = 3.06\ \text{s}.$$

(b) At the edge of the cliff,

$$v_{xi} = at = (15.0)(3.06) = 45.8\ \text{m/s}.$$

Substituting into $\Delta y = \frac{1}{2}a_y t^2$, we find

$$-100 = \frac{1}{2}(-9.80)t^2$$

$$t = 4.52\ \text{s}$$

$$\Delta x = v_{xi}t + \frac{1}{2}a_x t^2 = (45.8)(4.52\ \text{s}) + \frac{1}{2}(15.0)(4.52\ \text{s})^2.$$

Solving,

$$\Delta x = \boxed{360\ \text{m}}.$$

(c) For the Coyote's motion through the air

$$v_{xf} = v_{xi} + a_x t = 45.8 + 15(4.52) = \boxed{114\ \text{m/s}}$$

$$v_{yf} = v_{yi} + a_y t = 0 - 9.80(4.52) = \boxed{-44.3\ \text{m/s}}.$$

**P4.66** Think of shaking down the mercury in an old fever thermometer. Swing your hand through a circular arc, quickly reversing direction at the bottom end. Suppose your hand moves through one-quarter of a circle of radius 60 cm in 0.1 s. Its speed is

$$\frac{\frac{1}{4}(2\pi)(0.6\ \text{m})}{0.1\ \text{s}} \cong 9\ \text{m/s}$$

and its centripetal acceleration is $\frac{v^2}{r} \cong \frac{(9\ \text{m/s})^2}{0.6\ \text{m}}\boxed{\sim 10^2\ \text{m/s}^2}$.

The tangential acceleration of stopping and reversing the motion will make the total acceleration somewhat larger, but will not affect its order of magnitude.

**P4.67** (a) $\Delta x = v_{xi}t$, $\Delta y = v_{yi}t + \frac{1}{2}gt^2$

$$d\cos 50.0° = (10.0\cos 15.0°)t$$

and

$$-d\sin 50.0° = (10.0\sin 15.0°)t + \frac{1}{2}(-9.80)t^2.$$

Solving, $d = \boxed{43.2\ \text{m}}$ and $t = 2.88\ \text{s}$.

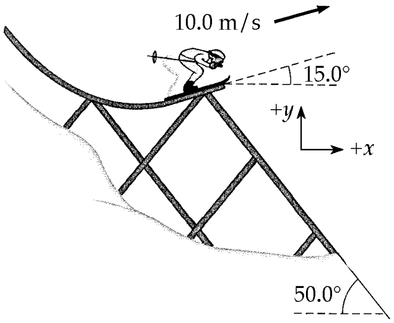


**FIG. P4.67**

(b) Since $a_x = 0$,

$$v_{xf} = v_{xi} = 10.0\cos 15.0° = \boxed{9.66\ \text{m/s}}$$

$$v_{yf} = v_{yi} + a_y t = 10.0\sin 15.0° - 9.80(2.88) = \boxed{-25.6\ \text{m/s}}.$$

Air resistance would decrease the values of the range and maximum height. As an airfoil, he can get some lift and increase his distance.

***P4.68** For one electron, we have

$$y = v_{iy}t,\ D = v_{ix}t + \frac{1}{2}a_x t^2 \cong \frac{1}{2}a_x t^2,\ v_{yf} = v_{yi},\ \text{and}\ v_{xf} = v_{xi} + a_x t \cong a_x t.$$

The angle its direction makes with the *x*-axis is given by

$$\theta = \tan^{-1}\frac{v_{yf}}{v_{xf}} = \tan^{-1}\frac{v_{yi}}{a_x t} = \tan^{-1}\frac{v_{yi}t}{a_x t^2} = \tan^{-1}\frac{y}{2D}.$$

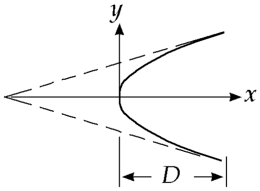


**FIG. P4.68**

Thus the horizontal distance from the aperture to the virtual source is 2*D*. The source is at coordinate $\boxed{x = -D}$.

***P4.69** (a) The ice chest floats downstream 2 km in time *t*, so that $2\ \text{km} = v_w t$. The upstream motion of the boat is described by $d = (v - v_w)15\ \text{min}$. The downstream motion is described by $d + 2\ \text{km} = (v + v_w)(t - 15\ \text{min})$. We eliminate $t = \frac{2\ \text{km}}{v_w}$ and *d* by substitution:

$$(v - v_w)15\ \text{min} + 2\ \text{km} = (v + v_w)\left(\frac{2\ \text{km}}{v_w} - 15\ \text{min}\right)$$

$$v(15\ \text{min}) - v_w(15\ \text{min}) + 2\ \text{km} = \frac{v}{v_w}2\ \text{km} + 2\ \text{km} - v(15\ \text{min}) - v_w(15\ \text{min})$$

$$v(30\ \text{min}) = \frac{v}{v_w}\ 2\ \text{km}$$

$$v_w = \frac{2\ \text{km}}{30\ \text{min}} = \boxed{4.00\ \text{km/h}}.$$

(b) In the reference frame of the water, the chest is motionless. The boat travels upstream for 15 min at speed *v*, and then downstream at the same speed, to return to the same point. Thus it travels for 30 min. During this time, the falls approach the chest at speed $v_w$, traveling 2 km. Thus

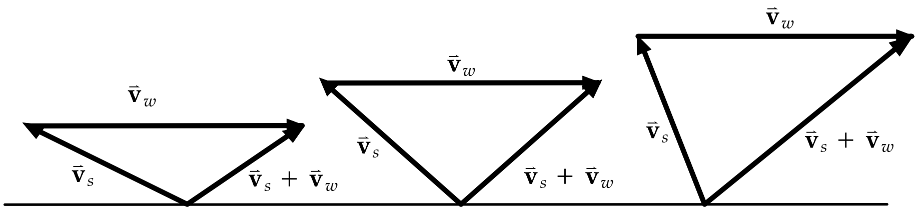
$$v_w = \frac{\Delta x}{\Delta t} = \frac{2\ \text{km}}{30\ \text{min}} = \boxed{4.00\ \text{km/h}}.$$

***P4.70** Let the river flow in the $x$ direction.

(a) To minimize time, $\boxed{\text{swim perpendicular to the banks}}$ in the $y$ direction. You are in the water for time $t$ in $\Delta y = v_y t$, $t = \dfrac{80\text{ m}}{1.5\text{ m/s}} = 53.3\text{ s}$.

(b) The water carries you downstream by $\Delta x = v_x t = (2.50\text{ m/s})53.3\text{ s} = \boxed{133\text{ m}}$.

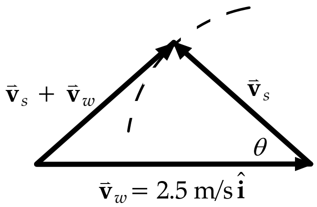
(c)



$\vec{\mathbf{v}}_w$
$\vec{\mathbf{v}}_s$
$\vec{\mathbf{v}}_s + \vec{\mathbf{v}}_w$
$\vec{\mathbf{v}}_w$
$\vec{\mathbf{v}}_s$
$\vec{\mathbf{v}}_s + \vec{\mathbf{v}}_w$
$\vec{\mathbf{v}}_w$
$\vec{\mathbf{v}}_s$
$\vec{\mathbf{v}}_s + \vec{\mathbf{v}}_w$

To minimize downstream drift, you should swim so that your resultant velocity $\vec{\mathbf{v}}_s + \vec{\mathbf{v}}_w$ is perpendicular to your swimming velocity $\vec{\mathbf{v}}_s$ relative to the water. This condition is shown in the middle picture. It maximizes the angle between the resultant velocity and the shore. The angle between $\vec{\mathbf{v}}_s$ and the shore is given by $\cos\theta = \dfrac{1.5\text{ m/s}}{2.5\text{ m/s}}$,

$\boxed{\theta = 53.1°}$.



$\vec{\mathbf{v}}_s + \vec{\mathbf{v}}_w$
$\vec{\mathbf{v}}_s$
$\theta$
$\vec{\mathbf{v}}_w = 2.5\text{ m/s}\,\hat{\mathbf{i}}$

(d) Now $v_y = v_s \sin\theta = 1.5\text{ m/s}\ \sin 53.1° = 1.20\text{ m/s}$

$$t = \frac{\Delta y}{v_y} = \frac{80\text{ m}}{1.2\text{ m/s}} = 66.7\text{ s}$$

$$\Delta x = v_x t = (2.5\text{ m/s} - 1.5\text{ m/s}\cos 53.1°)66.7\text{ s} = \boxed{107\text{ m}}.$$

***P4.71** Find the highest firing angle $\theta_H$ for which the projectile will clear the mountain peak; this will yield the range of the closest point of bombardment. Next find the lowest firing angle; this will yield the maximum range under these conditions if both $\theta_H$ and $\theta_L$ are $> 45°$; $x = 2500\text{ m}$, $y = 1800\text{ m}$, $v_i = 250\text{ m/s}$.

$$y_f = v_{yi}t - \frac{1}{2}gt^2 = v_i(\sin\theta)t - \frac{1}{2}gt^2$$
$$x_f = v_{xi}t = v_i(\cos\theta)t$$

Thus

$$t = \frac{x_f}{v_i\cos\theta}.$$

Substitute into the expression for $y_f$

$$y_f = v_i(\sin\theta)\frac{x_f}{v_i\cos\theta} - \frac{1}{2}g\left(\frac{x_f}{v_i\cos\theta}\right)^2 = x_f\tan\theta - \frac{gx_f^2}{2v_i^2\cos^2\theta}$$

but $\dfrac{1}{\cos^2\theta} = \tan^2\theta + 1$ so $y_f = x_f\tan\theta - \dfrac{gx_f^2}{2v_i^2}\left(\tan^2\theta + 1\right)$ and

$$0 = \frac{gx_f^2}{2v_i^2}\tan^2\theta - x_f\tan\theta + \frac{gx_f^2}{2v_i^2} + y_f.$$

Substitute values, use the quadratic formula and find

$$\tan\theta = 3.905 \text{ or } 1.197, \text{ which gives } \theta_H = 75.6° \text{ and } \theta_L = 50.1°.$$

$$\text{Range}\left(\text{at }\theta_H\right) = \frac{v_i^2\sin 2\theta_H}{g} = 3.07\times10^3\text{ m from enemy ship}$$

$$3.07\times10^3 - 2\,500 - 300 = 270\text{ m from shore.}$$

$$\text{Range}\left(\text{at }\theta_L\right) = \frac{v_i^2\sin 2\theta_L}{g} = 6.28\times10^3\text{ m from enemy ship}$$

$$6.28\times10^3 - 2\,500 - 300 = 3.48\times10^3\text{ from shore.}$$

Therefore, safe distance is $\boxed{< 270\text{ m}}$ or $\boxed{> 3.48\times10^3\text{ m}}$ from the shore.

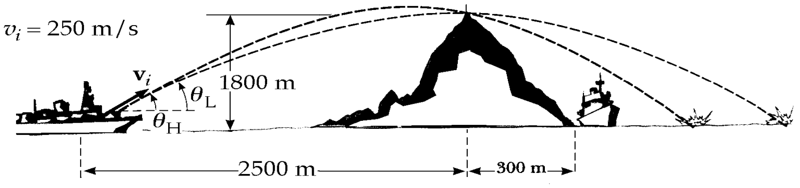


FIG. P4.71

***P4.72** We follow the steps outlined in Example 4.7, eliminating $t = \frac{d\cos\phi}{v_i\cos\theta}$ to find

$$\frac{v_i\sin\theta d\cos\phi}{v_i\cos\theta} - \frac{gd^2\cos^2\phi}{2v_i^2\cos^2\theta} = -d\sin\phi.$$

Clearing of fractions,

$$2v_i^2\cos\theta\sin\theta\cos\phi - gd\cos^2\phi = -2v_i^2\cos^2\theta\sin\phi.$$

To maximize $d$ as a function of $\theta$, we differentiate through with respect to $\theta$ and set $\frac{dd}{d\theta} = 0$:

$$2v_i^2\cos\theta\cos\theta\cos\phi + 2v_i^2\sin\theta(-\sin\theta)\cos\phi - g\frac{dd}{d\theta}\cos^2\phi = -2v_i^2 2\cos\theta(-\sin\theta)\sin\phi.$$

We use the trigonometric identities from Appendix B4 $\cos 2\theta = \cos^2\theta - \sin^2\theta$ and $\sin 2\theta = 2\sin\theta\cos\theta$ to find $\cos\phi\cos 2\theta = \sin 2\theta\sin\phi$. Next, $\frac{\sin\phi}{\cos\phi} = \tan\phi$ and $\cot 2\theta = \frac{1}{\tan 2\theta}$ give $\cot 2\phi = \tan\phi = \tan(90° - 2\theta)$ so $\phi = 90° - 2\theta$ and $\theta = 45° - \frac{\phi}{2}$.

## ANSWERS TO EVEN PROBLEMS

**P4.2** (a) $\mathbf{r} = 18.0t\hat{\mathbf{i}} + \left(4.00t - 4.90t^2\right)\hat{\mathbf{j}}$;
(b) $\mathbf{v} = 18.0\,\hat{\mathbf{i}} + (4.00 - 9.80t)\,\hat{\mathbf{j}}$;
(c) $\mathbf{a} = \left(-9.80\ \text{m/s}^2\right)\hat{\mathbf{j}}$;
(d) $(54.0\ \text{m})\,\hat{\mathbf{i}} - (32.1\ \text{m})\,\hat{\mathbf{j}}$;
(e) $(18.0\ \text{m/s})\,\hat{\mathbf{i}} - (25.4\ \text{m/s})\,\hat{\mathbf{j}}$;
(f) $\left(-9.80\ \text{m/s}^2\right)\hat{\mathbf{j}}$

**P4.4** (a) $\mathbf{v} = \left(-5.00\omega\,\hat{\mathbf{i}} + 0\hat{\mathbf{j}}\right)\ \text{m/s}$;
$\mathbf{a} = \left(0\hat{\mathbf{i}} + 5.00\omega^2\hat{\mathbf{j}}\right)\ \text{m/s}^2$;
(b) $\mathbf{r} = 4.00\ \text{m}\ \hat{\mathbf{j}}$
$+5.00\ \text{m}\left(-\sin\omega t\,\hat{\mathbf{i}} - \cos\omega t\,\hat{\mathbf{j}}\right)$;
$\mathbf{v} = 5.00\ \text{m}\ \omega\left(-\cos\omega t\,\hat{\mathbf{i}} + \sin\omega t\,\hat{\mathbf{j}}\right)$;
$\mathbf{a} = 5.00\ \text{m}\ \omega^2\left(\sin\omega t\,\hat{\mathbf{i}} + \cos\omega t\,\hat{\mathbf{j}}\right)$;
(c) a circle of radius 5.00 m centered at $(0,\ 4.00\ \text{m})$

**P4.6** (a) $\mathbf{v} = -12.0t\,\hat{\mathbf{j}}\ \text{m/s}$; $\mathbf{a} = -12.0\hat{\mathbf{j}}\ \text{m/s}^2$;
(b) $\mathbf{r} = \left(3.00\hat{\mathbf{i}} - 6.00\hat{\mathbf{j}}\right)$ m; $\mathbf{v} = -12.0\hat{\mathbf{j}}\ \text{m/s}$

**P4.8** (a) $\mathbf{r} = \left(5.00t\hat{\mathbf{i}} + 1.50t^2\hat{\mathbf{j}}\right)$ m;
$\mathbf{v} = \left(5.00\hat{\mathbf{i}} + 3.00t\hat{\mathbf{j}}\right)\ \text{m/s}$;
(b) $\mathbf{r} = \left(10.0\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}}\right)$ m; $7.81\ \text{m/s}$

**P4.10** $\left(7.23\times10^3\ \text{m},\ 1.68\times10^3\ \text{m}\right)$

**P4.12** (a) $d\sqrt{\frac{g}{2h}}$ horizontally;
(b) $\tan^{-1}\left(\frac{2h}{d}\right)$ below the horizontal

**P4.14** $0.600\ \text{m/s}^2$

**P4.16** (a) 76.0°; (b) the same; (c) $\frac{17d}{8}$

**P4.18** $25.8\ \text{m/s}$

**P4.20** $d\tan\theta_i - \frac{gd^2}{\left(2v_i^2\cos^2\theta_i\right)}$

| | |
|---|---|
| **P4.22** | 33.5° below the horizontal |
| **P4.24** | (a) 0.852 s; (b) 3.29 $\text{m/s}$; (c) 4.03 $\text{m/s}$;<br>(d) 50.8°; (e) 1.12 s |
| **P4.26** | $\tan^{-1}\left(\frac{\sqrt{2gh}}{v}\right)$ |
| **P4.28** | $0.033\,7\ \text{m/s}^2$ toward the center of the Earth |
| **P4.30** | 0.281 rev/s |
| **P4.32** | $7.58\times10^3\ \text{m/s}$; $5.80\times10^3\ \text{s}$ |
| **P4.34** | (a) $0.600\ \text{m/s}^2$ forward;<br>(b) $0.800\ \text{m/s}^2$ inward;<br>(c) $1.00\ \text{m/s}^2$ forward and 53.1° inward |
| **P4.36** | (a) see the solution; (b) $29.7\ \text{m/s}^2$;<br>(c) 6.67 m/s at 36.9° above the horizontal |
| **P4.38** | (a) 26.9 m/s; (b) 67.3 m;<br>(c) $\left(2.00\hat{\mathbf{i}}-5.00\hat{\mathbf{j}}\right)\ \text{m/s}^2$ |
| **P4.40** | 18.0 s |
| **P4.42** | 153 km/h at 11.3° north of west |
| **P4.44** | (a) $10.1\ \text{m/s}^2$ at 14.3° south from the vertical; (b) $9.80\ \text{m/s}^2$ vertically downward |
| **P4.46** | 27.7° east of north |
| **P4.48** | $2v_i t\cos\theta_i$ |
| **P4.50** | (a) see the solution;<br>(b) $\theta_i = 45° + \frac{\phi}{2}$; $d_{\max} = \frac{v_i^2(1-\sin\phi)}{g\cos^2\phi}$ |
| **P4.52** | (a) 1.69 km/s; (b) $6.47\times10^3\ \text{s}$ |
| **P4.54** | 10.7 m/s |
| **P4.56** | $\frac{R}{2}$ |
| **P4.58** | 7.50 m/s in the direction the ball was thrown |
| **P4.60** | (a) 19.6 cm; (b) 0.0561°;<br>(c) aim low 3.68 cm; (d) aim low 3.68 cm;<br>(e) aim high 6.12 cm; (f) aim low;<br>(g) aim low |
| **P4.62** | (a) $\sqrt{gR}$; (b) $\left(\sqrt{2}-1\right)R$ |
| **P4.64** | (18.8 m; −17.3 m) |
| **P4.66** | see the solution; $\sim 10^2\ \text{m/s}^2$ |
| **P4.68** | $x = -D$ |
| **P4.70** | (a) at 90° to the bank; (b) 133 m;<br>(c) upstream at 53.1° to the bank; (d) 107 m |
| **P4.72** | see the solution |

# 5

# The Laws of Motion

**CHAPTER OUTLINE**

5.1 The Concept of Force
5.2 Newton's First Law and Inertial Frames
5.3 Mass
5.4 Newton's Second Law
5.5 The Gravitational Force and Weight
5.6 Newton's Third Law
5.7 Some Applications of Newton's Laws
5.8 Forces of Friction

## ANSWERS TO QUESTIONS

**Q5.1** (a) The force due to gravity of the earth pulling down on the ball—the reaction force is the force due to gravity of the ball pulling up on the earth. The force of the hand pushing up on the ball—reaction force is ball pushing down on the hand.

(b) The only force acting on the ball in free-fall is the gravity due to the earth -the reaction force is the gravity due to the ball pulling on the earth.

**Q5.2** The resultant force is zero, as the acceleration is zero.

**Q5.3** Mistake one: The car might be momentarily at rest, in the process of (suddenly) reversing forward into backward motion. In this case, the forces on it add to a (large) backward resultant.

Mistake two: There are no cars in interstellar space. If the car is remaining at rest, there are some large forces on it, including its weight and some force or forces of support.

Mistake three: The statement reverses cause and effect, like a politician who thinks that his getting elected was the reason for people to vote for him.

**Q5.4** When the bus starts moving, the mass of Claudette is accelerated by the force of the back of the seat on her body. Clark is standing, however, and the only force on him is the friction between his shoes and the floor of the bus. Thus, when the bus starts moving, his feet start accelerating forward, but the rest of his body experiences almost no accelerating force (only that due to his being attached to his accelerating feet!). As a consequence, his body tends to stay almost at rest, according to Newton's first law, relative to the ground. Relative to Claudette, however, he is moving toward her and falls into her lap. (Both performers won Academy Awards.)

**Q5.5** First ask, "Was the bus moving forward or backing up?" If it was moving forward, the passenger is lying. A fast stop would make the suitcase fly toward the front of the bus, not toward the rear. If the bus was backing up at any reasonable speed, a sudden stop could not make a suitcase fly far. Fine her for malicious litigiousness.

**Q5.6** It would be smart for the explorer to gently push the rock back into the storage compartment. Newton's 3rd law states that the rock will apply the same size force on her that she applies on it. The harder she pushes on the rock, the larger her resulting acceleration.

**Q5.7** The molecules of the floor resist the ball on impact and push the ball back, upward. The actual force acting is due to the forces between molecules that allow the floor to keep its integrity and to prevent the ball from passing through. Notice that for a ball passing through a window, the molecular forces weren't strong enough.

**Q5.8** While a football is in flight, the force of gravity and air resistance act on it. When a football is in the process of being kicked, the foot pushes forward on the ball and the ball pushes backward on the foot. At this time and while the ball is in flight, the Earth pulls down on the ball (gravity) and the ball pulls up on the Earth. The moving ball pushes forward on the air and the air backward on the ball.

**Q5.9** It is impossible to string a horizontal cable without its sagging a bit. Since the cable has a mass, gravity pulls it downward. A vertical component of the tension must balance the weight for the cable to be in equilibrium. If the cable were completely horizontal, then there would be no vertical component of the tension to balance the weight.

Some physics teachers demonstrate this by asking a beefy student to pull on the ends of a cord supporting a can of soup at its center. Some get two burly young men to pull on opposite ends of a strong rope, while the smallest person in class gleefully mashes the center of the rope down to the table. Point out the beauty of sagging suspension-bridge cables. With a laser and an optical lever, demonstrate that the mayor makes the courtroom table sag when he sits on it, and the judge bends the bench. Give them "I make the floor sag" buttons, available to instructors using this manual. Estimate the cost of an infinitely strong cable, and the truth will always win.

**Q5.10** As the barbell goes through the bottom of a cycle, the lifter exerts an upward force on it, and the scale reads the larger upward force that the floor exerts on them together. Around the top of the weight's motion, the scale reads less than average. If the iron is moving upward, the lifter can declare that she has thrown it, just by letting go of it for a moment, so our answer applies also to this case.

**Q5.11** As the sand leaks out, the acceleration increases. With the same driving force, a decrease in the mass causes an increase in the acceleration.

**Q5.12** As the rocket takes off, it burns fuel, pushing the gases from the combustion out the back of the rocket. Since the gases have mass, the total remaining mass of the rocket, fuel, and oxidizer decreases. With a constant thrust, a decrease in the mass results in an increasing acceleration.

**Q5.13** The friction of the road pushing on the tires of a car causes an automobile to move. The push of the air on the propeller moves the airplane. The push of the water on the oars causes the rowboat to move.

**Q5.14** As a man takes a step, the action is the force his foot exerts on the Earth; the reaction is the force of the Earth on his foot. In the second case, the action is the force exerted on the girl's back by the snowball; the reaction is the force exerted on the snowball by the girl's back. The third action is the force of the glove on the ball; the reaction is the force of the ball on the glove. The fourth action is the force exerted on the window by the air molecules; the reaction is the force on the air molecules exerted by the window. We could in each case interchange the terms 'action' and 'reaction.'

**Q5.15** The tension in the rope must be 9 200 N. Since the rope is moving at a constant speed, then the resultant force on it must be zero. The 49ers are pulling with a force of 9 200 N. If the 49ers were winning with the rope steadily moving in their direction or if the contest was even, then the tension would still be 9 200 N. In all of these case, the acceleration is zero, and so must be the resultant force on the rope. To win the tug-of-war, a team must exert a larger force on the ground than their opponents do.

**Q5.16** The tension in the rope when pulling the car is twice that in the tug-of-war. One could consider the car as behaving like another team of twenty more people.

**Q5.17** This statement contradicts Newton's 3rd law. The force that the locomotive exerted on the wall is the same as that exerted by the wall on the locomotive. The wall temporarily exerted on the locomotive a force greater than the force that the wall could exert without breaking.

**Q5.18** The sack of sand moves up with the athlete, regardless of how quickly the athlete climbs. Since the athlete and the sack of sand have the same weight, the acceleration of the system must be zero.

**Q5.19** The resultant force doesn't always add to zero. If it did, nothing could ever accelerate. If we choose a single object as our system, action and reaction forces can never add to zero, as they act on different objects.

**Q5.20** An object cannot exert a force on itself. If it could, then objects would be able to accelerate themselves, without interacting with the environment. You cannot lift yourself by tugging on your bootstraps.

**Q5.21** To get the box to slide, you must push harder than the maximum static frictional force. Once the box is moving, you need to push with a force equal to the kinetic frictional force to maintain the box's motion.

**Q5.22** The stopping distance will be the same if the mass of the truck is doubled. The stopping distance will decrease by a factor of four if the initial speed is cut in half.

**Q5.23** If you slam on the brakes, your tires will skid on the road. The force of kinetic friction between the tires and the road is less than the maximum static friction force. Anti-lock brakes work by "pumping" the brakes (much more rapidly that you can) to minimize skidding of the tires on the road.

**Q5.24** With friction, it takes longer to come down than to go up. On the way up, the frictional force and the component of the weight down the plane are in the same direction, giving a large acceleration. On the way down, the forces are in opposite directions, giving a relatively smaller acceleration. If the incline is frictionless, it takes the same amount of time to go up as it does to come down.

**Q5.25** (a) The force of static friction between the crate and the bed of the truck causes the crate to accelerate. Note that the friction force on the crate is in the direction of its motion relative to the ground (but opposite to the direction of possible sliding motion of the crate relative to the truck bed).

(b) It is most likely that the crate would slide forward relative to the bed of the truck.

**Q5.26** In Question 25, part (a) is an example of such a situation. Any situation in which friction is the force that accelerates an object from rest is an example. As you pull away from a stop light, friction is the force that accelerates forward a box of tissues on the level floor of the car. At the same time, friction of the ground on the tires of the car accelerates the car forward.

## SOLUTIONS TO PROBLEMS

The following problems cover Sections 5.1–5.6.

Section 5.1 **The Concept of Force**

Section 5.2 **Newton's First Law and Inertial Frames**

Section 5.3 **Mass**

Section 5.4 **Newton's Second Law**

Section 5.5 **The Gravitational Force and Weight**

Section 5.6 **Newton's Third Law**

**P5.1** For the same force $F$, acting on different masses

$$F = m_1 a_1$$

and

$$F = m_2 a_2$$

(a) $\dfrac{m_1}{m_2} = \dfrac{a_2}{a_1} = \boxed{\dfrac{1}{3}}$

(b) $F = (m_1 + m_2)a = 4m_1 a = m_1\left(3.00\ \text{m/s}^2\right)$

$a = \boxed{0.750\ \text{m/s}^2}$

***P5.2** $v_f = 880\ \text{m/s},\ m = 25.8\ \text{kg},\ x_f = 6\ \text{m}$

$v_f^2 = 2ax_f = 2x_f\left(\dfrac{F}{m}\right)$

$F = \dfrac{mv_f^2}{2x_f} = \boxed{1.66\times 10^6\ \text{N forward}}$

**P5.3** $m = 3.00\ \text{kg}$

$\mathbf{a} = \left(2.00\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}}\right)\ \text{m/s}^2$

$\sum \mathbf{F} = m\mathbf{a} = \boxed{\left(6.00\hat{\mathbf{i}} + 15.0\hat{\mathbf{j}}\right)\ \text{N}}$

$\left|\sum \mathbf{F}\right| = \sqrt{(6.00)^2 + (15.0)^2}\ \text{N} = \boxed{16.2\ \text{N}}$

**P5.4** $F_g = \text{weight of ball} = mg$

$v_{\text{release}} = v$ and time to accelerate $= t$ :

$$\mathbf{a} = \frac{\Delta v}{\Delta t} = \frac{v}{t} = \frac{v}{t}\hat{\mathbf{i}}$$

(a) Distance $x = \bar{v}t$:

$$x = \left(\frac{v}{2}\right)t = \boxed{\frac{vt}{2}}$$

(b) $\mathbf{F}_p - F_g\hat{\mathbf{j}} = \frac{F_g v}{gt}\hat{\mathbf{i}}$

$$\mathbf{F}_p = \boxed{\frac{F_g v}{gt}\hat{\mathbf{i}} + F_g\hat{\mathbf{j}}}$$

**P5.5** $m = 4.00\text{ kg}$, $\mathbf{v}_i = 3.00\hat{\mathbf{i}}\text{ m/s}$, $\mathbf{v}_8 = \left(8.00\hat{\mathbf{i}} + 10.0\hat{\mathbf{j}}\right)\text{ m/s}$, $t = 8.00\text{ s}$

$$\mathbf{a} = \frac{\Delta\mathbf{v}}{t} = \frac{5.00\hat{\mathbf{i}} + 10.0\hat{\mathbf{j}}}{8.00}\text{ m/s}^2$$

$$\mathbf{F} = m\mathbf{a} = \boxed{\left(2.50\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}}\right)\text{ N}}$$

$$F = \sqrt{(2.50)^2 + (5.00)^2} = \boxed{5.59\text{ N}}$$

**P5.6** (a) Let the $x$-axis be in the original direction of the molecule's motion.

$$v_f = v_i + at:\quad -670\text{ m/s} = 670\text{ m/s} + a\left(3.00\times10^{-13}\text{ s}\right)$$

$$a = \boxed{-4.47\times10^{15}\text{ m/s}^2}$$

(b) For the molecule, $\sum\mathbf{F} = m\mathbf{a}$. Its weight is negligible.

$$\mathbf{F}_{\text{wall on molecule}} = 4.68\times10^{-26}\text{ kg}\left(-4.47\times10^{15}\text{ m/s}^2\right) = -2.09\times10^{-10}\text{ N}$$

$$\vec{\mathbf{F}}_{\text{molecule on wall}} = \boxed{+2.09\times10^{-10}\text{ N}}$$

**P5.7** (a) $\sum F = ma$ and $v_f^2 = v_i^2 + 2ax_f$ or $a = \dfrac{v_f^2 - v_i^2}{2x_f}$.

Therefore,

$$\sum F = m\frac{\left(v_f^2 - v_i^2\right)}{2x_f}$$

$$\sum F = 9.11 \times 10^{-31}\ \text{kg}\frac{\left[\left(7.00 \times 10^5\ \text{m/s}^2\right)^2 - \left(3.00 \times 10^5\ \text{m/s}^2\right)^2\right]}{2(0.050\,0\ \text{m})} = \boxed{3.64 \times 10^{-18}\ \text{N}}.$$

(b) The weight of the electron is

$$F_g = mg = \left(9.11 \times 10^{-31}\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right) = 8.93 \times 10^{-30}\ \text{N}$$

The accelerating force is $\boxed{4.08 \times 10^{11}\ \text{times the weight of the electron.}}$

**P5.8** (a) $F_g = mg = 120\ \text{lb} = (4.448\ \text{N/lb})(120\ \text{lb}) = \boxed{534\ \text{N}}$

(b) $m = \dfrac{F_g}{g} = \dfrac{534\ \text{N}}{9.80\ \text{m/s}^2} = \boxed{54.5\ \text{kg}}$

**P5.9** $F_g = mg = 900\ \text{N}$, $m = \dfrac{900\ \text{N}}{9.80\ \text{m/s}^2} = 91.8\ \text{kg}$

$$\left(F_g\right)_{\text{on Jupiter}} = 91.8\ \text{kg}\left(25.9\ \text{m/s}^2\right) = \boxed{2.38\ \text{kN}}$$

**P5.10** Imagine a quick trip by jet, on which you do not visit the rest room and your perspiration is just canceled out by a glass of tomato juice. By subtraction, $\left(F_g\right)_p = mg_p$ and $\left(F_g\right)_C = mg_C$ give

$$\Delta F_g = m\left(g_p - g_C\right).$$

For a person whose mass is 88.7 kg, the change in weight is

$$\Delta F_g = 88.7\ \text{kg}(9.809\,5 - 9.780\,8) = \boxed{2.55\ \text{N}}.$$

A precise balance scale, as in a doctor's office, reads the same in different locations because it compares you with the standard masses on its beams. A typical bathroom scale is not precise enough to reveal this difference.

**P5.11** (a) $\sum \mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 = \left(20.0\hat{\mathbf{i}} + 15.0\hat{\mathbf{j}}\right)\text{ N}$

$$\sum \mathbf{F} = m\mathbf{a}:\quad 20.0\hat{\mathbf{i}} + 15.0\hat{\mathbf{j}} = 5.00\mathbf{a}$$

$$\mathbf{a} = \left(4.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}}\right)\text{ m/s}^2$$

or

$$\boxed{a = 5.00\text{ m/s}^2\text{ at }\theta = 36.9^\circ}$$

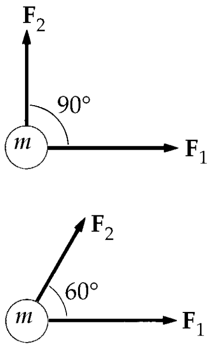


**FIG. P5.11**

(b)

$$F_{2x} = 15.0\cos 60.0^\circ = 7.50\text{ N}$$
$$F_{2y} = 15.0\sin 60.0^\circ = 13.0\text{ N}$$
$$\mathbf{F}_2 = \left(7.50\hat{\mathbf{i}} + 13.0\hat{\mathbf{j}}\right)\text{ N}$$
$$\sum \mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 = \left(27.5\hat{\mathbf{i}} + 13.0\hat{\mathbf{j}}\right)\text{ N} = m\mathbf{a} = 5.00\mathbf{a}$$
$$\mathbf{a} = \boxed{\left(5.50\hat{\mathbf{i}} + 2.60\hat{\mathbf{j}}\right)\text{ m/s}^2 = 6.08\text{ m/s}^2\text{ at }25.3^\circ}$$

**P5.12** We find acceleration:

$$\mathbf{r}_f - \mathbf{r}_i = \mathbf{v}_i t + \frac{1}{2}\mathbf{a}t^2$$
$$4.20\text{ m}\hat{\mathbf{i}} - 3.30\text{ m}\hat{\mathbf{j}} = 0 + \frac{1}{2}\mathbf{a}(1.20\text{ s})^2 = 0.720\text{ s}^2\mathbf{a}$$
$$\mathbf{a} = \left(5.83\hat{\mathbf{i}} - 4.58\hat{\mathbf{j}}\right)\text{ m/s}^2.$$

Now $\sum \mathbf{F} = m\mathbf{a}$ becomes

$$\mathbf{F}_g + \mathbf{F}_2 = m\mathbf{a}$$
$$\mathbf{F}_2 = 2.80\text{ kg}\left(5.83\hat{\mathbf{i}} - 4.58\hat{\mathbf{j}}\right)\text{ m/s}^2 + (2.80\text{ kg})\left(9.80\text{ m/s}^2\right)\hat{\mathbf{j}}$$
$$\mathbf{F}_2 = \boxed{\left(16.3\hat{\mathbf{i}} + 14.6\hat{\mathbf{j}}\right)\text{ N}}.$$

**P5.13** (a) You and the earth exert equal forces on each other: $m_y g = M_e a_e$. If your mass is 70.0 kg,

$$a_e = \frac{(70.0\text{ kg})\left(9.80\text{ m/s}^2\right)}{5.98\times 10^{24}\text{ kg}} = \boxed{\sim 10^{-22}\text{ m/s}^2}.$$

(b) You and the planet move for equal times intervals according to $x = \frac{1}{2}at^2$. If the seat is 50.0 cm high,

$$\sqrt{\frac{2x_y}{a_y}} = \sqrt{\frac{2x_e}{a_e}}$$

$$x_e = \frac{a_e}{a_y}x_y = \frac{m_y}{m_e}x_y = \frac{70.0\text{ kg}(0.500\text{ m})}{5.98\times 10^{24}\text{ kg}}\boxed{\sim 10^{-23}\text{ m}}.$$

**P5.14** $\sum \mathbf{F} = m\mathbf{a}$ reads

$$\left(-2.00\hat{\mathbf{i}} + 2.00\hat{\mathbf{j}} + 5.00\hat{\mathbf{i}} - 3.00\hat{\mathbf{j}} - 45.0\hat{\mathbf{i}}\right) \text{ N} = m\left(3.75 \text{ m/s}^2\right)\hat{\mathbf{a}}$$

where $\hat{\mathbf{a}}$ represents the direction of **a**

$$\left(-42.0\hat{\mathbf{i}} - 1.00\hat{\mathbf{j}}\right) \text{ N} = m\left(3.75 \text{ m/s}^2\right)\hat{\mathbf{a}}$$

$$\sum \mathbf{F} = \sqrt{(42.0)^2 + (1.00)^2} \text{ N at } \tan^{-1}\left(\frac{1.00}{42.0}\right) \text{ below the } -x\text{-axis}$$

$$\sum \mathbf{F} = 42.0 \text{ N at } 181° = m\left(3.75 \text{ m/s}^2\right)\hat{\mathbf{a}}.$$

For the vectors to be equal, their magnitudes and their directions must be equal.

(a) $\therefore \boxed{\hat{\mathbf{a}} \text{ is at } 181°}$ counterclockwise from the $x$-axis

(b) $m = \dfrac{42.0 \text{ N}}{3.75 \text{ m/s}^2} = \boxed{11.2 \text{ kg}}$

(d) $\mathbf{v}_f = \mathbf{v}_i + \mathbf{a}t = 0 + \left(3.75 \text{ m/s}^2 \text{ at } 181°\right)10.0 \text{ s so } \mathbf{v}_f = 37.5 \text{ m/s at } 181°$

$\mathbf{v}_f = 37.5 \text{ m/s } \cos 181° \hat{\mathbf{i}} + 37.5 \text{ m/s } \sin 181° \hat{\mathbf{j}} \text{ so } \mathbf{v}_f = \boxed{\left(-37.5\hat{\mathbf{i}} - 0.893\hat{\mathbf{j}}\right) \text{ m/s}}$

(c) $\left|\mathbf{v}_f\right| = \sqrt{37.5^2 + 0.893^2} \text{ m/s} = \boxed{37.5 \text{ m/s}}$

**P5.15** (a) $\boxed{15.0 \text{ lb up}}$

(b) $\boxed{5.00 \text{ lb up}}$

(c) $\boxed{0}$

---

## Section 5.7 **Some Applications of Newton's Laws**

**P5.16** $v_x = \dfrac{dx}{dt} = 10t,\ v_y = \dfrac{dy}{dt} = 9t^2$

$a_x = \dfrac{dv_x}{dt} = 10,\ a_y = \dfrac{dv_y}{dt} = 18t$

At $t = 2.00$ s, $a_x = 10.0 \text{ m/s}^2$, $a_y = 36.0 \text{ m/s}^2$

$\sum F_x = ma_x$: $3.00 \text{ kg}\left(10.0 \text{ m/s}^2\right) = 30.0 \text{ N}$

$\sum F_y = ma_y$: $3.00 \text{ kg}\left(36.0 \text{ m/s}^2\right) = 108 \text{ N}$

$$\sum F = \sqrt{F_x^2 + F_y^2} = \boxed{112 \text{ N}}$$

**P5.17**

$$m = 1.00 \text{ kg}$$
$$mg = 9.80 \text{ N}$$
$$\tan\alpha = \frac{0.200 \text{ m}}{25.0 \text{ m}}$$
$$\alpha = 0.458°$$

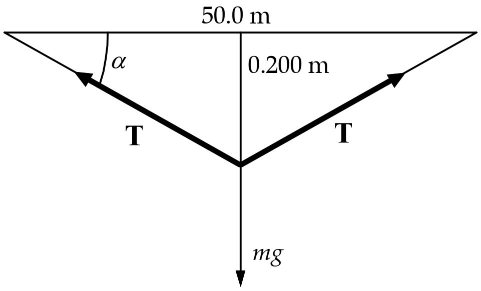


FIG. P5.17

Balance forces,

$$2T\sin\alpha = mg$$
$$T = \frac{9.80 \text{ N}}{2\sin\alpha} = \boxed{613 \text{ N}}$$

**P5.18**

$$T_3 = F_g \tag{1}$$
$$T_1\sin\theta_1 + T_2\sin\theta_2 = F_g \tag{2}$$
$$T_1\cos\theta_1 = T_2\cos\theta_2 \tag{3}$$

Eliminate $T_2$ and solve for $T_1$

$$T_1 = \frac{F_g\cos\theta_2}{(\sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2)} = \frac{F_g\cos\theta_2}{\sin(\theta_1+\theta_2)}$$
$$T_3 = F_g = \boxed{325 \text{ N}}$$
$$T_1 = F_g\left(\frac{\cos 25.0°}{\sin 85.0°}\right) = \boxed{296 \text{ N}}$$
$$T_2 = T_1\left(\frac{\cos\theta_1}{\cos\theta_2}\right) = 296 \text{ N}\left(\frac{\cos 60.0°}{\cos 25.0°}\right) = \boxed{163 \text{ N}}$$

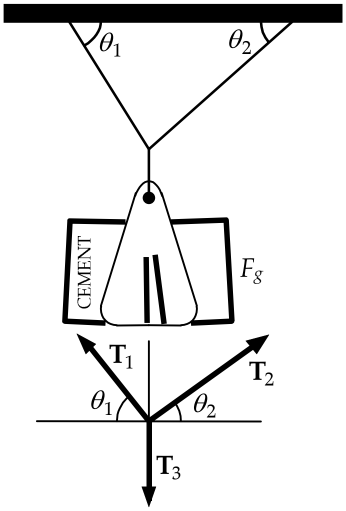


FIG. P5.18

**P5.19** See the solution for $T_1$ in Problem 5.18.

**P5.20** (a) An explanation proceeding from fundamental physical principles will be best for the parents and for you. Consider forces on the bit of string touching the weight hanger as shown in the free-body diagram:

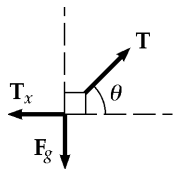


**FIG. P5.20**

Horizontal Forces: $\sum F_x = ma_x$: $-T_x + T\cos\theta = 0$

Vertical Forces: $\sum F_y = ma_y$: $-F_g + T\sin\theta = 0$

You need only the equation for the vertical forces to find that the tension in the string is given by $T = \boxed{\frac{F_g}{\sin\theta}}$. The force the child feels gets smaller, changing from $T$ to $T\cos\theta$, while the counterweight hangs on the string. On the other hand, the kite does not notice what you are doing and the tension in the main part of the string stays constant. You do not need a level, since you learned in physics lab to sight to a horizontal line in a building. Share with the parents your estimate of the experimental uncertainty, which you make by thinking critically about the measurement, by repeating trials, practicing in advance and looking for variations and improvements in technique, including using other observers. You will then be glad to have the parents themselves repeat your measurements.

(b) $$T = \frac{F_g}{\sin\theta} = \frac{0.132\ \text{kg}\left(9.80\ \text{m/s}^2\right)}{\sin 46.3°} = \boxed{1.79\ \text{N}}$$

**P5.21** (a) Isolate either mass

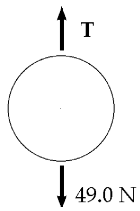


**FIG. P5.21(a)**

$$T + mg = ma = 0$$
$$|T| = |mg|.$$

The scale reads the tension $T$,

so

$$T = mg = 5.00\ \text{kg}\left(9.80\ \text{m/s}^2\right) = \boxed{49.0\ \text{N}}.$$

(b) Isolate the pulley

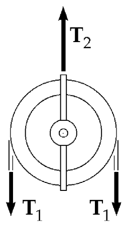


**FIG. P5.21(b)**

$$\mathbf{T}_2 + 2\mathbf{T}_1 = 0$$
$$T_2 = 2|T_1| = 2mg = \boxed{98.0\ \text{N}}.$$

(c) $\sum \mathbf{F} = \mathbf{n} + \mathbf{T} + m\mathbf{g} = 0$

Take the component along the incline

$$\mathbf{n}_x + \mathbf{T}_x + m\mathbf{g}_x = 0$$

or

$$0 + T - mg\sin 30.0° = 0$$

$$T = mg\sin 30.0° = \frac{mg}{2} = \frac{5.00(9.80)}{2}$$
$$= \boxed{24.5\ \text{N}}.$$

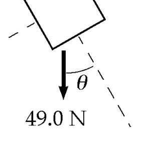


**FIG. P5.21(c)**

**P5.22** The two forces acting on the block are the normal force, $n$, and the weight, $mg$. If the block is considered to be a point mass and the $x$-axis is chosen to be parallel to the plane, then the free body diagram will be as shown in the figure to the right. The angle $\theta$ is the angle of inclination of the plane. Applying Newton's second law for the accelerating system (and taking the direction up the plane as the positive $x$ direction) we have

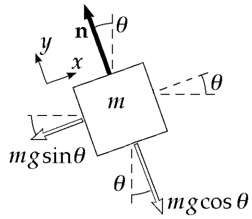


**FIG. P5.22**

$$\sum F_y = n - mg\cos\theta = 0:\ n = mg\cos\theta$$
$$\sum F_x = -mg\sin\theta = ma:\ a = -g\sin\theta$$

(a) When $\theta = 15.0°$

$$a = \boxed{-2.54\ \text{m/s}^2}$$

(b) Starting from rest

$$v_f^2 = v_i^2 + 2a\left(x_f - x_i\right) = 2ax_f$$
$$\left|v_f\right| = \sqrt{2ax_f} = \sqrt{2\left(-2.54\ \text{m/s}^2\right)(-2.00\ \text{m})} = \boxed{3.18\ \text{m/s}}$$

**P5.23** Choose a coordinate system with $\hat{\mathbf{i}}$ East and $\hat{\mathbf{j}}$ North.

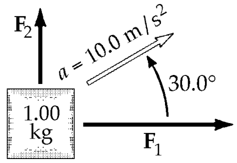


**FIG. P5.23**

$$\sum \mathbf{F} = m\mathbf{a} = 1.00\ \text{kg}\left(10.0\ \text{m/s}^2\right)\ \text{at}\ 30.0°$$
$$(5.00\ \text{N})\hat{\mathbf{j}} + \mathbf{F}_1 = (10.0\ \text{N})\angle 30.0° = (5.00\ \text{N})\hat{\mathbf{j}} + (8.66\ \text{N})\hat{\mathbf{i}}$$
$$\therefore F_1 = \boxed{8.66\ \text{N (East)}}$$

***P5.24** First, consider the block moving along the horizontal. The only force in the direction of movement is $T$. Thus, $\sum F_x = ma$

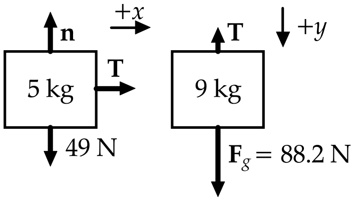


**FIG. P5.24**

$$T = (5\ \text{kg})a \qquad (1)$$

Next consider the block that moves vertically. The forces on it are the tension $T$ and its weight, 88.2 N.

We have $\sum F_y = ma$

$$88.2\ \text{N} - T = (9\ \text{kg})a \qquad (2)$$

Note that both blocks must have the same magnitude of acceleration. Equations (1) and (2) can be added to give $88.2\ \text{N} = (14\ \text{kg})a$. Then

$$\boxed{a = 6.30\ \text{m/s}^2\ \text{and}\ T = 31.5\ \text{N}}.$$

**P5.25** After it leaves your hand, the block's speed changes only because of one component of its weight:

$$\sum F_x = ma_x \quad -mg\sin 20.0° = ma$$

$$v_f^2 = v_i^2 + 2a\left(x_f - x_i\right).$$

Taking $v_f = 0$, $v_i = 5.00$ m/s, and $a = -g\sin(20.0°)$ gives

$$0 = (5.00)^2 - 2(9.80)\sin(20.0°)\left(x_f - 0\right)$$

or

$$x_f = \frac{25.0}{2(9.80)\sin(20.0°)} = \boxed{3.73\text{ m}}.$$

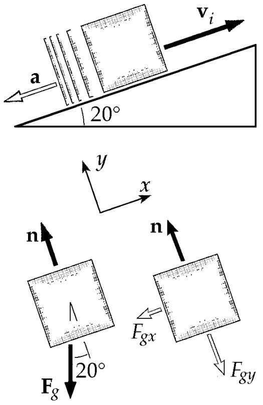


FIG. P5.25

**P5.26** $m_1 = 2.00$ kg, $m_2 = 6.00$ kg, $\theta = 55.0°$

(a) $\sum F_x = m_2 g\sin\theta - T = m_2 a$

and

$$T - m_1 g = m_1 a$$

$$a = \frac{m_2 g\sin\theta - m_1 g}{m_1 + m_2} = \boxed{3.57\text{ m/s}^2}$$

(b) $T = m_1(a + g) = \boxed{26.7\text{ N}}$

(c) Since $v_i = 0$, $v_f = at = \left(3.57\text{ m/s}^2\right)(2.00\text{ s}) = \boxed{7.14\text{ m/s}}$.

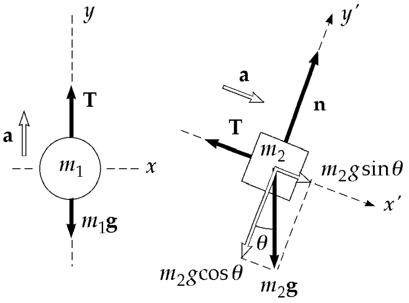


FIG. P5.26

***P5.27** We assume the vertical bar is in compression, pushing up on the pin with force $A$, and the tilted bar is in tension, exerting force $B$ on the pin at $-50°$.

$$\sum F_x = 0: \quad -2\,500\text{ N}\cos 30° + B\cos 50° = 0$$

$$\boxed{B = 3.37\times 10^3\text{ N}}$$

$$\sum F_y = 0: \quad -2\,500\text{ N}\sin 30° + A - 3.37\times 10^3\text{ N}\,\sin 50° = 0$$

$$\boxed{A = 3.83\times 10^3\text{ N}}$$

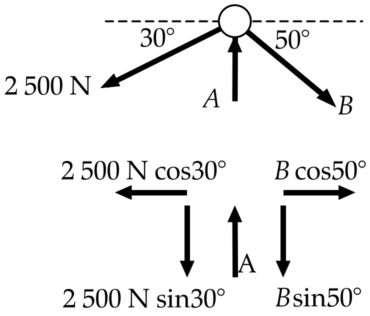


FIG. P5.27

Positive answers confirm that

$$\boxed{B\text{ is in tension and }A\text{ is in compression.}}$$

**P5.28** First, consider the 3.00 kg rising mass. The forces on it are the tension, $T$, and its weight, 29.4 N. With the upward direction as positive, the second law becomes

$$\sum F_y = ma_y: T - 29.4\text{ N} = (3.00\text{ kg})a \qquad (1)$$

The forces on the falling 5.00 kg mass are its weight and $T$, and its acceleration is the same as that of the rising mass. Calling the positive direction down for this mass, we have

$$\sum F_y = ma_y: 49\text{ N} - T = (5.00\text{ kg})a \qquad (2)$$

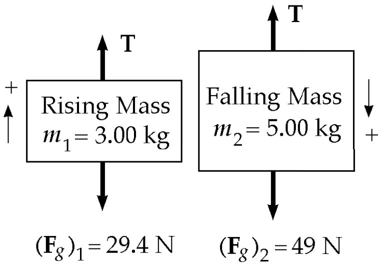


FIG. P5.28

Equations (1) and (2) can be solved simultaneously by adding them:

$$T - 29.4\text{ N} + 49.0\text{ N} - T = (3.00\text{ kg})a + (5.00\text{ kg})a$$

(b) This gives the acceleration as

$$a = \frac{19.6\text{ N}}{8.00\text{ kg}} = \boxed{2.45\text{ m/s}^2}.$$

(a) Then

$$T - 29.4\text{ N} = (3.00\text{ kg})(2.45\text{ m/s}^2) = 7.35\text{ N}.$$

The tension is

$$T = \boxed{36.8\text{ N}}.$$

(c) Consider either mass. We have

$$y = v_i t + \frac{1}{2}at^2 = 0 + \frac{1}{2}(2.45\text{ m/s}^2)(1.00\text{ s})^2 = \boxed{1.23\text{ m}}.$$

***P5.29** As the man rises steadily the pulley turns steadily and the tension in the rope is the same on both sides of the pulley. Choose man-pulley-and-platform as the system:

$$\sum F_y = ma_y$$
$$+T - 950\text{ N} = 0$$
$$T = 950\text{ N}.$$

The worker must pull on the rope with force $\boxed{950\text{ N}}$.

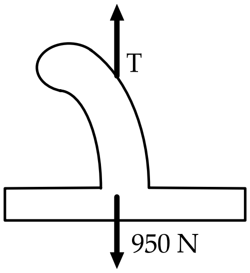


FIG. P5.29

***P5.30** Both blocks move with acceleration $a=\left(\frac{m_2-m_1}{m_2+m_1}\right)g$:

$$a=\left(\frac{7\text{ kg}-2\text{ kg}}{7\text{ kg}+2\text{ kg}}\right)9.8\text{ m/s}^2=5.44\text{ m/s}^2.$$

(a) Take the upward direction as positive for $m_1$.

$$v_{xf}^2=v_{xi}^2+2a_x\left(x_f-x_i\right):\quad 0=\left(-2.4\text{ m/s}\right)^2+2\left(5.44\text{ m/s}^2\right)\left(x_f-0\right)$$

$$x_f=-\frac{5.76\text{ m}^2/\text{s}^2}{2\left(5.44\text{ m/s}^2\right)}=-0.529\text{ m}$$

$$x_f=\boxed{0.529\text{ m below its initial level}}$$

(b) $v_{xf}=v_{xi}+a_xt$: $v_{xf}=-2.40\text{ m/s}+\left(5.44\text{ m/s}^2\right)(1.80\text{ s})$

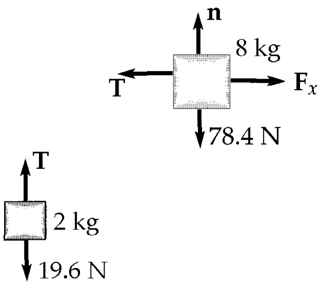
$$v_{xf}=\boxed{7.40\text{ m/s upward}}$$

**P5.31** Forces acting on 2.00 kg block:

$$T-m_1g=m_1a \qquad (1)$$

Forces acting on 8.00 kg block:

$$F_x-T=m_2a \qquad (2)$$



(a) Eliminate $T$ and solve for $a$:

$$a=\frac{F_x-m_1g}{m_1+m_2}$$

$$\boxed{a>0\text{ for }F_x>m_1g=19.6\text{ N}}.$$

(b) Eliminate $a$ and solve for $T$:

$$T=\frac{m_1}{m_1+m_2}\left(F_x+m_2g\right)$$

$$\boxed{T=0\text{ for }F_x\le-m_2g=-78.4\text{ N}}.$$

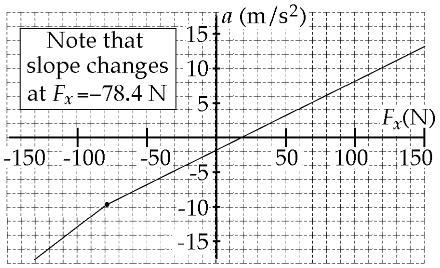


**FIG. P5.31**

(c)

| $F_x$, N | −100 | −78.4 | −50.0 | 0 | 50.0 | 100 |
|---|---|---|---|---|---|---|
| $a_x$, $\text{m/s}^2$ | −12.5 | −9.80 | −6.96 | −1.96 | 3.04 | 8.04 |

***P5.32** (a) For force components along the incline, with the upward direction taken as positive,

$$\sum F_x = ma_x: \quad -mg\sin\theta = ma_x$$
$$a_x = -g\sin\theta = -\left(9.8\ \text{m/s}^2\right)\sin 35° = -5.62\ \text{m/s}^2.$$

For the upward motion,

$$v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$$
$$0 = (5\ \text{m/s})^2 + 2\left(-5.62\ \text{m/s}^2\right)\left(x_f - 0\right)$$
$$x_f = \frac{25\ \text{m}^2/\text{s}^2}{2\left(5.62\ \text{m/s}^2\right)} = \boxed{2.22\ \text{m}}.$$

(b) The time to slide down is given by

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$
$$0 = 2.22\ \text{m} + 0 + \frac{1}{2}\left(-5.62\ \text{m/s}^2\right)t^2$$
$$t = \sqrt{\frac{2(2.22\ \text{m})}{5.62\ \text{m/s}^2}} = 0.890\ \text{s}.$$

For the second particle,

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$
$$0 = 10\ \text{m} + v_{xi}(0.890\ \text{s}) + \left(-5.62\ \text{m/s}^2\right)(0.890\ \text{s})^2$$
$$v_{xi} = \frac{-10\ \text{m} + 2.22\ \text{m}}{0.890\ \text{s}} = -8.74\ \text{m/s}$$
$$\text{speed} = \boxed{8.74\ \text{m/s}}.$$

**P5.33** First, we will compute the needed accelerations:

(1) Before it starts to move: $a_y = 0$

(2) During the first 0.800 s: $a_y = \dfrac{v_{yf} - v_{yi}}{t} = \dfrac{1.20 \text{ m/s} - 0}{0.800 \text{ s}} = 1.50 \text{ m/s}^2$

(3) While moving at constant velocity: $a_y = 0$

(4) During the last 1.50 s: $a_y = \dfrac{v_{yf} - v_{yi}}{t} = \dfrac{0 - 1.20 \text{ m/s}}{1.50 \text{ s}} = -0.800 \text{ m/s}^2$

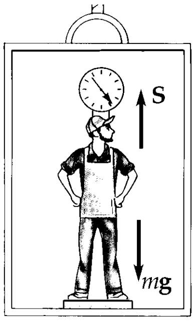


**FIG. P5.33**

Newton's second law is: $\sum F_y = ma_y$

$$+S - (72.0 \text{ kg})(9.80 \text{ m/s}^2) = (72.0 \text{ kg})a_y$$
$$S = 706 \text{ N} + (72.0 \text{ kg})a_y.$$

(a) When $a_y = 0$, $S = \boxed{706 \text{ N}}$.

(b) When $a_y = 1.50 \text{ m/s}^2$, $S = \boxed{814 \text{ N}}$.

(c) When $a_y = 0$, $S = \boxed{706 \text{ N}}$.

(d) When $a_y = -0.800 \text{ m/s}^2$, $S = \boxed{648 \text{ N}}$.

**P5.34** (a) Pulley $P_1$ has acceleration $a_2$.
Since $m_1$ moves *twice* the distance $P_1$ moves in the same time, $m_1$ has twice the acceleration of $P_1$, i.e., $\boxed{a_1 = 2a_2}$.

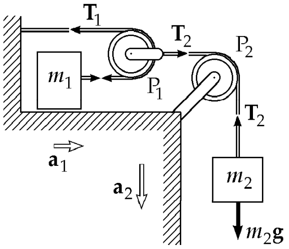


**FIG. P5.34**

(b) From the figure, and using

$$\sum F = ma: \quad m_2g - T_2 = m_2a_2 \qquad (1)$$
$$T_1 = m_1a_1 = 2m_1a_2 \qquad (2)$$
$$T_2 - 2T_1 = 0 \qquad (3)$$

Equation (1) becomes $m_2g - 2T_1 = m_2a_2$. This equation combined with Equation (2) yields

$$\frac{T_1}{m_1}\left(2m_1 + \frac{m_2}{2}\right) = m_2g$$

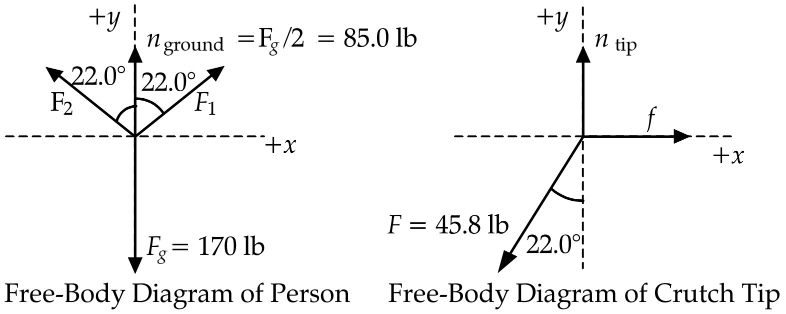
$$\boxed{T_1 = \frac{m_1m_2}{2m_1 + \frac{1}{2}m_2}g} \text{ and } \boxed{T_2 = \frac{m_1m_2}{m_1 + \frac{1}{4}m_2}g}.$$

(c) From the values of $T_1$ and $T_2$ we find that

$$a_1 = \frac{T_1}{m_1} = \boxed{\frac{m_2g}{2m_1 + \frac{1}{2}m_2}} \text{ and } a_2 = \frac{1}{2}a_1 = \boxed{\frac{m_2g}{4m_1 + m_2}}.$$

## Section 5.8 **Forces of Friction**

***P5.35**



+y
$n_{\text{ground}} = F_g/2 = 85.0$ lb
22.0°
22.0°
$F_2$
$F_1$
+x
$F_g = 170$ lb
Free-Body Diagram of Person
+y
n tip
f
+x
F = 45.8 lb
22.0°
Free-Body Diagram of Crutch Tip

**FIG. P5.35**

From the free-body diagram of the person,

$$\sum F_x = F_1\sin(22.0°) - F_2\sin(22.0°) = 0,$$

which gives

$$F_1 = F_2 = F.$$

Then, $\sum F_y = 2F\cos 22.0° + 85.0 \text{ lbs} - 170 \text{ lbs} = 0$ yields $F = 45.8$ lb.

(a) Now consider the free-body diagram of a crutch tip.

$$\sum F_x = f - (45.8 \text{ lb})\sin 22.0° = 0,$$

or

$$f = 17.2 \text{ lb}.$$

$$\sum F_y = n_{\text{tip}} - (45.8 \text{ lb})\cos 22.0° = 0,$$

which gives

$$n_{\text{tip}} = 42.5 \text{ lb}.$$

For minimum coefficient of friction, the crutch tip will be on the verge of slipping, so $f = (f_s)_{\max} = \mu_s n_{\text{tip}}$ and $\mu_s = \dfrac{f}{n_{\text{tip}}} = \dfrac{17.2 \text{ lb}}{42.5 \text{ lb}} = \boxed{0.404}$.

(b) As found above, the compression force in each crutch is

$$F_1 = F_2 = F = \boxed{45.8 \text{ lb}}.$$

**P5.36** For equilibrium: $f = F$ and $n = F_g$. Also, $f = \mu n$ i.e.,

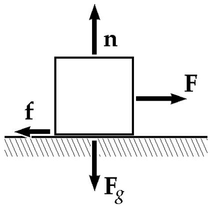


FIG. P5.36

$$\mu = \frac{f}{n} = \frac{F}{F_g}$$

$$\mu_s = \frac{75.0\text{ N}}{25.0(9.80)\text{ N}} = \boxed{0.306}$$

and

$$\mu_k = \frac{60.0\text{ N}}{25.0(9.80)\text{ N}} = \boxed{0.245}.$$

**P5.37** $\sum F_y = ma_y$: $+n - mg = 0$

$f_s \le \mu_s n = \mu_s mg$

This maximum magnitude of static friction acts so long as the tires roll without skidding.

$$\sum F_x = ma_x: \quad -f_s = ma$$

The maximum acceleration is

$$a = -\mu_s g.$$

The initial and final conditions are: $x_i = 0$, $v_i = 50.0\text{ mi/h} = 22.4\text{ m/s}$, $v_f = 0$

$$v_f^2 = v_i^2 + 2a\left(x_f - x_i\right): \quad -v_i^2 = -2\mu_s g x_f$$

(a) $$x_f = \frac{v_i^2}{2\mu g}$$

$$x_f = \frac{(22.4\text{ m/s})^2}{2(0.100)\left(9.80\text{ m/s}^2\right)} = \boxed{256\text{ m}}$$

(b) $$x_f = \frac{v_i^2}{2\mu g}$$

$$x_f = \frac{(22.4\text{ m/s})^2}{2(0.600)\left(9.80\text{ m/s}^2\right)} = \boxed{42.7\text{ m}}$$

**P5.38** If all the weight is on the rear wheels,

(a) $F = ma$: $\mu_s mg = ma$

But

$$\Delta x = \frac{at^2}{2} = \frac{\mu_s g t^2}{2}$$

so $\mu_s = \dfrac{2\Delta x}{gt^2}$:

$$\mu_s = \frac{2(0.250\text{ mi})(1\,609\text{ m/mi})}{\left(9.80\text{ m/s}^2\right)(4.96\text{ s})^2} = \boxed{3.34}.$$

(b) Time would increase, as the wheels would skid and only kinetic friction would act; or perhaps the car would flip over.

***P5.39** (a) The person pushes backward on the floor. The floor pushes forward on the person with a force of friction. This is the only horizontal force on the person. If the person's shoe is on the point of slipping the static friction force has its maximum value.

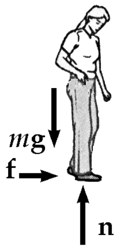


**FIG. P5.39**

$$\sum F_x = ma_x: \qquad f = \mu_s n = ma_x$$
$$\sum F_y = ma_y: \qquad n - mg = 0$$
$$ma_x = \mu_s mg \qquad a_x = \mu_s g = 0.5\left(9.8\text{ m/s}^2\right) = 4.9\text{ m/s}^2$$
$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 \qquad 3\text{ m} = 0 + 0 + \frac{1}{2}\left(4.9\text{ m/s}^2\right)t^2$$
$$t = \boxed{1.11\text{ s}}$$

(b) $x_f = \dfrac{1}{2}\mu_s g t^2$, $t = \sqrt{\dfrac{2x_f}{\mu_s g}} = \sqrt{\dfrac{2(3\text{ m})}{(0.8)\left(9.8\text{ m/s}^2\right)}} = \boxed{0.875\text{ s}}$

**P5.40** $m_{\text{suitcase}} = 20.0\text{ kg}$, $F = 35.0\text{ N}$

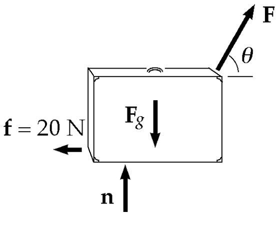


**FIG. P5.40**

$$\sum F_x = ma_x: \quad -20.0\text{ N} + F\cos\theta = 0$$
$$\sum F_y = ma_y: \quad +n + F\sin\theta - F_g = 0$$

(a) $F\cos\theta = 20.0\text{ N}$

$$\cos\theta = \frac{20.0\text{ N}}{35.0\text{ N}} = 0.571$$
$$\boxed{\theta = 55.2°}$$

(b) $n = F_g - F\sin\theta = [196 - 35.0(0.821)]\text{ N}$

$$\boxed{n = 167\text{ N}}$$

**P5.41** $m = 3.00\text{ kg}$, $\theta = 30.0°$, $x = 2.00\text{ m}$, $t = 1.50\text{ s}$

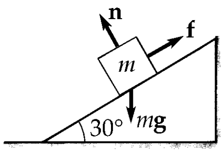


FIG. P5.41

(a) $x = \frac{1}{2}at^2$:

$$2.00\text{ m} = \frac{1}{2}a(1.50\text{ s})^2$$
$$a = \frac{4.00}{(1.50)^2} = \boxed{1.78\text{ m/s}^2}$$

$\sum \mathbf{F} = \mathbf{n} + \mathbf{f} + m\mathbf{g} = m\mathbf{a}$:

$$\begin{aligned} \text{Along } x\text{:}\quad & 0 - f + mg\sin 30.0° = ma \\ & f = m(g\sin 30.0° - a) \\ \text{Along } y\text{:}\quad & n + 0 - mg\cos 30.0° = 0 \\ & n = mg\cos 30.0° \end{aligned}$$

(b) $\mu_k = \frac{f}{n} = \frac{m(g\sin 30.0° - a)}{mg\cos 30.0°}$, $\mu_k = \tan 30.0° - \frac{a}{g\cos 30.0°} = \boxed{0.368}$

(c) $f = m(g\sin 30.0° - a)$, $f = 3.00(9.80\sin 30.0° - 1.78) = \boxed{9.37\text{ N}}$

(d) $v_f^2 = v_i^2 + 2a(x_f - x_i)$

where

$$x_f - x_i = 2.00\text{ m}$$

$$v_f^2 = 0 + 2(1.78)(2.00) = 7.11\text{ m}^2/\text{s}^2$$
$$v_f = \sqrt{7.11\text{ m}^2/\text{s}^2} = \boxed{2.67\text{ m/s}}$$

***P5.42** First we find the coefficient of friction:

$$\sum F_y = 0: \qquad +n - mg = 0$$

$$f = \mu_s n = \mu_s mg$$

$$\sum F_x = ma_x: \qquad v_f^2 = v_i^2 + 2a_x \Delta x = 0$$

$$-\mu_s mg = -\frac{mv_i^2}{2\Delta x}$$

$$\mu_s = \frac{v_i^2}{2g\Delta x} = \frac{(88\ \text{ft/s})^2}{2(32.1\ \text{ft/s}^2)(123\ \text{ft})} = 0.981$$

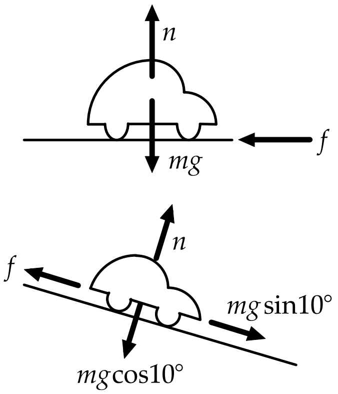


FIG. P5.42

Now on the slope

$$\sum F_y = 0: \qquad +n - mg\cos 10° = 0$$

$$f_s = \mu_s n = \mu_s mg\cos 10°$$

$$\sum F_x = ma_x: \quad -\mu_s mg\cos 10° + mg\sin 10° = -\frac{mv_i^2}{2\Delta x}$$

$$\Delta x = \frac{v_i^2}{2g(\mu_s\cos 10° - \sin 10°)}$$

$$= \frac{(88\ \text{ft/s})^2}{2(32.1\ \text{ft/s}^2)(0.981\cos 10° - \sin 10°)} = \boxed{152\ \text{ft}}.$$

**P5.43** $T - f_k = 5.00a$ (for 5.00 kg mass)

$9.00g - T = 9.00a$ (for 9.00 kg mass)

Adding these two equations gives:

$$9.00(9.80) - 0.200(5.00)(9.80) = 14.0a$$

$$a = 5.60\ \text{m/s}^2$$

$$\therefore T = 5.00(5.60) + 0.200(5.00)(9.80)$$

$$= \boxed{37.8\ \text{N}}$$

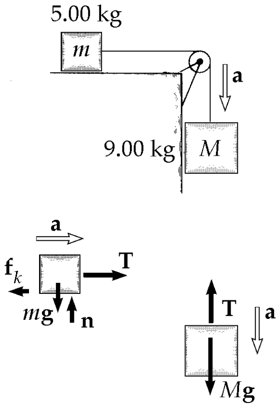


FIG. P5.43

**P5.44** Let $a$ represent the positive magnitude of the acceleration $-a\hat{\mathbf{j}}$ of $m_1$, of the acceleration $-a\hat{\mathbf{i}}$ of $m_2$, and of the acceleration $+a\hat{\mathbf{j}}$ of $m_3$. Call $T_{12}$ the tension in the left rope and $T_{23}$ the tension in the cord on the right.

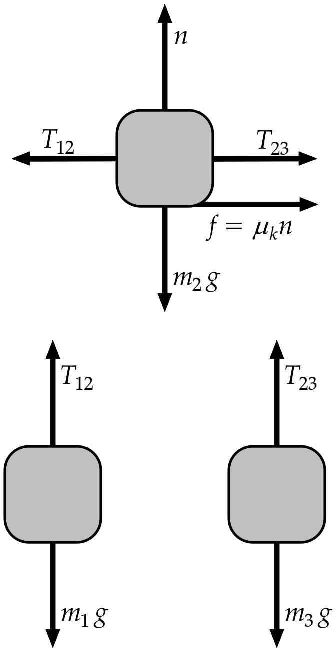


FIG. P5.44

For $m_1$, $\quad \sum F_y = ma_y \quad +T_{12} - m_1 g = -m_1 a$

For $m_2$, $\quad \sum F_x = ma_x \quad -T_{12} + \mu_k n + T_{23} = -m_2 a$

and $\quad \sum F_y = ma_y \quad n - m_2 g = 0$

for $m_3$, $\quad \sum F_y = ma_y \quad T_{23} - m_3 g = +m_3 a$

we have three simultaneous equations

$$-T_{12} + 39.2\text{ N} = (4.00\text{ kg})a$$
$$+T_{12} - 0.350(9.80\text{ N}) - T_{23} = (1.00\text{ kg})a$$
$$+T_{23} - 19.6\text{ N} = (2.00\text{ kg})a.$$

(a) Add them up:

$$+39.2\text{ N} - 3.43\text{ N} - 19.6\text{ N} = (7.00\text{ kg})a$$

$$a = \boxed{2.31\text{ m/s}^2\text{, down for } m_1\text{, left for } m_2\text{, and up for } m_3}.$$

(b) Now $-T_{12} + 39.2\text{ N} = (4.00\text{ kg})(2.31\text{ m/s}^2)$

$$\boxed{T_{12} = 30.0\text{ N}}$$

and $T_{23} - 19.6\text{ N} = (2.00\text{ kg})(2.31\text{ m/s}^2)$

$$\boxed{T_{23} = 24.2\text{ N}}.$$

**P5.45** (a) See Figure to the right

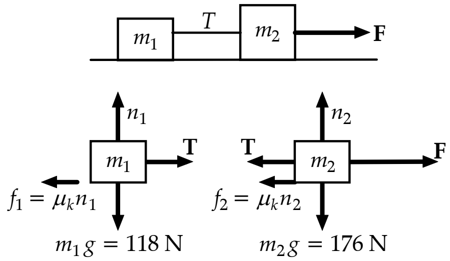


FIG. P5.45

(b) $68.0 - T - \mu m_2 g = m_2 a$ (Block #2)
$T - \mu m_1 g = m_1 a$ (Block #1)

Adding,

$$68.0 - \mu(m_1 + m_2)g = (m_1 + m_2)a$$
$$a = \frac{68.0}{(m_1 + m_2)} - \mu g = \boxed{1.29\text{ m/s}^2}$$
$$T = m_1 a + \mu m_1 g = \boxed{27.2\text{ N}}$$

**P5.46** (Case 1, impending upward motion)
Setting

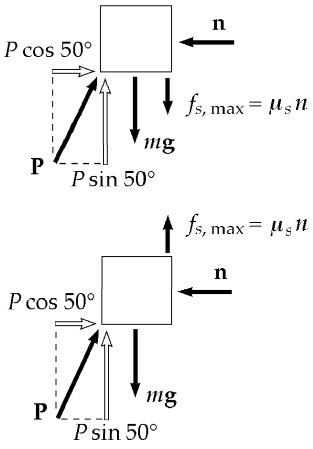


**FIG. P5.46**

$$\begin{aligned} \sum F_x &= 0: & P\cos 50.0° - n &= 0 \\ f_{s,\max} &= \mu_s n: & f_{s,\max} &= \mu_s P\cos 50.0° \\ & & &= 0.250(0.643)P = 0.161P \end{aligned}$$

Setting

$$\sum F_y = 0: \quad P\sin 50.0° - 0.161P - 3.00(9.80) = 0$$
$$P_{\max} = \boxed{48.6\text{ N}}$$

(Case 2, impending downward motion)
As in Case 1,

$$f_{s,\max} = 0.161P$$

Setting

$$\sum F_y = 0: \quad P\sin 50.0° + 0.161P - 3.00(9.80) = 0$$
$$P_{\min} = \boxed{31.7\text{ N}}$$

***P5.47** When the sled is sliding uphill

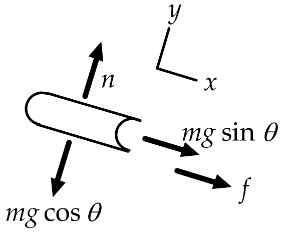


**FIG. P5.47**

$$\begin{aligned} \sum F_y = ma_y:& \quad +n - mg\cos\theta = 0 \\ & \quad f = \mu_k n = \mu_k mg\cos\theta \\ \sum F_x = ma_x:& \quad +mg\sin\theta + \mu_k mg\cos\theta = ma_{\text{up}} \\ & \quad v_f = 0 = v_i + a_{\text{up}} t_{\text{up}} \\ & \quad v_i = -a_{\text{up}} t_{\text{up}} \end{aligned}$$

$$\Delta x = \frac{1}{2}\left(v_i + v_f\right)t_{\text{up}}$$
$$\Delta x = \frac{1}{2}\left(a_{\text{up}} t_{\text{up}} + 0\right)t_{\text{up}} = \frac{1}{2} a_{\text{up}} t_{\text{up}}^2$$

When the sled is sliding down, the direction of the friction force is reversed:

$$mg\sin\theta - \mu_k mg\cos\theta = ma_{\text{down}}$$
$$\Delta x = \frac{1}{2} a_{\text{down}} t_{\text{down}}^2 .$$

Now

$$t_{\text{down}} = 2t_{\text{up}}$$
$$\frac{1}{2} a_{\text{up}} t_{\text{up}}^2 = \frac{1}{2} a_{\text{down}} \left(2t_{\text{up}}\right)^2$$
$$a_{\text{up}} = 4a_{\text{down}}$$
$$g\sin\theta + \mu_k g\cos\theta = 4\left(g\sin\theta - \mu_k g\cos\theta\right)$$
$$5\mu_k\cos\theta = 3\sin\theta$$
$$\boxed{\mu_k = \left(\frac{3}{5}\right)\tan\theta}$$

***P5.48** Since the board is in equilibrium, $\sum F_x = 0$ and we see that the normal forces must be the same on both sides of the board. Also, if the minimum normal forces (compression forces) are being applied, the board is on the verge of slipping and the friction force on each side is

$$f = (f_s)_{\max} = \mu_s n.$$

The board is also in equilibrium in the vertical direction, so

$$\sum F_y = 2f - F_g = 0, \text{ or } f = \frac{F_g}{2}.$$

The minimum compression force needed is then

$$n = \frac{f}{\mu_s} = \frac{F_g}{2\mu_s} = \frac{95.5 \text{ N}}{2(0.663)} = \boxed{72.0 \text{ N}}.$$

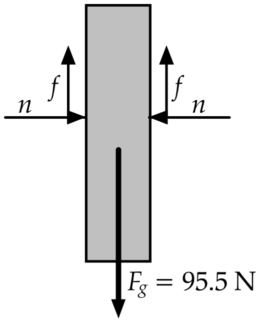


FIG. P5.48

***P5.49** (a) $n + F\sin 15° - (75 \text{ N})\cos 25° = 0$

$\therefore n = 67.97 - 0.259F$

$f_{s,\,\max} = \mu_s n = 24.67 - 0.094F$

For equilibrium: $F\cos 15° + 24.67 - 0.094F - 75\sin 25° = 0$.

This gives $\boxed{F = 8.05 \text{ N}}$.

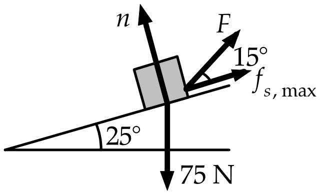


FIG. P5.49(a)

(b) $F\cos 15° - (24.67 - 0.094F) - 75\sin 25° = 0$.

This gives $\boxed{F = 53.2 \text{ N}}$.

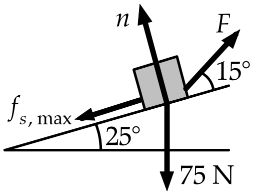


FIG. P5.49(b)

(c) $f_k = \mu_k n = 10.6 - 0.040F$. Since the velocity is constant, the net force is zero:

$$F\cos 15° - (10.6 - 0.040F) - 75\sin 25° = 0.$$

This gives $\boxed{F = 42.0 \text{ N}}$.

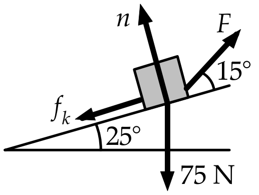


FIG. P5.49(c)

***P5.50** We must consider separately the disk when it is in contact with the roof and when it has gone over the top into free fall. In the first case, we take $x$ and $y$ as parallel and perpendicular to the surface of the roof:

$$\sum F_y = ma_y: \quad +n - mg\cos\theta = 0$$
$$n = mg\cos\theta$$

then friction is $f_k = \mu_k n = \mu_k mg\cos\theta$

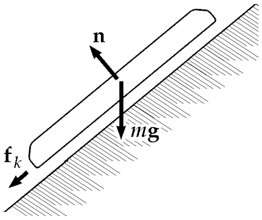


**FIG. P5.50**

$$\sum F_x = ma_x: \quad -f_k - mg\sin\theta = ma_x$$
$$a_x = -\mu_k g\cos\theta - g\sin\theta = (-0.4\cos 37° - \sin 37°)9.8\ \text{m/s}^2 = -9.03\ \text{m/s}^2$$

The Frisbee goes ballistic with speed given by

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i) = (15\ \text{m/s})^2 + 2(-9.03\ \text{m/s}^2)(10\ \text{m} - 0) = 44.4\ \text{m}^2/\text{s}^2$$
$$v_{xf} = 6.67\ \text{m/s}$$

For the free fall, we take $x$ and $y$ horizontal and vertical:

$$v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$$
$$0 = (6.67\ \text{m/s}\ \sin 37°)^2 + 2(-9.8\ \text{m/s}^2)(y_f - 10\ \text{m}\ \sin 37°)$$
$$y_f = 6.02\ \text{m} + \frac{(4.01\ \text{m/s})^2}{19.6\ \text{m/s}^2} = \boxed{6.84\ \text{m}}$$

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## Additional Problems

**P5.51** (a) see figure to the right

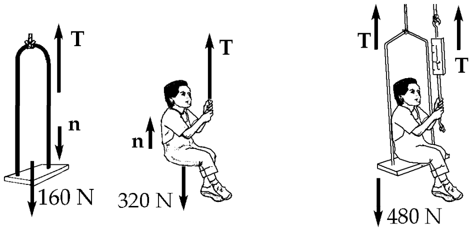


**FIG. P5.51**

(b) First consider Pat and the chair as the system. Note that *two* ropes support the system, and $T = 250$ N in each rope. Applying $\sum F = ma$

$$2T - 480 = ma, \text{ where } m = \frac{480}{9.80} = 49.0\ \text{kg}.$$

Solving for $a$ gives

$$a = \frac{500 - 480}{49.0} = \boxed{0.408\ \text{m/s}^2}.$$

(c) $\sum F = ma$ on Pat:

$$\sum F = n + T - 320 = ma, \text{ where } m = \frac{320}{9.80} = 32.7\ \text{kg}$$

$$n = ma + 320 - T = 32.7(0.408) + 320 - 250 = \boxed{83.3\ \text{N}}.$$

**P5.52** $\sum \mathbf{F} = m\mathbf{a}$ gives the object's acceleration

$$\mathbf{a} = \frac{\sum F}{m} = \frac{\left(8.00\hat{\mathbf{i}} - 4.00t\hat{\mathbf{j}}\right)\text{ N}}{2.00\text{ kg}}$$

$$\mathbf{a} = \left(4.00\text{ m/s}^2\right)\hat{\mathbf{i}} - \left(2.00\text{ m/s}^3\right)t\hat{\mathbf{j}} = \frac{d\mathbf{v}}{dt}.$$

Its velocity is

$$\int_{v_i}^{v} d\mathbf{v} = \mathbf{v} - \mathbf{v}_i = \mathbf{v} - 0 = \int_0^t \mathbf{a}dt$$

$$\mathbf{v} = \int_0^t \left[\left(4.00\text{ m/s}^2\right)\hat{\mathbf{i}} - \left(2.00\text{ m/s}^3\right)t\hat{\mathbf{j}}\right]dt$$

$$\mathbf{v} = \left(4.00t\text{ m/s}^2\right)\hat{\mathbf{i}} - \left(1.00t^2\text{ m/s}^3\right)\hat{\mathbf{j}}.$$

(a) We require $|\mathbf{v}| = 15.0\text{ m/s}$, $|\mathbf{v}|^2 = 225\text{ m}^2/\text{s}^2$

$$16.0t^2\text{ m}^2/\text{s}^4 + 1.00t^4\text{ m}^2/\text{s}^6 = 225\text{ m}^2/\text{s}^2$$

$$1.00t^4 + 16.0\text{ s}^2t^2 - 225\text{ s}^4 = 0$$

$$t^2 = \frac{-16.0 \pm \sqrt{(16.0)^2 - 4(-225)}}{2.00} = 9.00\text{ s}^2$$

$$t = \boxed{3.00\text{ s}}.$$

Take $\mathbf{r}_i = 0$ at $t = 0$. The position is

$$\mathbf{r} = \int_0^t \mathbf{v}dt = \int_0^t \left[\left(4.00t\text{ m/s}^2\right)\hat{\mathbf{i}} - \left(1.00t^2\text{ m/s}^3\right)\hat{\mathbf{j}}\right]dt$$

$$\mathbf{r} = \left(4.00\text{ m/s}^2\right)\frac{t^2}{2}\hat{\mathbf{i}} - \left(1.00\text{ m/s}^3\right)\frac{t^3}{3}\hat{\mathbf{j}}$$

at $t = 3$ s we evaluate.

(c) $\mathbf{r} = \boxed{\left(18.0\hat{\mathbf{i}} - 9.00\hat{\mathbf{j}}\right)\text{ m}}$

(b) So $|\mathbf{r}| = \sqrt{(18.0)^2 + (9.00)^2}\text{ m} = \boxed{20.1\text{ m}}$

***P5.53** (a) Situation A

$$\sum F_x = ma_x: \quad F_A + \mu_s n - mg\sin\theta = 0$$
$$\sum F_y = ma_y: \quad +n - mg\cos\theta = 0$$

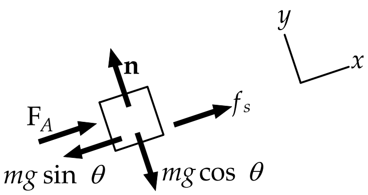


**FIG. P5.53(a)**

Eliminate $n = mg\cos\theta$ to solve for

$$\boxed{F_A = mg(\sin\theta - \mu_s\cos\theta)}.$$

(b) Situation B

$$\sum F_x = ma_x: \quad F_B\cos\theta + \mu_s n - mg\sin\theta = 0$$
$$\sum F_y = ma_y: \quad -F_B\sin\theta + n - mg\cos\theta = 0$$

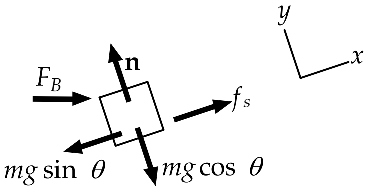


**FIG. P5.53(b)**

Substitute $n = mg\cos\theta + F_B\sin\theta$ to find

$$F_B\cos\theta + \mu_s mg\cos\theta + \mu_s F_B\sin\theta - mg\sin\theta = 0$$
$$\boxed{F_B = \frac{mg(\sin\theta - \mu_s\cos\theta)}{\cos\theta + \mu_s\sin\theta}}$$

(c) $F_A = 2\text{ kg } 9.8\text{ m/s}^2(\sin 25° - 0.16\cos 25°) = 5.44\text{ N}$

$$F_B = \frac{19.6\text{ N}(0.278)}{\cos 25° + 0.16\sin 25°} = 5.59\text{ N}$$

Student $\boxed{A}$ need exert less force.

(d) $$F_B = \frac{F_A}{\cos 25° + 0.38\sin 25°} = \frac{F_A}{1.07}$$

Student $\boxed{B}$ need exert less force.

**P5.54**

$$18\text{ N} - P = (2\text{ kg})a$$
$$P - Q = (3\text{ kg})a$$
$$Q = (4\text{ kg})a$$

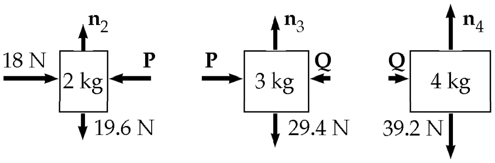


**FIG. P5.54**

Adding gives $18\text{ N} = (9\text{ kg})a$ so

$$a = \boxed{2.00\text{ m/s}^2}.$$

(b) $Q = 4\text{ kg}(2\text{ m/s}^2) = \boxed{8.00\text{ N net force on the 4 kg}}$

$P - 8\text{ N} = 3\text{ kg}(2\text{ m/s}^2) = \boxed{6.00\text{ N net force on the 3 kg}}$ and $P = 14\text{ N}$

$18\text{ N} - 14\text{ N} = 2\text{ kg}(2\text{ m/s}^2) = \boxed{4.00\text{ N net force on the 2 kg}}$

*continued on next page*

(c) From above, $Q = \boxed{8.00\ \text{N}}$ and $P = \boxed{14.0\ \text{N}}$.

(d) The 3-kg block models the heavy block of wood. The contact force on your back is represented by $Q$, which is much less than the force $F$. The difference between $F$ and $Q$ is the net force causing acceleration of the 5-kg pair of objects. The acceleration is real and nonzero, but lasts for so short a time that it never is associated with a large velocity. The frame of the building and your legs exert forces, small relative to the hammer blow, to bring the partition, block, and you to rest again over a time large relative to the hammer blow. This problem lends itself to interesting lecture demonstrations. One person can hold a lead brick in one hand while another hits the brick with a hammer.

**P5.55** (a) First, we note that $F = T_1$. Next, we focus on the mass $M$ and write $T_5 = Mg$. Next, we focus on the bottom pulley and write $T_5 = T_2 + T_3$. Finally, we focus on the top pulley and write $T_4 = T_1 + T_2 + T_3$.

Since the pulleys are not starting to rotate and are frictionless, $T_1 = T_3$, and $T_2 = T_3$. From this information, we have $T_5 = 2T_2$, so $T_2 = \frac{Mg}{2}$.

Then $\boxed{T_1 = T_2 = T_3 = \frac{Mg}{2}}$, and $\boxed{T_4 = \frac{3Mg}{2}}$, and $\boxed{T_5 = Mg}$.

(b) Since $F = T_1$, we have $\boxed{F = \frac{Mg}{2}}$.

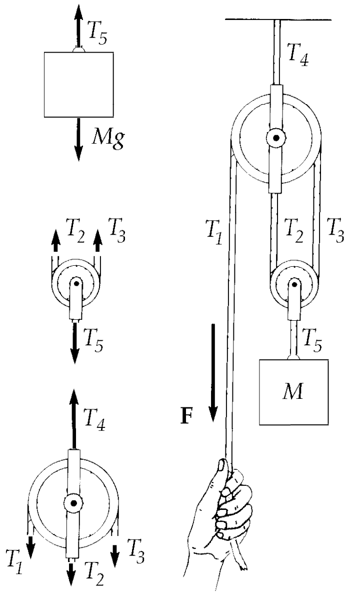


**FIG. P5.55**

**P5.56** We find the diver's impact speed by analyzing his free-fall motion:

$$v_f^2 = v_i^2 + 2ax = 0 + 2\left(-9.80\ \text{m/s}^2\right)(-10.0\ \text{m}) \text{ so } v_f = -14.0\ \text{m/s}.$$

Now for the 2.00 s of stopping, we have $v_f = v_i + at$:

$$0 = -14.0\ \text{m/s} + a(2.00\ \text{s})$$
$$a = +7.00\ \text{m/s}^2.$$

Call the force exerted by the water on the diver $R$. Using $\sum F_y = ma$,

$$+R - 70.0\ \text{kg}\left(9.80\ \text{m/s}^2\right) = 70.0\ \text{kg}\left(7.00\ \text{m/s}^2\right)$$
$$R = \boxed{1.18\ \text{kN}}.$$

**P5.57** (a) The crate is in equilibrium, just before it starts to move. Let the normal force acting on it be $n$ and the friction force, $f_s$.

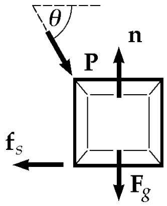


**FIG. P5.57**

Resolving vertically:

$$n = F_g + P\sin\theta$$

Horizontally:

$$P\cos\theta = f_s$$

But,

$$f_s \le \mu_s n$$

i.e.,

$$P\cos\theta \le \mu_s\left(F_g + P\sin\theta\right)$$

or

$$P(\cos\theta - \mu_s\sin\theta) \le \mu_s F_g.$$

Divide by $\cos\theta$:

$$P(1 - \mu_s\tan\theta) \le \mu_s F_g\sec\theta.$$

Then

$$\boxed{P_{\text{minimum}} = \frac{\mu_s F_g \sec\theta}{1 - \mu_s\tan\theta}}.$$

(b) $P = \dfrac{0.400(100\text{ N})\sec\theta}{1 - 0.400\tan\theta}$

| $\theta(\text{deg})$ | 0.00 | 15.0 | 30.0 | 45.0 | 60.0 |
|---|---|---|---|---|---|
| $P(N)$ | 40.0 | 46.4 | 60.1 | 94.3 | 260 |

If the angle were 68.2° or more, the expression for $P$ would go to infinity and motion would become impossible.

**P5.58** (a) Following the in-chapter Example about a block on a frictionless incline, we have

$$a = g\sin\theta = (9.80\ \text{m/s}^2)\sin 30.0°$$

$$\boxed{a = 4.90\ \text{m/s}^2}$$

(b) The block slides distance $x$ on the incline, with $\sin 30.0° = \dfrac{0.500\ \text{m}}{x}$

$x = 1.00\ \text{m}$: $v_f^2 = v_i^2 + 2a(x_f - x_i) = 0 + 2(4.90\ \text{m/s}^2)(1.00\ \text{m})$

$v_f = \boxed{3.13\ \text{m/s}}$ after time $t_s = \dfrac{2x_f}{v_f} = \dfrac{2(1.00\ \text{m})}{3.13\ \text{m/s}} = 0.639\ \text{s}$.

(c) Now in free fall $y_f - y_i = v_{yi}t + \dfrac{1}{2}a_y t^2$:

$$-2.00 = (-3.13\ \text{m/s})\sin 30.0°\,t - \frac{1}{2}(9.80\ \text{m/s}^2)t^2$$

$$(4.90\ \text{m/s}^2)t^2 + (1.56\ \text{m/s})t - 2.00\ \text{m} = 0$$

$$t = \frac{-1.56\ \text{m/s} \pm \sqrt{(1.56\ \text{m/s})^2 - 4(4.90\ \text{m/s}^2)(-2.00\ \text{m})}}{9.80\ \text{m/s}^2}$$

Only one root is physical

$$t = 0.499\ \text{s}$$

$$x_f = v_x t = \left[(3.13\ \text{m/s})\cos 30.0°\right](0.499\ \text{s}) = \boxed{1.35\ \text{m}}$$

(d) total time $= t_s + t = 0.639\ \text{s} + 0.499\ \text{s} = \boxed{1.14\ \text{s}}$

(e) The mass of the block makes no difference.

**P5.59** With motion impending,

$$n + T\sin\theta - mg = 0$$

$$f = \mu_s\left(mg - T\sin\theta\right)$$

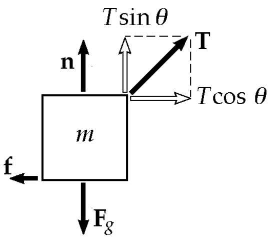


**FIG. P5.59**

and

$$T\cos\theta - \mu_s mg + \mu_s T\sin\theta = 0$$

so

$$T = \frac{\mu_s mg}{\cos\theta + \mu_s\sin\theta}.$$

To minimize $T$, we maximize $\cos\theta + \mu_s\sin\theta$

$$\frac{d}{d\theta}\left(\cos\theta + \mu_s\sin\theta\right) = 0 = -\sin\theta + \mu_s\cos\theta.$$

(a) $\theta = \tan^{-1}\mu_s = \tan^{-1}0.350 = \boxed{19.3°}$

(b) $T = \dfrac{0.350(1.30\text{ kg})\left(9.80\text{ m/s}^2\right)}{\cos 19.3° + 0.350\sin 19.3°} = \boxed{4.21\text{ N}}$

***P5.60** (a) See Figure (a) to the right.

(b) See Figure (b) to the right.

(c) For the pin,

$$\sum F_y = ma_y:\quad C\cos\theta - 357\text{ N} = 0$$

$$C = \frac{357\text{ N}}{\cos\theta}.$$

For the foot,

$m\mathbf{g} = (36.4\text{ kg})\left(9.8\text{ m/s}^2\right) = 357\text{ N}$

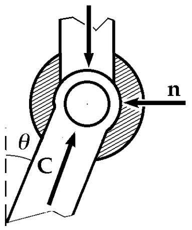


**FIG. P5.60(a)**

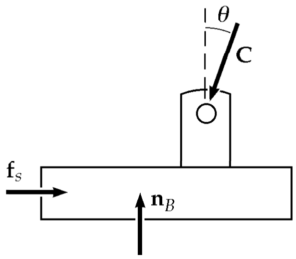


**FIG. P5.60(b)**

$$\sum F_y = ma_y:\quad +n_B - C\cos\theta = 0$$

$$n_B = \boxed{357\text{ N}}.$$

(d) For the foot with motion impending,

$$\sum F_x = ma_x:\quad +f_s - C\sin\theta_s = 0$$

$$\mu_s n_B = C\sin\theta_s$$

$$\mu_s = \frac{C\sin\theta_s}{n_B} = \frac{\left(357\text{ N}/\cos\theta_s\right)\sin\theta_s}{357\text{ N}} = \tan\theta_s.$$

(e) The maximum coefficient is

$$\mu_s = \tan\theta_s = \tan 50.2° = \boxed{1.20}.$$

**P5.61** $\sum F = ma$

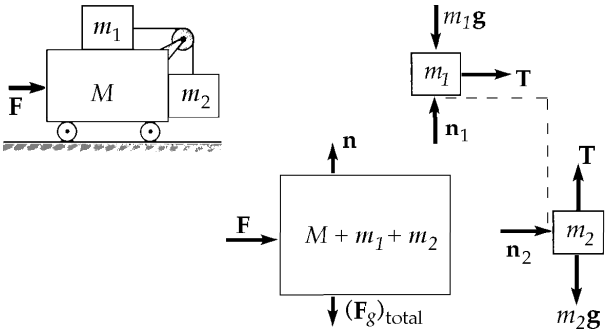


**FIG. P5.61**

For $m_1$: $T = m_1 a$

For $m_2$: $T - m_2 g = 0$

Eliminating $T$,

$$a = \frac{m_2 g}{m_1}$$

For all 3 blocks:

$$F = (M + m_1 + m_2)a = \boxed{(M + m_1 + m_2)\left(\frac{m_2 g}{m_1}\right)}$$

**P5.62**

| $t$(s) | $t^2\left(\text{s}^2\right)$ | $x$(m) |
|---|---|---|
| 0 | 0 | 0 |
| 1.02 | 1.040 | 0.100 |
| 1.53 | 2.341 | 0.200 |
| 2.01 | 4.040 | 0.350 |
| 2.64 | 6.970 | 0.500 |
| 3.30 | 10.89 | 0.750 |
| 3.75 | 14.06 | 1.00 |

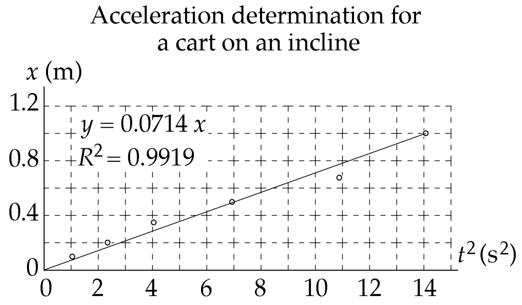


**FIG. P5.62**

From $x = \frac{1}{2}at^2$ the slope of a graph of $x$ versus $t^2$ is $\frac{1}{2}a$, and

$$a = 2 \times \text{slope} = 2\left(0.0714 \text{ m/s}^2\right) = \boxed{0.143 \text{ m/s}^2}.$$

From $a' = g\sin\theta$,

$$a' = 9.80 \text{ m/s}^2\left(\frac{1.774}{127.1}\right) = 0.137 \text{ m/s}^2\text{, different by 4\%.}$$

The difference is accounted for by the uncertainty in the data, which we may estimate from the third point as

$$\frac{0.350 - (0.0714)(4.04)}{0.350} = 18\%.$$

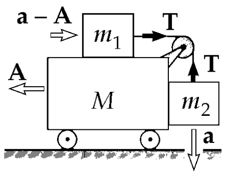


FIG. P5.63

**P5.63** (1) $m_1(a-A)=T \Rightarrow a=\dfrac{T}{m_1}+A$

(2) $MA=R_x=T \Rightarrow A=\dfrac{T}{M}$

(3) $m_2a=m_2g-T \Rightarrow T=m_2(g-a)$

(a) Substitute the value for $a$ from (1) into (3) and solve for $T$:

$$T=m_2\left[g-\left(\frac{T}{m_1}+A\right)\right].$$

Substitute for $A$ from (2):

$$T=m_2\left[g-\left(\frac{T}{m_1}+\frac{T}{M}\right)\right]=\boxed{m_2g\left[\frac{m_1M}{m_1M+m_2(m_1+M)}\right]}.$$

(b) Solve (3) for $a$ and substitute value of $T$:

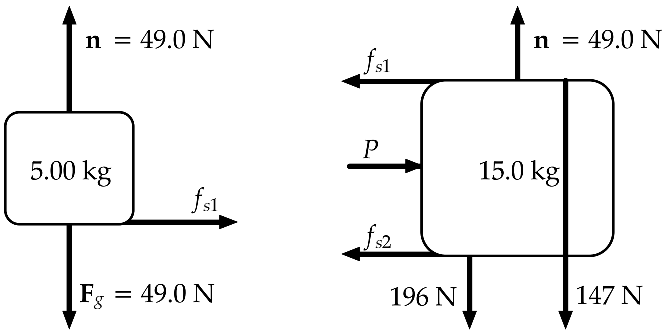
$$\boxed{a=\frac{m_2g(m_1+M)}{m_1M+m_2(M+m_1)}}.$$

(c) From (2), $A=\dfrac{T}{M}$, Substitute the value of $T$:

$$\boxed{A=\frac{m_1m_2g}{m_1M+m_2(m_1+M)}}.$$

(d) $\boxed{a-A=\dfrac{Mm_2g}{m_1M+m_2(m_1+M)}}$

**P5.64** (a), (b) Motion impending



$f_{s1} = \mu n = 14.7\text{ N}$ $\qquad f_{s2} = 0.500(196\text{ N}) = 98.0\text{ N}$

**FIG. P5.64**

$$P = f_{s1} + f_{s2} = 14.7\text{ N} + 98.0\text{ N} = \boxed{113\text{ N}}$$

(c) Once motion starts, kinetic friction acts.

$$112.7\text{ N} - 0.100(49.0\text{ N}) - 0.400(196\text{ N}) = (15.0\text{ kg})a_2$$

$$a_2 = \boxed{1.96\text{ m/s}^2}$$

$$0.100(49.0\text{ N}) = (5.00\text{ kg})a_1$$

$$a_1 = \boxed{0.980\text{ m/s}^2}$$

***P5.65** (a) Let $x$ represent the position of the glider along the air track. Then $z^2 = x^2 + h_0^2$, $x = \left(z^2 - h_0^2\right)^{1/2}$, $v_x = \frac{dx}{dt} = \frac{1}{2}\left(z^2 - h_0^2\right)^{-1/2}(2z)\frac{dz}{dt}$. Now $\frac{dz}{dt}$ is the rate at which string passes over the pulley, so it is equal to $v_y$ of the counterweight.

$$v_x = z\left(z^2 - h_0^2\right)^{-1/2} v_y = uv_y$$

(b) $a_x = \frac{dv_x}{dt} = \frac{d}{dt}uv_y = u\frac{dv_y}{dt} + v_y\frac{du}{dt}$ at release from rest, $v_y = 0$ and $a_x = ua_y$.

(c) $\sin 30.0° = \frac{80.0\text{ cm}}{z}$, $z = 1.60\text{ m}$, $u = \left(z^2 - h_0^2\right)^{-1/2} z = \left(1.6^2 - 0.8^2\right)^{-1/2}(1.6) = 1.15$.

For the counterweight

$$\sum F_y = ma_y: \quad T - 0.5\text{ kg } 9.8\text{ m/s}^2 = -0.5\text{ kg}a_y$$

$$a_y = -2T + 9.8$$

For the glider

$$\sum F_x = ma_x: \quad T\cos 30° = 1.00\text{ kg } a_x = 1.15a_y = 1.15(-2T + 9.8) = -2.31T + 11.3\text{ N}$$

$$3.18T = 11.3\text{ N}$$

$$T = \boxed{3.56\text{ N}}$$

***P5.66** The upward acceleration of the rod is described by

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$1\times 10^{-3}\ \text{m} = 0 + 0 + \frac{1}{2}a_y\left(8\times 10^{-3}\ \text{s}\right)^2$$

$$a_y = 31.2\ \text{m/s}^2$$

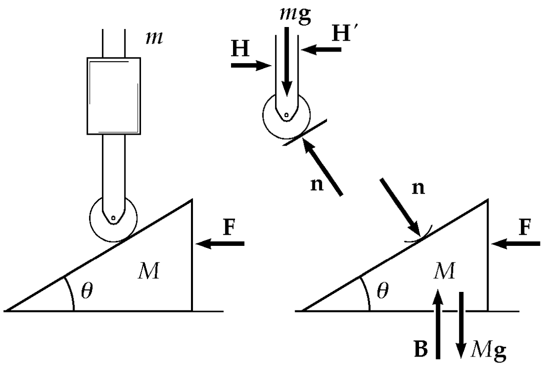


**FIG. P5.66**

The distance $y$ moved by the rod and the distance $x$ moved by the wedge in the same time are related by $\tan 15° = \frac{y}{x} \Rightarrow x = \frac{y}{\tan 15°}$. Then their speeds and accelerations are related by

$$\frac{dx}{dt} = \frac{1}{\tan 15°}\frac{dy}{dt}$$

and

$$\frac{d^2x}{dt^2} = \frac{1}{\tan 15°}\frac{d^2y}{dt^2} = \left(\frac{1}{\tan 15°}\right)31.2\ \text{m/s}^2 = 117\ \text{m/s}^2.$$

The free body diagram for the rod is shown. Here $H$ and $H'$ are forces exerted by the guide.

$$\sum F_y = ma_y: \qquad n\cos 15° - mg = ma_y$$

$$n\cos 15° - 0.250\ \text{kg}\left(9.8\ \text{m/s}^2\right) = 0.250\ \text{kg}\left(31.2\ \text{m/s}^2\right)$$

$$n = \frac{10.3\ \text{N}}{\cos 15°} = 10.6\ \text{N}$$

For the wedge,

$$\sum F_x = Ma_x: \quad -n\sin 15° + F = 0.5\ \text{kg}\left(117\ \text{m/s}^2\right)$$

$$F = (10.6\ \text{N})\sin 15° + 58.3\ \text{N} = \boxed{61.1\ \text{N}}$$

***P5.67** (a) Consider forces on the midpoint of the rope. It is nearly in equilibrium just before the car begins to move. Take the $y$-axis in the direction of the force you exert:

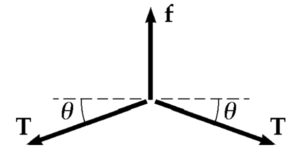


**FIG. P5.67**

$$\sum F_y = ma_y: \quad -T\sin\theta + f - T\sin\theta = 0$$

$$\boxed{T = \frac{f}{2\sin\theta}}.$$

(b) $T = \dfrac{100\ \text{N}}{2\sin 7°} = \boxed{410\ \text{N}}$

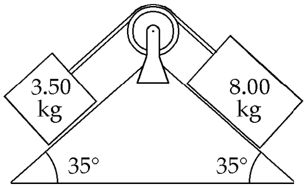


**FIG. P5.68**

**P5.68** Since it has a larger mass, we expect the 8.00-kg block to move down the plane. The acceleration for both blocks should have the same magnitude since they are joined together by a non-stretching string. Define up the left hand plane as positive for the 3.50-kg object and down the right hand plane as positive for the 8.00-kg object.

$$\sum F_1 = m_1 a_1: \quad -m_1 g \sin 35.0° + T = m_1 a$$
$$\sum F_2 = m_2 a_2: \quad m_2 g \sin 35.0° - T = m_2 a$$

and

$$-(3.50)(9.80)\sin 35.0° + T = 3.50a$$
$$(8.00)(9.80)\sin 35.0° - T = 8.00a.$$

Adding, we obtain

$$+45.0 \text{ N} - 19.7 \text{ N} = (11.5 \text{ kg})a.$$

(b) Thus the acceleration is

$$\boxed{a = 2.20 \text{ m/s}^2}.$$

By substitution,

$$-19.7 \text{ N} + T = (3.50 \text{ kg})(2.20 \text{ m/s}^2) = 7.70 \text{ N}.$$

(a) The tension is

$$\boxed{T = 27.4 \text{ N}}.$$

**P5.69** Choose the $x$-axis pointing down the slope.

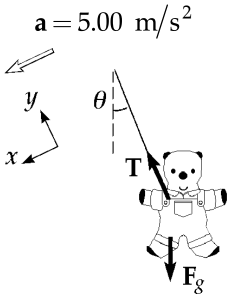


**FIG. P5.69**

$$v_f = v_i + at: \quad 30.0 \text{ m/s} = 0 + a(6.00 \text{ s})$$
$$a = 5.00 \text{ m/s}^2.$$

Consider forces on the toy.

$$\sum F_x = ma_x: \quad mg \sin\theta = m(5.00 \text{ m/s}^2)$$
$$\theta = \boxed{30.7°}$$
$$\sum F_y = ma_y: \quad -mg\cos\theta + T = 0$$
$$T = mg\cos\theta = (0.100)(9.80)\cos 30.7°$$
$$T = \boxed{0.843 \text{ N}}$$

***P5.70** Throughout its up and down motion after release the block has

$$\sum F_y = ma_y: \quad +n - mg\cos\theta = 0$$
$$n = mg\cos\theta.$$

Let $\mathbf{R} = R_x\hat{\mathbf{i}} + R_y\hat{\mathbf{j}}$ represent the force of table on incline. We have

$$\sum F_x = ma_x: \qquad +R_x - n\sin\theta = 0$$
$$R_x = mg\cos\theta\sin\theta$$
$$\sum F_y = ma_y: \quad -Mg - n\cos\theta + R_y = 0$$
$$R_y = Mg + mg\cos^2\theta.$$

$$\boxed{\mathbf{R} = mg\cos\theta\sin\theta \text{ to the right} + \left(M + m\cos^2\theta\right)g \text{ upward}}$$

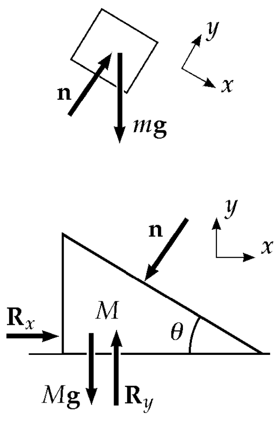


**FIG. P5.70**

***P5.71** Take $+x$ in the direction of motion of the tablecloth. For the mug:

$$\sum F_x = ma_x \quad 0.1\text{ N} = 0.2\text{ kg}\,a_x$$
$$a_x = 0.5\text{ m/s}^2.$$

Relative to the tablecloth, the acceleration of the mug is $0.5\text{ m/s}^2 - 3\text{ m/s}^2 = -2.5\text{ m/s}^2$. The mug reaches the edge of the tablecloth after time given by

$$\Delta x = v_{xi}t + \frac{1}{2}a_x t^2$$
$$-0.3\text{ m} = 0 + \frac{1}{2}\left(-2.5\text{ m/s}^2\right)t^2$$
$$t = 0.490\text{ s}.$$

The motion of the mug relative to tabletop is over distance

$$\frac{1}{2}a_x t^2 = \frac{1}{2}\left(0.5\text{ m/s}^2\right)(0.490\text{ s})^2 = \boxed{0.060\,0\text{ m}}.$$

The tablecloth slides 36 cm over the table in this process.

**P5.72** $\sum F_y = ma_y: \; n - mg\cos\theta = 0$

or

$$n = 8.40(9.80)\cos\theta$$
$$n = (82.3\text{ N})\cos\theta$$

$\sum F_x = ma_x: \; mg\sin\theta = ma$

or

$$a = g\sin\theta$$
$$a = \left(9.80\text{ m/s}^2\right)\sin\theta$$

| $\theta$, deg | $n$, N | $a$, m/s$^2$ |
|---|---|---|
| 0.00 | 82.3 | 0.00 |
| 5.00 | 82.0 | 0.854 |
| 10.0 | 81.1 | 1.70 |
| 15.0 | 79.5 | 2.54 |
| 20.0 | 77.4 | 3.35 |
| 25.0 | 74.6 | 4.14 |
| 30.0 | 71.3 | 4.90 |
| 35.0 | 67.4 | 5.62 |
| 40.0 | 63.1 | 6.30 |
| 45.0 | 58.2 | 6.93 |
| 50.0 | 52.9 | 7.51 |
| 55.0 | 47.2 | 8.03 |
| 60.0 | 41.2 | 8.49 |
| 65.0 | 34.8 | 8.88 |
| 70.0 | 28.2 | 9.21 |
| 75.0 | 21.3 | 9.47 |
| 80.0 | 14.3 | 9.65 |
| 85.0 | 7.17 | 9.76 |
| 90.0 | 0.00 | 9.80 |

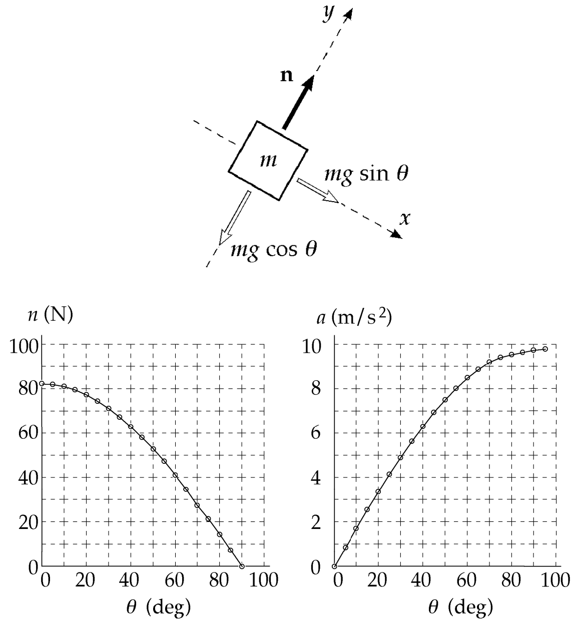


**FIG. P5.72**

At 0°, the normal force is the full weight and the acceleration is zero. At 90°, the mass is in free fall next to the vertical incline.

**P5.73** (a) Apply Newton's second law to two points where butterflies are attached on either half of mobile (other half the same, by symmetry)

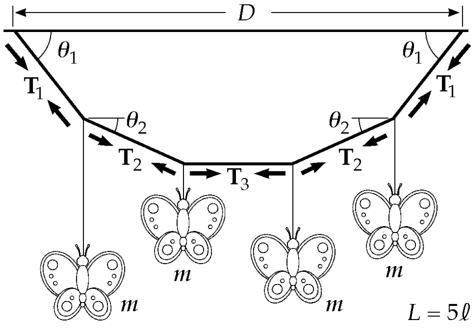


FIG. P5.69

(1) $T_2\cos\theta_2 - T_1\cos\theta_1 = 0$
(2) $T_1\sin\theta_1 - T_2\sin\theta_2 - mg = 0$
(3) $T_2\cos\theta_2 - T_3 = 0$
(4) $T_2\sin\theta_2 - mg = 0$

Substituting (4) into (2) for $T_2\sin\theta_2$,

$$T_1\sin\theta_1 - mg - mg = 0.$$

Then

$$\boxed{T_1 = \frac{2mg}{\sin\theta_1}}.$$

Substitute (3) into (1) for $T_2\cos\theta_2$:

$$T_3 - T_1\cos\theta_1 = 0,\ T_3 = T_1\cos\theta_1$$

Substitute value of $T_1$:

$$T_3 = 2mg\frac{\cos\theta_1}{\sin\theta_1} = \boxed{\frac{2mg}{\tan\theta_1} = T_3}.$$

From Equation (4),

$$T_2 = \frac{mg}{\sin\theta_2}.$$

(b) Divide (4) by (3):

$$\frac{T_2\sin\theta_2}{T_2\cos\theta_2} = \frac{mg}{T_3}.$$

Substitute value of $T_3$:

$$\tan\theta_2 = \frac{mg\tan\theta_1}{2mg},\ \boxed{\theta_2 = \tan^{-1}\left(\frac{\tan\theta_1}{2}\right)}.$$

Then we can finish answering part (a):

$$\boxed{T_2 = \frac{mg}{\sin\left[\tan^{-1}\left(\frac{1}{2}\tan\theta_1\right)\right]}}.$$

(c) *D* is the horizontal distance between the points at which the two ends of the string are attached to the ceiling.

$$D = 2\ell\cos\theta_1 + 2\ell\cos\theta_2 + \ell \text{ and } L = 5\ell$$

$$D = \frac{L}{5}\left\{2\cos\theta_1 + 2\cos\left[\tan^{-1}\left(\frac{1}{2}\tan\theta_1\right)\right] + 1\right\}$$

## ANSWERS TO EVEN PROBLEMS

| | |
|---|---|
| **P5.2** | $1.66\times10^6$ N forward |
| **P5.4** | (a) $\frac{vt}{2}$; (b) $\left(\frac{F_g v}{gt}\right)\hat{\mathbf{i}}+F_g\hat{\mathbf{j}}$ |
| **P5.6** | (a) $4.47\times10^{15}$ m/s$^2$ away from the wall;<br>(b) $2.09\times10^{-10}$ N toward the wall |
| **P5.8** | (a) 534 N down; (b) 54.5 kg |
| **P5.10** | 2.55 N for an 88.7 kg person |
| **P5.12** | $\left(16.3\hat{\mathbf{i}}+14.6\hat{\mathbf{j}}\right)$ N |
| **P5.14** | (a) 181°; (b) 11.2 kg; (c) 37.5 m/s;<br>(d) $\left(-37.5\hat{\mathbf{i}}-0.893\hat{\mathbf{j}}\right)$ m/s |
| **P5.16** | 112 N |
| **P5.18** | $T_1=296$ N; $T_2=163$ N; $T_3=325$ N |
| **P5.20** | (a) see the solution; (b) 1.79 N |
| **P5.22** | (a) 2.54 m/s$^2$ down the incline;<br>(b) 3.18 m/s |
| **P5.24** | see the solution; 6.30 m/s$^2$; 31.5 N |
| **P5.26** | (a) 3.57 m/s$^2$; (b) 26.7 N; (c) 7.14 m/s |
| **P5.28** | (a) 36.8 N; (b) 2.45 m/s$^2$; (c) 1.23 m |
| **P5.30** | (a) 0.529 m; (b) 7.40 m/s upward |
| **P5.32** | (a) 2.22 m; (b) 8.74 m/s |
| **P5.34** | (a) $a_1=2a_2$;<br>(b) $T_1=\frac{m_1m_2g}{2m_1+\frac{m_2}{2}}$; $T_2=\frac{m_1m_2g}{m_1+\frac{m_2}{4}}$;<br>(c) $a_1=\frac{m_2g}{2m_1+\frac{m_2}{2}}$; $a_2=\frac{m_2g}{4m_1+m_2}$ |
| **P5.36** | $\mu_s=0.306$; $\mu_k=0.245$ |
| **P5.38** | (a) 3.34; (b) Time would increase |
| **P5.40** | (a) 55.2°; (b) 167 N |
| **P5.42** | 152 ft |
| **P5.44** | (a) 2.31 m/s$^2$ down for $m_1$, left for $m_2$ and up for $m_3$; (b) 30.0 N and 24.2 N |
| **P5.46** | Any value between 31.7 N and 48.6 N |
| **P5.48** | 72.0 N |
| **P5.50** | 6.84 m |
| **P5.52** | (a) 3.00 s; (b) 20.1 m; (c) $\left(18.0\hat{\mathbf{i}}-9.00\hat{\mathbf{j}}\right)$ m |
| **P5.54** | (a) 2.00 m/s$^2$ to the right;<br>(b) 8.00 N right on 4 kg;<br>6.00 N right on 3 kg; 4 N right on 2 kg;<br>(c) 8.00 N between 4 kg and 3 kg;<br>14.0 N between 2 kg and 3 kg;<br>(d) see the solution |
| **P5.56** | 1.18 kN |
| **P5.58** | (a) 4.90 m/s$^2$; (b) 3.13 m/s at 30.0° below the horizontal; (c) 1.35 m; (d) 1.14 s; (e) No |
| **P5.60** | (a) and (b) see the solution; (c) 357 N;<br>(d) see the solution; (e) 1.20 |
| **P5.62** | see the solution; 0.143 m/s$^2$ agrees with 0.137 m/s$^2$ |
| **P5.64** | (a) see the solution;<br>(b) on block one:<br>$49.0\text{ N}\,\hat{\mathbf{j}}-49.0\text{ N}\,\hat{\mathbf{j}}+14.7\text{ N}\,\hat{\mathbf{i}}$;<br>on block two: $-49.0\text{ N}\,\hat{\mathbf{j}}-14.7\text{ N}\,\hat{\mathbf{i}}-147\text{ N}\,\hat{\mathbf{j}}$<br>$+196\text{ N}\,\hat{\mathbf{j}}-98.0\text{ N}\,\hat{\mathbf{i}}+113\text{ N}\,\hat{\mathbf{i}}$;<br>(c) for block one: $0.980\hat{\mathbf{i}}$ m/s$^2$;<br>for block two: 1.96 m/s$^2$ $\hat{\mathbf{i}}$ |
| **P5.66** | 61.1 N |
| **P5.68** | (a) 2.20 m/s$^2$; (b) 27.4 N |
| **P5.70** | $mg\cos\theta\sin\theta$ to the right<br>$+\left(M+m\cos^2\theta\right)g$ upward |
| **P5.72** | see the solution |

# 6

# Circular Motion and Other Applications of Newton's Laws

**CHAPTER OUTLINE**

6.1 Newton's Second Law Applied to Uniform Circular Motion
6.2 Nonuniform Circular Motion
6.3 Motion in Accelerated Frames
6.4 Motion in the Presence of Resistive Forces
6.5 Numerical Modeling in Particle Dynamics

## ANSWERS TO QUESTIONS

**Q6.1** Mud flies off a rapidly spinning tire because the resultant force is not sufficient to keep it moving in a circular path. In this case, the force that plays a major role is the adhesion between the mud and the tire.

**Q6.2** The spring will stretch. In order for the object to move in a circle, the force exerted on the object by the spring must have a size of $\frac{mv^2}{r}$. Newton's third law says that the force exerted on the object by the spring has the same size as the force exerted by the object on the spring. It is the force exerted on the spring that causes the spring to stretch.

**Q6.3** Driving in a circle at a constant speed requires a centripetal acceleration but no tangential acceleration.

**Q6.4** (a) The object will move in a circle at a constant speed.

(b) The object will move in a straight line at a changing speed.

**Q6.5** The speed changes. The tangential force component causes tangential acceleration.

**Q6.6** Consider the force required to keep a rock in the Earth's crust moving in a circle. The size of the force is proportional to the radius of the circle. If that rock is at the Equator, the radius of the circle through which it moves is about 6400 km. If the rock is at the north pole, the radius of the circle through which it moves is zero!

**Q6.7** Consider standing on a bathroom scale. The resultant force on you is your actual weight minus the normal force. The scale reading shows the size of the normal force, and is your 'apparent weight.' If you are at the North or South Pole, it can be precisely equal to your actual weight. If you are at the equator, your apparent weight must be less, so that the resultant force on you can be a downward force large enough to cause your centripetal acceleration as the Earth rotates.

**Q6.8** A torque is exerted by the thrust force of the water times the distance between the nozzles.

**Q6.9** I would not accept that statement for two reasons. First, to be "beyond the pull of gravity," one would have to be infinitely far away from all other matter. Second, astronauts in orbit are moving in a circular path. It is the gravitational pull of Earth on the astronauts that keeps them in orbit. In the space shuttle, just above the atmosphere, gravity is only slightly weaker than at the Earth's surface. Gravity does its job most clearly on an orbiting spacecraft, because the craft feels no other forces and is in free fall.

**Q6.10** This is the same principle as the centrifuge. All the material inside the cylinder tends to move along a straight-line path, but the walls of the cylinder exert an inward force to keep everything moving around in a circular path.

**Q6.11** The ball would not behave as it would when dropped on the Earth. As the astronaut holds the ball, she and the ball are moving with the same angular velocity. The ball, however, being closer to the center of rotation, is moving with a slower tangential velocity. Once the ball is released, it acts according to Newton's first law, and simply drifts with constant velocity in the original direction of its velocity when released—it is no longer "attached" to the rotating space station. Since the ball follows a straight line and the astronaut follows a circular path, it will appear to the astronaut that the ball will "fall to the floor". But other dramatic effects will occur. Imagine that the ball is held so high that it is just slightly away from the center of rotation. Then, as the ball is released, it will move very slowly along a straight line. Thus, the astronaut may make several full rotations around the circular path before the ball strikes the floor. This will result in three obvious variations with the Earth drop. First, the time to fall will be much larger than that on the Earth, even though the feet of the astronaut are pressed into the floor with a force that suggests the same force of gravity as on Earth. Second, the ball may actually appear to bob up and down if several rotations are made while it "falls". As the ball moves in a straight line while the astronaut rotates, sometimes she is on the side of the circle on which the ball is moving toward her and other times she is on the other side, where the ball is moving away from her. The third effect is that the ball will not drop straight down to her feet. In the extreme case we have been imagining, it may actually strike the surface while she is on the opposite side, so it looks like it ended up "falling up". In the least extreme case, in which only a portion of a rotation is made before the ball strikes the surface, the ball will appear to move backward relative to the astronaut as it falls.

**Q6.12** The water has inertia. The water tends to move along a straight line, but the bucket pulls it in and around in a circle.

**Q6.13** There is no such force. If the passenger slides outward across the slippery car seat, it is because the passenger is moving forward in a straight line while the car is turning under him. If the passenger pushes hard against the outside door, the door is exerting an inward force on him. No object is exerting an outward force on him, but he should still buckle his seatbelt.

**Q6.14** Blood pressure cannot supply the force necessary both to balance the gravitational force and to provide the centripetal acceleration, to keep blood flowing up to the pilot's brain.

**Q6.15** The person in the elevator is in an accelerating reference frame. The apparent acceleration due to gravity, "$g$," is changed inside the elevator. "$g$" $= g \pm a$

**Q6.16** When you are not accelerating, the normal force and your weight are equal in size. Your body interprets the force of the floor pushing up on you as your weight. When you accelerate in an elevator, this normal force changes so that you accelerate with the elevator. In free fall, you are never weightless since the Earth's gravity and your mass do not change. It is the normal force—your apparent weight—that is zero.

**Q6.17** From the proportionality of the drag force to the speed squared and from Newton's second law, we derive the equation that describes the motion of the skydiver:

$$m\frac{dv_y}{dt} = mg - \frac{D\rho A}{2}v_y^2$$

where $D$ is the coefficient of drag of the parachutist, and $A$ is the projected area of the parachutist's body. At terminal speed,

$$a_y = \frac{dv_y}{dt} = 0 \text{ and } V_T\left(\frac{2mg}{D\rho A}\right)^{1/2}.$$

When the parachute opens, the coefficient of drag $D$ and the effective area $A$ both increase, thus reducing the speed of the skydiver.

Modern parachutes also add a third term, lift, to change the equation to

$$m\frac{dv_y}{dt} = mg - \frac{D\rho A}{2}v_y^2 - \frac{L\rho A}{2}v_x^2$$

where $v_y$ is the vertical velocity, and $v_x$ is the horizontal velocity. The effect of lift is clearly seen in the "paraplane," an ultralight airplane made from a fan, a chair, and a parachute.

**Q6.18** The larger drop has higher terminal speed. In the case of spheres, the text demonstrates that terminal speed is proportional to the square root of radius. When moving with terminal speed, an object is in equilibrium and has zero acceleration.

**Q6.19** Lower air density reduces air resistance, so a tank-truck-load of fuel takes you farther.

**Q6.20** Suppose the rock is moving rapidly when it enters the water. The speed of the rock decreases until it reaches terminal velocity. The acceleration, which is upward, decreases to zero as the rock approaches terminal velocity.

**Q6.21** The thesis is false. The moment of decay of a radioactive atomic nucleus (for example) cannot be predicted. Quantum mechanics implies that the future is indeterminate. On the other hand, our sense of free will, of being able to make choices for ourselves that can appear to be random, may be an illusion. It may have nothing to do with the subatomic randomness described by quantum mechanics.

## SOLUTIONS TO PROBLEMS

### Section 6.1 **Newton's Second Law Applied to Uniform Circular Motion**

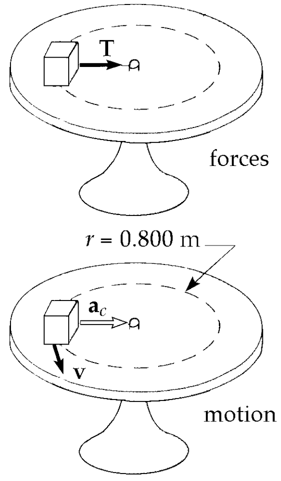


**FIG. P6.1**

**P6.1** $m = 3.00\text{ kg}$, $r = 0.800\text{ m}$. The string will break if the tension exceeds the weight corresponding to 25.0 kg, so

$$T_{\max} = Mg = 25.0(9.80) = 245\text{ N}.$$

When the 3.00 kg mass rotates in a horizontal circle, the tension causes the centripetal acceleration,

so $$T = \frac{mv^2}{r} = \frac{(3.00)v^2}{0.800}.$$

Then $$v^2 = \frac{rT}{m} = \frac{(0.800)T}{3.00} \le \frac{(0.800)T_{\max}}{3.00} = \frac{0.800(245)}{3.00} = 65.3\text{ m}^2/\text{s}^2$$

and $$0 \le v \le \sqrt{65.3}$$

or $$\boxed{0 \le v \le 8.08\text{ m/s}}.$$

**P6.2** In $\sum F = m\frac{v^2}{r}$, both $m$ and $r$ are unknown but remain constant. Therefore, $\sum F$ is proportional to $v^2$ and increases by a factor of $\left(\frac{18.0}{14.0}\right)^2$ as $v$ increases from 14.0 m/s to 18.0 m/s. The total force at the higher speed is then

$$\sum F_{\text{fast}} = \left(\frac{18.0}{14.0}\right)^2 (130\text{ N}) = 215\text{ N}.$$

Symbolically, write $\sum F_{\text{slow}} = \left(\frac{m}{r}\right)(14.0\text{ m/s})^2$ and $\sum F_{\text{fast}} = \left(\frac{m}{r}\right)(18.0\text{ m/s})^2$.

Dividing gives $\frac{\sum F_{\text{fast}}}{\sum F_{\text{slow}}} = \left(\frac{18.0}{14.0}\right)^2$, or

$$\sum F_{\text{fast}} = \left(\frac{18.0}{14.0}\right)^2 \sum F_{\text{slow}} = \left(\frac{18.0}{14.0}\right)^2 (130\text{ N}) = \boxed{215\text{ N}}.$$

This force must be $\boxed{\text{horizontally inward}}$ to produce the driver's centripetal acceleration.

**P6.3** (a) $F = \frac{mv^2}{r} = \frac{\left(9.11 \times 0^{-31}\ \text{kg}\right)\left(2.20 \times 10^{6}\ \text{m/s}\right)^2}{0.530 \times 10^{-10}\ \text{m}} = \boxed{8.32 \times 10^{-8}\ \text{N inward}}$

(b) $a = \frac{v^2}{r} = \frac{\left(2.20 \times 10^{6}\ \text{m/s}\right)^2}{0.530 \times 10^{-10}\ \text{m}} = \boxed{9.13 \times 10^{22}\ \text{m/s}^2\ \text{inward}}$

**P6.4** Neglecting relativistic effects. $F = ma_c = \frac{mv^2}{r}$

$$F = \left(2 \times 1.661 \times 10^{-27}\ \text{kg}\right)\frac{\left(2.998 \times 10^{7}\ \text{m/s}\right)^2}{(0.480\ \text{m})} = \boxed{6.22 \times 10^{-12}\ \text{N}}$$

**P6.5** (a) $\boxed{\text{static friction}}$

(b) $ma\hat{\mathbf{i}} = f\,\hat{\mathbf{i}} + n\hat{\mathbf{j}} + mg\left(-\hat{\mathbf{j}}\right)$

$\sum F_y = 0 = n - mg$

thus $n = mg$ and $\sum F_r = m\frac{v^2}{r} = f = \mu n = \mu mg$.

Then $\mu = \frac{v^2}{rg} = \frac{(50.0\ \text{cm/s})^2}{(30.0\ \text{cm})\left(980\ \text{cm/s}^2\right)} = \boxed{0.085\,0}$.

**P6.6** (a) $\sum F_y = ma_y$, $mg_{\text{moon}}$ down $= \frac{mv^2}{r}$ down

$v = \sqrt{g_{\text{moon}} r} = \sqrt{\left(1.52\ \text{m/s}^2\right)\left(1.7 \times 10^{6}\ \text{m} + 100 \times 10^{3}\ \text{m}\right)} = \boxed{1.65 \times 10^{3}\ \text{m/s}}$

(b) $v = \frac{2\pi r}{T}$, $T = \frac{2\pi\left(1.8 \times 10^{6}\ \text{m}\right)}{1.65 \times 10^{3}\ \text{m/s}} = \boxed{6.84 \times 10^{3}\ \text{s}} = 1.90\ \text{h}$

**P6.7** $n = mg$ since $a_y = 0$

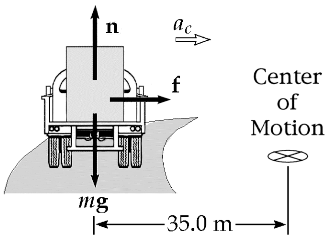


**FIG. P6.7**

The force causing the centripetal acceleration is the frictional force *f*.

From Newton's second law $f = ma_c = \frac{mv^2}{r}$.

But the friction condition is $f \le \mu_s n$

i.e., $\frac{mv^2}{r} \le \mu_s mg$

$v \le \sqrt{\mu_s rg} = \sqrt{0.600(35.0\ \text{m})\left(9.80\ \text{m/s}^2\right)}$ $\qquad v \le \boxed{14.3\ \text{m/s}}$

**P6.8** $$a = \frac{v^2}{r} = \frac{\left[(86.5\ \text{km/h})\left(\frac{1\ \text{h}}{3\,600\ \text{s}}\right)\left(\frac{1\,000\ \text{m}}{1\ \text{km}}\right)\right]^2}{61.0\ \text{m}}\left(\frac{1\ g}{9.80\ \text{m/s}^2}\right) = \boxed{0.966g}$$

**P6.9** $$T\cos 5.00° = mg = (80.0\ \text{kg})(9.80\ \text{m/s}^2)$$

(a) $T = 787\ \text{N}$: $\mathbf{T} = \boxed{(68.6\ \text{N})\hat{\mathbf{i}} + (784\ \text{N})\hat{\mathbf{j}}}$

(b) $T\sin 5.00° = ma_c$: $\boxed{a_c = 0.857\ \text{m/s}^2}$ toward the center of the circle.

The length of the wire is unnecessary information. We could, on the other hand, use it to find the radius of the circle, the speed of the bob, and the period of the motion.

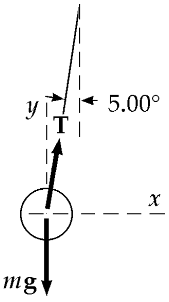


FIG. P6.9

**P6.10** (b) $$v = \frac{235\ \text{m}}{36.0\ \text{s}} = \boxed{6.53\ \text{m/s}}$$

The radius is given by $\frac{1}{4}2\pi r = 235\ \text{m}$

$$r = 150\ \text{m}$$

(a) $$\mathbf{a}_r = \left(\frac{v^2}{r}\right) \text{ toward center}$$

$$= \frac{(6.53\ \text{m/s})^2}{150\ \text{m}} \text{ at } 35.0° \text{ north of west}$$

$$= (0.285\ \text{m/s}^2)\left(\cos 35.0°\left(-\hat{\mathbf{i}}\right) + \sin 35.0°\,\hat{\mathbf{j}}\right)$$

$$= \boxed{-0.233\ \text{m/s}^2\,\hat{\mathbf{i}} + 0.163\ \text{m/s}^2\,\hat{\mathbf{j}}}$$

(c) $$\bar{\mathbf{a}} = \frac{(\mathbf{v}_f - \mathbf{v}_i)}{t}$$

$$= \frac{\left(6.53\ \text{m/s}\,\hat{\mathbf{j}} - 6.53\ \text{m/s}\,\hat{\mathbf{i}}\right)}{36.0\ \text{s}}$$

$$= \boxed{-0.181\ \text{m/s}^2\,\hat{\mathbf{i}} + 0.181\ \text{m/s}^2\,\hat{\mathbf{j}}}$$

***P6.11** $F_g = mg = (4\text{ kg})\left(9.8\text{ m/s}^2\right) = 39.2\text{ N}$

$$\sin\theta = \frac{1.5\text{ m}}{2\text{ m}}$$

$\theta = 48.6°$

$r = (2\text{ m})\cos 48.6° = 1.32\text{ m}$

$$\sum F_x = ma_x = \frac{mv^2}{r}$$

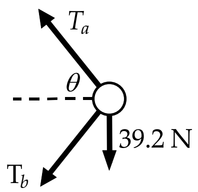
$$T_a\cos 48.6° + T_b\cos 48.6° = \frac{(4\text{ kg})(6\text{ m/s})^2}{1.32\text{ m}}$$

$$T_a + T_b = \frac{109\text{ N}}{\cos 48.6°} = 165\text{ N}$$

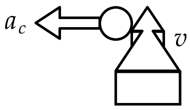
$$\sum F_y = ma_y$$

$$+T_a\sin 48.6° - T_b\sin 48.6° - 39.2\text{ N} = 0$$

$$T_a - T_b = \frac{39.2\text{ N}}{\sin 48.6°} = 52.3\text{ N}$$



forces



motion

**FIG. P6.11**

(a) To solve simultaneously, we add the equations in $T_a$ and $T_b$:

$$T_a + T_b + T_a - T_b = 165\text{ N} + 52.3\text{ N}$$

$$T_a = \frac{217\text{ N}}{2} = \boxed{108\text{ N}}$$

(b) $T_b = 165\text{ N} - T_a = 165\text{ N} - 108\text{ N} = \boxed{56.2\text{ N}}$

***P6.12** $a_c = \dfrac{v^2}{r}$. Let $f$ represent the rotation rate. Each revolution carries each bit of metal through distance $2\pi r$, so $v = 2\pi r f$ and

$$a_c = \frac{v^2}{r} = 4\pi^2 r f^2 = 100\,g.$$

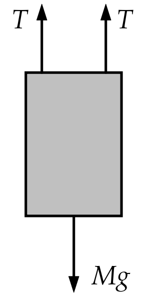
A smaller radius implies smaller acceleration. To meet the criterion for each bit of metal we consider the minimum radius:

$$f = \left(\frac{100\,g}{4\pi^2 r}\right)^{1/2} = \left(\frac{100\cdot 9.8\text{ m/s}^2}{4\pi^2(0.021\text{ m})}\right)^{1/2} = 34.4\ \frac{1}{\text{s}}\left(\frac{60\text{ s}}{1\text{ min}}\right) = \boxed{2.06\times 10^3\ \text{rev/min}}.$$

## Section 6.2 **Nonuniform Circular Motion**

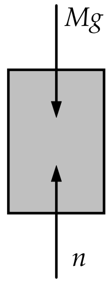
**P6.13** $M = 40.0\text{ kg},\ R = 3.00\text{ m},\ T = 350\text{ N}$

(a)
$$\sum F = 2T - Mg = \frac{Mv^2}{R}$$
$$v^2 = (2T - Mg)\left(\frac{R}{M}\right)$$
$$v^2 = [700 - (40.0)(9.80)]\left(\frac{3.00}{40.0}\right) = 23.1\ (\text{m}^2/\text{s}^2)$$
$$\boxed{v = 4.81\ \text{m/s}}$$



child + seat

**FIG. P6.13(a)**



child alone

**FIG. P6.13(b)**

(b)
$$n - Mg = F = \frac{Mv^2}{R}$$
$$n = Mg + \frac{Mv^2}{R} = 40.0\left(9.80 + \frac{23.1}{3.00}\right) = \boxed{700\text{ N}}$$

**P6.14** (a) Consider the forces acting on the system consisting of the child *and* the seat:

$$\sum F_y = ma_y \Rightarrow 2T - mg = m\frac{v^2}{R}$$
$$v^2 = R\left(\frac{2T}{m} - g\right)$$
$$v = \boxed{\sqrt{R\left(\frac{2T}{m} - g\right)}}$$

(b) Consider the forces acting on the child alone:

$$\sum F_y = ma_y \Rightarrow n = m\left(g + \frac{v^2}{R}\right)$$

and from above, $v^2 = R\left(\frac{2T}{m} - g\right)$, so

$$n = m\left(g + \frac{2T}{m} - g\right) = \boxed{2T}.$$

**P6.15** Let the tension at the lowest point be $T$.

$$\sum F = ma: \quad T - mg = ma_c = \frac{mv^2}{r}$$
$$T = m\left(g + \frac{v^2}{r}\right)$$
$$T = (85.0\text{ kg})\left[9.80\ \text{m/s}^2 + \frac{(8.00\ \text{m/s})^2}{10.0\text{ m}}\right] = 1.38\text{ kN} > 1\,000\text{ N}$$

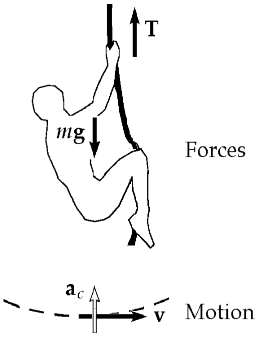


**FIG. P6.15**

$\boxed{\text{He doesn't make it across the river because the vine breaks.}}$

**P6.16** (a) $a_c = \frac{v^2}{r} = \frac{(4.00\text{ m/s})^2}{12.0\text{ m}} = \boxed{1.33\text{ m/s}^2}$

(b) $a = \sqrt{a_c^2 + a_t^2}$

$a = \sqrt{(1.33)^2 + (1.20)^2} = \boxed{1.79\text{ m/s}^2}$

at an angle $\theta = \tan^{-1}\left(\frac{a_c}{a_t}\right) = \boxed{48.0°\text{ inward}}$

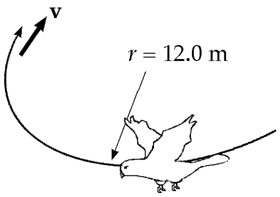


**FIG. P6.16**

**P6.17** $\sum F_y = \frac{mv^2}{r} = mg + n$

But $n = 0$ at this minimum speed condition, so

$\frac{mv^2}{r} = mg \Rightarrow v = \sqrt{gr} = \sqrt{\left(9.80\text{ m/s}^2\right)(1.00\text{ m})} = \boxed{3.13\text{ m/s}}$.

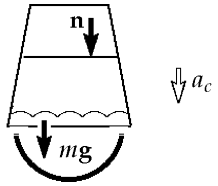


**FIG. P6.17**

**P6.18** At the top of the vertical circle,

$$T = m\frac{v^2}{R} - mg$$

or $T = (0.400)\frac{(4.00)^2}{0.500} - (0.400)(9.80) = \boxed{8.88\text{ N}}$

**P6.19** (a) $v = 20.0\text{ m/s}$,

$n =$ force of track on roller coaster, and

$R = 10.0\text{ m}$.

$$\sum F = \frac{Mv^2}{R} = n - Mg$$

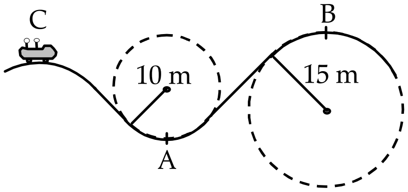


**FIG. P6.19**

From this we find

$$n = Mg + \frac{Mv^2}{R} = (500\text{ kg})\left(9.80\text{ m/s}^2\right) + \frac{(500\text{ kg})\left(20.0\text{ m/s}^2\right)}{10.0\text{ m}}$$

$$n = 4\,900\text{ N} + 20\,000\text{ N} = \boxed{2.49 \times 10^4\text{ N}}$$

(b) At B, $n - Mg = -\frac{Mv^2}{R}$

The max speed at B corresponds to

$$n = 0$$

$$-Mg = -\frac{Mv_{\text{max}}^2}{R} \Rightarrow v_{\text{max}} = \sqrt{Rg} = \sqrt{15.0(9.80)} = \boxed{12.1\text{ m/s}}$$

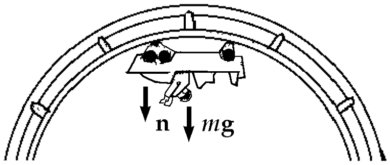


FIG. P6.20

**P6.20** (a) $a_c = \dfrac{v^2}{r}$ $\quad r = \dfrac{v^2}{a_c} = \dfrac{(13.0\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)} = \boxed{8.62\ \text{m}}$

(b) Let $n$ be the force exerted by the rail.

Newton's law gives

$$Mg + n = \frac{Mv^2}{r}$$

$$n = M\left(\frac{v^2}{r} - g\right) = M(2g - g) = \boxed{Mg\text{, downward}}$$

(c) $a_c = \dfrac{v^2}{r}$ $\quad a_c = \dfrac{(13.0\ \text{m/s})^2}{20.0\ \text{m}} = \boxed{8.45\ \text{m/s}^2}$

If the force exerted by the rail is $n_1$

then $\quad n_1 + Mg = \dfrac{Mv^2}{r} = Ma_c$

$$n_1 = M(a_c - g) \text{ which is} < 0 \text{ since } a_c = 8.45\ \text{m/s}^2$$

Thus, the normal force would have to point away from the center of the curve. Unless they have belts, the riders will fall from the cars. To be safe we must require $n_1$ to be positive. Then $a_c > g$. We need

$$\frac{v^2}{r} > g \text{ or } v > \sqrt{rg} = \sqrt{(20.0\ \text{m})(9.80\ \text{m/s}^2)},\ v > 14.0\ \text{m/s}.$$

## Section 6.3 Motion in Accelerated Frames

**P6.21** (a) $\sum F_x = Ma$, $a = \dfrac{T}{M} = \dfrac{18.0\ \text{N}}{5.00\ \text{kg}} = \boxed{3.60\ \text{m/s}^2}$ to the right.

(b) If $v =$ const, $a = 0$, so $\boxed{T = 0}$ (This is also an equilibrium situation.)

(c) Someone in the car (noninertial observer) claims that the forces on the mass along $x$ are $T$ and a fictitious force ($-Ma$). Someone at rest outside the car (inertial observer) claims that $T$ is the only force on $M$ in the $x$-direction.

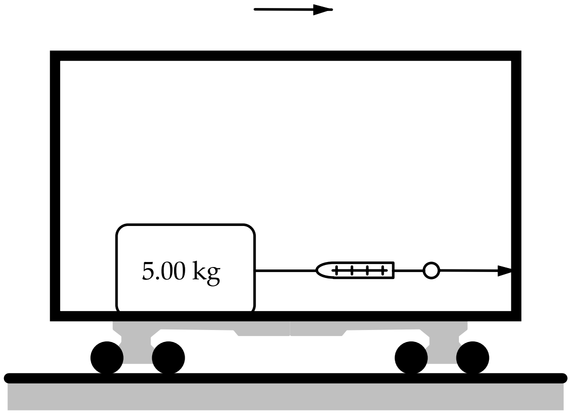


FIG. P6.21

***P6.22** We adopt the view of an inertial observer. If it is on the verge of sliding, the cup is moving on a circle with its centripetal acceleration caused by friction.

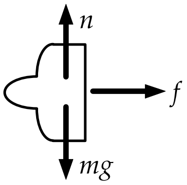


**FIG. P6.22**

$$\sum F_y = ma_y: \quad +n - mg = 0$$

$$\sum F_x = ma_x: \quad f = \frac{mv^2}{r} = \mu_s n = \mu_s mg$$

$$v = \sqrt{\mu_s g r} = \sqrt{0.8\left(9.8 \text{ m/s}^2\right)(30 \text{ m})} = \boxed{15.3 \text{ m/s}}$$

If you go too fast the cup will begin sliding $\boxed{\text{straight across the dashboard to the left.}}$

**P6.23** The only forces acting on the suspended object are the force of gravity $m\mathbf{g}$ and the force of tension $\mathbf{T}$, as shown in the free-body diagram. Applying Newton's second law in the $x$ and $y$ directions,

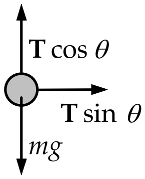


**FIG. P6.23**

$$\sum F_x = T\sin\theta = ma \qquad (1)$$

$$\sum F_y = T\cos\theta - mg = 0$$

or $$T\cos\theta = mg \qquad (2)$$

(a) Dividing equation (1) by (2) gives

$$\tan\theta = \frac{a}{g} = \frac{3.00 \text{ m/s}^2}{9.80 \text{ m/s}^2} = 0.306.$$

Solving for $\theta$, $\theta = \boxed{17.0°}$

(b) From Equation (1),

$$T = \frac{ma}{\sin\theta} = \frac{(0.500 \text{ kg})\left(3.00 \text{ m/s}^2\right)}{\sin(17.0°)} = \boxed{5.12 \text{ N}}.$$

***P6.24** The water moves at speed

$$v = \frac{2\pi r}{T} = \frac{2\pi(0.12 \text{ m})}{7.25 \text{ s}} = 0.104 \text{ m/s}.$$

The top layer of water feels a downward force of gravity $mg$ and an outward fictitious force in the turntable frame of reference,

$$\frac{mv^2}{r} = \frac{m(0.104 \text{ m/s})^2}{0.12 \text{ m}} = m9.01 \times 10^{-2} \text{ m/s}^2.$$

It behaves as if it were stationary in a gravity field pointing downward and outward at

$$\tan^{-1}\frac{0.0901 \text{ m/s}^2}{9.8 \text{ m/s}^2} = \boxed{0.527°}.$$

Its surface slopes upward toward the outside, making this angle with the horizontal.

**P6.25** $F_{\max} = F_g + ma = 591\text{ N}$

$F_{\min} = F_g - ma = 391\text{ N}$

(a) Adding, $2F_g = 982\text{ N}$, $F_g = \boxed{491\text{ N}}$

(b) Since $F_g = mg$, $m = \dfrac{491\text{ N}}{9.80\text{ m/s}^2} = \boxed{50.1\text{ kg}}$

(c) Subtracting the above equations,

$2ma = 200\text{ N}$ $\quad\therefore a = \boxed{2.00\text{ m/s}^2}$

**P6.26** (a)

$$\sum F_r = ma_r$$

$$mg = \frac{mv^2}{R} = \frac{m}{R}\left(\frac{2\pi R}{T}\right)^2$$

$$g = \frac{4\pi^2 R}{T^2}$$

$$T = \sqrt{\frac{4\pi^2 R}{g}} = 2\pi\sqrt{\frac{6.37\times10^6\text{ m}}{9.80\text{ m/s}^2}} = 5.07\times10^3\text{ s} = \boxed{1.41\text{ h}}$$

(b)

$$\text{speed increase factor} = \frac{v_{\text{new}}}{v_{\text{current}}} = \frac{2\pi R}{T_{\text{new}}}\left(\frac{T_{\text{current}}}{2\pi R}\right) = \frac{T_{\text{current}}}{T_{\text{new}}} = \frac{24.0\text{ h}}{1.41\text{ h}} = \boxed{17.1}$$

***P6.27** The car moves to the right with acceleration $a$. We find the acceleration of $a_b$ of the block relative to the Earth. The block moves to the right also.

$$\sum F_y = ma_y:\quad +n - mg = 0,\ n = mg,\ f = \mu_k mg$$
$$\sum F_x = ma_x:\quad +\mu_k mg = ma_b,\ a_b = \mu_k g$$

The acceleration of the block relative to the car is $a_b - a = \mu_k g - a$. In this frame the block starts from rest and undergoes displacement $-\ell$ and gains speed according to

$$v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$$
$$v_{xf}^2 = 0 + 2(\mu_k g - a)(-\ell - 0) = 2\ell(a - \mu_k g).$$

(a) $\boxed{v = \left(2\ell(a-\mu_k g)\right)^{1/2}}$ to the left

*continued on next page*

(b) The time for which the box slides is given by

$$\begin{aligned}\Delta x &= \frac{1}{2}\left(v_{xi}+v_{xf}\right)t\\ -\ell &= \frac{1}{2}\left[0-\left(2\ell\left(a-\mu_k g\right)\right)^{1/2}\right]t\\ t &= \left(\frac{2\ell}{a-\mu_k g}\right)^{1/2}.\end{aligned}$$

The car in the Earth frame acquires finals speed $v_{xf}=v_{xi}+at=0+a\left(\frac{2\ell}{a-\mu_k g}\right)^{1/2}$. The speed of the box in the Earth frame is then

$$\begin{aligned}v_{be}=v_{bc}+v_{ce} &= -\left[2\ell\left(a-\mu_k g\right)\right]^{1/2}+a\left(\frac{2\ell}{a-\mu_k g}\right)^{1/2}\\ &= \frac{-(2\ell)^{1/2}\left(a-\mu_k g\right)+(2\ell)^{1/2}a}{\left(a-\mu_k g\right)^{1/2}}=\boxed{\frac{\mu_k g(2\ell)^{1/2}}{\left(a-\mu_k g\right)^{1/2}}}\\ &= \frac{\mu_k g 2\ell}{\left[2\ell\left(a-\mu_k g\right)\right]^{1/2}}=\frac{2\mu_k g\ell}{v}.\end{aligned}$$

***P6.28** Consider forces on the backpack as it slides in the Earth frame of reference.

$$\begin{aligned}&\sum F_y=ma_y:\quad +n-mg=ma,\ n=m(g+a),\ f_k=\mu_k m(g+a)\\ &\sum F_x=ma_x:\quad -\mu_k m(g+a)=ma_x\end{aligned}$$

The motion across the floor is described by $L=vt+\frac{1}{2}a_x t^2=vt-\frac{1}{2}\mu_k(g+a)t^2$.

We solve for $\mu_k$: $vt-L=\frac{1}{2}\mu_k(g+a)t^2$, $\boxed{\frac{2(vt-L)}{(g+a)t^2}=\mu_k}$.

**P6.29** In an inertial reference frame, the girl is accelerating horizontally inward at

$$\frac{v^2}{r}=\frac{(5.70\ \text{m/s})^2}{2.40\ \text{m}}=13.5\ \text{m/s}^2$$

In her own non-inertial frame, her head feels a horizontally outward fictitious force equal to its mass times this acceleration. Together this force and the weight of her head add to have a magnitude equal to the mass of her head times an acceleration of

$$\sqrt{g^2+\left(\frac{v^2}{r}\right)^2}=\sqrt{(9.80)^2+(13.5)^2}\ \text{m/s}^2=16.7\ \text{m/s}^2$$

This is larger than $g$ by a factor of $\frac{16.7}{9.80}=1.71$.

Thus, the force required to lift her head is larger by this factor, or the required force is

$$F=1.71(55.0\ \text{N})=\boxed{93.8\ \text{N}}.$$

***P6.30** (a) The chunk is at radius $r = \dfrac{0.137\text{ m} + 0.080\text{ m}}{4} = 0.054\,2\text{ m}$. Its speed is

$$v = \frac{2\pi r}{T} = 2\pi(0.054\,2\text{ m})\frac{20\,000}{60\text{ s}} = 114\text{ m/s}$$

and its acceleration

$$a_c = \frac{v^2}{r} = \frac{(114\text{ m/s})^2}{0.054\,2\text{ m}} = \boxed{2.38\times10^5\text{ m/s}^2\text{ horizontally inward}}$$

$$= 2.38\times10^5\text{ m/s}^2\left(\frac{g}{9.8\text{ m/s}^2}\right) = \boxed{2.43\times10^4 g}.$$

(b) In the frame of the turning cone, the chunk feels a horizontally outward force of $\dfrac{mv^2}{r}$. In this frame its acceleration is up along the cone, at $\tan^{-1}\dfrac{3.3\text{ cm}}{\frac{(13.7-8)\text{ cm}}{2}} = 49.2°$.

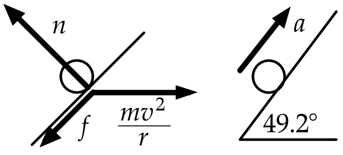


**FIG. P6.30(b)**

Take the $y$ axis perpendicular to the cone:

$$\sum F_y = ma_y: \quad +n - \frac{mv^2}{r}\sin 49.2° = 0$$

$$n = \left(2\times10^{-3}\text{ kg}\right)\left(2.38\times10^5\text{ m/s}^2\right)\sin 49.2° = \boxed{360\text{ N}}$$

(c) $f = \mu_k n = 0.6(360\text{ N}) = 216\text{ N}$

$$\sum F_x = ma_x: \quad \frac{mv^2}{r}\cos 49.2° - f = ma_x$$

$$\left(2\times10^{-3}\text{ kg}\right)\left(2.38\times10^5\text{ m/s}^2\right)\cos 49.2° - 216\text{ N} = \left(2\times10^{-3}\text{ kg}\right)a_x$$

$$a_x = \boxed{47.5\times10^4\text{ m/s}^2\text{ radially up the wall of the cone}}$$

**P6.31** $a_r = \left(\dfrac{4\pi^2 R_e}{T^2}\right)\cos 35.0° = 0.027\,6\text{ m/s}^2$

We take the $y$ axis along the local vertical.

$(a_{\text{net}})_y = 9.80 - (a_r)_y = 9.78\text{ m/s}^2$

$(a_{\text{net}})_x = 0.015\,8\text{ m/s}^2$

$\theta = \arctan\dfrac{a_x}{a_y} = \boxed{0.092\,8°}$

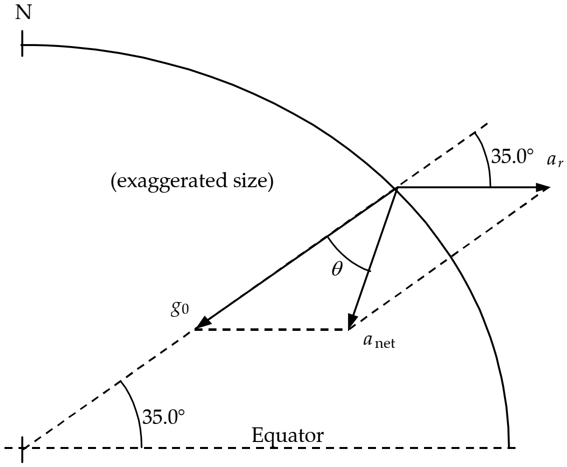


**FIG. P6.31**

## Section 6.4 **Motion in the Presence of Resistive Forces**

**P6.32** $m = 80.0\text{ kg}$, $v_T = 50.0\text{ m/s}$, $mg = \dfrac{D\rho A v_T^2}{2}$ $\therefore \dfrac{D\rho A}{2} = \dfrac{mg}{v_T^2} = 0.314\text{ kg/m}$

(a) At $v = 30.0\text{ m/s}$

$$a = g - \frac{\frac{D\rho A v^2}{2}}{m} = 9.80 - \frac{(0.314)(30.0)^2}{80.0} = \boxed{6.27\text{ m/s}^2\text{ downward}}$$

(b) At $v = 50.0\text{ m/s}$, terminal velocity has been reached.

$$\sum F_y = 0 = mg - R$$

$$\Rightarrow R = mg = (80.0\text{ kg})(9.80\text{ m/s}^2) = \boxed{784\text{ N directed up}}$$

(c) At $v = 30.0\text{ m/s}$

$$\frac{D\rho A v^2}{2} = (0.314)(30.0)^2 = \boxed{283\text{ N}}\text{ upward}$$

**P6.33** (a) $a = g - bv$

When $v = v_T$, $a = 0$ and $g = bv_T$ $\qquad b = \dfrac{g}{v_T}$

The Styrofoam falls 1.50 m at constant speed $v_T$ in 5.00 s.

Thus, $\qquad v_T = \dfrac{y}{t} = \dfrac{1.50\text{ m}}{5.00\text{ s}} = 0.300\text{ m/s}$

Then $\qquad b = \dfrac{9.80\text{ m/s}^2}{0.300\text{ m/s}} = \boxed{32.7\text{ s}^{-1}}$

(b) At $t = 0$, $v = 0$ and $\qquad a = g = \boxed{9.80\text{ m/s}^2}$ down

(c) When $v = 0.150\text{ m/s}$, $\quad a = g - bv = 9.80\text{ m/s}^2 - (32.7\text{ s}^{-1})(0.150\text{ m/s}) = \boxed{4.90\text{ m/s}^2}$ down

**P6.34** (a) $\rho = \dfrac{m}{V}$, $A = 0.020\,1\text{ m}^2$, $R = \dfrac{1}{2}\rho_{\text{air}} A D v_T^2 = mg$

$$m = \rho_{\text{bead}} V = 0.830\text{ g/cm}^3\left[\frac{4}{3}\pi(8.00\text{ cm})^3\right] = 1.78\text{ kg}$$

Assuming a drag coefficient of $D = 0.500$ for this spherical object, and taking the density of air at 20°C from the endpapers, we have

$$v_T = \sqrt{\frac{2(1.78\text{ kg})(9.80\text{ m/s}^2)}{0.500(1.20\text{ kg/m}^3)(0.020\,1\text{ m}^2)}} = \boxed{53.8\text{ m/s}}$$

(b) $v_f^2 = v_i^2 + 2gh = 0 + 2gh$: $h = \dfrac{v_f^2}{2g} = \dfrac{(53.8\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = \boxed{148\text{ m}}$

**P6.35** Since the upward velocity is constant, the resultant force on the ball is zero. Thus, the upward applied force equals the sum of the gravitational and drag forces (both downward): $F = mg + bv$.

The mass of the copper ball is

$$m = \frac{4\pi\rho r^3}{3} = \left(\frac{4}{3}\right)\pi\left(8.92\times10^3\ \text{kg/m}^3\right)\left(2.00\times10^{-2}\ \text{m}\right)^3 = 0.299\ \text{kg}.$$

The applied force is then

$$F = mg + bv = (0.299)(9.80) + (0.950)\left(9.00\times10^{-2}\right) = \boxed{3.01\ \text{N}}.$$

**P6.36**

$$\sum F_y = ma_y$$
$$+T\cos 40.0° - mg = 0$$
$$T = \frac{(620\ \text{kg})\left(9.80\ \text{m/s}^2\right)}{\cos 40.0°} = 7.93\times10^3\ \text{N}$$
$$\sum F_x = ma_x$$
$$-R + T\sin 40.0° = 0$$
$$R = \left(7.93\times10^3\ \text{N}\right)\sin 40.0° = 5.10\times10^3\ \text{N} = \frac{1}{2}D\rho Av^2$$
$$D = \frac{2R}{\rho Av^2} = \frac{2\left(5.10\times10^3\ \text{N}\right)\left(\frac{\text{kg m/s}^2}{\text{N}}\right)}{\left(1.20\ \text{kg/m}^2\right)\left(3.80\ \text{m}^2\right)\left(40.0\ \text{m/s}\right)^2} = \boxed{1.40}$$

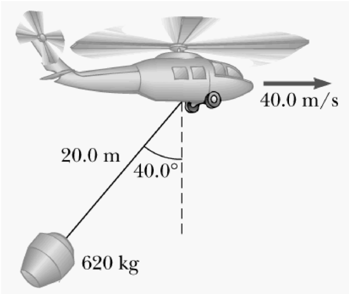


**FIG. P6.36**

**P6.37** (a) At terminal velocity, $R = v_T b = mg$

$$\therefore b = \frac{mg}{v_T} = \frac{\left(3.00\times10^{-3}\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)}{2.00\times10^{-2}\ \text{m/s}} = \boxed{1.47\ \text{N}\cdot\text{s/m}}$$

(b) In the equation describing the time variation of the velocity, we have

$$v = v_T\left(1 - e^{-bt/m}\right) \qquad v = 0.632v_T \text{ when } e^{-bt/m} = 0.368$$

or at time $t = -\left(\frac{m}{b}\right)\ln(0.368) = \boxed{2.04\times10^{-3}\ \text{s}}$

(c) At terminal velocity, $R = v_T b = mg = \boxed{2.94\times10^{-2}\ \text{N}}$

**P6.38** The resistive force is

$$R = \frac{1}{2}D\rho Av^2 = \frac{1}{2}(0.250)\left(1.20\ \text{kg/m}^3\right)\left(2.20\ \text{m}^2\right)\left(27.8\ \text{m/s}\right)^2$$
$$R = 255\ \text{N}$$
$$a = -\frac{R}{m} = -\frac{255\ \text{N}}{1200\ \text{kg}} = \boxed{-0.212\ \text{m/s}^2}$$

**P6.39** (a) $v(t)=v_i e^{-ct}$ $\qquad v(20.0\text{ s})=5.00=v_i e^{-20.0c}$, $v_i=10.0\text{ m/s}$.

So $5.00=10.0e^{-20.0c}$ and $-20.0c=\ln\left(\frac{1}{2}\right)$ $\qquad c=-\frac{\ln\left(\frac{1}{2}\right)}{20.0}=\boxed{3.47\times10^{-2}\text{ s}^{-1}}$

(b) At $t=40.0$ s $\qquad v=(10.0\text{ m/s})e^{-40.0c}=(10.0\text{ m/s})(0.250)=\boxed{2.50\text{ m/s}}$

(c) $v=v_i e^{-ct}$ $\qquad s=\frac{dv}{dt}=-cv_i e^{-ct}=\boxed{-cv}$

**P6.40**

$$\sum F=ma$$

$$-kmv^2=m\frac{dv}{dt}$$

$$-kdt=\frac{dv}{v^2}$$

$$-k\int_0^t dt=\int_{v_0}^{v} v^{-2}dv$$

$$-k(t-0)=\left.\frac{v^{-1}}{-1}\right|_{v_0}^{v}=-\frac{1}{v}+\frac{1}{v_0}$$

$$\frac{1}{v}=\frac{1}{v_0}+kt=\frac{1+v_0kt}{v_0}$$

$$v=\frac{v_0}{1+v_0kt}$$

***P6.41** (a) From Problem 40,

$$v=\frac{dx}{dt}=\frac{v_0}{1+v_0kt}$$

$$\int_0^x dx=\int_0^t v_0\frac{dt}{1+v_0kt}=\frac{1}{k}\int_0^t\frac{v_0kdt}{1+v_0kt}$$

$$x\Big|_0^x=\frac{1}{k}\ln(1+v_0kt)\Big|_0^t$$

$$x-0=\frac{1}{k}\left[\ln(1+v_0kt)-\ln1\right]$$

$$\boxed{x=\frac{1}{k}\ln(1+v_0kt)}$$

(b) We have $\ln(1+v_0kt)=kx$

$1+v_0kt=e^{kx}$ so $v=\frac{v_0}{1+v_0kt}=\frac{v_0}{e^{kx}}=\boxed{v_0e^{-kx}=v}$

***P6.42** We write $-kmv^2=-\frac{1}{2}D\rho Av^2$ so

$$k=\frac{D\rho A}{2m}=\frac{0.305\left(1.20\text{ kg/m}^3\right)\left(4.2\times10^{-3}\text{ m}^2\right)}{2(0.145\text{ kg})}=5.3\times10^{-3}/\text{m}$$

$$v=v_0e^{-kx}=(40.2\text{ m/s})e^{-\left(5.3\times10^{-3}/\text{m}\right)(18.3\text{ m})}=\boxed{36.5\text{ m/s}}$$

**P6.43** In $R = \frac{1}{2}D\rho A v^2$, we estimate that $D = 1.00$, $\rho = 1.20\ \text{kg/m}^3$, $A = (0.100\ \text{m})(0.160\ \text{m}) = 1.60 \times 10^{-2}\ \text{m}^2$ and $v = 27.0\ \text{m/s}$. The resistance force is then

$$R = \frac{1}{2}(1.00)\left(1.20\ \text{kg/m}^3\right)\left(1.60 \times 10^{-2}\ \text{m}^2\right)(27.0\ \text{m/s})^2 = 7.00\ \text{N}$$

or

$$R \sim \boxed{10^1\ \text{N}}$$

## Section 6.5 **Numerical Modeling in Particle Dynamics**

*Note*: In some problems we compute each new position as $x(t+\Delta t) = x(t) + v(t+\Delta t)\Delta t$, rather than $x(t+\Delta t) = x(t) + v(t)\Delta t$ as quoted in the text. This method has the same theoretical validity as that presented in the text, and in practice can give quicker convergence.

**P6.44** (a) At $v = v_T$, $a = 0$, $-mg + bv_T = 0$ $\qquad v_T = \frac{mg}{b} = \frac{\left(3.00 \times 10^{-3}\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)}{3.00 \times 10^{-2}\ \text{kg/s}} = \boxed{0.980\ \text{m/s}}$

(b)

| $t$(s) | $x$(m) | $v$(m/s) | $F$(mN) | $a\left(\text{m/s}^2\right)$ |
|---|---|---|---|---|
| 0 | 2 | 0 | –29.4 | –9.8 |
| 0.005 | 2 | –0.049 | –27.93 | –9.31 |
| 0.01 | 1.999 755 | –0.095 55 | –26.534 | –8.844 5 |
| 0.015 | 1.999 3 | –0.139 77 | –25.2 | –8.40 |

. . . we list the result after each tenth iteration

| $t$(s) | $x$(m) | $v$(m/s) | $F$(mN) | $a\left(\text{m/s}^2\right)$ |
|---|---|---|---|---|
| 0.05 | 1.990 | –0.393 | –17.6 | –5.87 |
| 0.1 | 1.965 | –0.629 | –10.5 | –3.51 |
| 0.15 | 1.930 | –0.770 | –6.31 | –2.10 |
| 0.2 | 1.889 | –0.854 | –3.78 | –1.26 |
| 0.25 | 1.845 | –0.904 | –2.26 | –0.754 |
| 0.3 | 1.799 | –0.935 | –1.35 | –0.451 |
| 0.35 | 1.752 | –0.953 | –0.811 | –0.270 |
| 0.4 | 1.704 | –0.964 | –0.486 | –0.162 |
| 0.45 | 1.65 | –0.970 | –0.291 | –0.096 9 |
| 0.5 | 1.61 | –0.974 | –0.174 | –0.058 0 |
| 0.55 | 1.56 | –0.977 | –0.110 | –0.034 7 |
| 0.6 | 1.51 | –0.978 | –0.062 4 | –0.020 8 |
| 0.65 | 1.46 | –0.979 | –0.037 4 | –0.012 5 |

Terminal velocity is never reached. The leaf is at 99.9% of $v_T$ after 0.67 s. The fall to the ground takes about 2.14 s. Repeating with $\Delta t = 0.001\ \text{s}$, we find the fall takes 2.14 s.

**P6.45** (a) When $v = v_T$, $a = 0$, $\sum F = -mg + Cv_T^2 = 0$

$$v_T = -\sqrt{\frac{mg}{C}} = -\sqrt{\frac{\left(4.80\times10^{-4}\text{ kg}\right)\left(9.80\text{ m/s}^2\right)}{2.50\times10^{-5}\text{ kg/m}}} = \boxed{-13.7\text{ m/s}}$$

(b)

| $t$(s) | $x$(m) | $v$(m/s) | $F$(mN) | $a\left(\text{m/s}^2\right)$ |
|---|---|---|---|---|
| 0 | 0 | 0 | – 4.704 | –9.8 |
| 0.2 | 0 | –1.96 | – 4.608 | –9.599 9 |
| 0.4 | –0.392 | –3.88 | – 4.327 6 | –9.015 9 |
| 0.6 | –1.168 | –5.683 2 | –3.896 5 | –8.117 8 |
| 0.8 | –2.30 | –7.306 8 | –3.369 3 | –7.019 3 |
| 1.0 | –3.77 | –8.710 7 | –2.807 1 | –5.848 1 |
| 1.2 | –5.51 | –9.880 3 | –2.263 5 | –4.715 6 |
| 1.4 | –7.48 | –10.823 | –1.775 3 | –3.698 6 |
| 1.6 | –9.65 | –11.563 | –1.361 6 | –2.836 6 |
| 1.8 | –11.96 | –12.13 | –1.03 | –2.14 |
| 2 | –14.4 | –12.56 | –0.762 | –1.59 |

. . . listing results after each fifth step

| $t$(s) | $x$(m) | $v$(m/s) | $F$(mN) | $a\left(\text{m/s}^2\right)$ |
|---|---|---|---|---|
| 3 | –27.4 | –13.49 | –0.154 | –0.321 |
| 4 | –41.0 | –13.67 | –0.029 1 | –0.060 6 |
| 5 | –54.7 | –13.71 | –0.005 42 | –0.011 3 |

The hailstone reaches 99% of $v_T$ after 3.3 s, 99.95% of $v_T$ after 5.0 s, 99.99% of $v_T$ after 6.0 s, 99.999% of $v_T$ after 7.4 s.

**P6.46** (a) At terminal velocity, $\sum F = 0 = -mg + Cv_T^2$

$$C = \frac{mg}{v_T^2} = \frac{(0.142\text{ kg})\left(9.80\text{ m/s}^2\right)}{(42.5\text{ m/s})^2} = \boxed{7.70\times10^{-4}\text{ kg/m}}$$

(b) $Cv^2 = \left(7.70\times10^{-4}\text{ kg/m}\right)(36.0\text{ m/s})^2 = \boxed{0.998\text{ N}}$

(c)

| Elapsed Time (s) | Altitude (m) | Speed (m/s) | Resistance Force (N) | Net Force (N) | Acceleration $\left(\text{m/s}^2\right)$ |
|---|---|---|---|---|---|
| 0.000 00 | 0.000 00 | 36.000 00 | –0.998 49 | –2.390 09 | –16.831 58 |
| 0.050 00 | 1.757 92 | 35.158 42 | –0.952 35 | –2.343 95 | –16.506 67 |
| … | | | | | |
| 2.950 00 | 48.623 27 | 0.824 94 | –0.000 52 | –1.392 12 | –9.803 69 |
| 3.000 00 | 48.640 00 | 0.334 76 | –0.000 09 | –1.391 69 | –9.800 61 |
| 3.050 00 | 48.632 24 | –0.155 27 | 0.000 02 | –1.391 58 | –9.799 87 |
| … | | | | | |
| 6.250 00 | 1.250 85 | –26.852 97 | 0.555 55 | –0.836 05 | –5.887 69 |
| 6.300 00 | –0.106 52 | –27.147 36 | 0.567 80 | –0.823 80 | –5.801 44 |

Maximum height is about $\boxed{49\text{ m}}$. It returns to the ground after about $\boxed{6.3\text{ s}}$ with a speed of approximately $\boxed{27\text{ m/s}}$.

**P6.47** (a) At constant velocity $\sum F = 0 = -mg + Cv_T^2$

$$v_T = -\sqrt{\frac{mg}{C}} = -\sqrt{\frac{(50.0\text{ kg})\left(9.80\text{ m/s}^2\right)}{0.200\text{ kg/m}}} = \boxed{-49.5\text{ m/s}}$$ with chute closed and

$$v_T = -\sqrt{\frac{(50.0\text{ kg})(9.80\text{ m/s})}{20.0\text{ kg/m}}} = \boxed{-4.95\text{ m/s}}$$ with chute open.

(b) We use time increments of 0.1 s for $0 < t < 10\text{ s}$, then 0.01 s for $10\text{ s} < t < 12\text{ s}$, and then 0.1 s again.

| time(s) | height(m) | velocity(m/s) |
|---|---|---|
| 0 | 1000 | 0 |
| 1 | 995 | –9.7 |
| 2 | 980 | –18.6 |
| 4 | 929 | –32.7 |
| 7 | 812 | –43.7 |
| 10 | 674 | –47.7 |
| 10.1 | 671 | –16.7 |
| 10.3 | 669 | –8.02 |
| 11 | 665 | –5.09 |
| 12 | 659 | –4.95 |
| 50 | 471 | –4.95 |
| 100 | 224 | –4.95 |
| 145 | 0 | –4.95 |

**6.48** (a) We use a time increment of 0.01 s.

| time(s) | $x$(m) | $y$(m) |
|---|---|---|
| 0 | 0 | 0 |
| 0.100 | 7.81 | 5.43 |
| 0.200 | 14.9 | 10.2 |
| 0.400 | 27.1 | 18.3 |
| 1.00 | 51.9 | 32.7 |
| 1.92 | 70.0 | 38.5 |
| 2.00 | 70.9 | 38.5 |
| 4.00 | 80.4 | 26.7 |
| 5.00 | 81.4 | 17.7 |
| 6.85 | 81.8 | 0 |

| with $\theta$ | we find range |
|---|---|
| 30.0° | 86.410 m |
| 35.0° | 81.8 m |
| 25.0° | 90.181 m |
| 20.0° | 92.874 m |
| 15.0° | 93.812 m |
| 10.0° | 90.965 m |
| 17.0° | 93.732 m |
| 16.0° | 93.839 8 m |
| 15.5° | 93.829 m |
| 15.8° | 93.839 m |
| 16.1° | 93.838 m |
| 15.9° | 93.840 2 m |

(b) range $= \boxed{81.8\text{ m}}$

(c) So we have maximum range at $\theta = \boxed{15.9°}$

**P6.49** (a) At terminal speed, $\sum F = -mg + Cv^2 = 0$. Thus,

$$C = \frac{mg}{v^2} = \frac{(0.046\,0 \text{ kg})(9.80 \text{ m/s}^2)}{(44.0 \text{ m/s})^2} = \boxed{2.33 \times 10^{-4} \text{ kg/m}}$$

(b) We set up a spreadsheet to calculate the motion, try different initial speeds, and home in on $\boxed{53 \text{ m/s}}$ as that required for horizontal range of 155 m, thus:

| Time $t$ (s) | $x$ (m) | $v_x$ (m/s) | $a_x$ $(\text{m/s}^2)$ | $y$ (m) | $v_y$ (m/s) | $a_y$ $(\text{m/s}^2)$ | $v=\sqrt{v_x^2+v_y^2}$ (m/s) | $\tan^{-1}\left(\frac{v_y}{v_x}\right)$ (deg) |
|---|---|---|---|---|---|---|---|---|
| 0.000 0 | 0.000 0 | 45.687 0 | –10.565 9 | 0.000 0 | 27.451 5 | –13.614 6 | 53.300 0 | 31.000 0 |
| 0.002 7 | 0.121 1 | 45.659 0 | –10.552 9 | 0.072 7 | 27.415 5 | –13.604 6 | 53.257 4 | 30.982 2 |
| … | | | | | | | | |
| 2.501 6 | 90.194 6 | 28.937 5 | –4.238 8 | 32.502 4 | 0.023 5 | –9.800 0 | 28.937 5 | 0.046 6 |
| 2.504 3 | 90.271 3 | 28.926 3 | –4.235 5 | 32.502 4 | –0.002 4 | –9.800 0 | 28.926 3 | –0.004 8 |
| 2.506 9 | 90.348 0 | 28.915 0 | –4.232 2 | 32.502 4 | –0.028 4 | –9.800 0 | 28.915 1 | –0.056 3 |
| … | | | | | | | | |
| 3.423 8 | 115.229 8 | 25.492 6 | –3.289 6 | 28.397 2 | –8.890 5 | –9.399 9 | 26.998 4 | –19.226 2 |
| 3.426 5 | 115.297 4 | 25.483 9 | –3.287 4 | 28.373 6 | –8.915 4 | –9.397 7 | 26.998 4 | –19.282 2 |
| 3.429 1 | 115.364 9 | 25.475 1 | –3.285 1 | 28.350 0 | –8.940 3 | –9.395 4 | 26.998 4 | –19.338 2 |
| … | | | | | | | | |
| 5.151 6 | 154.996 8 | 20.843 8 | –2.199 2 | 0.005 9 | –23.308 7 | –7.049 8 | 31.269 2 | –48.195 4 |
| 5.154 3 | 155.052 0 | 20.838 0 | –2.198 0 | –0.055 9 | –23.327 4 | –7.045 4 | 31.279 2 | –48.226 2 |

(c) Similarly, the initial speed is $\boxed{42 \text{ m/s}}$. The motion proceeds thus:

| Time $t$ (s) | $x$ (m) | $v_x$ (m/s) | $a_x$ $(\text{m/s}^2)$ | $y$ (m) | $v_y$ (m/s) | $a_y$ $(\text{m/s}^2)$ | $v=\sqrt{v_x^2+v_y^2}$ (m/s) | $\tan^{-1}\left(\frac{v_y}{v_x}\right)$ (deg) |
|---|---|---|---|---|---|---|---|---|
| 0.000 0 | 0.000 0 | 28.746 2 | –4.182 9 | 0.000 0 | 30.826 6 | –14.610 3 | 42.150 0 | 47.000 0 |
| 0.003 5 | 0.100 6 | 28.731 6 | –4.178 7 | 0.107 9 | 30.775 4 | –14.594 3 | 42.102 6 | 46.967 1 |
| … | | | | | | | | |
| 2.740 5 | 66.307 8 | 20.548 4 | –2.137 4 | 39.485 4 | 0.026 0 | –9.800 0 | 20.548 5 | 0.072 5 |
| 2.744 0 | 66.379 7 | 20.541 0 | –2.135 8 | 39.485 5 | –0.008 3 | –9.800 0 | 20.541 0 | –0.023 1 |
| 2.747 5 | 66.451 6 | 20.533 5 | –2.134 3 | 39.485 5 | –0.042 6 | –9.800 0 | 20.533 5 | –0.118 8 |
| … | | | | | | | | |
| 3.146 5 | 74.480 5 | 19.715 6 | –1.967 6 | 38.696 3 | –3.942 3 | –9.721 3 | 20.105 8 | –11.307 7 |
| 3.150 0 | 74.549 5 | 19.708 7 | –1.966 2 | 38.682 5 | –3.976 4 | –9.720 0 | 20.105 8 | –11.406 7 |
| 3.153 5 | 74.618 5 | 19.701 8 | –1.964 9 | 38.668 6 | –4.010 4 | –9.718 6 | 20.105 8 | –11.505 6 |
| … | | | | | | | | |
| 5.677 0 | 118.969 7 | 15.739 4 | –1.254 0 | 0.046 5 | –25.260 0 | –6.570 1 | 29.762 3 | –58.073 1 |
| 5.680 5 | 119.024 8 | 15.735 0 | –1.253 3 | –0.041 9 | –25.283 0 | –6.564 2 | 29.779 5 | –58.103 7 |

The trajectory in (c) reaches maximum height 39 m, as opposed to 33 m in (b). In both, the ball reaches maximum height when it has covered about 57% of its range. Its speed is a minimum somewhat later. The impact speeds are both about 30 m/s.

## Additional Problems

***P6.50** When the cloth is at a lower angle $\theta$, the radial component of $\sum F = ma$ reads

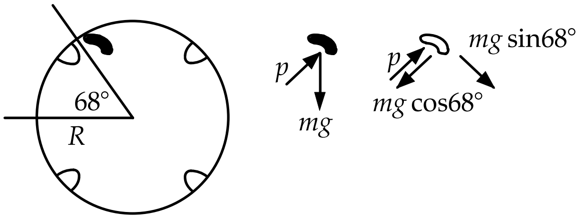


**FIG. P6.50**

$$n + mg\sin\theta = \frac{mv^2}{r}.$$

At $\theta = 68.0°$, the normal force drops to zero and $g\sin 68° = \frac{v^2}{r}$.

$$v = \sqrt{rg\sin 68°} = \sqrt{(0.33\text{ m})\left(9.8\text{ m/s}^2\right)\sin 68°} = 1.73\text{ m/s}$$

The rate of revolution is

$$\text{angular speed} = (1.73\text{ m/s})\left(\frac{1\text{ rev}}{2\pi r}\right)\left(\frac{2\pi r}{2\pi(0.33\text{ m})}\right) = \boxed{0.835\text{ rev/s}} = 50.1\text{ rev/min}.$$

***P6.51** (a)

$$v = (30\text{ km/h})\left(\frac{1\text{ h}}{3\,600\text{ s}}\right)\left(\frac{1\,000\text{ m}}{1\text{ km}}\right) = 8.33\text{ m/s}$$

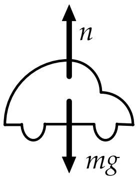


**FIG. P6.51**

$$\sum F_y = ma_y:\ +n - mg = -\frac{mv^2}{r}$$

$$n = m\left(g - \frac{v^2}{r}\right) = 1\,800\text{ kg}\left[9.8\text{ m/s}^2 - \frac{(8.33\text{ m/s})^2}{20.4\text{ m}}\right]$$

$$= \boxed{1.15\times 10^4\text{ N up}}$$

(b) Take $n = 0$. Then $mg = \frac{mv^2}{r}$.

$$v = \sqrt{gr} = \sqrt{\left(9.8\text{ m/s}^2\right)(20.4\text{ m})} = \boxed{14.1\text{ m/s}} = 50.9\text{ km/h}$$

**P6.52** (a)

$$\sum F_y = ma_y = \frac{mv^2}{R}$$

$$mg - n = \frac{mv^2}{R} \qquad n = \boxed{mg - \frac{mv^2}{R}}$$

(b) When $n = 0$, $mg = \frac{mv^2}{R}$

Then, $v = \boxed{\sqrt{gR}}$.

***P6.53** (a) $\text{slope} = \dfrac{0.160\text{ N} - 0}{9.9\text{ m}^2/\text{s}^2} = \boxed{0.016\,2\text{ kg/m}}$

(b) $\text{slope} = \dfrac{R}{v^2} = \dfrac{\frac{1}{2}D\rho A v^2}{v^2} = \boxed{\dfrac{1}{2}D\rho A}$

(c) $\dfrac{1}{2}D\rho A = 0.016\,2\text{ kg/m}$

$$D = \frac{2(0.016\,2\text{ kg/m})}{\left(1.20\text{ kg/m}^3\right)\pi(0.105\text{ m})^2} = \boxed{0.778}$$

(d) From the table, the eighth point is at force $mg = 8\left(1.64\times10^{-3}\text{ kg}\right)\left(9.8\text{ m/s}^2\right) = 0.129\text{ N}$ and horizontal coordinate $(2.80\text{ m/s})^2$. The vertical coordinate of the line is here $(0.016\,2\text{ kg/m})(2.8\text{ m/s})^2 = 0.127\text{ N}$. The scatter percentage is $\dfrac{0.129\text{ N} - 0.127\text{ N}}{0.127\text{ N}} = 1.5\%$.

(e) The interpretation of the graph can be stated thus: For stacked coffee filters falling at terminal speed, a graph of air resistance force as a function of squared speed demonstrates that the force is proportional to the speed squared within the experimental uncertainty estimated as $2\%$. This proportionality agrees with that described by the theoretical equation $R = \dfrac{1}{2}D\rho A v^2$. The value of the constant slope of the graph implies that the drag coefficient for coffee filters is $D = 0.78 \pm 2\%$.

**P6.54** (a) While the car negotiates the curve, the accelerometer is at the angle $\theta$.

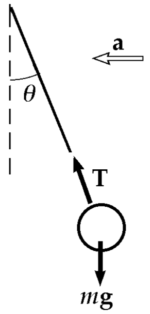


**FIG. P6.54**

Horizontally: $T\sin\theta = \dfrac{mv^2}{r}$

Vertically: $T\cos\theta = mg$

where $r$ is the radius of the curve, and $v$ is the speed of the car.

By division, $\tan\theta = \dfrac{v^2}{rg}$

Then $a_c = \dfrac{v^2}{r} = g\tan\theta$: $a_c = \left(9.80\text{ m/s}^2\right)\tan 15.0°$

$$a_c = \boxed{2.63\text{ m/s}^2}$$

(b) $r = \dfrac{v^2}{a_c}$ $\qquad r = \dfrac{(23.0\text{ m/s})^2}{2.63\text{ m/s}^2} = \boxed{201\text{ m}}$

(c) $v^2 = rg\tan\theta = (201\text{ m})\left(9.80\text{ m/s}^2\right)\tan 9.00°$ $\qquad v = \boxed{17.7\text{ m/s}}$

**P6.55** Take $x$-axis up the hill

$$\sum F_x = ma_x: \quad +T\sin\theta - mg\sin\phi = ma$$
$$a = \frac{T}{m}\sin\theta - g\sin\phi$$
$$\sum F_y = ma_y: \quad +T\cos\theta - mg\cos\phi = 0$$
$$T = \frac{mg\cos\phi}{\cos\theta}$$
$$a = \frac{g\cos\phi\sin\theta}{\cos\theta} - g\sin\phi$$
$$a = \boxed{g(\cos\phi\tan\theta - \sin\phi)}$$

***P6.56** (a) The speed of the bag is $\frac{2\pi(7.46\text{ m})}{38\text{ s}} = 1.23\text{ m/s}$. The total force on it must add to

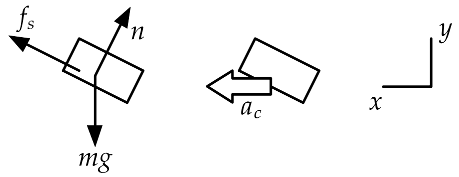


**FIG. P6.56**

$$ma_c = \frac{(30\text{ kg})(1.23\text{ m/s})^2}{7.46\text{ m}} = 6.12\text{ N}$$

$$\sum F_x = ma_x: \quad f_s\cos 20 - n\sin 20 = 6.12\text{ N}$$
$$\sum F_y = ma_y: \quad f_s\sin 20 + n\cos 20 - (30\text{ kg})\left(9.8\text{ m/s}^2\right) = 0$$
$$n = \frac{f_s\cos 20 - 6.12\text{ N}}{\sin 20}$$

Substitute:

$$f_s\sin 20 + f_s\frac{\cos^2 20}{\sin 20} - (6.12\text{ N})\frac{\cos 20}{\sin 20} = 294\text{ N}$$
$$f_s(2.92) = 294\text{ N} + 16.8\text{ N}$$
$$f_s = \boxed{106\text{ N}}$$

(b)

$$v = \frac{2\pi(7.94\text{ m})}{34\text{ s}} = 1.47\text{ m/s}$$
$$ma_c = \frac{(30\text{ kg})(1.47\text{ m/s})^2}{7.94\text{ m}} = 8.13\text{ N}$$
$$f_s\cos 20 - n\sin 20 = 8.13\text{ N}$$
$$f_s\sin 20 + n\cos 20 = 294\text{ N}$$
$$n = \frac{f_s\cos 20 - 8.13\text{ N}}{\sin 20}$$
$$f_s\sin 20 + f_s\frac{\cos^2 20}{\sin 20} - (8.13\text{ N})\frac{\cos 20}{\sin 20} = 294\text{ N}$$
$$f_s(2.92) = 294\text{ N} + 22.4\text{ N}$$
$$f_s = 108\text{ N}$$
$$n = \frac{(108\text{ N})\cos 20 - 8.13\text{ N}}{\sin 20} = 273\text{ N}$$
$$\mu_s = \frac{f_s}{n} = \frac{108\text{ N}}{273\text{ N}} = \boxed{0.396}$$

**P6.57** (a) Since the centripetal acceleration of a person is downward (toward the axis of the earth), it is equivalent to the effect of a falling elevator. Therefore,

$$F_g' = F_g - \frac{mv^2}{r} \text{ or } \boxed{F_g > F_g'}$$

(b) At the poles $v = 0$ and $F_g' = F_g = mg = 75.0(9.80) = \boxed{735\text{ N}}$ down.

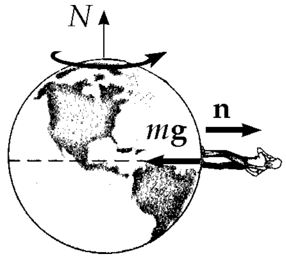


**FIG. P6.57**

At the equator, $F_g' = F_g - ma_c = 735\text{ N} - 75.0(0.033\,7)\text{ N} = \boxed{732\text{ N}}$ down.

**P6.58** (a) Since the object of mass $m_2$ is in equilibrium, $\sum F_y = T - m_2 g = 0$

or $T = \boxed{m_2 g}$.

(b) The tension in the string provides the required centripetal acceleration of the puck.

Thus, $F_c = T = \boxed{m_2 g}$.

(c) From $F_c = \dfrac{m_1 v^2}{R}$

we have $v = \sqrt{\dfrac{RF_c}{m_1}} = \boxed{\sqrt{\left(\dfrac{m_2}{m_1}\right) gR}}$.

**P6.59** (a) $v = (300\text{ mi/h})\left(\dfrac{88.0\text{ ft/s}}{60.0\text{ mi/h}}\right) = 440\text{ ft/s}$

At the lowest point, his seat exerts an upward force; therefore, his weight seems to increase. His apparent weight is

$$F_g' = mg + m\frac{v^2}{r} = 160 + \left(\frac{160}{32.0}\right)\frac{(440)^2}{1\,200} = \boxed{967\text{ lb}}.$$

(b) At the highest point, the force of the seat on the pilot is directed down and

$$F_g' = mg - m\frac{v^2}{r} = \boxed{-647\text{ lb}}.$$

Since the plane is upside down, the seat exerts this downward force.

(c) When $F_g' = 0$, then $mg = \dfrac{mv^2}{R}$. If we vary the aircraft's $R$ and $v$ such that the above is true, then the pilot feels weightless.

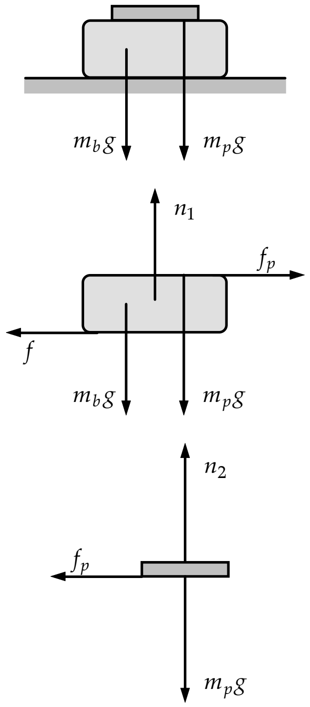


**FIG. P6.60**

**P6.60** For the block to remain stationary, $\sum F_y = 0$ and $\sum F_x = ma_r$.

$$n_1 = \left(m_p + m_b\right)g \text{ so } f \le \mu_{s1} n_1 = \mu_{s1}\left(m_p + m_b\right)g.$$

At the point of slipping, the required centripetal force equals the maximum friction force:

$$\therefore \left(m_p + m_b\right)\frac{v_{\max}^2}{r} = \mu_{s1}\left(m_p + m_b\right)g$$

or $v_{\max} = \sqrt{\mu_{s1} r g} = \sqrt{(0.750)(0.120)(9.80)} = 0.939 \text{ m/s}$.

For the penny to remain stationary on the block:

$$\sum F_y = 0 \Rightarrow n_2 - m_p g = 0 \text{ or } n_2 = m_p g$$

and $\sum F_x = ma_r \Rightarrow f_p = m_p \dfrac{v^2}{r}$.

When the penny is about to slip on the block, $f_p = f_{p,\max} = \mu_{s2} n_2$

or $\mu_{s2} m_p g = m_p \dfrac{v_{\max}^2}{r}$

$$v_{\max} = \sqrt{\mu_{s2} r g} = \sqrt{(0.520)(0.120)(9.80)} = 0.782 \text{ m/s}$$

This is less than the maximum speed for the block, so the penny slips before the block starts to slip. The maximum rotation frequency is

$$\text{Max rpm} = \frac{v_{\max}}{2\pi r} = (0.782 \text{ m/s})\left[\frac{1 \text{ rev}}{2\pi(0.120 \text{ m})}\right]\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = \boxed{62.2 \text{ rev/min}}.$$

**P6.61** $v = \dfrac{2\pi r}{T} = \dfrac{2\pi(9.00 \text{ m})}{(15.0 \text{ s})} = 3.77 \text{ m/s}$

(a) $a_r = \dfrac{v^2}{r} = \boxed{1.58 \text{ m/s}^2}$

(b) $F_{\text{low}} = m\left(g + a_r\right) = \boxed{455 \text{ N}}$

(c) $F_{\text{high}} = m\left(g - a_r\right) = \boxed{328 \text{ N}}$

(d) $F_{\text{mid}} = m\sqrt{g^2 + a_r^2} = \boxed{397 \text{ N upward and}}$ at $\theta = \tan^{-1}\dfrac{a_r}{g} = \tan^{-1}\dfrac{1.58}{9.8} = \boxed{9.15° \text{ inward}}$.

**P6.62** Standing on the inner surface of the rim, and moving with it, each person will feel a normal force exerted by the rim. This inward force causes the $3.00 \text{ m/s}^2$ centripetal acceleration:

$a_c = \dfrac{v^2}{r}$: $\quad v = \sqrt{a_c r} = \sqrt{\left(3.00 \text{ m/s}^2\right)(60.0 \text{ m})} = 13.4 \text{ m/s}$

The period of rotation comes from $v = \dfrac{2\pi r}{T}$: $\quad T = \dfrac{2\pi r}{v} = \dfrac{2\pi(60.0 \text{ m})}{13.4 \text{ m/s}} = 28.1 \text{ s}$

so the frequency of rotation is $\quad f = \dfrac{1}{T} = \dfrac{1}{28.1 \text{ s}} = \dfrac{1}{28.1 \text{ s}}\left(\dfrac{60 \text{ s}}{1 \text{ min}}\right) = \boxed{2.14 \text{ rev/min}}$.

**P6.63** (a) The mass at the end of the chain is in vertical equilibrium. Thus $T\cos\theta = mg$.

Horizontally $T\sin\theta = ma_r = \dfrac{mv^2}{r}$

$$r = (2.50\sin\theta + 4.00)\text{ m}$$
$$r = (2.50\sin 28.0° + 4.00)\text{ m} = 5.17\text{ m}$$

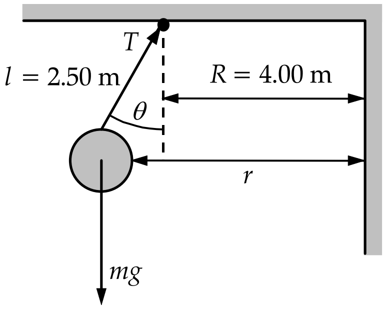


**FIG. P6.63**

Then $a_r = \dfrac{v^2}{5.17\text{ m}}$.

By division $\tan\theta = \dfrac{a_r}{g} = \dfrac{v^2}{5.17g}$

$$v^2 = 5.17g\tan\theta = (5.17)(9.80)(\tan 28.0°)\ \text{m}^2/\text{s}^2$$
$$v = \boxed{5.19\ \text{m/s}}$$

(b) $T\cos\theta = mg$

$$T = \frac{mg}{\cos\theta} = \frac{(50.0\text{ kg})(9.80\text{ m/s}^2)}{\cos 28.0°} = \boxed{555\text{ N}}$$

**P6.64** (a) The putty, when dislodged, rises and returns to the original level in time *t*. To find *t*, we use $v_f = v_i + at$: i.e., $-v = +v - gt$ or $t = \dfrac{2v}{g}$ where *v* is the speed of a point on the rim of the wheel.

If *R* is the radius of the wheel, $v = \dfrac{2\pi R}{t}$, so $t = \dfrac{2v}{g} = \dfrac{2\pi R}{v}$.

Thus, $v^2 = \pi Rg$ and $\boxed{v = \sqrt{\pi Rg}}$.

(b) The putty is dislodged when *F*, the force holding it to the wheel is

$$F = \frac{mv^2}{R} = \boxed{m\pi g}.$$

**P6.65** (a)

$$n = \frac{mv^2}{R} \qquad f - mg = 0$$

$$f = \mu_s n \qquad v = \frac{2\pi R}{T}$$

$$T = \boxed{\sqrt{\frac{4\pi^2 R\mu_s}{g}}}$$

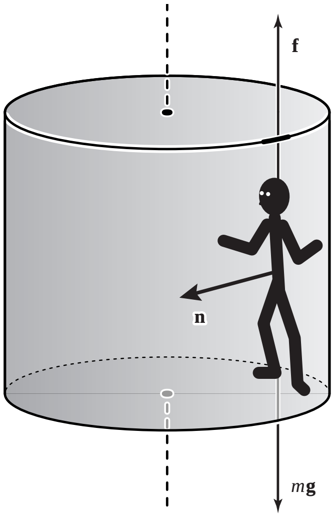


**FIG. P6.65**

(b) $T = \boxed{2.54\text{ s}}$

$$\#\ \frac{\text{rev}}{\text{min}} = \frac{1\text{ rev}}{2.54\text{ s}}\left(\frac{60\text{ s}}{\text{min}}\right) = \boxed{23.6\ \frac{\text{rev}}{\text{min}}}$$

**P6.66** Let the $x$–axis point eastward, the $y$-axis upward, and the $z$-axis point southward.

(a) The range is $Z = \frac{v_i^2 \sin 2\theta_i}{g}$

The initial speed of the ball is therefore

$$v_i = \sqrt{\frac{gZ}{\sin 2\theta_i}} = \sqrt{\frac{(9.80)(285)}{\sin 96.0°}} = 53.0 \text{ m/s}$$

The time the ball is in the air is found from $\Delta y = v_{iy}t + \frac{1}{2}a_y t^2$ as

$$0 = (53.0 \text{ m/s})(\sin 48.0°)t - \left(4.90 \text{ m/s}^2\right)t^2$$

giving $t = \boxed{8.04 \text{ s}}$.

(b) $$v_{ix} = \frac{2\pi R_e \cos\phi_i}{86\,400 \text{ s}} = \frac{2\pi\left(6.37\times 10^6 \text{ m}\right)\cos 35.0°}{86\,400 \text{ s}} = \boxed{379 \text{ m/s}}$$

(c) 360° of latitude corresponds to a distance of $2\pi R_e$, so 285 m is a change in latitude of

$$\Delta\phi = \left(\frac{S}{2\pi R_e}\right)(360°) = \left(\frac{285 \text{ m}}{2\pi\left(6.37\times 10^6 \text{ m}\right)}\right)(360°) = 2.56\times 10^{-3} \text{ degrees}$$

The final latitude is then $\phi_f = \phi_i - \Delta\phi = 35.0° – 0.002\,56° = 34.997\,4°$.

The cup is moving eastward at a speed $v_{fx} = \frac{2\pi R_e \cos\phi_f}{86\,400 \text{ s}}$, which is larger than the eastward velocity of the tee by

$$\begin{aligned}\Delta v_x = v_{fx} - v_{fi} &= \frac{2\pi R_e}{86\,400 \text{ s}}\left[\cos\phi_f - \cos\phi_i\right] = \frac{2\pi R_e}{86\,400 \text{ s}}\left[\cos(\phi_i - \Delta\phi) - \cos\phi_i\right] \\ &= \frac{2\pi R_e}{86\,400 \text{ s}}\left[\cos\phi_i \cos\Delta\phi + \sin\phi_i \sin\Delta\phi - \cos\phi_i\right]\end{aligned}$$

Since $\Delta\phi$ is such a small angle, $\cos\Delta\phi \approx 1$ and $\Delta v_x \approx \frac{2\pi R_e}{86\,400 \text{ s}}\sin\phi_i \sin\Delta\phi$.

$$\Delta v_x \approx \frac{2\pi\left(6.37\times 10^6 \text{ m}\right)}{86\,400 \text{ s}}\sin 35.0° \sin 0.002\,56° = \boxed{1.19\times 10^{-2} \text{ m/s}}$$

(d) $$\Delta x = (\Delta v_x)t = \left(1.19\times 10^{-2} \text{ m/s}\right)(8.04 \text{ s}) = 0.095\,5 \text{ m} = \boxed{9.55 \text{ cm}}$$

**P6.67** (a) If the car is about to slip *down* the incline, *f* is directed up the incline.

$\sum F_y = n\cos\theta + f\sin\theta - mg = 0$ where $f = \mu_s n$ gives

$$n = \frac{mg}{\cos\theta(1+\mu_s\tan\theta)} \text{ and } f = \frac{\mu_s mg}{\cos\theta(1+\mu_s\tan\theta)}.$$

Then, $\sum F_x = n\sin\theta - f\cos\theta = m\frac{v_{\text{min}}^2}{R}$ yields

$$\boxed{v_{\text{min}} = \sqrt{\frac{Rg(\tan\theta-\mu_s)}{1+\mu_s\tan\theta}}}.$$

When the car is about to slip *up* the incline, *f* is directed down the incline. Then, $\sum F_y = n\cos\theta - f\sin\theta - mg = 0$ with $f = \mu_s n$ yields

$$n = \frac{mg}{\cos\theta(1-\mu_s\tan\theta)} \text{ and } f = \frac{\mu_s mg}{\cos\theta(1-\mu_s\tan\theta)}.$$

In this case, $\sum F_x = n\sin\theta + f\cos\theta = m\frac{v_{\text{max}}^2}{R}$, which gives

$$\boxed{v_{\text{max}} = \sqrt{\frac{Rg(\tan\theta+\mu_s)}{1-\mu_s\tan\theta}}}.$$

(b) If $v_{\text{min}} = \sqrt{\frac{Rg(\tan\theta-\mu_s)}{1+\mu_s\tan\theta}} = 0$, then $\boxed{\mu_s = \tan\theta}$.

(c)

$$v_{\text{min}} = \sqrt{\frac{(100\text{ m})\left(9.80\text{ m/s}^2\right)(\tan 10.0°-0.100)}{1+(0.100)\tan 10.0°}} = \boxed{8.57\text{ m/s}}$$

$$v_{\text{max}} = \sqrt{\frac{(100\text{ m})\left(9.80\text{ m/s}^2\right)(\tan 10.0°+0.100)}{1-(0.100)\tan 10.0°}} = \boxed{16.6\text{ m/s}}$$

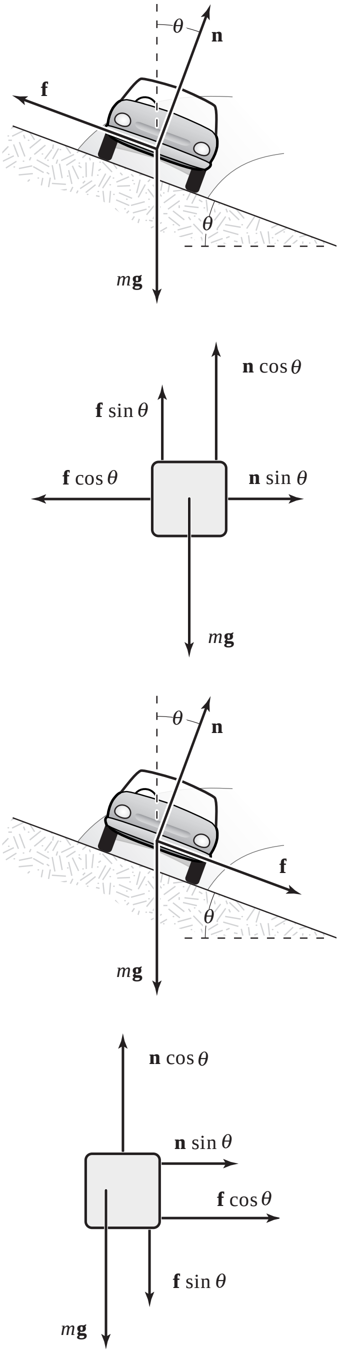


FIG. P6.67

**P6.68** (a) The bead moves in a circle with radius $v = R\sin\theta$ at a speed of

$$v = \frac{2\pi r}{T} = \frac{2\pi R\sin\theta}{T}$$

The normal force has
an inward radial component of $n\sin\theta$
and an upward component of $n\cos\theta$

$$\sum F_y = ma_y: \quad n\cos\theta - mg = 0$$

or

$$n = \frac{mg}{\cos\theta}$$

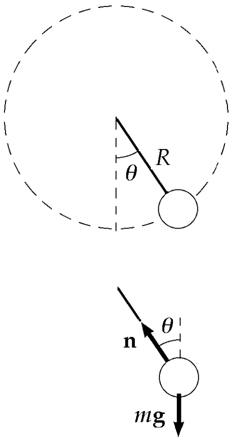


FIG. P6.68(a)

Then $\sum F_x = n\sin\theta = m\frac{v^2}{r}$ becomes $\left(\frac{mg}{\cos\theta}\right)\sin\theta = \frac{m}{R\sin\theta}\left(\frac{2\pi R\sin\theta}{T}\right)^2$

which reduces to $\frac{g\sin\theta}{\cos\theta} = \frac{4\pi^2 R\sin\theta}{T^2}$

This has two solutions: $\sin\theta = 0 \Rightarrow \theta = 0°$ (1)

and $\cos\theta = \frac{gT^2}{4\pi^2 R}$ (2)

If $R = 15.0$ cm and $T = 0.450$ s, the second solution yields

$$\cos\theta = \frac{(9.80\ \text{m/s}^2)(0.450\ \text{s})^2}{4\pi^2(0.150\ \text{m})} = 0.335 \text{ and } \theta = 70.4°$$

Thus, in this case, the bead can ride at two positions $\boxed{\theta = 70.4°}$ and $\boxed{\theta = 0°}$.

(b) At this slower rotation, solution (2) above becomes

$$\cos\theta = \frac{(9.80\ \text{m/s}^2)(0.850\ \text{s})^2}{4\pi^2(0.150\ \text{m})} = 1.20\text{, which is impossible.}$$

In this case, the bead can ride only at the bottom of the loop, $\boxed{\theta = 0°}$. The loop's rotation must be faster than a certain threshold value in order for the bead to move away from the lowest position.

**P6.69** At terminal velocity, the accelerating force of gravity is balanced by frictional drag: $mg = arv + br^2v^2$

(a) $mg = \left(3.10\times10^{-9}\right)v + \left(0.870\times10^{-10}\right)v^2$

For water, $m = \rho V = 1\,000\ \text{kg/m}^3\left[\frac{4}{3}\pi\left(10^{-5}\ \text{m}\right)^3\right]$

$$4.11\times10^{-11} = \left(3.10\times10^{-9}\right)v + \left(0.870\times10^{-10}\right)v^2$$

Assuming $v$ is small, ignore the second term on the right hand side: $\boxed{v = 0.013\,2\ \text{m/s}}$.

(b) $mg = \left(3.10\times10^{-8}\right)v + \left(0.870\times10^{-8}\right)v^2$

Here we cannot ignore the second term because the coefficients are of nearly equal magnitude.

$$4.11\times10^{-8} = \left(3.10\times10^{-8}\right)v + \left(0.870\times10^{-8}\right)v^2$$

$$v = \frac{-3.10 \pm \sqrt{(3.10)^2 + 4(0.870)(4.11)}}{2(0.870)} = \boxed{1.03\ \text{m/s}}$$

(c) $mg = \left(3.10\times10^{-7}\right)v + \left(0.870\times10^{-6}\right)v^2$

Assuming $v > 1\ \text{m/s}$, and ignoring the first term:

$4.11\times10^{-5} = \left(0.870\times10^{-6}\right)v^2$ $\quad v = \boxed{6.87\ \text{m/s}}$

**P6.70** $v = \left(\frac{mg}{b}\right)\left[1 - \exp\left(\frac{-bt}{m}\right)\right]$ where $\exp(x) = e^x$ is the exponential function.

At $t \to \infty$, $\quad v \to v_T = \frac{mg}{b}$

At $t = 5.54$ s $\quad 0.500v_T = v_T\left[1 - \exp\left(\frac{-b(5.54\ \text{s})}{9.00\ \text{kg}}\right)\right]$

$$\exp\left(\frac{-b(5.54\ \text{s})}{9.00\ \text{kg}}\right) = 0.500;$$

$$\frac{-b(5.54\ \text{s})}{9.00\ \text{kg}} = \ln 0.500 = -0.693;$$

$$b = \frac{(9.00\ \text{kg})(0.693)}{5.54\ \text{s}} = 1.13\ \text{m/s}$$

(a) $v_T = \frac{mg}{b}$ $\quad v_T = \frac{(9.00\ \text{kg})\left(9.80\ \text{m/s}^2\right)}{1.13\ \text{kg/s}} = \boxed{78.3\ \text{m/s}}$

(b) $0.750v_T = v_T\left[1 - \exp\left(\frac{-1.13t}{9.00\ \text{s}}\right)\right]$ $\quad \exp\left(\frac{-1.13t}{9.00\ \text{s}}\right) = 0.250$

$$t = \frac{9.00(\ln 0.250)}{-1.13}\ \text{s} = \boxed{11.1\ \text{s}}$$

*continued on next page*

(c) $\frac{dx}{dt}=\left(\frac{mg}{b}\right)\left[1-\exp\left(-\frac{bt}{m}\right)\right]$; $\int_{x_0}^{x} dx=\int_0^t\left(\frac{mg}{b}\right)\left[1-\exp\left(\frac{-bt}{m}\right)\right]dt$

$$x-x_0=\frac{mgt}{b}+\left(\frac{m^2g}{b^2}\right)\exp\left(\frac{-bt}{m}\right)\Bigg|_0^t=\frac{mgt}{b}+\left(\frac{m^2g}{b^2}\right)\left[\exp\left(\frac{-bt}{m}\right)-1\right]$$

At $t=5.54\text{ s}$, $x=9.00\text{ kg}\left(9.80\text{ m/s}^2\right)\frac{5.54\text{ s}}{1.13\text{ kg/s}}+\left(\frac{(9.00\text{ kg})^2\left(9.80\text{ m/s}^2\right)}{(1.13\text{ m/s})^2}\right)\left[\exp(-0.693)-1\right]$

$$x=434\text{ m}+626\text{ m}(-0.500)=\boxed{121\text{ m}}$$

**P6.71** $\sum F_y=L_y-T_y-mg=L\cos 20.0°-T\sin 20.0°-7.35\text{ N}=ma_y=0$

$\sum F_x=L_x+T_x=L\sin 20.0°+T\cos 20.0°=m\frac{v^2}{r}$

$m\frac{v^2}{r}=0.750\text{ kg}\frac{(35.0\text{ m/s})^2}{(60.0\text{ m})\cos 20.0°}=16.3\text{ N}$

$\therefore L\sin 20.0°+T\cos 20.0°=16.3\text{ N}$

$L\cos 20.0°-T\sin 20.0°=7.35\text{ N}$

$L+T\frac{\cos 20.0°}{\sin 20.0°}=\frac{16.3\text{ N}}{\sin 20.0°}$

$L-T\frac{\sin 20.0°}{\cos 20.0°}=\frac{7.35\text{ N}}{\cos 20.0°}$

$T(\cot 20.0°+\tan 20.0°)=\frac{16.3\text{ N}}{\sin 20.0°}-\frac{7.35\text{ N}}{\cos 20.0°}$

$T(3.11)=39.8\text{ N}$

$T=\boxed{12.8\text{ N}}$

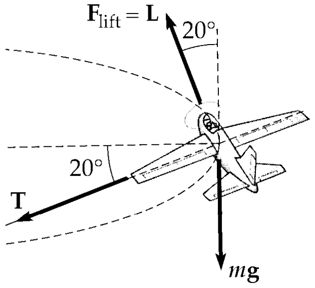
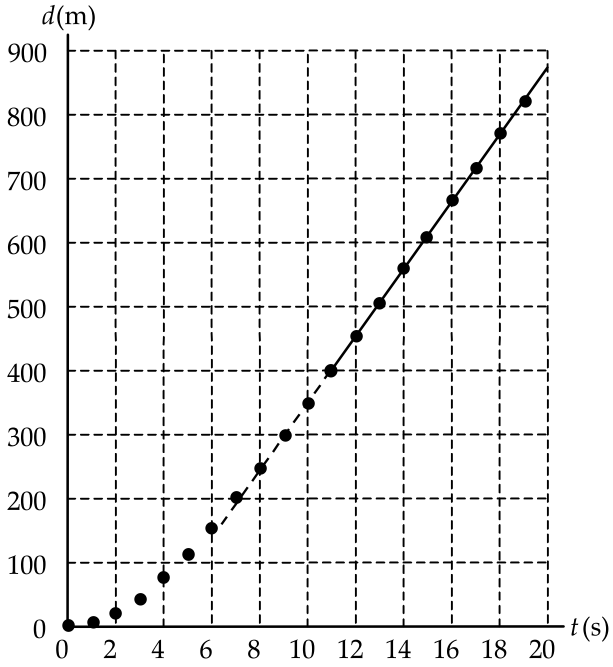


FIG. P6.71

**P6.72** (a)

| $t$(s) | $d$(m) |
|---|---|
| 1.00 | 4.88 |
| 2.00 | 18.9 |
| 3.00 | 42.1 |
| 4.00 | 73.8 |
| 5.00 | 112 |
| 6.00 | 154 |
| 7.00 | 199 |
| 8.00 | 246 |
| 9.00 | 296 |
| 10.0 | 347 |
| 11.0 | 399 |
| 12.0 | 452 |
| 13.0 | 505 |
| 14.0 | 558 |
| 15.0 | 611 |
| 16.0 | 664 |
| 17.0 | 717 |
| 18.0 | 770 |
| 19.0 | 823 |
| 20.0 | 876 |

(b)



(c) A straight line fits the points from $t = 11.0\text{ s}$ to 20.0 s quite precisely. Its slope is the terminal speed.

$$v_T = \text{slope} = \frac{876\text{ m} - 399\text{ m}}{20.0\text{ s} - 11.0\text{ s}} = \boxed{53.0\text{ m/s}}$$

***P6.73** $v = v_i - kx$ implies the acceleration is $a = \dfrac{dv}{dt} = 0 - k\dfrac{dx}{dt} = -kv$

Then the total force is $\sum F = ma = m(-kv)$

The resistive force is opposite to the velocity: $\boxed{\sum \mathbf{F} = -km\mathbf{v}}$.

## ANSWERS TO EVEN PROBLEMS

**P6.2** 215 N horizontally inward

**P6.4** $6.22 \times 10^{-12}$ N

**P6.6** (a) 1.65 km/s; (b) $6.84 \times 10^{3}$ s

**P6.8** 0.966 $g$

**P6.10** (a) $(-0.233\,\hat{\mathbf{i}} + 0.163\,\hat{\mathbf{j}})\text{ m/s}^2$; (b) 6.53 m/s; (c) $(-0.181\,\hat{\mathbf{i}} + 0.181\,\hat{\mathbf{j}})\text{ m/s}^2$

**P6.12** $2.06 \times 10^{3}$ rev/min

**P6.14** (a) $\sqrt{R\left(\dfrac{2T}{m} - g\right)}$; (b) $2T$ upward

**P6.16** (a) $1.33\text{ m/s}^2$; (b) $1.79\text{ m/s}^2$ forward and 48.0° inward

**P6.18** 8.88 N

| | |
|---|---|
| **P6.20** | (a) 8.62 m; (b) $Mg$ downward;<br>(c) 8.45 $\text{m/s}^2$, Unless they are belted in, the riders will fall from the cars. |
| **P6.22** | 15.3 m/s Straight across the dashboard to the left |
| **P6.24** | 0.527° |
| **P6.26** | (a) 1.41 h; (b) 17.1 |
| **P6.28** | $\mu_k = \dfrac{2(vt-L)}{(g+a)t^2}$ |
| **P6.30** | (a) $2.38\times10^5\ \text{m/s}^2$ horizontally inward $= 2.43\times10^4 g$; (b) 360 N inward perpendicular to the cone;<br>(c) $47.5\times10^4\ \text{m/s}^2$ |
| **P6.32** | (a) 6.27 $\text{m/s}^2$ downward; (b) 784 N up;<br>(c) 283 N up |
| **P6.34** | (a) 53.8 m/s; (b) 148 m |
| **P6.36** | 1.40 |
| **P6.38** | $-0.212\ \text{m/s}^2$ |
| **P6.40** | see the solution |
| **P6.42** | 36.5 m/s |
| **P6.44** | (a) 0.980 m/s; (b) see the solution |
| **P6.46** | (a) $7.70\times10^{-4}$ kg/m; (b) 0.998 N;<br>(c) The ball reaches maximum height 49 m. Its flight lasts 6.3 s and its impact speed is 27 m/s. |
| **P6.48** | (a) see the solution; (b) 81.8 m; (c) 15.9° |
| **P6.50** | 0.835 rev/s |
| **P6.52** | (a) $mg - \dfrac{mv^2}{R}$; (b) $v = \sqrt{gR}$ |
| **P6.54** | (a) 2.63 $\text{m/s}^2$; (b) 201 m; (c) 17.7 m/s |
| **P6.56** | (a) 106 N; (b) 0.396 |
| **P6.58** | (a) $m_2g$; (b) $m_2g$; (c) $\sqrt{\left(\dfrac{m_2}{m_1}\right)gR}$ |
| **P6.60** | 62.2 rev/min |
| **P6.62** | 2.14 rev/min |
| **P6.64** | (a) $v = \sqrt{\pi Rg}$; (b) $m\pi g$ |
| **P6.66** | (a) 8.04 s; (b) 379 m/s; (c) 1.19 cm/s;<br>(d) 9.55 cm |
| **P6.68** | (a) either 70.4° or 0°; (b) 0° |
| **P6.70** | (a) 78.3 m/s; (b) 11.1 s; (c) 121 m |
| **P6.72** | (a) and (b) see the solution; (c) 53.0 m/s |

# 7

# Energy and Energy Transfer

## CHAPTER OUTLINE

7.1 Systems and Environments
7.2 Work Done by a Constant Force
7.3 The Scalar Product of Two Vectors
7.4 Work Done by a Varying Force
7.5 Kinetic Energy and the Work-Kinetic Energy Theorem
7.6 The Non-Isolated System—Conservation of Energy
7.7 Situations Involving Kinetic Friction
7.8 Power
7.9 Energy and the Automobile

## ANSWERS TO QUESTIONS

**Q7.1** The force is perpendicular to every increment of displacement. Therefore, $\mathbf{F} \cdot \Delta\mathbf{r} = 0$.

**Q7.2** (a) Positive work is done by the chicken on the dirt.

(b) No work is done, although it may seem like there is.

(c) Positive work is done on the bucket.

(d) Negative work is done on the bucket.

(e) Negative work is done on the person's torso.

**Q7.3** Yes. Force times distance over which the toe is in contact with the ball. No, he is no longer applying a force. Yes, both air friction and gravity do work.

**Q7.4** Force of tension on a ball rotating on the end of a string. Normal force and gravitational force on an object at rest or moving across a level floor.

**Q7.5** (a) Tension (b) Air resistance

(c) Positive in increasing velocity on the downswing.
Negative in decreasing velocity on the upswing.

**Q7.6** No. The vectors might be in the third and fourth quadrants, but if the angle between them is less than 90° their dot product is positive.

**Q7.7** The scalar product of two vectors is positive if the angle between them is between 0 and 90°. The scalar product is negative when $90° < \theta < 180°$.

**Q7.8** If the coils of the spring are initially in contact with one another, as the load increases from zero, the graph would be an upwardly curved arc. After the load increases sufficiently, the graph will be linear, described by Hooke's Law. This linear region will be quite large compared to the first region. The graph will then be a downward curved arc as the coiled spring becomes a completely straight wire. As the load increases with a straight wire, the graph will become a straight line again, with a significantly smaller slope. Eventually, the wire would break.

**Q7.9** $k' = 2k$. To stretch the smaller piece one meter, each coil would have to stretch twice as much as one coil in the original long spring, since there would be half as many coils. Assuming that the spring is ideal, twice the stretch requires twice the force.

**Q7.10** Kinetic energy is always positive. Mass and squared speed are both positive. A moving object can always do positive work in striking another object and causing it to move along the same direction of motion.

**Q7.11** Work is only done in accelerating the ball from rest. The work is done over the effective length of the pitcher's arm—the distance his hand moves through windup and until release.

**Q7.12** Kinetic energy is proportional to mass. The first bullet has twice as much kinetic energy.

**Q7.13** The longer barrel will have the higher muzzle speed. Since the accelerating force acts over a longer distance, the change in kinetic energy will be larger.

**Q7.14** (a) Kinetic energy is proportional to squared speed. Doubling the speed makes an object's kinetic energy four times larger.

(b) If the total work on an object is zero in some process, its speed must be the same at the final point as it was at the initial point.

**Q7.15** The larger engine is unnecessary. Consider a 30 minute commute. If you travel the same speed in each car, it will take the same amount of time, expending the same amount of energy. The extra power available from the larger engine isn't used.

**Q7.16** If the instantaneous power output by some agent changes continuously, its average power in a process must be equal to its instantaneous power at least one instant. If its power output is constant, its instantaneous power is always equal to its average power.

**Q7.17** It decreases, as the force required to lift the car decreases.

**Q7.18** As you ride an express subway train, a backpack at your feet has no kinetic energy as measured by you since, according to you, the backpack is not moving. In the frame of reference of someone on the side of the tracks as the train rolls by, the backpack is moving and has mass, and thus has kinetic energy.

**Q7.19** The rock increases in speed. The farther it has fallen, the more force it might exert on the sand at the bottom; but it might instead make a deeper crater with an equal-size average force. The farther it falls, the more work it will do in stopping. Its kinetic energy is increasing due to the work that the gravitational force does on it.

**Q7.20** The normal force does no work because the angle between the normal force and the direction of motion is usually 90°. Static friction usually does no work because there is no distance through which the force is applied.

**Q7.21** An argument for: As a glider moves along an airtrack, the only force that the track applies on the glider is the normal force. Since the angle between the direction of motion and the normal force is 90°, the work done must be zero, even if the track is not level.
Against: An airtrack has bumpers. When a glider bounces from the bumper at the end of the airtrack, it loses a bit of energy, as evidenced by a decreased speed. The airtrack does negative work.

**Q7.22** Gaspard de Coriolis first stated the work-kinetic energy theorem. Jean Victor Poncelet, an engineer who invaded Russia with Napoleon, is most responsible for demonstrating its wide practical applicability, in his 1829 book *Industrial Mechanics*. Their work came remarkably late compared to the elucidation of momentum conservation in collisions by Descartes and to Newton's *Mathematical Principles of the Philosophy of Nature*, both in the 1600's.

## SOLUTIONS TO PROBLEMS

### Section 7.1 **Systems and Environments**

### Section 7.2 **Work Done by a Constant Force**

**P7.1** (a) $W = F\Delta r\cos\theta = (16.0\text{ N})(2.20\text{ m})\cos 25.0° = \boxed{31.9\text{ J}}$

(b), (c) The normal force and the weight are both at 90° to the displacement in any time interval. Both do $\boxed{0}$ work.

(d) $\sum W = 31.9\text{ J} + 0 + 0 = \boxed{31.9\text{ J}}$

**P7.2** The component of force along the direction of motion is

$$F\cos\theta = (35.0\text{ N})\cos 25.0° = 31.7\text{ N}.$$

The work done by this force is

$$W = (F\cos\theta)\Delta r = (31.7\text{ N})(50.0\text{ m}) = \boxed{1.59\times 10^3\text{ J}}.$$

**P7.3** **Method One.**

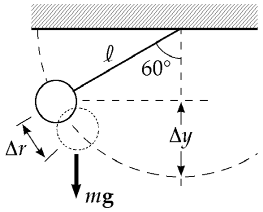


**FIG. P7.3**

Let $\phi$ represent the instantaneous angle the rope makes with the vertical as it is swinging up from $\phi_i = 0$ to $\phi_f = 60°$. In an incremental bit of motion from angle $\phi$ to $\phi + d\phi$, the definition of radian measure implies that $\Delta r = (12\text{ m})d\phi$. The angle $\theta$ between the incremental displacement and the force of gravity is $\theta = 90° + \phi$. Then $\cos\theta = \cos(90° + \phi) = -\sin\phi$. The work done by the gravitational force on Batman is

$$W = \int_i^f F\cos\theta dr = \int_{\phi=0}^{\phi=60°} mg(-\sin\phi)(12\text{ m})d\phi$$

$$= -mg(12\text{ m})\int_0^{60°}\sin\phi\, d\phi = (-80\text{ kg})\left(9.8\text{ m/s}^2\right)(12\text{ m})(-\cos\phi)\Big|_0^{60°}$$

$$= (-784\text{ N})(12\text{ m})(-\cos 60° + 1) = \boxed{-4.70\times 10^3\text{ J}}$$

**Method Two.**

The force of gravity on Batman is $mg = (80\text{ kg})\left(9.8\text{ m/s}^2\right) = 784\text{ N}$ down. Only his vertical displacement contributes to the work gravity does. His original $y$-coordinate below the tree limb is –12 m. His final $y$-coordinate is $(-12\text{ m})\cos 60° = -6\text{ m}$. His change in elevation is $-6\text{ m} - (-12\text{ m}) = 6\text{ m}$. The work done by gravity is

$$W = F\Delta r\cos\theta = (784\text{ N})(6\text{ m})\cos 180° = \boxed{-4.70\text{ kJ}}.$$

**P7.4** (a) $W = mgh = \left(3.35\times10^{-5}\right)(9.80)(100)\text{ J} = \boxed{3.28\times10^{-2}\text{ J}}$

(b) Since $R = mg$, $W_{\text{air resistance}} = \boxed{-3.28\times10^{-2}\text{ J}}$

## Section 7.3 **The Scalar Product of Two Vectors**

**P7.5** $A = 5.00$; $B = 9.00$; $\theta = 50.0°$

$\mathbf{A}\cdot\mathbf{B} = AB\cos\theta = (5.00)(9.00)\cos50.0° = \boxed{28.9}$

**P7.6**

$$\mathbf{A}\cdot\mathbf{B} = \left(A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}\right)\cdot\left(B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}\right)$$

$$\begin{aligned}\mathbf{A}\cdot\mathbf{B} = {} & A_xB_x\left(\hat{\mathbf{i}}\cdot\hat{\mathbf{i}}\right) + A_xB_y\left(\hat{\mathbf{i}}\cdot\hat{\mathbf{j}}\right) + A_xB_z\left(\hat{\mathbf{i}}\cdot\hat{\mathbf{k}}\right)\\ & + A_yB_x\left(\hat{\mathbf{j}}\cdot\hat{\mathbf{i}}\right) + A_yB_y\left(\hat{\mathbf{j}}\cdot\hat{\mathbf{j}}\right) + A_yB_z\left(\hat{\mathbf{j}}\cdot\hat{\mathbf{k}}\right)\\ & + A_zB_x\left(\hat{\mathbf{k}}\cdot\hat{\mathbf{i}}\right) + A_zB_y\left(\hat{\mathbf{k}}\cdot\hat{\mathbf{j}}\right) + A_zB_z\left(\hat{\mathbf{k}}\cdot\hat{\mathbf{k}}\right)\end{aligned}$$

$$\mathbf{A}\cdot\mathbf{B} = \boxed{A_xB_x + A_yB_y + A_zB_z}$$

**P7.7** (a) $W = \mathbf{F}\cdot\Delta\mathbf{r} = F_x x + F_y y = (6.00)(3.00)\text{ N}\cdot\text{m} + (-2.00)(1.00)\text{ N}\cdot\text{m} = \boxed{16.0\text{ J}}$

(b) $$\theta = \cos^{-1}\left(\frac{\mathbf{F}\cdot\Delta\mathbf{r}}{F\Delta r}\right) = \cos^{-1}\frac{16}{\sqrt{\left((6.00)^2 + (-2.00)^2\right)\left((3.00)^2 + (1.00)^2\right)}} = \boxed{36.9°}$$

**P7.8** We must first find the angle between the two vectors. It is:

$$\theta = 360° - 118° - 90.0° - 132° = 20.0°$$

Then

$$\mathbf{F}\cdot\mathbf{v} = Fv\cos\theta = (32.8\text{ N})(0.173\text{ m/s})\cos20.0°$$

or $\mathbf{F}\cdot\mathbf{v} = 5.33\dfrac{\text{N}\cdot\text{m}}{\text{s}} = 5.33\dfrac{\text{J}}{\text{s}} = \boxed{5.33\text{ W}}$

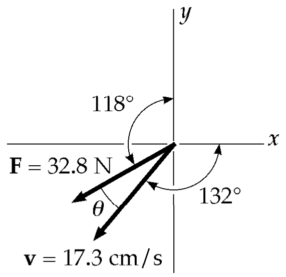


**FIG. P7.8**

**P7.9** (a) $\mathbf{A} = 3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}$

$\mathbf{B} = 4.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}}$ $\qquad \theta = \cos^{-1}\dfrac{\mathbf{A}\cdot\mathbf{B}}{AB} = \cos^{-1}\dfrac{12.0 + 8.00}{\sqrt{(13.0)(32.0)}} = \boxed{11.3°}$

(b) $\mathbf{B} = 3.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}} + 2.00\hat{\mathbf{k}}$

$\mathbf{A} = -2.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}}$ $\qquad \cos\theta = \dfrac{\mathbf{A}\cdot\mathbf{B}}{AB} = \dfrac{-6.00 - 16.0}{\sqrt{(20.0)(29.0)}}$ $\qquad \theta = \boxed{156°}$

(c) $\mathbf{A} = \hat{\mathbf{i}} - 2.00\hat{\mathbf{j}} + 2.00\hat{\mathbf{k}}$

$\mathbf{B} = 3.00\hat{\mathbf{j}} + 4.00\hat{\mathbf{k}}$ $\qquad \theta = \cos^{-1}\left(\dfrac{\mathbf{A}\cdot\mathbf{B}}{AB}\right) = \cos^{-1}\left(\dfrac{-6.00 + 8.00}{\sqrt{9.00}\cdot\sqrt{25.0}}\right) = \boxed{82.3°}$

**P7.10** $\mathbf{A}-\mathbf{B}=\left(3.00\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}\right)-\left(-\hat{\mathbf{i}}+2.00\hat{\mathbf{j}}+5.00\hat{\mathbf{k}}\right)$

$$\mathbf{A}-\mathbf{B}=4.00\hat{\mathbf{i}}-\hat{\mathbf{j}}-6.00\hat{\mathbf{k}}$$

$$\mathbf{C}\cdot(\mathbf{A}-\mathbf{B})=\left(2.00\hat{\mathbf{j}}-3.00\hat{\mathbf{k}}\right)\cdot\left(4.00\hat{\mathbf{i}}-\hat{\mathbf{j}}-6.00\hat{\mathbf{k}}\right)=0+(-2.00)+(+18.0)=\boxed{16.0}$$

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## Section 7.4 Work Done by a Varying Force

**P7.11** $W=\int_i^f F\,dx=$ area under curve from $x_i$ to $x_f$

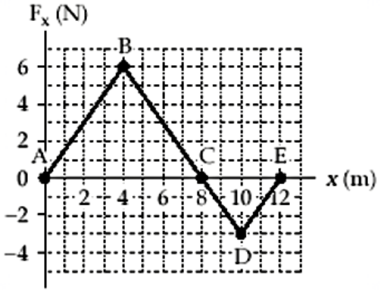

FIG. P7.11

(a) $x_i=0 \qquad x_f=8.00\text{ m}$

$$W=\text{ area of triangle } ABC=\left(\frac{1}{2}\right)AC\times\text{ altitude,}$$

$$W_{0\to 8}=\left(\frac{1}{2}\right)\times 8.00\text{ m}\times 6.00\text{ N}=\boxed{24.0\text{ J}}$$

(b) $x_i=8.00\text{ m} \qquad x_f=10.0\text{ m}$

$$W=\text{ area of }\Delta CDE=\left(\frac{1}{2}\right)CE\times\text{ altitude,}$$

$$W_{8\to 10}=\left(\frac{1}{2}\right)\times(2.00\text{ m})\times(-3.00\text{ N})=\boxed{-3.00\text{ J}}$$

(c) $W_{0\to 10}=W_{0\to 8}+W_{8\to 10}=24.0+(-3.00)=\boxed{21.0\text{ J}}$

**P7.12** $F_x=(8x-16)\text{ N}$

(a) See figure to the right

(b) $$W_{\text{net}}=\frac{-(2.00\text{ m})(16.0\text{ N})}{2}+\frac{(1.00\text{ m})(8.00\text{ N})}{2}=\boxed{-12.0\text{ J}}$$

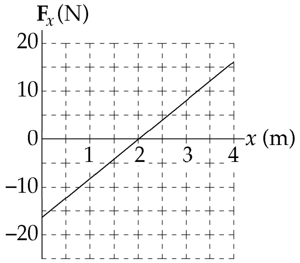

FIG. P7.12

**P7.13** $W = \int F_x dx$

and $W$ equals the area under the Force-Displacement curve

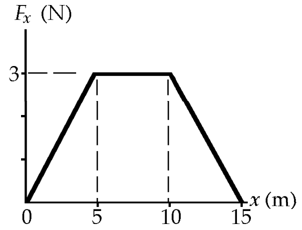

**FIG. P7.13**

(a) For the region $0 \le x \le 5.00\ \text{m}$,

$$W = \frac{(3.00\ \text{N})(5.00\ \text{m})}{2} = \boxed{7.50\ \text{J}}$$

(b) For the region $5.00 \le x \le 10.0$,

$$W = (3.00\ \text{N})(5.00\ \text{m}) = \boxed{15.0\ \text{J}}$$

(c) For the region $10.0 \le x \le 15.0$,

$$W = \frac{(3.00\ \text{N})(5.00\ \text{m})}{2} = \boxed{7.50\ \text{J}}$$

(d) For the region $0 \le x \le 15.0$

$$W = (7.50 + 7.50 + 15.0)\ \text{J} = \boxed{30.0\ \text{J}}$$

**P7.14** $W = \int_i^f \mathbf{F} \cdot d\mathbf{r} = \int_0^{5\ \text{m}} \left(4x\hat{\mathbf{i}} + 3y\hat{\mathbf{j}}\right) \text{N} \cdot dx\hat{\mathbf{i}}$

$$\int_0^{5\ \text{m}} (4\ \text{N/m})x\,dx + 0 = (4\ \text{N/m})\left.\frac{x^2}{2}\right|_0^{5\ \text{m}} = \boxed{50.0\ \text{J}}$$

**P7.15** $k = \frac{F}{y} = \frac{Mg}{y} = \frac{(4.00)(9.80)\ \text{N}}{2.50 \times 10^{-2}\ \text{m}} = 1.57 \times 10^3\ \text{N/m}$

(a) For 1.50 kg mass $y = \frac{mg}{k} = \frac{(1.50)(9.80)}{1.57 \times 10^3} = \boxed{0.938\ \text{cm}}$

(b) Work $= \frac{1}{2}ky^2$

Work $= \frac{1}{2}\left(1.57 \times 10^3\ \text{N} \cdot \text{m}\right)\left(4.00 \times 10^{-2}\ \text{m}\right)^2 = \boxed{1.25\ \text{J}}$

**P7.16** (a) Spring constant is given by $F = kx$

$$k = \frac{F}{x} = \frac{(230\ \text{N})}{(0.400\ \text{m})} = \boxed{575\ \text{N/m}}$$

(b) Work $= F_{\text{avg}}x = \frac{1}{2}(230\ \text{N})(0.400\ \text{m}) = \boxed{46.0\ \text{J}}$

***P7.17** (a)

$$F_{\text{applied}} = k_{\text{leaf}}x_\ell + k_{\text{helper}}x_h = k_\ell x_\ell + k_h(x_\ell - y_0)$$

$$5\times10^5\ \text{N} = 5.25\times10^5\ \frac{\text{N}}{\text{m}}x_\ell + 3.60\times10^5\ \frac{\text{N}}{\text{m}}(x_\ell - 0.5\ \text{m})$$

$$x_\ell = \frac{6.8\times10^5\ \text{N}}{8.85\times10^5\ \text{N/m}} = \boxed{0.768\ \text{m}}$$

(b)

$$W = \frac{1}{2}k_\ell x_\ell^2 + \frac{1}{2}k_h x_h^2 = \frac{1}{2}\left(5.25\times10^5\ \frac{\text{N}}{\text{m}}\right)(0.768\ \text{m})^2 + \frac{1}{2}3.60\times10^5\ \frac{\text{N}}{\text{m}}(0.268\ \text{m})^2$$

$$= \boxed{1.68\times10^5\ \text{J}}$$

**P7.18** (a)

$$W = \int_i^f \mathbf{F}\cdot d\mathbf{r}$$

$$W = \int_0^{0.600\ \text{m}} \left(15\,000\ \text{N} + 10\,000x\ \text{N/m} - 25\,000x^2\ \text{N/m}^2\right)dx\cos 0°$$

$$W = 15\,000x + \frac{10\,000x^2}{2} - \frac{25\,000x^3}{3}\bigg|_0^{0.600\ \text{m}}$$

$$W = 9.00\ \text{kJ} + 1.80\ \text{kJ} - 1.80\ \text{kJ} = \boxed{9.00\ \text{kJ}}$$

(b) Similarly,

$$W = (15.0\ \text{kN})(1.00\ \text{m}) + \frac{(10.0\ \text{kN/m})(1.00\ \text{m})^2}{2} - \frac{(25.0\ \text{kN/m}^2)(1.00\ \text{m})^3}{3}$$

$W = \boxed{11.7\ \text{kJ}}$, larger by 29.6%

**P7.19** $4.00\ \text{J} = \frac{1}{2}k(0.100\ \text{m})^2$

$\therefore k = 800\ \text{N/m}$ and to stretch the spring to 0.200 m requires

$$\Delta W = \frac{1}{2}(800)(0.200)^2 - 4.00\ \text{J} = \boxed{12.0\ \text{J}}$$

**P7.20** (a) The radius to the object makes angle $\theta$ with the horizontal, so its weight makes angle $\theta$ with the negative side of the *x*-axis, when we take the *x*–axis in the direction of motion tangent to the cylinder.

$$\sum F_x = ma_x$$

$$F - mg\cos\theta = 0$$

$$F = \boxed{mg\cos\theta}$$

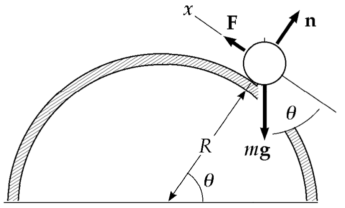

**FIG. P7.20**

(b)

$$W = \int_i^f \mathbf{F}\cdot d\mathbf{r}$$

We use radian measure to express the next bit of displacement as $dr = Rd\theta$ in terms of the next bit of angle moved through:

$$W = \int_0^{\pi/2} mg\cos\theta R d\theta = mgR\sin\theta\Big|_0^{\pi/2}$$

$$W = mgR(1-0) = \boxed{mgR}$$

***P7.21** The same force makes both light springs stretch.

(a) The hanging mass moves down by

$$x = x_1 + x_2 = \frac{mg}{k_1} + \frac{mg}{k_2} = mg\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

$$= 1.5 \text{ kg } 9.8 \text{ m/s}^2 \left(\frac{1 \text{ m}}{1\,200 \text{ N}} + \frac{1 \text{ m}}{1\,800 \text{ N}}\right) = \boxed{2.04 \times 10^{-2} \text{ m}}$$

(b) We define the effective spring constant as

$$k = \frac{F}{x} = \frac{mg}{mg(1/k_1 + 1/k_2)} = \left(\frac{1}{k_1} + \frac{1}{k_2}\right)^{-1}$$

$$= \left(\frac{1 \text{ m}}{1\,200 \text{ N}} + \frac{1 \text{ m}}{1\,800 \text{ N}}\right)^{-1} = \boxed{720 \text{ N/m}}$$

***P7.22** See the solution to problem 7.21.

(a) $x = mg\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$

(b) $k = \left(\frac{1}{k_1} + \frac{1}{k_2}\right)^{-1}$

**P7.23** $[k] = \left[\frac{F}{x}\right] = \frac{\text{N}}{\text{m}} = \frac{\text{kg} \cdot \text{m/s}^2}{\text{m}} = \boxed{\frac{\text{kg}}{\text{s}^2}}$

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Section 7.5 **Kinetic Energy and the Work-Kinetic Energy Theorem**

Section 7.6 **The Non-Isolated System—Conservation of Energy**

**P7.24** (a) $K_A = \frac{1}{2}(0.600 \text{ kg})(2.00 \text{ m/s})^2 = \boxed{1.20 \text{ J}}$

(b) $\frac{1}{2}mv_B^2 = K_B$: $v_B = \sqrt{\frac{2K_B}{m}} = \sqrt{\frac{(2)(7.50)}{0.600}} = \boxed{5.00 \text{ m/s}}$

(c) $\sum W = \Delta K = K_B - K_A = \frac{1}{2}m\left(v_B^2 - v_A^2\right) = 7.50 \text{ J} - 1.20 \text{ J} = \boxed{6.30 \text{ J}}$

**P7.25** (a) $K = \frac{1}{2}mv^2 = \frac{1}{2}(0.300 \text{ kg})(15.0 \text{ m/s})^2 = \boxed{33.8 \text{ J}}$

(b) $K = \frac{1}{2}(0.300)(30.0)^2 = \frac{1}{2}(0.300)(15.0)^2(4) = 4(33.8) = \boxed{135 \text{ J}}$

**P7.26** $\mathbf{v}_i = \left(6.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}\right) = \text{ m/s}$

(a) $v_i = \sqrt{v_{ix}^2 + v_{iy}^2} = \sqrt{40.0}\text{ m/s}$

$$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(3.00\text{ kg})\left(40.0\text{ m}^2/\text{s}^2\right) = \boxed{60.0\text{ J}}$$

(b) $\mathbf{v}_f = 8.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}}$

$$v_f^2 = \mathbf{v}_f \cdot \mathbf{v}_f = 64.0 + 16.0 = 80.0\text{ m}^2/\text{s}^2$$

$$\Delta K = K_f - K_i = \frac{1}{2}m\left(v_f^2 - v_i^2\right) = \frac{3.00}{2}(80.0) - 60.0 = \boxed{60.0\text{ J}}$$

**P7.27** Consider the work done on the pile driver from the time it starts from rest until it comes to rest at the end of the fall. Let $d = 5.00\text{ m}$ represent the distance over which the driver falls freely, and $h = 0.12\text{ m}$ the distance it moves the piling.

$$\sum W = \Delta K: \quad W_{\text{gravity}} + W_{\text{beam}} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

so $(mg)(h+d)\cos 0° + \left(\overline{F}\right)(d)\cos 180° = 0 - 0$.

Thus, $\overline{F} = \dfrac{(mg)(h+d)}{d} = \dfrac{(2\,100\text{ kg})\left(9.80\text{ m/s}^2\right)(5.12\text{ m})}{0.120\text{ m}} = \boxed{8.78\times 10^5\text{ N}}$. The force on the pile driver is $\boxed{\text{upward}}$.

**P7.28** (a) $\Delta K = K_f - K_i = \dfrac{1}{2}mv_f^2 - 0 = \sum W = $ (area under curve from $x = 0$ to $x = 5.00\text{ m}$)

$$v_f = \sqrt{\frac{2(\text{area})}{m}} = \sqrt{\frac{2(7.50\text{ J})}{4.00\text{ kg}}} = \boxed{1.94\text{ m/s}}$$

(b) $\Delta K = K_f - K_i = \dfrac{1}{2}mv_f^2 - 0 = \sum W = $ (area under curve from $x = 0$ to $x = 10.0\text{ m}$)

$$v_f = \sqrt{\frac{2(\text{area})}{m}} = \sqrt{\frac{2(22.5\text{ J})}{4.00\text{ kg}}} = \boxed{3.35\text{ m/s}}$$

(c) $\Delta K = K_f - K_i = \dfrac{1}{2}mv_f^2 - 0 = \sum W = $ (area under curve from $x = 0$ to $x = 15.0\text{ m}$)

$$v_f = \sqrt{\frac{2(\text{area})}{m}} = \sqrt{\frac{2(30.0\text{ J})}{4.00\text{ kg}}} = \boxed{3.87\text{ m/s}}$$

**P7.29** (a) $K_i + \sum W = K_f = \dfrac{1}{2}mv_f^2$

$$0 + \sum W = \frac{1}{2}\left(15.0\times 10^{-3}\text{ kg}\right)(780\text{ m/s})^2 = \boxed{4.56\text{ kJ}}$$

(b) $F = \dfrac{W}{\Delta r\cos\theta} = \dfrac{4.56\times 10^3\text{ J}}{(0.720\text{ m})\cos 0°} = \boxed{6.34\text{ kN}}$

(c) $a = \dfrac{v_f^2 - v_i^2}{2x_f} = \dfrac{(780\text{ m/s})^2 - 0}{2(0.720\text{ m})} = \boxed{422\text{ km/s}^2}$

(d) $\sum F = ma = \left(15\times 10^{-3}\text{ kg}\right)\left(422\times 10^3\text{ m/s}^2\right) = \boxed{6.34\text{ kN}}$

**P7.30** (a) $v_f = 0.096\left(3\times10^{8}\text{ m/s}\right) = 2.88\times10^{7}\text{ m/s}$

$$K_f = \frac{1}{2}mv_f^2 = \frac{1}{2}\left(9.11\times10^{-31}\text{ kg}\right)\left(2.88\times10^{7}\text{ m/s}\right)^2 = \boxed{3.78\times10^{-16}\text{ J}}$$

(b) $K_i + W = K_f$: $\quad 0 + F\Delta r\cos\theta = K_f$

$$F(0.028\text{ m})\cos 0° = 3.78\times10^{-16}\text{ J}$$

$$F = \boxed{1.35\times10^{-14}\text{ N}}$$

(c) $\sum F = ma$; $\quad a = \dfrac{\sum F}{m} = \dfrac{1.35\times10^{-14}\text{ N}}{9.11\times10^{-31}\text{ kg}} = \boxed{1.48\times10^{+16}\text{ m/s}^2}$

(d) $v_{xf} = v_{xi} + a_x t \quad 2.88\times10^{7}\text{ m/s} = 0 + \left(1.48\times10^{16}\text{ m/s}^2\right)t$

$$t = \boxed{1.94\times10^{-9}\text{ s}}$$

Check: $\quad x_f = x_i + \dfrac{1}{2}\left(v_{xi} + v_{xf}\right)t$

$$0.028\text{ m} = 0 + \frac{1}{2}\left(0 + 2.88\times10^{7}\text{ m/s}\right)t$$

$$t = 1.94\times10^{-9}\text{ s}$$

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## Section 7.7 **Situations Involving Kinetic Friction**

**P7.31** $\sum F_y = ma_y$: $\quad n - 392\text{ N} = 0$

$$n = 392\text{ N}$$

$$f_k = \mu_k n = (0.300)(392\text{ N}) = 118\text{ N}$$

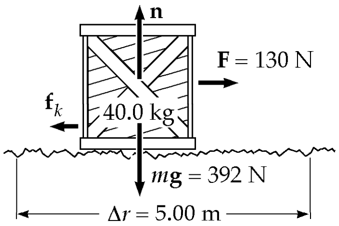

**FIG. P7.31**

(a) $W_F = F\Delta r\cos\theta = (130)(5.00)\cos 0° = \boxed{650\text{ J}}$

(b) $\Delta E_{\text{int}} = f_k\Delta x = (118)(5.00) = \boxed{588\text{ J}}$

(c) $W_n = n\Delta r\cos\theta = (392)(5.00)\cos 90° = \boxed{0}$

(d) $W_g = mg\Delta r\cos\theta = (392)(5.00)\cos(-90°) = \boxed{0}$

(e) $\Delta K = K_f - K_i = \sum W_{\text{other}} - \Delta E_{\text{int}}$

$$\frac{1}{2}mv_f^2 - 0 = 650\text{ J} - 588\text{ J} + 0 + 0 = \boxed{62.0\text{ J}}$$

(f) $v_f = \sqrt{\dfrac{2K_f}{m}} = \sqrt{\dfrac{2(62.0\text{ J})}{40.0\text{ kg}}} = \boxed{1.76\text{ m/s}}$

**P7.32** (a)

$$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 = \frac{1}{2}(500)\left(5.00 \times 10^{-2}\right)^2 - 0 = 0.625 \text{ J}$$

$$W_s = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2 - 0$$

so $$v_f = \sqrt{\frac{2\left(\sum W\right)}{m}} = \sqrt{\frac{2(0.625)}{2.00}} \text{ m/s} = \boxed{0.791 \text{ m/s}}$$

(b)

$$\frac{1}{2}mv_i^2 - f_k\Delta x + W_s = \frac{1}{2}mv_f^2$$

$$0 - (0.350)(2.00)(9.80)(0.050\,0) \text{ J} + 0.625 \text{ J} = \frac{1}{2}mv_f^2$$

$$0.282 \text{ J} = \frac{1}{2}(2.00 \text{ kg})v_f^2$$

$$v_f = \sqrt{\frac{2(0.282)}{2.00}} \text{ m/s} = \boxed{0.531 \text{ m/s}}$$

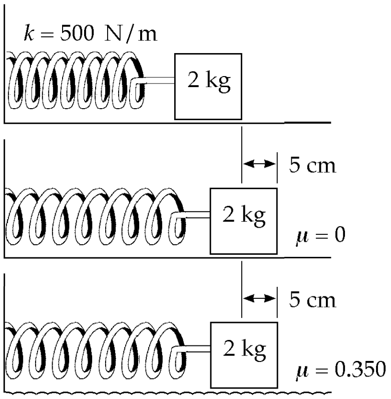

FIG. P7.32

**P7.33** (a)

$$W_g = mg\ell\cos(90.0° + \theta)$$

$$W_g = (10.0 \text{ kg})\left(9.80 \text{ m/s}^2\right)(5.00 \text{ m})\cos 110° = \boxed{-168 \text{ J}}$$

(b)

$$f_k = \mu_k n = \mu_k mg\cos\theta$$

$$\Delta E_{\text{int}} = \ell f_k = \ell\mu_k mg\cos\theta$$

$$\Delta E_{\text{int}} = (5.00 \text{ m})(0.400)(10.0)(9.80)\cos 20.0° = \boxed{184 \text{ J}}$$

(c)

$$W_F = F\ell = (100)(5.00) = \boxed{500 \text{ J}}$$

(d)

$$\Delta K = \sum W_{\text{other}} - \Delta E_{\text{int}} = W_F + W_g - \Delta E_{\text{int}} = \boxed{148 \text{ J}}$$

(e)

$$\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$v_f = \sqrt{\frac{2(\Delta K)}{m} + v_i^2} = \sqrt{\frac{2(148)}{10.0} + (1.50)^2} = \boxed{5.65 \text{ m/s}}$$

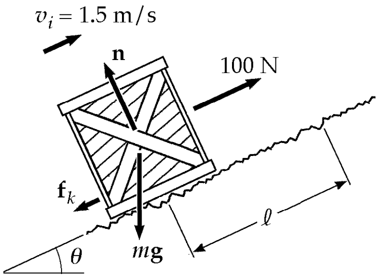

FIG. P7.33

**P7.34** $\sum F_y = ma_y$: $n + (70.0 \text{ N})\sin 20.0° - 147 \text{ N} = 0$

$n = 123$ N

$f_k = \mu_k n = 0.300(123 \text{ N}) = 36.9 \text{ N}$

(a)

$$W = F\Delta r\cos\theta = (70.0 \text{ N})(5.00 \text{ m})\cos 20.0° = \boxed{329 \text{ J}}$$

(b)

$$W = F\Delta r\cos\theta = (123 \text{ N})(5.00 \text{ m})\cos 90.0° = \boxed{0 \text{ J}}$$

(c)

$$W = F\Delta r\cos\theta = (147 \text{ N})(5.00 \text{ m})\cos 90.0° = \boxed{0}$$

(d)

$$\Delta E_{\text{int}} = F\Delta x = (36.9 \text{ N})(5.00 \text{ m}) = \boxed{185 \text{ J}}$$

(e)

$$\Delta K = K_f - K_i = \sum W - \Delta E_{\text{int}} = 329 \text{ J} - 185 \text{ J} = \boxed{+144 \text{ J}}$$

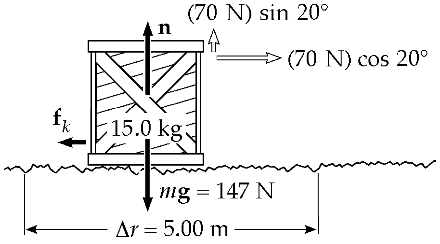

FIG. P7.34

**P7.35** $v_i = 2.00\ \text{m/s}$ $\qquad \mu_k = 0.100$

$$K_i - f_k\Delta x + W_{\text{other}} = K_f: \qquad \frac{1}{2}mv_i^2 - f_k\Delta x = 0$$

$$\frac{1}{2}mv_i^2 = \mu_k mg\Delta x \qquad \Delta x = \frac{v_i^2}{2\mu_k g} = \frac{(2.00\ \text{m/s})^2}{2(0.100)(9.80)} = \boxed{2.04\ \text{m}}$$

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## Section 7.8 **Power**

***P7.36** $$\mathcal{P}_{\text{av}} = \frac{W}{\Delta t} = \frac{K_f}{\Delta t} = \frac{mv^2}{2\Delta t} = \frac{0.875\ \text{kg}(0.620\ \text{m/s})^2}{2\left(21\times 10^{-3}\ \text{s}\right)} = \boxed{8.01\ \text{W}}$$

**P7.37** $$\text{Power} = \frac{W}{t} \qquad \mathcal{P} = \frac{mgh}{t} = \frac{(700\ \text{N})(10.0\ \text{m})}{8.00\ \text{s}} = \boxed{875\ \text{W}}$$

**P7.38** A 1 300-kg car speeds up from rest to 55.0 mi/h = 24.6 m/s in 15.0 s. The output work of the engine is equal to its final kinetic energy,

$$\frac{1}{2}(1\,300\ \text{kg})(24.6\ \text{m/s})^2 = 390\ \text{kJ}$$

with power $\mathcal{P} = \dfrac{390\,000\ \text{J}}{15.0\ \text{s}}\boxed{\sim 10^4\ \text{W}}$ around 30 horsepower.

**P7.39** (a) $\sum W = \Delta K$, but $\Delta K = 0$ because he moves at constant speed. The skier rises a vertical distance of $(60.0\ \text{m})\sin 30.0° = 30.0\ \text{m}$. Thus,

$$W_{\text{in}} = -W_g = (70.0\ \text{kg})\left(9.8\ \text{m/s}^2\right)(30.0\ \text{m}) = \boxed{2.06\times 10^4\ \text{J}} = \boxed{20.6\ \text{kJ}}.$$

(b) The time to travel 60.0 m at a constant speed of 2.00 m/s is 30.0 s. Thus,

$$\mathcal{P}_{\text{input}} = \frac{W}{\Delta t} = \frac{2.06\times 10^4\ \text{J}}{30.0\ \text{s}} = \boxed{686\ \text{W}} = 0.919\ \text{hp}.$$

**P7.40** (a) The distance moved upward in the first 3.00 s is

$$\Delta y = \bar{v}t = \left[\frac{0 + 1.75\ \text{m/s}}{2}\right](3.00\ \text{s}) = 2.63\ \text{m}.$$

The motor and the earth's gravity do work on the elevator car:

$$\frac{1}{2}mv_i^2 + W_{\text{motor}} + mg\Delta y\cos 180° = \frac{1}{2}mv_f^2$$

$$W_{\text{motor}} = \frac{1}{2}(650\ \text{kg})(1.75\ \text{m/s})^2 - 0 + (650\ \text{kg})g(2.63\ \text{m}) = 1.77\times 10^4\ \text{J}$$

Also, $W = \bar{\mathcal{P}}t$ so $\bar{\mathcal{P}} = \dfrac{W}{t} = \dfrac{1.77\times 10^4\ \text{J}}{3.00\ \text{s}} = \boxed{5.91\times 10^3\ \text{W}} = 7.92\ \text{hp}.$

(b) When moving upward at constant speed $(v = 1.75\ \text{m/s})$ the applied force equals the weight $= (650\ \text{kg})\left(9.80\ \text{m/s}^2\right) = 6.37\times 10^3\ \text{N}$. Therefore,

$$\mathcal{P} = Fv = \left(6.37\times 10^3\ \text{N}\right)(1.75\ \text{m/s}) = \boxed{1.11\times 10^4\ \text{W}} = 14.9\ \text{hp}.$$

**P7.41** *energy* = *power* × *time*

For the 28.0 W bulb:

$$\text{Energy used} = (28.0\text{ W})\left(1.00\times 10^{4}\text{ h}\right) = 280\text{ kilowatt}\cdot\text{hrs}$$
$$\text{total cost} = \$17.00 + (280\text{ kWh})(\$0.080/\text{kWh}) = \$39.40$$

For the 100 W bulb:

$$\text{Energy used} = (100\text{ W})\left(1.00\times 10^{4}\text{ h}\right) = 1.00\times 10^{3}\text{ kilowatt}\cdot\text{hrs}$$
$$\#\text{ bulb used} = \frac{1.00\times 10^{4}\text{ h}}{750\text{ h/bulb}} = 13.3$$
$$\text{total cost} = 13.3(\$0.420) + \left(1.00\times 10^{3}\text{ kWh}\right)(\$0.080/\text{kWh}) = \$85.60$$

Savings with energy-efficient bulb $= \$85.60 - \$39.40 = \boxed{\$46.20}$

***P7.42** (a) Burning 1 lb of fat releases energy $1\text{ lb}\left(\frac{454\text{ g}}{1\text{ lb}}\right)\left(\frac{9\text{ kcal}}{1\text{ g}}\right)\left(\frac{4\,186\text{ J}}{1\text{ kcal}}\right) = 1.71\times 10^{7}\text{ J}.$

The mechanical energy output is $\left(1.71\times 10^{7}\text{ J}\right)(0.20) = nF\Delta r\cos\theta.$

Then
$$3.42\times 10^{6}\text{ J} = nmg\Delta y\cos 0°$$
$$3.42\times 10^{6}\text{ J} = n(50\text{ kg})\left(9.8\text{ m/s}^{2}\right)(80\text{ steps})(0.150\text{ m})$$
$$3.42\times 10^{6}\text{ J} = n\left(5.88\times 10^{3}\text{ J}\right)$$

where the number of times she must climb the steps is $n = \frac{3.42\times 10^{6}\text{ J}}{5.88\times 10^{3}\text{ J}} = \boxed{582}.$

This method is impractical compared to limiting food intake.

(b) Her mechanical power output is

$$\mathcal{P} = \frac{W}{t} = \frac{5.88\times 10^{3}\text{ J}}{65\text{ s}} = \boxed{90.5\text{ W}} = 90.5\text{ W}\left(\frac{1\text{ hp}}{746\text{ W}}\right) = \boxed{0.121\text{ hp}}.$$

***P7.43** (a) The fuel economy for walking is $\frac{1\text{ h}}{220\text{ kcal}}\left(\frac{3\text{ mi}}{\text{h}}\right)\left(\frac{1\text{ kcal}}{4\,186\text{ J}}\right)\left(\frac{1.30\times 10^{8}\text{ J}}{1\text{ gal}}\right) = \boxed{423\text{ mi/gal}}.$

(b) For bicycling $\frac{1\text{ h}}{400\text{ kcal}}\left(\frac{10\text{ mi}}{\text{h}}\right)\left(\frac{1\text{ kcal}}{4\,186\text{ J}}\right)\left(\frac{1.30\times 10^{8}\text{ J}}{1\text{ gal}}\right) = \boxed{776\text{ mi/gal}}.$

## Section 7.9 **Energy and the Automobile**

**P7.44** At a speed of 26.8 m/s (60.0 mph), the car described in Table 7.2 delivers a power of $\mathscr{P}_1 = 18.3\ \text{kW}$ to the wheels. If an additional load of 350 kg is added to the car, a larger output power of

$$\mathscr{P}_2 = \mathscr{P}_1 + \text{(power input to move 350 kg at speed } v\text{)}$$

will be required. The additional power output needed to move 350 kg at speed $v$ is:

$$\Delta\mathscr{P}_{\text{out}} = (\Delta f)v = (\mu_r mg)v.$$

Assuming a coefficient of rolling friction of $\mu_r = 0.016\,0$, the power output now needed from the engine is

$$\mathscr{P}_2 = \mathscr{P}_1 + (0.016\,0)(350\ \text{kg})\left(9.80\ \text{m/s}^2\right)(26.8\ \text{m/s}) = 18.3\ \text{kW} + 1.47\ \text{kW}.$$

With the assumption of constant efficiency of the engine, the input power must increase by the same factor as the output power. Thus, the fuel economy must decrease by this factor:

$$(\text{fuel economy})_2 = \left(\frac{\mathscr{P}_1}{\mathscr{P}_2}\right)(\text{fuel economy})_1 = \left(\frac{18.3}{18.3+1.47}\right)(6.40\ \text{km/L})$$

or $(\text{fuel economy})_2 = \boxed{5.92\ \text{km/L}}$.

**P7.45** (a) 
$$\text{fuel needed} = \frac{\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2}{\text{useful energy per gallon}} = \frac{\frac{1}{2}mv_f^2 - 0}{\text{eff.}\times(\text{energy content of fuel})}$$
$$= \frac{\frac{1}{2}(900\ \text{kg})(24.6\ \text{m/s})^2}{(0.150)\left(1.34\times10^8\ \text{J/gal}\right)} = \boxed{1.35\times10^{-2}\ \text{gal}}$$

(b) $\boxed{73.8}$

(c) 
$$\text{power} = \left(\frac{1\ \text{gal}}{38.0\ \text{mi}}\right)\left(\frac{55.0\ \text{mi}}{1.00\ \text{h}}\right)\left(\frac{1.00\ \text{h}}{3\,600\ \text{s}}\right)\left(\frac{1.34\times10^8\ \text{J}}{1\ \text{gal}}\right)(0.150) = \boxed{8.08\ \text{kW}}$$

## Additional Problems

**P7.46** At start, $\mathbf{v} = (40.0\ \text{m/s})\cos 30.0°\,\hat{\mathbf{i}} + (40.0\ \text{m/s})\sin 30.0°\,\hat{\mathbf{j}}$

At apex, $\mathbf{v} = (40.0\ \text{m/s})\cos 30.0°\,\hat{\mathbf{i}} + 0\hat{\mathbf{j}} = (34.6\ \text{m/s})\hat{\mathbf{i}}$

And $K = \frac{1}{2}mv^2 = \frac{1}{2}(0.150\ \text{kg})(34.6\ \text{m/s})^2 = \boxed{90.0\ \text{J}}$

**P7.47** Concentration of Energy output $= (0.600\ \text{J/kg}\cdot\text{step})(60.0\ \text{kg})\left(\dfrac{1\ \text{step}}{1.50\ \text{m}}\right) = 24.0\ \text{J/m}$

$$F = (24.0\ \text{J/m})(1\ \text{N}\cdot\text{m/J}) = 24.0\ \text{N}$$
$$\mathscr{P} = Fv$$
$$70.0\ \text{W} = (24.0\ \text{N})v$$
$$v = \boxed{2.92\ \text{m/s}}$$

**P7.48** (a) $\mathbf{A}\cdot\hat{\mathbf{i}} = (A)(1)\cos\alpha$. But also, $\mathbf{A}\cdot\hat{\mathbf{i}} = A_x$.

Thus, $(A)(1)\cos\alpha = A_x$ or $\boxed{\cos\alpha = \dfrac{A_x}{A}}$.

Similarly, $\boxed{\cos\beta = \dfrac{A_y}{A}}$

and $\boxed{\cos\gamma = \dfrac{A_z}{A}}$

where $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$.

(b) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \left(\dfrac{A_x}{A}\right)^2 + \left(\dfrac{A_y}{A}\right)^2 + \left(\dfrac{A_z}{A}\right)^2 = \dfrac{A^2}{A^2} = 1$

**P7.49** (a) $x = t + 2.00t^3$

Therefore,

$$v = \frac{dx}{dt} = 1 + 6.00t^2$$
$$K = \frac{1}{2}mv^2 = \frac{1}{2}(4.00)\left(1 + 6.00t^2\right)^2 = \boxed{\left(2.00 + 24.0t^2 + 72.0t^4\right)\ \text{J}}$$

(b) $a = \dfrac{dv}{dt} = \boxed{(12.0t)\ \text{m/s}^2}$

$F = ma = 4.00(12.0t) = \boxed{(48.0t)\ \text{N}}$

(c) $\mathscr{P} = Fv = (48.0t)\left(1 + 6.00t^2\right) = \boxed{\left(48.0t + 288t^3\right)\ \text{W}}$

(d) $W = \displaystyle\int_0^{2.00} \mathscr{P}dt = \int_0^{2.00}\left(48.0t + 288t^3\right)dt = \boxed{1\,250\ \text{J}}$

***P7.50** (a) We write

$$F = ax^b$$
$$1\,000 \text{ N} = a(0.129 \text{ m})^b$$
$$5\,000 \text{ N} = a(0.315 \text{ m})^b$$
$$5 = \left(\frac{0.315}{0.129}\right)^b = 2.44^b$$
$$\ln 5 = b \ln 2.44$$
$$b = \frac{\ln 5}{\ln 2.44} = \boxed{1.80 = b}$$
$$a = \frac{1\,000 \text{ N}}{(0.129 \text{ m})^{1.80}} = \boxed{4.01 \times 10^4 \text{ N/m}^{1.8} = a}$$

(b)
$$W = \int_0^{0.25 \text{ m}} F dx = \int_0^{0.25 \text{ m}} 4.01 \times 10^4 \frac{\text{N}}{\text{m}^{1.8}} x^{1.8} dx$$
$$= 4.01 \times 10^4 \frac{\text{N}}{\text{m}^{1.8}} \frac{x^{2.8}}{2.8}\bigg|_0^{0.25 \text{ m}} = 4.01 \times 10^4 \frac{\text{N}}{\text{m}^{1.8}} \frac{(0.25 \text{ m})^{2.8}}{2.8}$$
$$= \boxed{294 \text{ J}}$$

***P7.51** The work done by the applied force is

$$W = \int_i^f F_{\text{applied}} dx = \int_0^{x_{\max}} -\left[-\left(k_1 x + k_2 x^2\right)\right] dx$$
$$= \int_0^{x_{\max}} k_1 x\, dx + \int_0^{x_{\max}} k_2 x^2 dx = k_1 \frac{x^2}{2}\bigg|_0^{x_{\max}} + k_2 \frac{x^3}{3}\bigg|_0^{x_{\max}}$$
$$= \boxed{k_1 \frac{x_{\max}^2}{2} + k_2 \frac{x_{\max}^3}{3}}$$

**P7.52** (a) The work done by the traveler is $mgh_sN$ where $N$ is the number of steps he climbs during the ride.

$$N = (\text{time on escalator})(n)$$

where
$$(\text{time on escalator}) = \frac{h}{\text{vertical velocity of person}}$$

and
$$\text{vertical velocity of person} = v + nh_s$$

Then,
$$N = \frac{nh}{v + nh_s}$$

and the work done by the person becomes $W_{\text{person}} = \boxed{\dfrac{mgnhh_s}{v + nh_s}}$

*continued on next page*

(b) The work done by the escalator is

$$W_e = (\text{power})(\text{time}) = \left[(\text{force exerted})(\text{speed})(\text{time})\right] = mgvt$$

where $t = \dfrac{h}{v + nh_s}$ as above.

Thus, $W_e = \boxed{\dfrac{mgvh}{v + nh_s}}$.

As a check, the total work done on the person's body must add up to *mgh*, the work an elevator would do in lifting him.

It does add up as follows: $\sum W = W_{\text{person}} + W_e = \dfrac{mgnhh_s}{v + nh_s} + \dfrac{mgvh}{v + nh_s} = \dfrac{mgh(nh_s + v)}{v + nh_s} = mgh$

**P7.53** (a) $\Delta K = \dfrac{1}{2}mv^2 - 0 = \sum W$, so

$v^2 = \dfrac{2W}{m}$ and $v = \boxed{\sqrt{\dfrac{2W}{m}}}$

(b) $W = \mathbf{F}\cdot\mathbf{d} = F_x d \Rightarrow F_x = \boxed{\dfrac{W}{d}}$

***P7.54** During its whole motion from $y = 10.0\text{ m}$ to $y = -3.20\text{ mm}$, the force of gravity and the force of the plate do work on the ball. It starts and ends at rest

$$\begin{aligned}
&K_i + \sum W = K_f \\
&0 + F_g \Delta y \cos 0° + F_p \Delta x \cos 180° = 0 \\
&mg(10.003\ 2\text{ m}) - F_p(0.003\ 20\text{ m}) = 0 \\
&F_p = \frac{5\text{ kg}(9.8\text{ m/s}^2)(10\text{ m})}{3.2 \times 10^{-3}\text{ m}} = \boxed{1.53 \times 10^5\text{ N upward}}
\end{aligned}$$

**P7.55** (a) $\mathcal{P} = Fv = F(v_i + at) = F\left(0 + \dfrac{F}{m}t\right) = \boxed{\left(\dfrac{F^2}{m}\right)t}$

(b) $\mathcal{P} = \left[\dfrac{(20.0\text{ N})^2}{5.00\text{ kg}}\right](3.00\text{ s}) = \boxed{240\text{ W}}$

***P7.56** (a) $$W_1=\int_i^f F_1 dx=\int_{x_{i1}}^{x_{i1}+x_a} k_1 x\,dx=\frac{1}{2}k_1\left[\left(x_{i1}+x_a\right)^2-x_{i1}^2\right]=\frac{1}{2}k_1\left(x_a^2+2x_a x_{i1}\right)$$

(b) $$W_2=\int_{-x_{i2}}^{-x_{i2}+x_a} k_2 x\,dx=\frac{1}{2}k_2\left[\left(-x_{i2}+x_a\right)^2-x_{i2}^2\right]=\frac{1}{2}k_2\left(x_a^2-2x_a x_{i2}\right)$$

(c) Before the horizontal force is applied, the springs exert equal forces: $k_1x_{i1}=k_2x_{i2}$

$$x_{i2}=\frac{k_1x_{i1}}{k_2}$$

(d) $$\begin{aligned}W_1+W_2&=\frac{1}{2}k_1x_a^2+k_1x_ax_{i1}+\frac{1}{2}k_2x_a^2-k_2x_ax_{i2}\\&=\frac{1}{2}k_1x_a^2+\frac{1}{2}k_2x_a^2+k_1x_ax_{i1}-k_2x_a\frac{k_1x_{i1}}{k_2}\\&=\frac{1}{2}(k_1+k_2)x_a^2\end{aligned}$$

***P7.57** (a) $$v=\int_0^t a\,dt=\int_0^t\left(1.16t-0.21t^2+0.24t^3\right)dt$$

$$=1.16\frac{t^2}{2}-0.21\frac{t^3}{3}+0.24\frac{t^4}{4}\bigg|_0^t=0.58t^2-0.07t^3+0.06t^4$$

At $t=0$, $v_i=0$. At $t=2.5\text{ s}$,

$$v_f=\left(0.58\text{ m/s}^3\right)(2.5\text{ s})^2-\left(0.07\text{ m/s}^4\right)(2.5\text{ s})^3+\left(0.06\text{ m/s}^5\right)(2.5\text{ s})^4=4.88\text{ m/s}$$

$$K_i+W=K_f$$

$$0+W=\frac{1}{2}mv_f^2=\frac{1}{2}1\,160\text{ kg}(4.88\text{ m/s})^2=\boxed{1.38\times10^4\text{ J}}$$

(b) At $t=2.5\text{ s}$,

$$a=\left(1.16\text{ m/s}^3\right)2.5\text{ s}-\left(0.210\text{ m/s}^4\right)(2.5\text{ s})^2+\left(0.240\text{ m/s}^5\right)(2.5\text{ s})^3=5.34\text{ m/s}^2.$$

Through the axles the wheels exert on the chassis force

$$\sum F=ma=1\,160\text{ kg }5.34\text{ m/s}^2=6.19\times10^3\text{ N}$$

and inject power

$$\mathcal{P}=Fv=6.19\times10^3\text{ N}(4.88\text{ m/s})=\boxed{3.02\times10^4\text{ W}}.$$

**P7.58** (a) The new length of each spring is $\sqrt{x^2+L^2}$, so its extension is $\sqrt{x^2+L^2}-L$ and the force it exerts is $k\left(\sqrt{x^2+L^2}-L\right)$ toward its fixed end. The $y$ components of the two spring forces add to zero. Their $x$ components add to

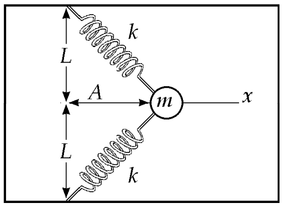

**FIG. P7.58**

$$\mathbf{F} = -2\hat{\mathbf{i}}k\left(\sqrt{x^2+L^2}-L\right)\frac{x}{\sqrt{x^2+L^2}} = \boxed{-2kx\hat{\mathbf{i}}\left(1-\frac{L}{\sqrt{x^2+L^2}}\right)}.$$

(b)

$$W = \int_i^f F_x dx \qquad W = \int_A^0 -2kx\left(1-\frac{L}{\sqrt{x^2+L^2}}\right)dx$$

$$W = -2k\int_A^0 x\,dx + kL\int_A^0 \left(x^2+L^2\right)^{-1/2} 2x\,dx \qquad W = -2k\frac{x^2}{2}\bigg|_A^0 + kL\frac{\left(x^2+L^2\right)^{1/2}}{(1/2)}\Bigg|_A^0$$

$$W = -0 + kA^2 + 2kL^2 - 2kL\sqrt{A^2+L^2} \qquad W = \boxed{2kL^2 + kA^2 - 2kL\sqrt{A^2+L^2}}$$

***P7.59** For the rocket falling at terminal speed we have

$$\sum F = ma$$
$$+R - Mg = 0$$
$$Mg = \frac{1}{2}D\rho A v_T^2$$

(a) For the rocket with engine exerting thrust $T$ and flying up at the same speed,

$$\sum F = ma$$
$$+T - Mg - R = 0$$
$$T = 2Mg$$

The engine power is $\mathcal{P} = Fv = Tv_T = \boxed{2Mgv_T}$.

(b) For the rocket with engine exerting thrust $T_b$ and flying down steadily at $3v_T$,

$R_b = \frac{1}{2}D\rho A(3v_T)^2 = 9Mg$

$$\sum F = ma$$
$$-T_b - Mg + 9Mg = 0$$
$$T_b = 8Mg$$

The engine power is $\mathcal{P} = Tv = 8Mg3v_T = \boxed{24Mgv_T}$.

**P7.60** (a)

$$\mathbf{F}_1 = (25.0\text{ N})\left(\cos 35.0°\,\hat{\mathbf{i}} + \sin 35.0°\,\hat{\mathbf{j}}\right) = \boxed{\left(20.5\hat{\mathbf{i}} + 14.3\hat{\mathbf{j}}\right)\text{ N}}$$

$$\mathbf{F}_2 = (42.0\text{ N})\left(\cos 150°\,\hat{\mathbf{i}} + \sin 150°\,\hat{\mathbf{j}}\right) = \boxed{\left(-36.4\hat{\mathbf{i}} + 21.0\hat{\mathbf{j}}\right)\text{ N}}$$

(b)

$$\sum \mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 = \boxed{\left(-15.9\hat{\mathbf{i}} + 35.3\hat{\mathbf{j}}\right)\text{ N}}$$

(c)

$$\mathbf{a} = \frac{\sum \mathbf{F}}{m} = \boxed{\left(-3.18\hat{\mathbf{i}} + 7.07\hat{\mathbf{j}}\right)\text{ m/s}^2}$$

(d)

$$\mathbf{v}_f = \mathbf{v}_i + \mathbf{a}t = \left(4.00\hat{\mathbf{i}} + 2.50\hat{\mathbf{j}}\right)\text{ m/s} + \left(-3.18\hat{\mathbf{i}} + 7.07\hat{\mathbf{j}}\right)\left(\text{m/s}^2\right)(3.00\text{ s})$$

$$\mathbf{v}_f = \boxed{\left(-5.54\hat{\mathbf{i}} + 23.7\hat{\mathbf{j}}\right)\text{ m/s}}$$

(e)

$$\mathbf{r}_f = \mathbf{r}_i + \mathbf{v}_i t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{r}_f = 0 + \left(4.00\hat{\mathbf{i}} + 2.50\hat{\mathbf{j}}\right)(\text{m/s})(3.00\text{ s}) + \frac{1}{2}\left(-3.18\hat{\mathbf{i}} + 7.07\hat{\mathbf{j}}\right)\left(\text{m/s}^2\right)(3.00\text{ s})^2$$

$$\Delta\mathbf{r} = \mathbf{r}_f = \boxed{\left(-2.30\hat{\mathbf{i}} + 39.3\hat{\mathbf{j}}\right)\text{ m}}$$

(f)

$$K_f = \frac{1}{2}mv_f^2 = \frac{1}{2}(5.00\text{ kg})\left[(5.54)^2 + (23.7)^2\right]\left(\text{m/s}^2\right) = \boxed{1.48\text{ kJ}}$$

(g)

$$K_f = \frac{1}{2}mv_i^2 + \sum \mathbf{F}\cdot\Delta\mathbf{r}$$

$$K_f = \frac{1}{2}(5.00\text{ kg})\left[(4.00)^2 + (2.50)^2\right](\text{m/s})^2 + \left[(-15.9\text{ N})(-2.30\text{ m}) + (35.3\text{ N})(39.3\text{ m})\right]$$

$$K_f = 55.6\text{ J} + 1\,426\text{ J} = \boxed{1.48\text{ kJ}}$$

**P7.61** (a)

$$\sum W = \Delta K: \qquad W_s + W_g = 0$$

$$\frac{1}{2}kx_i^2 - 0 + mg\Delta x\cos(90°+60°) = 0$$

$$\frac{1}{2}\left(1.40\times 10^3\text{ N/m}\right)\times(0.100)^2 - (0.200)(9.80)(\sin 60.0°)\Delta x = 0$$

$$\Delta x = \boxed{4.12\text{ m}}$$

(b)

$$\sum W = \Delta K + \Delta E_{\text{int}}: \qquad W_s + W_g - \Delta E_{\text{int}} = 0$$

$$\frac{1}{2}kx_i^2 + mg\Delta x\cos 150° - \mu_k mg\cos 60°\,\Delta x = 0$$

$$\frac{1}{2}\left(1.40\times 10^3\text{ N/m}\right)\times(0.100)^2 - (0.200)(9.80)(\sin 60.0°)\Delta x - (0.200)(9.80)(0.400)(\cos 60.0°)\Delta x = 0$$

$$\Delta x = \boxed{3.35\text{ m}}$$

**P7.62** (a)

| $F$(N) | $L$(mm) | $F$(N) | $L$(mm) |
|---|---|---|---|
| 2.00 | 15.0 | 14.0 | 112 |
| 4.00 | 32.0 | 16.0 | 126 |
| 6.00 | 49.0 | 18.0 | 149 |
| 8.00 | 64.0 | 20.0 | 175 |
| 10.0 | 79.0 | 22.0 | 190 |
| 12.0 | 98.0 | | |

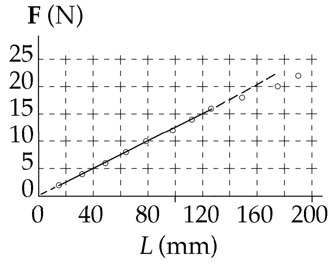

**FIG. P7.62**

(b) A straight line fits the first eight points, together with the origin. By least-square fitting, its slope is

$$0.125\ \text{N/mm} \pm 2\% = \boxed{125\ \text{N/m}} \pm 2\%$$

In $F = kx$, the spring constant is $k = \frac{F}{x}$, the same as the slope of the $F$-versus-$x$ graph.

(c) $F = kx = (125\ \text{N/m})(0.105\ \text{m}) = \boxed{13.1\ \text{N}}$

**P7.63**

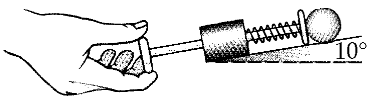

**FIG. P7.63**

$$K_i + W_s + W_g = K_f$$

$$\frac{1}{2}mv_i^2 + \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 + mg\Delta x\cos\theta = \frac{1}{2}mv_f^2$$

$$0 + \frac{1}{2}kx_i^2 - 0 + mgx_i\cos 100° = \frac{1}{2}mv_f^2$$

$$\frac{1}{2}(1.20\ \text{N/cm})(5.00\ \text{cm})(0.050\,0\ \text{m}) - (0.100\ \text{kg})(9.80\ \text{m/s}^2)(0.050\,0\ \text{m})\sin 10.0° = \frac{1}{2}(0.100\ \text{kg})v^2$$

$$0.150\ \text{J} - 8.51\times 10^{-3}\ \text{J} = (0.050\,0\ \text{kg})v^2$$

$$v = \sqrt{\frac{0.141}{0.050\,0}} = \boxed{1.68\ \text{m/s}}$$

**P7.64** (a) $\Delta E_{\text{int}} = -\Delta K = -\frac{1}{2}m\left(v_f^2 - v_i^2\right)$: $\Delta E_{\text{int}} = -\frac{1}{2}(0.400\ \text{kg})\left((6.00)^2 - (8.00)^2\right)(\text{m/s})^2 = \boxed{5.60\ \text{J}}$

(b) $\Delta E_{\text{int}} = f\Delta r = \mu_k mg(2\pi r)$: $5.60\ \text{J} = \mu_k(0.400\ \text{kg})(9.80\ \text{m/s}^2)2\pi(1.50\ \text{m})$

Thus, $\mu_k = \boxed{0.152}$.

(c) After $N$ revolutions, the object comes to rest and $K_f = 0$.

Thus, $\Delta E_{\text{int}} = -\Delta K = -0 + K_i = \frac{1}{2}mv_i^2$

or $\mu_k mg[N(2\pi r)] = \frac{1}{2}mv_i^2$.

This gives $N = \dfrac{\frac{1}{2}mv_i^2}{\mu_k mg(2\pi r)} = \dfrac{\frac{1}{2}(8.00\ \text{m/s})^2}{(0.152)(9.80\ \text{m/s}^2)2\pi(1.50\ \text{m})} = \boxed{2.28\ \text{rev}}$.

**P7.65** If positive $F$ represents an outward force, (same as direction as $r$), then

$$W=\int_i^f \mathbf{F}\cdot d\mathbf{r}=\int_{r_i}^{r_f}\left(2F_0\sigma^{13}r^{-13}-F_0\sigma^7r^{-7}\right)dr$$

$$W=\left.\frac{2F_0\sigma^{13}r^{-12}}{-12}-\frac{F_0\sigma^7r^{-6}}{-6}\right|_{r_i}^{r_f}$$

$$W=\frac{-F_0\sigma^{13}\left(r_f^{-12}-r_i^{-12}\right)}{6}+\frac{F_0\sigma^7\left(r_f^{-6}-r_i^{-6}\right)}{6}=\frac{F_0\sigma^7}{6}\left[r_f^{-6}-r_i^{-6}\right]-\frac{F_0\sigma^{13}}{6}\left[r_f^{-12}-r_i^{-12}\right]$$

$$W=1.03\times10^{-77}\left[r_f^{-6}-r_i^{-6}\right]-1.89\times10^{-134}\left[r_f^{-12}-r_i^{-12}\right]$$

$$W=1.03\times10^{-77}\left[1.88\times10^{-6}-2.44\times10^{-6}\right]10^{60}-1.89\times10^{-134}\left[3.54\times10^{-12}-5.96\times10^{-8}\right]10^{120}$$

$$W=-2.49\times10^{-21}\text{ J}+1.12\times10^{-21}\text{ J}=\boxed{-1.37\times10^{-21}\text{ J}}$$

**P7.66** $$\mathcal{P}\Delta t=W=\Delta K=\frac{(\Delta m)v^2}{2}$$

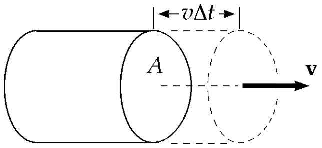

**FIG. P7.66**

The density is $$\rho=\frac{\Delta m}{\text{vol}}=\frac{\Delta m}{A\Delta x}.$$

Substituting this into the first equation and solving for $\mathcal{P}$, since $\dfrac{\Delta x}{\Delta t}=v$,

for a constant speed, we get $$\boxed{\mathcal{P}=\frac{\rho Av^3}{2}}.$$

Also, since $\mathcal{P}=Fv$, $$\boxed{F=\frac{\rho Av^2}{2}}.$$

Our model predicts the same proportionalities as the empirical equation, and gives $D=1$ for the drag coefficient. Air actually slips around the moving object, instead of accumulating in front of it. For this reason, the drag coefficient is not necessarily unity. It is typically less than one for a streamlined object and can be greater than one if the airflow around the object is complicated.

**P7.67** We evaluate $\displaystyle\int_{12.8}^{23.7}\frac{375dx}{x^3+3.75x}$ by calculating

$$\frac{375(0.100)}{(12.8)^3+3.75(12.8)}+\frac{375(0.100)}{(12.9)^3+3.75(12.9)}+\ldots\frac{375(0.100)}{(23.6)^3+3.75(23.6)}=0.806$$

and

$$\frac{375(0.100)}{(12.9)^3+3.75(12.9)}+\frac{375(0.100)}{(13.0)^3+3.75(13.0)}+\ldots\frac{375(0.100)}{(23.7)^3+3.75(23.7)}=0.791.$$

The answer must be between these two values. We may find it more precisely by using a value for $\Delta x$ smaller than 0.100. Thus, we find the integral to be $\boxed{0.799\text{ N}\cdot\text{m}}$.

***P7.68** $\mathscr{P} = \frac{1}{2}D\rho\pi r^2 v^3$

(a) $\mathscr{P}_a = \frac{1}{2}1\left(1.20\ \text{kg/m}^3\right)\pi(1.5\ \text{m})^2(8\ \text{m/s})^3 = \boxed{2.17\times10^3\ \text{W}}$

(b) $\frac{\mathscr{P}_b}{\mathscr{P}_a} = \frac{v_b^3}{v_a^3} = \left(\frac{24\ \text{m/s}}{8\ \text{m/s}}\right)^3 = 3^3 = 27$

$\mathscr{P}_b = 27\left(2.17\times10^3\ \text{W}\right) = \boxed{5.86\times10^4\ \text{W}}$

**P7.69** (a) The suggested equation $\mathscr{P}\Delta t = bwd$ implies all of the following cases:

(1) $\mathscr{P}\Delta t = b\left(\frac{w}{2}\right)(2d)$ (2) $\mathscr{P}\left(\frac{\Delta t}{2}\right) = b\left(\frac{w}{2}\right)d$

(3) $\mathscr{P}\left(\frac{\Delta t}{2}\right) = bw\left(\frac{d}{2}\right)$ and (4) $\left(\frac{\mathscr{P}}{2}\right)\Delta t = b\left(\frac{w}{2}\right)d$

These are all of the proportionalities Aristotle lists.

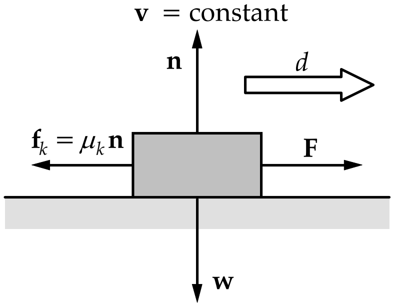

**FIG. P7.69**

(b) For one example, consider a horizontal force $F$ pushing an object of weight $w$ at constant velocity across a horizontal floor with which the object has coefficient of friction $\mu_k$.

$\sum\mathbf{F} = m\mathbf{a}$ implies that:

$$+n - w = 0 \text{ and } F - \mu_k n = 0$$

so that $F = \mu_k w$

As the object moves a distance $d$, the agent exerting the force does work

$$W = Fd\cos\theta = Fd\cos0° = \mu_k wd \text{ and puts out power } \mathscr{P} = \frac{W}{\Delta t}$$

This yields the equation $\mathscr{P}\Delta t = \mu_k wd$ which represents Aristotle's theory with $b = \mu_k$.

Our theory is more general than Aristotle's. Ours can also describe accelerated motion.

***P7.70** (a) So long as the spring force is greater than the friction force, the block will be gaining speed. The block slows down when the friction force becomes the greater. It has maximum speed when $kx_a - f_k = ma = 0$.

$\left(1.0\times10^3\ \text{N/m}\right)x_a - 4.0\ \text{N} = 0$ $\boxed{x = -4.0\times10^{-3}\ \text{m}}$

(b) By the same logic,

$\left(1.0\times10^3\ \text{N/m}\right)x_b - 10.0\ \text{N} = 0$ $\boxed{x = -1.0\times10^{-2}\ \text{m}}$

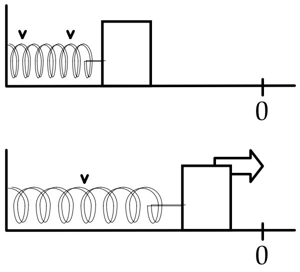

**FIG. P7.70**

## ANSWERS TO EVEN PROBLEMS

| | |
|---|---|
| **P7.2** | $1.59 \times 10^3$ J |
| **P7.4** | (a) $3.28 \times 10^{-2}$ J; (b) $-3.28 \times 10^{-2}$ J |
| **P7.6** | see the solution |
| **P7.8** | 5.33 W |
| **P7.10** | 16.0 |
| **P7.12** | (a) see the solution; (b) −12.0 J |
| **P7.14** | 50.0 J |
| **P7.16** | (a) 575 N/m; (b) 46.0 J |
| **P7.18** | (a) 9.00 kJ; (b) 11.7 kJ, larger by 29.6% |
| **P7.20** | (a) see the solution; (b) $mgR$ |
| **P7.22** | (a) $\frac{mg}{k_1} + \frac{mg}{k_2}$; (b) $\left(\frac{1}{k_1} + \frac{1}{k_2}\right)^{-1}$ |
| **P7.24** | (a) 1.20 J; (b) 5.00 m/s; (c) 6.30 J |
| **P7.26** | (a) 60.0 J; (b) 60.0 J |
| **P7.28** | (a) 1.94 m/s; (b) 3.35 m/s; (c) 3.87 m/s |
| **P7.30** | (a) $3.78 \times 10^{-16}$ J; (b) $1.35 \times 10^{-14}$ N;<br>(c) $1.48 \times 10^{+16}$ m/s$^2$; (d) 1.94 ns |
| **P7.32** | (a) 0.791 m/s; (b) 0.531 m/s |
| **P7.34** | (a) 329 J; (b) 0; (c) 0; (d) 185 J; (e) 144 J |
| **P7.36** | 8.01 W |
| **P7.38** | $\sim 10^4$ W |
| **P7.40** | (a) 5.91 kW; (b) 11.1 kW |
| **P7.42** | No. (a) 582; (b) 90.5 W = 0.121 hp |
| **P7.44** | 5.92 km/L |
| **P7.46** | 90.0 J |
| **P7.48** | (a) $\cos\alpha = \frac{A_x}{A}$; $\cos\beta = \frac{A_y}{A}$; $\cos\gamma = \frac{A_z}{A}$;<br>(b) see the solution |
| **P7.50** | (a) $a = \frac{40.1\text{ kN}}{m^{1.8}}$; $b = 1.80$; (b) 294 J |
| **P7.52** | (a) $\frac{mgnhh_s}{v + nh_s}$; (b) $\frac{mgvh}{v + nh_s}$ |
| **P7.54** | $1.53 \times 10^5$ N upward |
| **P7.56** | see the solution |
| **P7.58** | (a) see the solution;<br>(b) $2kL^2 + kA^2 - 2kL\sqrt{A^2 + L^2}$ |
| **P7.60** | (a) $\mathbf{F}_1 = \left(20.5\hat{\mathbf{i}} + 14.3\hat{\mathbf{j}}\right)$ N;<br>$\mathbf{F}_2 = \left(-36.4\hat{\mathbf{i}} + 21.0\hat{\mathbf{j}}\right)$ N;<br>(b) $\left(-15.9\hat{\mathbf{i}} + 35.3\hat{\mathbf{j}}\right)$ N;<br>(c) $\left(-3.18\hat{\mathbf{i}} + 7.07\hat{\mathbf{j}}\right)$ m/s$^2$;<br>(d) $\left(-5.54\hat{\mathbf{i}} + 23.7\hat{\mathbf{j}}\right)$ m/s;<br>(e) $\left(-2.30\hat{\mathbf{i}} + 39.3\hat{\mathbf{j}}\right)$ m; (f) 1.48 kJ; (g) 1.48 kJ |
| **P7.62** | (a) see the solution; (b) 125 N/m ± 2%;<br>(c) 13.1 N |
| **P7.64** | (a) 5.60 J; (b) 0.152; (c) 2.28 rev |
| **P7.66** | see the solution |
| **P7.68** | (a) 2.17 kW; (b) 58.6 kW |
| **P7.70** | (a) $x = -4.0$ mm; (b) −1.0 cm |

# 8

# Potential Energy

CHAPTER OUTLINE

8.1 Potential Energy of a System
8.2 The Isolated System—Conservation of Mechanical Energy
8.3 Conservative and Nonconservative Forces
8.4 Changes in Mechanical Energy for Nonconservative Forces
8.5 Relationship Between Conservative Forces and Potential Energy
8.6 Energy Diagrams and the Equilibrium of a System

## ANSWERS TO QUESTIONS

**Q8.1** The final speed of the children will not depend on the slide length or the presence of bumps if there is no friction. If there is friction, a longer slide will result in a lower final speed. Bumps will have the same effect as they effectively lengthen the distance over which friction can do work, to decrease the total mechanical energy of the children.

**Q8.2** Total energy is the sum of kinetic and potential energies. Potential energy can be negative, so the sum of kinetic plus potential can also be negative.

**Q8.3** Both agree on the *change* in potential energy, and the kinetic energy. They may disagree on the value of gravitational potential energy, depending on their choice of a zero point.

**Q8.4** (a) $mgh$ is provided by the muscles.

(b) No further energy is supplied to the object-Earth system, but some chemical energy must be supplied to the muscles as they keep the weight aloft.

(c) The object loses energy $mgh$, giving it back to the muscles, where most of it becomes internal energy.

**Q8.5** Lift a book from a low shelf to place it on a high shelf. The net change in its kinetic energy is zero, but the book-Earth system increases in gravitational potential energy. Stretch a rubber band to encompass the ends of a ruler. It increases in elastic energy. Rub your hands together or let a pearl drift down at constant speed in a bottle of shampoo. Each system (two hands; pearl and shampoo) increases in internal energy.

**Q8.6** Three potential energy terms will appear in the expression of total mechanical energy, one for each conservative force. If you write an equation with initial energy on one side and final energy on the other, the equation contains six potential-energy terms.

**Q8.7** (a) It does if it makes the object's speed change, but not if it only makes the direction of the velocity change.

(b) Yes, according to Newton's second law.

**Q8.8** The original kinetic energy of the skidding can be degraded into kinetic energy of random molecular motion in the tires and the road: it is internal energy. If the brakes are used properly, the same energy appears as internal energy in the brake shoes and drums.

**Q8.9** All the energy is supplied by foodstuffs that gained their energy from the sun.

**Q8.10** Elastic potential energy of plates under stress plus gravitational energy is released when the plates "slip". It is carried away by mechanical waves.

**Q8.11** The total energy of the ball-Earth system is conserved. Since the system initially has gravitational energy *mgh* and no kinetic energy, the ball will again have zero kinetic energy when it returns to its original position. Air resistance will cause the ball to come back to a point slightly below its initial position. On the other hand, if anyone gives a forward push to the ball anywhere along its path, the demonstrator will have to duck.

**Q8.12** Using switchbacks requires no less work, as it does not change the *change* in potential energy from top to bottom. It does, however, require less force (of static friction on the rolling drive wheels of a car) to propel the car up the gentler slope. Less power is required if the work can be done over a longer period of time.

**Q8.13** There is no work done since there is no change in kinetic energy. In this case, air resistance must be negligible since the acceleration is zero.

**Q8.14** There is no violation. Choose the book as the system. You did work and the earth did work on the book. The average force you exerted just counterbalanced the weight of the book. The total work on the book is zero, and is equal to its overall change in kinetic energy.

**Q8.15** Kinetic energy is greatest at the starting point. Gravitational energy is a maximum at the top of the flight of the ball.

**Q8.16** Gravitational energy is proportional to mass, so it doubles.

**Q8.17** In stirring cake batter and in weightlifting, your body returns to the same conformation after each stroke. During each stroke chemical energy is irreversibly converted into output work (and internal energy). This observation proves that muscular forces are nonconservative.

**Q8.18** Let the gravitational energy be zero at the lowest point in the motion. If you start the vibration by pushing down on the block (2), its kinetic energy becomes extra elastic potential energy in the spring ($U_s$). After the block starts moving up at its lower turning point (3), this energy becomes both kinetic energy ($K$) and gravitational potential energy ($U_g$), and then just gravitational energy when the block is at its greatest height (1). The energy then turns back into kinetic and elastic potential energy, and the cycle repeats.

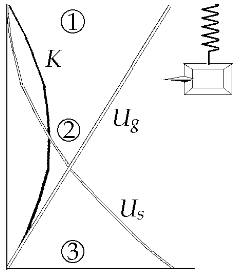

**FIG. Q8.18**

**Q8.19** (a) Kinetic energy of the running athlete is transformed into elastic potential energy of the bent pole. This potential energy is transformed to a combination of kinetic energy and gravitational potential energy of the athlete and pole as the athlete approaches the bar. The energy is then all gravitational potential of the pole and the athlete as the athlete hopefully clears the bar. This potential energy then turns to kinetic energy as the athlete and pole fall to the ground. It immediately becomes internal energy as their macroscopic motion stops.

(b) Rotational kinetic energy of the athlete and shot is transformed into translational kinetic energy of the shot. As the shot goes through its trajectory as a projectile, the kinetic energy turns to a mix of kinetic and gravitational potential. The energy becomes internal energy as the shot comes to rest.

(c) Kinetic energy of the running athlete is transformed to a mix of kinetic and gravitational potential as the athlete becomes projectile going over a bar. This energy turns back into kinetic as the athlete falls down, and becomes internal energy as he stops on the ground.

The ultimate source of energy for all of these sports is the sun. See question 9.

**Q8.20** Chemical energy in the fuel turns into internal energy as the fuel burns. Most of this leaves the car by heat through the walls of the engine and by matter transfer in the exhaust gases. Some leaves the system of fuel by work done to push down the piston. Of this work, a little results in internal energy in the bearings and gears, but most becomes work done on the air to push it aside. The work on the air immediately turns into internal energy in the air. If you use the windshield wipers, you take energy from the crankshaft and turn it into extra internal energy in the glass and wiper blades and wiper-motor coils. If you turn on the air conditioner, your end effect is to put extra energy out into the surroundings. You must apply the brakes at the end of your trip. As soon as the sound of the engine has died away, all you have to show for it is thermal pollution.

**Q8.21** A graph of potential energy versus position is a straight horizontal line for a particle in neutral equilibrium. The graph represents a constant function.

**Q8.22** The ball is in neutral equilibrium.

**Q8.23** The ball is in stable equilibrium when it is directly below the pivot point. The ball is in unstable equilibrium when it is vertically above the pivot.

## SOLUTIONS TO PROBLEMS

### Section 8.1 Potential Energy of a System

**P8.1** (a) With our choice for the zero level for potential energy when the car is at point B,

$$\boxed{U_B = 0}.$$

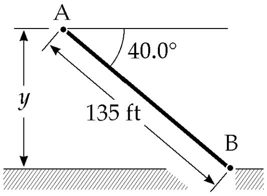

**FIG. P8.1**

When the car is at point A, the potential energy of the car-Earth system is given by

$$U_A = mgy$$

where $y$ is the vertical height above zero level. With $135 \text{ ft} = 41.1 \text{ m}$, this height is found as:

$$y = (41.1 \text{ m})\sin 40.0° = 26.4 \text{ m}.$$

Thus,

$$U_A = (1\,000 \text{ kg})\left(9.80 \text{ m/s}^2\right)(26.4 \text{ m}) = \boxed{2.59 \times 10^5 \text{ J}}.$$

The change in potential energy as the car moves from A to B is

$$U_B - U_A = 0 - 2.59 \times 10^5 \text{ J} = \boxed{-2.59 \times 10^5 \text{ J}}.$$

(b) With our choice of the zero level when the car is at point A, we have $\boxed{U_A = 0}$. The potential energy when the car is at point B is given by $U_B = mgy$ where $y$ is the vertical distance of point B below point A. In part (a), we found the magnitude of this distance to be 26.5 m. Because this distance is now below the zero reference level, it is a negative number.

Thus,

$$U_B = (1\,000 \text{ kg})\left(9.80 \text{ m/s}^2\right)(-26.5 \text{ m}) = \boxed{-2.59 \times 10^5 \text{ J}}.$$

The change in potential energy when the car moves from A to B is

$$U_B - U_A = -2.59 \times 10^5 \text{ J} - 0 = \boxed{-2.59 \times 10^5 \text{ J}}.$$

**P8.2** (a) We take the zero configuration of system potential energy with the child at the lowest point of the arc. When the string is held horizontal initially, the initial position is 2.00 m above the zero level. Thus,

$$U_g = mgy = (400\text{ N})(2.00\text{ m}) = \boxed{800\text{ J}}.$$

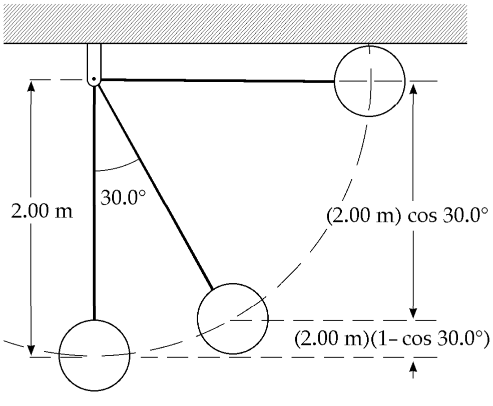

**FIG. P8.2**

(b) From the sketch, we see that at an angle of 30.0° the child is at a vertical height of $(2.00\text{ m})(1-\cos 30.0°)$ above the lowest point of the arc. Thus,

$$U_g = mgy = (400\text{ N})(2.00\text{ m})(1-\cos 30.0°) = \boxed{107\text{ J}}.$$

(c) The zero level has been selected at the lowest point of the arc. Therefore, $\boxed{U_g = 0}$ at this location.

***P8.3** The volume flow rate is the volume of water going over the falls each second:

$$3\text{ m}(0.5\text{ m})(1.2\text{ m/s}) = 1.8\text{ m}^3/\text{s}$$

The mass flow rate is $\dfrac{m}{t} = \rho\dfrac{V}{t} = \left(1\,000\text{ kg/m}^3\right)\left(1.8\text{ m}^3/\text{s}\right) = 1\,800\text{ kg/s}$

If the stream has uniform width and depth, the speed of the water below the falls is the same as the speed above the falls. Then no kinetic energy, but only gravitational energy is available for conversion into internal and electric energy.

The input power is $\mathcal{P}_{\text{in}} = \dfrac{\text{energy}}{t} = \dfrac{mgy}{t} = \dfrac{m}{t}gy = (1\,800\text{ kg/s})\left(9.8\text{ m/s}^2\right)(5\text{ m}) = 8.82\times10^4\text{ J/s}$

The output power is $\mathcal{P}_{\text{useful}} = (\text{efficiency})\mathcal{P}_{\text{in}} = 0.25\left(8.82\times10^4\text{ W}\right) = \boxed{2.20\times10^4\text{ W}}$

The efficiency of electric generation at Hoover Dam is about 85%, with a head of water (vertical drop) of 174 m. Intensive research is underway to improve the efficiency of low head generators.

## Section 8.2 The Isolated System—Conservation of Mechanical Energy

***P8.4** (a) One child in one jump converts chemical energy into mechanical energy in the amount that her body has as gravitational energy at the top of her jump: $mgy = 36\text{ kg}\left(9.81\text{ m/s}^2\right)(0.25\text{ m}) = 88.3\text{ J}$. For all of the jumps of the children the energy is $12\left(1.05\times10^6\right)88.3\text{ J} = \boxed{1.11\times10^9\text{ J}}$.

(b) The seismic energy is modeled as $E = \dfrac{0.01}{100}1.11\times10^9\text{ J} = 1.11\times10^5\text{ J}$, making the Richter magnitude $\dfrac{\log E - 4.8}{1.5} = \dfrac{\log 1.11\times10^5 - 4.8}{1.5} = \dfrac{5.05-4.8}{1.5} = \boxed{0.2}$.

**P8.5** $U_i + K_i = U_f + K_f$: $mgh + 0 = mg(2R) + \frac{1}{2}mv^2$

$$g(3.50R) = 2g(R) + \frac{1}{2}v^2$$

$$\boxed{v = \sqrt{3.00gR}}$$

$\sum F = m\frac{v^2}{R}$: $n + mg = m\frac{v^2}{R}$

$$n = m\left[\frac{v^2}{R} - g\right] = m\left[\frac{3.00gR}{R} - g\right] = 2.00mg$$

$$n = 2.00\left(5.00\times10^{-3}\text{ kg}\right)\left(9.80\text{ m/s}^2\right)$$

$$= \boxed{0.098\,0\text{ N downward}}$$

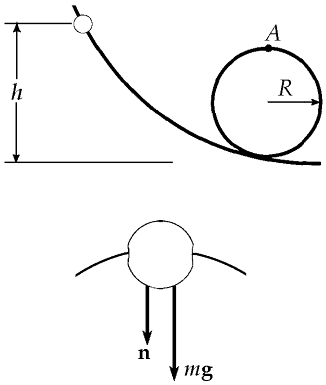

FIG. P8.5

**P8.6** From leaving ground to the highest point, $K_i + U_i = K_f + U_f$

$$\frac{1}{2}m(6.00\text{ m/s})^2 + 0 = 0 + m\left(9.80\text{ m/s}^2\right)y$$

The mass makes no difference: $\therefore y = \frac{(6.00\text{ m/s})^2}{(2)\left(9.80\text{ m/s}^2\right)} = \boxed{1.84\text{ m}}$

***P8.7** (a) $\frac{1}{2}mv_i^2 + \frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}kx_f^2$

$$0 + \frac{1}{2}(10\text{ N/m})(-0.18\text{ m})^2 = \frac{1}{2}(0.15\text{ kg})v_f^2 + 0$$

$$v_f = (0.18\text{ m})\sqrt{\left(\frac{10\text{ N}}{0.15\text{ kg}\cdot\text{m}}\right)\left(\frac{1\text{ kg}\cdot\text{m}}{1\text{ N}\cdot\text{s}^2}\right)} = \boxed{1.47\text{ m/s}}$$

(b) $K_i + U_{si} = K_f + U_{sf}$

$$0 + \frac{1}{2}(10\text{ N/m})(-0.18\text{ m})^2 = \frac{1}{2}(0.15\text{ kg})v_f^2 + \frac{1}{2}(10\text{ N/m})(0.25\text{ m} - 0.18\text{ m})^2$$

$$0.162\text{ J} = \frac{1}{2}(0.15\text{ kg})v_f^2 + 0.024\,5\text{ J}$$

$$v_f = \sqrt{\frac{2(0.138\text{ J})}{0.15\text{ kg}}} = \boxed{1.35\text{ m/s}}$$

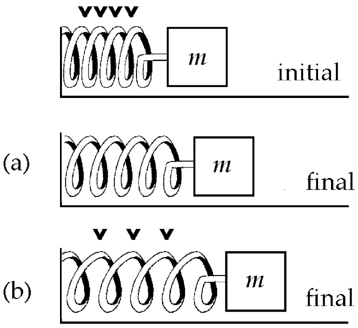

FIG. P8.7

***P8.8** The energy of the car is $E=\frac{1}{2}mv^2+mgy$

$E=\frac{1}{2}mv^2+mgd\sin\theta$ where $d$ is the distance it has moved along the track.

$$\mathcal{P}=\frac{dE}{dt}=mv\frac{dv}{dt}+mgv\sin\theta$$

(a) When speed is constant,

$$\mathcal{P}=mgv\sin\theta=950\text{ kg}\left(9.80\text{ m/s}^2\right)(2.20\text{ m/s})\sin 30°=\boxed{1.02\times10^4\text{ W}}$$

(b) $\frac{dv}{dt}=a=\frac{2.2\text{ m/s}-0}{12\text{ s}}=0.183\text{ m/s}^2$

Maximum power is injected just before maximum speed is attained:

$$\mathcal{P}=mva+mgv\sin\theta=950\text{ kg}(2.2\text{ m/s})\left(0.183\text{ m/s}^2\right)+1.02\times10^4\text{ W}=\boxed{1.06\times10^4\text{ W}}$$

(c) At the top end,

$$\frac{1}{2}mv^2+mgd\sin\theta=950\text{ kg}\left(\frac{1}{2}(2.20\text{ m/s})^2+\left(9.80\text{ m/s}^2\right)1\,250\text{ m}\sin 30°\right)=\boxed{5.82\times10^6\text{ J}}$$

***P8.9** (a) Energy of the object-Earth system is conserved as the object moves between the release point and the lowest point. We choose to measure heights from $y=0$ at the top end of the string.

$$\left(K+U_g\right)_i=\left(K+U_g\right)_f:\qquad 0+mgy_i=\frac{1}{2}mv_f^2+mgy_f$$

$$\left(9.8\text{ m/s}^2\right)(-2\text{ m}\cos 30°)=\frac{1}{2}v_f^2+\left(9.8\text{ m/s}^2\right)(-2\text{ m})$$

$$v_f=\sqrt{2\left(9.8\text{ m/s}^2\right)(2\text{ m})(1-\cos 30°)}=\boxed{2.29\text{ m/s}}$$

(b) Choose the initial point at $\theta=30°$ and the final point at $\theta=15°$:

$$0+mg(-L\cos 30°)=\frac{1}{2}mv_f^2+mg(-L\cos 15°)$$

$$v_f=\sqrt{2gL(\cos 15°-\cos 30°)}=\sqrt{2\left(9.8\text{ m/s}^2\right)(2\text{ m})(\cos 15°-\cos 30°)}=\boxed{1.98\text{ m/s}}$$

**P8.10** Choose the zero point of gravitational potential energy of the object-spring-Earth system as the configuration in which the object comes to rest. Then because the incline is frictionless, we have $E_B=E_A$:

$$K_B+U_{gB}+U_{sB}=K_A+U_{gA}+U_{sA}$$

or

$$0+mg(d+x)\sin\theta+0=0+0+\frac{1}{2}kx^2.$$

Solving for $d$ gives

$$d=\boxed{\frac{kx^2}{2mg\sin\theta}-x}.$$

**P8.11** From conservation of energy for the block-spring-Earth system,

$$U_{gt} = U_{si},$$

or

$$(0.250\ \text{kg})(9.80\ \text{m/s}^2)h = \left(\frac{1}{2}\right)(5\,000\ \text{N/m})(0.100\ \text{m})^2$$

This gives a maximum height $h = \boxed{10.2\ \text{m}}$.

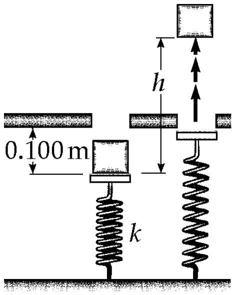

**FIG. P8.11**

**P8.12** (a) The force needed to hang on is equal to the force $F$ the trapeze bar exerts on the performer.

From the free-body diagram for the performer's body, as shown,

$$F - mg\cos\theta = m\frac{v^2}{\ell}$$

or

$$F = mg\cos\theta + m\frac{v^2}{\ell}$$

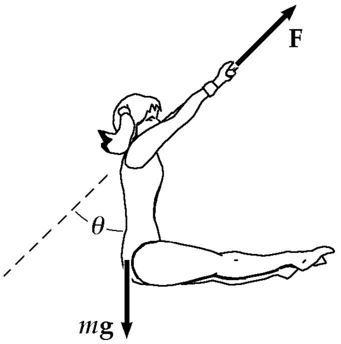

**FIG. P8.12**

Apply conservation of mechanical energy of the performer-Earth system as the performer moves between the starting point and any later point:

$$mg(\ell - \ell\cos\theta_i) = mg(\ell - \ell\cos\theta) + \frac{1}{2}mv^2$$

Solve for $\frac{mv^2}{\ell}$ and substitute into the force equation to obtain $F = \boxed{mg(3\cos\theta - 2\cos\theta_i)}$.

(b) At the bottom of the swing, $\theta = 0°$ so

$$F = mg(3 - 2\cos\theta_i)$$
$$F = 2mg = mg(3 - 2\cos\theta_i)$$

which gives

$$\theta_i = \boxed{60.0°}.$$

**P8.13** Using conservation of energy for the system of the Earth and the two objects

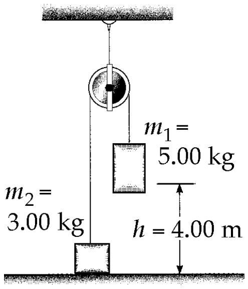

**FIG. P8.13**

(a) $$(5.00\text{ kg})g(4.00\text{ m}) = (3.00\text{ kg})g(4.00\text{ m}) + \frac{1}{2}(5.00+3.00)v^2$$

$$v = \sqrt{19.6} = \boxed{4.43\text{ m/s}}$$

(b) Now we apply conservation of energy for the system of the 3.00 kg object and the Earth during the time interval between the instant when the string goes slack and the instant at which the 3.00 kg object reaches its highest position in its free fall.

$$\frac{1}{2}(3.00)v^2 = mg\,\Delta y = 3.00g\Delta y$$

$$\Delta y = 1.00\text{ m}$$

$$y_{\max} = 4.00\text{ m} + \Delta y = \boxed{5.00\text{ m}}$$

**P8.14** $m_1 > m_2$

(a) $$m_1gh = \frac{1}{2}(m_1+m_2)v^2 + m_2gh$$

$$\boxed{v = \sqrt{\frac{2(m_1-m_2)gh}{(m_1+m_2)}}}$$

(b) Since $m_2$ has kinetic energy $\frac{1}{2}m_2v^2$, it will rise an additional height $\Delta h$ determined from

$$m_2g\,\Delta h = \frac{1}{2}m_2v^2$$

or from (a),

$$\Delta h = \frac{v^2}{2g} = \frac{(m_1-m_2)h}{(m_1+m_2)}$$

The total height $m_2$ reaches is $h + \Delta h = \boxed{\dfrac{2m_1h}{m_1+m_2}}$.

**P8.15** The force of tension and subsequent force of compression in the rod do no work on the ball, since they are perpendicular to each step of displacement. Consider energy conservation of the ball-Earth system between the instant just after you strike the ball and the instant when it reaches the top. The speed at the top is zero if you hit it just hard enough to get it there.

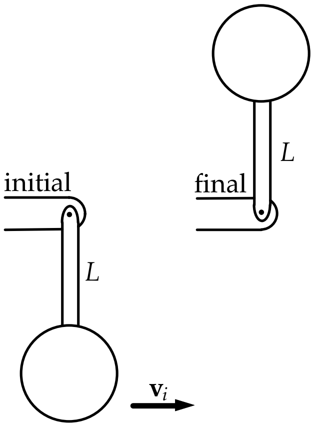

**FIG. P8.15**

$K_i + U_{gi} = K_f + U_{gf}$: $$\frac{1}{2}mv_i^2 + 0 = 0 + mg(2L)$$

$$v_i = \sqrt{4gL} = \sqrt{4(9.80)(0.770)}$$

$$v_i = \boxed{5.49\text{ m/s}}$$

***P8.16** $$\text{efficiency} = \frac{\text{useful output energy}}{\text{total input energy}} = \frac{\text{useful output power}}{\text{total input power}}$$

$$e = \frac{m_{\text{water}}gy/t}{(1/2)m_{\text{air}}\left(v^2/t\right)} = \frac{2\rho_{\text{water}}\left(v_{\text{water}}/t\right)gy}{\rho_{\text{air}}\pi r^2\left(\ell v^2/t\right)} = \frac{2\rho_{\text{w}}\left(v_{\text{w}}/t\right)gy}{\rho_{\text{a}}\pi r^2 v^3}$$

where $\ell$ is the length of a cylinder of air passing through the mill and $v_{\text{w}}$ is the volume of water pumped in time $t$. We need inject negligible kinetic energy into the water because it starts and ends at rest.

$$\frac{v_{\text{w}}}{t} = \frac{e\rho_{\text{a}}\pi r^2 v^3}{2\rho_{\text{w}}gy} = \frac{0.275\left(1.20\ \text{kg/m}^3\right)\pi(1.15\ \text{m})^2(11\ \text{m/s})^3}{2\left(1\,000\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)35\ \text{m}}$$

$$= 2.66\times10^{-3}\ \text{m}^3/\text{s}\left(\frac{1\,000\ \text{L}}{1\ \text{m}^3}\right)\left(\frac{60\ \text{s}}{1\ \text{min}}\right) = \boxed{160\ \text{L/min}}$$

**P8.17** (a) $K_i + U_{gi} = K_f + U_{gf}$

$$\frac{1}{2}mv_i^2 + 0 = \frac{1}{2}mv_f^2 + mgy_f$$
$$\frac{1}{2}mv_{xi}^2 + \frac{1}{2}mv_{yi}^2 = \frac{1}{2}mv_{xf}^2 + mgy_f$$

But $v_{xi} = v_{xf}$, so for the first ball

$$y_f = \frac{v_{yi}^2}{2g} = \frac{(1\,000\sin 37.0°)^2}{2(9.80)} = \boxed{1.85\times10^4\ \text{m}}$$

and for the second

$$y_f = \frac{(1\,000)^2}{2(9.80)} = \boxed{5.10\times10^4\ \text{m}}$$

(b) The total energy of each is constant with value

$$\frac{1}{2}(20.0\ \text{kg})(1\,000\ \text{m/s})^2 = \boxed{1.00\times10^7\ \text{J}}.$$

**P8.18** In the swing down to the breaking point, energy is conserved:

$$mgr\cos\theta = \frac{1}{2}mv^2$$

at the breaking point consider radial forces

$$\sum F_r = ma_r$$

$$+T_{\max} - mg\cos\theta = m\frac{v^2}{r}$$

Eliminate $\frac{v^2}{r} = 2g\cos\theta$

$$T_{\max} - mg\cos\theta = 2mg\cos\theta$$

$$T_{\max} = 3mg\cos\theta$$

$$\theta = \cos^{-1}\left(\frac{T_{\max}}{3mg}\right) = \cos^{-1}\left(\frac{44.5\text{ N}}{3(2.00\text{ kg})(9.80\text{ m/s}^2)}\right)$$

$$\theta = \boxed{40.8°}$$

***P8.19** (a) For a 5-m cord the spring constant is described by $F = kx$, $mg = k(1.5\text{ m})$. For a longer cord of length $L$ the stretch distance is longer so the spring constant is smaller in inverse proportion:

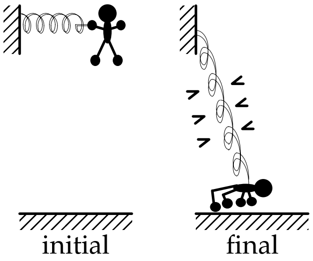

**FIG. P8.19(a)**

$$k = \frac{5\text{ m}}{L}\frac{mg}{1.5\text{ m}} = 3.33\,mg/L$$

$$\left(K + U_g + U_s\right)_i = \left(K + U_g + U_s\right)_f$$

$$0 + mgy_i + 0 = 0 + mgy_f + \frac{1}{2}kx_f^2$$

$$mg\left(y_i - y_f\right) = \frac{1}{2}kx_f^2 = \frac{1}{2}3.33\frac{mg}{L}x_f^2$$

here $y_i - y_f = 55\text{ m} = L + x_f$

$$55.0\text{ m}L = \frac{1}{2}3.33(55.0\text{ m} - L)^2$$

$$55.0\text{ m}L = 5.04\times10^3\text{ m}^2 - 183\text{ m}L + 1.67L^2$$

$$0 = 1.67L^2 - 238L + 5.04\times10^3 = 0$$

$$L = \frac{238 \pm \sqrt{238^2 - 4(1.67)\left(5.04\times10^3\right)}}{2(1.67)} = \frac{238\pm152}{3.33} = \boxed{25.8\text{ m}}$$

only the value of $L$ less than 55 m is physical.

(b) $k = 3.33\frac{mg}{25.8\text{ m}}$ $\qquad x_{\max} = x_f = 55.0\text{ m} - 25.8\text{ m} = 29.2\text{ m}$

$\sum F = ma$ $\qquad +kx_{\max} - mg = ma$

$$3.33\frac{mg}{25.8\text{ m}}29.2\text{ m} - mg = ma$$

$$a = 2.77g = \boxed{27.1\text{ m/s}^2}$$

***P8.20** When block B moves up by 1 cm, block A moves down by 2 cm and the separation becomes 3 cm. We then choose the final point to be when B has moved up by $\frac{h}{3}$ and has speed $\frac{v_A}{2}$. Then A has moved down $\frac{2h}{3}$ and has speed $v_A$:

$$\left(K_A + K_B + U_g\right)_i = \left(K_A + K_B + U_g\right)_f$$

$$0 + 0 + 0 = \frac{1}{2}mv_A^2 + \frac{1}{2}m\left(\frac{v_A}{2}\right)^2 + \frac{mgh}{3} - \frac{mg2h}{3}$$

$$\frac{mgh}{3} = \frac{5}{8}mv_A^2$$

$$\boxed{v_A = \sqrt{\frac{8gh}{15}}}$$

## Section 8.3 **Conservative and Nonconservative Forces**

**P8.21** $F_g = mg = (4.00\text{ kg})(9.80\text{ m/s}^2) = 39.2\text{ N}$

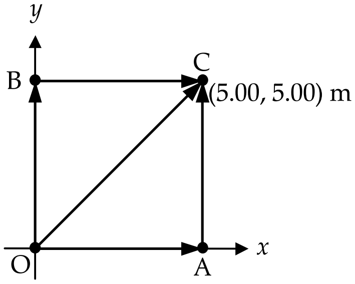

**FIG. P8.21**

(a) Work along OAC = work along OA + work along AC

$= F_g(\text{OA})\cos 90.0° + F_g(\text{AC})\cos 180°$

$= (39.2\text{ N})(5.00\text{ m}) + (39.2\text{ N})(5.00\text{ m})(-1)$

$= \boxed{-196\text{ J}}$

(b) $W$ along OBC = $W$ along OB + $W$ along BC

$= (39.2\text{ N})(5.00\text{ m})\cos 180° + (39.2\text{ N})(5.00\text{ m})\cos 90.0°$

$= \boxed{-196\text{ J}}$

(c) Work along OC $= F_g(\text{OC})\cos 135°$

$$= (39.2\text{ N})\left(5.00 \times \sqrt{2}\text{ m}\right)\left(-\frac{1}{\sqrt{2}}\right) = \boxed{-196\text{ J}}$$

The results should all be the same, since gravitational forces are conservative.

**P8.22** (a) $W = \int \mathbf{F} \cdot d\mathbf{r}$ and if the force is constant, this can be written as

$W = \mathbf{F} \cdot \int d\mathbf{r} = \boxed{\mathbf{F} \cdot \left(\mathbf{r}_f - \mathbf{r}_i\right)\text{, which depends only on end points, not path.}}$

(b) $$W = \int \mathbf{F} \cdot d\mathbf{r} = \int \left(3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}\right) \cdot \left(dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}}\right) = (3.00\text{ N})\int_0^{5.00\text{ m}} dx + (4.00\text{ N})\int_0^{5.00\text{ m}} dy$$

$$W = (3.00\text{ N})x\Big|_0^{5.00\text{ m}} + (4.00\text{ N})y\Big|_0^{5.00\text{ m}} = 15.0\text{ J} + 20.0\text{ J} = \boxed{35.0\text{ J}}$$

The same calculation applies for all paths.

**P8.23** (a)

$$W_{OA}=\int_0^{5.00\text{ m}} dx\hat{\mathbf{i}}\cdot\left(2y\hat{\mathbf{i}}+x^2\hat{\mathbf{j}}\right)=\int_0^{5.00\text{ m}} 2y\,dx$$

and since along this path, $y=0$ $\quad W_{OA}=0$

$$W_{AC}=\int_0^{5.00\text{ m}} dy\hat{\mathbf{j}}\cdot\left(2y\hat{\mathbf{i}}+x^2\hat{\mathbf{j}}\right)=\int_0^{5.00\text{ m}} x^2dy$$

For $x=5.00$ m, $\quad W_{AC}=125\text{ J}$

and $\quad W_{OAC}=0+125=\boxed{125\text{ J}}$

(b)

$$W_{OB}=\int_0^{5.00\text{ m}} dy\hat{\mathbf{j}}\cdot\left(2y\hat{\mathbf{i}}+x^2\hat{\mathbf{j}}\right)=\int_0^{5.00\text{ m}} x^2dy$$

since along this path, $x=0$, $\quad W_{OB}=0$

$$W_{BC}=\int_0^{5.00\text{ m}} dx\hat{\mathbf{i}}\cdot\left(2y\hat{\mathbf{i}}+x^2\hat{\mathbf{j}}\right)=\int_0^{5.00\text{ m}} 2y\,dx$$

since $y=5.00$ m, $\quad W_{BC}=50.0\text{ J}$

$$W_{OBC}=0+50.0=\boxed{50.0\text{ J}}$$

(c)

$$W_{OC}=\int\left(dx\hat{\mathbf{i}}+dy\hat{\mathbf{j}}\right)\cdot\left(2y\hat{\mathbf{i}}+x^2\hat{\mathbf{j}}\right)=\int\left(2y\,dx+x^2dy\right)$$

Since $x=y$ along $OC$, $\quad W_{OC}=\int_0^{5.00\text{ m}}\left(2x+x^2\right)dx=\boxed{66.7\text{ J}}$

(d) $F$ is $\boxed{\text{nonconservative}}$ since the work done is path dependent.

**P8.24** (a)

$$(\Delta K)_{A\to B}=\sum W=W_g=mg\Delta h=mg(5.00-3.20)$$

$$\frac{1}{2}mv_B^2-\frac{1}{2}mv_A^2=m(9.80)(1.80)$$

$$v_B=\boxed{5.94\text{ m/s}}$$

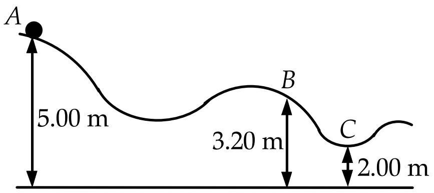

**FIG. P8.24**

Similarly, $v_C=\sqrt{v_A^2+2g(5.00-2.00)}=\boxed{7.67\text{ m/s}}$

(b) $\left.W_g\right|_{A\to C}=mg(3.00\text{ m})=\boxed{147\text{ J}}$

**P8.25** (a) $\mathbf{F} = \left(3.00\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}}\right)\text{ N}$

$m = 4.00\text{ kg}$

$\mathbf{r} = \left(2.00\hat{\mathbf{i}} - 3.00\hat{\mathbf{j}}\right)\text{ m}$

$W = 3.00(2.00) + 5.00(-3.00) = \boxed{-9.00\text{ J}}$

The result does not depend on the path since the force is conservative.

(b) $W = \Delta K$

$$-9.00 = \frac{4.00v^2}{2} - 4.00\left(\frac{(4.00)^2}{2}\right)$$

so $v = \sqrt{\dfrac{32.0 - 9.00}{2.00}} = \boxed{3.39\text{ m/s}}$

(c) $\Delta U = -W = \boxed{9.00\text{ J}}$

## Section 8.4 Changes in Mechanical Energy for Nonconservative Forces

**P8.26** (a) $U_f = K_i - K_f + U_i$ $\quad U_f = 30.0 - 18.0 + 10.0 = \boxed{22.0\text{ J}}$

$\boxed{E = 40.0\text{ J}}$

(b) Yes, $\Delta E_{\text{mech}} = \Delta K + \Delta U$ is not equal to zero. For conservative forces $\Delta K + \Delta U = 0$.

**P8.27** The distance traveled by the ball from the top of the arc to the bottom is $\pi R$. The work done by the non-conservative force, the force exerted by the pitcher,

is $\quad \Delta E = F\Delta r\cos 0° = F(\pi R)$.

We shall assign the gravitational energy of the ball-Earth system to be zero with the ball at the bottom of the arc.

Then $\quad \Delta E_{\text{mech}} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + mgy_f - mgy_i$

becomes $\quad \frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + mgy_i + F(\pi R)$

or $\quad v_f = \sqrt{v_i^2 + 2gy_i + \dfrac{2F(\pi R)}{m}} = \sqrt{(15.0)^2 + 2(9.80)(1.20) + \dfrac{2(30.0)\pi(0.600)}{0.250}}$

$v_f = \boxed{26.5\text{ m/s}}$

***P8.28** The useful output energy is

$$120\text{ Wh}(1 - 0.60) = mg\left(y_f - y_i\right) = F_g\Delta y$$

$$\Delta y = \frac{120\text{ W}(3\,600\text{ s})0.40}{890\text{ N}}\left(\frac{\text{J}}{\text{W}\cdot\text{s}}\right)\left(\frac{\text{N}\cdot\text{m}}{\text{J}}\right) = \boxed{194\text{ m}}$$

***P8.29** As the locomotive moves up the hill at constant speed, its output power goes into internal energy plus gravitational energy of the locomotive-Earth system:

$$\mathcal{P}t = mgy + f\Delta r = mg\Delta r \sin\theta + f\Delta r \qquad \mathcal{P} = mgv_f \sin\theta + fv_f$$

As the locomotive moves on level track,

$$\mathcal{P} = fv_i \qquad 1\,000 \text{ hp}\left(\frac{746 \text{ W}}{1 \text{ hp}}\right) = f(27 \text{ m/s}) \qquad f = 2.76\times10^4 \text{ N}$$

Then also $746\,000 \text{ W} = (160\,000 \text{ kg})(9.8 \text{ m/s}^2)v_f\left(\frac{5 \text{ m}}{100 \text{ m}}\right) + (2.76\times10^4 \text{ N})v_f$

$$v_f = \frac{746\,000 \text{ W}}{1.06\times10^5 \text{ N}} = \boxed{7.04 \text{ m/s}}$$

**P8.30** We shall take the zero level of gravitational potential energy to be at the lowest level reached by the diver under the water, and consider the energy change from when the diver started to fall until he came to rest.

$$\Delta E = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + mgy_f - mgy_i = f_k d\cos 180°$$

$$0 - 0 - mg(y_i - y_f) = -f_k d$$

$$f_k = \frac{mg(y_i - y_f)}{d} = \frac{(70.0 \text{ kg})(9.80 \text{ m/s}^2)(10.0 \text{ m} + 5.00 \text{ m})}{5.00 \text{ m}} = \boxed{2.06 \text{ kN}}$$

**P8.31** $U_i + K_i + \Delta E_{\text{mech}} = U_f + K_f$: $\quad m_2gh - fh = \frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2$

$$f = \mu n = \mu m_1 g$$

$$m_2gh - \mu m_1 gh = \frac{1}{2}(m_1 + m_2)v^2$$

$$v^2 = \frac{2(m_2 - \mu m_1)(hg)}{m_1 + m_2}$$

$$v = \sqrt{\frac{2(9.80 \text{ m/s}^2)(1.50 \text{ m})[5.00 \text{ kg} - 0.400(3.00 \text{ kg})]}{8.00 \text{ kg}}} = \boxed{3.74 \text{ m/s}}$$

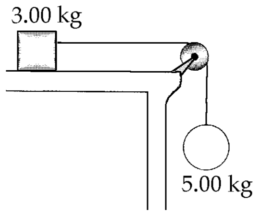

FIG. P8.31

**P8.32** $\Delta E_{\text{mech}} = (K_f - K_i) + (U_{gf} - U_{gi})$

But $\Delta E_{\text{mech}} = W_{\text{app}} - f\Delta x$, where $W_{\text{app}}$ is the work the boy did pushing forward on the wheels.

Thus, $\quad W_{\text{app}} = (K_f - K_i) + (U_{gf} - U_{gi}) + f\Delta x$

or $\quad W_{\text{app}} = \frac{1}{2}m(v_f^2 - v_i^2) + mg(-h) + f\Delta x$

$$W_{\text{app}} = \frac{1}{2}(47.0)[(6.20)^2 - (1.40)^2] - (47.0)(9.80)(2.60) + (41.0)(12.4)$$

$$W_{\text{app}} = \boxed{168 \text{ J}}$$

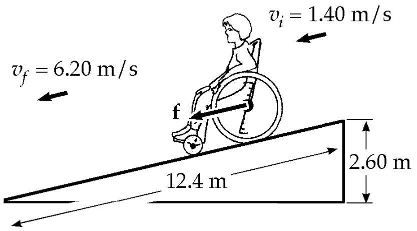

FIG. P8.32

**P8.33** (a) $\Delta K = \frac{1}{2}m\left(v_f^2 - v_i^2\right) = -\frac{1}{2}mv_i^2 = \boxed{-160\text{ J}}$

(b) $\Delta U = mg(3.00\text{ m})\sin 30.0° = \boxed{73.5\text{ J}}$

(c) The mechanical energy converted due to friction is 86.5 J

$$f = \frac{86.5\text{ J}}{3.00\text{ m}} = \boxed{28.8\text{ N}}$$

(d) $f = \mu_k n = \mu_k mg\cos 30.0° = 28.8\text{ N}$

$$\mu_k = \frac{28.8\text{ N}}{(5.00\text{ kg})\left(9.80\text{ m/s}^2\right)\cos 30.0°} = \boxed{0.679}$$

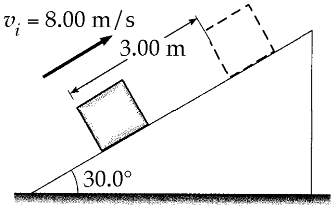

FIG. P8.33

**P8.34** Consider the whole motion: $K_i + U_i + \Delta E_{\text{mech}} = K_f + U_f$

(a) $0 + mgy_i - f_1\Delta x_1 - f_2\Delta x_2 = \frac{1}{2}mv_f^2 + 0$

$$(80.0\text{ kg})\left(9.80\text{ m/s}^2\right)1\,000\text{ m} - (50.0\text{ N})(800\text{ m}) - (3\,600\text{ N})(200\text{ m}) = \frac{1}{2}(80.0\text{ kg})v_f^2$$

$$784\,000\text{ J} - 40\,000\text{ J} - 720\,000\text{ J} = \frac{1}{2}(80.0\text{ kg})v_f^2$$

$$v_f = \sqrt{\frac{2(24\,000\text{ J})}{80.0\text{ kg}}} = \boxed{24.5\text{ m/s}}$$

(b) $\boxed{\text{Yes}}$ this is too fast for safety.

(c) Now in the same energy equation as in part (a), $\Delta x_2$ is unknown, and $\Delta x_1 = 1\,000\text{ m} - \Delta x_2$:

$$784\,000\text{ J} - (50.0\text{ N})(1\,000\text{ m} - \Delta x_2) - (3\,600\text{ N})\Delta x_2 = \frac{1}{2}(80.0\text{ kg})(5.00\text{ m/s})^2$$

$$784\,000\text{ J} - 50\,000\text{ J} - (3\,550\text{ N})\Delta x_2 = 1\,000\text{ J}$$

$$\Delta x_2 = \frac{733\,000\text{ J}}{3\,550\text{ N}} = \boxed{206\text{ m}}$$

(d) Really the air drag will depend on the skydiver's speed. It will be larger than her 784 N weight only after the chute is opened. It will be nearly equal to 784 N before she opens the chute and again before she touches down, whenever she moves near terminal speed.

**P8.35** (a) $(K+U)_i+\Delta E_{\text{mech}}=(K+U)_f$:

$$0+\frac{1}{2}kx^2-f\Delta x=\frac{1}{2}mv^2+0$$

$$\frac{1}{2}(8.00\text{ N/m})\left(5.00\times10^{-2}\text{ m}\right)^2-\left(3.20\times10^{-2}\text{ N}\right)(0.150\text{ m})=\frac{1}{2}\left(5.30\times10^{-3}\text{ kg}\right)v^2$$

$$v=\sqrt{\frac{2\left(5.20\times10^{-3}\text{ J}\right)}{5.30\times10^{-3}\text{ kg}}}=\boxed{1.40\text{ m/s}}$$

(b) When the spring force just equals the friction force, the ball will stop speeding up. Here $|\mathbf{F}_s|=kx$; the spring is compressed by

$$\frac{3.20\times10^{-2}\text{ N}}{8.00\text{ N/m}}=0.400\text{ cm}$$

and the ball has moved

$$5.00\text{ cm}-0.400\text{ cm}=\boxed{4.60\text{ cm from the start.}}$$

(c) Between start and maximum speed points,

$$\frac{1}{2}kx_i^2-f\Delta x=\frac{1}{2}mv^2+\frac{1}{2}kx_f^2$$

$$\frac{1}{2}8.00\left(5.00\times10^{-2}\right)^2-\left(3.20\times10^{-2}\right)\left(4.60\times10^{-2}\right)=\frac{1}{2}\left(5.30\times10^{-3}\right)v^2+\frac{1}{2}8.00\left(4.00\times10^{-3}\right)^2$$

$$v=\boxed{1.79\text{ m/s}}$$

**P8.36**

$$\sum F_y=n-mg\cos37.0°=0$$

$$\therefore n=mg\cos37.0°=400\text{ N}$$

$$f=\mu n=0.250(400\text{ N})=100\text{ N}$$

$$-f\Delta x=\Delta E_{\text{mech}}$$

$$(-100)(20.0)=\Delta U_A+\Delta U_B+\Delta K_A+\Delta K_B$$

$$\Delta U_A=m_Ag\left(h_f-h_i\right)=(50.0)(9.80)(20.0\sin37.0°)=5.90\times10^3$$

$$\Delta U_B=m_Bg\left(h_f-h_i\right)=(100)(9.80)(-20.0)=-1.96\times10^4$$

$$\Delta K_A=\frac{1}{2}m_A\left(v_f^2-v_i^2\right)$$

$$\Delta K_B=\frac{1}{2}m_B\left(v_f^2-v_i^2\right)=\frac{m_B}{m_A}\Delta K_A=2\Delta K_A$$

Adding and solving, $\Delta K_A=\boxed{3.92\text{ kJ}}$.

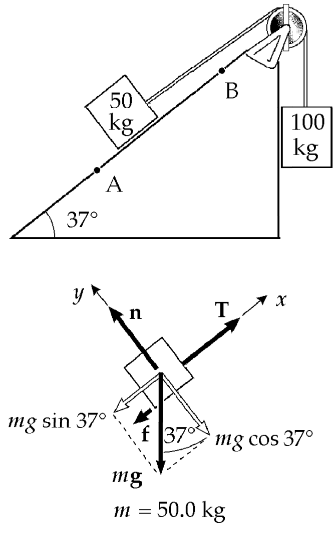

FIG. P8.36

**P8.37** (a) The object moved down distance $1.20 \text{ m} + x$. Choose $y = 0$ at its lower point.

$$K_i + U_{gi} + U_{si} + \Delta E_{\text{mech}} = K_f + U_{gf} + U_{sf}$$

$$0 + mgy_i + 0 + 0 = 0 + 0 + \frac{1}{2}kx^2$$

$$(1.50 \text{ kg})\left(9.80 \text{ m/s}^2\right)(1.20 \text{ m} + x) = \frac{1}{2}(320 \text{ N/m})x^2$$

$$0 = (160 \text{ N/m})x^2 - (14.7 \text{ N})x - 17.6 \text{ J}$$

$$x = \frac{14.7 \text{ N} \pm \sqrt{(-14.7 \text{ N})^2 - 4(160 \text{ N/m})(-17.6 \text{ N}\cdot\text{m})}}{2(160 \text{ N/m})}$$

$$x = \frac{14.7 \text{ N} \pm 107 \text{ N}}{320 \text{ N/m}}$$

The negative root tells how high the object will rebound if it is instantly glued to the spring. We want

$$x = \boxed{0.381 \text{ m}}$$

(b) From the same equation,

$$(1.50 \text{ kg})\left(1.63 \text{ m/s}^2\right)(1.20 \text{ m} + x) = \frac{1}{2}(320 \text{ N/m})x^2$$

$$0 = 160x^2 - 2.44x - 2.93$$

The positive root is $x = \boxed{0.143 \text{ m}}$.

(c) The equation expressing the energy version of the nonisolated system model has one more term:

$$mgy_i - f\Delta x = \frac{1}{2}kx^2$$

$$(1.50 \text{ kg})\left(9.80 \text{ m/s}^2\right)(1.20 \text{ m} + x) - 0.700 \text{ N}(1.20 \text{ m} + x) = \frac{1}{2}(320 \text{ N/m})x^2$$

$$17.6 \text{ J} + 14.7 \text{ N}x - 0.840 \text{ J} - 0.700 \text{ N}x = 160 \text{ N/m}\,x^2$$

$$160x^2 - 14.0x - 16.8 = 0$$

$$x = \frac{14.0 \pm \sqrt{(14.0)^2 - 4(160)(-16.8)}}{320}$$

$$x = \boxed{0.371 \text{ m}}$$

**P8.38** The total mechanical energy of the skysurfer-Earth system is

$$E_{\text{mech}} = K + U_g = \frac{1}{2}mv^2 + mgh.$$

Since the skysurfer has constant speed,

$$\frac{dE_{\text{mech}}}{dt} = mv\frac{dv}{dt} + mg\frac{dh}{dt} = 0 + mg(-v) = -mgv.$$

The rate the system is losing mechanical energy is then

$$\left|\frac{dE_{\text{mech}}}{dt}\right| = mgv = (75.0\text{ kg})\left(9.80\text{ m/s}^2\right)(60.0\text{ m/s}) = \boxed{44.1\text{ kW}}.$$

***P8.39** (a) Let *m* be the mass of the whole board. The portion on the rough surface has mass $\frac{mx}{L}$. The normal force supporting it is $\frac{mxg}{L}$ and the frictional force is $\frac{\mu_k mgx}{L} = ma$. Then $\boxed{a = \frac{\mu_k gx}{L}\text{ opposite to the motion.}}$

(b) In an incremental bit of forward motion *dx*, the kinetic energy converted into internal energy is $f_k dx = \frac{\mu_k mgx}{L}dx$. The whole energy converted is

$$\frac{1}{2}mv^2 = \int_0^L \frac{\mu_k mgx}{L}dx = \frac{\mu_k mg}{L}\frac{x^2}{2}\bigg|_0^L = \frac{\mu_k mgL}{2}$$

$$\boxed{v = \sqrt{\mu_k gL}}$$

## Section 8.5 Relationship Between Conservative Forces and Potential Energy

**P8.40** (a) $U = -\int_0^x \left(-Ax + Bx^2\right)dx = \boxed{\frac{Ax^2}{2} - \frac{Bx^3}{3}}$

(b) $\Delta U = -\int_{2.00\text{ m}}^{3.00\text{ m}} F dx = \frac{A\left[\left(3.00^2\right) - (2.00)^2\right]}{2} - \frac{B\left[(3.00)^3 - (2.00)^3\right]}{3} = \boxed{\frac{5.00}{2}A - \frac{19.0}{3}B}$

$\Delta K = \boxed{\left(-\frac{5.00}{2}A + \frac{19.0}{3}B\right)}$

**P8.41** (a) $W = \int F_x dx = \int_1^{5.00\text{ m}} (2x+4)dx = \left(\frac{2x^2}{2} + 4x\right)_1^{5.00\text{ m}} = 25.0 + 20.0 - 1.00 - 4.00 = \boxed{40.0\text{ J}}$

(b) $\Delta K + \Delta U = 0$ $\qquad \Delta U = -\Delta K = -W = \boxed{-40.0\text{ J}}$

(c) $\Delta K = K_f - \frac{mv_1^2}{2}$ $\qquad K_f = \Delta K + \frac{mv_1^2}{2} = \boxed{62.5\text{ J}}$

**P8.42** $$F_x = -\frac{\partial U}{\partial x} = -\frac{\partial\left(3x^3y-7x\right)}{\partial x} = -\left(9x^2y-7\right) = 7-9x^2y$$

$$F_y = -\frac{\partial U}{\partial y} = -\frac{\partial\left(3x^3y-7x\right)}{\partial y} = -\left(3x^3-0\right) = -3x^3$$

Thus, the force acting at the point $(x,\ y)$ is $\mathbf{F} = F_x\hat{\mathbf{i}} + F_y\hat{\mathbf{j}} = \boxed{\left(7-9x^2y\right)\hat{\mathbf{i}} - 3x^3\hat{\mathbf{j}}}$.

**P8.43** $$U(r) = \frac{A}{r}$$

$F_r = -\frac{\partial U}{\partial r} = -\frac{d}{dr}\left(\frac{A}{r}\right) = \boxed{\frac{A}{r^2}}$. The positive value indicates a force of repulsion.

## Section 8.6 Energy Diagrams and the Equilibrium of a System

**P8.44**

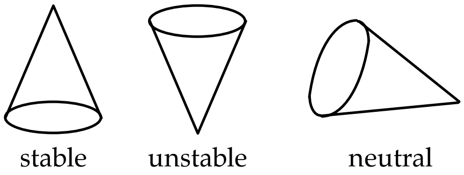

**FIG. P8.44**

**P8.45** (a) $F_x$ is zero at points A, C and E; $F_x$ is positive at point B and negative at point D.

(b) A and E are unstable, and C is stable.

(c)

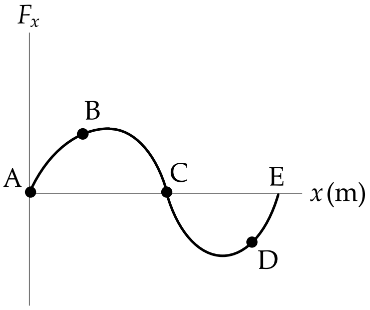

**FIG. P8.45**

**P8.46** (a) There is an equilibrium point wherever the graph of potential energy is horizontal:

At $r = 1.5$ mm and 3.2 mm, the equilibrium is stable.
At $r = 2.3$ mm, the equilibrium is unstable.
A particle moving out toward $r \to \infty$ approaches neutral equilibrium.

(b) The system energy $E$ cannot be less than –5.6 J. The particle is bound if $\boxed{-5.6\text{ J} \le E < 1\text{ J}}$.

(c) If the system energy is –3 J, its potential energy must be less than or equal to –3 J. Thus, the particle's position is limited to $\boxed{0.6\text{ mm} \le r \le 3.6\text{ mm}}$.

(d) $K + U = E$. Thus, $K_{\text{max}} = E - U_{\text{min}} = -3.0\text{ J} - (-5.6\text{ J}) = \boxed{2.6\text{ J}}$.

(e) Kinetic energy is a maximum when the potential energy is a minimum, at $\boxed{r = 1.5\text{ mm}}$.

(f) $-3\text{ J} + W = 1\text{ J}$. Hence, the binding energy is $W = \boxed{4\text{ J}}$.

**P8.47** (a) When the mass moves distance $x$, the length of each spring changes from $L$ to $\sqrt{x^2 + L^2}$, so each exerts force $k\left(\sqrt{x^2 + L^2} - L\right)$ towards its fixed end. The $y$-components cancel out and the $x$ components add to:

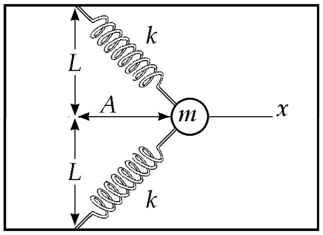

**FIG. P8.47(a)**

$$F_x = -2k\left(\sqrt{x^2 + L^2} - L\right)\left(\frac{x}{\sqrt{x^2 + L^2}}\right) = -2kx + \frac{2kLx}{\sqrt{x^2 + L^2}}$$

Choose $U = 0$ at $x = 0$. Then at any point the potential energy of the system is

$$U(x) = -\int_0^x F_x dx = -\int_0^x \left(-2kx + \frac{2kLx}{\sqrt{x^2 + L^2}}\right) dx = 2k\int_0^x x dx - 2kL\int_0^x \frac{x}{\sqrt{x^2 + L^2}} dx$$

$$U(x) = \boxed{kx^2 + 2kL\left(L - \sqrt{x^2 + L^2}\right)}$$

(b) $U(x) = 40.0x^2 + 96.0\left(1.20 - \sqrt{x^2 + 1.44}\right)$

For negative $x$, $U(x)$ has the same value as for positive $x$. The only equilibrium point (i.e., where $F_x = 0$) is $\boxed{x = 0}$.

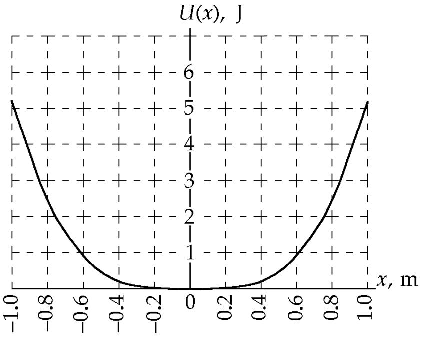

**FIG. P8.47(b)**

(c) $K_i + U_i + \Delta E_{\text{mech}} = K_f + U_f$

$0 + 0.400\text{ J} + 0 = \frac{1}{2}(1.18\text{ kg})v_f^2 + 0$

$v_f = \boxed{0.823\text{ m/s}}$

## Additional Problems

**P8.48** The potential energy of the block-Earth system is $mgh$.

An amount of energy $\mu_k mgd\cos\theta$ is converted into internal energy due to friction on the incline.

Therefore the final height $y_{\max}$ is found from

$$mgy_{\max} = mgh - \mu_k mgd\cos\theta$$

where

$$d = \frac{y_{\max}}{\sin\theta}$$

$$\therefore mgy_{\max} = mgh - \mu_k mgy_{\max}\cot\theta$$

Solving,

$$y_{\max} = \frac{h}{1+\mu_k\cot\theta}.$$

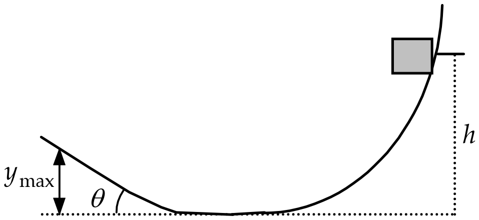

**FIG. P8.48**

**P8.49** At a pace I could keep up for a half-hour exercise period, I climb two stories up, traversing forty steps each 18 cm high, in 20 s. My output work becomes the final gravitational energy of the system of the Earth and me,

$$mgy = (85\text{ kg})\left(9.80\text{ m/s}^2\right)(40\times 0.18\text{ m}) = 6\,000\text{ J}$$

making my sustainable power $\frac{6\,000\text{ J}}{20\text{ s}} = \boxed{\sim 10^2\text{ W}}$.

**P8.50** $v = 100\text{ km/h} = 27.8\text{ m/s}$

The retarding force due to air resistance is

$$R = \frac{1}{2}D\rho Av^2 = \frac{1}{2}(0.330)\left(1.20\text{ kg/m}^3\right)\left(2.50\text{ m}^2\right)(27.8\text{ m/s})^2 = 382\text{ N}$$

Comparing the energy of the car at two points along the hill,

$$K_i + U_{gi} + \Delta E = K_f + U_{gf}$$

or $$K_i + U_{gi} + \Delta W_e - R(\Delta s) = K_f + U_{gf}$$

where $\Delta W_e$ is the work input from the engine. Thus,

$$\Delta W_e = R(\Delta s) + \left(K_f - K_i\right) + \left(U_{gf} - U_{gi}\right)$$

Recognizing that $K_f = K_i$ and dividing by the travel time $\Delta t$ gives the required power input from the engine as

$$\mathcal{P} = \left(\frac{\Delta W_e}{\Delta t}\right) = R\left(\frac{\Delta s}{\Delta t}\right) + mg\left(\frac{\Delta y}{\Delta t}\right) = Rv + mgv\sin\theta$$

$$\mathcal{P} = (382\text{ N})(27.8\text{ m/s}) + (1\,500\text{ kg})\left(9.80\text{ m/s}^2\right)(27.8\text{ m/s})\sin 3.20°$$

$$\mathcal{P} = \boxed{33.4\text{ kW} = 44.8\text{ hp}}$$

**P8.51** $m$ = mass of pumpkin

$R$ = radius of silo top

$\sum F_r = ma_r \Rightarrow n - mg\cos\theta = -m\dfrac{v^2}{R}$

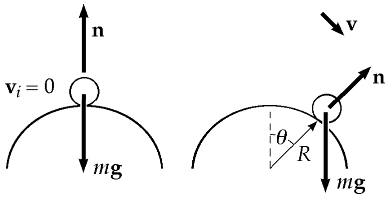

FIG. P8.51

When the pumpkin first loses contact with the surface, $n = 0$. Thus, at the point where it leaves the surface: $v^2 = Rg\cos\theta$.

Choose $U_g = 0$ in the $\theta = 90.0°$ plane. Then applying conservation of energy for the pumpkin-Earth system between the starting point and the point where the pumpkin leaves the surface gives

$$K_f + U_{gf} = K_i + U_{gi}$$

$$\frac{1}{2}mv^2 + mgR\cos\theta = 0 + mgR$$

Using the result from the force analysis, this becomes

$\dfrac{1}{2}mRg\cos\theta + mgR\cos\theta = mgR$, which reduces to

$\cos\theta = \dfrac{2}{3}$, and gives $\theta = \cos^{-1}(2/3) = \boxed{48.2°}$

as the angle at which the pumpkin will lose contact with the surface.

**P8.52**

(a) $U_A = mgR = (0.200\text{ kg})(9.80\text{ m/s}^2)(0.300\text{ m}) = \boxed{0.588\text{ J}}$

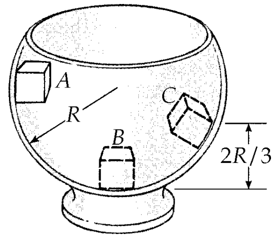

FIG. P8.52

(b) $K_A + U_A = K_B + U_B$

$K_B = K_A + U_A - U_B = mgR = \boxed{0.588\text{ J}}$

(c) $v_B = \sqrt{\dfrac{2K_B}{m}} = \sqrt{\dfrac{2(0.588\text{ J})}{0.200\text{ kg}}} = \boxed{2.42\text{ m/s}}$

(d) $U_C = mgh_C = (0.200\text{ kg})(9.80\text{ m/s}^2)(0.200\text{ m}) = \boxed{0.392\text{ J}}$

$K_C = K_A + U_A - U_C = mg(h_A - h_C)$

$K_C = (0.200\text{ kg})(9.80\text{ m/s}^2)(0.300 - 0.200)\text{ m} = \boxed{0.196\text{ J}}$

**P8.53**

(a) $K_B = \dfrac{1}{2}mv_B^2 = \dfrac{1}{2}(0.200\text{ kg})(1.50\text{ m/s})^2 = \boxed{0.225\text{ J}}$

(b)
$$\begin{aligned}\Delta E_{\text{mech}} &= \Delta K + \Delta U = K_B - K_A + U_B - U_A \\ &= K_B + mg(h_B - h_A) \\ &= 0.225\text{ J} + (0.200\text{ kg})(9.80\text{ m/s}^2)(0 - 0.300\text{ m}) \\ &= 0.225\text{ J} - 0.588\text{ J} = \boxed{-0.363\text{ J}}\end{aligned}$$

(c) It's possible to find an effective coefficient of friction, but not the actual value of $\mu$ since $n$ and $f$ vary with position.

**P8.54** The gain in internal energy due to friction represents a loss in mechanical energy that must be equal to the change in the kinetic energy plus the change in the potential energy.

Therefore,

$$-\mu_k mgx\cos\theta = \Delta K + \frac{1}{2}kx^2 - mgx\sin\theta$$

and since $v_i = v_f = 0$, $\Delta K = 0$.

Thus,

$$-\mu_k(2.00)(9.80)(\cos 37.0°)(0.200) = \frac{(100)(0.200)^2}{2} - (2.00)(9.80)(\sin 37.0°)(0.200)$$

and we find $\mu_k = \boxed{0.115}$. Note that in the above we had a *gain* in elastic potential energy for the spring and a *loss* in gravitational potential energy.

**P8.55** (a) Since no nonconservative work is done, $\Delta E = 0$

Also $\Delta K = 0$

therefore, $U_i = U_f$

where $U_i = (mg\sin\theta)x$

and $U_f = \frac{1}{2}kx^2$

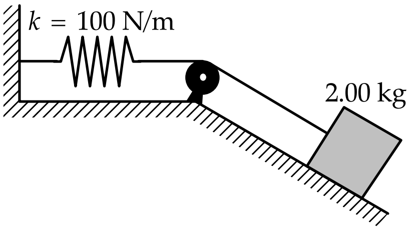

**FIG. P8.55**

Substituting values yields $(2.00)(9.80)\sin 37.0° = (100)\frac{x}{2}$ and solving we find

$$x = \boxed{0.236\text{ m}}$$

(b) $\sum F = ma$. Only gravity and the spring force act on the block, so

$$-kx + mg\sin\theta = ma$$

For $x = 0.236$ m,

$a = \boxed{-5.90\text{ m/s}^2}$. The negative sign indicates $a$ is up the incline.

The $\boxed{\text{acceleration depends on position}}$.

(c) $U$(gravity) decreases monotonically as the height decreases.

$U$(spring) increases monotonically as the spring is stretched.

$K$ initially increases, but then goes back to zero.

**P8.56** $k = 2.50 \times 10^4 \text{ N/m}$, $m = 25.0 \text{ kg}$

$x_A = -0.100 \text{ m}$, $U_g\big|_{x=0} = U_s\big|_{x=0} = 0$

(a) $E_{\text{mech}} = K_A + U_{gA} + U_{sA}$

$$E_{\text{mech}} = 0 + mgx_A + \frac{1}{2}kx_A^2$$

$$E_{\text{mech}} = (25.0 \text{ kg})(9.80 \text{ m/s}^2)(-0.100 \text{ m}) + \frac{1}{2}(2.50 \times 10^4 \text{ N/m})(-0.100 \text{ m})^2$$

$$E_{\text{mech}} = -24.5 \text{ J} + 125 \text{ J} = \boxed{100 \text{ J}}$$

(b) Since only conservative forces are involved, the total energy of the child-pogo-stick-Earth system at point *C* is the same as that at point *A*.

$K_C + U_{gC} + U_{sC} = K_A + U_{gA} + U_{sA}$: $0 + (25.0 \text{ kg})(9.80 \text{ m/s}^2)x_C + 0 = 0 - 24.5 \text{ J} + 125 \text{ J}$

$$x_C = \boxed{0.410 \text{ m}}$$

(c) $K_B + U_{gB} + U_{sB} = K_A + U_{gA} + U_{sA}$: $\frac{1}{2}(25.0 \text{ kg})v_B^2 + 0 + 0 = 0 + (-24.5 \text{ J}) + 125 \text{ J}$

$$v_B = \boxed{2.84 \text{ m/s}}$$

(d) *K* and *v* are at a maximum when $a = \sum F/m = 0$ (i.e., when the magnitude of the upward spring force equals the magnitude of the downward gravitational force).

This occurs at $x < 0$ where $k|x| = mg$

or $$|x| = \frac{(25.0 \text{ kg})(9.8 \text{ m/s}^2)}{2.50 \times 10^4 \text{ N/m}} = 9.80 \times 10^{-3} \text{ m}$$

Thus, $K = K_{\max}$ at $x = \boxed{-9.80 \text{ mm}}$

(e) $$K_{\max} = K_A + \left(U_{gA} - U_g\big|_{x=-9.80 \text{ mm}}\right) + \left(U_{sA} - U_s\big|_{x=-9.80 \text{ mm}}\right)$$

or $$\frac{1}{2}(25.0 \text{ kg})v_{\max}^2 = (25.0 \text{ kg})(9.80 \text{ m/s}^2)\left[(-0.100 \text{ m}) - (-0.009\,8 \text{ m})\right] + \frac{1}{2}(2.50 \times 10^4 \text{ N/m})\left[(-0.100 \text{ m})^2 - (-0.009\,8 \text{ m})^2\right]$$

yielding $v_{\max} = \boxed{2.85 \text{ m/s}}$

**P8.57** $$\Delta E_{\text{mech}} = -f\Delta x$$

$$E_f - E_i = -f \cdot d_{BC}$$

$$\frac{1}{2}kx^2 - mgh = -\mu mgd_{BC}$$

$$\mu = \frac{mgh - \frac{1}{2}kx^2}{mgd_{BC}} = \boxed{0.328}$$

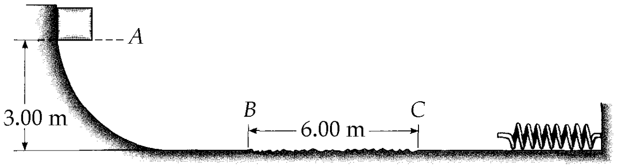

**FIG. P8.57**

**P8.58** (a) $\mathbf{F} = -\dfrac{d}{dx}\left(-x^3 + 2x^2 + 3x\right)\hat{\mathbf{i}} = \boxed{\left(3x^2 - 4x - 3\right)\hat{\mathbf{i}}}$

(b) $F = 0$

when $x = \boxed{1.87 \text{ and } -0.535}$

(c) The stable point is at

$$x = -0.535 \text{ point of minimum } U(x).$$

The unstable point is at

$$x = 1.87 \text{ maximum in } U(x).$$

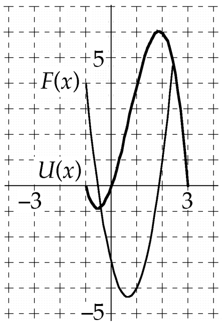

**FIG. P8.58**

**P8.59** $(K+U)_i = (K+U)_f$

$$0 + (30.0 \text{ kg})\left(9.80 \text{ m/s}^2\right)(0.200 \text{ m}) + \frac{1}{2}(250 \text{ N/m})(0.200 \text{ m})^2$$
$$= \frac{1}{2}(50.0 \text{ kg})v^2 + (20.0 \text{ kg})\left(9.80 \text{ m/s}^2\right)(0.200 \text{ m})\sin 40.0°$$
$$58.8 \text{ J} + 5.00 \text{ J} = (25.0 \text{ kg})v^2 + 25.2 \text{ J}$$
$$\boxed{v = 1.24 \text{ m/s}}$$

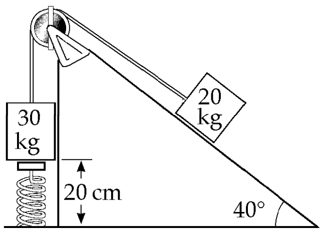

**FIG. P8.59**

**P8.60** (a) Between the second and the third picture, $\Delta E_{\text{mech}} = \Delta K + \Delta U$

$$-\mu mgd = -\frac{1}{2}mv_i^2 + \frac{1}{2}kd^2$$
$$\frac{1}{2}(50.0 \text{ N/m})d^2 + 0.250(1.00 \text{ kg})\left(9.80 \text{ m/s}^2\right)d - \frac{1}{2}(1.00 \text{ kg})\left(3.00 \text{ m/s}^2\right) = 0$$
$$d = \frac{[-2.45 \pm 21.25] \text{ N}}{50.0 \text{ N/m}} = \boxed{0.378 \text{ m}}$$

(b) Between picture two and picture four, $\Delta E_{\text{mech}} = \Delta K + \Delta U$

$$-f(2d) = \frac{1}{2}mv^2 - \frac{1}{2}mv_i^2$$
$$v = \sqrt{(3.00 \text{ m/s})^2 - \frac{2}{(1.00 \text{ kg})}(2.45 \text{ N})(2)(0.378 \text{ m})}$$
$$= \boxed{2.30 \text{ m/s}}$$

(c) For the motion from picture two to picture five, $\Delta E_{\text{mech}} = \Delta K + \Delta U$

$$-f(D + 2d) = -\frac{1}{2}(1.00 \text{ kg})(3.00 \text{ m/s})^2$$
$$D = \frac{9.00 \text{ J}}{2(0.250)(1.00 \text{ kg})\left(9.80 \text{ m/s}^2\right)} - 2(0.378 \text{ m}) = \boxed{1.08 \text{ m}}$$

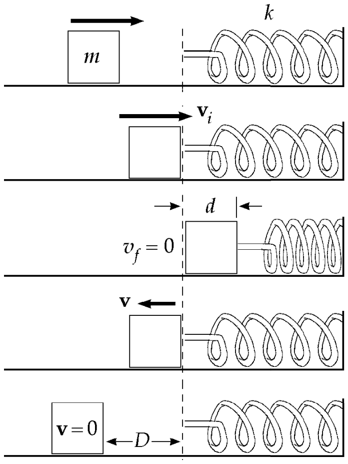

**FIG. P8.60**

**P8.61** (a) Initial compression of spring: $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$

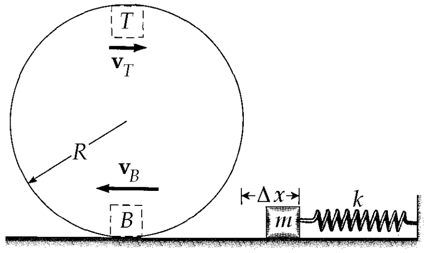

**FIG. P8.61**

$$\frac{1}{2}(450\ \text{N/m})(\Delta x)^2 = \frac{1}{2}(0.500\ \text{kg})(12.0\ \text{m/s})^2$$

$$\therefore \Delta x = \boxed{0.400\ \text{m}}$$

(b) Speed of block at top of track: $\Delta E_{\text{mech}} = -f\Delta x$

$$\left(mgh_T + \frac{1}{2}mv_T^2\right) - \left(mgh_B + \frac{1}{2}mv_B^2\right) = -f(\pi R)$$

$$(0.500\ \text{kg})\left(9.80\ \text{m/s}^2\right)(2.00\ \text{m}) + \frac{1}{2}(0.500\ \text{kg})v_T^2 - \frac{1}{2}(0.500\ \text{kg})(12.0\ \text{m/s})^2$$
$$= -(7.00\ \text{N})(\pi)(1.00\ \text{m})$$

$$0.250v_T^2 = 4.21$$

$$\therefore v_T = \boxed{4.10\ \text{m/s}}$$

(c) Does block fall off at or before top of track? Block falls if $a_c < g$

$$a_c = \frac{v_T^2}{R} = \frac{(4.10)^2}{1.00} = 16.8\ \text{m/s}^2$$

Therefore $a_c > g$ and the $\boxed{\text{block stays on the track}}$.

**P8.62** Let $\lambda$ represent the mass of each one meter of the chain and $T$ represent the tension in the chain at the table edge. We imagine the edge to act like a frictionless and massless pulley.

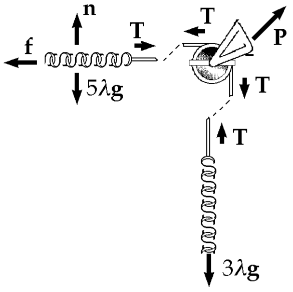

**FIG. P8.62**

(a) For the five meters on the table with motion impending,

$$\sum F_y = 0: \quad +n - 5\lambda g = 0 \qquad n = 5\lambda g$$

$$f_s \le \mu_s n = 0.6(5\lambda g) = 3\lambda g$$

$$\sum F_x = 0: \quad +T - f_s = 0 \qquad T = f_s \qquad T \le 3\lambda g$$

The maximum value is barely enough to support the hanging segment according to

$$\sum F_y = 0: \quad +T - 3\lambda g = 0 \qquad T = 3\lambda g$$

so it is at this point that the chain starts to slide.

*continued on next page*

(b) Let $x$ represent the variable distance the chain has slipped since the start.

Then length $(5-x)$ remains on the table, with now

$$\sum F_y = 0: \quad +n-(5-x)\lambda g = 0 \qquad n=(5-x)\lambda g$$

$$f_k = \mu_k n = 0.4(5-x)\lambda g = 2\lambda g - 0.4x\lambda g$$

Consider energies of the chain-Earth system at the initial moment when the chain starts to slip, and a final moment when $x=5$, when the last link goes over the brink. Measure heights above the final position of the leading end of the chain. At the moment the final link slips off, the center of the chain is at $y_f = 4$ meters.

Originally, 5 meters of chain is at height 8 m and the middle of the dangling segment is at height $8-\frac{3}{2}=6.5\text{ m}$.

$$K_i + U_i + \Delta E_{\text{mech}} = K_f + U_f: \qquad 0+\left(m_1 g y_1 + m_2 g y_2\right)_i - \int_i^f f_k dx = \left(\frac{1}{2}mv^2 + mgy\right)_f$$

$$(5\lambda g)8 + (3\lambda g)6.5 - \int_0^5 (2\lambda g - 0.4x\lambda g)dx = \frac{1}{2}(8\lambda)v^2 + (8\lambda g)4$$

$$40.0g + 19.5g - 2.00g\int_0^5 dx + 0.400g\int_0^5 x\,dx = 4.00v^2 + 32.0g$$

$$27.5g - 2.00gx\Big|_0^5 + 0.400g\frac{x^2}{2}\Bigg|_0^5 = 4.00v^2$$

$$27.5g - 2.00g(5.00) + 0.400g(12.5) = 4.00v^2$$

$$22.5g = 4.00v^2$$

$$v = \sqrt{\frac{(22.5\text{ m})\left(9.80\text{ m/s}^2\right)}{4.00}} = \boxed{7.42\text{ m/s}}$$

**P8.63** Launch speed is found from

$$mg\left(\frac{4}{5}h\right) = \frac{1}{2}mv^2: \qquad v = \sqrt{2g\left(\frac{4}{5}\right)h}$$

$$v_y = v\sin\theta$$

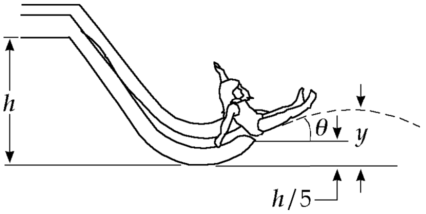

**FIG. P8.63**

The height $y$ above the water (by conservation of energy for the child-Earth system) is found from

$$mgy = \frac{1}{2}mv_y^2 + mg\frac{h}{5} \qquad \text{(since } \frac{1}{2}mv_x^2 \text{ is constant in projectile motion)}$$

$$y = \frac{1}{2g}v_y^2 + \frac{h}{5} = \frac{1}{2g}v^2\sin^2\theta + \frac{h}{5}$$

$$y = \frac{1}{2g}\left[2g\left(\frac{4}{5}h\right)\right]\sin^2\theta + \frac{h}{5} = \boxed{\frac{4}{5}h\sin^2\theta + \frac{h}{5}}$$

***P8.64** (a) The length of string between glider and pulley is given by $\ell^2 = x^2 + h_0^2$. Then $2\ell\frac{d\ell}{dt} = 2x\frac{dx}{dt} + 0$.

Now $\frac{d\ell}{dt}$ is the rate at which string goes over the pulley: $\frac{d\ell}{dt} = v_y = \frac{x}{\ell}v_x = (\cos\theta)v_x$.

(b) $\left(K_A + K_B + U_g\right)_i = \left(K_A + K_B + U_g\right)_f$

$$0 + 0 + m_B g(y_{30} - y_{45}) = \frac{1}{2}m_A v_x^2 + \frac{1}{2}m_B v_y^2$$

Now $y_{30} - y_{45}$ is the amount of string that has gone over the pulley, $\ell_{30} - \ell_{45}$. We have $\sin 30° = \frac{h_0}{\ell_{30}}$ and $\sin 45° = \frac{h_0}{\ell_{45}}$, so $\ell_{30} - \ell_{45} = \frac{h_0}{\sin 30°} - \frac{h_0}{\sin 45°} = 0.40\text{ m}\left(2 - \sqrt{2}\right) = 0.234\text{ m}$.

From the energy equation

$$0.5\text{ kg } 9.8\text{ m/s}^2\ 0.234\text{ m} = \frac{1}{2}1.00\text{ kg } v_x^2 + \frac{1}{2}0.500\text{ kg } v_x^2\cos^2 45°$$

$$v_x = \sqrt{\frac{1.15\text{ J}}{0.625\text{ kg}}} = \boxed{1.35\text{ m/s}}$$

(c) $v_y = v_x\cos\theta = (1.35\text{ m/s})\cos 45° = \boxed{0.958\text{ m/s}}$

(d) The acceleration of neither glider is constant, so knowing distance and acceleration at one point is not sufficient to find speed at another point.

**P8.65** The geometry reveals $D = L\sin\theta + L\sin\phi$, $50.0\text{ m} = 40.0\text{ m}(\sin 50° + \sin\phi)$, $\phi = 28.9°$

(a) From takeoff to alighting for the Jane-Earth system

$$\left(K + U_g\right)_i + W_{\text{wind}} = \left(K + U_g\right)_f$$

$$\frac{1}{2}mv_i^2 + mg(-L\cos\theta) + FD(-1) = 0 + mg(-L\cos\phi)$$

$$\frac{1}{2}50\text{ kg } v_i^2 + 50\text{ kg}\left(9.8\text{ m/s}^2\right)(-40\text{ m}\cos 50°) - 110\text{ N}(50\text{ m}) = 50\text{ kg}\left(9.8\text{ m/s}^2\right)(-40\text{ m}\cos 28.9°)$$

$$\frac{1}{2}50\text{ kg } v_i^2 - 1.26\times 10^4\text{ J} - 5.5\times 10^3\text{ J} = -1.72\times 10^4\text{ J}$$

$$v_i = \sqrt{\frac{2(947\text{ J})}{50\text{ kg}}} = \boxed{6.15\text{ m/s}}$$

(b) For the swing back

$$\frac{1}{2}mv_i^2 + mg(-L\cos\phi) + FD(+1) = 0 + mg(-L\cos\theta)$$

$$\frac{1}{2}130\text{ kg } v_i^2 + 130\text{ kg}\left(9.8\text{ m/s}^2\right)(-40\text{ m}\cos 28.9°) + 110\text{ N}(50\text{ m})$$

$$= 130\text{ kg}\left(9.8\text{ m/s}^2\right)(-40\text{ m}\cos 50°)$$

$$\frac{1}{2}130\text{ kg } v_i^2 - 4.46\times 10^4\text{ J} + 5\,500\text{ J} = -3.28\times 10^4\text{ J}$$

$$v_i = \sqrt{\frac{2(6\,340\text{ J})}{130\text{ kg}}} = \boxed{9.87\text{ m/s}}$$

**P8.66** Case I: Surface is frictionless $\frac{1}{2}mv^2 = \frac{1}{2}kx^2$

$$k = \frac{mv^2}{x^2} = \frac{(5.00\text{ kg})(1.20\text{ m/s})^2}{10^{-2}\text{ m}^2} = 7.20\times10^2\text{ N/m}$$

Case II: Surface is rough, $\mu_k = 0.300$

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 - \mu_k mgx$$

$$\frac{5.00\text{ kg}}{2}v^2 = \frac{1}{2}\left(7.20\times10^2\text{ N/m}\right)\left(10^{-1}\text{ m}\right)^2 - (0.300)(5.00\text{ kg})\left(9.80\text{ m/s}^2\right)\left(10^{-1}\text{ m}\right)$$

$$\boxed{v = 0.923\text{ m/s}}$$

***P8.67** (a) $\left(K+U_g\right)_A = \left(K+U_g\right)_B$

$$0 + mgy_A = \frac{1}{2}mv_B^2 + 0 \qquad v_B = \sqrt{2gy_A} = \sqrt{2\left(9.8\text{ m/s}^2\right)6.3\text{ m}} = \boxed{11.1\text{ m/s}}$$

(b) $$a_c = \frac{v^2}{r} = \frac{(11.1\text{ m/s})^2}{6.3\text{ m}} = \boxed{19.6\text{ m/s}^2\text{ up}}$$

(c) $\sum F_y = ma_y \qquad +n_B - mg = ma_c$

$$n_B = 76\text{ kg}\left(9.8\text{ m/s}^2 + 19.6\text{ m/s}^2\right) = \boxed{2.23\times10^3\text{ N up}}$$

(d) $$W = F\Delta r\cos\theta = 2.23\times10^3\text{ N}(0.450\text{ m})\cos 0° = \boxed{1.01\times10^3\text{ J}}$$

(e) $\left(K+U_g\right)_B + W = \left(K+U_g\right)_D$

$$\frac{1}{2}mv_B^2 + 0 + 1.01\times10^3\text{ J} = \frac{1}{2}mv_D^2 + mg(y_D - y_B)$$

$$\frac{1}{2}76\text{ kg}(11.1\text{ m/s})^2 + 1.01\times10^3\text{ J} = \frac{1}{2}76\text{ kg }v_D^2 + 76\text{ kg}\left(9.8\text{ m/s}^2\right)6.3\text{ m}$$

$$\sqrt{\frac{\left(5.70\times10^3\text{ J} - 4.69\times10^3\text{ J}\right)2}{76\text{ kg}}} = v_D = \boxed{5.14\text{ m/s}}$$

(f) $\left(K+U_g\right)_D = \left(K+U_g\right)_E$ where $E$ is the apex of his motion

$$\frac{1}{2}mv_D^2 + 0 = 0 + mg(y_E - y_D) \qquad y_E - y_D = \frac{v_D^2}{2g} = \frac{(5.14\text{ m/s})^2}{2\left(9.8\text{ m/s}^2\right)} = \boxed{1.35\text{ m}}$$

(g) Consider the motion with constant acceleration between takeoff and touchdown. The time is the positive root of

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_yt^2$$

$$-2.34\text{ m} = 0 + 5.14\text{ m/s}\,t + \frac{1}{2}\left(-9.8\text{ m/s}^2\right)t^2$$

$$4.9t^2 - 5.14t - 2.34 = 0$$

$$t = \frac{5.14 \pm \sqrt{5.14^2 - 4(4.9)(-2.34)}}{9.8} = \boxed{1.39\text{ s}}$$

***P8.68** If the spring is just barely able to lift the lower block from the table, the spring lifts it through no noticeable distance, but exerts on the block a force equal to its weight $Mg$. The extension of the spring, from $|\mathbf{F}_s| = kx$, must be $Mg/k$. Between an initial point at release and a final point when the moving block first comes to rest, we have

$$K_i + U_{gi} + U_{si} = K_f + U_{gf} + U_{sf}: \qquad 0 + mg\left(-\frac{4mg}{k}\right) + \frac{1}{2}k\left(\frac{4mg}{k}\right)^2 = 0 + mg\left(\frac{Mg}{k}\right) + \frac{1}{2}k\left(\frac{Mg}{k}\right)^2$$

$$-\frac{4m^2g^2}{k} + \frac{8m^2g^2}{k} = \frac{mMg^2}{k} + \frac{M^2g^2}{2k}$$

$$4m^2 = mM + \frac{M^2}{2}$$

$$\frac{M^2}{2} + mM - 4m^2 = 0$$

$$M = \frac{-m \pm \sqrt{m^2 - 4\left(\frac{1}{2}\right)\left(-4m^2\right)}}{2\left(\frac{1}{2}\right)} = -m \pm \sqrt{9m^2}$$

Only a positive mass is physical, so we take $M = m(3-1) = \boxed{2m}$.

**P8.69** (a) Take the original point where the ball is released and the final point where its upward swing stops at height $H$ and horizontal displacement

$$x = \sqrt{L^2 - (L-H)^2} = \sqrt{2LH - H^2}$$

Since the wind force is purely horizontal, it does work

$$W_{\text{wind}} = \int \mathbf{F} \cdot d\mathbf{s} = F\int dx = F\sqrt{2LH - H^2}$$

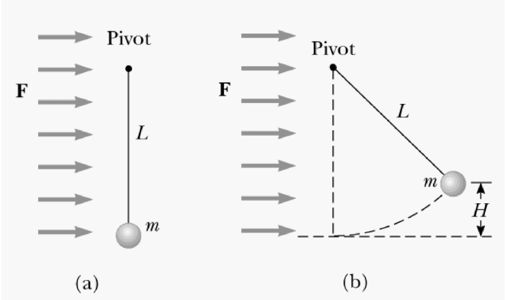

**FIG. P8.69**

The work-energy theorem can be written:

$K_i + U_{gi} + W_{\text{wind}} = K_f + U_{gf}$, or

$0 + 0 + F\sqrt{2LH - H^2} = 0 + mgH$ giving $F^2 2LH - F^2H^2 = m^2g^2H^2$

Here $H = 0$ represents the lower turning point of the ball's oscillation, and the upper limit is at $F^2(2L) = \left(F^2 + m^2g^2\right)H$. Solving for $H$ yields

$$H = \frac{2LF^2}{F^2 + m^2g^2} = \boxed{\frac{2L}{1 + (mg/F)^2}}$$

As $F \to 0$, $H \to 0$ as is reasonable.
As $F \to \infty$, $H \to 2L$, which would be hard to approach experimentally.

(b) $$H = \frac{2(2.00\text{ m})}{1 + \left[(2.00\text{ kg})\left(9.80\text{ m/s}^2\right)/14.7\text{ N}\right]^2} = \boxed{1.44\text{ m}}$$

*continued on next page*

(c) Call $\theta$ the equilibrium angle with the vertical.

$$\sum F_x = 0 \Rightarrow T\sin\theta = F, \text{ and}$$
$$\sum F_y = 0 \Rightarrow T\cos\theta = mg$$

Dividing: $\tan\theta = \frac{F}{mg} = \frac{14.7\text{ N}}{19.6\text{ N}} = 0.750$, or $\theta = 36.9°$

Therefore, $H_{\text{eq}} = L(1-\cos\theta) = (2.00\text{ m})(1-\cos 36.9°) = \boxed{0.400\text{ m}}$

(d) As $F \to \infty$, $\tan\theta \to \infty$, $\theta \to 90.0°$ and $H_{\text{eq}} \to L$

A very strong wind pulls the string out horizontal, parallel to the ground. Thus, $\boxed{\left(H_{\text{eq}}\right)_{\max} = L}$.

**P8.70** Call $\phi = 180° - \theta$ the angle between the upward vertical and the radius to the release point. Call $v_r$ the speed here. By conservation of energy

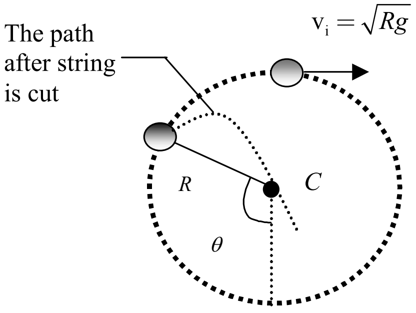

**FIG. P8.70**

$$K_i + U_i + \Delta E = K_r + U_r$$
$$\frac{1}{2}mv_i^2 + mgR + 0 = \frac{1}{2}mv_r^2 + mgR\cos\phi$$
$$gR + 2gR = v_r^2 + 2gR\cos\phi$$
$$v_r = \sqrt{3gR - 2gR\cos\phi}$$

The components of velocity at release are $v_x = v_r\cos\phi$ and $v_y = v_r\sin\phi$ so for the projectile motion we have

$$x = v_x t \qquad R\sin\phi = v_r\cos\phi t$$
$$y = v_y t - \frac{1}{2}gt^2 \qquad -R\cos\phi = v_r\sin\phi t - \frac{1}{2}gt^2$$

By substitution

$$-R\cos\phi = v_r\sin\phi\frac{R\sin\phi}{v_r\cos\phi} - \frac{g}{2}\frac{R^2\sin^2\phi}{v_r^2\cos^2\phi}$$

with $\sin^2\phi + \cos^2\phi = 1$,

$$gR\sin^2\phi = 2v_r^2\cos\phi = 2\cos\phi(3gR - 2gR\cos\phi)$$
$$\sin^2\phi = 6\cos\phi - 4\cos^2\phi = 1 - \cos^2\phi$$
$$3\cos^2\phi - 6\cos\phi + 1 = 0$$
$$\cos\phi = \frac{6 \pm \sqrt{36-12}}{6}$$

Only the – sign gives a value for $\cos\phi$ that is less than one:

$$\cos\phi = 0.183\,5 \qquad \phi = 79.43° \qquad \text{so } \theta = \boxed{100.6°}$$

**P8.71** Applying Newton's second law at the bottom (b) and top (t) of the circle gives

$$T_b - mg = \frac{mv_b^2}{R} \text{ and } -T_t - mg = -\frac{mv_t^2}{R}$$

Adding these gives $T_b = T_t + 2mg + \dfrac{m\left(v_b^2 - v_t^2\right)}{R}$

Also, energy must be conserved and $\Delta U + \Delta K = 0$

So, $\dfrac{m\left(v_b^2 - v_t^2\right)}{2} + (0 - 2mgR) = 0$ and $\dfrac{m\left(v_b^2 - v_t^2\right)}{R} = 4mg$

Substituting into the above equation gives $\boxed{T_b = T_t + 6mg}$.

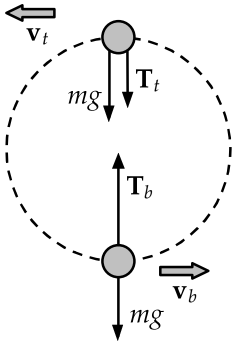

FIG. P8.71

**P8.72** (a) Energy is conserved in the swing of the pendulum, and the stationary peg does no work. So the ball's speed does not change when the string hits or leaves the peg, and the ball swings equally high on both sides.

(b) Relative to the point of suspension,

$$U_i = 0,\, U_f = -mg\left[d - (L - d)\right]$$

From this we find that

$$-mg(2d - L) + \frac{1}{2}mv^2 = 0$$

Also for centripetal motion,

$$mg = \frac{mv^2}{R} \text{ where } R = L - d.$$

Upon solving, we get $\boxed{d = \dfrac{3L}{5}}$.

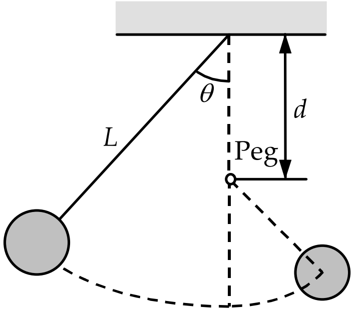

FIG. P8.72

***P8.73** (a) At the top of the loop the car and riders are in free fall:

$$\sum F_y = ma_y: \qquad mg \text{ down} = \frac{mv^2}{R} \text{ down}$$

$$v = \sqrt{Rg}$$

Energy of the car-riders-Earth system is conserved between release and top of loop:

$$K_i + U_{gi} = K_f + U_{gf}: \qquad 0 + mgh = \frac{1}{2}mv^2 + mg(2R)$$

$$gh = \frac{1}{2}Rg + g(2R)$$

$$\boxed{h = 2.50R}$$

(b) Let $h$ now represent the height $\geq 2.5R$ of the release point. At the bottom of the loop we have

$$mgh = \frac{1}{2}mv_b^2 \qquad \text{or} \qquad v_b^2 = 2gh$$

$$\sum F_y = ma_y: \qquad n_b - mg = \frac{mv_b^2}{R}(\text{up})$$

$$n_b = mg + \frac{m(2gh)}{R}$$

At the top of the loop, $mgh = \frac{1}{2}mv_t^2 + mg(2R)$

$$v_t^2 = 2gh - 4gR$$

$$\sum F_y = ma_y: \qquad -n_t - mg = -\frac{mv_t^2}{R}$$

$$n_t = -mg + \frac{m}{R}(2gh - 4gR)$$

$$n_t = \frac{m(2gh)}{R} - 5mg$$

Then the normal force at the bottom is larger by

$$n_b - n_t = mg + \frac{m(2gh)}{R} - \frac{m(2gh)}{R} + 5mg = \boxed{6mg}.$$

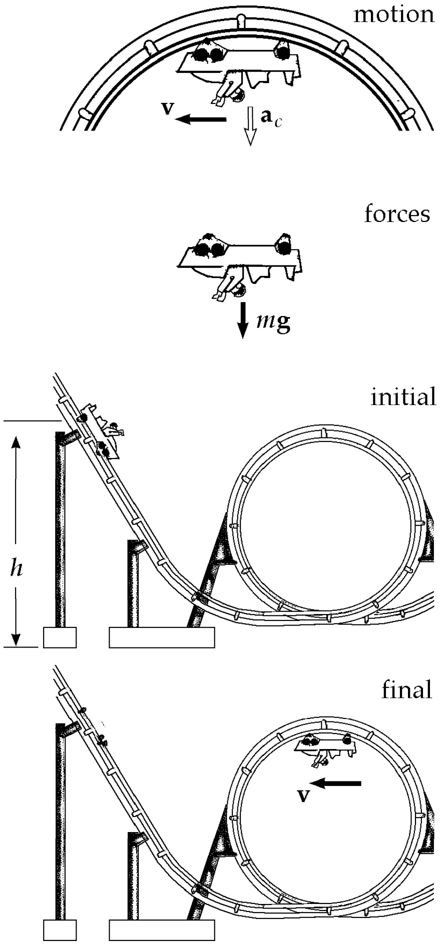

FIG. P8.73

***P8.74** (a) Conservation of energy for the sled-rider-Earth system, between A and C:

$$K_i + U_{gi} = K_f + U_{gf}$$

$$\frac{1}{2}m(2.5\ \text{m/s})^2 + m\left(9.80\ \text{m/s}^2\right)(9.76\ \text{m}) = \frac{1}{2}mv_C^2 + 0$$

$$v_C = \sqrt{(2.5\ \text{m/s})^2 + 2\left(9.80\ \text{m/s}^2\right)(9.76\ \text{m})} = \boxed{14.1\ \text{m/s}}$$

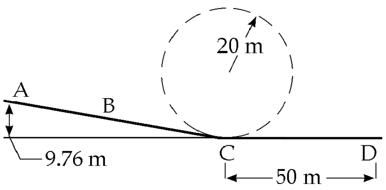

**FIG. P8.74(a)**

(b) Incorporating the loss of mechanical energy during the portion of the motion in the water, we have, for the entire motion between A and D (the rider's stopping point),

$$K_i + U_{gi} - f_k\Delta x = K_f + U_{gf}: \quad \frac{1}{2}(80\ \text{kg})(2.5\ \text{m/s})^2 + (80\ \text{kg})\left(9.80\ \text{m/s}^2\right)(9.76\ \text{m}) - f_k\Delta x = 0 + 0$$

$$-f_k\Delta x = \boxed{-7.90\times 10^3\ \text{J}}$$

(c) The water exerts a frictional force $f_k = \dfrac{7.90\times 10^3\ \text{J}}{\Delta x} = \dfrac{7.90\times 10^3\ \text{N}\cdot\text{m}}{50\ \text{m}} = 158\ \text{N}$

and also a normal force of $n = mg = (80\ \text{kg})\left(9.80\ \text{m/s}^2\right) = 784\ \text{N}$

The magnitude of the water force is $\sqrt{(158\ \text{N})^2 + (784\ \text{N})^2} = \boxed{800\ \text{N}}$

(d) The angle of the slide is

$$\theta = \sin^{-1}\frac{9.76\ \text{m}}{54.3\ \text{m}} = 10.4°$$

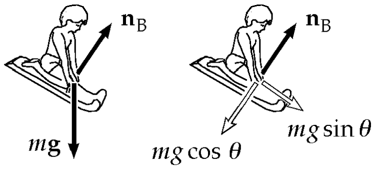

**FIG. P8.74(d)**

For forces perpendicular to the track at B,

$$\sum F_y = ma_y: \quad n_B - mg\cos\theta = 0$$

$$n_B = (80.0\ \text{kg})\left(9.80\ \text{m/s}^2\right)\cos 10.4° = \boxed{771\ \text{N}}$$

(e)

$$\sum F_y = ma_y: \quad +n_C - mg = \frac{mv_C^2}{r}$$

$$n_C = (80.0\ \text{kg})\left(9.80\ \text{m/s}^2\right) + \frac{(80.0\ \text{kg})(14.1\ \text{m/s})^2}{20\ \text{m}}$$

$$n_C = \boxed{1.57\times 10^3\ \text{N up}}$$

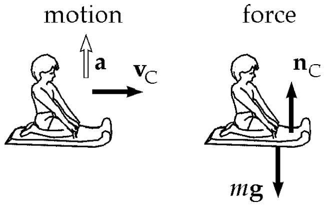

**FIG. P8.74(e)**

The rider pays for the thrills of a giddy height at A, and a high speed and tremendous splash at C. As a bonus, he gets the quick change in direction and magnitude among the forces we found in parts (d), (e), and (c).

## ANSWERS TO EVEN PROBLEMS

| | |
|---|---|
| **P8.2** | (a) 800 J; (b) 107 J; (c) 0 |
| **P8.4** | (a) $1.11\times10^{9}$ J; (b) 0.2 |
| **P8.6** | 1.84 m |
| **P8.8** | (a) 10.2 kW; (b) 10.6 kW; (c) $5.82\times10^{6}$ J |
| **P8.10** | $d=\dfrac{kx^2}{2mg\sin\theta}-x$ |
| **P8.12** | (a) see the solution; (b) 60.0° |
| **P8.14** | (a) $\sqrt{\dfrac{2(m_1-m_2)gh}{(m_1+m_2)}}$; (b) $\dfrac{2m_1h}{m_1+m_2}$ |
| **P8.16** | 160 L/min |
| **P8.18** | 40.8° |
| **P8.20** | $\left(\dfrac{8gh}{15}\right)^{1/2}$ |
| **P8.22** | (a) see the solution; (b) 35.0 J |
| **P8.24** | (a) $v_B=5.94$ m/s; $v_C=7.67$ m/s; (b) 147 J |
| **P8.26** | (a) $U_f=22.0$ J; $E=40.0$ J; (b) Yes. The total mechanical energy changes. |
| **P8.28** | 194 m |
| **P8.30** | 2.06 kN up |
| **P8.32** | 168 J |
| **P8.34** | (a) 24.5 m/s; (b) yes; (c) 206 m; (d) Air drag depends strongly on speed. |
| **P8.36** | 3.92 kJ |
| **P8.38** | 44.1 kW |
| **P8.40** | (a) $\dfrac{Ax^2}{2}-\dfrac{Bx^3}{3}$;<br>(b) $\Delta U=\dfrac{5A}{2}-\dfrac{19B}{3}$; $\Delta K=\dfrac{19B}{3}-\dfrac{5A}{2}$ |
| **P8.42** | $\left(7-9x^2y\right)\hat{\mathbf{i}}-3x^3\hat{\mathbf{j}}$ |
| **P8.44** | see the solution |
| **P8.46** | (a) $r=1.5$ mm and 3.2 mm, stable; 2.3 mm and unstable; $r\to\infty$ neutral;<br>(b) $-5.6\text{ J}\le E<1\text{ J}$; (c) $0.6\text{ mm}\le r\le3.6\text{ mm}$;<br>(d) 2.6 J; (e) 1.5 mm; (f) 4 J |
| **P8.48** | see the solution |
| **P8.50** | 33.4 kW |
| **P8.52** | (a) 0.588 J; (b) 0.588 J; (c) 2.42 m/s;<br>(d) 0.196 J; 0.392 J |
| **P8.54** | 0.115 |
| **P8.56** | (a) 100 J; (b) 0.410 m; (c) 2.84 m/s;<br>(d) −9.80 mm; (e) 2.85 m/s |
| **P8.58** | (a) $\left(3x^2-4x-3\right)\hat{\mathbf{i}}$; (b) 1.87; −0.535;<br>(c) see the solution |
| **P8.60** | (a) 0.378 m; (b) 2.30 m/s; (c) 1.08 m |
| **P8.62** | (a) see the solution; (b) 7.42 m/s |
| **P8.64** | (a) see the solution; (b) 1.35 m/s;<br>(c) 0.958 m/s; (d) see the solution |
| **P8.66** | 0.923 m/s |
| **P8.68** | $2m$ |
| **P8.70** | 100.6° |
| **P8.72** | see the solution |
| **P8.74** | (a) 14.1 m/s; (b) −7.90 J; (c) 800 N;<br>(d) 771 N; (e) 1.57 kN up |

# 9

# Linear Momentum and Collisions

**CHAPTER OUTLINE**

9.1 Linear Momentum and Its Conservation
9.2 Impulse and Momentum
9.3 Collisions in One Dimension
9.4 Two-Dimensional Collisions
9.5 The Center of Mass
9.6 Motion of a System of Particles
9.7 Rocket Propulsion

## ANSWERS TO QUESTIONS

**Q9.1** No. Impulse, $F\Delta t$, depends on the force and the time for which it is applied.

**Q9.2** The momentum doubles since it is proportional to the speed. The kinetic energy quadruples, since it is proportional to the speed-squared.

**Q9.3** The momenta of two particles will only be the same if the masses of the particles of the same.

**Q9.4** (a) It does not carry force, for if it did, it could accelerate itself.

(b) It cannot deliver more kinetic energy than it possesses. This would violate the law of energy conservation.

(c) It can deliver more momentum in a collision than it possesses in its flight, by bouncing from the object it strikes.

**Q9.5** Provided there is some form of potential energy in the system, the parts of an isolated system can move if the system is initially at rest. Consider two air-track gliders on a horizontal track. If you compress a spring between them and then tie them together with a string, it is possible for the system to start out at rest. If you then burn the string, the potential energy stored in the spring will be converted into kinetic energy of the gliders.

**Q9.6** No. Only in a precise head-on collision with momenta with equal magnitudes and opposite directions can both objects wind up at rest. Yes. Assume that ball 2, originally at rest, is struck squarely by an equal-mass ball 1. Then ball 2 will take off with the velocity of ball 1, leaving ball 1 at rest.

**Q9.7** Interestingly, mutual gravitation brings the ball and the Earth together. As the ball moves downward, the Earth moves upward, although with an acceleration $10^{25}$ times smaller than that of the ball. The two objects meet, rebound, and separate. Momentum of the ball-Earth system is conserved.

**Q9.8** (a) Linear momentum is conserved since there are no external forces acting on the system.

(b) Kinetic energy is not conserved because the chemical potential energy initially in the explosive is converted into kinetic energy of the pieces of the bomb.

**Q9.9** Momentum conservation is not violated if we make our system include the Earth along with the clay. When the clay receives an impulse backwards, the Earth receives the same size impulse forwards. The resulting acceleration of the Earth due to this impulse is significantly smaller than the acceleration of the clay, but the planet absorbs all of the momentum that the clay loses.

**Q9.10** Momentum conservation is not violated if we choose as our system the planet along with you. When you receive an impulse forward, the Earth receives the same size impulse backwards. The resulting acceleration of the Earth due to this impulse is significantly smaller than your acceleration forward, but the planet's backward momentum is equal in magnitude to your forward momentum.

**Q9.11** As a ball rolls down an incline, the Earth receives an impulse of the same size and in the opposite direction as that of the ball. If you consider the Earth-ball system, momentum conservation is not violated.

**Q9.12** Suppose car and truck move along the same line. If one vehicle overtakes the other, the faster-moving one loses more energy than the slower one gains. In a head-on collision, if the speed of the truck is less than $\frac{m_T+3m_c}{3m_T+m_c}$ times the speed of the car, the car will lose more energy.

**Q9.13** The rifle has a much lower speed than the bullet and much less kinetic energy. The butt distributes the recoil force over an area much larger than that of the bullet.

**Q9.14** His impact speed is determined by the acceleration of gravity and the distance of fall, in $v_f^2 = v_i^2 - 2g(0-y_i)$. The force exerted by the pad depends also on the unknown stiffness of the pad.

**Q9.15** The product of the mass flow rate and velocity of the water determines the force the firefighters must exert.

**Q9.16** The sheet stretches and pulls the two students toward each other. These effects are larger for a faster-moving egg. The time over which the egg stops is extended so that the force stopping it is never too large.

**Q9.17** (c) In this case, the impulse on the Frisbee is largest. According to Newton's third law, the impulse on the skater and thus the final speed of the skater will also be largest.

**Q9.18** Usually but not necessarily. In a one-dimensional collision between two identical particles with the same initial speed, the kinetic energy of the particles will not change.

**Q9.19** $g$ downward.

**Q9.20** As one finger slides towards the center, the normal force exerted by the sliding finger on the ruler increases. At some point, this normal force will increase enough so that static friction between the sliding finger and the ruler will stop their relative motion. At this moment the other finger starts sliding along the ruler towards the center. This process repeats until the fingers meet at the center of the ruler.

**Q9.21** The planet is in motion around the sun, and thus has momentum and kinetic energy of its own. The spacecraft is directed to cross the planet's orbit behind it, so that the planet's gravity has a component pulling forward on the spacecraft. Since this is an elastic collision, and the velocity of the planet remains nearly unchanged, the probe must both increase speed and change direction for both momentum and kinetic energy to be conserved.

**Q9.22** No—an external force of gravity acts on the moon. Yes, because its speed is constant.

**Q9.23** The impulse given to the egg is the same regardless of how it stops. If you increase the impact time by dropping the egg onto foam, you will decrease the impact force.

**Q9.24** Yes. A boomerang, a kitchen stool.

**Q9.25** The center of mass of the balls is in free fall, moving up and then down with the acceleration due to gravity, during the 40% of the time when the juggler's hands are empty. During the 60% of the time when the juggler is engaged in catching and tossing, the center of mass must accelerate up with a somewhat smaller average acceleration. The center of mass moves around in a little circle, making three revolutions for every one revolution that one ball makes. Letting $T$ represent the time for one cycle and $F_g$ the weight of one ball, we have $F_J 0.60T = 3F_g T$ and $F_J = 5F_g$. The average force exerted by the juggler is five times the weight of one ball.

**Q9.26** In empty space, the center of mass of a rocket-plus-fuel system does not accelerate during a burn, because no outside force acts on this system. According to the text's 'basic expression for rocket propulsion,' the change in speed of the rocket body will be larger than the speed of the exhaust relative to the rocket, if the final mass is less than 37% of the original mass.

**Q9.27** The gun recoiled.

**Q9.28** Inflate a balloon and release it. The air escaping from the balloon gives the balloon an impulse.

**Q9.29** There was a time when the English favored position (a), the Germans position (b), and the French position (c). A Frenchman, Jean D'Alembert, is most responsible for showing that each theory is consistent with the others. All are equally correct. Each is useful for giving a mathematically simple solution for some problems.

## SOLUTIONS TO PROBLEMS

### Section 9.1 **Linear Momentum and Its Conservation**

**P9.1** $m = 3.00\text{ kg}$, $\mathbf{v} = \left(3.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}}\right)\text{ m/s}$

(a) $\mathbf{p} = m\mathbf{v} = \left(9.00\hat{\mathbf{i}} - 12.0\hat{\mathbf{j}}\right)\text{ kg}\cdot\text{m/s}$

Thus, $\boxed{p_x = 9.00\text{ kg}\cdot\text{m/s}}$

and $\boxed{p_y = -12.0\text{ kg}\cdot\text{m/s}}$

(b) $p = \sqrt{p_x^2 + p_y^2} = \sqrt{(9.00)^2 + (12.0)^2} = \boxed{15.0\text{ kg}\cdot\text{m/s}}$

$\theta = \tan^{-1}\left(\dfrac{p_y}{p_x}\right) = \tan^{-1}(-1.33) = \boxed{307°}$

**P9.2** (a) At maximum height $\mathbf{v} = 0$, so $\mathbf{p} = \boxed{0}$.

(b) Its original kinetic energy is its constant total energy,

$$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(0.100)\text{kg}(15.0\ \text{m/s})^2 = 11.2\ \text{J}.$$

At the top all of this energy is gravitational. Halfway up, one-half of it is gravitational and the other half is kinetic:

$$K = 5.62\ \text{J} = \frac{1}{2}(0.100\ \text{kg})v^2$$

$$v = \sqrt{\frac{2 \times 5.62\ \text{J}}{0.100\ \text{kg}}} = 10.6\ \text{m/s}$$

Then $\mathbf{p} = m\mathbf{v} = (0.100\ \text{kg})(10.6\ \text{m/s})\hat{\mathbf{j}}$

$$\mathbf{p} = \boxed{1.06\ \text{kg}\cdot\text{m/s}\,\hat{\mathbf{j}}}.$$

**P9.3** I have mass 85.0 kg and can jump to raise my center of gravity 25.0 cm. I leave the ground with speed given by

$$v_f^2 - v_i^2 = 2a(x_f - x_i): \qquad 0 - v_i^2 = 2(-9.80\ \text{m/s}^2)(0.250\ \text{m})$$

$$v_i = 2.20\ \text{m/s}$$

Total momentum of the system of the Earth and me is conserved as I push the earth down and myself up:

$$0 = (5.98 \times 10^{24}\ \text{kg})v_e + (85.0\ \text{kg})(2.20\ \text{m/s})$$

$$v_e \sim \boxed{10^{-23}\ \text{m/s}}$$

**P9.4** (a) For the system of two blocks $\Delta p = 0$,

or $p_i = p_f$

Therefore, $0 = Mv_m + (3M)(2.00\ \text{m/s})$

Solving gives $v_m = \boxed{-6.00\ \text{m/s}}$ (motion toward the left).

(b) $\frac{1}{2}kx^2 = \frac{1}{2}Mv_M^2 + \frac{1}{2}(3M)v_{3M}^2 = \boxed{8.40\ \text{J}}$

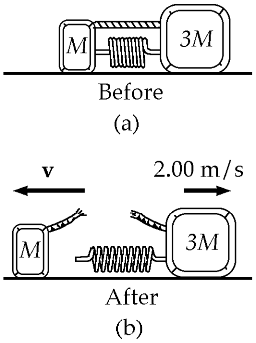

**FIG. P9.4**

**P9.5** (a) The momentum is $p = mv$, so $v = \frac{p}{m}$ and the kinetic energy is $K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{p}{m}\right)^2 = \boxed{\frac{p^2}{2m}}$.

(b) $K = \frac{1}{2}mv^2$ implies $v = \sqrt{\frac{2K}{m}}$, so $p = mv = m\sqrt{\frac{2K}{m}} = \boxed{\sqrt{2mK}}$.

---

## Section 9.2 **Impulse and Momentum**

***P9.6** From the impulse-momentum theorem, $\overline{F}(\Delta t) = \Delta p = mv_f - mv_i$, the average force required to hold onto the child is

$$\overline{F} = \frac{m(v_f - v_i)}{(\Delta t)} = \frac{(12\text{ kg})(0 - 60\text{ mi/h})}{0.050\text{ s} - 0}\left(\frac{1\text{ m/s}}{2.237\text{ mi/h}}\right) = -6.44 \times 10^3\text{ N}.$$

Therefore, the magnitude of the needed retarding force is $\boxed{6.44 \times 10^3\text{ N}}$, or 1 400 lb. A person cannot exert a force of this magnitude and a safety device should be used.

**P9.7** (a) $I = \int F dt =$ area under curve

$$I = \frac{1}{2}\left(1.50 \times 10^{-3}\text{ s}\right)(18\,000\text{ N}) = \boxed{13.5\text{ N}\cdot\text{s}}$$

(b) $F = \frac{13.5\text{ N}\cdot\text{s}}{1.50 \times 10^{-3}\text{ s}} = \boxed{9.00\text{ kN}}$

(c) From the graph, we see that $F_{\max} = \boxed{18.0\text{ kN}}$

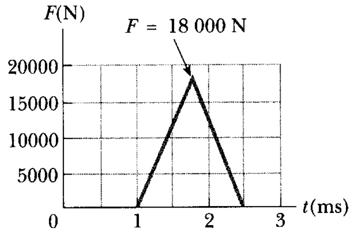

**FIG. P9.7**

***P9.8** The impact speed is given by $\frac{1}{2}mv_1^2 = mgy_1$. The rebound speed is given by $mgy_2 = \frac{1}{2}mv_2^2$. The impulse of the floor is the change in momentum,

$$\begin{aligned} mv_2\text{ up} - mv_1\text{ down} &= m(v_2 + v_1)\text{ up} \\ &= m\left(\sqrt{2gh_2} + \sqrt{2gh_1}\right)\text{ up} \\ &= 0.15\text{ kg}\sqrt{2\left(9.8\text{ m/s}^2\right)}\left(\sqrt{0.960\text{ m}} + \sqrt{1.25\text{ m}}\right)\text{ up} \\ &= \boxed{1.39\text{ kg}\cdot\text{m/s upward}} \end{aligned}$$

**P9.9**

$$\Delta \mathbf{p} = \mathbf{F}\Delta t$$

$$\Delta p_y = m\left(v_{fy} - v_{iy}\right) = m(v\cos 60.0°) - mv\cos 60.0° = 0$$

$$\begin{aligned}\Delta p_x &= m(-v\sin 60.0° - v\sin 60.0°) = -2mv\sin 60.0°\\ &= -2(3.00\text{ kg})(10.0\text{ m/s})(0.866)\\ &= -52.0\text{ kg}\cdot\text{m/s}\end{aligned}$$

$$F_{\text{ave}} = \frac{\Delta p_x}{\Delta t} = \frac{-52.0\text{ kg}\cdot\text{m/s}}{0.200\text{ s}} = \boxed{-260\text{ N}}$$

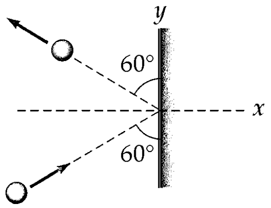

**FIG. P9.9**

**P9.10** Assume the initial direction of the ball in the $-x$ direction.

(a) Impulse, $\mathbf{I} = \Delta\mathbf{p} = \mathbf{p}_f - \mathbf{p}_i = (0.060\,0)(40.0)\hat{\mathbf{i}} - (0.060\,0)(50.0)\left(-\hat{\mathbf{i}}\right) = \boxed{5.40\hat{\mathbf{i}}\text{ N}\cdot\text{s}}$

(b) $\text{Work} = K_f - K_i = \dfrac{1}{2}(0.060\,0)\left[(40.0)^2 - (50.0)^2\right] = \boxed{-27.0\text{ J}}$

**P9.11** Take $x$-axis toward the pitcher

(a) $p_{ix} + I_x = p_{fx}$: $\quad (0.200\text{ kg})(15.0\text{ m/s})(-\cos 45.0°) + I_x = (0.200\text{ kg})(40.0\text{ m/s})\cos 30.0°$

$I_x = 9.05\text{ N}\cdot\text{s}$

$p_{iy} + I_y = p_{fy}$: $\quad (0.200\text{ kg})(15.0\text{ m/s})(-\sin 45.0°) + I_y = (0.200\text{ kg})(40.0\text{ m/s})\sin 30.0°$

$$\mathbf{I} = \boxed{\left(9.05\hat{\mathbf{i}} + 6.12\hat{\mathbf{j}}\right)\text{ N}\cdot\text{s}}$$

(b) $\mathbf{I} = \dfrac{1}{2}(0 + \mathbf{F}_m)(4.00\text{ ms}) + \mathbf{F}_m(20.0\text{ ms}) + \dfrac{1}{2}\mathbf{F}_m(4.00\text{ ms})$

$\mathbf{F}_m \times 24.0 \times 10^{-3}\text{ s} = \left(9.05\hat{\mathbf{i}} + 6.12\hat{\mathbf{j}}\right)\text{ N}\cdot\text{s}$

$\mathbf{F}_m = \boxed{\left(377\hat{\mathbf{i}} + 255\hat{\mathbf{j}}\right)\text{ N}}$

**P9.12** If the diver starts from rest and drops vertically into the water, the velocity just before impact is found from

$$K_f + U_{gf} = K_i + U_{gi}$$

$$\frac{1}{2}mv_{\text{impact}}^2 + 0 = 0 + mgh \Rightarrow v_{\text{impact}} = \sqrt{2gh}$$

With the diver at rest after an impact time of $\Delta t$, the average force during impact is given by

$$\overline{F} = \frac{m\left(0 - v_{\text{impact}}\right)}{\Delta t} = \frac{-m\sqrt{2gh}}{\Delta t} \text{ or } \overline{F} = \frac{m\sqrt{2gh}}{\Delta t} \text{ (directed upward).}$$

Assuming a mass of 55 kg and an impact time of $\approx 1.0$ s, the magnitude of this average force is

$$\left|\overline{F}\right| = \frac{(55\text{ kg})\sqrt{2\left(9.8\text{ m/s}^2\right)(10\text{ m})}}{1.0\text{ s}} = 770\text{ N, or } \boxed{\sim 10^3\text{ N}}.$$

**P9.13** The force exerted on the water by the hose is

$$F=\frac{\Delta p_{\text{water}}}{\Delta t}=\frac{mv_f-mv_i}{\Delta t}=\frac{(0.600\text{ kg})(25.0\text{ m/s})-0}{1.00\text{ s}}=\boxed{15.0\text{ N}}.$$

According to Newton's third law, the water exerts a force of equal magnitude back on the hose. Thus, the gardener must apply a 15.0 N force (in the direction of the velocity of the exiting water stream) to hold the hose stationary.

***P9.14** (a) Energy is conserved for the spring-mass system:

$$K_i+U_{si}=K_f+U_{sf}:\qquad 0+\frac{1}{2}kx^2=\frac{1}{2}mv^2+0$$

$$v=x\sqrt{\frac{k}{m}}$$

(b) From the equation, a $\boxed{\text{smaller}}$ value of $m$ makes $v=x\sqrt{\frac{k}{m}}$ larger.

(c) $I=\left|\mathbf{p}_f-\mathbf{p}_i\right|=mv_f=0=mx\sqrt{\frac{k}{m}}=x\sqrt{km}$

(d) From the equation, a $\boxed{\text{larger}}$ value of $m$ makes $I=x\sqrt{km}$ larger.

(e) For the glider, $W=K_f-K_i=\frac{1}{2}mv^2-0=\frac{1}{2}kx^2$

The mass makes $\boxed{\text{no difference}}$ to the work.

## Section 9.3 **Collisions in One Dimension**

**P9.15** $(200\text{ g})(55.0\text{ m/s})=(46.0\text{ g})v+(200\text{ g})(40.0\text{ m/s})$

$v=\boxed{65.2\text{ m/s}}$

***P9.16** $(m_1v_1+m_2v_2)_i=(m_1v_1+m_2v_2)_f$

$22.5\text{ g}(35\text{ m/s})+300\text{ g}(-2.5\text{ m/s})=22.5\text{ g}v_{1f}+0$

$$v_{1f}=\frac{37.5\text{ g}\cdot\text{m/s}}{22.5\text{ g}}=\boxed{1.67\text{ m/s}}$$

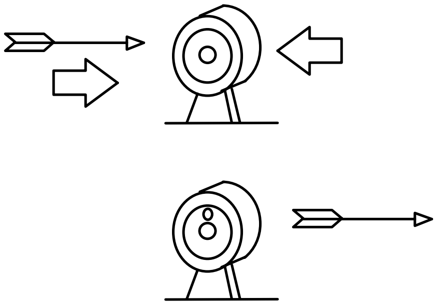

FIG. P9.16

**P9.17** Momentum is conserved

$$\left(10.0\times10^{-3}\ \text{kg}\right)v=(5.01\ \text{kg})(0.600\ \text{m/s})$$

$$v=\boxed{301\ \text{m/s}}$$

**P9.18** (a) $mv_{1i}+3mv_{2i}=4mv_f$ where $m=2.50\times10^4$ kg

$$v_f=\frac{4.00+3(2.00)}{4}=\boxed{2.50\ \text{m/s}}$$

(b) $K_f-K_i=\frac{1}{2}(4m)v_f^2-\left[\frac{1}{2}mv_{1i}^2+\frac{1}{2}(3m)v_{2i}^2\right]=\left(2.50\times10^4\right)(12.5-8.00-6.00)=\boxed{-3.75\times10^4\ \text{J}}$

**P9.19** (a) The internal forces exerted by the actor do not change the total momentum of the system of the four cars and the movie actor

$$(4m)v_i=(3m)(2.00\ \text{m/s})+m(4.00\ \text{m/s})$$

$$v_i=\frac{6.00\ \text{m/s}+4.00\ \text{m/s}}{4}=\boxed{2.50\ \text{m/s}}$$

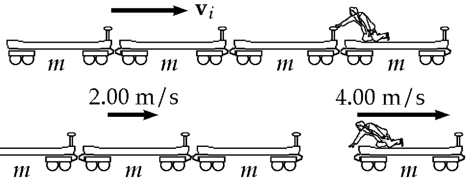

FIG. P9.19

(b) $W_{\text{actor}}=K_f-K_i=\frac{1}{2}\left[(3m)(2.00\ \text{m/s})^2+m(4.00\ \text{m/s})^2\right]-\frac{1}{2}(4\ \text{m})(2.50\ \text{m/s})^2$

$$W_{\text{actor}}=\frac{\left(2.50\times10^4\ \text{kg}\right)}{2}(12.0+16.0-25.0)(\text{m/s})^2=\boxed{37.5\ \text{kJ}}$$

(c) The event considered here is the time reversal of the perfectly inelastic collision in the previous problem. The same momentum conservation equation describes both processes.

**P9.20** $v_1$, speed of $m_1$ at B before collision.

$$\frac{1}{2}m_1v_1^2=m_1gh$$

$$v_1=\sqrt{2(9.80)(5.00)}=9.90\ \text{m/s}$$

$v_{1f}$, speed of $m_1$ at B just after collision.

$$v_{1f}=\frac{m_1-m_2}{m_1+m_2}v_1=-\frac{1}{3}(9.90)\ \text{m/s}=-3.30\ \text{m/s}$$

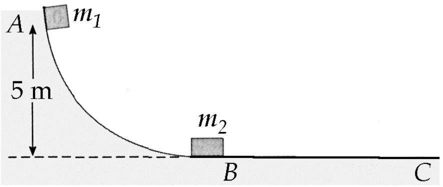

FIG. P9.20

At the highest point (after collision)

$$m_1gh_{\max}=\frac{1}{2}m_1(-3.30)^2 \qquad h_{\max}=\frac{(-3.30\ \text{m/s})^2}{2\left(9.80\ \text{m/s}^2\right)}=\boxed{0.556\ \text{m}}$$

**P9.21** (a), (b) Let $v_g$ and $v_p$ be the velocity of the girl and the plank relative to the ice surface. Then we may say that $v_g - v_p$ is the velocity of the girl relative to the plank, so that

$$v_g - v_p = 1.50 \qquad (1)$$

But also we must have $m_g v_g + m_p v_p = 0$, since total momentum of the girl-plank system is zero relative to the ice surface. Therefore

$$45.0 v_g + 150 v_p = 0\text{, or } v_g = -3.33 v_p$$

Putting this into the equation (1) above gives

$$-3.33 v_p - v_p = 1.50 \text{ or } v_p = \boxed{-0.346 \text{ m/s}}$$

Then $v_g = -3.33(-0.346) = \boxed{1.15 \text{ m/s}}$

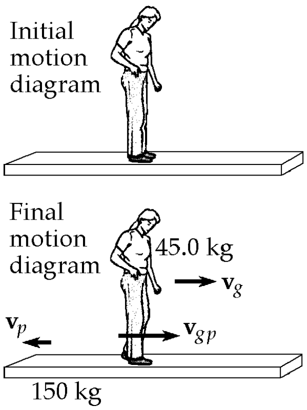

**FIG. P9.21**

***P9.22** For the car-truck-driver-driver system, momentum is conserved:

$\mathbf{p}_{1i} + \mathbf{p}_{2i} = \mathbf{p}_{1f} + \mathbf{p}_{2f}$: $\quad (4\,000 \text{ kg})(8 \text{ m/s})\hat{\mathbf{i}} + (800 \text{ kg})(8 \text{ m/s})(-\hat{\mathbf{i}}) = (4\,800 \text{ kg}) v_f \hat{\mathbf{i}}$

$$v_f = \frac{25\,600 \text{ kg}\cdot\text{m/s}}{4\,800 \text{ kg}} = 5.33 \text{ m/s}$$

For the driver of the truck, the impulse-momentum theorem is

$\mathbf{F}\Delta t = \mathbf{p}_f - \mathbf{p}_i$: $\quad \mathbf{F}(0.120 \text{ s}) = (80 \text{ kg})(5.33 \text{ m/s})\hat{\mathbf{i}} - (80 \text{ kg})(8 \text{ m/s})\hat{\mathbf{i}}$

$$\mathbf{F} = \boxed{1.78 \times 10^3 \text{ N}(-\hat{\mathbf{i}}) \text{ on the truck driver}}$$

For the driver of the car, $\quad \mathbf{F}(0.120 \text{ s}) = (80 \text{ kg})(5.33 \text{ m/s})\hat{\mathbf{i}} - (80 \text{ kg})(8 \text{ m/s})(-\hat{\mathbf{i}})$

$$\mathbf{F} = \boxed{8.89 \times 10^3 \text{ N}\hat{\mathbf{i}} \text{ on the car driver}}\text{, 5 times larger.}$$

**P9.23** (a) According to the Example in the chapter text, the fraction of total kinetic energy transferred to the moderator is

$$f_2 = \frac{4m_1 m_2}{(m_1 + m_2)^2}$$

where $m_2$ is the moderator nucleus and in this case, $m_2 = 12m_1$

$$f_2 = \frac{4m_1(12m_1)}{(13m_1)^2} = \frac{48}{169} = \boxed{0.284 \text{ or } 28.4\%}$$

of the neutron energy is transferred to the carbon nucleus.

(b) $K_C = (0.284)(1.6 \times 10^{-13} \text{ J}) = \boxed{4.54 \times 10^{-14} \text{ J}}$

$K_n = (0.716)(1.6 \times 10^{-13} \text{ J}) = \boxed{1.15 \times 10^{-13} \text{ J}}$

**P9.24** Energy is conserved for the bob-Earth system between bottom and top of swing. At the top the stiff rod is in compression and the bob nearly at rest.

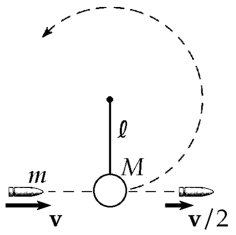

**FIG. P9.24**

$$K_i + U_i = K_f + U_f: \qquad \frac{1}{2}Mv_b^2 + 0 = 0 + Mg2\ell$$

$$v_b^2 = g4\ell \text{ so } v_b = 2\sqrt{g\ell}$$

Momentum of the bob-bullet system is conserved in the collision:

$$mv = m\frac{v}{2} + M\left(2\sqrt{g\ell}\right) \qquad \boxed{v = \frac{4M}{m}\sqrt{g\ell}}$$

**P9.25** At impact, momentum of the clay-block system is conserved, so:

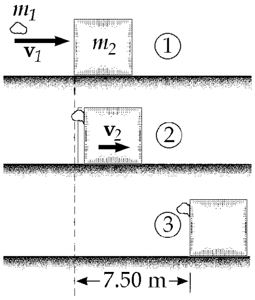

**FIG. P9.25**

$$mv_1 = (m_1 + m_2)v_2$$

After impact, the change in kinetic energy of the clay-block-surface system is equal to the increase in internal energy:

$$\frac{1}{2}(m_1 + m_2)v_2^2 = f_f d = \mu(m_1 + m_2)gd$$

$$\frac{1}{2}(0.112 \text{ kg})v_2^2 = 0.650(0.112 \text{ kg})\left(9.80 \text{ m/s}^2\right)(7.50 \text{ m})$$

$$v_2^2 = 95.6 \text{ m}^2/\text{s}^2 \qquad v_2 = 9.77 \text{ m/s}$$

$$\left(12.0\times10^{-3} \text{ kg}\right)v_1 = (0.112 \text{ kg})(9.77 \text{ m/s}) \qquad v_1 = \boxed{91.2 \text{ m/s}}$$

**P9.26** We assume equal firing speeds $v$ and equal forces $F$ required for the two bullets to push wood fibers apart. These equal forces act backward on the two bullets.

For the first, $K_i + \Delta E_{\text{mech}} = K_f$ $\qquad \frac{1}{2}\left(7.00\times10^{-3} \text{ kg}\right)v^2 - F\left(8.00\times10^{-2} \text{ m}\right) = 0$

For the second, $p_i = p_f$ $\qquad \left(7.00\times10^{-3} \text{ kg}\right)v = (1.014 \text{ kg})v_f$

$$v_f = \frac{\left(7.00\times10^{-3}\right)v}{1.014}$$

Again, $K_i + \Delta E_{\text{mech}} = K_f$: $\qquad \frac{1}{2}\left(7.00\times10^{-3} \text{ kg}\right)v^2 - Fd = \frac{1}{2}(1.014 \text{ kg})v_f^2$

Substituting for $v_f$, $\qquad \frac{1}{2}\left(7.00\times10^{-3} \text{ kg}\right)v^2 - Fd = \frac{1}{2}(1.014 \text{ kg})\left(\frac{7.00\times10^{-3}v}{1.014}\right)^2$

$$Fd = \frac{1}{2}\left(7.00\times10^{-3}\right)v^2 - \frac{1}{2}\frac{\left(7.00\times10^{-3}\right)^2}{1.014}v^2$$

Substituting for $v$, $\qquad Fd = F\left(8.00\times10^{-2} \text{ m}\right)\left(1 - \frac{7.00\times10^{-3}}{1.014}\right) \qquad d = \boxed{7.94 \text{ cm}}$

***P9.27** (a) Using conservation of momentum, $\left(\sum \mathbf{p}\right)_{\text{after}} = \left(\sum \mathbf{p}\right)_{\text{before}}$, gives

$$\left[(4.0+10+3.0)\text{ kg}\right]v = (4.0\text{ kg})(5.0\text{ m/s}) + (10\text{ kg})(3.0\text{ m/s}) + (3.0\text{ kg})(-4.0\text{ m/s}).$$

Therefore, $v = +2.24\text{ m/s}$, or $\boxed{2.24\text{ m/s toward the right}}$.

(b) $\boxed{\text{No}}$. For example, if the 10-kg and 3.0-kg mass were to stick together first, they would move with a speed given by solving

$$(13\text{ kg})v_1 = (10\text{ kg})(3.0\text{ m/s}) + (3.0\text{ kg})(-4.0\text{ m/s}),\text{ or } v_1 = +1.38\text{ m/s}.$$

Then when this 13 kg combined mass collides with the 4.0 kg mass, we have

$$(17\text{ kg})v = (13\text{ kg})(1.38\text{ m/s}) + (4.0\text{ kg})(5.0\text{ m/s}),\text{ and } v = +2.24\text{ m/s}$$

just as in part (a). Coupling order makes no difference.

## Section 9.4 Two-Dimensional Collisions

**P9.28** (a) First, we conserve momentum for the system of two football players in the *x* direction (the direction of travel of the fullback).

$$(90.0\text{ kg})(5.00\text{ m/s}) + 0 = (185\text{ kg})V\cos\theta$$

where $\theta$ is the angle between the direction of the final velocity $V$ and the *x* axis. We find

$$V\cos\theta = 2.43\text{ m/s} \qquad (1)$$

Now consider conservation of momentum of the system in the *y* direction (the direction of travel of the opponent).

$$(95.0\text{ kg})(3.00\text{ m/s}) + 0 = (185\text{ kg})(V\sin\theta)$$

which gives, $V\sin\theta = 1.54\text{ m/s}$ (2)

Divide equation (2) by (1) $\tan\theta = \dfrac{1.54}{2.43} = 0.633$

From which $\boxed{\theta = 32.3°}$

Then, either (1) or (2) gives $V = \boxed{2.88\text{ m/s}}$

(b) $K_i = \frac{1}{2}(90.0\text{ kg})(5.00\text{ m/s})^2 + \frac{1}{2}(95.0\text{ kg})(3.00\text{ m/s})^2 = 1.55\times10^3\text{ J}$

$K_f = \frac{1}{2}(185\text{ kg})(2.88\text{ m/s})^2 = 7.67\times10^2\text{ J}$

Thus, the kinetic energy lost is $\boxed{\text{783 J into internal energy.}}$

**P9.29**

$$p_{xf} = p_{xi}$$

$$mv_{\mathrm{O}} \cos 37.0° + mv_{\mathrm{Y}} \cos 53.0° = m(5.00 \text{ m/s})$$

$$0.799v_{\mathrm{O}} + 0.602v_{\mathrm{Y}} = 5.00 \text{ m/s} \qquad (1)$$

$$p_{yf} = p_{yi}$$

$$mv_{\mathrm{O}} \sin 37.0° - mv_{\mathrm{Y}} \sin 53.0° = 0$$

$$0.602v_{\mathrm{O}} = 0.799v_{\mathrm{Y}} \qquad (2)$$

Solving (1) and (2) simultaneously,

$$\boxed{v_{\mathrm{O}} = 3.99 \text{ m/s}} \text{ and } \boxed{v_{\mathrm{Y}} = 3.01 \text{ m/s}}.$$

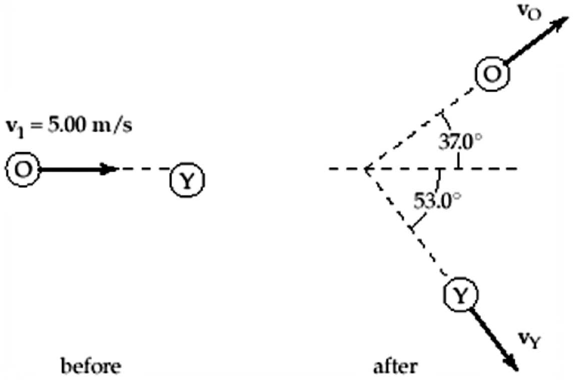

FIG. P9.29

**P9.30**

$$p_{xf} = p_{xi}: \qquad mv_{\mathrm{O}} \cos\theta + mv_{\mathrm{Y}} \cos(90.0° - \theta) = mv_i$$

$$v_{\mathrm{O}} \cos\theta + v_{\mathrm{Y}} \sin\theta = v_i \qquad (1)$$

$$p_{yf} = p_{yi}: \qquad mv_{\mathrm{O}} \sin\theta - mv_{\mathrm{Y}} \sin(90.0° - \theta) = 0$$

$$v_{\mathrm{O}} \sin\theta = v_{\mathrm{Y}} \cos\theta \qquad (2)$$

From equation (2),

$$v_{\mathrm{O}} = v_{\mathrm{Y}} \left( \frac{\cos\theta}{\sin\theta} \right) \qquad (3)$$

Substituting into equation (1),

$$v_{\mathrm{Y}} \left( \frac{\cos^2\theta}{\sin\theta} \right) + v_{\mathrm{Y}} \sin\theta = v_i$$

so $v_{\mathrm{Y}} \left( \cos^2\theta + \sin^2\theta \right) = v_i \sin\theta$, and $\boxed{v_{\mathrm{Y}} = v_i \sin\theta}$.

Then, from equation (3), $\boxed{v_{\mathrm{O}} = v_i \cos\theta}$.

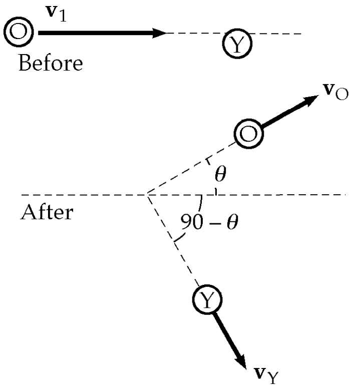

FIG. P9.30

We did not need to write down an equation expressing conservation of mechanical energy. In the problem situation, the requirement of perpendicular final velocities is equivalent to the condition of elasticity.

**P9.31** The initial momentum of the system is 0. Thus,

$$(1.20m)v_{Bi} = m(10.0\ \text{m/s})$$

and $v_{Bi} = 8.33\ \text{m/s}$

$$K_i = \frac{1}{2}m(10.0\ \text{m/s})^2 + \frac{1}{2}(1.20m)(8.33\ \text{m/s})^2 = \frac{1}{2}m\left(183\ \text{m}^2/\text{s}^2\right)$$

$$K_f = \frac{1}{2}m(v_G)^2 + \frac{1}{2}(1.20m)(v_B)^2 = \frac{1}{2}\left(\frac{1}{2}m\left(183\ \text{m}^2/\text{s}^2\right)\right)$$

or $v_G^2 + 1.20v_B^2 = 91.7\ \text{m}^2/\text{s}^2$ (1)

From conservation of momentum,

$$mv_G = (1.20m)v_B$$

or $v_G = 1.20v_B$ (2)

Solving (1) and (2) simultaneously, we find

$\boxed{v_G = 7.07\ \text{m/s}}$ (speed of green puck after collision)

and $\boxed{v_B = 5.89\ \text{m/s}}$ (speed of blue puck after collision)

**P9.32** We use conservation of momentum for the system of two vehicles for both northward and eastward components.

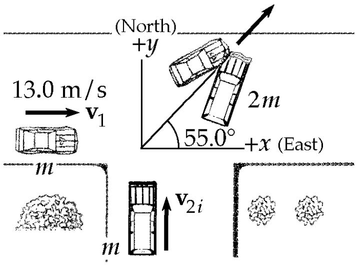

**FIG. P9.32**

For the eastward direction:

$$M(13.0\ \text{m/s}) = 2MV_f \cos 55.0°$$

For the northward direction:

$$Mv_{2i} = 2MV_f \sin 55.0°$$

Divide the northward equation by the eastward equation to find:

$$v_{2i} = (13.0\ \text{m/s})\tan 55.0° = 18.6\ \text{m/s} = \boxed{41.5\ \text{mi/h}}$$

Thus, the driver of the north bound car was untruthful.

**P9.33** By conservation of momentum for the system of the two billiard balls (with all masses equal),

$$5.00\ \text{m/s} + 0 = (4.33\ \text{m/s})\cos 30.0° + v_{2fx}$$
$$v_{2fx} = 1.25\ \text{m/s}$$
$$0 = (4.33\ \text{m/s})\sin 30.0° + v_{2fy}$$
$$v_{2fy} = -2.16\ \text{m/s}$$
$$\mathbf{v}_{2f} = \boxed{2.50\ \text{m/s at } -60.0°}$$

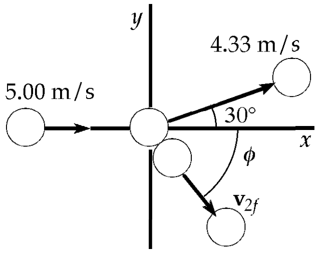

**FIG. P9.33**

Note that we did not need to use the fact that the collision is perfectly elastic.

**P9.34** (a) $\mathbf{p}_i = \mathbf{p}_f$ so $p_{xi} = p_{xf}$

and $p_{yi} = p_{yf}$

$$mv_i = mv\cos\theta + mv\cos\phi \qquad (1)$$
$$0 = mv\sin\theta + mv\sin\phi \qquad (2)$$

From (2), $\sin\theta = -\sin\phi$

so $\theta = -\phi$

Furthermore, energy conservation for the system of two protons requires

$$\frac{1}{2}mv_i^2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

so $\boxed{v = \dfrac{v_i}{\sqrt{2}}}$

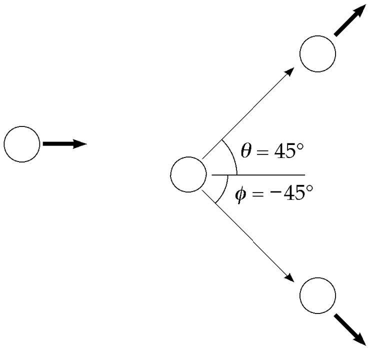

**FIG. P9.34**

(b) Hence, (1) gives $v_i = \dfrac{2v_i\cos\theta}{\sqrt{2}}$ $\qquad \theta = \boxed{45.0°}$ $\qquad \phi = \boxed{-45.0°}$

**P9.35** $m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = (m_1 + m_2)\mathbf{v}_f$: $\quad 3.00(5.00)\hat{\mathbf{i}} - 6.00\hat{\mathbf{j}} = 5.00\mathbf{v}$

$$\mathbf{v} = \boxed{\left(3.00\hat{\mathbf{i}} - 1.20\hat{\mathbf{j}}\right)\ \text{m/s}}$$

**P9.36** $x$-component of momentum for the system of the two objects:

$p_{1ix} + p_{2ix} = p_{1fx} + p_{2fx}$: $\qquad -mv_i + 3mv_i = 0 + 3mv_{2x}$

$y$-component of momentum of the system: $\qquad 0 + 0 = -mv_{1y} + 3mv_{2y}$

by conservation of energy of the system: $\qquad +\frac{1}{2}mv_i^2 + \frac{1}{2}3mv_i^2 = \frac{1}{2}mv_{1y}^2 + \frac{1}{2}3m\left(v_{2x}^2 + v_{2y}^2\right)$

we have $\qquad v_{2x} = \dfrac{2v_i}{3}$

also $\qquad v_{1y} = 3v_{2y}$

So the energy equation becomes $\qquad 4v_i^2 = 9v_{2y}^2 + \dfrac{4v_i^2}{3} + 3v_{2y}^2$

$$\frac{8v_i^2}{3} = 12v_{2y}^2$$

or $\qquad v_{2y} = \dfrac{\sqrt{2}v_i}{3}$

*continued on next page*

(a) The object of mass *m* has final speed $v_{1y} = 3v_{2y} = \boxed{\sqrt{2}v_i}$

and the object of mass 3 *m* moves at $\sqrt{v_{2x}^2 + v_{2y}^2} = \sqrt{\frac{4v_i^2}{9} + \frac{2v_i^2}{9}}$

$$\sqrt{v_{2x}^2 + v_{2y}^2} = \boxed{\sqrt{\frac{2}{3}}v_i}$$

(b) $\theta = \tan^{-1}\left(\frac{v_{2y}}{v_{2x}}\right)$ $\qquad$ $\theta = \tan^{-1}\left(\frac{\sqrt{2}v_i}{3}\frac{3}{2v_i}\right) = \boxed{35.3°}$

**P9.37** $m_0 = 17.0 \times 10^{-27}$ kg $\qquad$ $\mathbf{v}_i = 0$ (the parent nucleus)

$m_1 = 5.00 \times 10^{-27}$ kg $\qquad$ $\mathbf{v}_1 = 6.00 \times 10^6 \hat{\mathbf{j}}$ m/s

$m_2 = 8.40 \times 10^{-27}$ kg $\qquad$ $\mathbf{v}_2 = 4.00 \times 10^6 \hat{\mathbf{i}}$ m/s

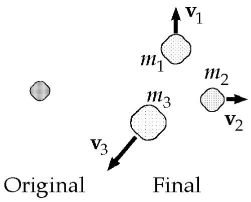

**FIG. P9.37**

(a) $m_1\mathbf{v}_1 + m_2\mathbf{v}_2 + m_3\mathbf{v}_3 = 0$

where $m_3 = m_0 - m_1 - m_2 = 3.60 \times 10^{-27}$ kg

$$\left(5.00 \times 10^{-27}\right)\left(6.00 \times 10^6 \hat{\mathbf{j}}\right) + \left(8.40 \times 10^{-27}\right)\left(4.00 \times 10^6 \hat{\mathbf{i}}\right) + \left(3.60 \times 10^{-27}\right)\mathbf{v}_3 = 0$$

$$\mathbf{v}_3 = \boxed{\left(-9.33 \times 10^6 \hat{\mathbf{i}} - 8.33 \times 10^6 \hat{\mathbf{j}}\right) \text{ m/s}}$$

(b) $E = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{1}{2}m_3v_3^2$

$$E = \frac{1}{2}\left[\left(5.00 \times 10^{-27}\right)\left(6.00 \times 10^6\right)^2 + \left(8.40 \times 10^{-27}\right)\left(4.00 \times 10^6\right)^2 + \left(3.60 \times 10^{-27}\right)\left(12.5 \times 10^6\right)^2\right]$$

$$\boxed{E = 4.39 \times 10^{-13} \text{ J}}$$

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## Section 9.5 **The Center of Mass**

**P9.38** The *x*-coordinate of the center of mass is

$$x_{\text{CM}} = \frac{\sum m_i x_i}{\sum m_i} = \frac{0+0+0+0}{(2.00 \text{ kg} + 3.00 \text{ kg} + 2.50 \text{ kg} + 4.00 \text{ kg})}$$

$$\boxed{x_{\text{CM}} = 0}$$

and the *y*-coordinate of the center of mass is

$$y_{\text{CM}} = \frac{\sum m_i y_i}{\sum m_i} = \frac{(2.00 \text{ kg})(3.00 \text{ m}) + (3.00 \text{ kg})(2.50 \text{ m}) + (2.50 \text{ kg})(0) + (4.00 \text{ kg})(-0.500 \text{ m})}{2.00 \text{ kg} + 3.00 \text{ kg} + 2.50 \text{ kg} + 4.00 \text{ kg}}$$

$$\boxed{y_{\text{CM}} = 1.00 \text{ m}}$$

**P9.39** Take $x$-axis starting from the oxygen nucleus and pointing toward the middle of the **V**.

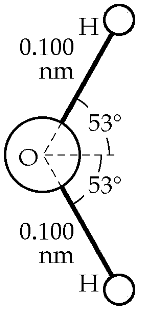

**FIG. P9.39**

Then $\boxed{y_{\text{CM}} = 0}$

and $x_{\text{CM}} = \dfrac{\sum m_i x_i}{\sum m_i} =$

$$x_{\text{CM}} = \frac{0 + 1.008\ \text{u}(0.100\ \text{nm})\cos 53.0° + 1.008\ \text{u}(0.100\ \text{nm})\cos 53.0°}{15.999\ \text{u} + 1.008\ \text{u} + 1.008\ \text{u}}$$

$$\boxed{x_{\text{CM}} = 0.006\,73\ \text{nm from the oxygen nucleus}}$$

***P9.40** Let the $x$ axis start at the Earth's center and point toward the Moon.

$$x_{\text{CM}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{5.98 \times 10^{24}\ \text{kg}\ 0 + 7.36 \times 10^{22}\ \text{kg}\left(3.84 \times 10^{8}\ \text{m}\right)}{6.05 \times 10^{24}\ \text{kg}}$$

$$= \boxed{4.67 \times 10^{6}\ \text{m from the Earth's center}}$$

The center of mass is within the Earth, which has radius $6.37 \times 10^{6}$ m.

**P9.41** Let $A_1$ represent the area of the bottom row of squares, $A_2$ the middle square, and $A_3$ the top pair.

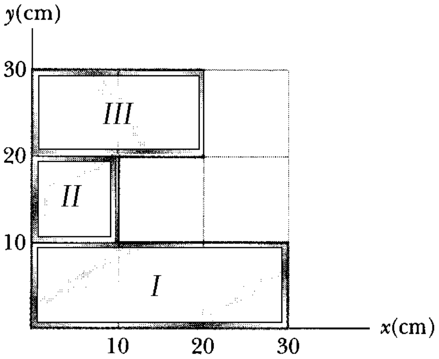

**FIG. P9.41**

$$A = A_1 + A_2 + A_3$$

$$M = M_1 + M_2 + M_3$$

$$\frac{M_1}{A_1} = \frac{M}{A}$$

$$A_1 = 300\ \text{cm}^2,\ A_2 = 100\ \text{cm}^2,\ A_3 = 200\ \text{cm}^2,\ A = 600\ \text{cm}^2$$

$$M_1 = M\left(\frac{A_1}{A}\right) = \frac{300\ \text{cm}^2}{600\ \text{cm}^2} M = \frac{M}{2}$$

$$M_2 = M\left(\frac{A_2}{A}\right) = \frac{100\ \text{cm}^2}{600\ \text{cm}^2} M = \frac{M}{6}$$

$$M_3 = M\left(\frac{A_3}{A}\right) = \frac{200\ \text{cm}^2}{600\ \text{cm}^2} M = \frac{M}{3}$$

$$x_{\text{CM}} = \frac{x_1 M_1 + x_2 M_2 + x_3 M_3}{M} = \frac{15.0\ \text{cm}\left(\frac{1}{2}M\right) + 5.00\ \text{cm}\left(\frac{1}{6}M\right) + 10.0\ \text{cm}\left(\frac{1}{3}M\right)}{M}$$

$$x_{\text{CM}} = \boxed{11.7\ \text{cm}}$$

$$y_{\text{CM}} = \frac{\frac{1}{2}M(5.00\ \text{cm}) + \frac{1}{6}M(15.0\ \text{cm}) + \left(\frac{1}{3}M\right)(25.0\ \text{cm})}{M} = 13.3\ \text{cm}$$

$$y_{\text{CM}} = \boxed{13.3\ \text{cm}}$$

***P9.42** (a) Represent the height of a particle of mass $dm$ within the object as $y$. Its contribution to the gravitational energy of the object-Earth system is $(dm)gy$. The total gravitational energy is $U_g = \int_{\text{all mass}} gy\,dm = g\int y\,dm$. For the center of mass we have $y_{\text{CM}} = \frac{1}{M}\int y\,dm$, so $U_g = gMy_{\text{CM}}$.

(b) The volume of the ramp is $\frac{1}{2}(3.6\text{ m})(15.7\text{ m})(64.8\text{ m}) = 1.83\times10^3\text{ m}^3$. Its mass is $\rho V = \left(3\,800\text{ kg/m}^3\right)\left(1.83\times10^3\text{ m}^3\right) = 6.96\times10^6\text{ kg}$. Its center of mass is above its base by one-third of its height, $y_{\text{CM}} = \frac{1}{3}15.7\text{ m} = 5.23\text{ m}$. Then

$U_g = Mgy_{\text{CM}} = 6.96\times10^6\text{ kg}\left(9.8\text{ m/s}^2\right)5.23\text{ m} = \boxed{3.57\times10^8\text{ J}}$.

**P9.43** (a)

$$M = \int_0^{0.300\text{ m}} \lambda dx = \int_0^{0.300\text{ m}} \left[50.0\text{ g/m} + 20.0x\text{ g/m}^2\right]dx$$

$$M = \left[50.0x\text{ g/m} + 10.0x^2\text{ g/m}^2\right]_0^{0.300\text{ m}} = \boxed{15.9\text{ g}}$$

(b)

$$x_{\text{CM}} = \frac{\int_{\text{all mass}} x\,dm}{M} = \frac{1}{M}\int_0^{0.300\text{ m}} \lambda\,x\,dx = \frac{1}{M}\int_0^{0.300\text{ m}} \left[50.0x\text{ g/m} + 20.0x^2\text{ g/m}^2\right]dx$$

$$x_{\text{CM}} = \frac{1}{15.9\text{ g}}\left[25.0x^2\text{ g/m} + \frac{20x^3\text{ g/m}^2}{3}\right]_0^{0.300\text{ m}} = \boxed{0.153\text{ m}}$$

***P9.44** Take the origin at the center of curvature. We have $L = \frac{1}{4}2\pi r$, $r = \frac{2L}{\pi}$. An incremental bit of the rod at angle $\theta$ from the $x$ axis has mass given by $\frac{dm}{r d\theta} = \frac{M}{L}$, $dm = \frac{Mr}{L}d\theta$ where we have used the definition of radian measure. Now

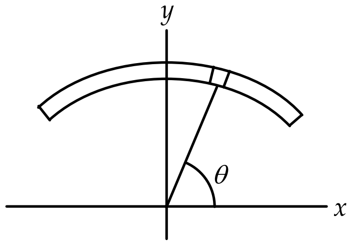

**FIG. P9.44**

$$y_{\text{CM}} = \frac{1}{M}\int_{\text{all mass}} y\,dm = \frac{1}{M}\int_{\theta=45^\circ}^{135^\circ} r\sin\theta\frac{Mr}{L}d\theta = \frac{r^2}{L}\int_{45^\circ}^{135^\circ}\sin\theta\,d\theta$$

$$= \left(\frac{2L}{\pi}\right)^2\frac{1}{L}(-\cos\theta)\Big|_{45^\circ}^{135^\circ} = \frac{4L}{\pi^2}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = \frac{4\sqrt{2}L}{\pi^2}$$

The top of the bar is above the origin by $r = \frac{2L}{\pi}$, so the center of mass is below the middle of the bar by $\frac{2L}{\pi} - \frac{4\sqrt{2}L}{\pi^2} = \frac{2}{\pi}\left(1 - \frac{2\sqrt{2}}{\pi}\right)L = \boxed{0.063\,5L}$.

## Section 9.6 **Motion of a System of Particles**

**P9.45** (a)

$$\mathbf{v}_{\text{CM}}=\frac{\sum m_i\mathbf{v}_i}{M}=\frac{m_1\mathbf{v}_1+m_2\mathbf{v}_2}{M}$$

$$=\frac{(2.00\text{ kg})\left(2.00\hat{\mathbf{i}}\text{ m/s}-3.00\hat{\mathbf{j}}\text{ m/s}\right)+(3.00\text{ kg})\left(1.00\hat{\mathbf{i}}\text{ m/s}+6.00\hat{\mathbf{j}}\text{ m/s}\right)}{5.00\text{ kg}}$$

$$\mathbf{v}_{\text{CM}}=\boxed{\left(1.40\hat{\mathbf{i}}+2.40\hat{\mathbf{j}}\right)\text{ m/s}}$$

(b) $\mathbf{p}=M\mathbf{v}_{\text{CM}}=(5.00\text{ kg})\left(1.40\hat{\mathbf{i}}+2.40\hat{\mathbf{j}}\right)\text{ m/s}=\boxed{\left(7.00\hat{\mathbf{i}}+12.0\hat{\mathbf{j}}\right)\text{ kg}\cdot\text{m/s}}$

**P9.46** (a) See figure to the right.

(b) Using the definition of the position vector at the center of mass,

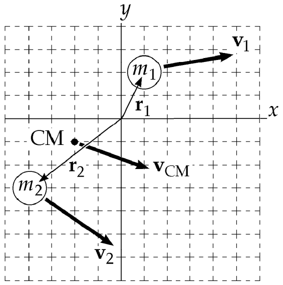

**FIG. P9.46**

$$\mathbf{r}_{\text{CM}}=\frac{m_1\mathbf{r}_1+m_2\mathbf{r}_2}{m_1+m_2}$$

$$\mathbf{r}_{\text{CM}}=\frac{(2.00\text{ kg})(1.00\text{ m},\ 2.00\text{ m})+(3.00\text{ kg})(-4.00\text{ m},\ -3.00\text{ m})}{2.00\text{ kg}+3.00\text{ kg}}$$

$$\mathbf{r}_{\text{CM}}=\boxed{\left(-2.00\hat{\mathbf{i}}-1.00\hat{\mathbf{j}}\right)\text{ m}}$$

(c) The velocity of the center of mass is

$$\mathbf{v}_{\text{CM}}=\frac{\mathbf{P}}{M}=\frac{m_1\mathbf{v}_1+m_2\mathbf{v}_2}{m_1+m_2}=\frac{(2.00\text{ kg})(3.00\text{ m/s},\ 0.50\text{ m/s})+(3.00\text{ kg})(3.00\text{ m/s},\ -2.00\text{ m/s})}{(2.00\text{ kg}+3.00\text{ kg})}$$

$$\mathbf{v}_{\text{CM}}=\boxed{\left(3.00\hat{\mathbf{i}}-1.00\hat{\mathbf{j}}\right)\text{ m/s}}$$

(d) The total linear momentum of the system can be calculated as $\mathbf{P}=M\mathbf{v}_{\text{CM}}$

or as $\mathbf{P}=m_1\mathbf{v}_1+m_2\mathbf{v}_2$

Either gives $\mathbf{P}=\boxed{\left(15.0\hat{\mathbf{i}}-5.00\hat{\mathbf{j}}\right)\text{ kg}\cdot\text{m/s}}$

**P9.47** Let $x=$ distance from shore to center of boat

$\ell=$ length of boat

$x'=$ distance boat moves as Juliet moves toward Romeo

The center of mass stays fixed.

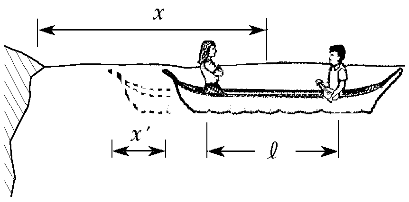

**FIG. P9.47**

Before: $$x_{\text{CM}}=\frac{\left[M_bx+M_J\left(x-\frac{\ell}{2}\right)+M_R\left(x+\frac{\ell}{2}\right)\right]}{\left(M_B+M_J+M_R\right)}$$

After: $$x_{\text{CM}}=\frac{\left[M_B(x-x')+M_J\left(x+\frac{\ell}{2}-x'\right)+M_R\left(x+\frac{\ell}{2}-x'\right)\right]}{\left(M_B+M_J+M_R\right)}$$

$$\ell\left(-\frac{55.0}{2}+\frac{77.0}{2}\right)=x'(-80.0-55.0-77.0)+\frac{\ell}{2}(55.0+77.0)$$

$$x'=\frac{55.0\ell}{212}=\frac{55.0(2.70)}{212}=\boxed{0.700\text{ m}}$$

**P9.48** (a) Conservation of momentum for the two-ball system gives us:

$$0.200\text{ kg}(1.50\text{ m/s})+0.300\text{ kg}(-0.400\text{ m/s})=0.200\text{ kg }v_{1f}+0.300\text{ kg }v_{2f}$$

Relative velocity equation:

$$v_{2f}-v_{1f}=1.90\text{ m/s}$$

Then $0.300-0.120=0.200v_{1f}+0.300\left(1.90+v_{1f}\right)$

$v_{1f}=-0.780\text{ m/s}$ $\quad v_{2f}=1.12\text{ m/s}$

$\boxed{\mathbf{v}_{1f}=-0.780\hat{\mathbf{i}}\text{ m/s}}$ $\quad \boxed{\mathbf{v}_{2f}=1.12\hat{\mathbf{i}}\text{ m/s}}$

(b) Before, $\mathbf{v}_{\text{CM}}=\dfrac{(0.200\text{ kg})(1.50\text{ m/s})\hat{\mathbf{i}}+(0.300\text{ kg})(-0.400\text{ m/s})\hat{\mathbf{i}}}{0.500\text{ kg}}$

$\boxed{\mathbf{v}_{\text{CM}}=(0.360\text{ m/s})\hat{\mathbf{i}}}$

Afterwards, the center of mass must move at the same velocity, as momentum of the system is conserved.

## Section 9.7 **Rocket Propulsion**

**P9.49** (a) Thrust $=\left|v_e\dfrac{dM}{dt}\right|$ $\quad$ Thrust $=\left(2.60\times10^3\text{ m/s}\right)\left(1.50\times10^4\text{ kg/s}\right)=\boxed{3.90\times10^7\text{ N}}$

(b) $\sum F_y=\text{Thrust}-Mg=Ma$: $\quad 3.90\times10^7-\left(3.00\times10^6\right)(9.80)=\left(3.00\times10^6\right)a$

$a=\boxed{3.20\text{ m/s}^2}$

***P9.50** (a) The fear burns at a rate $\quad \dfrac{dM}{dt}=\dfrac{12.7\text{ g}}{1.90\text{ s}}=6.68\times10^{-3}\text{ kg/s}$

Thrust $=v_e\dfrac{dM}{dt}$: $\quad 5.26\text{ N}=v_e\left(6.68\times10^{-3}\text{ kg/s}\right)$

$v_e=\boxed{787\text{ m/s}}$

(b) $v_f-v_i=v_e\ln\left(\dfrac{M_i}{M_f}\right)$: $\quad v_f-0=(797\text{ m/s})\ln\left(\dfrac{53.5\text{ g}+25.5\text{ g}}{53.5\text{ g}+25.5\text{ g}-12.7\text{ g}}\right)$

$v_f=\boxed{138\text{ m/s}}$

**P9.51** $v=v_e\ln\dfrac{M_i}{M_f}$

(a) $M_i=e^{v/v_e}M_f$ $\quad M_i=e^5\left(3.00\times10^3\text{ kg}\right)=4.45\times10^5\text{ kg}$

The mass of fuel and oxidizer is $\quad \Delta M=M_i-M_f=(445-3.00)\times10^3\text{ kg}=\boxed{442\text{ metric tons}}$

(b) $\Delta M=e^2(3.00\text{ metric tons})-3.00\text{ metric tons}=\boxed{19.2\text{ metric tons}}$

Because of the exponential, a relatively small increase in fuel and/or engine efficiency causes a large change in the amount of fuel and oxidizer required.

**P9.52** (a) From Equation 9.41, $v - 0 = v_e \ln\left(\frac{M_i}{M_f}\right) = -v_e \ln\left(\frac{M_f}{M_i}\right)$

Now, $M_f = M_i - kt$, so $v = -v_e \ln\left(\frac{M_i - kt}{M_i}\right) = -v_e \ln\left(1 - \frac{k}{M_i}t\right)$

With the definition, $T_p \equiv \frac{M_i}{k}$, this becomes

$$v(t) = \boxed{-v_e \ln\left(1 - \frac{t}{T_p}\right)}$$

(b) With $v_e = 1\,500$ m/s, and $T_p = 144$ s, $v = -(1\,500 \text{ m/s})\ln\left(1 - \frac{t}{144 \text{ s}}\right)$

| $t(s)$ | $v(\text{m/s})$ |
|---|---|
| 0 | 0 |
| 20 | 224 |
| 40 | 488 |
| 60 | 808 |
| 80 | 1220 |
| 100 | 1780 |
| 120 | 2690 |
| 132 | 3730 |

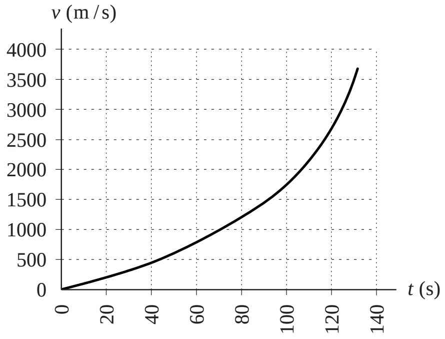

**FIG. P9.52(b)**

(c) $a(t) = \frac{dv}{dt} = \frac{d\left[-v_e \ln\left(1 - \frac{t}{T_p}\right)\right]}{dt} = -v_e\left(\frac{1}{1 - \frac{t}{T_p}}\right)\left(-\frac{1}{T_p}\right) = \left(\frac{v_e}{T_p}\right)\left(\frac{1}{1 - \frac{t}{T_p}}\right)$, or

$$a(t) = \boxed{\frac{v_e}{T_p - t}}$$

(d) With $v_e = 1\,500$ m/s, and $T_p = 144$ s, $a = \frac{1\,500 \text{ m/s}}{144 \text{ s} - t}$

| $t(s)$ | $a(\text{m/s}^2)$ |
|---|---|
| 0 | 10.4 |
| 20 | 12.1 |
| 40 | 14.4 |
| 60 | 17.9 |
| 80 | 23.4 |
| 100 | 34.1 |
| 120 | 62.5 |
| 132 | 125 |

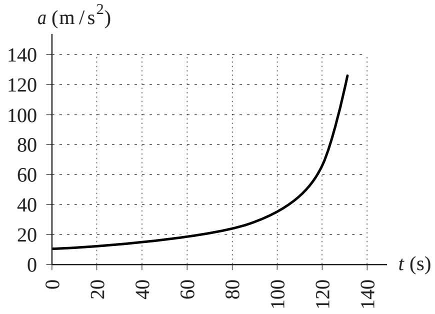

**FIG. P9.52(d)**

*continued on next page*

(e) $$x(t) = 0 + \int_0^t v\,dt = \int_0^t \left[ -v_e \ln\left(1 - \frac{t}{T_p}\right) \right] dt = v_e T_p \int_0^t \ln\left[1 - \frac{t}{T_p}\right]\left(-\frac{dt}{T_p}\right)$$

$$x(t) = v_e T_p \left[ \left(1 - \frac{t}{T_p}\right) \ln\left(1 - \frac{t}{T_p}\right) - \left(1 - \frac{t}{T_p}\right) \right]_0^t$$

$$x(t) = \boxed{v_e\left(T_p - t\right)\ln\left(1 - \frac{t}{T_p}\right) + v_e t}$$

(f) With $v_e = 1\,500\ \text{m/s} = 1.50\ \text{km/s}$, and $T_p = 144\ \text{s}$,

$$x = 1.50(144 - t)\ln\left(1 - \frac{t}{144}\right) + 1.50t$$

| $t$(s) | $x$(km) |
|---|---|
| 0 | 0 |
| 20 | 2.19 |
| 40 | 9.23 |
| 60 | 22.1 |
| 80 | 42.2 |
| 100 | 71.7 |
| 120 | 115 |
| 132 | 153 |

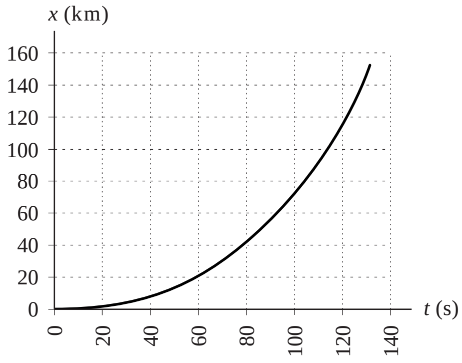

**FIG. P9.52(f)**

***P9.53** The thrust acting on the spacecraft is

$\sum F = ma$: $$\sum F = (3\,500\ \text{kg})\left(2.50 \times 10^{-6}\right)\left(9.80\ \text{m/s}^2\right) = 8.58 \times 10^{-2}\ \text{N}$$

thrust $= \left(\frac{dM}{dt}\right)v_e$: $$8.58 \times 10^{-2}\ \text{N} = \left(\frac{\Delta M}{3\,600\ \text{s}}\right)(70\ \text{m/s})$$

$$\Delta M = \boxed{4.41\ \text{kg}}$$

## Additional Problems

**P9.54** (a) When the spring is fully compressed, each cart moves with same velocity **v**. Apply conservation of momentum for the system of two gliders

$$\mathbf{p}_i = \mathbf{p}_f: \qquad m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = (m_1+m_2)\mathbf{v} \qquad \boxed{\mathbf{v} = \frac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2}{m_1+m_2}}$$

(b) Only conservative forces act, therefore $\Delta E = 0$. $\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}(m_1+m_2)v^2 + \frac{1}{2}kx_m^2$

Substitute for $v$ from (a) and solve for $x_m$.

$$x_m^2 = \frac{(m_1+m_2)m_1v_1^2 + (m_1+m_2)m_2v_2^2 - (m_1v_1)^2 - (m_2v_2)^2 - 2m_1m_2v_1v_2}{k(m_1+m_2)}$$

$$x_m = \sqrt{\frac{m_1m_2\left(v_1^2+v_2^2-2v_1v_2\right)}{k(m_1+m_2)}} = \boxed{(v_1-v_2)\sqrt{\frac{m_1m_2}{k(m_1+m_2)}}}$$

(c) $m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = m_1\mathbf{v}_{1f} + m_2\mathbf{v}_{2f}$

Conservation of momentum: $$m_1\left(\mathbf{v}_1 - \mathbf{v}_{1f}\right) = m_2\left(\mathbf{v}_{2f} - \mathbf{v}_2\right) \tag{1}$$

Conservation of energy: $$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$$

which simplifies to: $$m_1\left(v_1^2 - v_{1f}^2\right) = m_2\left(v_{2f}^2 - v_2^2\right)$$

Factoring gives $$m_1\left(\mathbf{v}_1 - \mathbf{v}_{1f}\right)\cdot\left(\mathbf{v}_1 + \mathbf{v}_{1f}\right) = m_2\left(\mathbf{v}_{2f} - \mathbf{v}_2\right)\cdot\left(\mathbf{v}_{2f} + \mathbf{v}_2\right)$$

and with the use of the momentum equation (equation (1)),

this reduces to $$\left(\mathbf{v}_1 + \mathbf{v}_{1f}\right) = \left(\mathbf{v}_{2f} + \mathbf{v}_2\right)$$

or $$\mathbf{v}_{1f} = \mathbf{v}_{2f} + \mathbf{v}_2 - \mathbf{v}_1 \tag{2}$$

Substituting equation (2) into equation (1) and simplifying yields:

$$\mathbf{v}_{2f} = \boxed{\left(\frac{2m_1}{m_1+m_2}\right)\mathbf{v}_1 + \left(\frac{m_2-m_1}{m_1+m_2}\right)\mathbf{v}_2}$$

Upon substitution of this expression for $\mathbf{v}_{2f}$ into equation 2, one finds

$$\mathbf{v}_{1f} = \boxed{\left(\frac{m_1-m_2}{m_1+m_2}\right)\mathbf{v}_1 + \left(\frac{2m_2}{m_1+m_2}\right)\mathbf{v}_2}$$

Observe that these results are the same as Equations 9.20 and 9.21, which should have been expected since this is a perfectly elastic collision in one dimension.

**P9.55** (a) $(60.0\text{ kg})4.00\text{ m/s} = (120 + 60.0)\text{ kg}v_f$

$$\mathbf{v}_f = \boxed{1.33\text{ m/s}\,\hat{\mathbf{i}}}$$

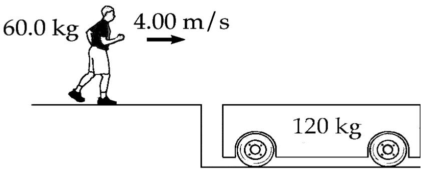

FIG. P9.55

(b) $\sum F_y = 0$: $n - (60.0\text{ kg})9.80\text{ m/s}^2 = 0$

$$f_k = \mu_k n = 0.400(588\text{ N}) = 235\text{ N}$$

$$\mathbf{f}_k = \boxed{-235\text{ N}\hat{\mathbf{i}}}$$

(c) For the person, $p_i + I = p_f$

$$mv_i + Ft = mv_f$$

$$(60.0\text{ kg})4.00\text{ m/s} - (235\text{ N})t = (60.0\text{ kg})1.33\text{ m/s}$$

$$t = \boxed{0.680\text{ s}}$$

(d) person: $m\mathbf{v}_f - m\mathbf{v}_i = 60.0\text{ kg}(1.33 - 4.00)\text{ m/s} = \boxed{-160\text{ N}\cdot\text{s}\hat{\mathbf{i}}}$

cart: $120\text{ kg}(1.33\text{ m/s}) - 0 = \boxed{+160\text{ N}\cdot\text{s}\hat{\mathbf{i}}}$

(e) $x_f - x_i = \frac{1}{2}(v_i + v_f)t = \frac{1}{2}[(4.00 + 1.33)\text{ m/s}]0.680\text{ s} = \boxed{1.81\text{ m}}$

(f) $x_f - x_i = \frac{1}{2}(v_i + v_f)t = \frac{1}{2}(0 + 1.33\text{ m/s})0.680\text{ s} = \boxed{0.454\text{ m}}$

(g) $\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}60.0\text{ kg}(1.33\text{ m/s})^2 - \frac{1}{2}60.0\text{ kg}(4.00\text{ m/s})^2 = \boxed{-427\text{ J}}$

(h) $\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}120.0\text{ kg}(1.33\text{ m/s})^2 - 0 = \boxed{107\text{ J}}$

(i) The force exerted by the person on the cart must equal in magnitude and opposite in direction to the force exerted by the cart on the person. The changes in momentum of the two objects must be equal in magnitude and must add to zero. Their changes in kinetic energy are different in magnitude and do not add to zero. The following represent two ways of thinking about 'why.' The distance the cart moves is different from the distance moved by the point of application of the friction force to the cart. The total change in mechanical energy for both objects together, $-320$ J, becomes $+320$ J of additional internal energy in this perfectly inelastic collision.

**P9.56** The equation for the horizontal range of a projectile is $R = \frac{v_i^2 \sin 2\theta}{g}$. Thus, with $\theta = 45.0°$, the initial velocity is

$$v_i = \sqrt{Rg} = \sqrt{(200\text{ m})(9.80\text{ m/s}^2)} = 44.3\text{ m/s}$$

$$I = \overline{F}(\Delta t) = \Delta p = mv_i - 0$$

Therefore, the magnitude of the average force acting on the ball during the impact is:

$$\overline{F} = \frac{mv_i}{\Delta t} = \frac{(46.0 \times 10^{-3}\text{ kg})(44.3\text{ m/s})}{7.00 \times 10^{-3}\text{ s}} = \boxed{291\text{ N}}.$$

**P9.57** We hope the momentum of the wrench provides enough recoil so that the astronaut can reach the ship before he loses life support! We might expect the elapsed time to be on the order of several minutes based on the description of the situation.

No external force acts on the system (astronaut plus wrench), so the total momentum is constant. Since the final momentum (wrench plus astronaut) must be zero, we have final momentum = initial momentum = 0.

$$m_{\text{wrench}}v_{\text{wrench}} + m_{\text{astronaut}}v_{\text{astronaut}} = 0$$

Thus $v_{\text{astronaut}} = -\frac{m_{\text{wrench}}v_{\text{wrench}}}{m_{\text{astronaut}}} = -\frac{(0.500\text{ kg})(20.0\text{ m/s})}{80.0\text{ kg}} = -0.125\text{ m/s}$

At this speed, the time to travel to the ship is

$$t = \frac{30.0\text{ m}}{0.125\text{ m/s}} = \boxed{240\text{ s}} = 4.00\text{ minutes}$$

The astronaut is fortunate that the wrench gave him sufficient momentum to return to the ship in a reasonable amount of time! In this problem, we were told that the astronaut was not drifting away from the ship when he threw the wrench. However, this is not quite possible since he did not encounter an external force that would reduce his velocity away from the ship (there is no air friction beyond earth's atmosphere). If this were a real-life situation, the astronaut would have to throw the wrench hard enough to overcome his momentum caused by his original push away from the ship.

**P9.58** Using conservation of momentum from just before to just after the impact of the bullet with the block:

$$mv_i = (M+m)v_f$$

or $$v_i = \left(\frac{M+m}{m}\right)v_f \qquad (1)$$

The speed of the block and embedded bullet just after impact may be found using kinematic equations:

$$d = v_f t \text{ and } h = \frac{1}{2}gt^2$$

Thus, $t = \sqrt{\frac{2h}{g}}$ and $v_f = \frac{d}{t} = d\sqrt{\frac{g}{2h}} = \sqrt{\frac{gd^2}{2h}}$

Substituting into (1) from above gives $v_i = \boxed{\left(\frac{M+m}{m}\right)\sqrt{\frac{gd^2}{2h}}}$.

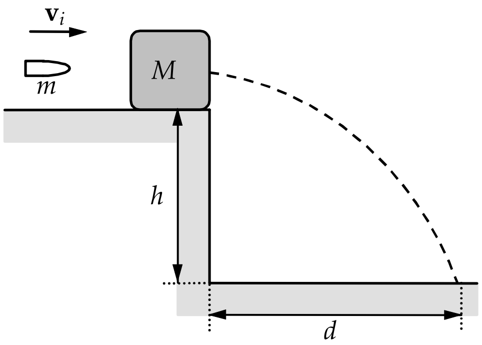

**FIG. P9.58**

***P9.59** (a) Conservation of momentum:

$$0.5\text{ kg}\left(2\hat{\mathbf{i}}-3\hat{\mathbf{j}}+1\hat{\mathbf{k}}\right)\text{ m/s}+1.5\text{ kg}\left(-1\hat{\mathbf{i}}+2\hat{\mathbf{j}}-3\hat{\mathbf{k}}\right)\text{ m/s}$$

$$=0.5\text{ kg}\left(-1\hat{\mathbf{i}}+3\hat{\mathbf{j}}-8\hat{\mathbf{k}}\right)\text{ m/s}+1.5\text{ kg }\mathbf{v}_{2f}$$

$$\mathbf{v}_{2f}=\frac{\left(-0.5\hat{\mathbf{i}}+1.5\hat{\mathbf{j}}-4\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}+\left(0.5\hat{\mathbf{i}}-1.5\hat{\mathbf{j}}+4\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}}{1.5\text{ kg}}=\boxed{0}$$

The original kinetic energy is

$$\frac{1}{2}0.5\text{ kg}\left(2^2+3^2+1^2\right)\text{ m}^2/\text{s}^2+\frac{1}{2}1.5\text{ kg}\left(1^2+2^2+3^2\right)\text{ m}^2/\text{s}^2=14.0\text{ J}$$

The final kinetic energy is $\frac{1}{2}0.5\text{ kg}\left(1^2+3^2+8^2\right)\text{ m}^2/\text{s}^2+0=18.5\text{ J}$ different from the original energy so the collision is $\boxed{\text{inelastic}}$.

(b) We follow the same steps as in part (a):

$$\left(-0.5\hat{\mathbf{i}}+1.5\hat{\mathbf{j}}-4\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}=0.5\text{ kg}\left(-0.25\hat{\mathbf{i}}+0.75\hat{\mathbf{j}}-2\hat{\mathbf{k}}\right)\text{ m/s}+1.5\text{ kg }\mathbf{v}_{2f}$$

$$\mathbf{v}_{2f}=\frac{\left(-0.5\hat{\mathbf{i}}+1.5\hat{\mathbf{j}}-4\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}+\left(0.125\hat{\mathbf{i}}-0.375\hat{\mathbf{j}}+1\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}}{1.5\text{ kg}}$$

$$=\boxed{\left(-0.250\hat{\mathbf{i}}+0.750\hat{\mathbf{j}}-2.00\hat{\mathbf{k}}\right)\text{ m/s}}$$

We see $\mathbf{v}_{2f}=\mathbf{v}_{1f}$, so the collision is $\boxed{\text{perfectly inelastic}}$.

(c) Conservation of momentum:

$$\left(-0.5\hat{\mathbf{i}}+1.5\hat{\mathbf{j}}-4\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}=0.5\text{ kg}\left(-1\hat{\mathbf{i}}+3\hat{\mathbf{j}}+a\hat{\mathbf{k}}\right)\text{ m/s}+1.5\text{ kg }\mathbf{v}_{2f}$$

$$\mathbf{v}_{2f}=\frac{\left(-0.5\hat{\mathbf{i}}+1.5\hat{\mathbf{j}}-4\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}+\left(0.5\hat{\mathbf{i}}-1.5\hat{\mathbf{j}}-0.5a\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}}{1.5\text{ kg}}$$

$$=\boxed{(-2.67-0.333a)\hat{\mathbf{k}}\text{ m/s}}$$

Conservation of energy:

$$14.0\text{ J}=\frac{1}{2}0.5\text{ kg}\left(1^2+3^2+a^2\right)\text{ m}^2/\text{s}^2+\frac{1}{2}1.5\text{ kg}(2.67+0.333a)^2\text{ m}^2/\text{s}^2$$

$$=2.5\text{ J}+0.25a^2+5.33\text{ J}+1.33a+0.083\,3a^2$$

$$0=0.333a^2+1.33a-6.167$$

$$a=\frac{-1.33\pm\sqrt{1.33^2-4(0.333)(-6.167)}}{0.667}$$

$a=2.74$ or $-6.74$. Either value is possible.

$$\therefore\ \boxed{a=2.74},\quad \mathbf{v}_{2f}=\left(-2.67-0.333(2.74)\right)\hat{\mathbf{k}}\text{ m/s}=\boxed{-3.58\hat{\mathbf{k}}\text{ m/s}}$$

$$\therefore\ \boxed{a=-6.74},\quad \mathbf{v}_{2f}=\left(-2.67-0.333(-6.74)\right)\hat{\mathbf{k}}\text{ m/s}=\boxed{-0.419\hat{\mathbf{k}}\text{ m/s}}$$

**P9.60** (a) The initial momentum of the system is zero, which remains constant throughout the motion. Therefore, when $m_1$ leaves the wedge, we must have

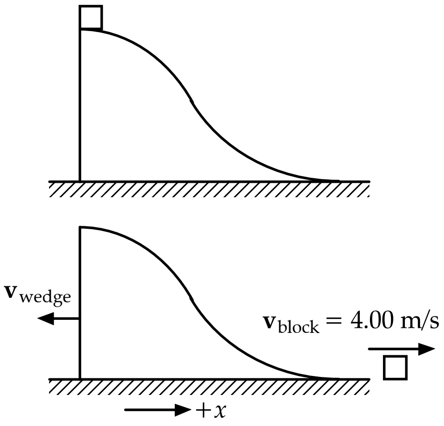
$\mathbf{v}_{\text{wedge}}$
$\mathbf{v}_{\text{block}} = 4.00$ m/s
$+x$

**FIG. P9.60**

$$m_2 v_{\text{wedge}} + m_1 v_{\text{block}} = 0$$

or $$(3.00\text{ kg})v_{\text{wedge}} + (0.500\text{ kg})(+4.00\text{ m/s}) = 0$$

so $$v_{\text{wedge}} = \boxed{-0.667\text{ m/s}}$$

(b) Using conservation of energy for the block-wedge-Earth system as the block slides down the smooth (frictionless) wedge, we have

$$\left[K_{\text{block}} + U_{\text{system}}\right]_i + \left[K_{\text{wedge}}\right]_i = \left[K_{\text{block}} + U_{\text{system}}\right]_f + \left[K_{\text{wedge}}\right]_f$$

or $$\left[0 + m_1 g h\right] + 0 = \left[\frac{1}{2}m_1(4.00)^2 + 0\right] + \frac{1}{2}m_2(-0.667)^2 \text{ which gives } \boxed{h = 0.952\text{ m}}.$$

***P9.61** (a) Conservation of the $x$ component of momentum for the cart-bucket-water system:

$$m v_i + 0 = (m + \rho V)v \qquad \boxed{v_i = \frac{m + \rho V}{m}v}$$

(b) Raindrops with zero $x$-component of momentum stop in the bucket and slow its horizontal motion. When they drip out, they carry with them horizontal momentum. Thus the cart slows with constant acceleration.

**P9.62** Consider the motion of the firefighter during the three intervals:

(1) before, (2) during, and (3) after collision with the platform.

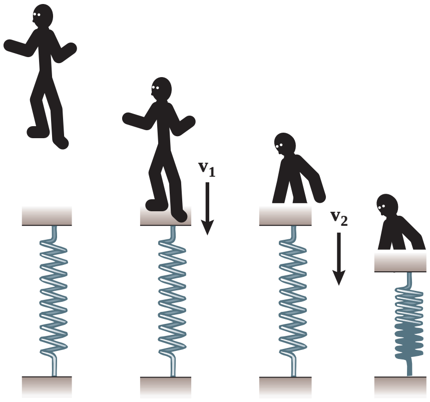

**FIG. P9.62**

(a) While falling a height of 4.00 m, his speed changes from $v_i = 0$ to $v_1$ as found from

$$\Delta E = \left(K_f + U_f\right) - \left(K_i - U_i\right), \text{ or}$$
$$K_f = \Delta E - U_f + K_i + U_i$$

When the initial position of the platform is taken as the zero level of gravitational potential, we have

$$\frac{1}{2}mv_1^2 = fh\cos(180°) - 0 + 0 + mgh$$

Solving for $v_1$ gives

$$v_1 = \sqrt{\frac{2(-fh + mgh)}{m}} = \sqrt{\frac{2(-300(4.00) + 75.0(9.80)4.00)}{75.0}} = \boxed{6.81 \text{ m/s}}$$

(b) During the inelastic collision, momentum is conserved; and if $v_2$ is the speed of the firefighter and platform just after collision, we have $mv_1 = (m+M)v_2$ or

$$v_2 = \frac{m_1 v_1}{m+M} = \frac{75.0(6.81)}{75.0 + 20.0} = 5.38 \text{ m/s}$$

Following the collision and again solving for the work done by non-conservative forces, using the distances as labeled in the figure, we have (with the zero level of gravitational potential at the initial position of the platform):

$$\Delta E = K_f + U_{fg} + U_{fs} - K_i - U_{ig} - U_{is}, \text{ or}$$
$$-fs = 0 + (m+M)g(-s) + \frac{1}{2}ks^2 - \frac{1}{2}(m+M)v^2 - 0 - 0$$

This results in a quadratic equation in $s$:

$$2\,000s^2 - (931)s + 300s - 1\,375 = 0 \text{ or } \boxed{s = 1.00 \text{ m}}$$

***P9.63** (a) Each object swings down according to

$$mgR = \frac{1}{2}mv_1^2 \qquad MgR = \frac{1}{2}Mv_1^2 \qquad v_1 = \sqrt{2gR}$$

The collision: $-mv_1 + Mv_1 = +(m+M)v_2$

$$v_2 = \frac{M-m}{M+m}v_1$$

Swinging up: $\frac{1}{2}(M+m)v_2^2 = (M+m)gR(1-\cos 35°)$

$$v_2 = \sqrt{2gR(1-\cos 35°)}$$

$$\sqrt{2gR(1-\cos 35°)}(M+m) = (M-m)\sqrt{2gR}$$

$$0.425M + 0.425m = M - m$$

$$1.425m = 0.575M$$

$$\boxed{\frac{m}{M} = 0.403}$$

(b) No change is required if the force is different. The nature of the forces within the system of colliding objects does not affect the total momentum of the system. With strong magnetic attraction, the heavier object will be moving somewhat faster and the lighter object faster still. Their extra kinetic energy will all be immediately converted into extra internal energy when the objects latch together. Momentum conservation guarantees that none of the extra kinetic energy remains after the objects join to make them swing higher.

**P9.64** (a) Use conservation of the horizontal component of momentum for the system of the shell, the cannon, and the carriage, from just before to just after the cannon firing.

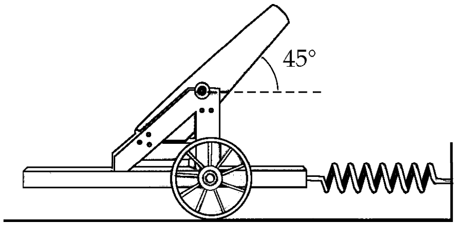

**FIG. P9.64**

$p_{xf} = p_{xi}$: $\quad m_{\text{shell}}v_{\text{shell}}\cos 45.0° + m_{\text{cannon}}v_{\text{recoil}} = 0$

$$(200)(125)\cos 45.0° + (5\,000)v_{\text{recoil}} = 0$$

or $\quad v_{\text{recoil}} = \boxed{-3.54 \text{ m/s}}$

(b) Use conservation of energy for the system of the cannon, the carriage, and the spring from right after the cannon is fired to the instant when the cannon comes to rest.

$K_f + U_{gf} + U_{sf} = K_i + U_{gi} + U_{si}$: $\quad 0 + 0 + \frac{1}{2}kx_{\max}^2 = \frac{1}{2}mv_{\text{recoil}}^2 + 0 + 0$

$$x_{\max} = \sqrt{\frac{mv_{\text{recoil}}^2}{k}} = \sqrt{\frac{(5\,000)(-3.54)^2}{2.00\times 10^4}}\text{ m} = \boxed{1.77 \text{ m}}$$

(c) $|F_{s,\,\max}| = kx_{\max}$ $\qquad |F_{s,\,\max}| = (2.00\times 10^4 \text{ N/m})(1.77 \text{ m}) = \boxed{3.54\times 10^4 \text{ N}}$

(d) No. The rail exerts a vertical external force (the normal force) on the cannon and prevents it from recoiling vertically. Momentum is not conserved in the vertical direction. The spring does not have time to stretch during the cannon firing. Thus, no external horizontal force is exerted on the system (cannon, carriage, and shell) from just before to just after firing. Momentum of this system is conserved in the horizontal direction during this interval.

**P9.65** (a) Utilizing conservation of momentum,

$$m_1 v_{1A} = (m_1 + m_2)v_B$$

$$v_{1A} = \frac{m_1 + m_2}{m_1}\sqrt{2gh}$$

$$v_{1A} \cong \boxed{6.29\ \text{m/s}}$$

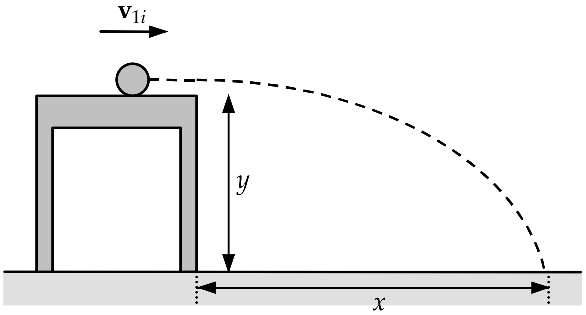

**FIG. P9.65**

(b) Utilizing the two equations,

$$\frac{1}{2}gt^2 = y \text{ and } x = v_{1A}t$$

we combine them to find

$$v_{1A} = \frac{x}{\sqrt{\frac{2y}{g}}}$$

From the data, $v_{1A} = \boxed{6.16\ \text{m/s}}$

Most of the 2% difference between the values for speed is accounted for by the uncertainty in the data, estimated as $\frac{0.01}{8.68} + \frac{0.1}{68.8} + \frac{1}{263} + \frac{1}{257} + \frac{0.1}{85.3} = 1.1\%$.

***P9.66** The ice cubes leave the track with speed determined by $mgy_i = \frac{1}{2}mv^2$;

$$v = \sqrt{2\left(9.8\ \text{m/s}^2\right)1.5\ \text{m}} = 5.42\ \text{m/s}.$$

Its speed at the apex of its trajectory is $5.42\ \text{m/s}\cos 40° = 4.15\ \text{m/s}$. For its collision with the wall we have

$$mv_i + F\Delta t = mv_f$$

$$0.005\ \text{kg}\ 4.15\ \text{m/s} + F\Delta t = 0.005\ \text{kg}\left(-\frac{1}{2}4.15\ \text{m/s}\right)$$

$$F\Delta t = -3.12 \times 10^{-2}\ \text{kg}\cdot\text{m/s}$$

The impulse exerted by the cube on the wall is to the right, $+3.12 \times 10^{-2}\ \text{kg}\cdot\text{m/s}$. Here $F$ could refer to a large force over a short contact time. It can also refer to the average force if we interpret $\Delta t$ as $\frac{1}{10}$ s, the time between one cube's tap and the next's.

$$F_{\text{av}} = \frac{3.12 \times 10^{-2}\ \text{kg}\cdot\text{m/s}}{0.1\ \text{s}} = \boxed{0.312\ \text{N to the right}}$$

**P9.67** (a) Find the speed when the bullet emerges from the block by using momentum conservation:

$$mv_i = MV_i + mv$$

The block moves a distance of 5.00 cm. Assume for an approximation that the block quickly reaches its maximum velocity, $V_i$, and the bullet kept going with a constant velocity, $v$. The block then compresses the spring and stops.

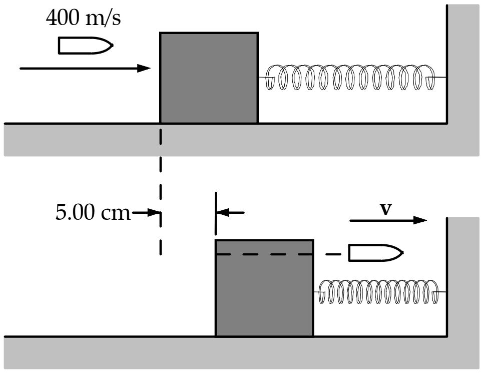

**FIG. P9.67**

$$\frac{1}{2}MV_i^2 = \frac{1}{2}kx^2$$

$$V_i = \sqrt{\frac{(900\ \text{N/m})(5.00\times10^{-2}\ \text{m})^2}{1.00\ \text{kg}}} = 1.50\ \text{m/s}$$

$$v = \frac{mv_i - MV_i}{m} = \frac{(5.00\times10^{-3}\ \text{kg})(400\ \text{m/s}) - (1.00\ \text{kg})(1.50\ \text{m/s})}{5.00\times10^{-3}\ \text{kg}}$$

$$v = \boxed{100\ \text{m/s}}$$

(b) $$\Delta E = \Delta K + \Delta U = \frac{1}{2}(5.00\times10^{-3}\ \text{kg})(100\ \text{m/s})^2 - \frac{1}{2}(5.00\times10^{-3}\ \text{kg})(400\ \text{m/s})^2 + \frac{1}{2}(900\ \text{N/m})(5.00\times10^{-2}\ \text{m})^2$$

$\Delta E = -374$ J, or there is an energy loss of $\boxed{374\ \text{J}}$.

***P9.68** The orbital speed of the Earth is

$$v_{\text{E}} = \frac{2\pi r}{T} = \frac{2\pi 1.496\times10^{11}\ \text{m}}{3.156\times10^{7}\ \text{s}} = 2.98\times10^{4}\ \text{m/s}$$

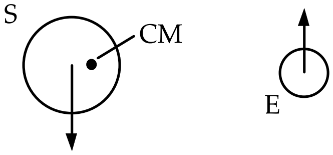

**FIG. P9.68**

In six months the Earth reverses its direction, to undergo momentum change

$$m_{\text{E}}|\Delta \mathbf{v}_{\text{E}}| = 2m_{\text{E}}v_{\text{E}} = 2(5.98\times10^{24}\ \text{kg})(2.98\times10^{4}\ \text{m/s}) = 3.56\times10^{25}\ \text{kg}\cdot\text{m/s}.$$

Relative to the center of mass, the sun always has momentum of the same magnitude in the opposite direction. Its 6-month momentum change is the same size, $m_{\text{S}}|\Delta \mathbf{v}_{\text{S}}| = 3.56\times10^{25}\ \text{kg}\cdot\text{m/s}$.

Then $|\Delta \mathbf{v}_{\text{S}}| = \dfrac{3.56\times10^{25}\ \text{kg}\cdot\text{m/s}}{1.991\times10^{30}\ \text{kg}} = \boxed{0.179\ \text{m/s}}$.

**P9.69** (a) $\mathbf{p}_i + \mathbf{F}t = \mathbf{p}_f$: $(3.00\text{ kg})(7.00\text{ m/s})\hat{\mathbf{j}} + (12.0\text{ N}\hat{\mathbf{i}})(5.00\text{ s}) = (3.00\text{ kg})\mathbf{v}_f$

$$\mathbf{v}_f = \boxed{(20.0\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}})\text{ m/s}}$$

(b) $\mathbf{a} = \dfrac{\mathbf{v}_f - \mathbf{v}_i}{t}$: $\mathbf{a} = \dfrac{(20.0\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}} - 7.00\hat{\mathbf{j}})\text{ m/s}}{5.00\text{ s}} = \boxed{4.00\hat{\mathbf{i}}\text{ m/s}^2}$

(c) $\mathbf{a} = \dfrac{\sum \mathbf{F}}{m}$: $\mathbf{a} = \dfrac{12.0\text{ N}\hat{\mathbf{i}}}{3.00\text{ kg}} = \boxed{4.00\hat{\mathbf{i}}\text{ m/s}^2}$

(d) $\Delta\mathbf{r} = \mathbf{v}_i t + \frac{1}{2}\mathbf{a}t^2$: $\Delta\mathbf{r} = (7.00\text{ m/s}\,\hat{\mathbf{j}})(5.00\text{ s}) + \frac{1}{2}(4.00\text{ m/s}^2\,\hat{\mathbf{i}})(5.00\text{ s})^2$

$$\Delta\mathbf{r} = \boxed{(50.0\hat{\mathbf{i}} + 35.0\hat{\mathbf{j}})\text{ m}}$$

(e) $W = \mathbf{F}\cdot\Delta\mathbf{r}$: $W = (12.0\text{ N}\hat{\mathbf{i}})\cdot(50.0\text{ m}\hat{\mathbf{i}} + 35.0\text{ m}\hat{\mathbf{j}}) = \boxed{600\text{ J}}$

(f) $\frac{1}{2}mv_f^2 = \frac{1}{2}(3.00\text{ kg})(20.0\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}})\cdot(20.0\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}})\text{ m}^2/\text{s}^2$

$$\frac{1}{2}mv_f^2 = (1.50\text{ kg})(449\text{ m}^2/\text{s}^2) = \boxed{674\text{ J}}$$

(g) $\frac{1}{2}mv_i^2 + W = \frac{1}{2}(3.00\text{ kg})(7.00\text{ m/s})^2 + 600\text{ J} = \boxed{674\text{ J}}$

**P9.70** We find the mass from $M = 360\text{ kg} - (2.50\text{ kg/s})t$.

We find the acceleration from $a = \dfrac{\text{Thrust}}{M} = \dfrac{v_e|dM/dt|}{M} = \dfrac{(1\,500\text{ m/s})(2.50\text{ kg/s})}{M} = \dfrac{3\,750\text{ N}}{M}$

We find the velocity and position according to Euler,

from $v_{\text{new}} = v_{\text{old}} + a(\Delta t)$

and $x_{\text{new}} = x_{\text{old}} + v(\Delta t)$

If we take $\Delta t = 0.132\text{ s}$, a portion of the output looks like this:

| Time $t$(s) | Total mass (kg) | Acceleration $a(\text{m/s}^2)$ | Speed, $v$ (m/s) | Position $x$(m) |
|---|---|---|---|---|
| 0.000 | 360.00 | 10.4167 | 0.0000 | 0.0000 |
| 0.132 | 359.67 | 10.4262 | 1.3750 | 0.1815 |
| 0.264 | 359.34 | 10.4358 | 2.7513 | 0.54467 |
| … | | | | |
| 65.868 | 195.330 | 19.1983 | 916.54 | 27191 |
| 66.000 | 195.000 | 19.2308 | 919.08 | 27312 |
| 66.132 | 194.670 | 19.2634 | 921.61 | 27433 |
| … | | | | |
| 131.736 | 30.660 | 122.3092 | 3687.3 | 152382 |
| 131.868 | 30.330 | 123.6400 | 3703.5 | 152871 |
| 132.000 | 30.000 | 125.0000 | 3719.8 | 153362 |

(a) The final speed is $v_f = \boxed{3.7\text{ km/s}}$

(b) The rocket travels $\boxed{153\text{ km}}$

**P9.71** The force exerted by the table is equal to the change in momentum of each of the links in the chain.

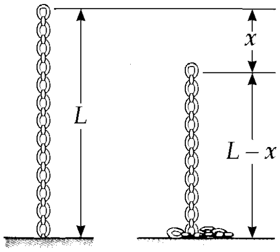

**FIG. P9.71**

By the calculus chain rule of derivatives,

$$F_1 = \frac{dp}{dt} = \frac{d(mv)}{dt} = v\frac{dm}{dt} + m\frac{dv}{dt}.$$

We choose to account for the change in momentum of each link by having it pass from our area of interest just before it hits the table, so that

$$v\frac{dm}{dt} \neq 0 \text{ and } m\frac{dv}{dt} = 0.$$

Since the mass per unit length is uniform, we can express each link of length *dx* as having a mass *dm*:

$$dm = \frac{M}{L}dx.$$

The magnitude of the force on the falling chain is the force that will be necessary to stop each of the elements *dm*.

$$F_1 = v\frac{dm}{dt} = v\left(\frac{M}{L}\right)\frac{dx}{dt} = \left(\frac{M}{L}\right)v^2$$

After falling a distance $x$, the square of the velocity of each link $v^2 = 2gx$ (from kinematics), hence

$$F_1 = \frac{2Mgx}{L}.$$

The links already on the table have a total length $x$, and their weight is supported by a force $F_2$:

$$F_2 = \frac{Mgx}{L}.$$

Hence, the *total* force on the chain is

$$F_{\text{total}} = F_1 + F_2 = \boxed{\frac{3Mgx}{L}}.$$

That is, *the total force is three times the weight of the chain on the table at that instant.*

**P9.72** A picture one second later differs by showing five extra kilograms of sand moving on the belt.

(a) $$\frac{\Delta p_x}{\Delta t}=\frac{(5.00\text{ kg})(0.750\text{ m/s})}{1.00\text{ s}}=\boxed{3.75\text{ N}}$$

(b) The only horizontal force on the sand is belt friction,

so from $p_{xi}+f\Delta t=p_{xf}$ this is $f=\frac{\Delta p_x}{\Delta t}=\boxed{3.75\text{ N}}$

(c) The belt is in equilibrium:

$\sum F_x=ma_x:\quad +F_{\text{ext}}-f=0$ and $F_{\text{ext}}=\boxed{3.75\text{ N}}$

(d) $W=F\Delta r\cos\theta=3.75\text{ N}(0.750\text{ m})\cos 0°=\boxed{2.81\text{ J}}$

(e) $\frac{1}{2}(\Delta m)v^2=\frac{1}{2}5.00\text{ kg}(0.750\text{ m/s})^2=\boxed{1.41\text{ J}}$

(f) $\boxed{\text{Friction between sand and belt converts half of the input work into extra internal energy.}}$

***P9.73** $$x_{\text{CM}}=\frac{\sum m_i x_i}{\sum m_i}=\frac{m_1\left(R+\frac{\ell}{2}\right)+m_2(0)}{m_1+m_2}=\boxed{\frac{m_1\left(R+\frac{\ell}{2}\right)}{m_1+m_2}}$$

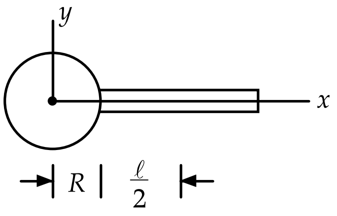

**FIG. P9.73**

## ANSWERS TO EVEN PROBLEMS

| | |
|---|---|
| **P9.2** | (a) 0; (b) 1.06 $\text{kg}\cdot\text{m/s}$; upward |
| **P9.4** | (a) 6.00 m/s to the left; (b) 8.40 J |
| **P9.6** | The force is 6.44 kN |
| **P9.8** | 1.39 $\text{kg}\cdot\text{m/s}$ upward |
| **P9.10** | (a) 5.40 $\text{N}\cdot\text{s}$ toward the net; (b) –27.0 J |
| **P9.12** | $\sim 10^3$ N upward |
| **P9.14** | (a) and (c) see the solution; (b) small; (d) large; (e) no difference |
| **P9.16** | 1.67 m/s |
| **P9.18** | (a) 2.50 m/s; (b) $3.75\times10^4$ J |
| **P9.20** | 0.556 m |
| **P9.22** | 1.78 kN on the truck driver; 8.89 kN in the opposite direction on the car driver |
| **P9.24** | $v=\frac{4M}{m}\sqrt{g\ell}$ |
| **P9.26** | 7.94 cm |
| **P9.28** | (a) 2.88 m/s at 32.3°; (b) 783 J becomes internal energy |
| **P9.30** | $v_Y=v_i\sin\theta;\ v_O=v_i\cos\theta$ |
| **P9.32** | No; his speed was 41.5 mi/h |
| **P9.34** | (a) $v=\frac{v_i}{\sqrt{2}}$; (b) 45.0° and –45.0° |

**P9.36** (a) $\sqrt{2}v_i$; $\sqrt{\frac{2}{3}}v_i$; (b) 35.3°

**P9.38** (0, 1.00 m)

**P9.40** $4.67 \times 10^6$ m from the Earth's center

**P9.42** (a) see the solution; (b) $3.57 \times 10^8$ J

**P9.44** $0.063\,5L$

**P9.46** (a) see the solution;
(b) $(-2.00\text{ m}, -1.00\text{ m})$;
(c) $(3.00\hat{\mathbf{i}} - 1.00\hat{\mathbf{j}})$ m/s;
(d) $(15.0\hat{\mathbf{i}} - 5.00\hat{\mathbf{j}})$ kg·m/s

**P9.48** (a) $-0.780\hat{\mathbf{i}}$ m/s; $1.12\hat{\mathbf{i}}$ m/s; (b) $0.360\hat{\mathbf{i}}$ m/s

**P9.50** (a) 787 m/s; (b) 138 m/s

**P9.52** see the solution

**P9.54** (a) $\frac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2}{m_1 + m_2}$;
(b) $(v_1 - v_2)\sqrt{\frac{m_1 m_2}{k(m_1 + m_2)}}$;
(c) $\mathbf{v}_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\mathbf{v}_1 + \left(\frac{2m_2}{m_1 + m_2}\right)\mathbf{v}_2$;
$\mathbf{v}_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right)\mathbf{v}_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)\mathbf{v}_2$

**P9.56** 291 N

**P9.58** $\left(\frac{M + m}{m}\right)\sqrt{\frac{gd^2}{2h}}$

**P9.60** (a) $-0.667$ m/s; (b) 0.952 m

**P9.62** (a) 6.81 m/s; (b) 1.00 m

**P9.64** (a) $-3.54$ m/s; (b) 1.77 m; (c) 35.4 kN;
(d) No. The rails exert a vertical force to change the momentum

**P9.66** 0.312 N to the right

**P9.68** 0.179 m/s

**P9.70** (a) 3.7 km/s; (b) 153 km

**P9.72** (a) 3.75 N to the right; (b) 3.75 N to the right; (c) 3.75 N; (d) 2.81 J; (e) 1.41 J;
(f) Friction between sand and belt converts half of the input work into extra internal energy.

# 10

# Rotation of a Rigid Object About a Fixed Axis

### CHAPTER OUTLINE

10.1 Angular Position, Velocity, and Acceleration
10.2 Rotational Kinematics: Rotational Motion with Constant Angular Acceleration
10.3 Angular and Linear Quantities
10.4 Rotational Energy
10.5 Calculation of Moments of Inertia
10.6 Torque
10.7 Relationship Between Torque and Angular Acceleration
10.8 Work, Power, and Energy in Rotational Motion
10.9 Rolling Motion of a Rigid Object

## ANSWERS TO QUESTIONS

**Q10.1** 1 rev/min, or $\frac{\pi}{30}$ rad/s. Into the wall (clockwise rotation). $\alpha = 0$.

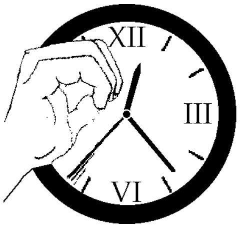

**FIG. Q10.1**

**Q10.2** $+\hat{\mathbf{k}}$, $-\hat{\mathbf{k}}$

**Q10.3** Yes, they are valid provided that $\omega$ is measured in degrees per second and $\alpha$ is measured in degrees per second-squared.

**Q10.4** The speedometer will be inaccurate. The speedometer measures the number of revolutions per second of the tires. A larger tire will travel more distance in one full revolution as $2\pi r$.

**Q10.5** Smallest $I$ is about $x$ axis and largest $I$ is about $y$ axis.

**Q10.6** The moment of inertia would no longer be $\frac{ML^2}{12}$ if the mass was nonuniformly distributed, nor could it be calculated if the mass distribution was not known.

**Q10.7** The object will start to rotate if the two forces act along different lines. Then the torques of the forces will not be equal in magnitude and opposite in direction.

**Q10.8** No horizontal force acts on the pencil, so its center of mass moves straight down.

**Q10.9** You could measure the time that it takes the hanging object, $m$, to fall a measured distance after being released from rest. Using this information, the linear acceleration of the mass can be calculated, and then the torque on the rotating object and its angular acceleration.

**Q10.10** You could use $\omega = \alpha t$ and $v = at$. The equation $v = R\omega$ is valid in this situation since $a = R\alpha$.

**Q10.11** The angular speed $\omega$ would decrease. The center of mass is farther from the pivot, but the moment of inertia increases also.

**Q10.12** The moment of inertia depends on the distribution of mass with respect to a given axis. If the axis is changed, then each bit of mass that makes up the object is a different distance from the axis. In example 10.6 in the text, the moment of inertia of a uniform rigid rod about an axis perpendicular to the rod and passing through the center of mass is derived. If you spin a pencil back and forth about this axis, you will get a feeling for its stubbornness against changing rotation. Now change the axis about which you rotate it by spinning it back and forth about the axis that goes down the middle of the graphite. Easier, isn't it? The moment of inertia about the graphite is much smaller, as the mass of the pencil is concentrated near this axis.

**Q10.13** Compared to an axis through the center of mass, any other parallel axis will have larger average squared distance from the axis to the particles of which the object is composed.

**Q10.14** A quick flip will set the hard–boiled egg spinning faster and more smoothly. The raw egg loses mechanical energy to internal fluid friction.

**Q10.15** $I_{\text{CM}} = MR^2$, $I_{\text{CM}} = MR^2$, $I_{\text{CM}} = \frac{1}{3}MR^2$, $I_{\text{CM}} = \frac{1}{2}MR^2$

**Q10.16** Yes. If you drop an object, it will gain translational kinetic energy from decreasing gravitational potential energy.

**Q10.17** No, just as an object need not be moving to have mass.

**Q10.18** No, only if its angular momentum changes.

**Q10.19** Yes. Consider a pendulum at its greatest excursion from equilibrium. It is momentarily at rest, but must have an angular acceleration or it would not oscillate.

**Q10.20** Since the source reel stops almost instantly when the tape stops playing, the friction on the source reel axle must be fairly large. Since the source reel appears to us to rotate at almost constant angular velocity, the angular acceleration must be very small. Therefore, the torque on the source reel due to the tension in the tape must almost exactly balance the frictional torque. In turn, the frictional torque is nearly constant because kinetic friction forces don't depend on velocity, and the radius of the axle where the friction is applied is constant. Thus we conclude that the torque exerted by the tape on the source reel is essentially constant in time as the tape plays.

As the source reel radius $R$ shrinks, the reel's angular speed $\omega = \frac{v}{R}$ must increase to keep the tape speed $v$ constant. But the biggest change is to the reel's moment of inertia. We model the reel as a roll of tape, ignoring any spool or platter carrying the tape. If we think of the roll of tape as a uniform disk, then its moment of inertia is $I = \frac{1}{2}MR^2$. But the roll's mass is proportional to its base area $\pi R^2$. Thus, on the whole the moment of inertia is proportional to $R^4$. The moment of inertia decreases very rapidly as the reel shrinks!

The tension in the tape coming into the read-and-write heads is normally dominated by balancing frictional torque on the source reel, according to $TR \approx \tau_{\text{friction}}$. Therefore, as the tape plays the tension is largest when the reel is smallest. However, in the case of a sudden jerk on the tape, the rotational dynamics of the source reel becomes important. If the source reel is full, then the moment of inertia, proportional to $R^4$, will be so large that higher tension in the tape will be required to give the source reel its angular acceleration. If the reel is nearly empty, then the same tape acceleration will require a smaller tension. Thus, the tape will be more likely to break when the source reel is nearly full. One sees the same effect in the case of paper towels; it is easier to snap a towel free when the roll is new than when it is nearly empty.

**Q10.21** The moment of inertia would decrease. This would result in a higher angular speed of the earth, shorter days, and more days in the year!

**Q10.22** There is very little resistance to motion that can reduce the kinetic energy of the rolling ball. Even though there is static friction between the ball and the floor (if there were none, then no rotation would occur and the ball would slide), there is no relative motion of the two surfaces—by the definition of "rolling"—and so no force of kinetic friction acts to reduce *K*. Air resistance and friction associated with deformation of the ball eventually stop the ball.

**Q10.23** In the frame of reference of the ground, no. Every point moves perpendicular to the line joining it to the instantaneous contact point. The contact point is not moving at all. The leading and trailing edges of the cylinder have velocities at 45° to the vertical as shown.

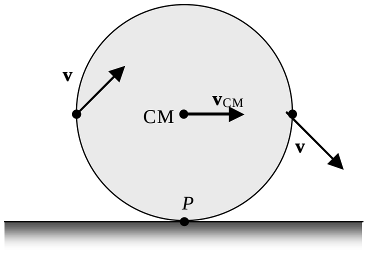

**FIG. Q10.23**

**Q10.24** The sphere would reach the bottom first; the hoop would reach the bottom last. If each object has the same mass and the same radius, they all have the same torque due to gravity acting on them. The one with the smallest moment of inertia will thus have the largest angular acceleration and reach the bottom of the plane first.

**Q10.25** To win the race, you want to decrease the moment of inertia of the wheels as much as possible. Small, light, solid disk-like wheels would be best!

## SOLUTIONS TO PROBLEMS

### Section 10.1 **Angular Position, Velocity, and Acceleration**

**P10.1** (a)

$$\theta\big|_{t=0} = \boxed{5.00\text{ rad}}$$

$$\omega\big|_{t=0} = \left.\frac{d\theta}{dt}\right|_{t=0} = 10.0 + 4.00t\big|_{t=0} = \boxed{10.0\text{ rad/s}}$$

$$\alpha_{t=0} = \left.\frac{d\omega}{dt}\right|_{t=0} = \boxed{4.00\text{ rad/s}^2}$$

(b)

$$\theta\big|_{t=3.00\text{ s}} = 5.00 + 30.0 + 18.0 = \boxed{53.0\text{ rad}}$$

$$\omega\big|_{t=3.00\text{ s}} = \left.\frac{d\theta}{dt}\right|_{t=3.00\text{ s}} = 10.0 + 4.00t\big|_{t=3.00\text{ s}} = \boxed{22.0\text{ rad/s}}$$

$$\alpha\big|_{t=3.00\text{ s}} = \left.\frac{d\omega}{dt}\right|_{t=3.00\text{ s}} = \boxed{4.00\text{ rad/s}^2}$$

## Section 10.2 **Rotational Kinematics: Rotational Motion with Constant Angular Acceleration**

***P10.2** $\omega_f = 2.51 \times 10^4 \text{ rev/min} = 2.63 \times 10^3 \text{ rad/s}$

(a) $$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{2.63 \times 10^3 \text{ rad/s} - 0}{3.2 \text{ s}} = \boxed{8.22 \times 10^2 \text{ rad/s}^2}$$

(b) $$\theta_f = \omega_i t + \frac{1}{2}\alpha t^2 = 0 + \frac{1}{2}\left(8.22 \times 10^2 \text{ rad/s}^2\right)(3.2 \text{ s})^2 = \boxed{4.21 \times 10^3 \text{ rad}}$$

**P10.3** (a) $$\alpha = \frac{\omega - \omega_i}{t} = \frac{12.0 \text{ rad/s}}{3.00 \text{ s}} = \boxed{4.00 \text{ rad/s}^2}$$

(b) $$\theta = \omega_i t + \frac{1}{2}\alpha t^2 = \frac{1}{2}\left(4.00 \text{ rad/s}^2\right)(3.00 \text{ s})^2 = \boxed{18.0 \text{ rad}}$$

**P10.4** $\omega_i = 2\,000 \text{ rad/s}$, $\alpha = -80.0 \text{ rad/s}^2$

(a) $$\omega_f = \omega_i + \alpha t = 2\,000 - (80.0)(10.0) = \boxed{1\,200 \text{ rad/s}}$$

(b) $$0 = \omega_i + \alpha t$$

$$t = \frac{\omega_i}{-\alpha} = \frac{2\,000}{80.0} = \boxed{25.0 \text{ s}}$$

**P10.5** $$\omega_i = \frac{100 \text{ rev}}{1.00 \text{ min}}\left(\frac{1 \text{ min}}{60.0 \text{ s}}\right)\left(\frac{2\pi \text{ rad}}{1.00 \text{ rev}}\right) = \frac{10\pi}{3} \text{ rad/s}, \; \omega_f = 0$$

(a) $$t = \frac{\omega_f - \omega_i}{\alpha} = \frac{0 - \frac{10\pi}{3}}{-2.00} \text{s} = \boxed{5.24 \text{ s}}$$

(b) $$\theta_f = \bar{\omega} t = \left(\frac{\omega_f + \omega_i}{2}\right)t = \left(\frac{10\pi}{6} \text{ rad/s}\right)\left(\frac{10\pi}{6} \text{ s}\right) = \boxed{27.4 \text{ rad}}$$

**P10.6** $\omega_i = 3\,600 \text{ rev/min} = 3.77 \times 10^2 \text{ rad/s}$

$\theta = 50.0 \text{ rev} = 3.14 \times 10^2 \text{ rad}$ and $\omega_f = 0$

$\omega_f^2 = \omega_i^2 + 2\alpha\theta$

$0 = \left(3.77 \times 10^2 \text{ rad/s}\right)^2 + 2\alpha\left(3.14 \times 10^2 \text{ rad}\right)$

$\alpha = \boxed{-2.26 \times 10^2 \text{ rad/s}^2}$

**P10.7** $\omega = 5.00 \text{ rev/s} = 10.0\pi \text{ rad/s}$. We will break the motion into two stages: (1) a period during which the tub speeds up and (2) a period during which it slows down.

While speeding up, $$\theta_1 = \bar{\omega} t = \frac{0 + 10.0\pi \text{ rad/s}}{2}(8.00 \text{ s}) = 40.0\pi \text{ rad}$$

While slowing down, $$\theta_2 = \bar{\omega} t = \frac{10.0\pi \text{ rad/s} + 0}{2}(12.0 \text{ s}) = 60.0\pi \text{ rad}$$

So, $$\theta_{\text{total}} = \theta_1 + \theta_2 = 100\pi \text{ rad} = \boxed{50.0 \text{ rev}}$$

**P10.8** $\theta_f - \theta_i = \omega_i t + \frac{1}{2}\alpha t^2$ and $\omega_f = \omega_i + \alpha t$ are two equations in two unknowns $\omega_i$ and $\alpha$

$\omega_i = \omega_f - \alpha t$: $\quad \theta_f - \theta_i = (\omega_f - \alpha t)t + \frac{1}{2}\alpha t^2 = \omega_f t - \frac{1}{2}\alpha t^2$

$$37.0\text{ rev}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = 98.0\text{ rad/s}(3.00\text{ s}) - \frac{1}{2}\alpha(3.00\text{ s})^2$$

$232\text{ rad} = 294\text{ rad} - (4.50\text{ s}^2)\alpha$: $\quad \alpha = \frac{61.5\text{ rad}}{4.50\text{ s}^2} = \boxed{13.7\text{ rad/s}^2}$

**P10.9** (a) $\omega = \frac{\Delta\theta}{\Delta t} = \frac{1\text{ rev}}{1\text{ day}} = \frac{2\pi\text{ rad}}{86\,400\text{ s}} = \boxed{7.27\times10^{-5}\text{ rad/s}}$

(b) $\Delta t = \frac{\Delta\theta}{\omega} = \frac{107°}{7.27\times10^{-5}\text{ rad/s}}\left(\frac{2\pi\text{ rad}}{360°}\right) = \boxed{2.57\times10^4\text{ s}}$ or 428 min

***P10.10** The location of the dog is described by $\theta_\text{d} = (0.750\text{ rad/s})t$. For the bone,

$$\theta_\text{b} = \frac{1}{3}2\pi\text{ rad} + \frac{1}{2}0.015\text{ rad/s}^2\, t^2.$$

We look for a solution to

$$0.75t = \frac{2\pi}{3} + 0.007\,5t^2$$
$$0 = 0.007\,5t^2 - 0.75t + 2.09 = 0$$
$$t = \frac{0.75 \pm \sqrt{0.75^2 - 4(0.007\,5)2.09}}{0.015} = 2.88\text{ s or }97.1\text{ s}$$

The dog and bone will also pass if $0.75t = \frac{2\pi}{3} - 2\pi + 0.007\,5t^2$ or if $0.75t = \frac{2\pi}{3} + 2\pi + 0.007\,5t^2$ that is, if either the dog or the turntable gains a lap on the other. The first equation has

$$t = \frac{0.75 \pm \sqrt{0.75^2 - 4(0.007\,5)(-4.19)}}{0.015} = 105\text{ s or }-5.30\text{ s}$$

only one positive root representing a physical answer. The second equation has

$$t = \frac{0.75 \pm \sqrt{0.75^2 - 4(0.007\,5)8.38}}{0.015} = 12.8\text{ s or }87.2\text{ s}.$$

In order, the dog passes the bone at $\boxed{2.88\text{ s}}$ after the merry-go-round starts to turn, and again at $\boxed{12.8\text{ s}}$ and 26.6 s, after gaining laps on the bone. The bone passes the dog at 73.4 s, 87.2 s, 97.1 s, 105 s, and so on, after the start.

## Section 10.3 **Angular and Linear Quantities**

**P10.11** Estimate the tire's radius at 0.250 m and miles driven as 10 000 per year.

$$\theta = \frac{s}{r} = \frac{1.00\times 10^4 \text{ mi}}{0.250 \text{ m}}\left(\frac{1\,609 \text{ m}}{1 \text{ mi}}\right) = 6.44\times 10^7 \text{ rad/yr}$$

$$\theta = 6.44\times 10^7 \text{ rad/yr}\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = 1.02\times 10^7 \text{ rev/yr or } \boxed{\sim 10^7 \text{ rev/yr}}$$

**P10.12** (a) $v = r\omega$; $\omega = \frac{v}{r} = \frac{45.0 \text{ m/s}}{250 \text{ m}} = \boxed{0.180 \text{ rad/s}}$

(b) $a_r = \frac{v^2}{r} = \frac{(45.0 \text{ m/s})^2}{250 \text{ m}} = \boxed{8.10 \text{ m/s}^2 \text{ toward the center of track}}$

**P10.13** Given $r = 1.00$ m, $\alpha = 4.00 \text{ rad/s}^2$, $\omega_i = 0$ and $\theta_i = 57.3° = 1.00$ rad

(a) $\omega_f = \omega_i + \alpha t = 0 + \alpha t$

At $t = 2.00$ s, $\omega_f = 4.00 \text{ rad/s}^2 (2.00 \text{ s}) = \boxed{8.00 \text{ rad/s}}$

(b) $v = r\omega = 1.00 \text{ m}(8.00 \text{ rad/s}) = \boxed{8.00 \text{ m/s}}$

$|a_r| = a_c = r\omega^2 = 1.00 \text{ m}(8.00 \text{ rad/s})^2 = 64.0 \text{ m/s}^2$

$a_t = r\alpha = 1.00 \text{ m}\left(4.00 \text{ rad/s}^2\right) = 4.00 \text{ m/s}^2$

The magnitude of the total acceleration is:

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{\left(64.0 \text{ m/s}^2\right)^2 + \left(4.00 \text{ m/s}^2\right)^2} = \boxed{64.1 \text{ m/s}^2}$$

The direction of the total acceleration vector makes an angle $\phi$ with respect to the radius to point $P$:

$$\phi = \tan^{-1}\left(\frac{a_t}{a_c}\right) = \tan^{-1}\left(\frac{4.00}{64.0}\right) = \boxed{3.58°}$$

(c) $\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2 = (1.00 \text{ rad}) + \frac{1}{2}\left(4.00 \text{ rad/s}^2\right)(2.00 \text{ s})^2 = \boxed{9.00 \text{ rad}}$

***P10.14** (a) Consider a tooth on the front sprocket. It gives this speed, relative to the frame, to the link of the chain it engages:

$$v = r\omega = \left(\frac{0.152\ \text{m}}{2}\right)76\ \text{rev/min}\left(\frac{2\pi\,\text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = \boxed{0.605\ \text{m/s}}$$

(b) Consider the chain link engaging a tooth on the rear sprocket:

$$\omega = \frac{v}{r} = \frac{0.605\ \text{m/s}}{\left(\frac{0.07\ \text{m}}{2}\right)} = \boxed{17.3\ \text{rad/s}}$$

(c) Consider the wheel tread and the road. A thread could be unwinding from the tire with this speed relative to the frame:

$$v = r\omega = \left(\frac{0.673\ \text{m}}{2}\right)17.3\ \text{rad/s} = \boxed{5.82\ \text{m/s}}$$

(d) We did not need to know the length of the pedal cranks, but we could use that information to find the linear speed of the pedals:

$$v = r\omega = 0.175\ \text{m}\ 7.96\ \text{rad/s}\left(\frac{1}{1\ \text{rad}}\right) = 1.39\ \text{m/s}$$

**P10.15** (a) $\omega = \frac{v}{r} = \frac{25.0\ \text{m/s}}{1.00\ \text{m}} = \boxed{25.0\ \text{rad/s}}$

(b) $\omega_f^2 = \omega_i^2 + 2\alpha(\Delta\theta)$

$$\alpha = \frac{\omega_f^2 - \omega_i^2}{2(\Delta\theta)} = \frac{(25.0\ \text{rad/s})^2 - 0}{2\left[(1.25\ \text{rev})(2\pi\ \text{rad/rev})\right]} = \boxed{39.8\ \text{rad/s}^2}$$

(c) $\Delta t = \frac{\Delta\omega}{\alpha} = \frac{25.0\ \text{rad/s}}{39.8\ \text{rad/s}^2} = \boxed{0.628\ \text{s}}$

**P10.16** (a) $s = \bar{v}t = (11.0\ \text{m/s})(9.00\ \text{s}) = 99.0\ \text{m}$

$$\theta = \frac{s}{r} = \frac{99.0\ \text{m}}{0.290\ \text{m}} = 341\ \text{rad} = \boxed{54.3\ \text{rev}}$$

(b) $\omega_f = \frac{v_f}{r} = \frac{22.0\ \text{m/s}}{0.290\ \text{m}} = 75.9\ \text{rad/s} = \boxed{12.1\ \text{rev/s}}$

**P10.17** (a) $\omega = 2\pi f = \dfrac{2\pi\,\text{rad}}{1\text{ rev}}\left(\dfrac{1\,200\text{ rev}}{60.0\text{ s}}\right) = \boxed{126\text{ rad/s}}$

(b) $v = \omega r = (126\text{ rad/s})\left(3.00\times 10^{-2}\text{ m}\right) = \boxed{3.77\text{ m/s}}$

(c) $a_c = \omega^2 r = (126)^2\left(8.00\times 10^{-2}\right) = 1\,260\text{ m/s}^2$ so $\mathbf{a}_r = \boxed{1.26\text{ km/s}^2\text{ toward the center}}$

(d) $s = r\theta = \omega r t = (126\text{ rad/s})\left(8.00\times 10^{-2}\text{ m}\right)(2.00\text{ s}) = \boxed{20.1\text{ m}}$

**P10.18** The force of static friction must act forward and then more and more inward on the tires, to produce both tangential and centripetal acceleration. Its tangential component is $m\left(1.70\text{ m/s}^2\right)$. Its radially inward component is $\dfrac{mv^2}{r}$. This takes the maximum value

$$m\omega_f^2 r = mr\left(\omega_i^2 + 2\alpha\Delta\theta\right) = mr\left(0 + 2\alpha\frac{\pi}{2}\right) = m\pi r\alpha = m\pi a_t = m\pi\left(1.70\text{ m/s}^2\right).$$

With skidding impending we have $\sum F_y = ma_y$, $+n - mg = 0$, $n = mg$

$$f_s = \mu_s n = \mu_s mg = \sqrt{m^2\left(1.70\text{ m/s}^2\right)^2 + m^2\pi^2\left(1.70\text{ m/s}^2\right)^2}$$

$$\mu_s = \frac{1.70\text{ m/s}^2}{g}\sqrt{1+\pi^2} = \boxed{0.572}$$

***P10.19** (a) Let $R_E$ represent the radius of the Earth. The base of the building moves east at $v_1 = \omega R_E$ where $\omega$ is one revolution per day. The top of the building moves east at $v_2 = \omega\left(R_E + h\right)$. Its eastward speed relative to the ground is $v_2 - v_1 = \omega h$. The object's time of fall is given by $\Delta y = 0 + \frac{1}{2}gt^2$, $t = \sqrt{\dfrac{2h}{g}}$. During its fall the object's eastward motion is unimpeded so its deflection distance is $\Delta x = \left(v_2 - v_1\right)t = \omega h\sqrt{\dfrac{2h}{g}} = \boxed{\omega h^{3/2}\left(\dfrac{2}{g}\right)^{1/2}}$.

(b) $\dfrac{2\pi\,\text{rad}}{86\,400\text{ s}}(50\text{ m})^{3/2}\left(\dfrac{2\text{ s}^2}{9.8\text{ m}}\right)^{1/2} = \boxed{1.16\text{ cm}}$

(c) The deflection is only 0.02% of the original height, so it is negligible in many practical cases.

## Section 10.4 **Rotational Energy**

**P10.20** $m_1 = 4.00\text{ kg},\ r_1 = |y_1| = 3.00\text{ m};$

$m_2 = 2.00\text{ kg},\ r_2 = |y_2| = 2.00\text{ m};$

$m_3 = 3.00\text{ kg},\ r_3 = |y_3| = 4.00\text{ m};$

$\omega = 2.00\ \text{rad/s about the } x\text{-axis}$

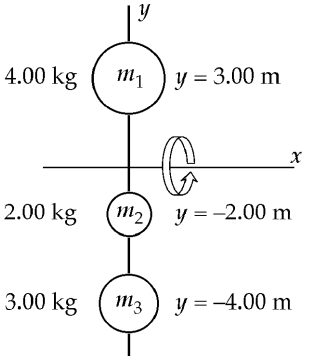

**FIG. P10.20**

(a) $$I_x = m_1r_1^2 + m_2r_2^2 + m_3r_3^2$$

$$I_x = 4.00(3.00)^2 + 2.00(2.00)^2 + 3.00(4.00)^2 = \boxed{92.0\text{ kg}\cdot\text{m}^2}$$

$$K_R = \frac{1}{2}I_x\omega^2 = \frac{1}{2}(92.0)(2.00)^2 = \boxed{184\text{ J}}$$

(b) $$v_1 = r_1\omega = 3.00(2.00) = \boxed{6.00\ \text{m/s}} \qquad K_1 = \frac{1}{2}m_1v_1^2 = \frac{1}{2}(4.00)(6.00)^2 = 72.0\text{ J}$$

$$v_2 = r_2\omega = 2.00(2.00) = \boxed{4.00\ \text{m/s}} \qquad K_2 = \frac{1}{2}m_2v_2^2 = \frac{1}{2}(2.00)(4.00)^2 = 16.0\text{ J}$$

$$v_3 = r_3\omega = 4.00(2.00) = \boxed{8.00\ \text{m/s}} \qquad K_3 = \frac{1}{2}m_3v_3^2 = \frac{1}{2}(3.00)(8.00)^2 = 96.0\text{ J}$$

$$K = K_1 + K_2 + K_3 = 72.0 + 16.0 + 96.0 = \boxed{184\text{ J}} = \frac{1}{2}I_x\omega^2$$

**P10.21** (a) $$I = \sum_j m_jr_j^2$$

In this case,

$$r_1 = r_2 = r_3 = r_4$$

$$r = \sqrt{(3.00\text{ m})^2 + (2.00\text{ m})^2} = \sqrt{13.0}\text{ m}$$

$$I = \left[\sqrt{13.0}\text{ m}\right]^2[3.00 + 2.00 + 2.00 + 4.00]\text{ kg}$$

$$= \boxed{143\text{ kg}\cdot\text{m}^2}$$

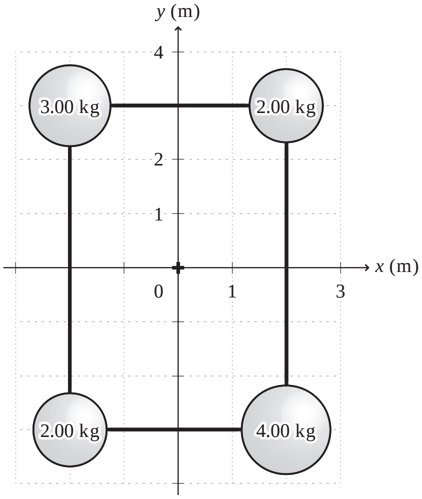

**FIG. P10.21**

(b) $$K_R = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(143\text{ kg}\cdot\text{m}^2\right)\left(6.00\ \text{rad/s}\right)^2$$

$$= \boxed{2.57\times10^3\text{ J}}$$

**P10.22** $I = Mx^2 + m(L-x)^2$

$\dfrac{dI}{dx} = 2Mx - 2m(L-x) = 0$ (for an extremum)

$\therefore x = \dfrac{mL}{M+m}$

$\dfrac{d^2I}{dx^2} = 2m + 2M$; therefore $I$ is minimum when the axis of rotation passes through $x = \dfrac{mL}{M+m}$ which is also the center of mass of the system. The moment of inertia about an axis passing through $x$ is

$$I_{\text{CM}} = M\left[\frac{mL}{M+m}\right]^2 + m\left[1 - \frac{m}{M+m}\right]^2 L^2 = \frac{Mm}{M+m}L^2 = \mu L^2$$

where $\mu = \dfrac{Mm}{M+m}$.

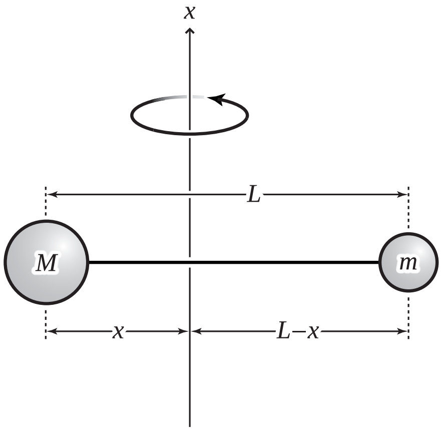

**FIG. P10.22**

## Section 10.5 **Calculation of Moments of Inertia**

**P10.23** We assume the rods are thin, with radius much less than $L$. Call the junction of the rods the origin of coordinates, and the axis of rotation the $z$-axis.

For the rod along the $y$-axis, $I = \dfrac{1}{3}mL^2$ from the table.

For the rod parallel to the $z$-axis, the parallel-axis theorem gives

$$I = \frac{1}{2}mr^2 + m\left(\frac{L}{2}\right)^2 \cong \frac{1}{4}mL^2$$

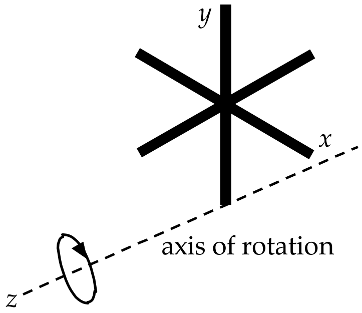

**FIG. P10.23**

In the rod along the $x$-axis, the bit of material between $x$ and $x + dx$ has mass $\left(\dfrac{m}{L}\right)dx$ and is at distance $r = \sqrt{x^2 + \left(\dfrac{L}{2}\right)^2}$ from the axis of rotation. The total rotational inertia is:

$$I_{\text{total}} = \frac{1}{3}mL^2 + \frac{1}{4}mL^2 + \int_{-L/2}^{L/2}\left(x^2 + \frac{L^2}{4}\right)\left(\frac{m}{L}\right)dx$$

$$= \frac{7}{12}mL^2 + \left(\frac{m}{L}\right)\frac{x^3}{3}\bigg|_{-L/2}^{L/2} + \frac{mL}{4}x\bigg|_{-L/2}^{L/2}$$

$$= \frac{7}{12}mL^2 + \frac{mL^2}{12} + \frac{mL^2}{4} = \boxed{\frac{11mL^2}{12}}$$

*Note*: The moment of inertia of the rod along the $x$ axis can also be calculated from the parallel-axis theorem as $\dfrac{1}{12}mL^2 + m\left(\dfrac{L}{2}\right)^2$.

**P10.24** Treat the tire as consisting of three parts. The two sidewalls are each treated as a hollow cylinder of inner radius 16.5 cm, outer radius 30.5 cm, and height 0.635 cm. The tread region is treated as a hollow cylinder of inner radius 30.5 cm, outer radius 33.0 cm, and height 20.0 cm.

Use $I=\frac{1}{2}m\left(R_1^2+R_2^2\right)$ for the moment of inertia of a hollow cylinder.

Sidewall:

$$m=\pi\left[(0.305\text{ m})^2-(0.165\text{ m})^2\right]\left(6.35\times10^{-3}\text{ m}\right)\left(1.10\times10^3\text{ kg/m}^3\right)=1.44\text{ kg}$$

$$I_{\text{side}}=\frac{1}{2}(1.44\text{ kg})\left[(0.165\text{ m})^2+(0.305\text{ m})^2\right]=8.68\times10^{-2}\text{ kg}\cdot\text{m}^2$$

Tread:

$$m=\pi\left[(0.330\text{ m})^2-(0.305\text{ m})^2\right](0.200\text{ m})\left(1.10\times10^3\text{ kg/m}^3\right)=11.0\text{ kg}$$

$$I_{\text{tread}}=\frac{1}{2}(11.0\text{ kg})\left[(0.330\text{ m})^2+(0.305\text{ m})^2\right]=1.11\text{ kg}\cdot\text{m}^2$$

Entire Tire:

$$I_{\text{total}}=2I_{\text{side}}+I_{\text{tread}}=2\left(8.68\times10^{-2}\text{ kg}\cdot\text{m}^2\right)+1.11\text{ kg}\cdot\text{m}^2=\boxed{1.28\text{ kg}\cdot\text{m}^2}$$

**P10.25** Every particle in the door could be slid straight down into a high-density rod across its bottom, without changing the particle's distance from the rotation axis of the door. Thus, a rod 0.870 m long with mass 23.0 kg, pivoted about one end, has the same rotational inertia as the door:

$$I=\frac{1}{3}ML^2=\frac{1}{3}(23.0\text{ kg})(0.870\text{ m})^2=\boxed{5.80\text{ kg}\cdot\text{m}^2}.$$

The $\boxed{\text{height of the door is unnecessary}}$ data.

**P10.26** Model your body as a cylinder of mass 60.0 kg and circumference 75.0 cm. Then its radius is

$$\frac{0.750\text{ m}}{2\pi}=0.120\text{ m}$$

and its moment of inertia is

$$\frac{1}{2}MR^2=\frac{1}{2}(60.0\text{ kg})(0.120\text{ m})^2=0.432\text{ kg}\cdot\text{m}^2\sim\boxed{10^0\text{ kg}\cdot\text{m}^2=1\text{ kg}\cdot\text{m}^2}.$$

**P10.27** For a spherical shell $dI = \frac{2}{3}dmr^2 = \frac{2}{3}\left[\left(4\pi r^2 dr\right)\rho\right]r^2$

$$I = \int dI = \int \frac{2}{3}\left(4\pi r^2\right)r^2\rho(r)dr$$

$$I = \int_0^R \frac{2}{3}\left(4\pi r^4\right)\left(14.2 - 11.6\frac{r}{R}\right)\left(10^3 \text{ kg/m}^3\right)dr$$

$$= \left(\frac{2}{3}\right)4\pi\left(14.2\times 10^3\right)\frac{R^5}{5} - \left(\frac{2}{3}\right)4\pi\left(11.6\times 10^3\right)\frac{R^5}{6}$$

$$I = \frac{8\pi}{3}\left(10^3\right)R^5\left(\frac{14.2}{5} - \frac{11.6}{6}\right)$$

$$M = \int dm = \int_0^R 4\pi r^2\left(14.2 - 11.6\frac{r}{R}\right)10^3\,dr$$

$$= 4\pi\times 10^3\left(\frac{14.2}{3} - \frac{11.6}{4}\right)R^3$$

$$\frac{I}{MR^2} = \frac{(8\pi/3)\left(10^3\right)R^5(14.2/5 - 11.6/6)}{4\pi\times 10^3 R^3R^2(14.2/3 - 11.6/4)} = \frac{2}{3}\left(\frac{.907}{1.83}\right) = 0.330$$

$$\therefore I = \boxed{0.330MR^2}$$

***P10.28** (a) By similar triangles, $\frac{y}{x} = \frac{h}{L}$, $y = \frac{hx}{L}$. The area of the front face is $\frac{1}{2}hL$. The volume of the plate is $\frac{1}{2}hLw$. Its density is $\rho = \frac{M}{V} = \frac{M}{\frac{1}{2}hLw} = \frac{2M}{hLw}$. The mass of the ribbon is

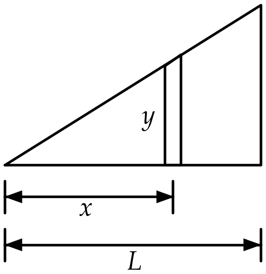

**FIG. P10.28**

$$dm = \rho dV = \rho ywdx = \frac{2Mywdx}{hLw} = \frac{2Mhx}{hLL}dx = \frac{2Mxdx}{L^2}.$$

The moment of inertia is

$$I = \int_{\text{all mass}} r^2dm = \int_{x=0}^{L} x^2\frac{2Mxdx}{L^2} = \frac{2M}{L^2}\int_0^L x^3dx = \frac{2M}{L^2}\frac{L^4}{4} = \boxed{\frac{ML^2}{2}}.$$

(b) From the parallel axis theorem $I = I_{\text{CM}} + M\left(\frac{2L}{3}\right)^2 = I_{\text{CM}} + \frac{4ML^2}{9}$ and $I_h = I_{\text{CM}} + M\left(\frac{L}{3}\right)^2 = I_{\text{CM}} + \frac{ML^2}{9}$. The two triangles constitute a rectangle with moment of inertia $I_{\text{CM}} + \frac{4ML^2}{9} + I_{\text{CM}} + \frac{ML^2}{9} = \frac{1}{3}(2M)L^2$. Then $2I_{\text{CM}} = \frac{1}{9}ML^2$

$$I = I_{\text{CM}} + \frac{4ML^2}{9} = \frac{1}{18}ML^2 + \frac{8}{18}ML^2 = \boxed{\frac{1}{2}ML^2}.$$

***P10.29** We consider the cam as the superposition of the original solid disk and a disk of negative mass cut from it. With half the radius, the cut-away part has one-quarter the face area and one-quarter the volume and one-quarter the mass $M_0$ of the original solid cylinder:

$$M_0 - \frac{1}{4}M_0 = M \qquad M_0 = \frac{4}{3}M.$$

By the parallel-axis theorem, the original cylinder had moment of inertia

$$I_{\text{CM}} + M_0\left(\frac{R}{2}\right)^2 = \frac{1}{2}M_0R^2 + M_0\frac{R^2}{4} = \frac{3}{4}M_0R^2.$$

The negative-mass portion has $I = \frac{1}{2}\left(-\frac{1}{4}M_0\right)\left(\frac{R}{2}\right)^2 = -\frac{M_0R^2}{32}$. The whole cam has

$$I = \frac{3}{4}M_0R^2 - \frac{M_0R^2}{32} = \frac{23}{32}M_0R^2 = \frac{23}{32}\frac{4}{3}MR^2 = \frac{23}{24}MR^2 \text{ and } K = \frac{1}{2}I\omega^2 = \frac{1}{2}\frac{23}{24}MR^2\omega^2 = \boxed{\frac{23}{48}MR^2\omega^2}.$$

## Section 10.6 **Torque**

**P10.30** Resolve the 100 N force into components perpendicular to and parallel to the rod, as

$$F_{\text{par}} = (100\text{ N})\cos 57.0° = 54.5\text{ N}$$

and $F_{\text{perp}} = (100\text{ N})\sin 57.0° = 83.9\text{ N}$

The torque of $F_{\text{par}}$ is zero since its line of action passes through the pivot point.

The torque of $F_{\text{perp}}$ is $\tau = 83.9\text{ N}(2.00\text{ m}) = \boxed{168\text{ N}\cdot\text{m}}$ (clockwise)

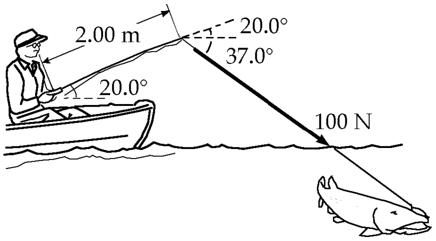

**FIG. P10.30**

**P10.31** $\sum\tau = 0.100\text{ m}(12.0\text{ N}) - 0.250\text{ m}(9.00\text{ N}) - 0.250\text{ m}(10.0\text{ N}) = \boxed{-3.55\text{ N}\cdot\text{m}}$

The thirty-degree angle is unnecessary information.

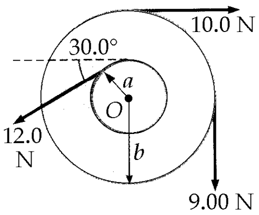

**FIG. P10.31**

**P10.32** The normal force exerted by the ground on each wheel is

$$n = \frac{mg}{4} = \frac{(1\,500\text{ kg})(9.80\text{ m/s}^2)}{4} = 3\,680\text{ N}$$

The torque of friction can be as large as

$$\tau_{\max} = f_{\max}r = (\mu_s n)r = (0.800)(3\,680\text{ N})(0.300\text{ m}) = \boxed{882\text{ N}\cdot\text{m}}$$

The torque of the axle on the wheel can be equally as large as the light wheel starts to turn without slipping.

**P10.33** In the previous problem we calculated the maximum torque that can be applied without skidding to be 882 N · m. This same torque is to be applied by the frictional force, *f*, between the brake pad and the rotor for this wheel. Since the wheel is slipping against the brake pad, we use the coefficient of kinetic friction to calculate the normal force.

$$\tau = fr = (\mu_k n)r\text{, so } n = \frac{\tau}{\mu_k r} = \frac{882\text{ N}\cdot\text{m}}{(0.500)(0.220\text{ m})} = 8.02\times10^3\text{ N} = \boxed{8.02\text{ kN}}$$

## Section 10.7 **Relationship Between Torque and Angular Acceleration**

**P10.34** (a)

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(2.00\text{ kg})(7.00\times10^{-2}\text{ m})^2 = 4.90\times10^{-3}\text{ kg}\cdot\text{m}^2$$

$$\alpha = \frac{\tau}{I} = \frac{0.600}{4.90\times10^{-3}} = 122\text{ rad/s}^2$$

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

$$\Delta t = \frac{\Delta\omega}{\alpha} = \frac{1\,200\left(\frac{2\pi}{60}\right)}{122} = \boxed{1.03\text{ s}}$$

(b)

$$\Delta\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}(122\text{ rad/s})(1.03\text{ s})^2 = 64.7\text{ rad} = \boxed{10.3\text{ rev}}$$

**P10.35** $m = 0.750\text{ kg}$, $F = 0.800\text{ N}$

(a) $\tau = rF = 30.0\text{ m}(0.800\text{ N}) = \boxed{24.0\text{ N}\cdot\text{m}}$

(b) $\alpha = \frac{\tau}{I} = \frac{rF}{mr^2} = \frac{24.0}{0.750(30.0)^2} = \boxed{0.035\,6\text{ rad/s}^2}$

(c) $a_t = \alpha r = 0.035\,6(30.0) = \boxed{1.07\text{ m/s}^2}$

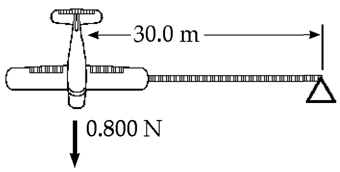

**FIG. P10.35**

**P10.36** $\omega_f = \omega_i + \alpha t$: $10.0\text{ rad/s} = 0 + \alpha(6.00\text{ s})$

$$\alpha = \frac{10.00}{6.00}\text{ rad/s}^2 = 1.67\text{ rad/s}^2$$

(a) $\sum\tau = 36.0\text{ N}\cdot\text{m} = I\alpha$: $I = \frac{\sum\tau}{\alpha} = \frac{36.0\text{ N}\cdot\text{m}}{1.67\text{ rad/s}^2} = \boxed{21.6\text{ kg}\cdot\text{m}^2}$

(b) $\omega_f = \omega_i + \alpha t$: $0 = 10.0 + \alpha(60.0)$

$$\alpha = -0.167\text{ rad/s}^2$$

$$|\tau| = |I\alpha| = (21.6\text{ kg}\cdot\text{m}^2)(0.167\text{ rad/s}^2) = \boxed{3.60\text{ N}\cdot\text{m}}$$

(c) Number of revolutions $\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$

During first 6.00 s $\theta_f = \frac{1}{2}(1.67)(6.00)^2 = 30.1\text{ rad}$

During next 60.0 s $\theta_f = 10.0(60.0) - \frac{1}{2}(0.167)(60.0)^2 = 299\text{ rad}$

$$\theta_{\text{total}} = 329\text{ rad}\left(\frac{1\text{ rev}}{2\pi\,\text{rad}}\right) = \boxed{52.4\text{ rev}}$$

**P10.37** For $m_1$,

$\sum F_y = ma_y$: $\quad +n - m_1 g = 0$

$n_1 = m_1 g = 19.6\text{ N}$

$f_{k1} = \mu_k n_1 = 7.06\text{ N}$

$\sum F_x = ma_x$: $\quad -7.06\text{ N} + T_1 = (2.00\text{ kg})a \qquad (1)$

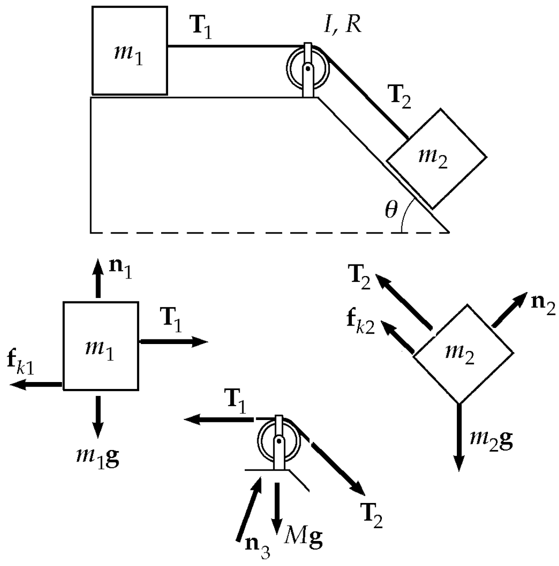

FIG. P10.37

For the pulley,

$\sum \tau = I\alpha$: $\quad -T_1 R + T_2 R = \frac{1}{2}MR^2\left(\frac{a}{R}\right)$

$-T_1 + T_2 = \frac{1}{2}(10.0\text{ kg})a$

$-T_1 + T_2 = (5.00\text{ kg})a \qquad (2)$

For $m_2$, $\quad +n_2 - m_2 g\cos\theta = 0$

$n_2 = 6.00\text{ kg}\left(9.80\text{ m/s}^2\right)(\cos 30.0°)$

$= 50.9\text{ N}$

$f_{k2} = \mu_k n_2$

$= 18.3\text{ N}$: $\quad -18.3\text{ N} - T_2 + m_2\sin\theta = m_2 a$

$-18.3\text{ N} - T_2 + 29.4\text{ N} = (6.00\text{ kg})a \qquad (3)$

(a) Add equations (1), (2), and (3):

$$-7.06\text{ N} - 18.3\text{ N} + 29.4\text{ N} = (13.0\text{ kg})a$$

$$a = \frac{4.01\text{ N}}{13.0\text{ kg}} = \boxed{0.309\text{ m/s}^2}$$

(b) $T_1 = 2.00\text{ kg}\left(0.309\text{ m/s}^2\right) + 7.06\text{ N} = \boxed{7.67\text{ N}}$

$T_2 = 7.67\text{ N} + 5.00\text{ kg}\left(0.309\text{ m/s}^2\right) = \boxed{9.22\text{ N}}$

**P10.38** $I = \frac{1}{2}mR^2 = \frac{1}{2}(100\text{ kg})(0.500\text{ m})^2 = 12.5\text{ kg}\cdot\text{m}^2$

$\omega_i = 50.0\text{ rev/min} = 5.24\text{ rad/s}$

$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{0 - 5.24\text{ rad/s}}{6.00\text{ s}} = -0.873\text{ rad/s}^2$

$\tau = I\alpha = 12.5\text{ kg}\cdot\text{m}^2\left(-0.873\text{ rad/s}^2\right) = -10.9\text{ N}\cdot\text{m}$

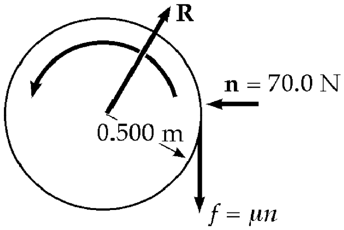

FIG. P10.38

The magnitude of the torque is given by $fR = 10.9\text{ N}\cdot\text{m}$, where $f$ is the force of friction.

Therefore, $\quad f = \frac{10.9\text{ N}\cdot\text{m}}{0.500\text{ m}} \quad$ and $\quad f = \mu_k n$

yields $\quad \mu_k = \frac{f}{n} = \frac{21.8\text{ N}}{70.0\text{ N}} = \boxed{0.312}$

***P10.39** $\sum \tau = I\alpha = \frac{1}{2}MR^2\alpha$

$$-135\text{ N}(0.230\text{ m}) + T(0.230\text{ m}) = \frac{1}{2}(80\text{ kg})\left(\frac{1.25}{2}\text{ m}\right)^2\left(-1.67\text{ rad/s}^2\right)$$

$T = \boxed{21.5\text{ N}}$

---

## Section 10.8 **Work, Power, and Energy in Rotational Motion**

**P10.40** The moment of inertia of a thin rod about an axis through one end is $I = \frac{1}{3}ML^2$. The total rotational kinetic energy is given as

$$K_R = \frac{1}{2}I_h\omega_h^2 + \frac{1}{2}I_m\omega_m^2$$

with $I_h = \frac{m_h L_h^2}{3} = \frac{60.0\text{ kg}(2.70\text{ m})^2}{3} = 146\text{ kg}\cdot\text{m}^2$

and $I_m = \frac{m_m L_m^2}{3} = \frac{100\text{ kg}(4.50\text{ m})^2}{3} = 675\text{ kg}\cdot\text{m}^2$

In addition, $\omega_h = \frac{2\pi\text{ rad}}{12\text{ h}}\left(\frac{1\text{ h}}{3\,600\text{ s}}\right) = 1.45\times10^{-4}\text{ rad/s}$

while $\omega_m = \frac{2\pi\text{ rad}}{1\text{ h}}\left(\frac{1\text{ h}}{3\,600\text{ s}}\right) = 1.75\times10^{-3}\text{ rad/s}$

Therefore, $K_R = \frac{1}{2}(146)\left(1.45\times10^{-4}\right)^2 + \frac{1}{2}(675)\left(1.75\times10^{-3}\right)^2 = \boxed{1.04\times10^{-3}\text{ J}}$

***P10.41** The power output of the bus is $\mathscr{P} = \frac{E}{\Delta t}$ where $E = \frac{1}{2}I\omega^2 = \frac{1}{2}\frac{1}{2}MR^2\omega^2$ is the stored energy and $\Delta t = \frac{\Delta x}{v}$ is the time it can roll. Then $\frac{1}{4}MR^2\omega^2 = \mathscr{P}\Delta t = \frac{\mathscr{P}\Delta x}{v}$ and

$$\Delta x = \frac{MR^2\omega^2 v}{4\mathscr{P}} = \frac{1\,600\text{ kg}(0.65\text{ m})^2\left(4\,000\cdot\frac{2\pi}{60\text{ s}}\right)^2 11.1\text{ m/s}}{4(18\cdot746\text{ W})} = \boxed{24.5\text{ km}}.$$

**P10.42** Work done $= F\Delta r = (5.57\text{ N})(0.800\text{ m}) = 4.46\text{ J}$

and Work $= \Delta K = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2$

(The last term is zero because the top starts from rest.)

Thus, $4.46\text{ J} = \frac{1}{2}\left(4.00\times10^{-4}\text{ kg}\cdot\text{m}^2\right)\omega_f^2$

and from this, $\omega_f = \boxed{149\text{ rad/s}}$.

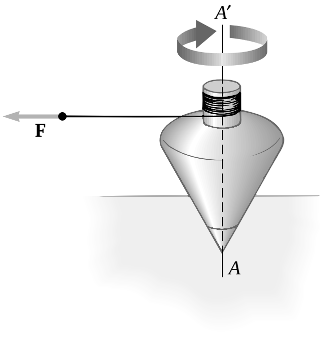

FIG. P10.42

***P10.43** (a) $I = \frac{1}{2}M\left(R_1^2 + R_2^2\right) = \frac{1}{2}(0.35\text{ kg})\left[(0.02\text{ m})^2 + (0.03\text{ m})^2\right] = 2.28 \times 10^{-4}\text{ kg}\cdot\text{m}^2$

$$\left(K_1 + K_2 + K_{\text{rot}} + U_{g2}\right)_i - f_k\Delta x = \left(K_1 + K_2 + K_{\text{rot}}\right)_f$$

$$\frac{1}{2}(0.850\text{ kg})(0.82\text{ m/s})^2 + \frac{1}{2}(0.42\text{ kg})(0.82\text{ m/s})^2 + \frac{1}{2}\left(2.28\times10^{-4}\text{ kg}\cdot\text{m}^2\right)\left(\frac{0.82\text{ m/s}}{0.03\text{ m}}\right)^2$$

$$+0.42\text{ kg}\left(9.8\text{ m/s}^2\right)(0.7\text{ m}) - 0.25(0.85\text{ kg})\left(9.8\text{ m/s}^2\right)(0.7\text{ m})$$

$$= \frac{1}{2}(0.85\text{ kg})v_f^2 + \frac{1}{2}(0.42\text{ kg})v_f^2 + \frac{1}{2}\left(2.28\times10^{-4}\text{ kg}\cdot\text{m}^2\right)\left(\frac{v_f}{0.03\text{ m}}\right)^2$$

$$0.512\text{ J} + 2.88\text{ J} - 1.46\text{ J} = (0.761\text{ kg})v_f^2$$

$$v_f = \sqrt{\frac{1.94\text{ J}}{0.761\text{ kg}}} = \boxed{1.59\text{ m/s}}$$

(b) $\omega = \frac{v}{r} = \frac{1.59\text{ m/s}}{0.03\text{ m}} = \boxed{53.1\text{ rad/s}}$

**P10.44** We assume the rod is thin. For the compound object

$$I = \frac{1}{3}M_{\text{rod}}L^2 + \left[\frac{2}{5}m_{\text{ball}}R^2 + M_{\text{ball}}D^2\right]$$

$$I = \frac{1}{3}1.20\text{ kg}(0.240\text{ m})^2 + \frac{2}{5}2.00\text{ kg}\left(4.00\times10^{-2}\text{ m}\right)^2 + 2.00\text{ kg}(0.280\text{ m})^2$$

$$I = 0.181\text{ kg}\cdot\text{m}^2$$

(a) $K_f + U_f = K_i + U_i + \Delta E$

$$\frac{1}{2}I\omega^2 + 0 = 0 + M_{\text{rod}}g\left(\frac{L}{2}\right) + M_{\text{ball}}g(L+R) + 0$$

$$\frac{1}{2}\left(0.181\text{ kg}\cdot\text{m}^2\right)\omega^2 = 1.20\text{ kg}\left(9.80\text{ m/s}^2\right)(0.120\text{ m}) + 2.00\text{ kg}\left(9.80\text{ m/s}^2\right)(0.280\text{ m})$$

$$\frac{1}{2}\left(0.181\text{ kg}\cdot\text{m}^2\right)\omega^2 = \boxed{6.90\text{ J}}$$

(b) $\omega = \boxed{8.73\text{ rad/s}}$

(c) $v = r\omega = (0.280\text{ m})8.73\text{ rad/s} = \boxed{2.44\text{ m/s}}$

(d) $v_f^2 = v_i^2 + 2a\left(y_f - y_i\right)$

$$v_f = \sqrt{0 + 2\left(9.80\text{ m/s}^2\right)(0.280\text{ m})} = 2.34\text{ m/s}$$

The speed it attains in swinging is greater by $\frac{2.44}{2.34} = \boxed{1.043\ 2\text{ times}}$

**P10.45** (a) For the counterweight,

$\sum F_y = ma_y$ becomes: $$50.0 - T = \left(\frac{50.0}{9.80}\right)a$$

For the reel $\sum \tau = I\alpha$ reads $TR = I\alpha = I\dfrac{a}{R}$

where $$I = \frac{1}{2}MR^2 = 0.093\,8\ \text{kg}\cdot\text{m}^2$$

We substitute to eliminate the acceleration:

$$50.0 - T = 5.10\left(\frac{TR^2}{I}\right)$$

$$T = \boxed{11.4\ \text{N}} \quad \text{and}$$

$$a = \frac{50.0 - 11.4}{5.10} = \boxed{7.57\ \text{m/s}^2}$$

$v_f^2 = v_i^2 + 2a\left(x_f - x_i\right)$: $$v_f = \sqrt{2(7.57)6.00} = \boxed{9.53\ \text{m/s}}$$

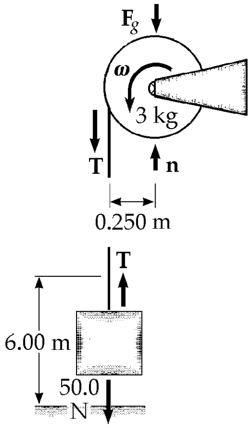

**FIG. P10.45**

(b) Use conservation of energy for the system of the object, the reel, and the Earth:

$(K+U)_i = (K+U)_f$: $$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$2mgh = mv^2 + I\left(\frac{v^2}{R^2}\right) = v^2\left(m + \frac{I}{R^2}\right)$$

$$v = \sqrt{\frac{2mgh}{m + \frac{I}{R^2}}} = \sqrt{\frac{2(50.0\ \text{N})(6.00\ \text{m})}{5.10\ \text{kg} + \frac{0.093\,8}{(0.250)^2}}} = \boxed{9.53\ \text{m/s}}$$

**P10.46** Choose the zero gravitational potential energy at the level where the masses pass.

$$K_f + U_{gf} = K_i + U_{gi} + \Delta E$$

$$\frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2 + \frac{1}{2}I\omega^2 = 0 + m_1gh_{1i} + m_2gh_{2i} + 0$$

$$\frac{1}{2}(15.0 + 10.0)v^2 + \frac{1}{2}\left[\frac{1}{2}(3.00)R^2\right]\left(\frac{v}{R}\right)^2 = 15.0(9.80)(1.50) + 10.0(9.80)(-1.50)$$

$$\frac{1}{2}(26.5\ \text{kg})v^2 = 73.5\ \text{J} \Rightarrow v = \boxed{2.36\ \text{m/s}}$$

**P10.47** From conservation of energy for the object-turntable-cylinder-Earth system,

$$\frac{1}{2}I\left(\frac{v}{r}\right)^2 + \frac{1}{2}mv^2 = mgh$$

$$I\frac{v^2}{r^2} = 2mgh - mv^2$$

$$I = \boxed{mr^2\left(\frac{2gh}{v^2} - 1\right)}$$

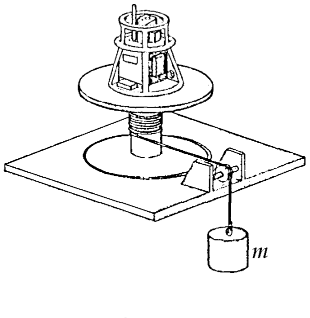

**FIG. P10.47**

**P10.48** The moment of inertia of the cylinder is

$$I = \frac{1}{2}mr^2 = \frac{1}{2}(81.6\text{ kg})(1.50\text{ m})^2 = 91.8\text{ kg}\cdot\text{m}^2$$

and the angular acceleration of the merry-go-round is found as

$$\alpha = \frac{\tau}{I} = \frac{(Fr)}{I} = \frac{(50.0\text{ N})(1.50\text{ m})}{\left(91.8\text{ kg}\cdot\text{m}^2\right)} = 0.817\text{ rad/s}^2.$$

At $t = 3.00$ s, we find the angular velocity

$$\omega = \omega_i + \alpha t$$

$$\omega = 0 + \left(0.817\text{ rad/s}^2\right)(3.00\text{ s}) = 2.45\text{ rad/s}$$

and $K = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(91.8\text{ kg}\cdot\text{m}^2\right)(2.45\text{ rad/s})^2 = \boxed{276\text{ J}}$.

**P10.49** (a) Find the velocity of the CM

$$(K+U)_i = (K+U)_f$$

$$0 + mgR = \frac{1}{2}I\omega^2$$

$$\omega = \sqrt{\frac{2mgR}{I}} = \sqrt{\frac{2mgR}{\frac{3}{2}mR^2}}$$

$$v_{\text{CM}} = R\sqrt{\frac{4g}{3R}} = \boxed{2\sqrt{\frac{Rg}{3}}}$$

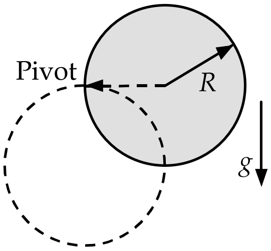

**FIG. P10.49**

(b) $$v_L = 2v_{\text{CM}} = \boxed{4\sqrt{\frac{Rg}{3}}}$$

(c) $$v_{\text{CM}} = \sqrt{\frac{2mgR}{2m}} = \boxed{\sqrt{Rg}}$$

***P10.50** (a) The moment of inertia of the cord on the spool is

$$\frac{1}{2}M\left(R_1^2 + R_2^2\right) = \frac{1}{2}0.1\text{ kg}\left((0.015\text{ m})^2 + (0.09\text{ m})^2\right) = 4.16\times10^{-4}\text{ kg}\cdot\text{m}^2.$$

The protruding strand has mass $\left(10^{-2}\text{ kg/m}\right)0.16\text{ m} = 1.6\times10^{-3}\text{ kg}$ and

$$I = I_{\text{CM}} + Md^2 = \frac{1}{12}ML^2 + Md^2 = 1.6\times10^{-3}\text{ kg}\left(\frac{1}{12}(0.16\text{ m})^2 + (0.09\text{ m} + 0.08\text{ m})^2\right)$$
$$= 4.97\times10^{-5}\text{ kg}\cdot\text{m}^2$$

For the whole cord, $I = 4.66\times10^{-4}\text{ kg}\cdot\text{m}^2$. In speeding up, the average power is

$$\mathcal{P} = \frac{E}{\Delta t} = \frac{\frac{1}{2}I\omega^2}{\Delta t} = \frac{4.66\times10^{-4}\text{ kg}\cdot\text{m}^2}{2(0.215\text{ s})}\left(\frac{2\,500\cdot2\pi}{60\text{ s}}\right)^2 = \boxed{74.3\text{ W}}$$

(b) $$\mathcal{P} = \tau\omega = (7.65\text{ N})(0.16\text{ m} + 0.09\text{ m})\left(\frac{2\,000\cdot2\pi}{60\text{ s}}\right) = \boxed{401\text{ W}}$$

## Section 10.9 **Rolling Motion of a Rigid Object**

**P10.51** (a) $K_{\text{trans}} = \frac{1}{2}mv^2 = \frac{1}{2}(10.0\text{ kg})(10.0\text{ m/s})^2 = \boxed{500\text{ J}}$

(b) $K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v^2}{r^2}\right) = \frac{1}{4}(10.0\text{ kg})(10.0\text{ m/s})^2 = \boxed{250\text{ J}}$

(c) $K_{\text{total}} = K_{\text{trans}} + K_{\text{rot}} = \boxed{750\text{ J}}$

**P10.52** $W = K_f - K_i = (K_{\text{trans}} + K_{\text{rot}})_f - (K_{\text{trans}} + K_{\text{rot}})_i$

$$W = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 - 0 - 0 = \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v}{R}\right)^2$$

or $W = \boxed{\left(\frac{7}{10}\right)Mv^2}$

**P10.53** (a)

$$\tau = I\alpha$$

$$mgR\sin\theta = \left(I_{CM} + mR^2\right)\alpha$$

$$a = \frac{mgR^2\sin\theta}{I_{\text{CM}} + mR^2}$$

$$a_{\text{hoop}} = \frac{mgR^2\sin\theta}{2mR^2} = \boxed{\frac{1}{2}g\sin\theta}$$

$$a_{\text{disk}} = \frac{mgR^2\sin\theta}{\frac{3}{2}mR^2} = \boxed{\frac{2}{3}g\sin\theta}$$

The disk moves with $\frac{4}{3}$ the acceleration of the hoop.

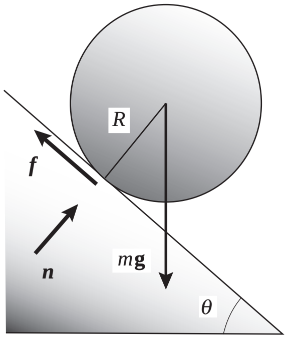

**FIG. P10.53**

(b)

$$Rf = I\alpha$$

$$f = \mu n = \mu mg\cos\theta$$

$$\mu = \frac{f}{mg\cos\theta} = \frac{\frac{I\alpha}{R}}{mg\cos\theta} = \frac{\left(\frac{2}{3}g\sin\theta\right)\left(\frac{1}{2}mR^2\right)}{R^2 mg\cos\theta} = \boxed{\frac{1}{3}\tan\theta}$$

**P10.54** $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}\left[m + \frac{I}{R^2}\right]v^2$ where $\omega = \frac{v}{R}$ since no slipping.

Also, $U_i = mgh$, $U_f = 0$, and $v_i = 0$

Therefore, $\frac{1}{2}\left[m + \frac{I}{R^2}\right]v^2 = mgh$

Thus, $v^2 = \frac{2gh}{\left[1 + \left(\frac{I}{mR^2}\right)\right]}$

For a disk, $I = \frac{1}{2}mR^2$

So $v^2 = \frac{2gh}{1 + \frac{1}{2}}$ or $\boxed{v_{\text{disk}} = \sqrt{\frac{4gh}{3}}}$

For a ring, $I = mR^2$ so $v^2 = \frac{2gh}{2}$ or $\boxed{v_{\text{ring}} = \sqrt{gh}}$

Since $v_{\text{disk}} > v_{\text{ring}}$, $\boxed{\text{the disk}}$ reaches the bottom first.

**P10.55** $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{3.00 \text{ m}}{1.50 \text{ s}} = 2.00 \text{ m/s} = \frac{1}{2}\left(0 + v_f\right)$

$$v_f = 4.00 \text{ m/s and } \omega_f = \frac{v_f}{r} = \frac{4.00 \text{ m/s}}{\left(6.38 \times 10^{-2} \text{ m}\right)/2} = \frac{8.00}{6.38 \times 10^{-2}} \text{ rad/s}$$

We ignore internal friction and suppose the can rolls without slipping.

$$\left(K_{\text{trans}} + K_{\text{rot}} + U_g\right)_i + \Delta E_{\text{mech}} = \left(K_{\text{trans}} + K_{\text{rot}} + U_g\right)_f$$

$$\left(0 + 0 + mgy_i\right) + 0 = \left(\frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2 + 0\right)$$

$$0.215 \text{ kg}\left(9.80 \text{ m/s}^2\right)\left[(3.00 \text{ m})\sin 25.0°\right] = \frac{1}{2}(0.215 \text{ kg})(4.00 \text{ m/s})^2 + \frac{1}{2}I\left(\frac{8.00}{6.38 \times 10^{-2}} \text{ rad/s}\right)^2$$

$$2.67 \text{ J} = 1.72 \text{ J} + \left(7\,860 \text{ s}^{-2}\right)t$$

$$I = \frac{0.951 \text{ kg} \cdot \text{m}^2/\text{s}^2}{7\,860 \text{ s}^{-2}} = \boxed{1.21 \times 10^{-4} \text{ kg} \cdot \text{m}^2}$$

The $\boxed{\text{height of the can}}$ is unnecessary data.

**P10.56** (a) Energy conservation for the system of the ball and the Earth between the horizontal section and top of loop:

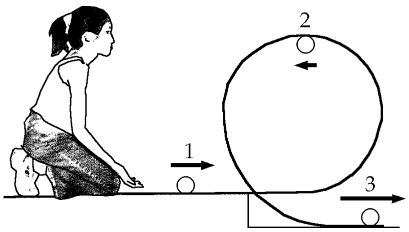

**FIG. P10.56**

$$\frac{1}{2}mv_2^2 + \frac{1}{2}I\omega_2^2 + mgy_2 = \frac{1}{2}mv_1^2 + \frac{1}{2}I\omega_1^2$$

$$\frac{1}{2}mv_2^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v_2}{r}\right)^2 + mgy_2 = \frac{1}{2}mv_1^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v_1}{r}\right)^2$$

$$\frac{5}{6}v_2^2 + gy_2 = \frac{5}{6}v_1^2$$

$$v_2 = \sqrt{v_1^2 - \frac{6}{5}gy_2} = \sqrt{(4.03 \text{ m/s})^2 - \frac{6}{5}\left(9.80 \text{ m/s}^2\right)(0.900 \text{ m})} = \boxed{2.38 \text{ m/s}}$$

The centripetal acceleration is $\frac{v_2^2}{r} = \frac{(2.38 \text{ m/s})^2}{0.450 \text{ m}} = 12.6 \text{ m/s}^2 > g$

Thus, the ball must be in contact with the track, with the track pushing downward on it.

(b)

$$\frac{1}{2}mv_3^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v_3}{r}\right)^2 + mgy_3 = \frac{1}{2}mv_1^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v_1}{r}\right)^2$$

$$v_3 = \sqrt{v_1^2 - \frac{6}{5}gy_3} = \sqrt{(4.03 \text{ m/s})^2 - \frac{6}{5}\left(9.80 \text{ m/s}^2\right)(-0.200 \text{ m})} = \boxed{4.31 \text{ m/s}}$$

(c)

$$\frac{1}{2}mv_2^2 + mgy_2 = \frac{1}{2}mv_1^2$$

$$v_2 = \sqrt{v_1^2 - 2gy_2} = \sqrt{(4.03 \text{ m/s})^2 - 2\left(9.80 \text{ m/s}^2\right)(0.900 \text{ m})} = \sqrt{-1.40 \text{ m}^2/\text{s}^2}$$

This result is imaginary. In the case where the ball does not roll, the ball starts with less energy than in part (a) and $\boxed{\text{never makes it to the top}}$ of the loop.

## Additional Problems

**P10.57**

$$mg\frac{\ell}{2}\sin\theta=\frac{1}{3}m\ell^2\alpha$$

$$\alpha=\frac{3}{2}\frac{g}{\ell}\sin\theta$$

$$a_t=\left(\frac{3}{2}\frac{g}{\ell}\sin\theta\right)r$$

Then $\left(\frac{3}{2}\frac{g}{\ell}\right)r>g\sin\theta$

for $r>\frac{2}{3}\ell$

$\therefore$ About $\boxed{\frac{1}{3}\text{ the length of the chimney}}$ will have a tangential acceleration greater than $g\sin\theta$.

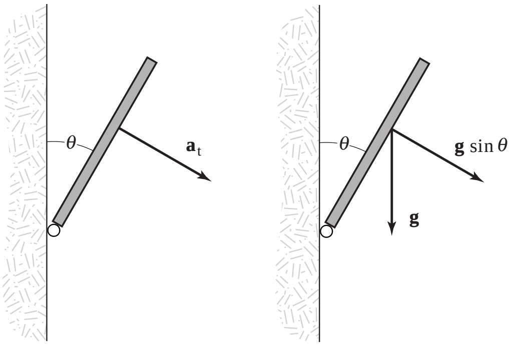

**FIG. P10.57**

**P10.58** The resistive force on each ball is $R=D\rho Av^2$. Here $v=r\omega$, where *r* is the radius of each ball's path. The resistive torque on each ball is $\tau=rR$, so the total resistive torque on the three ball system is $\tau_{\text{total}}=3rR$.

The power required to maintain a constant rotation rate is $\mathcal{P}=\tau_{\text{total}}\omega=3rR\omega$. This required power may be written as

$$\mathcal{P}=\tau_{\text{total}}\omega=3r\left[D\rho A(r\omega)^2\right]\omega=\left(3r^3DA\omega^3\right)\rho$$

With $$\omega=\frac{2\pi\,\text{rad}}{1\text{ rev}}\left(\frac{10^3\text{ rev}}{1\text{ min}}\right)\left(\frac{1\text{ min}}{60.0\text{ s}}\right)=\frac{1\,000\pi}{30.0}\text{ rad/s}$$

$$\mathcal{P}=3(0.100\text{ m})^3(0.600)\left(4.00\times10^{-4}\text{ m}^2\right)\left(\frac{1\,000\pi}{30.0\text{ s}}\right)^3\rho$$

or $\mathcal{P}=\left(0.827\text{ m}^5/\text{s}^3\right)\rho$, where $\rho$ is the density of the resisting medium.

(a) In air, $\rho=1.20\text{ kg/m}^3$,

and $\mathcal{P}=0.827\text{ m}^5/\text{s}^3\left(1.20\text{ kg/m}^3\right)=0.992\text{ N}\cdot\text{m/s}=\boxed{0.992\text{ W}}$

(b) In water, $\rho=1\,000\text{ kg/m}^3$ and $\mathcal{P}=\boxed{827\text{ W}}$.

**P10.59** (a)

$$W=\Delta K=\frac{1}{2}I\omega_f^2-\frac{1}{2}I\omega_i^2=\frac{1}{2}I\left(\omega_f^2-\omega_i^2\right)\quad\text{where}\quad I=\frac{1}{2}mR^2$$

$$=\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(1.00\text{ kg})(0.500\text{ m})^2\left[(8.00\text{ rad/s})^2-0\right]=\boxed{4.00\text{ J}}$$

(b)

$$t=\frac{\omega_f-0}{\alpha}=\frac{\omega r}{a}=\frac{(8.00\text{ rad/s})(0.500\text{ m})}{2.50\text{ m/s}^2}=\boxed{1.60\text{ s}}$$

(c)

$$\theta_f=\theta_i+\omega_it+\frac{1}{2}\alpha t^2;\ \theta_i=0;\ \omega_i=0$$

$$\theta_f=\frac{1}{2}\alpha t^2=\frac{1}{2}\left(\frac{2.50\text{ m/s}^2}{0.500\text{ m}}\right)(1.60\text{ s})^2=6.40\text{ rad}$$

$$s=r\theta=(0.500\text{ m})(6.40\text{ rad})=\boxed{3.20\text{ m}<4.00\text{ m Yes}}$$

***P10.60** The quantity of tape is constant. Then the area of the rings you see it fill is constant. This is expressed by $\pi r_t^2 - \pi r_s^2 = \pi r^2 - \pi r_s^2 + \pi r_2^2 - \pi r_s^2$ or $r_2 = \sqrt{r_t^2 + r_s^2 - r^2}$ is the outer radius of spool 2.

(a) Where the tape comes off spool 1, $\omega_1 = \frac{v}{r}$. Where the tape joins spool 2, $\omega_2 = \frac{v}{r_2} = v\left(r_s^2 + r_t^2 - r^2\right)^{-1/2}$.

(b) At the start, $r = r_t$ and $r_2 = r_s$ so $\omega_1 = \frac{v}{r_t}$ and $\omega_2 = \frac{v}{r_s}$. The takeup reel must spin at maximum speed. At the end, $r = r_s$ and $r_2 = r_t$ so $\omega_2 = \frac{v}{r_t}$ and $\omega_1 = \frac{v}{r_s}$. The angular speeds are just reversed.

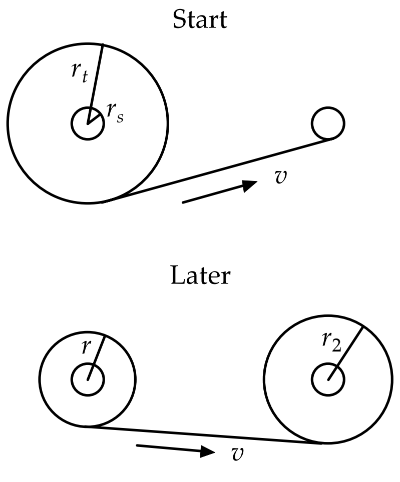

FIG. P10.60

**P10.61** (a) Since only conservative forces act within the system of the rod and the Earth,

$\Delta E = 0$ so $K_f + U_f = K_i + U_i$

$$\frac{1}{2}I\omega^2 + 0 = 0 + Mg\left(\frac{L}{2}\right)$$

where $I = \frac{1}{3}ML^2$

Therefore, $\omega = \boxed{\sqrt{\frac{3g}{L}}}$

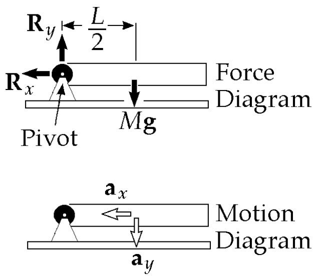

FIG. P10.61

(b) $\sum \tau = I\alpha$, so that in the horizontal orientation,

$$Mg\left(\frac{L}{2}\right) = \frac{ML^2}{3}\alpha$$

$$\alpha = \boxed{\frac{3g}{2L}}$$

(c) $a_x = a_r = -r\omega^2 = -\left(\frac{L}{2}\right)\omega^2 = \boxed{-\frac{3g}{2}}$ $a_y = -a_t = -r\alpha = -\alpha\left(\frac{L}{2}\right) = \boxed{-\frac{3g}{4}}$

(d) Using Newton's second law, we have

$$R_x = Ma_x = \boxed{-\frac{3Mg}{2}}$$

$R_y - Mg = Ma_y = -\frac{3Mg}{4}$ $R_y = \boxed{\frac{Mg}{4}}$

**P10.62** $\alpha = -10.0\ \text{rad/s}^2 - \left(5.00\ \text{rad/s}^3\right)t = \dfrac{d\omega}{dt}$

$$\int_{65.0}^{\omega} d\omega = \int_0^t [-10.0 - 5.00t]dt = -10.0t - 2.50t^2 = \omega - 65.0\ \text{rad/s}$$

$$\omega = \frac{d\theta}{dt} = 65.0\ \text{rad/s} - \left(10.0\ \text{rad/s}^2\right)t - \left(2.50\ \text{rad/s}^3\right)t^2$$

(a) At $t = 3.00$ s,

$$\omega = 65.0\ \text{rad/s} - \left(10.0\ \text{rad/s}^2\right)(3.00\ \text{s}) - \left(2.50\ \text{rad/s}^3\right)\left(9.00\ \text{s}^2\right) = \boxed{12.5\ \text{rad/s}}$$

(b) $\displaystyle\int_0^{\theta} d\theta = \int_0^t \omega\, dt = \int_0^t \left[65.0\ \text{rad/s} - \left(10.0\ \text{rad/s}^2\right)t - \left(2.50\ \text{rad/s}^3\right)t^2\right]dt$

$\theta = (65.0\ \text{rad/s})t - \left(5.00\ \text{rad/s}^2\right)t^2 - \left(0.833\ \text{rad/s}^3\right)t^3$

At $t = 3.00$ s,

$$\theta = (65.0\ \text{rad/s})(3.00\ \text{s}) - \left(5.00\ \text{rad/s}^2\right)9.00\ \text{s}^2 - \left(0.833\ \text{rad/s}^3\right)27.0\ \text{s}^3$$

$$\theta = \boxed{128\ \text{rad}}$$

**P10.63** The first drop has a velocity leaving the wheel given by $\frac{1}{2}mv_i^2 = mgh_1$, so

$$v_1 = \sqrt{2gh_1} = \sqrt{2\left(9.80\ \text{m/s}^2\right)(0.540\ \text{m})} = 3.25\ \text{m/s}$$

The second drop has a velocity given by

$$v_2 = \sqrt{2gh_2} = \sqrt{2\left(9.80\ \text{m/s}^2\right)(0.510\ \text{m})} = 3.16\ \text{m/s}$$

From $\omega = \dfrac{v}{r}$, we find

$$\omega_1 = \frac{v_1}{r} = \frac{3.25\ \text{m/s}}{0.381\ \text{m}} = 8.53\ \text{rad/s and } \omega_2 = \frac{v_2}{r} = \frac{3.16\ \text{m/s}}{0.381\ \text{m}} = 8.29\ \text{rad/s}$$

or

$$\alpha = \frac{\omega_2^2 - \omega_1^2}{2\theta} = \frac{(8.29\ \text{rad/s})^2 - (8.53\ \text{rad/s})^2}{4\pi} = \boxed{-0.322\ \text{rad/s}^2}$$

**P10.64** At the instant it comes off the wheel, the first drop has a velocity $v_1$, directed upward. The magnitude of this velocity is found from

$$K_i + U_{gi} = K_f + U_{gf}$$

$$\frac{1}{2}mv_1^2 + 0 = 0 + mgh_1 \text{ or } v_1 = \sqrt{2gh_1}$$

and the angular velocity of the wheel at the instant the first drop leaves is

$$\omega_1 = \frac{v_1}{R} = \sqrt{\frac{2gh_1}{R^2}}.$$

Similarly for the second drop: $v_2 = \sqrt{2gh_2}$ and $\omega_2 = \frac{v_2}{R} = \sqrt{\frac{2gh_2}{R^2}}$.

The angular acceleration of the wheel is then

$$a = \frac{\omega_2^2 - \omega_1^2}{2\theta} = \frac{\frac{2gh_2}{R^2} - \frac{2gh_1}{R^2}}{2(2\pi)} = \boxed{\frac{g(h_2 - h_1)}{2\pi R^2}}.$$

**P10.65** $K_f = \frac{1}{2}Mv_f^2 + \frac{1}{2}I\omega_f^2 : U_f = Mgh_f = 0;\ K_i = \frac{1}{2}Mv_i^2 + \frac{1}{2}I\omega_i^2 = 0$

$U_i = (Mgh)_i : f = \mu N = \mu Mg\cos\theta;\ \omega = \frac{v}{r};\ h = d\sin\theta$ and $I = \frac{1}{2}mr^2$

(a)

$$\Delta E = E_f - E_i \text{ or } -fd = K_f + U_f - K_i - U_i$$

$$-fd = \frac{1}{2}Mv_f^2 + \frac{1}{2}I\omega_f^2 - Mgh$$

$$-(\mu Mg\cos\theta)d = \frac{1}{2}Mv^2 + \left(\frac{mr^2}{2}\right)\frac{\frac{v^2}{r^2}}{2} - Mgd\sin\theta$$

$$\frac{1}{2}\left[M + \frac{m}{2}\right]v^2 = Mgd\sin\theta - (\mu Mg\cos\theta)d \text{ or}$$

$$v^2 = 2Mgd\frac{(\sin\theta - \mu\cos\theta)}{\frac{m}{2} + M}$$

$$v_d = \left[4gd\frac{M}{(m+2M)}(\sin\theta - \mu\cos\theta)\right]^{1/2}$$

(b)

$$v_f^2 = v_i^2 + 2a\Delta x,\ v_d^2 = 2ad$$

$$a = \frac{v_d^2}{2d} = \boxed{2g\left(\frac{M}{m+2M}\right)(\sin\theta - \mu\cos\theta)}$$

**P10.66** (a) $E=\frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\omega^2\right)$

$$E=\frac{1}{2}\cdot\frac{2}{5}\left(5.98\times10^{24}\right)\left(6.37\times10^{6}\right)^2\left(\frac{2\pi}{86\,400}\right)^2=\boxed{2.57\times10^{29}\text{ J}}$$

(b)
$$\frac{dE}{dt}=\frac{d}{dt}\left[\frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{2\pi}{T}\right)^2\right]$$

$$=\frac{1}{5}MR^2(2\pi)^2\left(-2T^{-3}\right)\frac{dT}{dt}$$

$$=\frac{1}{5}MR^2\left(\frac{2\pi}{T}\right)^2\left(\frac{-2}{T}\right)\frac{dT}{dt}$$

$$=\left(2.57\times10^{29}\text{ J}\right)\left(\frac{-2}{86\,400\text{ s}}\right)\left(\frac{10\times10^{-6}\text{ s}}{3.16\times10^{7}\text{ s}}\right)(86\,400\text{ s/day})$$

$$\frac{dE}{dt}=\boxed{-1.63\times10^{17}\text{ J/day}}$$

***P10.67** (a) $\omega_f=\omega_i+\alpha t$

$$\alpha=\frac{\omega_f-\omega_i}{t}=\frac{\frac{2\pi}{T_f}-\frac{2\pi}{T_i}}{t}=\frac{2\pi\left(T_i-T_f\right)}{T_iT_ft}$$

$$\sim\frac{2\pi\left(-10^{-3}\text{ s}\right)}{1\text{ d }1\text{ d }100\text{ yr}}\left(\frac{1\text{ d}}{86\,400\text{ s}}\right)^2\left(\frac{1\text{ yr}}{3.156\times10^{7}\text{ s}}\right)=\boxed{-10^{-22}\text{ s}^{-2}}$$

(b) The Earth, assumed uniform, has moment of inertia

$$I=\frac{2}{5}MR^2=\frac{2}{5}\left(5.98\times10^{24}\text{ kg}\right)\left(6.37\times10^{6}\text{ m}\right)^2=9.71\times10^{37}\text{ kg}\cdot\text{m}^2$$

$$\sum\tau=I\alpha\sim9.71\times10^{37}\text{ kg}\cdot\text{m}^2\left(-2.67\times10^{-22}\text{ s}^{-2}\right)=\boxed{-10^{16}\text{ N}\cdot\text{m}}$$

The negative sign indicates clockwise, to slow the planet's counterclockwise rotation.

(c) $|\tau|=Fd$. Suppose the person can exert a 900-N force.

$$d=\frac{|\tau|}{F}=\frac{2.59\times10^{16}\text{ N}\cdot\text{m}}{900\text{ N}}\sim\boxed{10^{13}\text{ m}}$$

This is the order of magnitude of the size of the planetary system.

**P10.68** $\Delta\theta = \omega t$

$$t = \frac{\Delta\theta}{\omega} = \frac{\left(\frac{31.0^\circ}{360^\circ}\right)\text{ rev}}{\frac{900\text{ rev}}{60\text{ s}}} = 0.005\,74\text{ s}$$

$$v = \frac{0.800\text{ m}}{0.005\,74\text{ s}} = \boxed{139\text{ m/s}}$$

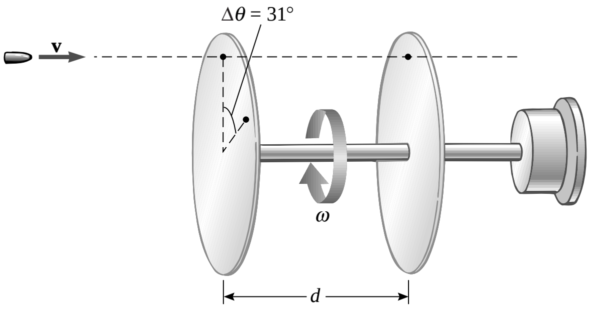

FIG. P10.68

**P10.69** $\tau_f$ will oppose the torque due to the hanging object:

$$\sum \tau = I\alpha = TR - \tau_f: \qquad \tau_f = TR - I\alpha \qquad (1)$$

Now find $T$, $I$ and $\alpha$ in given or known terms and substitute into equation (1).

$$\sum F_y = T - mg = -ma: \qquad T = m(g-a) \qquad (2)$$

also $\Delta y = v_i t + \dfrac{at^2}{2}$ $\qquad a = \dfrac{2y}{t^2} \qquad (3)$

and $\qquad \alpha = \dfrac{a}{R} = \dfrac{2y}{Rt^2}: \qquad (4)$

$$I = \frac{1}{2}M\left[R^2 + \left(\frac{R}{2}\right)^2\right] = \frac{5}{8}MR^2 \qquad (5)$$

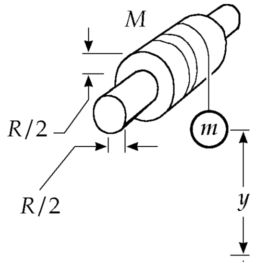

FIG. P10.69

Substituting (2), (3), (4), and (5) into (1),

we find $\qquad \tau_f = m\left(g - \dfrac{2y}{t^2}\right)R - \dfrac{5}{8}\dfrac{MR^2(2y)}{Rt^2} = \boxed{R\left[m\left(g - \dfrac{2y}{t^2}\right) - \dfrac{5}{4}\dfrac{My}{t^2}\right]}$

**P10.70** (a) $W = \Delta K + \Delta U$

$$W = K_f - K_i + U_f - U_i$$

$$0 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 - mgd\sin\theta - \frac{1}{2}kd^2$$

$$\frac{1}{2}\omega^2\left(I + mR^2\right) = mgd\sin\theta + \frac{1}{2}kd^2$$

$$\boxed{\omega = \sqrt{\frac{2mgd\sin\theta + kd^2}{I + mR^2}}}$$

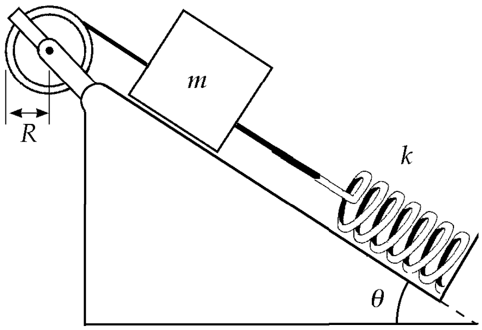

FIG. P10.70

(b) $$\omega = \sqrt{\frac{2(0.500\text{ kg})\left(9.80\text{ m/s}^2\right)(0.200\text{ m})(\sin 37.0^\circ) + 50.0\text{ N/m}(0.200\text{ m})^2}{1.00\text{ kg}\cdot\text{m}^2 + 0.500\text{ kg}(0.300\text{ m})^2}}$$

$$\omega = \sqrt{\frac{1.18 + 2.00}{1.05}} = \sqrt{3.04} = \boxed{1.74\text{ rad/s}}$$

**P10.71** (a)

$$m_2 g - T_2 = m_2 a$$

$$T_2 = m_2(g-a) = 20.0\text{ kg}\left(9.80\text{ m/s}^2 - 2.00\text{ m/s}^2\right) = \boxed{156\text{ N}}$$

$$T_1 - m_1 g\sin 37.0^\circ = m_1 a$$

$$T_1 = (15.0\text{ kg})(9.80\sin 37.0^\circ + 2.00)\text{ m/s}^2 = \boxed{118\text{ N}}$$

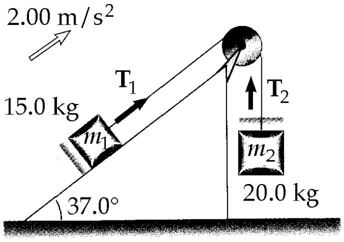

**FIG. P10.71**

(b)

$$(T_2 - T_1)R = I\alpha = I\left(\frac{a}{R}\right)$$

$$I = \frac{(T_2 - T_1)R^2}{a} = \frac{(156\text{ N} - 118\text{ N})(0.250\text{ m})^2}{2.00\text{ m/s}^2} = \boxed{1.17\text{ kg}\cdot\text{m}^2}$$

**P10.72** For the board just starting to move,

$$\sum\tau = I\alpha: \qquad mg\left(\frac{\ell}{2}\right)\cos\theta = \left(\frac{1}{3}m\ell^2\right)\alpha$$

$$\alpha = \frac{3}{2}\left(\frac{g}{\ell}\right)\cos\theta$$

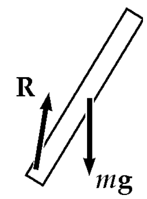

**FIG. P10.72**

The tangential acceleration of the end is $a_t = \ell\alpha = \frac{3}{2}g\cos\theta$

The vertical component is $a_y = a_t\cos\theta = \frac{3}{2}g\cos^2\theta$

If this is greater than $g$, the board will pull ahead of the ball falling:

(a) $\frac{3}{2}g\cos^2\theta \ge g$ gives $\cos^2\theta \ge \frac{2}{3}$ so $\cos\theta \ge \sqrt{\frac{2}{3}}$ and $\boxed{\theta \le 35.3^\circ}$

(b) When $\theta = 35.3^\circ$, the cup will land underneath the release-point of the ball if $r_c = \ell\cos\theta$

When $\ell = 1.00$ m, and $\theta = 35.3^\circ$ $\quad r_c = 1.00\text{ m}\sqrt{\frac{2}{3}} = 0.816\text{ m}$

so the cup should be $(1.00\text{ m} - 0.816\text{ m}) = \boxed{0.184\text{ m from the moving end}}$

**P10.73** At $t = 0$, $\omega = 3.50\text{ rad/s} = \omega_0 e^0$. Thus, $\omega_0 = 3.50\text{ rad/s}$

At $t = 9.30$ s, $\omega = 2.00\text{ rad/s} = \omega_0 e^{-\sigma(9.30\text{ s})}$, yielding $\sigma = 6.02\times10^{-2}\text{ s}^{-1}$

(a)

$$\alpha = \frac{d\omega}{dt} = \frac{d\left(\omega_0 e^{-\sigma t}\right)}{dt} = \omega_0(-\sigma)e^{-\sigma t}$$

At $t = 3.00$ s,

$$\alpha = (3.50\text{ rad/s})\left(-6.02\times10^{-2}\text{ s}^{-1}\right)e^{-3.00\left(6.02\times10^{-2}\right)} = \boxed{-0.176\text{ rad/s}^2}$$

(b)

$$\theta = \int_0^t \omega_0 e^{-\sigma t}dt = \frac{\omega_0}{-\sigma}\left[e^{-\sigma t} - 1\right] = \frac{\omega_0}{\sigma}\left[1 - e^{-\sigma t}\right]$$

At $t = 2.50$ s,

$$\theta = \frac{3.50\text{ rad/s}}{\left(6.02\times10^{-2}\right)1/\text{s}}\left[1 - e^{-\left(6.02\times10^{-2}\right)(2.50)}\right] = 8.12\text{ rad} = \boxed{1.29\text{ rev}}$$

(c)

$$\text{As } t\to\infty,\ \theta \to \frac{\omega_0}{\sigma}\left(1 - e^{-\infty}\right) = \frac{3.50\text{ rad/s}}{6.02\times10^{-2}\text{ s}^{-1}} = 58.2\text{ rad} = \boxed{9.26\text{ rev}}$$

**P10.74** Consider the total weight of each hand to act at the center of gravity (mid-point) of that hand. Then the total torque (taking CCW as positive) of these hands about the center of the clock is given by

$$\tau = -m_h g\left(\frac{L_h}{2}\right)\sin\theta_h - m_m g\left(\frac{L_m}{2}\right)\sin\theta_m = -\frac{g}{2}\left(m_h L_h \sin\theta_h + m_m L_m \sin\theta_m\right)$$

If we take $t = 0$ at 12 o'clock, then the angular positions of the hands at time $t$ are

$$\theta_h = \omega_h t,$$

where $\omega_h = \frac{\pi}{6}\ \text{rad/h}$

and $\theta_m = \omega_m t,$

where $\omega_m = 2\pi\ \text{rad/h}$

Therefore, $\tau = -4.90\ \text{m/s}^2\left[60.0\ \text{kg}(2.70\ \text{m})\sin\left(\frac{\pi t}{6}\right) + 100\ \text{kg}(4.50\ \text{m})\sin 2\pi t\right]$

or $\tau = -794\ \text{N}\cdot\text{m}\left[\sin\left(\frac{\pi t}{6}\right) + 2.78\sin 2\pi t\right]$, where $t$ is in hours.

(a) (i) At 3:00, $t = 3.00\ \text{h}$,

so $\tau = -794\ \text{N}\cdot\text{m}\left[\sin\left(\frac{\pi}{2}\right) + 2.78\sin 6\pi\right] = \boxed{-794\ \text{N}\cdot\text{m}}$

(ii) At 5:15, $t = 5\ \text{h} + \frac{15}{60}\ \text{h} = 5.25\ \text{h}$, and substitution gives:

$$\tau = \boxed{-2\,510\ \text{N}\cdot\text{m}}$$

(iii) At 6:00, $\tau = \boxed{0\ \text{N}\cdot\text{m}}$

(iv) At 8:20, $\tau = \boxed{-1\,160\ \text{N}\cdot\text{m}}$

(v) At 9:45, $\tau = \boxed{-2\,940\ \text{N}\cdot\text{m}}$

(b) The total torque is zero at those times when

$$\sin\left(\frac{\pi t}{6}\right) + 2.78\sin 2\pi t = 0$$

We proceed numerically, to find 0, 0.515 295 5, ..., corresponding to the times

12:00:00 12:30:55 12:58:19 1:32:31 1:57:01
2:33:25 2:56:29 3:33:22 3:56:55 4:32:24
4:58:14 5:30:52 6:00:00 6:29:08 7:01:46
7:27:36 8:03:05 8:26:38 9:03:31 9:26:35
10:02:59 10:27:29 11:01:41 11:29:05

***P10.75** (a) As the bicycle frame moves forward at speed $v$, the center of each wheel moves forward at the same speed and the wheels turn at angular speed $\omega = \frac{v}{R}$. The total kinetic energy of the bicycle is

$$K = K_{\text{trans}} + K_{\text{rot}}$$

or

$$K = \frac{1}{2}\left(m_{\text{frame}} + 2m_{\text{wheel}}\right)v^2 + 2\left(\frac{1}{2}I_{\text{wheel}}\omega^2\right) = \frac{1}{2}\left(m_{\text{frame}} + 2m_{\text{wheel}}\right)v^2 + \left(\frac{1}{2}m_{\text{wheel}}R^2\right)\left(\frac{v^2}{R^2}\right).$$

This yields

$$K = \frac{1}{2}\left(m_{\text{frame}} + 3m_{\text{wheel}}\right)v^2 = \frac{1}{2}\left[8.44\text{ kg} + 3(0.820\text{ kg})\right](3.35\text{ m/s})^2 = \boxed{61.2\text{ J}}.$$

(b) As the block moves forward with speed $v$, the top of each trunk moves forward at the same speed and the center of each trunk moves forward at speed $\frac{v}{2}$. The angular speed of each roller is $\omega = \frac{v}{2R}$. As in part (a), we have one object undergoing pure translation and two identical objects rolling without slipping. The total kinetic energy of the system of the stone and the trees is

$$K = K_{\text{trans}} + K_{\text{rot}}$$

or

$$K = \frac{1}{2}m_{\text{stone}}v^2 + 2\frac{1}{2}m_{\text{tree}}\left(\frac{v}{2}\right)^2 + 2\left(\frac{1}{2}I_{\text{tree}}\omega^2\right) = \frac{1}{2}\left(m_{\text{stone}} + \frac{1}{2}m_{\text{tree}}\right)v^2 + \left(\frac{1}{2}m_{\text{tree}}R^2\right)\left(\frac{v^2}{4R^2}\right).$$

This gives

$$K = \frac{1}{2}\left(m_{\text{stone}} + \frac{3}{4}m_{\text{tree}}\right)v^2 = \frac{1}{2}\left[844\text{ kg} + 0.75(82.0\text{ kg})\right](0.335\text{ m/s})^2 = \boxed{50.8\text{ J}}.$$

**P10.76** Energy is conserved so $\Delta U + \Delta K_{\text{rot}} + \Delta K_{\text{trans}} = 0$

$$mg(R-r)(\cos\theta - 1) + \left[\frac{1}{2}mv^2 - 0\right] + \frac{1}{2}\left[\frac{2}{5}mr^2\right]\omega^2 = 0$$

Since $r\omega = v$, this gives

$$\omega = \sqrt{\frac{10}{7}\frac{(R-r)(1-\cos\theta)g}{r^2}}$$

or $\omega = \boxed{\sqrt{\frac{10Rg(1-\cos\theta)}{7r^2}}}$ since $R >> r$.

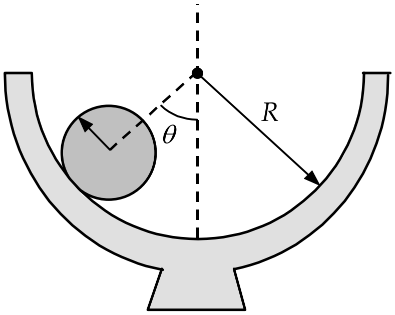

**FIG. P10.76**

**P10.77** $\sum F = T - Mg = -Ma$: $\sum \tau = TR = I\alpha = \frac{1}{2}MR^2\left(\frac{a}{R}\right)$

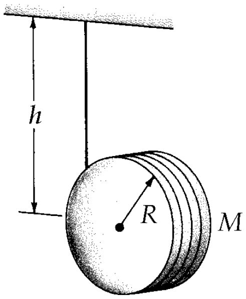

**FIG. P10.77**

(a) Combining the above two equations we find

$$T = M(g - a)$$

and

$$a = \frac{2T}{M}$$

thus $T = \boxed{\frac{Mg}{3}}$

(b) $a = \frac{2T}{M} = \frac{2}{M}\left(\frac{Mg}{3}\right) = \boxed{\frac{2}{3}g}$

(c) $v_f^2 = v_i^2 + 2a(x_f - x_i)$ $\quad v_f^2 = 0 + 2\left(\frac{2}{3}g\right)(h - 0)$

$$v_f = \boxed{\sqrt{\frac{4gh}{3}}}$$

For comparison, from conservation of energy for the system of the disk and the Earth we have

$$U_{gi} + K_{\text{rot}\,i} + K_{\text{trans}\,i} = U_{gf} + K_{\text{rot}\,f} + K_{\text{trans}\,f}: \quad Mgh + 0 + 0 = 0 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v_f}{R}\right)^2 + \frac{1}{2}Mv_f^{\,2}$$

$$v_f = \sqrt{\frac{4gh}{3}}$$

**P10.78** (a) $\sum F_x = F - f = Ma$: $\sum \tau = fR = I\alpha$

Using $I = \frac{1}{2}MR^2$ and $\alpha = \frac{a}{R}$, we find $a = \boxed{\frac{2F}{3M}}$

(b) When there is no slipping, $f = \mu Mg$.

Substituting this into the torque equation of part (a), we have

$$\mu MgR = \frac{1}{2}MRa \text{ and } \mu = \boxed{\frac{F}{3Mg}}.$$

**P10.79** (a) $\Delta K_{\text{rot}} + \Delta K_{\text{trans}} + \Delta U = 0$

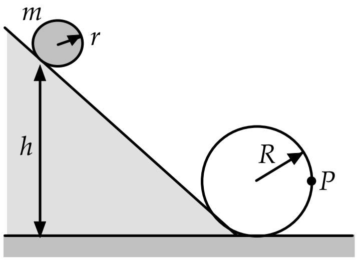

**FIG. P10.79**

Note that initially the center of mass of the sphere is a distance $h+r$ above the bottom of the loop; and as the mass reaches the top of the loop, this distance above the reference level is $2R-r$. The conservation of energy requirement gives

$$mg(h+r) = mg(2R-r) + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

For the sphere $I = \frac{2}{5}mr^2$ and $v = r\omega$ so that the expression becomes

$$gh + 2gr = 2gR + \frac{7}{10}v^2 \qquad (1)$$

Note that $h = h_{\min}$ when the speed of the sphere at the top of the loop satisfies the condition

$$\sum F = mg = \frac{mv^2}{(R-r)} \text{ or } v^2 = g(R-r)$$

Substituting this into Equation (1) gives

$$h_{\min} = 2(R-r) + 0.700(R-r) \text{ or } \boxed{h_{\min} = 2.70(R-r) = 2.70R}$$

(b) When the sphere is initially at $h = 3R$ and finally at point *P*, the conservation of energy equation gives

$$mg(3R+r) = mgR + \frac{1}{2}mv^2 + \frac{1}{5}mv^2, \text{ or}$$
$$v^2 = \frac{10}{7}(2R+r)g$$

Turning clockwise as it rolls without slipping past point *P*, the sphere is slowing down with counterclockwise angular acceleration caused by the torque of an upward force *f* of static friction. We have $\sum F_y = ma_y$ and $\sum \tau = I\alpha$ becoming $f - mg = -m\alpha r$ and $fr = \left(\frac{2}{5}\right)mr^2\alpha$.

Eliminating *f* by substitution yields $\alpha = \frac{5g}{7r}$ so that $\sum F_y = \boxed{-\frac{5}{7}mg}$

$$\sum F_x = -n = -\frac{mv^2}{R-r} = -\frac{\left(\frac{10}{7}\right)(2R+r)}{R-r}mg = \boxed{\frac{-20mg}{7}} \text{ (since } R >> r\text{)}$$

**P10.80** Consider the free-body diagram shown. The sum of torques about the chosen pivot is

$$\sum \tau = I\alpha \Rightarrow F\ell = \left(\frac{1}{3}ml^2\right)\left(\frac{a_{\text{CM}}}{\frac{l}{2}}\right) = \left(\frac{2}{3}ml\right)a_{\text{CM}} \qquad (1)$$

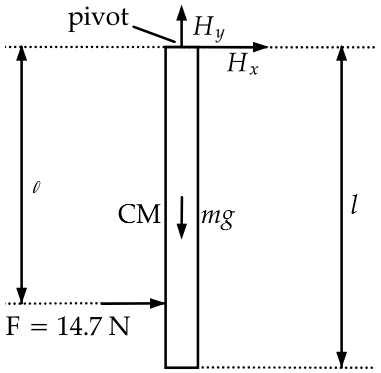

**FIG. P10.80**

(a) $\ell = l = 1.24\text{ m}$: In this case, Equation (1) becomes

$$a_{\text{CM}} = \frac{3F}{2m} = \frac{3(14.7\text{ N})}{2(0.630\text{ kg})} = 35.0\text{ m/s}^2$$

$\sum F_x = ma_{\text{CM}} \Rightarrow F + H_x = ma_{\text{CM}}$ or $H_x = ma_{\text{CM}} - F$

Thus, $H_x = (0.630\text{ kg})(35.0\text{ m/s}^2) - 14.7\text{ N} = +7.35\text{ N}$ or

$\mathbf{H}_x = \boxed{7.35\hat{\mathbf{i}}\text{ N}}$.

(b) $\ell = \frac{1}{2} = 0.620\text{ m}$: For this situation, Equation (1) yields

$$a_{\text{CM}} = \frac{3F}{4m} = \frac{3(14.7\text{ N})}{4(0.630\text{ kg})} = 17.5\text{ m/s}^2.$$

Again, $\sum F_x = ma_{\text{CM}} \Rightarrow H_x = ma_{\text{CM}} - F$, so

$$H_x = (0.630\text{ kg})(17.5\text{ m/s}^2) - 14.7\text{ N} = -3.68\text{ N or } \mathbf{H}_x = \boxed{-3.68\hat{\mathbf{i}}\text{ N}}.$$

(c) If $H_x = 0$, then $\sum F_x = ma_{\text{CM}} \Rightarrow F = ma_{\text{CM}}$, or $a_{\text{CM}} = \frac{F}{m}$.

Thus, Equation (1) becomes

$$F\ell = \left(\frac{2}{3}ml\right)\left(\frac{F}{m}\right) \text{ so } \ell = \frac{2}{3}l = \frac{2}{3}(1.24\text{ m}) = \boxed{0.827\text{ m (from the top)}}.$$

**P10.81** Let the ball have mass *m* and radius *r*. Then $I = \frac{2}{5}mr^2$. If the ball takes four seconds to go down twenty-meter alley, then $\bar{v} = 5\text{ m/s}$. The translational speed of the ball will decrease somewhat as the ball loses energy to sliding friction and some translational kinetic energy is converted to rotational kinetic energy; but its speed will always be on the order of $5.00\text{ m/s}$, including at the starting point.

As the ball slides, the kinetic friction force exerts a torque on the ball to increase the angular speed. When $\omega = \frac{v}{r}$, the ball has achieved pure rolling motion, and kinetic friction ceases. To determine the elapsed time before pure rolling motion is achieved, consider:

$$\sum \tau = I\alpha \Rightarrow (\mu_k mg)r = \left(\frac{2}{5}mr^2\right)\left[\frac{(5.00\text{ m/s})/r}{t}\right] \text{ which gives}$$

$$t = \frac{2(5.00\text{ m/s})}{5\mu_k g} = \frac{2.00\text{ m/s}}{\mu_k g}$$

Note that the mass and radius of the ball have canceled. If $\mu_k = 0.100$ for the polished alley, the sliding distance will be given by

$$\Delta x = \bar{v}t = (5.00\text{ m/s})\left[\frac{2.00\text{ m/s}}{(0.100)(9.80\text{ m/s}^2)}\right] = 10.2\text{ m or } \Delta x \sim \boxed{10^1\text{ m}}.$$

**P10.82** Conservation of energy between apex and the point where the grape leaves the surface:

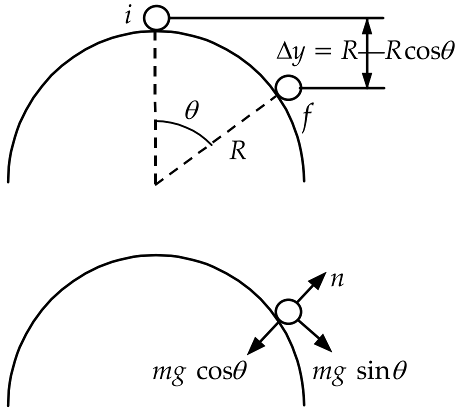

**FIG. P10.82**

$$mg\Delta y = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2$$

$$mgR(1-\cos\theta) = \frac{1}{2}mv_f^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v_f}{R}\right)^2$$

which gives $g(1-\cos\theta) = \frac{7}{10}\left(\frac{v_f^2}{R}\right)$ (1)

Consider the radial forces acting on the grape:

$$mg\cos\theta - n = \frac{mv_f^2}{R}.$$

At the point where the grape leaves the surface, $n \to 0$.

Thus, $mg\cos\theta = \frac{mv_f^2}{R}$ or $\frac{v_f^2}{R} = g\cos\theta$.

Substituting this into Equation (1) gives

$g - g\cos\theta = \frac{7}{10}g\cos\theta$ or $\cos\theta = \frac{10}{17}$ and $\theta = \boxed{54.0°}$.

**P10.83** (a) There are not any horizontal forces acting on the rod, so the center of mass will not move horizontally. Rather, the center of mass drops straight downward (distance *h*/2) with the rod rotating about the center of mass as it falls. From conservation of energy:

$$K_f + U_{gf} = K_i + U_{gi}$$

$$\frac{1}{2}Mv_{\text{CM}}^2 + \frac{1}{2}I\omega^2 + 0 = 0 + Mg\left(\frac{h}{2}\right) \text{ or}$$

$$\frac{1}{2}Mv_{\text{CM}}^2 + \frac{1}{2}\left(\frac{1}{12}Mh^2\right)\left(\frac{v_{\text{CM}}}{\frac{h}{2}}\right)^2 = Mg\left(\frac{h}{2}\right) \text{ which reduces to}$$

$$v_{\text{CM}} = \boxed{\sqrt{\frac{3gh}{4}}}$$

(b) In this case, the motion is a pure rotation about a fixed pivot point (the lower end of the rod) with the center of mass moving in a circular path of radius *h*/2. From conservation of energy:

$$K_f + U_{gf} = K_i + U_{gi}$$

$$\frac{1}{2}I\omega^2 + 0 = 0 + Mg\left(\frac{h}{2}\right) \text{ or}$$

$$\frac{1}{2}\left(\frac{1}{3}Mh^2\right)\left(\frac{v_{\text{CM}}}{\frac{h}{2}}\right)^2 = Mg\left(\frac{h}{2}\right) \text{ which reduces to}$$

$$v_{\text{CM}} = \boxed{\sqrt{\frac{3gh}{4}}}$$

**P10.84** (a) The mass of the roll decreases as it unrolls. We have $m=\frac{Mr^2}{R^2}$ where $M$ is the initial mass of the roll. Since $\Delta E=0$, we then have $\Delta U_g+\Delta K_{\text{trans}}+\Delta K_{\text{rot}}=0$. Thus, when $I=\frac{mr^2}{2}$,

$$(mgr-MgR)+\frac{mv^2}{2}+\left[\frac{mr^2}{2}\frac{\omega^2}{2}\right]=0$$

Since $\omega r=v$, this becomes $v=\boxed{\sqrt{\frac{4g\left(R^3-r^3\right)}{3r^2}}}$

(b) Using the given data, we find $v=\boxed{5.31\times10^4\ \text{m/s}}$

(c) We have assumed that $\Delta E=0$. When the roll gets to the end, we will have an inelastic collision with the surface. $\boxed{\text{The energy goes into internal energy}}$. With the assumption we made, there are problems with this question. It would take an infinite time to unwrap the tissue since $dr\to0$. Also, as $r$ approaches zero, the velocity of the center of mass approaches infinity, which is physically impossible.

**P10.85** (a)

$$\sum F_x=F+f=Ma_{\text{CM}}$$

$$\sum\tau=FR-fR=I\alpha$$

$$FR-(Ma_{\text{CM}}-F)R=\frac{Ia_{\text{CM}}}{R}\qquad\boxed{a_{\text{CM}}=\frac{4F}{3M}}$$

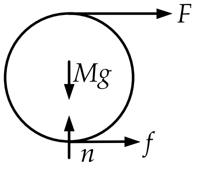

**FIG. P10.85**

(b) $f=Ma_{\text{CM}}-F=M\left(\frac{4F}{3M}\right)-F=\boxed{\frac{1}{3}F}$

(c)

$$v_f^2=v_i^2+2a\left(x_f-x_i\right)$$

$$v_f=\boxed{\sqrt{\frac{8Fd}{3M}}}$$

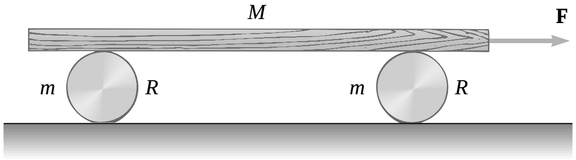

FIG. P10.86

**P10.86** Call $f_t$ the frictional force exerted by each roller backward on the plank. Name as $f_b$ the rolling resistance exerted backward by the ground on each roller. Suppose the rollers are equally far from the ends of the plank.

For the plank,

$$\sum F_x = ma_x \qquad 6.00\text{ N} - 2f_t = (6.00\text{ kg})a_p$$

The center of each roller moves forward only half as far as the plank. Each roller has acceleration $\frac{a_p}{2}$ and angular acceleration

$$\frac{a_p/2}{(5.00\text{ cm})} = \frac{a_p}{(0.100\text{ m})}$$

Then for each,

$$\sum F_x = ma_x \qquad +f_t - f_b = (2.00\text{ kg})\frac{a_p}{2}$$

$$\sum \tau = I\alpha \qquad f_t(5.00\text{ cm}) + f_b(5.00\text{ cm}) = \frac{1}{2}(2.00\text{ kg})(5.00\text{ cm})^2\frac{a_p}{10.0\text{ cm}}$$

So $\quad f_t + f_b = \left(\frac{1}{2}\text{ kg}\right)a_p$

Add to eliminate $f_b$:

$$2f_t = (1.50\text{ kg})a_p$$

(a) And $6.00\text{ N} - (1.50\text{ kg})a_p = (6.00\text{ kg})a_p$

$$a_p = \frac{(6.00\text{ N})}{(7.50\text{ kg})} = \boxed{0.800\text{ m/s}^2}$$

For each roller, $a = \frac{a_p}{2} = \boxed{0.400\text{ m/s}^2}$

(b) Substituting back, $2f_t = (1.50\text{ kg})0.800\text{ m/s}^2$

$$f_t = \boxed{0.600\text{ N}}$$

$$0.600\text{ N} + f_b = \frac{1}{2}\text{ kg}\left(0.800\text{ m/s}^2\right)$$

$$f_b = -0.200\text{ N}$$

The negative sign means that the horizontal force of ground on each roller is $\boxed{\text{0.200 N forward}}$ rather than backward as we assumed.

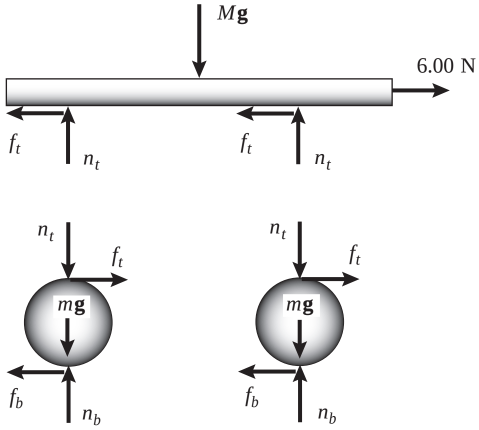

FIG. P10.86(b)

**P10.87** Rolling is instantaneous rotation about the contact point $P$. The weight and normal force produce no torque about this point.

Now $F_1$ produces $\boxed{\text{a clockwise torque}}$ about $P$ and makes the spool roll forward.

$\boxed{\text{Counterclockwise torques}}$ result from $F_3$ and $F_4$, making the spool roll to the left.

The force $F_2$ produces $\boxed{\text{zero torque}}$ about point $P$ and does not cause the spool to roll. If $F_2$ were strong enough, it would cause the spool to slide to the right, but not roll.

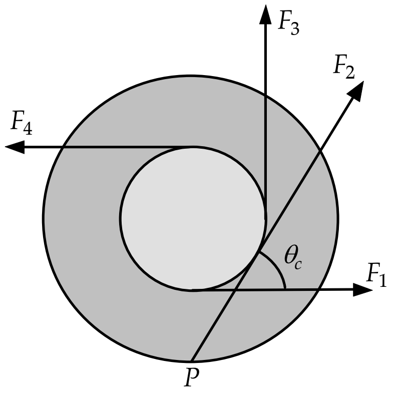

**FIG. P10.87**

**P10.88** The force applied at the critical angle exerts zero torque about the spool's contact point with the ground and so will not make the spool roll.

From the right triangle shown in the sketch, observe that $\theta_c = 90° - \phi = 90° - (90° - \gamma) = \gamma$.

Thus, $\boxed{\cos\theta_c = \cos\gamma = \dfrac{r}{R}}$.

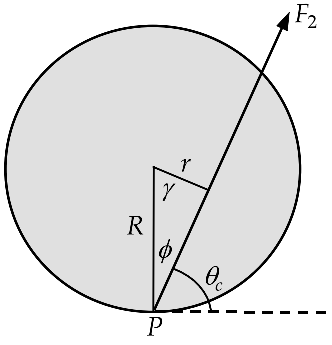

**FIG. P10.88**

**P10.89** (a) Consider motion starting from rest over distance $x$ along the incline:

$$(K_{\text{trans}} + K_{\text{rot}} + U)_i + \Delta E = (K_{\text{trans}} + K_{\text{rot}} + U)_f$$

$$0 + 0 + Mgx\sin\theta + 0 = \frac{1}{2}Mv^2 + 2\left(\frac{1}{2}mR^2\right)\left(\frac{v}{R}\right)^2 + 0$$

$$2Mgx\sin\theta = (M + 2m)v^2$$

Since acceleration is constant,

$$v^2 = v_i^2 + 2ax = 0 + 2ax, \text{ so}$$

$$2Mgx\sin\theta = (M + 2m)2ax$$

$$a = \boxed{\frac{Mg\sin\theta}{(M + 2m)}}$$

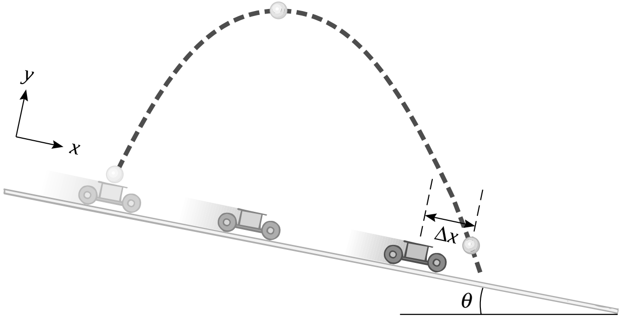

**FIG. P10.88**

*continued on next page*

(c) Suppose the ball is fired from a cart at rest. It moves with acceleration $g\sin\theta = a_x$ down the incline and $a_y = -g\cos\theta$ perpendicular to the incline. For its range along the ramp, we have

$$y - y_i = v_{yi}t - \frac{1}{2}g\cos\theta t^2 = 0 - 0$$

$$t = \frac{2v_{yi}}{g\cos\theta}$$

$$x - x_i = v_{xi}t + \frac{1}{2}a_x t^2$$

$$d = 0 + \frac{1}{2}g\sin\theta\left(\frac{4v_{yi}^2}{g^2\cos^2\theta}\right)$$

$$d = \boxed{\frac{2v_{yi}^2\sin\theta}{g\cos^2\theta}}$$

(b) In the same time the cart moves

$$x - x_i = v_{xi}t + \frac{1}{2}a_x t^2$$

$$d_c = 0 + \frac{1}{2}\left(\frac{g\sin\theta M}{(M+2m)}\right)\left(\frac{4v_{yi}^2}{g^2\cos^2\theta}\right)$$

$$d_c = \frac{2v_{yi}^2\sin\theta M}{g(M+2m)\cos^2\theta}$$

So the ball overshoots the cart by

$$\Delta x = d - d_c = \frac{2v_{yi}^2\sin\theta}{g\cos^2\theta} - \frac{2v_{yi}^2\sin\theta M}{g\cos^2\theta(M+2m)}$$

$$\Delta x = \frac{2v_{yi}^2\sin\theta M + 4v_{yi}^2\sin\theta m - 2v_{yi}^2\sin\theta M}{g\cos^2\theta(M+2m)}$$

$$\Delta x = \boxed{\frac{4mv_{yi}^2\sin\theta}{(M+2m)g\cos^2\theta}}$$

**P10.90** $\sum F_x = ma_x$ reads $-f + T = ma$. If we take torques around the center of mass, we can use $\sum \tau = I\alpha$, which reads $+fR_2 - TR_1 = I\alpha$. For rolling without slipping, $\alpha = \dfrac{a}{R_2}$. By substitution,

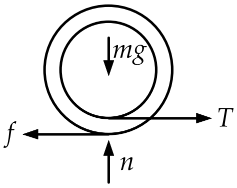

**FIG. P10.90**

$$\begin{aligned} fR_2 - TR_1 &= \frac{Ia}{R_2} = \frac{I}{R_2 m}(T - f) \\ fR_2^2 m - TR_1 R_2 m &= IT - If \\ f\left(I + mR_2^2\right) &= T\left(I + mR_1 R_2\right) \\ f &= \left(\frac{I + mR_1 R_2}{I + mR_2^2}\right)T \end{aligned}$$

Since the answer is positive, the friction force is confirmed to be $\boxed{\text{to the left.}}$

## ANSWERS TO EVEN PROBLEMS

**P10.2** (a) $822\ \text{rad/s}^2$; (b) $4.21 \times 10^3$ rad

**P10.4** (a) $1.20 \times 10^2$ rad/s; (b) 25.0 s

**P10.6** $-226\ \text{rad/s}^2$

**P10.8** $13.7\ \text{rad/s}^2$

**P10.10** (a) 2.88 s; (b) 12.8 s

**P10.12** (a) 0.180 rad/s;
(b) $8.10\ \text{m/s}^2$ toward the center of the track

**P10.14** (a) 0.605 m/s; (b) 17.3 rad/s; (c) 5.82 m/s;
(d) The crank length is unnecessary

**P10.16** (a) 54.3 rev; (b) 12.1 rev/s

**P10.18** 0.572

**P10.20** (a) $92.0\ \text{kg}\cdot\text{m}^2$; 184 J;
(b) 6.00 m/s; 4.00 m/s; 8.00 m/s; 184 J

**P10.22** see the solution

**P10.24** $1.28\ \text{kg}\cdot\text{m}^2$

**P10.26** $\sim 10^0\ \text{kg}\cdot\text{m}^2$

**P10.28** $\frac{1}{2}ML^2$

**P10.30** 168 N·m clockwise

**P10.32** 882 N·m

**P10.34** (a) 1.03 s; (b) 10.3 rev

**P10.36** (a) $21.6\ \text{kg}\cdot\text{m}^2$; (b) 3.60 N·m; (c) 52.4 rev

**P10.38** 0.312

**P10.40** $1.04 \times 10^{-3}$ J

**P10.42** 149 rad/s

**P10.44** (a) 6.90 J; (b) 8.73 rad/s; (c) 2.44 m/s;
(d) 1.043 2 times larger

**P10.46** 2.36 m/s

**P10.48** 276 J

**P10.50** (a) 74.3 W; (b) 401 W

**P10.52** $\dfrac{7Mv^2}{10}$

**P10.54** The disk; $\sqrt{\dfrac{4gh}{3}}$ versus $\sqrt{gh}$

| | |
|---|---|
| **P10.56** | (a) $2.38\ \text{m/s}$; (b) $4.31\ \text{m/s}$;<br>(c) It will not reach the top of the loop. |
| **P10.58** | (a) 0.992 W; (b) 827 W |
| **P10.60** | see the solution |
| **P10.62** | (a) $12.5\ \text{rad/s}$; (b) 128 rad |
| **P10.64** | $\dfrac{g(h_2-h_1)}{2\pi R^2}$ |
| **P10.66** | (a) $2.57\times10^{29}\ \text{J}$; (b) $-1.63\times10^{17}\ \text{J/day}$ |
| **P10.68** | $139\ \text{m/s}$ |
| **P10.70** | (a) $\sqrt{\dfrac{2mgd\sin\theta+kd^2}{I+mR^2}}$; (b) $1.74\ \text{rad/s}$ |
| **P10.72** | see the solution |
| **P10.74** | (a) $-794\ \text{N}\cdot\text{m}$; $-2\,510\ \text{N}\cdot\text{m}$; 0;<br>$-1\,160\ \text{N}\cdot\text{m}$; $-2\,940\ \text{N}\cdot\text{m}$;<br>(b) see the solution |
| **P10.76** | $\sqrt{\dfrac{10Rg(1-\cos\theta)}{7r^2}}$ |
| **P10.78** | see the solution |
| **P10.80** | (a) $35.0\ \text{m/s}^2$; $7.35\hat{\mathbf{i}}\ \text{N}$;<br>(b) $17.5\ \text{m/s}^2$; $-3.68\hat{\mathbf{i}}\ \text{N}$;<br>(c) At 0.827 m from the top. |
| **P10.82** | 54.0° |
| **P10.84** | (a) $\sqrt{\dfrac{4g\left(R^3-r^3\right)}{3r^2}}$; (b) $5.31\times10^4\ \text{m/s}$;<br>(c) It becomes internal energy. |
| **P10.86** | (a) $0.800\ \text{m/s}^2$; $0.400\ \text{m/s}^2$;<br>(b) 0.600 N between each cylinder and the plank; 0.200 N forward on each cylinder by the ground |
| **P10.88** | see the solution |
| **P10.90** | see the solution; to the left |

# 11

# Angular Momentum

## CHAPTER OUTLINE

11.1 The Vector Product and Torque
11.2 Angular Momentum
11.3 Angular Momentum of a Rotating Rigid Object
11.4 Conservation of Angular Momentum
11.5 The Motion of Gyroscopes and Tops
11.6 Angular Momentum as a Fundamental Quantity

## ANSWERS TO QUESTIONS

**Q11.1** No to both questions. An axis of rotation must be defined to calculate the torque acting on an object. The moment arm of each force is measured from the axis.

**Q11.2** $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C})$ is a scalar quantity, since $(\mathbf{B}\times\mathbf{C})$ is a vector. Since $\mathbf{A}\cdot\mathbf{B}$ is a scalar, and the cross product between a scalar and a vector is not defined, $(\mathbf{A}\cdot\mathbf{B})\times\mathbf{C}$ is undefined.

**Q11.3** (a) Down–cross–left is away from you: $-\hat{\mathbf{j}}\times\left(-\hat{\mathbf{i}}\right)=-\hat{\mathbf{k}}$

(b) Left–cross–down is toward you: $-\hat{\mathbf{i}}\times\left(-\hat{\mathbf{j}}\right)=\hat{\mathbf{k}}$

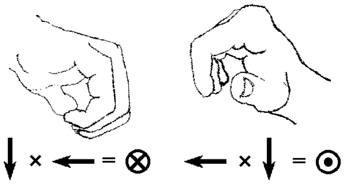

**FIG. Q11.3**

**Q11.4** The torque about the point of application of the force is zero.

**Q11.5** You cannot conclude anything about the magnitude of the angular momentum vector without first defining your axis of rotation. Its direction will be perpendicular to its velocity, but you cannot tell its direction in three-dimensional space until an axis is specified.

**Q11.6** Yes. If the particles are moving in a straight line, then the angular momentum of the particles about any point on the path is zero.

**Q11.7** Its angular momentum about that axis is constant in time. You cannot conclude anything about the magnitude of the angular momentum.

**Q11.8** No. The angular momentum about any axis that does not lie along the instantaneous line of motion of the ball is nonzero.

**Q11.9** There must be two rotors to balance the torques on the body of the helicopter. If it had only one rotor, the engine would cause the body of the helicopter to swing around rapidly with angular momentum opposite to the rotor.

**Q11.10** The angular momentum of the particle about the center of rotation is constant. The angular momentum about any point that does not lie along the axis through the center of rotation and perpendicular to the plane of motion of the particle is not constant in time.

**Q11.11** The long pole has a large moment of inertia about an axis along the rope. An unbalanced torque will then produce only a small angular acceleration of the performer-pole system, to extend the time available for getting back in balance. To keep the center of mass above the rope, the performer can shift the pole left or right, instead of having to bend his body around. The pole sags down at the ends to lower the system center of gravity.

**Q11.12** The diver leaves the platform with some angular momentum about a horizontal axis through her center of mass. When she draws up her legs, her moment of inertia decreases and her angular speed increases for conservation of angular momentum. Straightening out again slows her rotation.

**Q11.13** Suppose we look at the motorcycle moving to the right. Its drive wheel is turning clockwise. The wheel speeds up when it leaves the ground. No outside torque about its center of mass acts on the airborne cycle, so its angular momentum is conserved. As the drive wheel's clockwise angular momentum increases, the frame of the cycle acquires counterclockwise angular momentum. The cycle's front end moves up and its back end moves down.

**Q11.14** The angular speed must increase. Since gravity does not exert a torque on the system, its angular momentum remains constant as the gas contracts.

**Q11.15** Mass moves away from axis of rotation, so moment of inertia increases, angular speed decreases, and period increases.

**Q11.16** The turntable will rotate counterclockwise. Since the angular momentum of the mouse-turntable system is initially zero, as both are at rest, the turntable must rotate in the direction opposite to the motion of the mouse, for the angular momentum of the system to remain zero.

**Q11.17** Since the cat cannot apply an external torque to itself while falling, its angular momentum cannot change. Twisting in this manner changes the orientation of the cat to feet-down without changing the total angular momentum of the cat. Unfortunately, humans aren't flexible enough to accomplish this feat.

**Q11.18** The angular speed of the ball must increase. Since the angular momentum of the ball is constant, as the radius decreases, the angular speed must increase.

**Q11.19** Rotating the book about the axis that runs across the middle pages perpendicular to the binding—most likely where you put the rubber band—is the one that has the intermediate moment of inertia and gives unstable rotation.

**Q11.20** The suitcase might contain a spinning gyroscope. If the gyroscope is spinning about an axis that is oriented horizontally passing through the bellhop, the force he applies to turn the corner results in a torque that could make the suitcase swing away. If the bellhop turns quickly enough, anything at all could be in the suitcase and need not be rotating. Since the suitcase is massive, it will want to follow an inertial path. This could be perceived as the suitcase swinging away by the bellhop.

## SOLUTIONS TO PROBLEMS

### Section 11.1 The Vector Product and Torque

**P11.1** $$\mathbf{M}\times\mathbf{N}=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 6 & 2 & -1\\ 2 & -1 & -3\end{vmatrix}=\boxed{-7.00\hat{\mathbf{i}}+16.0\hat{\mathbf{j}}-10.0\hat{\mathbf{k}}}$$

**P11.2** (a) $$\text{area}=|\mathbf{A}\times\mathbf{B}|=AB\sin\theta=(42.0\text{ cm})(23.0\text{ cm})\sin(65.0°–15.0°)=\boxed{740\text{ cm}^2}$$

(b) $$\mathbf{A}+\mathbf{B}=\left[(42.0\text{ cm})\cos 15.0°+(23.0\text{ cm})\cos 65.0°\right]\hat{\mathbf{i}}+\left[(42.0\text{ cm})\sin 15.0°+(23.0\text{ cm})\sin 65.0°\right]\hat{\mathbf{j}}$$

$$\mathbf{A}+\mathbf{B}=(50.3\text{ cm})\hat{\mathbf{i}}+(31.7\text{ cm})\hat{\mathbf{j}}$$

$$\text{length}=|\mathbf{A}+\mathbf{B}|=\sqrt{(50.3\text{ cm})^2+(31.7\text{ cm})^2}=\boxed{59.5\text{ cm}}$$

**P11.3** (a) $$\mathbf{A}\times\mathbf{B}=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ -3 & 4 & 0\\ 2 & 3 & 0\end{vmatrix}=\boxed{-17.0\hat{\mathbf{k}}}$$

(b) $$|\mathbf{A}\times\mathbf{B}|=|\mathbf{A}||\mathbf{B}|\sin\theta$$

$$17=5\sqrt{13}\sin\theta$$

$$\theta=\arcsin\left(\frac{17}{5\sqrt{13}}\right)=\boxed{70.6°}$$

**P11.4** $$\mathbf{A}\cdot\mathbf{B}=-3.00(6.00)+7.00(-10.0)+(-4.00)(9.00)=-124$$

$$AB=\sqrt{(-3.00)^2+(7.00)^2+(-4.00)^2}\cdot\sqrt{(6.00)^2+(-10.0)^2+(9.00)^2}=127$$

(a) $$\cos^{-1}\left(\frac{\mathbf{A}\cdot\mathbf{B}}{AB}\right)=\cos^{-1}(-0.979)=\boxed{168°}$$

(b) $$\mathbf{A}\times\mathbf{B}=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ -3.00 & 7.00 & -4.00\\ 6.00 & -10.0 & 9.00\end{vmatrix}=23.0\hat{\mathbf{i}}+3.00\hat{\mathbf{j}}-12.0\hat{\mathbf{k}}$$

$$|\mathbf{A}\times\mathbf{B}|=\sqrt{(23.0)^2+(3.00)^2+(-12.0)^2}=26.1$$

$$\sin^{-1}\left(\frac{|\mathbf{A}\times\mathbf{B}|}{AB}\right)=\sin^{-1}(0.206)=\boxed{11.9°}\text{ or }168°$$

(c) Only $\boxed{\text{the first method}}$ gives the angle between the vectors unambiguously.

***P11.5** $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = 0.450\text{ m}(0.785\text{ N})\sin(90°-14°)\text{ up} \times \text{east}$

$= \boxed{0.343\text{ N}\cdot\text{m north}}$

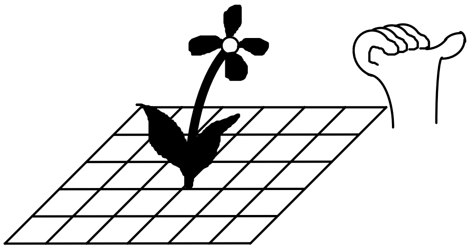

FIG. P11.5

**P11.6** The cross-product vector must be perpendicular to both of the factors, so its dot product with either factor must be zero:

Does $\left(2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}\right)\cdot\left(4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}\right) = 0$ ?

$$8 - 9 - 4 = -5 \neq 0$$

$\boxed{\text{No}}$. The cross product could not work out that way.

**P11.7** $|\mathbf{A} \times \mathbf{B}| = \mathbf{A}\cdot\mathbf{B} \Rightarrow AB\sin\theta = AB\cos\theta \Rightarrow \tan\theta = 1$ or

$\theta = \boxed{45.0°}$

**P11.8** (a) $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 3 & 0 \\ 3 & 2 & 0 \end{vmatrix} = \hat{\mathbf{i}}(0-0) - \hat{\mathbf{j}}(0-0) + \hat{\mathbf{k}}(2-9) = \boxed{(-7.00\text{ N}\cdot\text{m})\hat{\mathbf{k}}}$

(b) The particle's position vector relative to the new axis is $1\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 6\hat{\mathbf{j}} = 1\hat{\mathbf{i}} - 3\hat{\mathbf{j}}$.

$$\boldsymbol{\tau} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & -3 & 0 \\ 3 & 2 & 0 \end{vmatrix} = \boxed{(11.0\text{ N}\cdot\text{m})\hat{\mathbf{k}}}$$

**P11.9** $\boxed{|\mathbf{F}_3| = |\mathbf{F}_1| + |\mathbf{F}_2|}$

The torque produced by $\mathbf{F}_3$ depends on the perpendicular distance **OD**, therefore translating the point of application of $\mathbf{F}_3$ to any other point along **BC** $\boxed{\text{will not change the net torque}}$.

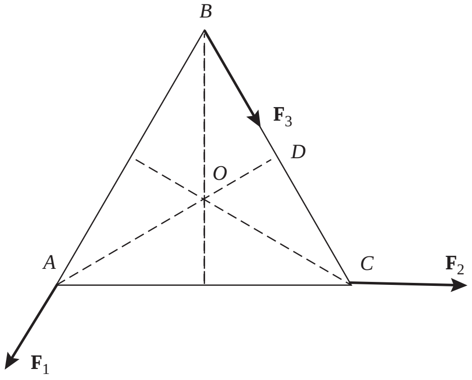

FIG. P11.9

***P11.10** $\left|\hat{\mathbf{i}}\times\hat{\mathbf{i}}\right| = 1\cdot 1\cdot\sin 0° = 0$

$\hat{\mathbf{j}}\times\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}\times\hat{\mathbf{k}}$ are zero similarly since the vectors being multiplied are parallel.

$\left|\hat{\mathbf{i}}\times\hat{\mathbf{j}}\right| = 1\cdot 1\cdot\sin 90° = 1$

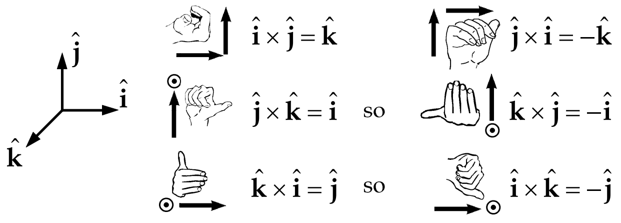
$\hat{\mathbf{j}}$
$\hat{\mathbf{i}}$
$\hat{\mathbf{k}}$
$\hat{\mathbf{i}}\times\hat{\mathbf{j}} = \hat{\mathbf{k}}$
$\hat{\mathbf{j}}\times\hat{\mathbf{i}} = -\hat{\mathbf{k}}$
$\hat{\mathbf{j}}\times\hat{\mathbf{k}} = \hat{\mathbf{i}}$ so
$\hat{\mathbf{k}}\times\hat{\mathbf{j}} = -\hat{\mathbf{i}}$
$\hat{\mathbf{k}}\times\hat{\mathbf{i}} = \hat{\mathbf{j}}$ so
$\hat{\mathbf{i}}\times\hat{\mathbf{k}} = -\hat{\mathbf{j}}$

**FIG. P11.10**

## Section 11.2 **Angular Momentum**

**P11.11** $L = \sum m_i v_i r_i$

$= (4.00\text{ kg})(5.00\text{ m/s})(0.500\text{ m}) + (3.00\text{ kg})(5.00\text{ m/s})(0.500\text{ m})$

$L = 17.5\text{ kg}\cdot\text{m}^2/\text{s}$, and

$$\boxed{\mathbf{L} = \left(17.5\text{ kg}\cdot\text{m}^2/\text{s}\right)\hat{\mathbf{k}}}$$

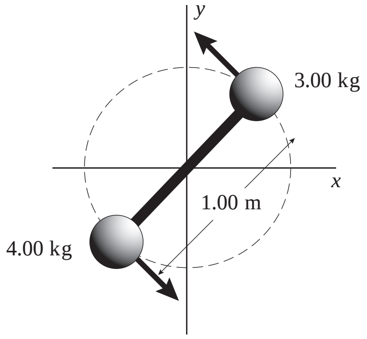

**FIG. P11.11**

**P11.12** $\mathbf{L} = \mathbf{r}\times\mathbf{p}$

$\mathbf{L} = \left(1.50\hat{\mathbf{i}} + 2.20\hat{\mathbf{j}}\right)\text{ m}\times(1.50\text{ kg})\left(4.20\hat{\mathbf{i}} - 3.60\hat{\mathbf{j}}\right)\text{ m/s}$

$\mathbf{L} = \left(-8.10\hat{\mathbf{k}} - 13.9\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m}^2/\text{s} = \boxed{\left(-22.0\text{ kg}\cdot\text{m}^2/\text{s}\right)\hat{\mathbf{k}}}$

**P11.13** $\mathbf{r} = \left(6.00\hat{\mathbf{i}} + 5.00t\hat{\mathbf{j}}\text{ m}\right)$ $\qquad$ $\mathbf{v} = \dfrac{d\mathbf{r}}{dt} = 5.00\hat{\mathbf{j}}\text{ m/s}$

so $\quad \mathbf{p} = m\mathbf{v} = 2.00\text{ kg}\left(5.00\hat{\mathbf{j}}\text{ m/s}\right) = 10.0\hat{\mathbf{j}}\text{ kg}\cdot\text{m/s}$

and $\quad \mathbf{L} = \mathbf{r}\times\mathbf{p} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 6.00 & 5.00t & 0 \\ 0 & 10.0 & 0 \end{vmatrix} = \boxed{\left(60.0\text{ kg}\cdot\text{m}^2/\text{s}\right)\hat{\mathbf{k}}}$

**P11.14** $\sum F_x = ma_x \qquad T\sin\theta = \dfrac{mv^2}{r}$

$\sum F_y = ma_y \qquad T\cos\theta = mg$

So $\dfrac{\sin\theta}{\cos\theta} = \dfrac{v^2}{rg} \qquad v = \sqrt{rg\dfrac{\sin\theta}{\cos\theta}}$

$$L = rmv\sin 90.0°$$

$$L = rm\sqrt{rg\frac{\sin\theta}{\cos\theta}}$$

$$L = \sqrt{m^2gr^3\frac{\sin\theta}{\cos\theta}}$$

$r = \ell\sin\theta$, so

$$L = \boxed{\sqrt{m^2g\ell^3\frac{\sin^4\theta}{\cos\theta}}}$$

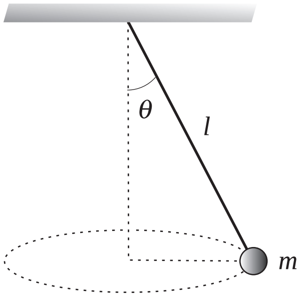

FIG. P11.14

**P11.15** The angular displacement of the particle around the circle is $\theta = \omega t = \dfrac{vt}{R}$.

The vector from the center of the circle to the mass is then $R\cos\theta\,\hat{\mathbf{i}} + R\sin\theta\,\hat{\mathbf{j}}$.

The vector from point $P$ to the mass is

$$\mathbf{r} = R\hat{\mathbf{i}} + R\cos\theta\,\hat{\mathbf{i}} + R\sin\theta\,\hat{\mathbf{j}}$$

$$\mathbf{r} = R\left[\left(1+\cos\left(\frac{vt}{R}\right)\right)\hat{\mathbf{i}} + \sin\left(\frac{vt}{R}\right)\hat{\mathbf{j}}\right]$$

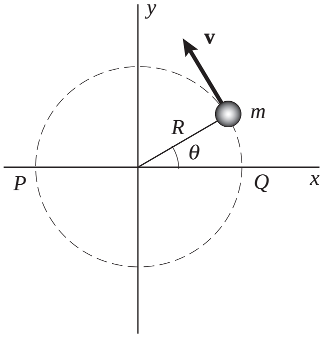

FIG. P11.15

The velocity is

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = -v\sin\left(\frac{vt}{R}\right)\hat{\mathbf{i}} + v\cos\left(\frac{vt}{R}\right)\hat{\mathbf{j}}$$

So $\mathbf{L} = \mathbf{r}\times m\mathbf{v}$

$$\mathbf{L} = mvR\left[(1+\cos\omega t)\hat{\mathbf{i}} + \sin\omega t\hat{\mathbf{j}}\right]\times\left[-\sin\omega t\hat{\mathbf{i}} + \cos\omega t\hat{\mathbf{j}}\right]$$

$$\mathbf{L} = \boxed{mvR\hat{\mathbf{k}}\left[\cos\left(\frac{vt}{R}\right)+1\right]}$$

**P11.16** (a) The net torque on the counterweight-cord-spool system is:

$$|\tau| = |\mathbf{r}\times\mathbf{F}| = 8.00\times10^{-2}\text{ m}(4.00\text{ kg})(9.80\text{ m/s}^2) = \boxed{3.14\text{ N}\cdot\text{m}}.$$

(b) $|\mathbf{L}| = |\mathbf{r}\times m\mathbf{v}| + I\omega \qquad |\mathbf{L}| = Rmv + \dfrac{1}{2}MR^2\left(\dfrac{v}{R}\right) = R\left(m+\dfrac{M}{2}\right)v = \boxed{(0.400\text{ kg}\cdot\text{m})v}$

(c) $\tau = \dfrac{dL}{dt} = (0.400\text{ kg}\cdot\text{m})a \qquad a = \dfrac{3.14\text{ N}\cdot\text{m}}{0.400\text{ kg}\cdot\text{m}} = \boxed{7.85\text{ m/s}^2}$

**P11.17** (a) $\boxed{\text{zero}}$

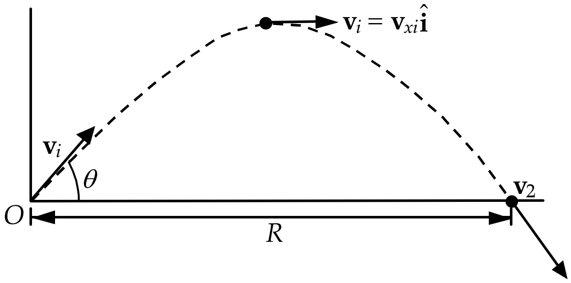

FIG. P11.17

(b) At the highest point of the trajectory,

$$x = \frac{1}{2}R = \frac{v_i^2 \sin 2\theta}{2g} \text{ and}$$

$$y = h_{\max} = \frac{(v_i \sin\theta)^2}{2g}$$

$$\begin{aligned}\mathbf{L}_1 &= \mathbf{r}_1 \times m\mathbf{v}_1 \\ &= \left[\frac{v_i^2 \sin 2\theta}{2g}\hat{\mathbf{i}} + \frac{(v_i \sin\theta)^2}{2g}\hat{\mathbf{j}}\right] \times mv_{xi}\hat{\mathbf{i}} \\ &= \boxed{\frac{-m(v_i \sin\theta)^2 v_i \cos\theta}{2g}\hat{\mathbf{k}}}\end{aligned}$$

(c) $$\begin{aligned}\mathbf{L}_2 &= R\hat{\mathbf{i}} \times m\mathbf{v}_2, \text{ where } R = \frac{v_i^2 \sin 2\theta}{g} \\ &= mR\hat{\mathbf{i}} \times \left(v_i \cos\theta\,\hat{\mathbf{i}} - v_i \sin\theta\,\hat{\mathbf{j}}\right) \\ &= -mRv_i \sin\theta\,\hat{\mathbf{k}} = \boxed{\frac{-mv_i^3 \sin 2\theta \sin\theta}{g}\hat{\mathbf{k}}}\end{aligned}$$

(d) The downward force of gravity exerts a torque in the $-z$ direction.

**P11.18** Whether we think of the Earth's surface as curved or flat, we interpret the problem to mean that the plane's line of flight extended is precisely tangent to the mountain at its peak, and nearly parallel to the wheat field. Let the positive $x$ direction be eastward, positive $y$ be northward, and positive $z$ be vertically upward.

(a) $$\mathbf{r} = (4.30 \text{ km})\hat{\mathbf{k}} = \left(4.30 \times 10^3 \text{ m}\right)\hat{\mathbf{k}}$$

$$\mathbf{p} = m\mathbf{v} = 12\,000 \text{ kg}\left(-175\hat{\mathbf{i}} \text{ m/s}\right) = -2.10 \times 10^6 \hat{\mathbf{i}} \text{ kg}\cdot\text{m/s}$$

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} = \left(4.30 \times 10^3 \hat{\mathbf{k}} \text{ m}\right) \times \left(-2.10 \times 10^6 \hat{\mathbf{i}} \text{ kg}\cdot\text{m/s}\right) = \boxed{\left(-9.03 \times 10^9 \text{ kg}\cdot\text{m}^2/\text{s}\right)\hat{\mathbf{j}}}$$

(b) $\boxed{\text{No}}$. $L = |\mathbf{r}||\mathbf{p}|\sin\theta = mv(r\sin\theta)$, and $r\sin\theta$ is the altitude of the plane. Therefore, $L =$ *constant* as the plane moves in level flight with constant velocity.

(c) $\boxed{\text{Zero}}$. The position vector from Pike's Peak to the plane is anti-parallel to the velocity of the plane. That is, it is directed along the same line and opposite in direction. Thus, $L = mvr\sin 180° = 0$.

**P11.19** The vector from $P$ to the falling ball is

$$\mathbf{r} = \mathbf{r}_i + \mathbf{v}_i t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{r} = \left(\ell\cos\theta\,\hat{\mathbf{i}} + \ell\sin\theta\,\hat{\mathbf{j}}\right) + 0 - \left(\frac{1}{2}gt^2\right)\hat{\mathbf{j}}$$

The velocity of the ball is

$$\mathbf{v} = \mathbf{v}_i + \mathbf{a}t = 0 - gt\hat{\mathbf{j}}$$

So $\mathbf{L} = \mathbf{r}\times m\mathbf{v}$

$$\mathbf{L} = m\left[\left(\ell\cos\theta\,\hat{\mathbf{i}} + \ell\sin\theta\,\hat{\mathbf{j}}\right) + 0 - \left(\frac{1}{2}gt^2\right)\hat{\mathbf{j}}\right]\times\left(-gt\hat{\mathbf{j}}\right)$$

$$\mathbf{L} = \boxed{-m\ell gt\cos\theta\,\hat{\mathbf{k}}}$$

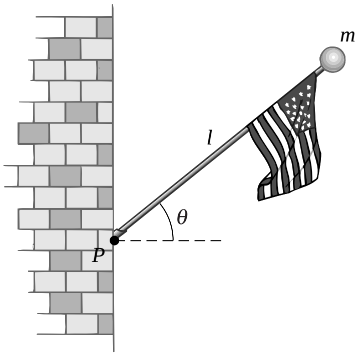

**FIG. P11.19**

**P11.20** In the vertical section of the hose, the water has zero angular momentum about our origin (point $O$ between the fireman's feet). As it leaves the nozzle, a parcel of mass $m$ has angular momentum:

$$L = |\mathbf{r}\times m\mathbf{v}| = mrv\sin 90.0° = m(1.30\text{ m})(12.5\text{ m/s})$$

$$L = \left(16.3\text{ m}^2/\text{s}\right)m$$

The torque on the hose is the rate of change in angular momentum. Thus,

$$\tau = \frac{dL}{dt} = \left(16.3\text{ m}^2/\text{s}\right)\frac{dm}{dt} = \left(16.3\text{ m}^2/\text{s}\right)(6.31\text{ kg/s}) = \boxed{103\text{ N}\cdot\text{m}}$$

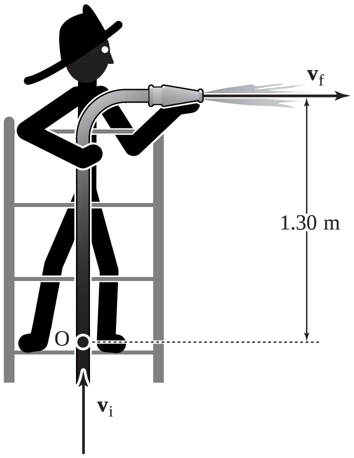

**FIG. P11.20**

## Section 11.3 **Angular Momentum of a Rotating Rigid Object**

***P11.21** $K = \dfrac{1}{2}I\omega^2 = \dfrac{1}{2}\dfrac{I^2\omega^2}{I} = \dfrac{L^2}{2I}$

**P11.22** The moment of inertia of the sphere about an axis through its center is

$$I = \frac{2}{5}MR^2 = \frac{2}{5}(15.0\text{ kg})(0.500\text{ m})^2 = 1.50\text{ kg}\cdot\text{m}^2$$

Therefore, the magnitude of the angular momentum is

$$L = I\omega = \left(1.50\text{ kg}\cdot\text{m}^2\right)(3.00\text{ rad/s}) = 4.50\text{ kg}\cdot\text{m}^2/\text{s}$$

Since the sphere rotates counterclockwise about the vertical axis, the angular momentum vector is directed upward in the $+z$ direction.

Thus, $\boxed{\mathbf{L} = \left(4.50\text{ kg}\cdot\text{m}^2/\text{s}\right)\hat{\mathbf{k}}}$.

**P11.23** (a) $L = I\omega = \left(\frac{1}{2}MR^2\right)\omega = \frac{1}{2}(3.00\text{ kg})(0.200\text{ m})^2(6.00\text{ rad/s}) = \boxed{0.360\text{ kg}\cdot\text{m}^2/\text{s}}$

(b) $L = I\omega = \left[\frac{1}{2}MR^2 + M\left(\frac{R}{2}\right)^2\right]\omega$

$$= \frac{3}{4}(3.00\text{ kg})(0.200\text{ m})^2(6.00\text{ rad/s}) = \boxed{0.540\text{ kg}\cdot\text{m}^2/\text{s}}$$

**P11.24** The total angular momentum about the center point is given by $L = I_h\omega_h + I_m\omega_m$

with $I_h = \frac{m_hL_h^2}{3} = \frac{60.0\text{ kg}(2.70\text{ m})^2}{3} = 146\text{ kg}\cdot\text{m}^2$

and $I_{m3} = \frac{m_mL_m^2}{3} = \frac{100\text{ kg}(4.50\text{ m})^2}{3} = 675\text{ kg}\cdot\text{m}^2$

In addition, $\omega_h = \frac{2\pi\text{ rad}}{12\text{ h}}\left(\frac{1\text{ h}}{3\,600\text{ s}}\right) = 1.45\times10^{-4}\text{ rad/s}$

while $\omega_m = \frac{2\pi\text{ rad}}{1\text{ h}}\left(\frac{1\text{ h}}{3\,600\text{ s}}\right) = 1.75\times10^{-3}\text{ rad/s}$

Thus, $L = 146\text{ kg}\cdot\text{m}^2\left(1.45\times10^{-4}\text{ rad/s}\right) + 675\text{ kg}\cdot\text{m}^2\left(1.75\times10^{-3}\text{ rad/s}\right)$

or $\boxed{L = 1.20\text{ kg}\cdot\text{m}^2/\text{s}}$

**P11.25** (a) $I = \frac{1}{12}m_1L^2 + m_2(0.500)^2 = \frac{1}{12}(0.100)(1.00)^2 + 0.400(0.500)^2 = 0.108\,3\text{ kg}\cdot\text{m}^2$

$L = I\omega = 0.108\,3(4.00) = \boxed{0.433\text{ kg}\cdot\text{m}^2/\text{s}}$

(b) $I = \frac{1}{3}m_1L^2 + m_2R^2 = \frac{1}{3}(0.100)(1.00)^2 + 0.400(1.00)^2 = 0.433$

$L = I\omega = 0.433(4.00) = \boxed{1.73\text{ kg}\cdot\text{m}^2/\text{s}}$

***P11.26** $\sum F_x = ma_x$: $+f_s = ma_x$

We must use the center of mass as the axis in

$\sum\tau = I\alpha$: $F_g(0) - n(77.5\text{ cm}) + f_s(88\text{ cm}) = 0$

$\sum F_y = ma_y$: $+n - F_g = 0$

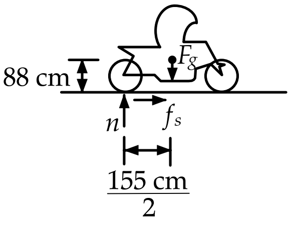

**FIG. P11.26**

We combine the equations by substitution:

$$-mg(77.5\text{ cm}) + ma_x(88\text{ cm}) = 0$$

$$a_x = \frac{\left(9.80\text{ m/s}^2\right)77.5\text{ cm}}{88\text{ cm}} = \boxed{8.63\text{ m/s}^2}$$

***P11.27** We require $a_c = g = \frac{v^2}{r} = \omega^2 r$

$$\omega = \sqrt{\frac{g}{r}} = \sqrt{\frac{\left(9.80\ \text{m/s}^2\right)}{100\ \text{m}}} = 0.313\ \text{rad/s}$$
$$I = Mr^2 = 5\times10^4\ \text{kg}(100\ \text{m})^2 = 5\times10^8\ \text{kg}\cdot\text{m}^2$$

(a) $L = I\omega = 5\times10^8\ \text{kg}\cdot\text{m}^2\ 0.313/\text{s} = \boxed{1.57\times10^8\ \text{kg}\cdot\text{m}^2/\text{s}}$

(c) $\sum \tau = I\alpha = \frac{I\left(\omega_f - \omega_i\right)}{\Delta t}$

$\sum \tau\Delta t = I\omega_f - I\omega_i = L_f - L_i$

This is the angular impulse-angular momentum theorem.

(b) $\Delta t = \frac{L_f - 0}{\sum \tau} = \frac{1.57\times10^8\ \text{kg}\cdot\text{m}^2/\text{s}}{2(125\ \text{N})(100\ \text{m})} = \boxed{6.26\times10^3\ \text{s}} = 1.74\ \text{h}$

## Section 11.4 **Conservation of Angular Momentum**

**P11.28** (a) From conservation of angular momentum for the system of two cylinders:

$(I_1 + I_2)\omega_f = I_1\omega_i$ or $\omega_f = \boxed{\frac{I_1}{I_1 + I_2}\omega_i}$

(b) $K_f = \frac{1}{2}(I_1 + I_2)\omega_f^2$ and $K_i = \frac{1}{2}I_1\omega_i^2$

so $\frac{K_f}{K_i} = \frac{\frac{1}{2}(I_1 + I_2)}{\frac{1}{2}I_1\omega_i^2}\left(\frac{I_1}{I_1 + I_2}\omega_i\right)^2 = \boxed{\frac{I_1}{I_1 + I_2}\ \text{which is less than 1}}$.

**P11.29** $I_i\omega_i = I_f\omega_f$: $\left(250\ \text{kg}\cdot\text{m}^2\right)(10.0\ \text{rev/min}) = \left[250\ \text{kg}\cdot\text{m}^2 + 25.0\ \text{kg}(2.00\ \text{m})^2\right]\omega_2$

$\omega_2 = \boxed{7.14\ \text{rev/min}}$

**P11.30** (a) The total angular momentum of the system of the student, the stool, and the weights about the axis of rotation is given by

$$I_{\text{total}} = I_{\text{weights}} + I_{\text{student}} = 2\left(mr^2\right) + 3.00\ \text{kg}\cdot\text{m}^2$$

Before: $r = 1.00\ \text{m}$.

Thus, $I_i = 2(3.00\ \text{kg})(1.00\ \text{m})^2 + 3.00\ \text{kg}\cdot\text{m}^2 = 9.00\ \text{kg}\cdot\text{m}^2$

After: $r = 0.300\ \text{m}$

Thus, $I_f = 2(3.00\ \text{kg})(0.300\ \text{m})^2 + 3.00\ \text{kg}\cdot\text{m}^2 = 3.54\ \text{kg}\cdot\text{m}^2$

We now use conservation of angular momentum.

$$I_f\omega_f = I_i\omega_i$$

or $$\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i = \left(\frac{9.00}{3.54}\right)(0.750\ \text{rad/s}) = \boxed{1.91\ \text{rad/s}}$$

(b) $$K_i = \frac{1}{2}I_i\omega_i^2 = \frac{1}{2}\left(9.00\ \text{kg}\cdot\text{m}^2\right)(0.750\ \text{rad/s})^2 = \boxed{2.53\ \text{J}}$$

$$K_f = \frac{1}{2}I_f\omega_f^2 = \frac{1}{2}\left(3.54\ \text{kg}\cdot\text{m}^2\right)(1.91\ \text{rad/s})^2 = \boxed{6.44\ \text{J}}$$

**P11.31** (a) Let $M =$ mass of rod and $m =$ mass of each bead. From $I_i\omega_i = I_f\omega_f$, we have

$$\left[\frac{1}{12}M\ell^2 + 2mr_1^2\right]\omega_i = \left[\frac{1}{12}M\ell^2 + 2mr_2^2\right]\omega_f$$

When $\ell = 0.500\ \text{m}$, $r_1 = 0.100\ \text{m}$, $r_2 = 0.250\ \text{m}$, and with other values as stated in the problem, we find

$$\boxed{\omega_f = 9.20\ \text{rad/s}}.$$

(b) Since there is no external torque on the rod,

$$L = \text{ constant and } \boxed{\omega \text{ is unchanged}}.$$

***P11.32** Let $M$ represent the mass of all the ribs together and $L$ the length of each. The original moment of inertia is $\frac{1}{3}ML^2$. The final effective length of each rib is $L\sin 22.5°$ and the final moment of inertia is $\frac{1}{3}M(L\sin 22.5°)^2$ angular momentum of the umbrella is conserved:

$$\frac{1}{3}ML^2\omega_i = \frac{1}{3}ML^2\sin^2 22.5°\,\omega_f$$

$$\omega_f = \frac{1.25\ \text{rad/s}}{\sin^2 22.5°} = \boxed{8.54\ \text{rad/s}}$$

**P11.33** (a) The table turns opposite to the way the woman walks, so its angular momentum cancels that of the woman. From conservation of angular momentum for the system of the woman and the turntable, we have $L_f = L_i = 0$

so, $$L_f = I_{\text{woman}}\omega_{\text{woman}} + I_{\text{table}}\omega_{\text{table}} = 0$$

and $$\omega_{\text{table}} = \left(-\frac{I_{\text{woman}}}{I_{\text{table}}}\right)\omega_{\text{woman}} = \left(-\frac{m_{\text{woman}}r^2}{I_{\text{table}}}\right)\left(\frac{v_{\text{woman}}}{r}\right) = -\frac{m_{\text{woman}}rv_{\text{woman}}}{I_{\text{table}}}$$

$$\omega_{\text{table}} = -\frac{60.0\text{ kg}(2.00\text{ m})(1.50\text{ m/s})}{500\text{ kg}\cdot\text{m}^2} = -0.360\text{ rad/s}$$

or $$\omega_{\text{table}} = \boxed{0.360\text{ rad/s (counterclockwise)}}$$

(b) $$\text{work done} = \Delta K = K_f - 0 = \frac{1}{2}m_{\text{woman}}v_{\text{woman}}^2 + \frac{1}{2}I\omega_{\text{table}}^2$$

$$W = \frac{1}{2}(60\text{ kg})(1.50\text{ m/s})^2 + \frac{1}{2}\left(500\text{ kg}\cdot\text{m}^2\right)(0.360\text{ rad/s})^2 = \boxed{99.9\text{ J}}$$

**P11.34** When they touch, the center of mass is distant from the center of the larger puck by

$$y_{\text{CM}} = \frac{0 + 80.0\text{ g}(4.00\text{ cm} + 6.00\text{ cm})}{120\text{ g} + 80.0\text{ g}} = 4.00\text{ cm}$$

(a) $$L = r_1m_1v_1 + r_2m_2v_2 = 0 + \left(6.00\times10^{-2}\text{ m}\right)\left(80.0\times10^{-3}\text{ kg}\right)(1.50\text{ m/s}) = \boxed{7.20\times10^{-3}\text{ kg}\cdot\text{m}^2/\text{s}}$$

(b) The moment of inertia about the CM is

$$I = \left(\frac{1}{2}m_1r_1^2 + m_1d_1^2\right) + \left(\frac{1}{2}m_2r_2^2 + m_2d_2^2\right)$$
$$I = \frac{1}{2}(0.120\text{ kg})\left(6.00\times10^{-2}\text{ m}\right)^2 + (0.120\text{ kg})\left(4.00\times10^{-2}\right)^2$$
$$+\frac{1}{2}\left(80.0\times10^{-3}\text{ kg}\right)\left(4.00\times10^{-2}\text{ m}\right)^2 + \left(80.0\times10^{-3}\text{ kg}\right)\left(6.00\times10^{-2}\text{ m}\right)^2$$
$$I = 7.60\times10^{-4}\text{ kg}\cdot\text{m}^2$$

Angular momentum of the two-puck system is conserved: $L = I\omega$

$$\omega = \frac{L}{I} = \frac{7.20\times10^{-3}\text{ kg}\cdot\text{m}^2/\text{s}}{7.60\times10^{-4}\text{ kg}\cdot\text{m}^2} = \boxed{9.47\text{ rad/s}}$$

**P11.35** (a) $L_i = mv\ell \qquad \sum \tau_{\text{ext}} = 0$, so $L_f = L_i = \boxed{mv\ell}$

$$L_f = (m+M)v_f\ell$$

$$v_f = \left(\frac{m}{m+M}\right)v$$

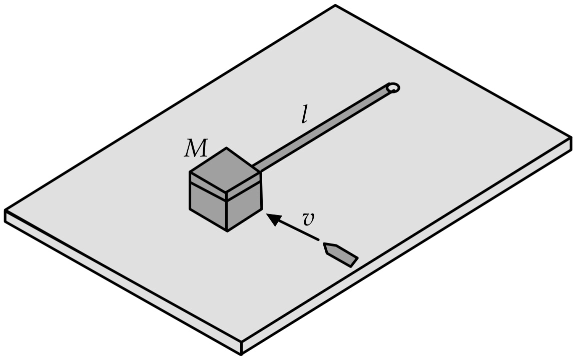

**FIG. P11.35**

(b) $K_i = \frac{1}{2}mv^2$

$$K_f = \frac{1}{2}(M+m)v_f^2$$

$v_f = \left(\frac{m}{M+m}\right)v \Rightarrow$ velocity of the bullet and block

Fraction of $K$ lost $= \dfrac{\frac{1}{2}mv^2 - \frac{1}{2}\frac{m^2}{M+m}v^2}{\frac{1}{2}mv^2} = \boxed{\dfrac{M}{M+m}}$

**P11.36** For one of the crew,

$\sum F_r = ma_r: \qquad n = \dfrac{mv^2}{r} = m\omega_i^2 r$

We require $\qquad n = mg$, so $\omega_i = \sqrt{\dfrac{g}{r}}$

Now, $\qquad I_i\omega_i = I_f\omega_f$

$$\left[5.00\times10^8\ \text{kg}\cdot\text{m}^2 + 150\times65.0\ \text{kg}\times(100\ \text{m})^2\right]\sqrt{\frac{g}{r}} = \left[5.00\times10^8\ \text{kg}\cdot\text{m}^2 + 50\times65.0\ \text{kg}(100\ \text{m})^2\right]\omega_f$$

$$\left(\frac{5.98\times10^8}{5.32\times10^8}\right)\sqrt{\frac{g}{r}} = \omega_f = 1.12\sqrt{\frac{g}{r}}$$

Now, $\qquad |a_r| = \omega_f^2 r = 1.26g = \boxed{12.3\ \text{m/s}^2}$

**P11.37** (a) Consider the system to consist of the wad of clay and the cylinder. No external forces acting on this system have a torque about the center of the cylinder. Thus, angular momentum of the system is conserved about the axis of the cylinder.

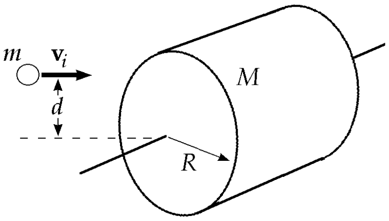

**FIG. P11.37**

$L_f = L_i: \qquad I\omega = mv_id$

or $\qquad \left[\frac{1}{2}MR^2 + mR^2\right]\omega = mv_id$

Thus, $\qquad \omega = \boxed{\dfrac{2mv_id}{(M+2m)R^2}}$.

(b) $\boxed{\text{No}}$. Some mechanical energy changes to internal energy in this perfectly inelastic collision.

***P11.38** (a) Let $\omega$ be the angular speed of the signboard when it is vertical.

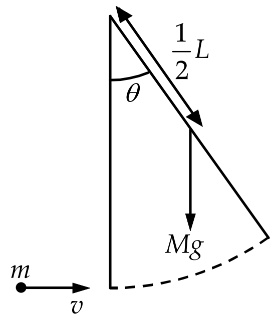

FIG. P11.38

$$\frac{1}{2}I\omega^2 = Mgh$$

$$\therefore \frac{1}{2}\left(\frac{1}{3}ML^2\right)\omega^2 = Mg\frac{1}{2}L(1-\cos\theta)$$

$$\therefore \omega = \sqrt{\frac{3g(1-\cos\theta)}{L}}$$

$$= \sqrt{\frac{3\left(9.80\text{ m/s}^2\right)(1-\cos 25.0°)}{0.50\text{ m}}}$$

$$= \boxed{2.35\text{ rad/s}}$$

(b) $I_f\omega_f = I_i\omega_i - mvL$ represents angular momentum conservation

$$\therefore \left(\frac{1}{3}ML^2 + mL^2\right)\omega_f = \frac{1}{3}ML^2\omega_i - mvL$$

$$\therefore \omega_f = \frac{\frac{1}{3}ML\omega_i - mv}{\left(\frac{1}{3}M + m\right)L}$$

$$= \frac{\frac{1}{3}(2.40\text{ kg})(0.5\text{ m})(2.347\text{ rad/s}) - (0.4\text{ kg})(1.6\text{ m/s})}{\left[\frac{1}{3}(2.40\text{ kg}) + 0.4\text{ kg}\right](0.5\text{ m})} = \boxed{0.498\text{ rad/s}}$$

(c) Let $h_{\text{CM}} =$ distance of center of mass from the axis of rotation.

$$h_{\text{CM}} = \frac{(2.40\text{ kg})(0.25\text{ m}) + (0.4\text{ kg})(0.50\text{ m})}{2.40\text{ kg} + 0.4\text{ kg}} = 0.285\,7\text{ m}.$$

Apply conservation of mechanical energy:

$$(M+m)gh_{\text{CM}}(1-\cos\theta) = \frac{1}{2}\left(\frac{1}{3}ML^2 + mL^2\right)\omega^2$$

$$\therefore \theta = \cos^{-1}\left[1 - \frac{\left(\frac{1}{3}M + m\right)L^2\omega^2}{2(M+m)gh_{\text{CM}}}\right]$$

$$= \cos^{-1}\left\{1 - \frac{\left[\frac{1}{3}(2.40\text{ kg}) + 0.4\text{ kg}\right](0.50\text{ m})^2(0.498\text{ rad/s})^2}{2(2.40\text{ kg} + 0.4\text{ kg})\left(9.80\text{ m/s}^2\right)(0.285\,7\text{ m})}\right\}$$

$$= \boxed{5.58°}$$

**P11.39** The meteor will slow the rotation of the Earth by the largest amount if its line of motion passes farthest from the Earth's axis. The meteor should be headed west and strike a point on the equator tangentially.

Let the $z$ axis coincide with the axis of the Earth with $+z$ pointing northward. Then, conserving angular momentum about this axis,

$$\sum \mathbf{L}_f = \sum \mathbf{L}_i \Rightarrow I\omega_f = I\omega_i + m\mathbf{v}\times\mathbf{r}$$

or
$$\frac{2}{5}MR^2\omega_f\hat{\mathbf{k}} = \frac{2}{5}MR^2\omega_i\hat{\mathbf{k}} - mvR\hat{\mathbf{k}}$$

Thus,
$$\omega_i - \omega_f = \frac{mvR}{\frac{2}{5}MR^2} = \frac{5mv}{2MR} \text{ or}$$

$$\omega_i - \omega_f = \frac{5\left(3.00\times10^{13}\text{ kg}\right)\left(30.0\times10^3\text{ m/s}\right)}{2\left(5.98\times10^{24}\text{ kg}\right)\left(6.37\times10^6\text{ m}\right)} = 5.91\times10^{-14}\text{ rad/s}$$

$$\boxed{\left|\Delta\omega_{\max}\right| \sim 10^{-13}\text{ rad/s}}$$

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## Section 11.5 The Motion of Gyroscopes and Tops

***P11.40** Angular momentum of the system of the spacecraft and the gyroscope is conserved. The gyroscope and spacecraft turn in opposite directions.

$$0 = I_1\omega_1 + I_2\omega_2: \quad -I_1\omega_1 = I_2\frac{\theta}{t}$$

$$-20\text{ kg}\cdot\text{m}^2(-100\text{ rad/s}) = 5\times10^5\text{ kg}\cdot\text{m}^2\left(\frac{30°}{t}\right)\left(\frac{\pi\text{ rad}}{180°}\right)$$

$$t = \frac{2.62\times10^5\text{ s}}{2\,000} = \boxed{131\text{ s}}$$

***P11.41**
$$I = \frac{2}{5}MR^2 = \frac{2}{5}\left(5.98\times10^{24}\text{ kg}\right)\left(6.37\times10^6\text{ m}\right)^2 = 9.71\times10^{37}\text{ kg}\cdot\text{m}^2$$

$$L = I\omega = 9.71\times10^{37}\text{ kg}\cdot\text{m}^2\left(\frac{2\pi\text{ rad}}{86\,400\text{ s}}\right) = 7.06\times10^{33}\text{ kg}\cdot\text{m}^2/\text{s}^2$$

$$\tau = L\omega_p = \left(7.06\times10^{33}\text{ kg}\cdot\text{m}^2/\text{s}\right)\left(\frac{2\pi\text{ rad}}{2.58\times10^4\text{ yr}}\right)\left(\frac{1\text{ yr}}{365.25\text{ d}}\right)\left(\frac{1\text{ d}}{86\,400\text{ s}}\right) = \boxed{5.45\times10^{22}\text{ N}\cdot\text{m}}$$

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## Section 11.6 Angular Momentum as a Fundamental Quantity

**P11.42** (a)
$$L = \frac{h}{2\pi} = mvr \text{ so } v = \frac{h}{2\pi mr} \qquad v = \frac{6.626\,1\times10^{-34}\text{ J}\cdot\text{s}}{2\pi\left(9.11\times10^{-31}\text{ kg}\right)\left(0.529\times10^{-10}\text{ m}\right)} = \boxed{2.19\times10^6\text{ m/s}}$$

(b)
$$K = \frac{1}{2}mv^2 = \frac{1}{2}\left(9.11\times10^{-31}\text{ kg}\right)\left(2.19\times10^6\text{ m/s}\right)^2 = \boxed{2.18\times10^{-18}\text{ J}}$$

(c)
$$\omega = \frac{L}{I} = \frac{\hbar}{mr^2} = \frac{1.055\times10^{-34}\text{ J}\cdot\text{s}}{\left(9.11\times10^{-31}\text{ kg}\right)\left(0.529\times10^{-10}\text{ m}\right)^2} = \boxed{4.13\times10^{16}\text{ rad/s}}$$

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## Additional Problems

***P11.43** First, we define the following symbols:

$I_P$ = moment of inertia due to mass of people on the equator
$I_E$ = moment of inertia of the Earth alone (without people)
$\omega$ = angular velocity of the Earth (due to rotation on its axis)
$T = \dfrac{2\pi}{\omega}$ = rotational period of the Earth (length of the day)
$R$ = radius of the Earth

The initial angular momentum of the system (before people start running) is

$$L_i = I_P\omega_i + I_E\omega_i = (I_P + I_E)\omega_i$$

When the Earth has angular speed $\omega$, the tangential speed of a point on the equator is $v_t = R\omega$. Thus, when the people run eastward along the equator at speed $v$ relative to the surface of the Earth, their tangential speed is $v_p = v_t + v = R\omega + v$ and their angular speed is $\omega_P = \dfrac{v_p}{R} = \omega + \dfrac{v}{R}$.

The angular momentum of the system after the people begin to run is

$$L_f = I_P\omega_p + I_E\omega = I_P\left(\omega + \frac{v}{R}\right) + I_E\omega = (I_P + I_E)\omega + \frac{I_P v}{R}.$$

Since no external torques have acted on the system, angular momentum is conserved $\left(L_f = L_i\right)$, giving $(I_P + I_E)\omega + \dfrac{I_P v}{R} = (I_P + I_E)\omega_i$. Thus, the final angular velocity of the Earth is $\omega = \omega_i - \dfrac{I_P v}{(I_P + I_E)R} = \omega_i(1 - x) =$, where $x \equiv \dfrac{I_P v}{(I_P + I_E)R\omega_i}$.

The new length of the day is $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{\omega_i(1-x)} = \dfrac{T_i}{1-x} \approx T_i(1+x)$, so the increase in the length of the day is $\Delta T = T - T_i \approx T_i x = T_i\left[\dfrac{I_P v}{(I_P + I_E)R\omega_i}\right]$. Since $\omega_i = \dfrac{2\pi}{T_i}$, this may be written as $\Delta T \approx \dfrac{T_i^2 I_P v}{2\pi(I_P + I_E)R}$.

To obtain a numeric answer, we compute

$$I_P = m_p R^2 = \left[\left(5.5\times10^9\right)(70\text{ kg})\right]\left(6.37\times10^6\text{ m}\right)^2 = 1.56\times10^{25}\text{ kg}\cdot\text{m}^2$$

and

$$I_E = \frac{2}{5}m_E R^2 = \frac{2}{5}\left(5.98\times10^{24}\text{ kg}\right)\left(6.37\times10^6\text{ m}\right)^2 = 9.71\times10^{37}\text{ kg}\cdot\text{m}^2.$$

Thus, $\Delta T \approx \dfrac{\left(8.64\times10^4\text{ s}\right)^2\left(1.56\times10^{25}\text{ kg}\cdot\text{m}^2\right)(2.5\text{ m/s})}{2\pi\left[\left(1.56\times10^{25} + 9.71\times10^{37}\right)\text{ kg}\cdot\text{m}^2\right]\left(6.37\times10^6\text{ m}\right)} = \boxed{7.50\times10^{-11}\text{ s}}$.

***P11.44** (a) $(K+U_s)_A=(K+U_s)_B$

$$0+mgy_A=\frac{1}{2}mv_B^2+0$$

$$v_B=\sqrt{2gy_A}=\sqrt{2\left(9.8\text{ m/s}^2\right)6.30\text{ m}}=\boxed{11.1\text{ m/s}}$$

(b) $L=mvr=76\text{ kg }11.1\text{ m/s }6.3\text{ m}=\boxed{5.32\times10^3\text{ kg}\cdot\text{m}^2/\text{s}}$ toward you along the axis of the channel.

(c) The wheels on his skateboard prevent any tangential force from acting on him. Then no torque about the axis of the channel acts on him and his angular momentum is constant. His legs convert chemical into mechanical energy. They do work to increase his kinetic energy. The normal force acts forward on his body on its rising trajectory, to increase his linear momentum.

(d) $L=mvr$ $\qquad v=\dfrac{5.32\times10^3\text{ kg}\cdot\text{m}^2/\text{s}}{76\text{ kg }5.85\text{ m}}=\boxed{12.0\text{ m/s}}$

(e) $\left(K+U_g\right)_B+W=\left(K+U_g\right)_C$

$$\frac{1}{2}76\text{ kg}(11.1\text{ m/s})^2+0+W=\frac{1}{2}76\text{ kg}(12.0\text{ m/s})^2+76\text{ kg }9.8\text{ m/s}^2\text{ }0.45\text{ m}$$

$$W=5.44\text{ kJ}-4.69\text{ kJ}+335\text{ J}=\boxed{1.08\text{ kJ}}$$

(f) $\left(K+U_g\right)_C=\left(K+U_g\right)_D$

$$\frac{1}{2}76\text{ kg}(12.0\text{ m/s})^2+0=\frac{1}{2}76\text{ kg}v_D^2+76\text{ kg }9.8\text{ m/s}^2\text{ }5.85\text{ m}$$

$$v_D=\boxed{5.34\text{ m/s}}$$

(g) Let point *E* be the apex of his flight:

$$\left(K+U_g\right)_D=\left(K+U_g\right)_E$$

$$\frac{1}{2}76\text{ kg}(5.34\text{ m/s})^2+0=0+76\text{ kg}\left(9.8\text{ m/s}^2\right)(y_E-y_D)$$

$$(y_E-y_D)=\boxed{1.46\text{ m}}$$

(h) For the motion between takeoff and touchdown

$$y_f=y_i+v_{yi}t+\frac{1}{2}a_yt^2$$

$$-2.34\text{ m}=0+5.34\text{ m/s}\,t-4.9\text{ m/s}^2\,t^2$$

$$t=\frac{-5.34\pm\sqrt{5.34^2+4(4.9)(2.34)}}{-9.8}=\boxed{1.43\text{ s}}$$

(i) This solution is more accurate. In chapter 8 we modeled the normal force as constant while the skateboarder stands up. Really it increases as the process goes on.

**P11.45** (a)

$$I=\sum m_i r_i^2$$
$$=m\left(\frac{4d}{3}\right)^2+m\left(\frac{d}{3}\right)^2+m\left(\frac{2d}{3}\right)^2$$
$$=\boxed{7m\frac{d^2}{3}}$$

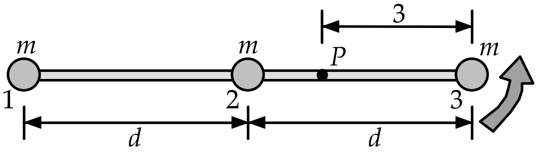


**FIG. P11.45**

(b) Think of the whole weight, $3mg$, acting at the center of gravity.

$$\boldsymbol{\tau}=\mathbf{r}\times\mathbf{F}=\left(\frac{d}{3}\right)\left(-\hat{\mathbf{i}}\right)\times 3mg\left(-\hat{\mathbf{j}}\right)=\boxed{(mgd)\hat{\mathbf{k}}}$$

(c) $$\alpha=\frac{\tau}{I}=\frac{3mgd}{7md^2}=\boxed{\frac{3g}{7d}\text{ counterclockwise}}$$

(d) $$a=\alpha r=\left(\frac{3g}{7d}\right)\left(\frac{2d}{3}\right)=\boxed{\frac{2g}{7}\text{ up}}$$

The angular acceleration is not constant, but energy is.

$$(K+U)_i+\Delta E=(K+U)_f$$
$$0+(3m)g\left(\frac{d}{3}\right)+0=\frac{1}{2}I\omega_f^2+0$$

(e) maximum kinetic energy $=\boxed{mgd}$

(f) $$\omega_f=\boxed{\sqrt{\frac{6g}{7d}}}$$

(g) $$L_f=I\omega_f=\frac{7md^2}{3}\sqrt{\frac{6g}{7d}}=\boxed{\left(\frac{14g}{3}\right)^{1/2}md^{3/2}}$$

(h) $$v_f=\omega_f r=\sqrt{\frac{6g}{7d}}\frac{d}{3}=\boxed{\sqrt{\frac{2gd}{21}}}$$

**P11.46** (a) The radial coordinate of the sliding mass is $r(t) = (0.012\,5\text{ m/s})t$. Its angular momentum is

$$L = mr^2\omega = (1.20\text{ kg})(2.50\text{ rev/s})(2\pi\text{ rad/rev})(0.012\,5\text{ m/s})^2 t^2$$

or $$L = \left(2.95\times10^{-3}\text{ kg}\cdot\text{m}^2/\text{s}^3\right)t^2$$

The drive motor must supply torque equal to the rate of change of this angular momentum:

$$\tau = \frac{dL}{dt} = \left(2.95\times10^{-3}\text{ kg}\cdot\text{m}^2/\text{s}^3\right)(2t) = \boxed{(0.005\,89\text{ W})t}$$

(b) $\tau_f = (0.005\,89\text{ W})(440\text{ s}) = \boxed{2.59\text{ N}\cdot\text{m}}$

(c) $\mathcal{P} = \tau\omega = (0.005\,89\text{ W})t(5\pi\text{ rad/s}) = \boxed{(0.092\,5\text{ W/s})t}$

(d) $\mathcal{P}_f = (0.092\,5\text{ W/s})(440\text{ s}) = \boxed{40.7\text{ W}}$

(e) $T = m\dfrac{v^2}{r} = mr\omega^2 = (1.20\text{ kg})(0.012\,5\text{ m/s})t(5\pi\text{ rad/s})^2 = \boxed{(3.70\text{ N/s})t}$

(f) $W = \displaystyle\int_0^{440\text{ s}} \mathcal{P}dt = \int_0^{440\text{ s}} (0.092\,5\text{ W/s})t\,dt = \frac{1}{2}\left(0.092\,5\text{ J/s}^2\right)(440\text{ s})^2 = \boxed{8.96\text{ kJ}}$

(g) The power the brake injects into the sliding block through the string is

$$\mathcal{P}_b = \mathbf{F}\cdot\mathbf{v} = Tv\cos180° = -(3.70\text{ N/s})t(0.012\,5\text{ m/s}) = -(0.046\,3\text{ W/s})t = \frac{dW_b}{dt}$$

$$W_b = \int_0^{440\text{ s}} \mathcal{P}_b dt = -\int_0^{440\text{ s}} (0.046\,3\text{ W/s})t\,dt$$

$$= -\frac{1}{2}(0.046\,3\text{ W/s})(440\text{ s})^2 = \boxed{-4.48\text{ kJ}}$$

(h) $\sum W = W + W_b = 8.96\text{ kJ} - 4.48\text{ kJ} = \boxed{4.48\text{ kJ}}$

Just half of the work required to increase the angular momentum goes into rotational kinetic energy. The other half becomes internal energy in the brake.

**P11.47** Using conservation of angular momentum, we have

$$L_{\text{aphelion}} = L_{\text{perihelion}} \text{ or } \left(mr_a^2\right)\omega_a = \left(mr_p^2\right)\omega_p.$$

Thus, $\left(mr_a^2\right)\dfrac{v_a}{r_a} = \left(mr_p^2\right)\dfrac{v_p}{r_p}$ giving

$$r_a v_a = r_p v_p \text{ or } v_a = \frac{r_p}{r_a}v_p = \frac{0.590\text{ AU}}{35.0\text{ AU}}(54.0\text{ km/s}) = \boxed{0.910\text{ km/s}}.$$

**P11.48** (a) $\sum \tau = MgR - MgR = \boxed{0}$

(b) $\sum \tau = \frac{dL}{dt}$, and since $\sum \tau = 0$, $L =$ constant.

Since the total angular momentum of the system is zero, the $\boxed{\text{monkey and bananas move upward with the same speed}}$ at any instant, and $\boxed{\text{he will not reach the bananas}}$ (until they get tangled in the pulley). Also, since the tension in the rope is the same on both sides, Newton's second law applied to the monkey and bananas give the same acceleration upwards.

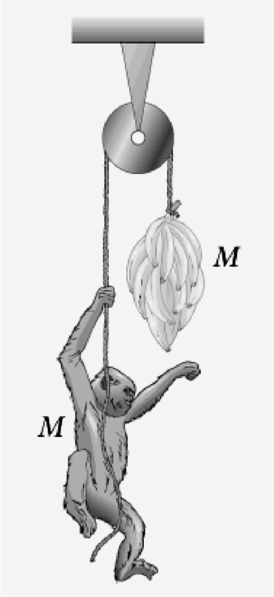


**FIG. P11.48**

**P11.49** (a) $\tau = |\mathbf{r} \times \mathbf{F}| = |\mathbf{r}||\mathbf{F}| \sin 180° = 0$

Angular momentum is conserved.

$$L_f = L_i$$
$$mrv = mr_i v_i$$
$$v = \boxed{\frac{r_i v_i}{r}}$$

(b) $T = \frac{mv^2}{r} = \boxed{\frac{m(r_i v_i)^2}{r^3}}$

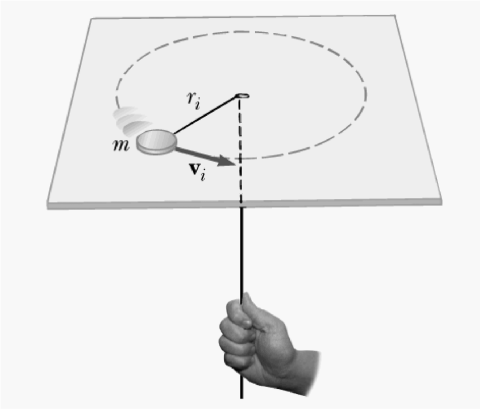


**FIG. P11.49**

(c) The work is done by the centripetal force in the *negative* direction.
Method 1:

$$W = \int F \cdot d\ell = -\int T dr' = -\int_{r_i}^{r} \frac{m(r_i v_i)^2}{(r')^3} dr' = \left. \frac{m(r_i v_i)^2}{2(r')^2} \right|_{r_i}^{r}$$

$$= \frac{m(r_i v_i)^2}{2} \left( \frac{1}{r^2} - \frac{1}{r_i^2} \right) = \boxed{\frac{1}{2} m v_i^2 \left( \frac{r_i^2}{r^2} - 1 \right)}$$

Method 2:

$$W = \Delta K = \frac{1}{2} m v^2 - \frac{1}{2} m v_i^2 = \boxed{\frac{1}{2} m v_i^2 \left( \frac{r_i^2}{r^2} - 1 \right)}$$

(d) Using the data given, we find

$$v = \boxed{4.50 \text{ m/s}} \qquad T = \boxed{10.1 \text{ N}} \qquad W = \boxed{0.450 \text{ J}}$$

**P11.50** (a) Angular momentum is conserved:

$$\frac{mv_i d}{2}=\left(\frac{1}{12}Md^2+m\left(\frac{d}{2}\right)^2\right)\omega$$

$$\omega=\boxed{\frac{6mv_i}{Md+3md}}$$

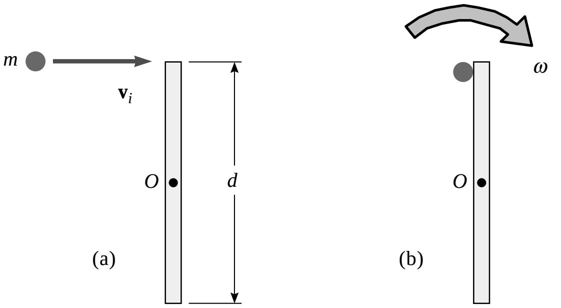


**FIG. P11.50**

(b) The original energy is $\frac{1}{2}mv_i^2$.

The final energy is

$$\frac{1}{2}I\omega^2=\frac{1}{2}\left(\frac{1}{12}Md^2+\frac{md^2}{4}\right)\frac{36m^2v_i^2}{(Md+3md)^2}=\frac{3m^2v_i^2d}{2(Md+3md)}.$$

The loss of energy is

$$\frac{1}{2}mv_i^2-\frac{3m^2v_i^2d}{2(Md+3md)}=\frac{mMv_i^2d}{2(Md+3md)}$$

and the fractional loss of energy is

$$\frac{mMv_i^2d2}{2(Md+3md)mv_i^2}=\boxed{\frac{M}{M+3m}}.$$

**P11.51** (a) $L_i=m_1v_{1i}r_{1i}+m_2v_{2i}r_{2i}=2mv\left(\frac{d}{2}\right)$

$L_i=2(75.0\text{ kg})(5.00\text{ m/s})(5.00\text{ m})$

$L_i=\boxed{3\,750\text{ kg}\cdot\text{m}^2/\text{s}}$

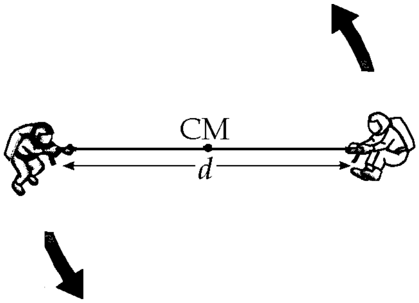


**FIG. P11.51**

(b) $K_i=\frac{1}{2}m_1v_{1i}^2+\frac{1}{2}m_2v_{2i}^2$

$K_i=2\left(\frac{1}{2}\right)(75.0\text{ kg})(5.00\text{ m/s})^2=\boxed{1.88\text{ kJ}}$

(c) Angular momentum is conserved: $L_f=L_i=\boxed{3\,750\text{ kg}\cdot\text{m}^2/\text{s}}$

(d) $v_f=\frac{L_f}{2(mr_f)}=\frac{3\,750\text{ kg}\cdot\text{m}^2/\text{s}}{2(75.0\text{ kg})(2.50\text{ m})}=\boxed{10.0\text{ m/s}}$

(e) $K_f=2\left(\frac{1}{2}\right)(75.0\text{ kg})(10.0\text{ m/s})^2=\boxed{7.50\text{ kJ}}$

(f) $W=K_f-K_i=\boxed{5.62\text{ kJ}}$

**P11.52** (a) $L_i = 2\left[Mv\left(\frac{d}{2}\right)\right] = \boxed{Mvd}$

(b) $K = 2\left(\frac{1}{2}Mv^2\right) = \boxed{Mv^2}$

(c) $L_f = L_i = \boxed{Mvd}$

(d) $v_f = \frac{L_f}{2Mr_f} = \frac{Mvd}{2M\left(\frac{d}{4}\right)} = \boxed{2v}$

(e) $K_f = 2\left(\frac{1}{2}Mv_f^2\right) = M(2v)^2 = \boxed{4Mv^2}$

(f) $W = K_f - K_i = \boxed{3Mv^2}$

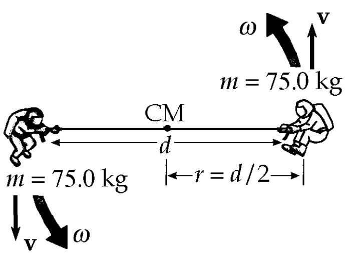


**FIG. P11.52**

***P11.53** The moment of inertia of the rest of the Earth is

$$I = \frac{2}{5}MR^2 = \frac{2}{5}5.98\times10^{24}\text{ kg}\left(6.37\times10^6\text{ m}\right)^2 = 9.71\times10^{37}\text{ kg}\cdot\text{m}^2.$$

For the original ice disks,

$$I = \frac{1}{2}Mr^2 = \frac{1}{2}2.30\times10^{19}\text{ kg}\left(6\times10^5\text{ m}\right)^2 = 4.14\times10^{30}\text{ kg}\cdot\text{m}^2.$$

For the final thin shell of water,

$$I = \frac{2}{3}Mr^2 = \frac{2}{3}2.30\times10^{19}\text{ kg}\left(6.37\times10^6\text{ m}\right)^2 = 6.22\times10^{32}\text{ kg}\cdot\text{m}^2.$$

Conservation of angular momentum for the spinning planet is expressed by $I_i\omega_i = I_f\omega_f$

$$\left(4.14\times10^{30} + 9.71\times10^{37}\right)\frac{2\pi}{86\,400\text{ s}} = \left(6.22\times10^{32} + 9.71\times10^{37}\right)\frac{2\pi}{(86\,400\text{ s} + \delta)}$$

$$\left(1 + \frac{\delta}{86\,400\text{ s}}\right)\left(1 + \frac{4.14\times10^{30}}{9.71\times10^{37}}\right) = \left(1 + \frac{6.22\times10^{32}}{9.71\times10^{37}}\right)$$

$$\frac{\delta}{86\,400\text{ s}} = \frac{6.22\times10^{32}}{9.71\times10^{37}} - \frac{4.14\times10^{30}}{9.71\times10^{37}}$$

$$\boxed{\delta = 0.550\text{ s}}$$

**P11.54** For the cube to tip over, the center of mass (CM) must rise so that it is over the axis of rotation AB. To do this, the CM must be raised a distance of $a\left(\sqrt{2}-1\right)$.

$$\therefore Mga\left(\sqrt{2}-1\right)=\frac{1}{2}I_{\text{cube}}\omega^2$$

From conservation of angular momentum,

$$\frac{4a}{3}mv=\left(\frac{8Ma^2}{3}\right)\omega$$

$$\omega=\frac{mv}{2Ma}$$

$$\frac{1}{2}\left(\frac{8Ma^2}{3}\right)\frac{m^2v^2}{4M^2a^2}=Mga\left(\sqrt{2}-1\right)$$

$$v=\boxed{\frac{M}{m}\sqrt{3ga\left(\sqrt{2}-1\right)}}$$

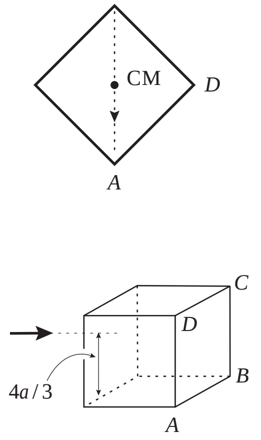


**FIG. P11.54**

**P11.55** Angular momentum is conserved during the inelastic collision.

$$Mva=I\omega$$

$$\omega=\frac{Mva}{I}=\frac{3v}{8a}$$

The condition, that the box falls off the table, is that the center of mass must reach its maximum height as the box rotates, $h_{\max}=a\sqrt{2}$. Using conservation of energy:

$$\frac{1}{2}I\omega^2=Mg\left(a\sqrt{2}-a\right)$$

$$\frac{1}{2}\left(\frac{8Ma^2}{3}\right)\left(\frac{3v}{8a}\right)^2=Mg\left(a\sqrt{2}-a\right)$$

$$v^2=\frac{16}{3}ga\left(\sqrt{2}-1\right)$$

$$v=\boxed{4\left[\frac{ga}{3}\left(\sqrt{2}-1\right)\right]^{1/2}}$$

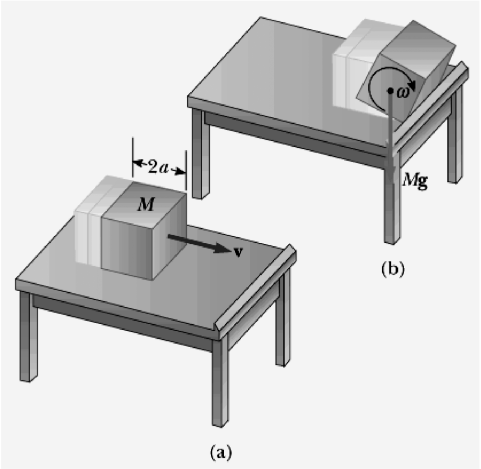


**FIG. P11.55**

**P11.56** (a) The net torque is zero at the point of contact, so the angular momentum before and after the collision must be equal.

$$\left(\frac{1}{2}MR^2\right)\omega_i=\left(\frac{1}{2}MR^2\right)\omega+\left(MR^2\right)\omega \qquad \omega=\boxed{\frac{\omega_i}{3}}$$

(b)
$$\frac{\Delta E}{E}=\frac{\frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{\omega_i}{3}\right)^2+\frac{1}{2}M\left(\frac{R\omega_i}{3}\right)^2-\frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega_i^2}{\frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega_i^2}=\boxed{-\frac{2}{3}}$$

**P11.57** (a) $\Delta t = \frac{\Delta p}{f} = \frac{Mv}{\mu Mg} = \frac{MR\omega}{\mu Mg} = \boxed{\frac{R\omega_i}{3\mu g}}$

(b) $W = \Delta K = \frac{1}{2}I\omega^2 = \frac{1}{18}MR^2\omega_i^2$ (See Problem 11.56)

$\mu Mgx = \frac{1}{18}MR^2\omega_i^2 \qquad \boxed{x = \frac{R^2\omega_i^2}{18\mu g}}$

## ANSWERS TO EVEN PROBLEMS

**P11.2** (a) 740 $\text{cm}^2$; (b) 59.5 cm

**P11.4** (a) 168°; (b) 11.9° principal value;
(c) Only the first is unambiguous.

**P11.6** No; see the solution

**P11.8** (a) $(-7.00\ \text{N}\cdot\text{m})\hat{\mathbf{k}}$; (b) $(11.0\ \text{N}\cdot\text{m})\hat{\mathbf{k}}$

**P11.10** see the solution

**P11.12** $\left(-22.0\ \text{kg}\cdot\text{m}^2/\text{s}\right)\hat{\mathbf{k}}$

**P11.14** see the solution

**P11.16** (a) $3.14\ \text{N}\cdot\text{m}$; (b) $(0.400\ \text{kg}\cdot\text{m})v$;
(c) $7.85\ \text{m/s}^2$

**P11.18** (a) $\left(+9.03\times10^{9}\ \text{kg}\cdot\text{m}^2/\text{s}\right)$ south; (b) No;
(c) 0

**P11.20** $103\ \text{N}\cdot\text{m}$

**P11.22** $\left(4.50\ \text{kg}\cdot\text{m}^2/\text{s}\right)$ up

**P11.24** $1.20\ \text{kg}\cdot\text{m}^2/\text{s}$ perpendicularly into the clock face

**P11.26** $8.63\ \text{m/s}^2$

**P11.28** (a) $\frac{I_1\omega_i}{I_1+I_2}$; (b) $\frac{K_f}{K_i} = \frac{I_1}{I_1+I_2}$

**P11.30** (a) $1.91\ \text{rad/s}$; (b) 2.53 J; 6.44 J

**P11.32** $8.54\ \text{rad/s}$

**P11.34** (a) $7.20\times10^{-3}\ \text{kg}\cdot\text{m}^2/\text{s}$; (b) $9.47\ \text{rad/s}$

**P11.36** $12.3\ \text{m/s}^2$

**P11.38** (a) $2.35\ \text{rad/s}$; (b) $0.498\ \text{rad/s}$; (c) 5.58°

**P11.40** 131 s

**P11.42** (a) $2.19\times10^{6}\ \text{m/s}$; (b) $2.18\times10^{-18}$ J;
(c) $4.13\times10^{16}\ \text{rad/s}$

**P11.44** (a) $11.1\ \text{m/s}$; (b) $5.32\times10^{3}\ \text{kg}\cdot\text{m}^2/\text{s}$;
(c) see the solution; (d) $12.0\ \text{m/s}$;
(e) 1.08 kJ; (f) $5.34\ \text{m/s}$; (g) 1.46 m;
(h) 1.43 s; (i) see the solution

**P11.46** (a) $(0.005\,89\ \text{W})t$; (b) $2.59\ \text{N}\cdot\text{m}$;
(c) $(0.092\,5\ \text{W/s})t$; (d) 40.7 W;
(e) $(3.70\ \text{N/s})t$; (f) 8.96 kJ; (g) −4.48 kJ
(h) +4.48 kJ

**P11.48** (a) 0; (b) 0; no

**P11.50** (a) $\frac{6mv_i}{Md+3md}$; (b) $\frac{M}{M+3m}$

**P11.52** (a) $Mvd$; (b) $Mv^2$; (c) $Mvd$; (d) $2v$;
(e) $4Mv^2$; (f) $3Mv^2$

**P11.54** $\frac{M}{m}\sqrt{3ga\left(\sqrt{2}-1\right)}$

**P11.56** (a) $\frac{\omega_i}{3}$; (b) $\frac{\Delta E}{E} = -\frac{2}{3}$

# 12

# Static Equilibrium and Elasticity

### CHAPTER OUTLINE

12.1 The Conditions for Equilibrium
12.2 More on the Center of Gravity
12.3 Examples of Rigid Objects in Static Equilibrium
12.4 Elastic Properties of Solids

## ANSWERS TO QUESTIONS

**Q12.1** When you bend over, your center of gravity shifts forward. Once your CG is no longer over your feet, gravity contributes to a nonzero net torque on your body and you begin to rotate.

**Q12.2** Yes, it can. Consider an object on a spring oscillating back and forth. In the center of the motion both the sum of the torques and the sum of the forces acting on the object are (separately) zero. Again, a meteoroid flying freely through interstellar space feels essentially no forces and keeps moving with constant velocity.

**Q12.3** No—one condition for equilibrium is that $\sum \mathbf{F} = 0$. For this to be true with only a single force acting on an object, that force would have to be of zero magnitude; so really no forces act on that object.

**Q12.4** (a) Consider pushing up with one hand on one side of a steering wheel and pulling down equally hard with the other hand on the other side. A pair of equal-magnitude oppositely-directed forces applied at different points is called a couple.

(b) An object in free fall has a non-zero net force acting on it, but a net torque of zero about its center of mass.

**Q12.5** No. If the torques are all in the same direction, then the net torque cannot be zero.

**Q12.6** (a) Yes, provided that its angular momentum is constant.

(b) Yes, provided that its linear momentum is constant.

**Q12.7** A V-shaped boomerang, a barstool, an empty coffee cup, a satellite dish, and a curving plastic slide at the edge of a swimming pool each have a center of mass that is not within the bulk of the object.

**Q12.8** Suspend the plywood from the nail, and hang the plumb bob from the nail. Trace on the plywood along the string of the plumb bob. Now suspend the plywood with the nail through a different point on the plywood, not along the first line you drew. Again hang the plumb bob from the nail and trace along the string. The center of gravity is located halfway through the thickness of the plywood under the intersection of the two lines you drew.

**Q12.9** The center of gravity must be directly over the point where the chair leg contacts the floor. That way, no torque is applied to the chair by gravity. The equilibrium is unstable.

**Q12.10** She can be correct. If the dog stands on a relatively thick scale, the dog's legs on the ground might support more of its weight than its legs on the scale. She can check for and if necessary correct for this error by having the dog stand like a bridge with two legs on the scale and two on a book of equal thickness—a physics textbook is a good choice.

**Q12.11** If their base areas are equal, the tall crate will topple first. Its center of gravity is higher off the incline than that of the shorter crate. The taller crate can be rotated only through a smaller angle before its center of gravity is no longer over its base.

**Q12.12** The free body diagram demonstrates that it is necessary to have friction on the ground to counterbalance the normal force of the wall and to keep the base of the ladder from sliding. Interestingly enough, if there is friction on the floor *and* on the wall, it is not possible to determine whether the ladder will slip from the equilibrium conditions alone.

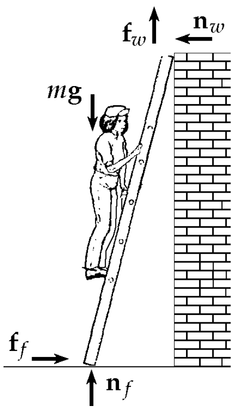


**FIG. Q12.12**

**Q12.13** When you lift a load with your back, your back muscles must supply the torque not only to rotate your upper body to a vertical position, but also to lift the load. Since the distance from the pivot—your hips—to the load—essentially your shoulders—is great, the force required to supply the lifting torque is very large. When lifting from your knees, your back muscles need only keep your back straight. The force required to do that is much smaller than when lifting with your back, as the torque required is small, because the moment arm of the load is small—the line of action of the load passes close to your hips. When you lift from your knees, your much stronger leg and hip muscles do the work.

**Q12.14** Shear deformation.

**Q12.15** The vertical columns experience simple compression due to gravity acting upon their mass. The horizontal slabs, however, suffer significant shear stress due to gravity. The bottom surface of a sagging lintel is under tension. Stone is much stronger under compression than under tension, so horizontal slabs are more likely to fail.

## SOLUTIONS TO PROBLEMS

### Section 12.1 **The Conditions for Equilibrium**

**P12.1** To hold the bat in equilibrium, the player must exert both a force and a torque on the bat to make

$$\sum F_x = \sum F_y = 0 \text{ and } \sum \tau = 0$$

$\sum F_y = 0 \Rightarrow F - 10.0\text{ N} = 0$, or the player must exert a net upward force of $F = \boxed{10.0\text{ N}}$

To satisfy the second condition of equilibrium, the player must exert an applied torque $\tau_a$ to make
$\sum \tau = \tau_a - (0.600\text{ m})(10.0\text{ N}) = 0$. Thus, the required torque is

$$\tau_a = +6.00\text{ N}\cdot\text{m} \text{ or } \boxed{6.00\text{ N}\cdot\text{m counterclockwise}}$$

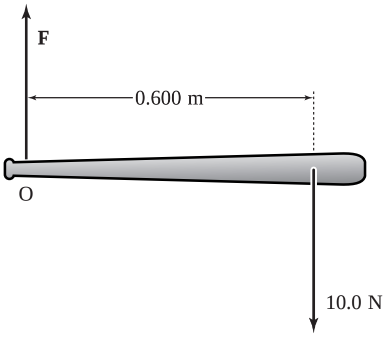


FIG. P12.1

**P12.2** Use distances, angles, and forces as shown. The conditions of equilibrium are:

$$\sum F_y = 0 \Rightarrow \boxed{F_y + R_y - F_g = 0}$$

$$\sum F_x = 0 \Rightarrow \boxed{F_x - R_x = 0}$$

$$\sum \tau = 0 \Rightarrow \boxed{F_y \ell \cos\theta - F_g\left(\frac{\ell}{2}\right)\cos\theta - F_x \ell \sin\theta = 0}$$

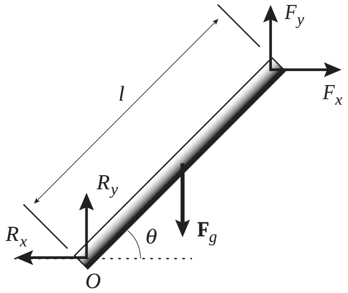


FIG. P12.2

**P12.3** Take torques about *P*.

$$\sum \tau_p = -n_0\left[\frac{\ell}{2} + d\right] + m_1 g\left[\frac{\ell}{2} + d\right] + m_b g d - m_2 g x = 0$$

We want to find *x* for which $n_0 = 0$.

$$x = \frac{(m_1 g + m_b g)d + m_1 g \frac{\ell}{2}}{m_2 g} = \boxed{\frac{(m_1 + m_b)d + m_1 \frac{\ell}{2}}{m_2}}$$

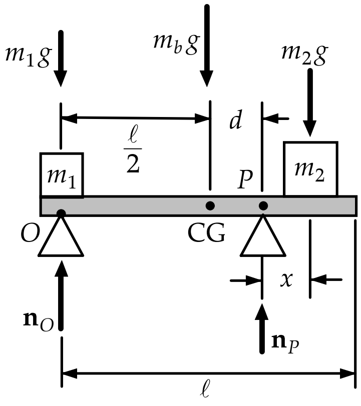


FIG. P12.3

## Section 12.2 **More on the Center of Gravity**

**P12.4** The hole we can count as negative mass

$$x_{\text{CG}} = \frac{m_1 x_1 - m_2 x_2}{m_1 - m_2}$$

Call $\sigma$ the mass of each unit of pizza area.

$$x_{\text{CG}} = \frac{\sigma \pi R^2 0 - \sigma \pi \left(\frac{R}{2}\right)^2 \left(-\frac{R}{2}\right)}{\sigma \pi R^2 - \sigma \pi \left(\frac{R}{2}\right)^2}$$

$$x_{\text{CG}} = \frac{\frac{R}{8}}{\frac{3}{4}} = \boxed{\frac{R}{6}}$$

**P12.5** The coordinates of the center of gravity of piece 1 are

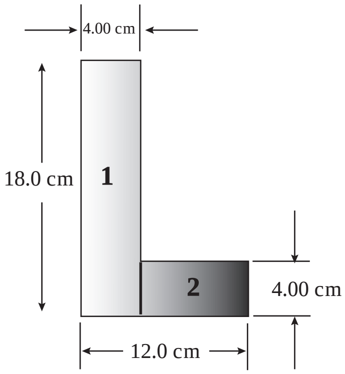


**FIG. P12.5**

$$x_1 = 2.00 \text{ cm} \text{ and } y_1 = 9.00 \text{ cm}.$$

The coordinates for piece 2 are

$$x_2 = 8.00 \text{ cm} \text{ and } y_2 = 2.00 \text{ cm}.$$

The area of each piece is

$$A_1 = 72.0 \text{ cm}^2 \text{ and } A_2 = 32.0 \text{ cm}^2.$$

And the mass of each piece is proportional to the area. Thus,

$$x_{\text{CG}} = \frac{\sum m_i x_i}{\sum m_i} = \frac{\left(72.0 \text{ cm}^2\right)(2.00 \text{ cm}) + \left(32.0 \text{ cm}^2\right)(8.00 \text{ cm})}{72.0 \text{ cm}^2 + 32.0 \text{ cm}^2} = \boxed{3.85 \text{ cm}}$$

and

$$y_{\text{CG}} = \frac{\sum m_i y_i}{\sum m_i} = \frac{\left(72.0 \text{ cm}^2\right)(9.00 \text{ cm}) + \left(32.0 \text{ cm}^2\right)(2.00 \text{ cm})}{104 \text{ cm}^2} = \boxed{6.85 \text{ cm}}.$$

**P12.6** Let $\sigma$ represent the mass-per-face area. A vertical strip at position $x$, with width $dx$ and height $\frac{(x-3.00)^2}{9}$ has mass

$$dm = \frac{\sigma(x-3.00)^2\,dx}{9}.$$

The total mass is

$$M = \int dm = \int_{x=0}^{3.00} \frac{\sigma(x-3)^2\,dx}{9}$$

$$M = \left(\frac{\sigma}{9}\right)\int_0^{3.00}\left(x^2-6x+9\right)dx$$

$$M = \left(\frac{\sigma}{9}\right)\left[\frac{x^3}{3}-\frac{6x^2}{2}+9x\right]_0^{3.00} = \sigma$$

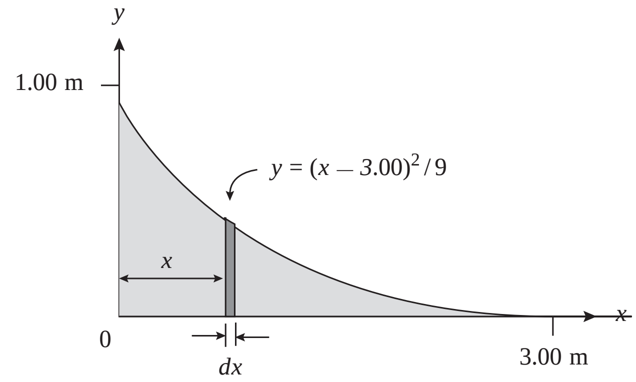


**FIG. P12.6**

The $x$-coordinate of the center of gravity is

$$x_{\mathrm{CG}} = \frac{\int x\,dm}{M} = \frac{1}{9\sigma}\int_0^{3.00}\sigma x(x-3)^2\,dx = \frac{\sigma}{9\sigma}\int_0^{3.00}\left(x^3-6x^2+9x\right)dx = \frac{1}{9}\left[\frac{x^4}{4}-\frac{6x^3}{3}+\frac{9x^2}{2}\right]_0^{3.00} = \frac{6.75\text{ m}}{9.00} = \boxed{0.750\text{ m}}$$

**P12.7** Let the fourth mass (8.00 kg) be placed at $(x, y)$, then

$$x_{\mathrm{CG}} = 0 = \frac{(3.00)(4.00)+m_4(x)}{12.0+m_4}$$

$$x = -\frac{12.0}{8.00} = \boxed{-1.50\text{ m}}$$

Similarly, $$y_{\mathrm{CG}} = 0 = \frac{(3.00)(4.00)+8.00(y)}{12.0+8.00}$$

$$y = \boxed{-1.50\text{ m}}$$

**P12.8** In a uniform gravitational field, the center of mass and center of gravity of an object coincide. Thus, the center of gravity of the triangle is located at $x = 6.67$ m, $y = 2.33$ m (see the Example on the center of mass of a triangle in Chapter 9).

The coordinates of the center of gravity of the three-object system are then:

$$x_{\mathrm{CG}} = \frac{\sum m_i x_i}{\sum m_i} = \frac{(6.00\text{ kg})(5.50\text{ m})+(3.00\text{ kg})(6.67\text{ m})+(5.00\text{ kg})(-3.50\text{ m})}{(6.00+3.00+5.00)\text{ kg}}$$

$$x_{\mathrm{CG}} = \frac{35.5\text{ kg}\cdot\text{m}}{14.0\text{ kg}} = \boxed{2.54\text{ m}}\text{ and}$$

$$y_{\mathrm{CG}} = \frac{\sum m_i y_i}{\sum m_i} = \frac{(6.00\text{ kg})(7.00\text{ m})+(3.00\text{ kg})(2.33\text{ m})+(5.00\text{ kg})(+3.50\text{ m})}{14.0\text{ kg}}$$

$$y_{\mathrm{CG}} = \frac{66.5\text{ kg}\cdot\text{m}}{14.0\text{ kg}} = \boxed{4.75\text{ m}}$$

## Section 12.3 **Examples of Rigid Objects in Static Equilibrium**

**P12.9** $\sum \tau = 0 = mg(3r) - Tr$

$$2T - Mg\sin 45.0° = 0$$

$$T = \frac{Mg\sin 45.0°}{2} = \frac{1\,500\text{ kg}(g)\sin 45.0°}{2}$$

$$= (530)(9.80)\text{ N}$$

$$m = \frac{T}{3g} = \frac{530g}{3g} = \boxed{177\text{ kg}}$$

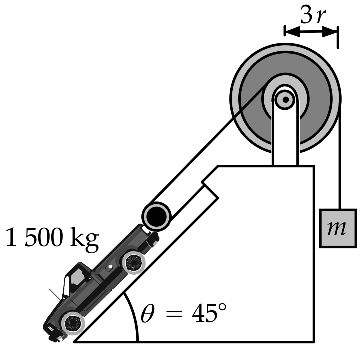


**FIG. P12.9**

***P12.10** (a) For rotational equilibrium of the lowest rod about its point of support, $\sum \tau = 0$.

$+12.0\text{ g } g\, 3\text{ cm} - m_1 g 4\text{ cm}$ $\boxed{m_1 = 9.00\text{ g}}$

(b) For the middle rod,

$+m_2 2\text{ cm} - (12.0\text{ g} + 9.0\text{ g})5\text{ cm} = 0$ $\boxed{m_2 = 52.5\text{ g}}$

(c) For the top rod,

$(52.5\text{ g} + 12.0\text{ g} + 9.0\text{ g})4\text{ cm} - m_3 6\text{ cm} = 0$ $\boxed{m_3 = 49.0\text{ g}}$

**P12.11** $F_g \rightarrow$ standard weight

$F_g' \rightarrow$ weight of goods sold

$$F_g(0.240) = F_g'(0.260)$$

$$F_g = F_g'\left(\frac{13}{12}\right)$$

$$\left(\frac{F_g - F_g'}{F_g'}\right)100 = \left(\frac{13}{12} - 1\right) \times 100 = \boxed{8.33\%}$$

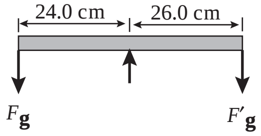


**FIG. P12.11**

***P12.12** (a) Consider the torques about an axis perpendicular to the page and through the left end of the horizontal beam.

$$\sum \tau = +(T\sin 30.0°)d - (196\text{ N})d = 0,$$

giving $T = \boxed{392\text{ N}}$.

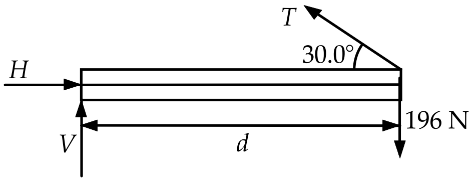


**FIG. P12.12**

(b) From $\sum F_x = 0$, $H - T\cos 30.0° = 0$, or $H = (392\text{ N})\cos 30.0° = \boxed{339\text{ N to the right}}$.

From $\sum F_y = 0$, $V + T\sin 30.0° – 200\text{ N} = 0$, or $V = 196\text{ N} - (392\text{ N})\sin 30.0° = \boxed{0}$.

**P12.13** (a) $\sum F_x = f - n_w = 0$

$\sum F_y = n_g - 800\text{ N} - 500\text{ N} = 0$

Taking torques about an axis at the foot of the ladder,

$$(800\text{ N})(4.00\text{ m})\sin 30.0° + (500\text{ N})(7.50\text{ m})\sin 30.0° - n_w(15.0\text{ cm})\cos 30.0° = 0$$

Solving the torque equation,

$$n_w = \frac{\left[(4.00\text{ m})(800\text{ N}) + (7.50\text{ m})(500\text{ N})\right]\tan 30.0°}{15.0\text{ m}} = 268\text{ N}.$$

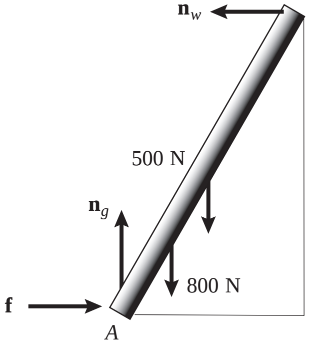


**FIG. P12.13**

Next substitute this value into the $F_x$ equation to find

$$f = n_w = \boxed{268\text{ N}} \text{ in the positive } x \text{ direction.}$$

Solving the equation $\sum F_y = 0$,

$$n_g = \boxed{1\,300\text{ N}} \text{ in the positive } y \text{ direction.}$$

(b) In this case, the torque equation $\sum \tau_A = 0$ gives:

$$(9.00\text{ m})(800\text{ N})\sin 30.0° + (7.50\text{ m})(500\text{ N})\sin 30.0° - (15.0\text{ m})(n_w)\sin 60.0° = 0$$

or $n_w = 421\text{ N}$.

Since $f = n_w = 421\text{ N}$ and $f = f_{\max} = \mu n_g$, we find

$$\mu = \frac{f_{\max}}{n_g} = \frac{421\text{ N}}{1\,300\text{ N}} = \boxed{0.324}.$$

**P12.14** (a) $\sum F_x = f - n_w = 0$ (1)

$\sum F_y = n_g - m_1 g - m_2 g = 0$ (2)

$$\sum \tau_A = -m_1 g\left(\frac{L}{2}\right)\cos\theta - m_2 g x\cos\theta + n_w L\sin\theta = 0$$

From the torque equation,

$$n_w = \left[\frac{1}{2}m_1 g + \left(\frac{x}{L}\right)m_2 g\right]\cot\theta$$

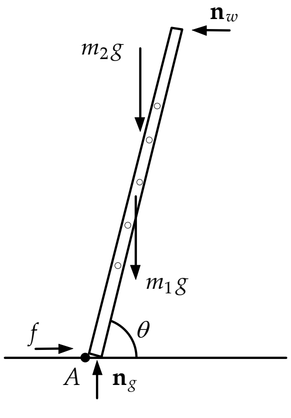


**FIG. P12.14**

Then, from equation (1): $f = n_w = \boxed{\left[\frac{1}{2}m_1 g + \left(\frac{x}{L}\right)m_2 g\right]\cot\theta}$

and from equation (2): $n_g = \boxed{(m_1 + m_2)g}$

(b) If the ladder is on the verge of slipping when $x = d$,

then $$\mu = \frac{f|_{x=d}}{n_g} = \boxed{\frac{\left(\frac{m_1}{2} + \frac{m_2 d}{L}\right)\cot\theta}{m_1 + m_2}}.$$

**P12.15** (a) Taking moments about $P$,

$$(R\sin 30.0°)0+(R\cos 30.0°)(5.00\text{ cm})-(150\text{ N})(30.0\text{ cm})=0$$
$$R=1\,039.2\text{ N}=1.04\text{ kN}$$

The force exerted by the hammer on the nail is equal in magnitude and opposite in direction:

$$\boxed{1.04\text{ kN at }60°\text{ upward and to the right.}}$$

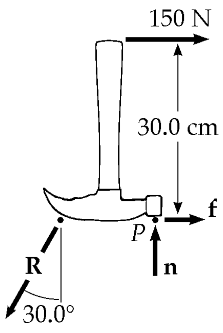


**FIG. P12.15**

(b) $f=R\sin 30.0°–150\text{ N}=370\text{ N}$
$n=R\cos 30.0°=900\text{ N}$

$$\boxed{\mathbf{F}_{\text{surface}}=(370\text{ N})\hat{\mathbf{i}}+(900\text{ N})\hat{\mathbf{j}}}$$

**P12.16** See the free-body diagram at the right.
When the plank is on the verge of tipping about point $P$, the normal force $n_1$ goes to zero. Then, summing torques about point $P$ gives

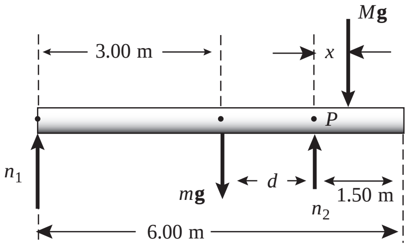


**FIG. P12.16**

$$\sum\tau_p=-mgd+Mgx=0 \qquad \text{or} \qquad x=\left(\frac{m}{M}\right)d\,.$$

From the dimensions given on the free-body diagram, observe that $d=1.50\text{ m}$ Thus, when the plank is about to tip,

$$x=\left(\frac{30.0\text{ kg}}{70.0\text{ kg}}\right)(1.50\text{ m})=\boxed{0.643\text{ m}}\,.$$

**P12.17** Torque about the front wheel is zero.

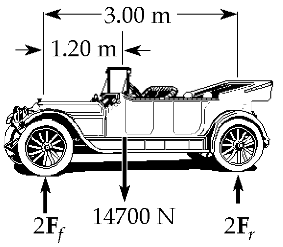


**FIG. P12.17**

$$0=(1.20\text{ m})(mg)-(3.00\text{ m})(2F_r)$$

Thus, the force at each rear wheel is

$$F_r=0.200mg=\boxed{2.94\text{ kN}}\,.$$

The force at each front wheel is then

$$F_f=\frac{mg-2F_r}{2}=\boxed{4.41\text{ kN}}\,.$$

**P12.18** $\sum F_x = F_b - F_t + 5.50\text{ N} = 0 \qquad (1)$

$\sum F_y = n - mg = 0$

Summing torques about point O,

$$\sum \tau_O = F_t(1.50\text{ m}) - (5.50\text{ m})(10.0\text{ m}) = 0$$

which yields $F_t = \boxed{36.7\text{ N to the left}}$

Then, from Equation (1),

$$F_b = 36.7\text{ N} - 5.50\text{ N} = \boxed{31.2\text{ N to the right}}$$

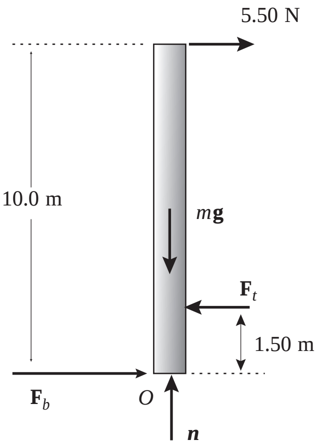


FIG. P12.18

**P12.19** (a) $T_e \sin 42.0° = 20.0\text{ N} \qquad \boxed{T_e = 29.9\text{ N}}$

(b) $T_e \cos 42.0° = T_m \qquad \boxed{T_m = 22.2\text{ N}}$

**P12.20** Relative to the hinge end of the bridge, the cable is attached horizontally out a distance $x = (5.00\text{ m})\cos 20.0° = 4.70\text{ m}$ and vertically down a distance $y = (5.00\text{ m})\sin 20.0° = 1.71\text{ m}$. The cable then makes the following angle with the horizontal:

$$\theta = \tan^{-1}\left[\frac{(12.0 + 1.71)\text{ m}}{4.70\text{ m}}\right] = 71.1° .$$

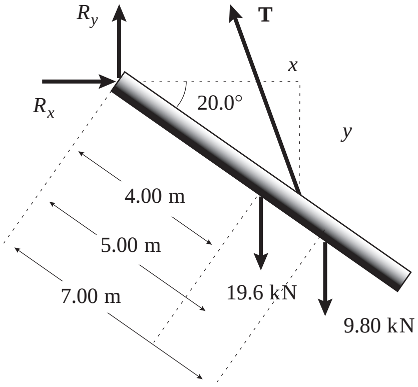


FIG. P12.20

(a) Take torques about the hinge end of the bridge:

$$\begin{aligned} R_x(0) + R_y(0) - 19.6\text{ kN}(4.00\text{ m})\cos 20.0° \\ -T\cos 71.1°(1.71\text{ m}) + T\sin 71.1°(4.70\text{ m}) \\ -9.80\text{ kN}(7.00\text{ m})\cos 20.0° = 0 \end{aligned}$$

which yields $T = \boxed{35.5\text{ kN}}$

(b) $\sum F_x = 0 \Rightarrow R_x - T\cos 71.1° = 0$

or $R_x = (35.5\text{ kN})\cos 71.1° = \boxed{11.5\text{ kN (right)}}$

(c) $\sum F_y = 0 \Rightarrow R_y - 19.6\text{ kN} + T\sin 71.1° - 9.80\text{ kN} = 0$

Thus,

$$\begin{aligned} R_y &= 29.4\text{ kN} - (35.5\text{ kN})\sin 71.1° = -4.19\text{ kN} \\ &= \boxed{4.19\text{ kN down}} \end{aligned}$$

***P12.21** (a) We model the horse as a particle. The drawbridge will fall out from under the horse.

$$\alpha = mg\frac{\frac{1}{2}\ell\cos\theta_0}{\frac{1}{3}m\ell^2} = \frac{3g}{2\ell}\cos\theta_0$$

$$= \frac{3\left(9.80\text{ m/s}^2\right)\cos 20.0°}{2(8.00\text{ m})} = \boxed{1.73\text{ rad/s}^2}$$

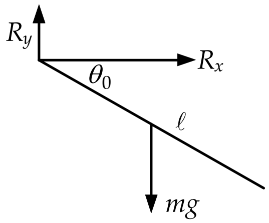


**FIG. P12.21(a)**

(b) $$\frac{1}{2}I\omega^2 = mgh$$

$$\therefore \frac{1}{2}\cdot\frac{1}{3}m\ell^2\omega^2 = mg\cdot\frac{1}{2}\ell(1-\sin\theta_0)$$

$$\therefore \omega = \sqrt{\frac{3g}{\ell}(1-\sin\theta_0)} = \sqrt{\frac{3\left(9.80\text{ m/s}^2\right)}{8.00\text{ m}}(1-\sin 20°)} = \boxed{1.56\text{ rad/s}}$$

(c) The linear acceleration of the bridge is:

$$a = \frac{1}{2}\ell\alpha = \frac{1}{2}(8.0\text{ m})\left(1.73\text{ rad/s}^2\right) = 6.907\text{ m/s}^2$$

The force at the hinge + the force of gravity produce the acceleration $a = 6.907\text{ m/s}^2$ at right angles to the bridge.

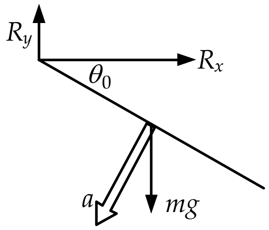


**FIG. P12.21(c)**

$$R_x = ma_x = (2\,000\text{ kg})\left(6.907\text{ m/s}^2\right)\cos 250° = -4.72\text{ kN}$$

$$R_y - mg = ma_y$$

$$\therefore R_y = m\left(g + a_y\right) = (2\,000\text{ kg})\left[9.80\text{ m/s}^2 + \left(6.907\text{ m/s}^2\right)\sin 250°\right] = 6.62\text{ kN}$$

Thus: $\boxed{\mathbf{R} = \left(-4.72\hat{\mathbf{i}} + 6.62\hat{\mathbf{j}}\right)\text{ kN}}$.

(d) $$R_x = 0$$

$$a = \omega^2\left(\frac{1}{2}\ell\right) = (1.56\text{ rad/s})^2(4.0\text{ m}) = 9.67\text{ m/s}^2$$

$$R_y - mg = ma$$

$$\therefore R_y = (2\,000\text{ kg})\left(9.8\text{ m/s}^2 + 9.67\text{ m/s}^2\right) = 38.9\text{ kN}$$

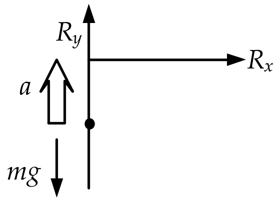
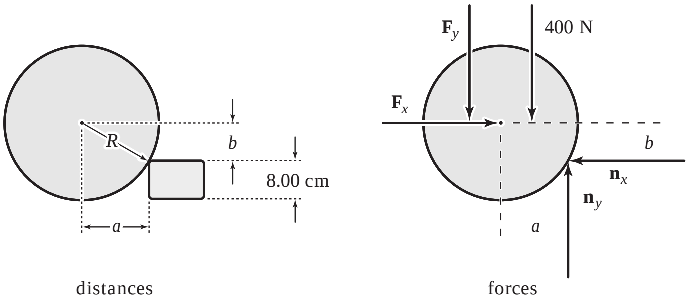


**FIG. P12.21(d)**

Thus: $\boxed{R_y = 38.9\hat{\mathbf{j}}\text{ kN}}$

**P12.22** Call the required force $F$, with components $F_x = F\cos 15.0°$ and $F_y = -F\sin 15.0°$, transmitted to the center of the wheel by the handles.

Just as the wheel leaves the ground, the ground exerts no force on it.



distances    forces

**FIG. P12.22**

$$\sum F_x = 0: \; F\cos 15.0° - n_x \qquad (1)$$

$$\sum F_y = 0: \; -F\sin 15.0° - 400 \text{ N} + n_y = 0 \quad (2)$$

Take torques about its contact point with the brick. The needed distances are seen to be:

$$b = R - 8.00 \text{ cm} = (20.0 - 8.00) \text{ cm} = 12.0 \text{ cm}$$

$$a = \sqrt{R^2 - b^2} = 16.0 \text{ cm}$$

(a) $\sum \tau = 0: \; -F_x b + F_y a + (400 \text{ N})a = 0$, or

$$F\left[-(12.0 \text{ cm})\cos 15.0° + (16.0 \text{ cm})\sin 15.0°\right] + (400 \text{ N})(16.0 \text{ cm}) = 0$$

so $F = \dfrac{6\,400 \text{ N}\cdot\text{cm}}{7.45 \text{ cm}} = \boxed{859 \text{ N}}$

(b) Then, using Equations (1) and (2),

$$n_x = (859 \text{ N})\cos 15.0° = 830 \text{ N} \;\text{ and}$$

$$n_y = 400 \text{ N} + (859 \text{ N})\sin 15.0° = 622 \text{ N}$$

$$n = \sqrt{n_x^2 + n_y^2} = \boxed{1.04 \text{ kN}}$$

$$\theta = \tan^{-1}\left(\frac{n_y}{n_x}\right) = \tan^{-1}(0.749) = \boxed{36.9° \text{ to the left and upward}}$$

***P12.23** When $x = x_{\text{min}}$, the rod is on the verge of slipping, so

$$f = (f_s)_{\text{max}} = \mu_s n = 0.50n\,.$$

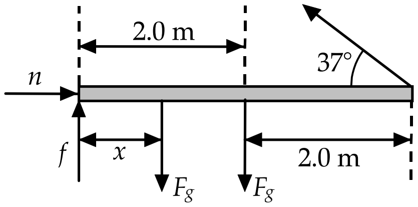


**FIG. P12.23**

From $\sum F_x = 0$, $n - T\cos 37° = 0$, or $n = 0.799T$.

Thus, $f = 0.50(0.799T) = 0.399T$

From $\sum F_y = 0$, $f + T\sin 37° - 2F_g = 0$, or $0.399T - 0.602T - 2F_g = 0$, giving $T = 2.00F_g$.

Using $\sum \tau = 0$ for an axis perpendicular to the page and through the left end of the beam gives $-F_g \cdot x_{\text{min}} - F_g(2.0 \text{ m}) + \left[(2F_g)\sin 37°\right](4.0 \text{ m}) = 0$, which reduces to $x_{\text{min}} = \boxed{2.82 \text{ m}}$.

**P12.24** $x = \boxed{\dfrac{3L}{4}}$

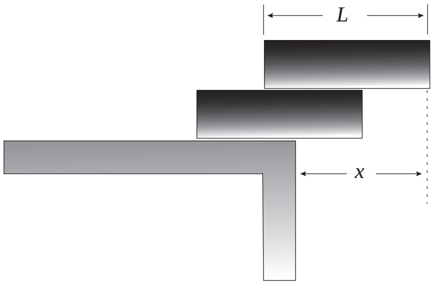


**FIG. P12.24**

If the CM of the two bricks does not lie over the edge, then the bricks balance.

If the lower brick is placed $\dfrac{L}{4}$ over the edge, then the second brick may be placed so that its end protrudes $\dfrac{3L}{4}$ over the edge.

**P12.25** To find $U$, measure distances and forces from point A. Then, balancing torques,

$$(0.750)U = 29.4(2.25) \qquad \boxed{U = 88.2\text{ N}}$$

To find $D$, measure distances and forces from point B. Then, balancing torques,

$$(0.750)D = (1.50)(29.4) \qquad \boxed{D = 58.8\text{ N}}$$

Also, notice that $U = D + F_g$, so $\sum F_y = 0$.

***P12.26** Consider forces and torques on the beam.

$$\sum F_x = 0: \quad R\cos\theta - T\cos 53° = 0$$
$$\sum F_y = 0: \quad R\sin\theta + T\sin 53° – 800\text{ N} = 0$$
$$\sum \tau = 0: \quad (T\sin 53°)8\text{ m} - (600\text{ N})x - (200\text{ N})4\text{ m} = 0$$

(a) Then $T = \dfrac{600\text{ N}x + 800\text{ N}\cdot\text{m}}{8\text{ m}\sin 53°} = (93.9\text{ N/m})x + 125\text{ N}$. As $x$ increases from 2 m, this expression grows larger.

(b) From substituting back,

$$R\cos\theta = [93.9x + 125]\cos 53°$$
$$R\sin\theta = 800\text{ N} - [93.9x + 125]\sin 53°$$

Dividing, $\tan\theta = \dfrac{R\sin\theta}{R\cos\theta} = -\tan 53° + \dfrac{800\text{ N}}{(93.9x + 125)\cos 53°}$

$$\tan\theta = \tan 53°\left(\frac{32}{3x+4} - 1\right)$$

As $x$ increases the fraction decreases and $\boxed{\theta\text{ decreases}}$.

*continued on next page*

(c) To find $R$ we can work out $R^2\cos^2\theta + R^2\sin^2\theta = R^2$. From the expressions above for $R\cos\theta$ and $R\sin\theta$,

$$R^2 = T^2\cos^2 53° + T^2\sin^2 53° - 1\,600\text{ N}T\sin 53° + (800\text{ N})^2$$
$$R^2 = T^2 - 1\,600T\sin 53° + 640\,000$$
$$R^2 = (93.9x + 125)^2 - 1\,278(93.9x + 125) + 640\,000$$
$$R = \left(8\,819x^2 - 96\,482x + 495\,678\right)^{1/2}$$

At $x = 0$ this gives $R = 704\text{ N}$. At $x = 2\text{ m}$, $R = 581\text{ N}$. At $x = 8\text{ m}$, $R = 537\text{ N}$. Over the range of possible values for $x$, the negative term $-96\,482x$ dominates the positive term $8\,819x^2$, and $\boxed{R\text{ decreases}}$ as $x$ increases.

## Section 12.4 **Elastic Properties of Solids**

**P12.27** $\dfrac{F}{A} = Y\dfrac{\Delta L}{L_i}$

$$\Delta L = \frac{FL_i}{AY} = \frac{(200)(9.80)(4.00)}{\left(0.200\times10^{-4}\right)\left(8.00\times10^{10}\right)} = \boxed{4.90\text{ mm}}$$

**P12.28** (a) stress $= \dfrac{F}{A} = \dfrac{F}{\pi r^2}$

$$F = (\text{stress})\pi\left(\frac{d}{2}\right)^2$$
$$F = \left(1.50\times10^8\text{ N/m}^2\right)\pi\left(\frac{2.50\times10^{-2}\text{ m}}{2}\right)^2$$
$$F = \boxed{73.6\text{ kN}}$$

(b) stress $= Y(\text{strain}) = \dfrac{Y\Delta L}{L_i}$

$$\Delta L = \frac{(\text{stress})L_i}{Y} = \frac{\left(1.50\times10^8\text{ N/m}^2\right)(0.250\text{ m})}{1.50\times10^{10}\text{ N/m}^2} = \boxed{2.50\text{ mm}}$$

***P12.29** The definition of $Y = \dfrac{\text{stress}}{\text{strain}}$ means that $Y$ is the slope of the graph:

$$Y = \frac{300\times10^6\text{ N/m}^2}{0.003} = \boxed{1.0\times10^{11}\text{ N/m}^2}.$$

**P12.30** Count the wires. If they are wrapped together so that all support nearly equal stress, the number should be

$$\frac{20.0\text{ kN}}{0.200\text{ kN}} = 100.$$

Since cross-sectional area is proportional to diameter squared, the diameter of the cable will be

$$(1\text{ mm})\sqrt{100}\ \boxed{\sim 1\text{ cm}}.$$

**P12.31** From the defining equation for the shear modulus, we find $\Delta x$ as

$$\Delta x = \frac{hf}{SA} = \frac{\left(5.00\times10^{-3}\text{ m}\right)(20.0\text{ N})}{\left(3.0\times10^{6}\text{ N/m}^2\right)\left(14.0\times10^{-4}\text{ m}^2\right)} = 2.38\times10^{-5}\text{ m}$$

or $\Delta x = \boxed{2.38\times10^{-2}\text{ mm}}$.

**P12.32** The force acting on the hammer changes its momentum according to

$$mv_i + \overline{F}(\Delta t) = mv_f \text{ so } \left|\overline{F}\right| = \frac{m\left|v_f - v_i\right|}{\Delta t}.$$

Hence, $\left|\overline{F}\right| = \dfrac{30.0\text{ kg}\left|-10.0\text{ m/s} - 20.0\text{ m/s}\right|}{0.110\text{ s}} = 8.18\times10^{3}\text{ N}$.

By Newton's third law, this is also the magnitude of the average force exerted on the spike by the hammer during the blow. Thus, the stress in the spike is:

$$\text{stress} = \frac{F}{A} = \frac{8.18\times10^{3}\text{ N}}{\pi\frac{(0.0230\text{ m})^2}{4}} = 1.97\times10^{7}\text{ N/m}^2$$

and the strain is: strain $= \dfrac{\text{stress}}{Y} = \dfrac{1.97\times10^{7}\text{ N/m}^2}{20.0\times10^{10}\text{ N/m}^2} = \boxed{9.85\times10^{-5}}$.

**P12.33** (a)

$$\begin{aligned} F &= (A)(\text{stress}) \\ &= \pi\left(5.00\times10^{-3}\text{ m}\right)^2\left(4.00\times10^{8}\text{ N/m}^2\right) \\ &= \boxed{3.14\times10^{4}\text{ N}} \end{aligned}$$

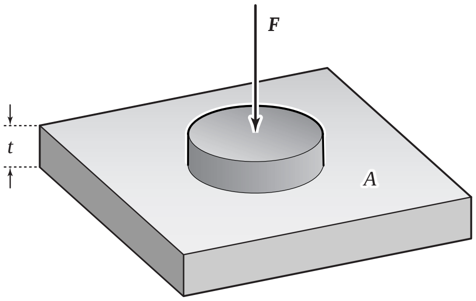


**FIG. P12.33**

(b) The area over which the shear occurs is equal to the circumference of the hole times its thickness. Thus,

$$\begin{aligned} A &= (2\pi r)t = 2\pi\left(5.00\times10^{-3}\text{ m}\right)\left(5.00\times10^{-3}\text{ m}\right) \\ &= 1.57\times10^{-4}\text{ m}^2 \end{aligned}$$

So, $F = (A)\text{Stress} = \left(1.57\times10^{-4}\text{ m}^2\right)\left(4.00\times10^{8}\text{ N/m}^2\right) = \boxed{6.28\times10^{4}\text{ N}}$.

**P12.34** Let the 3.00 kg mass be mass #1, with the 5.00 kg mass, mass # 2. Applying Newton's second law to each mass gives:

$$m_1 a = T - m_1 g \quad (1) \qquad \text{and} \qquad m_2 a = m_2 g - T \quad (2)$$

where $T$ is the tension in the wire.

Solving equation (1) for the acceleration gives: $a = \frac{T}{m_1} - g$,

and substituting this into equation (2) yields: $\frac{m_2}{m_1} T - m_2 g = m_2 g - T$.

Solving for the tension $T$ gives

$$T = \frac{2m_1 m_2 g}{m_2 + m_1} = \frac{2(3.00\text{ kg})(5.00\text{ kg})\left(9.80\text{ m/s}^2\right)}{8.00\text{ kg}} = 36.8\text{ N}.$$

From the definition of Young's modulus, $Y = \frac{FL_i}{A(\Delta L)}$, the elongation of the wire is:

$$\Delta L = \frac{TL_i}{YA} = \frac{(36.8\text{ N})(2.00\text{ m})}{\left(2.00 \times 10^{11}\text{ N/m}^2\right)\pi\left(2.00 \times 10^{-3}\text{ m}\right)^2} = \boxed{0.029\,3\text{ mm}}.$$

**P12.35** Consider recompressing the ice, which has a volume $1.09V_0$.

$$\Delta P = -B\left(\frac{\Delta V}{V_i}\right) = \frac{-\left(2.00 \times 10^9\text{ N/m}^2\right)(-0.090)}{1.09} = \boxed{1.65 \times 10^8\text{ N/m}^2}$$

***P12.36** $B = -\frac{\Delta P}{\frac{\Delta V}{V_i}} = -\frac{\Delta P V_i}{\Delta V}$

(a) $$\Delta V = -\frac{\Delta P V_i}{B} = -\frac{\left(1.13 \times 10^8\text{ N/m}^2\right)1\text{ m}^3}{0.21 \times 10^{10}\text{ N/m}^2} = \boxed{-0.053\,8\text{ m}^3}$$

(b) The quantity of water with mass $1.03 \times 10^3$ kg occupies volume at the bottom $1\text{ m}^3 - 0.053\,8\text{ m}^3 = 0.946\text{ m}^3$. So its density is $\frac{1.03 \times 10^3\text{ kg}}{0.946\text{ m}^3} = \boxed{1.09 \times 10^3\text{ kg/m}^3}$.

(c) With only a 5% volume change in this extreme case, liquid water is indeed nearly incompressible.

***P12.37** Part of the load force extends the cable and part compresses the column by the same distance $\Delta\ell$:

$$F = \frac{Y_A A_A \Delta\ell}{\ell_A} + \frac{Y_s A_s \Delta\ell}{\ell_s}$$

$$\Delta\ell = \frac{F}{\frac{Y_A A_A}{\ell_A} + \frac{Y_s A_s}{\ell_s}} = \frac{8\,500\text{ N}}{\frac{7\times10^{10}\,\pi\left(0.162\,4^2 - 0.161\,4^2\right)}{4(3.25)} + \frac{20\times10^{10}\,\pi(0.012\,7)^2}{4(5.75)}}$$

$$= \boxed{8.60 \times 10^{-4}\text{ m}}$$

## Additional Problems

***P12.38** (a) The beam is perpendicular to the wall, since $3^2+4^2=5^2$. Then $\sin\theta=\frac{4\text{ m}}{5\text{ m}}$; $\boxed{\theta=53.1°}$.

(b) $\sum\tau_{\text{hinge}}=0:\quad +T\sin\theta(3\text{ m})-250\text{ N}(10\text{ m})=0$

$$T=\frac{2\,500\text{ Nm}}{3\text{ m}\sin 53.1°}=\boxed{1.04\times10^3\text{ N}}$$

(c) $x=\frac{T}{k}=\frac{1.04\times10^3\text{ N}}{8.25\times10^3\text{ N/m}}=\boxed{0.126\text{ m}}$

The cable is 5.126 m long. From the law of cosines,

$$4^2=5.126^2+3^2-2(3)(5.126)\cos\theta$$

$$\theta=\cos^{-1}\frac{3^2+5.126^2-4^2}{2(3)(5.126)}=\boxed{51.2°}$$

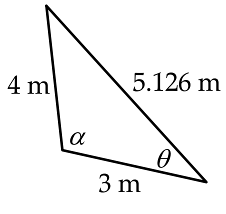


**FIG. P12.38**

(d) From the law of sines, the angle the hinge makes with the wall satisfies $\frac{\sin\alpha}{5.126\text{ m}}=\frac{\sin 51.2°}{4\text{ m}}$

$$\sin\alpha=0.998\,58$$

$$\sum\tau_{\text{hinge}}=0$$

$$+T(3\text{ m})\sin 51.2°–250\text{ N}(10\text{ m})(0.998\,58)=0$$

$$T=\boxed{1.07\times10^3\text{ N}}$$

(e) $x=\frac{1.07\times10^3\text{ N}}{8.25\times10^3\text{ N/m}}=\boxed{0.129\text{ m}}$

$$\theta=\cos^{-1}\frac{3^2+5.129^2-4^2}{2(3)(5.129)}=\boxed{51.1°}$$

(f) Now the answers are self-consistent:

$$\sin\alpha=5.129\text{ m}\frac{\sin 51.1°}{4\text{ m}}=0.998\,51$$

$$T(3\text{ m})\sin 51.1°–250\text{ N}(10\text{ m})(0.998\,51)=0$$

$$T=1.07\times10^3\text{ N}$$

$$x=0.129\,5\text{ m}$$

$$\boxed{\theta=51.1°}$$

**P12.39** Let $n_A$ and $n_B$ be the normal forces at the points of support.

Choosing the origin at point $A$ with $\sum F_y=0$ and $\sum\tau=0$, we find:

$$n_A+n_B-\left(8.00\times10^4\right)g-\left(3.00\times10^4\right)g=0\text{ and}$$

$$-\left(3.00\times10^4\right)(g)15.0-\left(8.00\times10^4\right)(g)25.0+n_B(50.0)=0$$

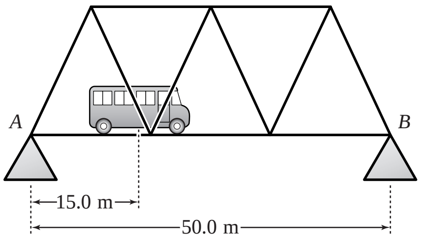


**FIG. P12.39**

The equations combine to give $n_A=\boxed{5.98\times10^5\text{ N}}$ and $b_B=\boxed{4.80\times10^5\text{ N}}$.

**P12.40** When the concrete has cured and the pre-stressing tension has been released, the rod presses in on the concrete and with equal force, $T_2$, the concrete produces tension in the rod.

(a) In the concrete: stress $= 8.00 \times 10^6\ \text{N/m}^2 = Y \cdot (\text{strain}) = Y\left(\dfrac{\Delta L}{L_i}\right)$

$$\text{Thus, } \Delta L = \frac{(\text{stress})L_i}{Y} = \frac{\left(8.00 \times 10^6\ \text{N/m}^2\right)(1.50\ \text{m})}{30.0 \times 10^9\ \text{N/m}^2}$$

$$\text{or} \quad \Delta L = 4.00 \times 10^{-4}\ \text{m} = \boxed{0.400\ \text{mm}}.$$

(b) In the concrete: stress $= \dfrac{T_2}{A_c} = 8.00 \times 10^6\ \text{N/m}^2$, so

$$T_2 = \left(8.00 \times 10^6\ \text{N/m}^2\right)\left(50.0 \times 10^{-4}\ \text{m}^2\right) = \boxed{40.0\ \text{kN}}$$

(c) For the rod: $\dfrac{T_2}{A_R} = \left(\dfrac{\Delta L}{L_i}\right)Y_{\text{steel}}$ so $\Delta L = \dfrac{T_2 L_i}{A_R Y_{\text{steel}}}$

$$\Delta L = \frac{\left(4.00 \times 10^4\ \text{N}\right)(1.50\ \text{m})}{\left(1.50 \times 10^{-4}\ \text{m}^2\right)\left(20.0 \times 10^{10}\ \text{N/m}^2\right)} = 2.00 \times 10^{-3}\ \text{m} = \boxed{2.00\ \text{mm}}$$

(d) The rod in the finished concrete is 2.00 mm longer than its unstretched length. To remove stress from the concrete, one must stretch the rod 0.400 mm farther, by a total of $\boxed{2.40\ \text{mm}}$.

(e) For the stretched rod around which the concrete is poured:

$$\frac{T_1}{A_R} = \left(\frac{\Delta L_{\text{total}}}{L_i}\right)Y_{\text{steel}} \quad \text{or} \quad T_1 = \left(\frac{\Delta L_{\text{total}}}{L_i}\right)A_R Y_{\text{steel}}$$

$$T_1 = \left(\frac{2.40 \times 10^{-3}\ \text{m}}{1.50\ \text{m}}\right)\left(1.50 \times 10^{-4}\ \text{m}^2\right)\left(20.0 \times 10^{10}\ \text{N/m}^2\right) = \boxed{48.0\ \text{kN}}$$

***P12.41** With $\ell$ as large as possible, $n_1$ and $n_2$ will both be large. The equality sign in $f_2 \le \mu_s n_2$ will be true, but the less-than sign in $f_1 < \mu_s n_1$. Take torques about the lower end of the pole.

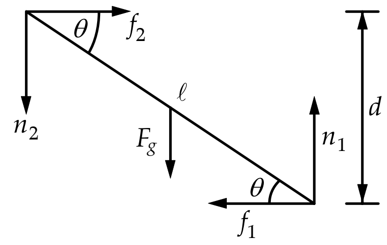


**FIG. P12.41**

$$n_2 \ell \cos\theta + F_g\left(\frac{1}{2}\ell\right)\cos\theta - f_2 \ell \sin\theta = 0$$

Setting $f_2 = 0.576 n_2$, the torque equation becomes

$$n_2(1 - 0.576\tan\theta) + \frac{1}{2}F_g = 0$$

Since $n_2 > 0$, it is necessary that

$$1 - 0.576\tan\theta < 0$$

$$\therefore \tan\theta > \frac{1}{0.576} = 1.736$$

$$\therefore \theta > 60.1°$$

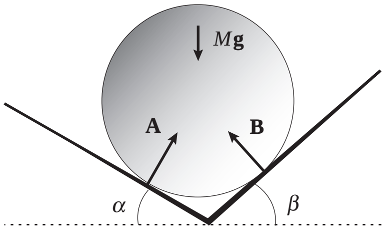
$$\therefore \ell = \frac{d}{\sin\theta} < \frac{7.80\ \text{ft}}{\sin 60.1°} = \boxed{9.00\ \text{ft}}$$

**P12.42** Call the normal forces $A$ and $B$. They make angles $\alpha$ and $\beta$ with the vertical.

$$\sum F_x = 0\colon \quad A\sin\alpha - B\sin\beta = 0$$
$$\sum F_y = 0\colon \quad A\cos\alpha - Mg + B\cos\beta = 0$$

Substitute $B = \dfrac{A\sin\alpha}{\sin\beta}$

$$A\cos\alpha + A\cos\beta\frac{\sin\alpha}{\sin\beta} = Mg$$
$$A(\cos\alpha\sin\beta + \sin\alpha\cos\beta) = Mg\sin\beta$$
$$A = \boxed{Mg\frac{\sin\beta}{\sin(\alpha+\beta)}}$$
$$B = \boxed{Mg\frac{\sin\alpha}{\sin(\alpha+\beta)}}$$



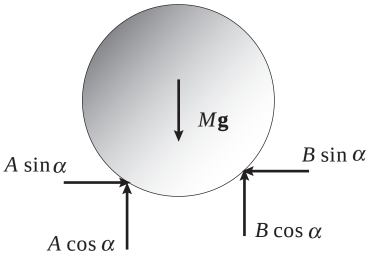


**FIG. P12.42**

**P12.43** (a) See the diagram.

(b) If $x = 1.00$ m, then

$$\sum\tau_O = (-700\text{ N})(1.00\text{ m}) - (200\text{ N})(3.00\text{ m})$$
$$-(80.0\text{ N})(6.00\text{ m})$$
$$+(T\sin 60.0°)(6.00\text{ m}) = 0$$

Solving for the tension gives: $T = \boxed{343\text{ N}}$.

From $\sum F_x = 0$, $R_x = T\cos 60.0° = \boxed{171\text{ N}}$.

From $\sum F_y = 0$, $R_y = 980\text{ N} - T\sin 60.0° = \boxed{683\text{ N}}$.

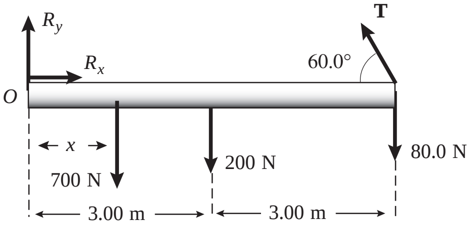


**FIG. P12.43**

(c) If $T = 900$ N:

$$\sum\tau_O = (-700\text{ N})x - (200\text{ N})(3.00\text{ m}) - (80.0\text{ N})(6.00\text{ m}) + [(900\text{ N})\sin 60.0°](6.00\text{ m}) = 0.$$

Solving for $x$ gives: $x = \boxed{5.13\text{ m}}$.

**P12.44** (a) Sum the torques about top hinge:

$$\sum \tau = 0:$$

$$\begin{aligned} &C(0)+D(0)+200\text{ N}\cos 30.0°(0) \\ &+200\text{ N}\sin 30.0°(3.00\text{ m}) \\ &-392\text{ N}(1.50\text{ m})+A(1.80\text{ m}) \\ &+B(0)=0 \end{aligned}$$

Giving $A=\boxed{160\text{ N (right)}}$.

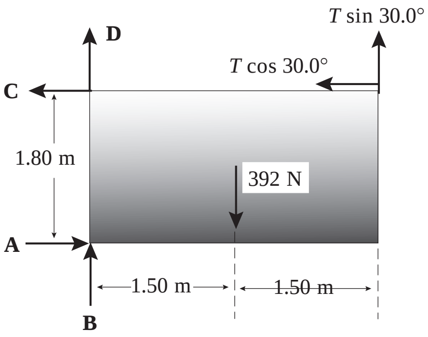


**FIG. P12.44**

(b) $\sum F_x = 0:$

$$\begin{aligned} -C-200\text{ N}\cos 30.0°+A&=0 \\ C=160\text{ N}-173\text{ N}&=-13.2\text{ N} \end{aligned}$$

In our diagram, this means $\boxed{13.2\text{ N to the right}}$.

(c) $\sum F_y = 0:\ +B+D-392\text{ N}+200\text{ N}\sin 30.0° = 0$

$B+D=392\text{ N}-100\text{ N}=\boxed{292\text{ N(up)}}$

(d) Given $C=0$: Take torques about bottom hinge to obtain

$$A(0)+B(0)+0(1.80\text{ m})+D(0)-392\text{ N}(1.50\text{ m})+T\sin 30.0°(3.00\text{ m})+T\cos 30.0°(1.80\text{ m})=0$$

so $T=\dfrac{588\text{ N}\cdot\text{m}}{(1.50\text{ m}+1.56\text{ m})}=\boxed{192\text{ N}}$.

**P12.45** Using $\sum F_x = \sum F_y = \sum \tau = 0$, choosing the origin at the left end of the beam, we have (neglecting the weight of the beam)

$$\sum F_x = R_x - T\cos\theta = 0,$$
$$\sum F_y = R_y + T\sin\theta - F_g = 0,$$

and $\sum \tau = -F_g(L+d)+T\sin\theta(2L+d)=0$.

Solving these equations, we find:

(a) $T=\boxed{\dfrac{F_g(L+d)}{\sin\theta(2L+d)}}$

(b) $R_x=\boxed{\dfrac{F_g(L+d)\cot\theta}{2L+d}}$ $\qquad R_y=\boxed{\dfrac{F_gL}{2L+d}}$

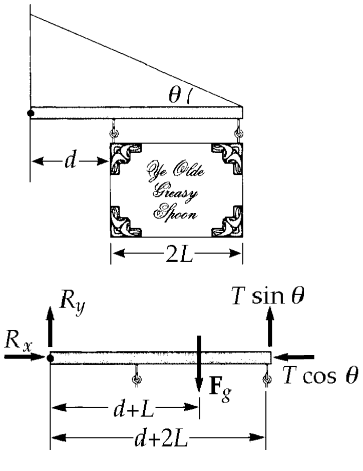


**FIG. P12.45**

**P12.46** $\sum \tau_{\text{point }0} = 0$ gives

$$(T\cos 25.0°)\left(\frac{3\ell}{4}\sin 65.0°\right)+(T\sin 25.0°)\left(\frac{3\ell}{4}\cos 65.0°\right)$$

$$=(2\,000\text{ N})(\ell\cos 65.0°)+(1\,200\text{ N})\left(\frac{\ell}{2}\cos 65.0°\right)$$

From which, $T = 1\,465\text{ N} = \boxed{1.46\text{ kN}}$

From $\sum F_x = 0$,

$$H = T\cos 25.0° = 1\,328\text{ N}(\text{toward right}) = \boxed{1.33\text{ kN}}$$

From $\sum F_y = 0$,

$$V = 3\,200\text{ N} - T\sin 25.0° = 2\,581\text{ N}(\text{upward}) = \boxed{2.58\text{ kN}}$$

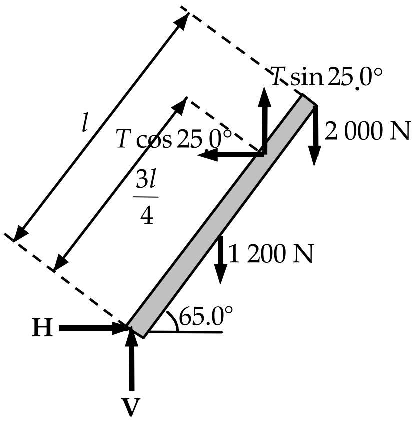


FIG. P12.46

**P12.47** We interpret the problem to mean that the support at point B is frictionless. Then the support exerts a force in the *x* direction and

$$\boxed{F_{By} = 0}$$

$$\sum F_x = F_{Bx} - F_{Ax} = 0$$

$$F_{Ay} - (3\,000 + 10\,000)g = 0$$

and $\sum \tau = -(3\,000g)(2.00) - (10\,000g)(6.00) + F_{Bx}(1.00) = 0$.

These equations combine to give

$$F_{Ax} = F_{Bx} = \boxed{6.47\times 10^5\text{ N}}$$

$$F_{Ay} = \boxed{1.27\times 10^5\text{ N}}$$

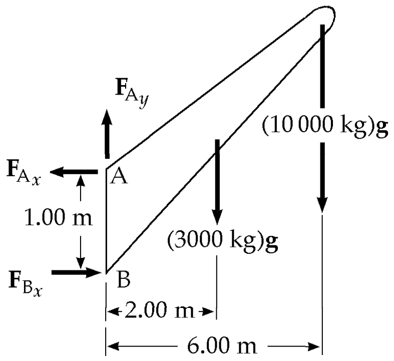


FIG. P12.47

**P12.48** $n = (M+m)g \quad H = f$

$$H_{\max} = f_{\max} = \mu_s(m+M)g$$

$$\sum \tau_A = 0 = \frac{mgL}{2}\cos 60.0° + Mgx\cos 60.0° - HL\sin 60.0°$$

$$\frac{x}{L} = \frac{H\tan 60.0°}{Mg} - \frac{m}{2M} = \frac{\mu_s(m+M)\tan 60.0°}{M} - \frac{m}{2M}$$

$$= \frac{3}{2}\mu_s\tan 60.0° - \frac{1}{4} = \boxed{0.789}$$

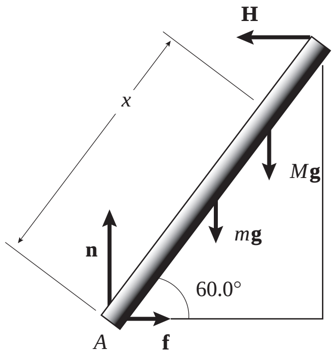


FIG. P12.48

**P12.49** From the free-body diagram, the angle **T** makes with the rod is

$$\theta = 60.0° + 20.0° = 80.0°$$

and the perpendicular component of **T** is $T\sin 80.0°$. Summing torques around the base of the rod,

$$\sum \tau = 0: \quad -(4.00\ \text{m})(10\,000\ \text{N})\cos 60° + T(4.00\ \text{m})\sin 80° = 0$$

$$T = \frac{(10\,000\ \text{N})\cos 60.0°}{\sin 80.0°} = \boxed{5.08 \times 10^3\ \text{N}}$$

$$\sum F_x = 0: \quad F_H - T\cos 20.0° = 0$$

$$F_H = T\cos 20.0° = \boxed{4.77 \times 10^3\ \text{N}}$$

$$\sum F_y = 0: \quad F_V + T\sin 20.0° - 10\,000\ \text{N} = 0$$

and $F_V = (10\,000\ \text{N}) - T\sin 20.0° = \boxed{8.26 \times 10^3\ \text{N}}$

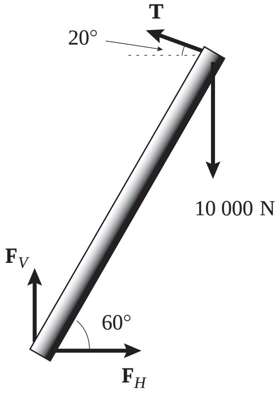


**FIG. P12.49**

**P12.50** Choosing the origin at **R**,

(1) $\sum F_x = +R\sin 15.0° - T\sin\theta = 0$

(2) $\sum F_y = 700 - R\cos 15.0° + T\cos\theta = 0$

(3) $\sum \tau = -700\cos\theta(0.180) + T(0.070\,0) = 0$

Solve the equations for $\theta$

from (3), $T = 1\,800\cos\theta$ from (1), $R = \dfrac{1\,800\sin\theta\cos\theta}{\sin 15.0°}$

Then (2) gives $700 - \dfrac{1\,800\sin\theta\cos\theta\cos 15.0°}{\sin 15.0°} + 1\,800\cos^2\theta = 0$

or $\cos^2\theta + 0.388\,9 - 3.732\sin\theta\cos\theta = 0$

Squaring, $\cos^4\theta - 0.880\,9\cos^2\theta + 0.010\,13 = 0$

Let $u = \cos^2\theta$ then using the quadratic equation,
$u = 0.011\,65$ or $0.869\,3$

Only the second root is physically possible,

$$\therefore \theta = \cos^{-1}\sqrt{0.869\,3} = \boxed{21.2°}$$

$$\therefore T = \boxed{1.68 \times 10^3\ \text{N}} \quad \text{and} \quad R = \boxed{2.34 \times 10^3\ \text{N}}$$

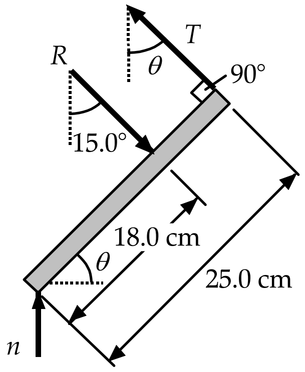


**FIG. P12.50**

**P12.51** Choosing torques about **R**, with $\sum \tau = 0$

$$-\frac{L}{2}(350\ \text{N}) + (T\sin 12.0°)\left(\frac{2L}{3}\right) - (200\ \text{N})L = 0.$$

From which, $T = \boxed{2.71\ \text{kN}}$.

Let $R_x$ = compression force along spine, and from $\sum F_x = 0$

$$R_x = T_x = T\cos 12.0° = \boxed{2.65\ \text{kN}}.$$

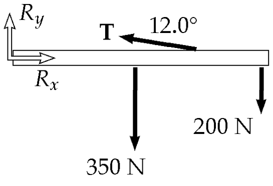


**FIG. P12.51**

**P12.52** (a) Just three forces act on the rod: forces perpendicular to the sides of the trough at A and B, and its weight. The lines of action of A and B will intersect at a point above the rod. They will have no torque about this point. The rod's weight will cause a torque about the point of intersection as in Figure 12.52(a), and the rod will not be in equilibrium unless the center of the rod lies vertically below the intersection point, as in Figure 12.52(b). All three forces must be concurrent. Then the line of action of the weight is a diagonal of the rectangle formed by the trough and the normal forces, and the rod's center of gravity is vertically above the bottom of the trough.

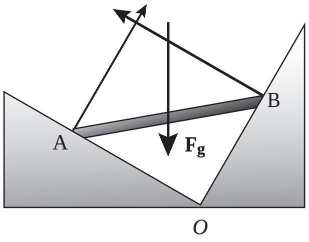


**FIG. P12.52(a)**

(b) In Figure (b), $\overline{AO}\cos 30.0° = \overline{BO}\cos 60.0°$ and

$$L^2 = \overline{AO}^2 + \overline{BO}^2 = \overline{AO}^2 + \overline{AO}^2\left(\frac{\cos^2 30.0°}{\cos^2 60.0°}\right)$$

$$\overline{AO} = \frac{L}{\sqrt{1+\frac{\cos^2 30.0°}{\cos^2 60.0°}}} = \frac{L}{2}$$

So $\cos\theta = \frac{\overline{AO}}{L} = \frac{1}{2}$ and $\theta = \boxed{60.0°}$.

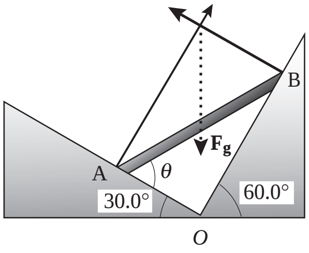


**FIG. P12.52(b)**

**P12.53** (a) Locate the origin at the bottom left corner of the cabinet and let $x =$ distance between the *resultant normal force* and the *front of the cabinet*. Then we have

$$\sum F_x = 200\cos 37.0° - \mu n = 0 \quad (1)$$

$$\sum F_y = 200\sin 37.0° + n - 400 = 0 \quad (2)$$

$$\sum \tau = n(0.600 - x) - 400(0.300) + 200\sin 37.0°(0.600)$$
$$-200\cos 37.0°(0.400) = 0 \quad (3)$$

From (2), $n = 400 - 200\sin 37.0° = 280\text{ N}$

From (3), $x = \frac{72.2 - 120 + 280(0.600) - 64.0}{280}$

$x = \boxed{20.1\text{ cm}}$ to the left of the front edge

From (1), $\mu_k = \frac{200\cos 37.0°}{280} = \boxed{0.571}$

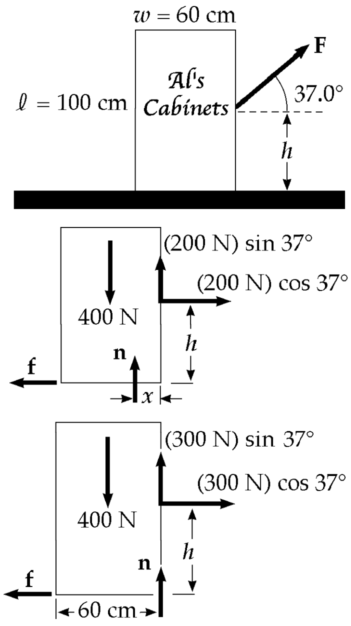


**FIG. P12.53**

(b) In this case, locate the origin $x = 0$ at the bottom right corner of the cabinet. Since the cabinet is about to tip, we can use $\sum \tau = 0$ to find $h$:

$$\sum \tau = 400(0.300) - (300\cos 37.0°)h = 0 \qquad h = \frac{120}{300\cos 37.0°} = \boxed{0.501\text{ m}}$$

**P12.54** (a), (b) Use the first diagram and sum the torques about the lower front corner of the cabinet.

$$\sum \tau = 0 \Rightarrow -F(1.00\text{ m}) + (400\text{ N})(0.300\text{ m}) = 0$$

yielding $F = \dfrac{(400\text{ N})(0.300\text{ m})}{1.00\text{ m}} = \boxed{120\text{ N}}$

$$\sum F_x = 0 \Rightarrow -f + 120\text{ N} = 0, \quad \text{or} \quad f = 120\text{ N}$$

$$\sum F_y = 0 \Rightarrow -400\text{ N} + n = 0, \quad \text{so} \quad n = 400\text{ N}$$

Thus, $\mu_s = \dfrac{f}{n} = \dfrac{120\text{ N}}{400\text{ N}} = \boxed{0.300}$.

(c) Apply $F'$ at the upper rear corner and directed so $\theta + \phi = 90.0°$ to obtain the largest possible lever arm.

$$\theta = \tan^{-1}\left(\frac{1.00\text{ m}}{0.600\text{ m}}\right) = 59.0°$$

Thus, $\phi = 90.0° – 59.0° = 31.0°$.

Sum the torques about the lower front corner of the cabinet:

$$-F'\sqrt{(1.00\text{ m})^2 + (0.600\text{ m})^2} + (400\text{ N})(0.300\text{ m}) = 0$$

so $\quad F' = \dfrac{120\text{ N}\cdot\text{m}}{1.17\text{ m}} = 103\text{ N}$.

Therefore, the minimum force required to tip the cabinet is

$\boxed{\text{103 N applied at 31.0° above the horizontal at the upper left corner}}$.

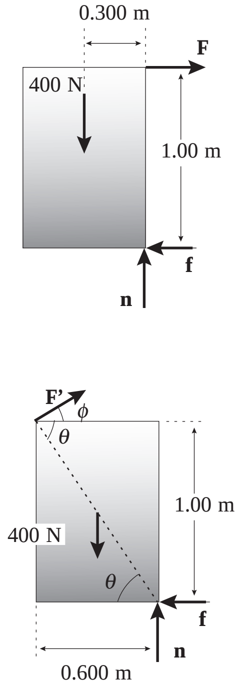


**FIG. P12.54**

**P12.55** (a) We can use $\sum F_x = \sum F_y = 0$ and $\sum \tau = 0$ with pivot point at the contact on the floor.

Then $\sum F_x = T - \mu_s n = 0$,

$\sum F_y = n - Mg - mg = 0$, and

$$\sum \tau = Mg(L\cos\theta) + mg\left(\frac{L}{2}\cos\theta\right) - T(L\sin\theta) = 0$$

Solving the above equations gives

$$M = \boxed{\frac{m}{2}\left(\frac{2\mu_s \sin\theta - \cos\theta}{\cos\theta - \mu_s \sin\theta}\right)}$$

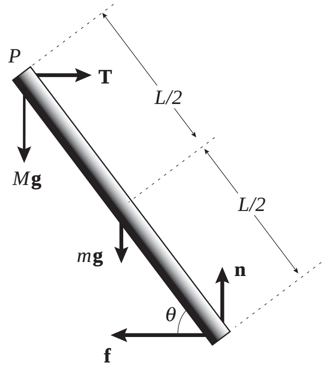


**FIG. P12.55**

This answer is the maximum vaue for $M$ if $\mu_s < \cot\theta$. If $\mu_s \geq \cot\theta$, the mass $M$ can increase without limit. It has no maximum value, and part (b) cannot be answered as stated either. In the case $\mu_s < \cot\theta$, we proceed.

(b) At the floor, we have the normal force in the $y$-direction and frictional force in the $x$-direction. The reaction force then is

$$R = \sqrt{n^2 + (\mu_s n)^2} = \boxed{(M+m)g\sqrt{1+\mu_s^2}}.$$

At point P, the force of the beam on the rope is

$$F = \sqrt{T^2 + (Mg)^2} = \boxed{g\sqrt{M^2 + \mu_s^2(M+m)^2}}.$$

**P12.56** (a) The height of pin B is

$$(10.0\text{ m})\sin 30.0° = 5.00\text{ m}.$$

The length of bar BC is then

$$\overline{BC} = \frac{5.00\text{ m}}{\sin 45.0°} = 7.07\text{ m}.$$

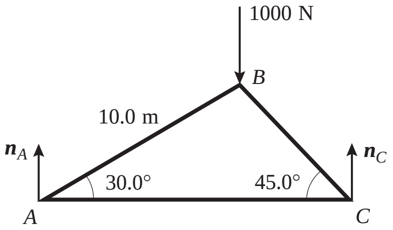


**FIG. P12.56(a)**

Consider the entire truss:

$$\sum F_y = n_A - 1\,000\text{ N} + n_C = 0$$
$$\sum \tau_A = -(1\,000\text{ N})10.0\cos 30.0° + n_C\left[10.0\cos 30.0° + 7.07\cos 45.0°\right] = 0$$

Which gives $\boxed{n_C = 634\text{ N}}$.

Then, $\boxed{n_A = 1\,000\text{ N} - n_C = 366\text{ N}}$.

(b) Suppose that a bar exerts on a pin a force not along the length of the bar. Then, the pin exerts on the bar a force with a component perpendicular to the bar. The only other force on the bar is the pin force on the other end. For $\sum \mathbf{F} = 0$, this force must also have a component perpendicular to the bar. Then, the total torque on the bar is not zero. The contradiction proves that the bar can only exert forces along its length.

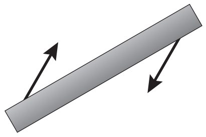


**FIG. P12.56(b)**

(c) Joint A:

$$\sum F_y = 0:\ -C_{AB}\sin 30.0° + 366\text{ N} = 0,$$

so $C_{AB} = \boxed{732\text{ N}}$

$$\sum F_x = 0:\ -C_{AB}\cos 30.0° + T_{AC} = 0$$
$$T_{AC} = (732\text{ N})\cos 30.0° = \boxed{634\text{ N}}$$

Joint B:

$$\sum F_x = 0:\ (732\text{ N})\cos 30.0° - C_{BC}\cos 45.0° = 0$$
$$C_{BC} = \frac{(732\text{ N})\cos 30.0°}{\cos 45.0°} = \boxed{897\text{ N}}$$

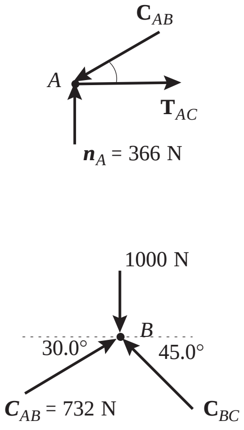


**FIG. P12.56(c)**

**P12.57** From geometry, observe that

$$\cos\theta = \frac{1}{4} \qquad \text{and} \qquad \theta = 75.5°$$

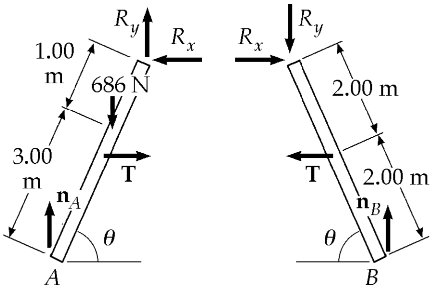


**FIG. P12.57**

For the left half of the ladder, we have

$$\sum F_x = T - R_x = 0 \qquad (1)$$
$$\sum F_y = R_y + n_A - 686\text{ N} = 0 \qquad (2)$$
$$\sum \tau_{\text{top}} = 686\text{ N}(1.00\cos 75.5°) + T(2.00\sin 75.5°) - n_A(4.00\cos 75.5°) = 0 \qquad (3)$$

For the right half of the ladder we have

$$\sum F_x = R_x - T = 0$$
$$\sum F_y = n_B - R_y = 0 \qquad (4)$$
$$\sum \tau_{\text{top}} = n_B(4.00\cos 75.5°) - T(2.00\sin 75.5°) = 0 \qquad (5)$$

Solving equations 1 through 5 simultaneously yields:

(a) $\boxed{T = 133\text{ N}}$

(b) $\boxed{n_A = 429\text{ N}}$ and $\boxed{n_B = 257\text{ N}}$

(c) $\boxed{R_x = 133\text{ N}}$ and $\boxed{R_y = 257\text{ N}}$

The force exerted by the left half of the ladder on the right half is to the right and downward.

**P12.58** (a)

$$x_{\text{CG}} = \frac{\sum m_i x_i}{\sum m_i}$$
$$= \frac{(1\,000\text{ kg})10.0\text{ m} + (125\text{ kg})0 + (125\text{ kg})0 + (125\text{ kg})20.0\text{ m}}{1\,375\text{ kg}} = \boxed{9.09\text{ m}}$$
$$y_{\text{CG}} = \frac{(1\,000\text{ kg})10.0\text{ m} + (125\text{ kg})20.0\text{ m} + (125\text{ kg})20.0\text{ m} + (125\text{ kg})0}{1\,375\text{ kg}}$$
$$= \boxed{10.9\text{ m}}$$

(b) By symmetry, $x_{\text{CG}} = \boxed{10.0\text{ m}}$

There is no change in $y_{\text{CG}} = \boxed{10.9\text{ m}}$

(c) $$v_{\text{CG}} = \left(\frac{10.0\text{ m} - 9.09\text{ m}}{8.00\text{ s}}\right) = \boxed{0.114\text{ m/s}}$$

**P12.59** Considering the torques about the point at the bottom of the bracket yields:

$$(0.050\,0\text{ m})(80.0\text{ N}) - F(0.060\,0\text{ m}) = 0 \text{ so } \boxed{F = 66.7\text{ N}}.$$

**P12.60** When it is on the verge of slipping, the cylinder is in equilibrium.

$\sum F_x = 0$: $\quad f_1 = n_2 = \mu_s n_1 \quad$ and $\quad f_2 = \mu_s n_2$

$\sum F_y = 0$: $\quad P + n_1 + f_2 = F_g$

$\sum \tau = 0$: $\quad P = f_1 + f_2$

As $P$ grows so do $f_1$ and $f_2$

Therefore, since $\mu_s = \frac{1}{2}$, $\quad f_1 = \frac{n_1}{2} \quad$ and $\quad f_2 = \frac{n_2}{2} = \frac{n_1}{4}$

then $\quad P + n_1 + \frac{n_1}{4} = F_g \quad (1) \quad$ and $\quad P = \frac{n_1}{2} + \frac{n_1}{4} = \frac{3}{4} n_1 \quad (2)$

So $\quad P + \frac{5}{4} n_1 = F_g \quad$ becomes $\quad P + \frac{5}{4}\left(\frac{4}{3} P\right) = F_g \quad$ or $\quad \frac{8}{3} P = F_g$

Therefore, $P = \boxed{\frac{3}{8} F_g}$

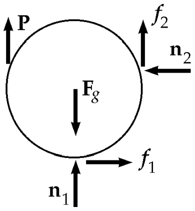


FIG. P12.60

**P12.61** (a) $|F| = k(\Delta L)$, Young's modulus is $Y = \frac{\frac{F}{A}}{\frac{\Delta L}{L_i}} = \frac{F L_i}{A(\Delta L)}$

Thus, $Y = \frac{k L_i}{A}$ and $k = \boxed{\frac{YA}{L_i}}$

(b) $W = -\int_0^{\Delta L} F dx = -\int_0^{\Delta L} (-kx) dx = \frac{YA}{L_i} \int_0^{\Delta L} x dx = \boxed{YA \frac{(\Delta L)^2}{2 L_i}}$

**P12.62** (a) Take both balls together. Their weight is 3.33 N and their CG is at their contact point.

$\sum F_x = 0: +P_3 - P_1 = 0$

$\sum F_y = 0: +P_2 - 3.33 \text{ N} = 0 \qquad P_2 = \boxed{3.33 \text{ N}}$

$\sum \tau_A = 0: -P_3 R + P_2 R - 3.33 \text{ N}(R + R\cos 45.0°)$
$\qquad + P_1(R + 2R\cos 45.0°) = 0$

Substituting,

$$-P_1 R + (3.33 \text{ N})R - (3.33 \text{ N})R(1 + \cos 45.0°)$$
$$+ P_1 R(1 + 2\cos 45.0°) = 0$$
$$(3.33 \text{ N})\cos 45.0° = 2P_1 \cos 45.0°$$

$P_1 = \boxed{1.67 \text{ N}}$ so $P_3 = \boxed{1.67 \text{ N}}$

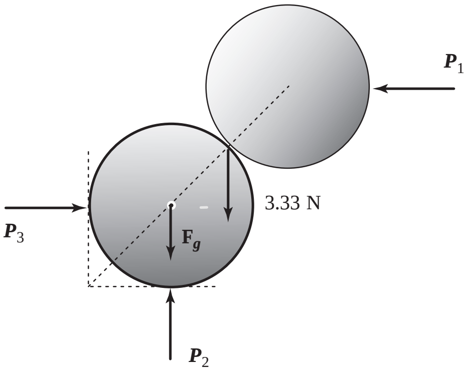


FIG. P12.62(a)

(b) Take the upper ball. The lines of action of its weight, of $P_1$, and of the normal force $n$ exerted by the lower ball all go through its center, so for rotational equilibrium there can be no frictional force.

$\sum F_x = 0: n\cos 45.0° - P_1 = 0$

$n = \frac{1.67 \text{ N}}{\cos 45.0°} = \boxed{2.36 \text{ N}}$

$\sum F_y = 0: n \sin 45.0° - 1.67 \text{ N} = 0$ gives the same result

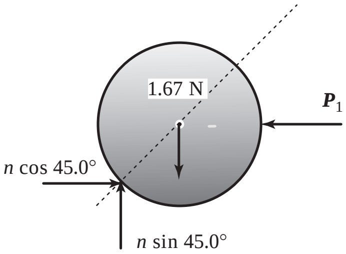


FIG. P12.62(b)

**P12.63** $\sum F_y = 0: \quad +380\text{ N} - F_g + 320\text{ N} = 0$

$F_g = 700\text{ N}$

Take torques about her feet:

$\sum \tau = 0: \quad -380\text{ N}(2.00\text{ m}) + (700\text{ N})x + (320\text{ N})0 = 0$

$x = \boxed{1.09\text{ m}}$

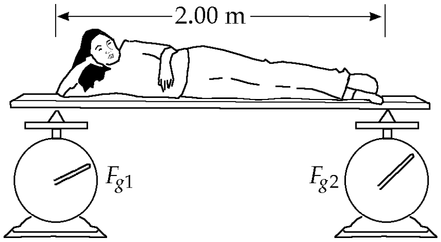


**FIG. P12.63**

**P12.64** The tension in this cable is not uniform, so this becomes a fairly difficult problem.

$$\frac{dL}{L} = \frac{F}{YA}$$

At any point in the cable, $F$ is the weight of cable below that point. Thus, $F = \mu g y$ where $\mu$ is the mass per unit length of the cable.

Then, $\Delta y = \int_0^{L_i}\left(\frac{dL}{L}\right)dy = \frac{\mu g}{YA}\int_0^{L_i} y\,dy = \frac{1}{2}\frac{\mu g L_i^2}{YA}$

$$\Delta y = \frac{1}{2}\frac{(2.40)(9.80)(500)^2}{\left(2.00\times10^{11}\right)\left(3.00\times10^{-4}\right)} = 0.049\,0\text{ m} = \boxed{4.90\text{ cm}}$$

**P12.65** (a) $F = m\left(\frac{\Delta v}{\Delta t}\right) = (1.00\text{ kg})\frac{(10.0 - 1.00)\text{ m/s}}{0.002\text{ s}} = \boxed{4\,500\text{ N}}$

(b) $\text{stress} = \frac{F}{A} = \frac{4\,500\text{ N}}{(0.010\text{ m})(0.100\text{ m})} = \boxed{4.50\times10^6\text{ N/m}^2}$

(c) $\boxed{\text{Yes}}$. This is more than sufficient to break the board.

**P12.66** The CG lies above the center of the bottom. Consider a disk of water at height $y$ above the bottom. Its radius is

$$25.0\text{ cm}+(35.0-25.0\text{ cm})\left(\frac{y}{30.0\text{ cm}}\right)=25.0\text{ cm}+\frac{y}{3}$$

Its area is $\pi\left(25.0\text{ cm}+\frac{y}{3}\right)^2$. Its volume is $\pi\left(25.0\text{ cm}+\frac{y}{3}\right)^2 dy$ and its mass is $\pi\rho\left(25.0\text{ cm}+\frac{y}{3}\right)^2 dy$. The whole mass of the water is

$$M=\int_{y=0}^{30.0\text{ cm}} dm=\int_0^{30.0\text{ cm}}\pi\rho\left(625+\frac{50.0y}{3}+\frac{y^2}{9}\right)dy$$

$$M=\pi\rho\left[625y+\frac{50.0y^2}{6}+\frac{y^3}{27}\right]_0^{30.0}$$

$$M=\pi\rho\left[625(30.0)+\frac{50.0(30.0)^2}{6}+\frac{(30.0)^3}{27}\right]$$

$$M=\pi\left(10^{-3}\text{ kg/cm}^3\right)\left(27\,250\text{ cm}^3\right)=85.6\text{ kg}$$

The height of the center of gravity is

$$y_{\text{CG}}=\int_{y=0}^{30.0\text{ cm}}\frac{y\,dm}{M}$$

$$=\pi\rho\int_0^{30.0\text{ cm}}\left(625y+\frac{50.0y^2}{3}+\frac{y^3}{9}\right)\frac{dy}{M}$$

$$=\frac{\pi\rho}{M}\left[\frac{625y^2}{2}+\frac{50.0y^3}{9}+\frac{y^4}{36}\right]_0^{30.0\text{ cm}}$$

$$=\frac{\pi\rho}{M}\left[\frac{625(30.0)^2}{2}+\frac{50.0(30.0)^3}{9}+\frac{(30.0)^4}{36}\right]$$

$$=\frac{\pi\left(10^{-3}\text{ kg/cm}^3\right)}{M}\left[453\,750\text{ cm}^4\right]$$

$$y_{\text{CG}}=\frac{1.43\times10^3\text{ kg}\cdot\text{cm}}{85.6\text{ kg}}=\boxed{16.7\text{ cm}}$$

**P12.67** Let $\theta$ represent the angle of the wire with the vertical. The radius of the circle of motion is $r = (0.850\text{ m})\sin\theta$.

For the mass:

$$\sum F_r = ma_r = m\frac{v^2}{r} = mr\omega^2$$

$$T\sin\theta = m\left[(0.850\text{ m})\sin\theta\right]\omega^2$$

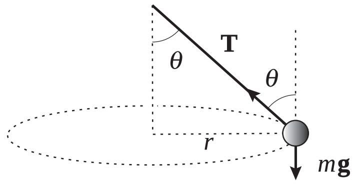


**FIG. P12.67**

Further, $\dfrac{T}{A} = Y\cdot(\text{strain})$ or $T = AY\cdot(\text{strain})$

Thus, $AY\cdot(\text{strain}) = m(0.850\text{ m})\omega^2$, giving

$$\omega = \sqrt{\frac{AY\cdot(\text{strain})}{m(0.850\text{ m})}} = \sqrt{\frac{\pi\left(3.90\times10^{-4}\text{ m}\right)^2\left(7.00\times10^{10}\text{ N/m}^2\right)\left(1.00\times10^{-3}\right)}{(1.20\text{ kg})(0.850\text{ m})}}$$

or $\omega = \boxed{5.73\text{ rad/s}}$.

**P12.68** For the bridge as a whole:

$$\sum\tau_A = n_A(0) - (13.3\text{ kN})(100\text{ m}) + n_E(200\text{ m}) = 0$$

so $n_E = \dfrac{(13.3\text{ kN})(100\text{ m})}{200\text{ m}} = \boxed{6.66\text{ kN}}$

$$\sum F_y = n_A - 13.3\text{ kN} + n_E = 0 \text{ gives}$$

$$n_A = 13.3\text{ kN} - n_E = \boxed{6.66\text{ kN}}$$

At Pin A:

$$\sum F_y = -F_{AB}\sin 40.0° + 6.66\text{ kN} = 0 \text{ or}$$

$$F_{AB} = \frac{6.66\text{ kN}}{\sin 40.0°} = \boxed{10.4\text{ kN (compression)}}$$

$$\sum F_x = F_{AC} - (10.4\text{ kN})\cos 40.0° = 0 \text{ so}$$

$$F_{AC} = (10.4\text{ kN})\cos 40.0° = \boxed{7.94\text{ kN (tension)}}$$

At Pin B:

$$\sum F_y = (10.4\text{ kN})\sin 40.0° - F_{BC}\sin 40.0° = 0$$

Thus, $F_{BC} = \boxed{10.4\text{ kN (tension)}}$

$$\sum F_x = F_{AB}\cos 40.0° + F_{BC}\cos 40.0° - F_{BD} = 0$$

$$F_{BD} = 2(10.4\text{ kN})\cos 40.0° = \boxed{15.9\text{ kN (compression)}}$$

By symmetry: $\boxed{F_{DE} = F_{AB} = 10.4\text{ kN (compression)}}$

$\boxed{F_{DC} = F_{BC} = 10.4\text{ kN (tension)}}$

and $\boxed{F_{EC} = F_{AC} = 7.94\text{ kN (tension)}}$

We can check by analyzing Pin C:

$$\sum F_x = +7.94\text{ kN} - 7.94\text{ kN} = 0 \text{ or } 0 = 0$$

$$\sum F_y = 2(10.4\text{ kN})\sin 40.0° - 13.3\text{ kN} = 0$$

which yields $0 = 0$.

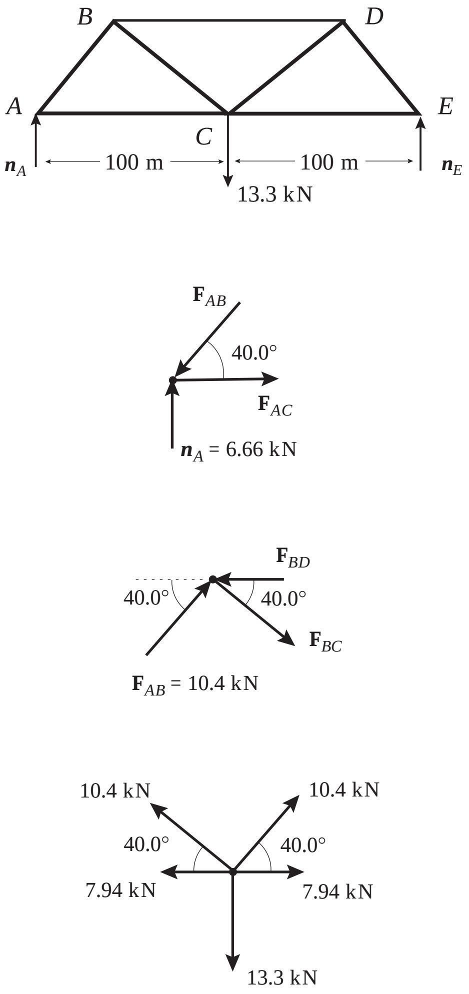
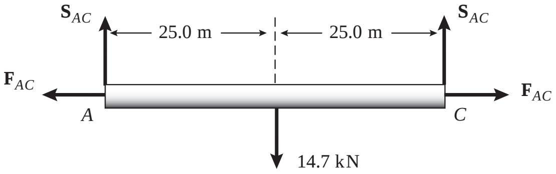


**FIG. P12.68**

**P12.69** Member *AC* is not in pure compression or tension. It also has shear forces present. It exerts a downward force $S_{AC}$ and a tension force $F_{AC}$ on Pin *A* and on Pin *C*. Still, this member is in equilibrium.



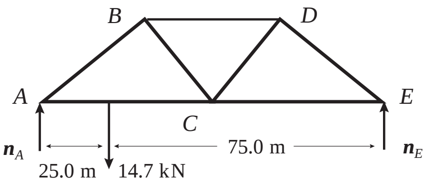
$$\sum F_x = F_{AC} - F'_{AC} = 0 \Rightarrow F_{AC} = F'_{AC}$$

$$\sum \tau_A = 0: \qquad -(14.7\text{ kN})(25.0\text{ m}) + S'_{AC}(50.0\text{ m}) = 0$$

or $\quad S'_{AC} = 7.35\text{ kN}$

$$\sum F_y = S_{AC} - 14.7\text{ kN} + 7.35\text{ kN} = 0 \Rightarrow S_{AC} = 7.35\text{ kN}$$

Then $S_{AC} = S'_{AC}$ and we have proved that the loading by the car is equivalent to one-half the weight of the car pulling down on each of pins *A* and *C*, so far as the rest of the truss is concerned.



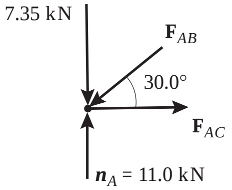
For the Bridge as a whole: $\sum \tau_A = 0$:

$$-(14.7\text{ kN})(25.0\text{ m}) + n_E(100\text{ m}) = 0$$

$$\boxed{n_E = 3.67\text{ kN}}$$

$$\sum F_y = n_A - 14.7\text{ kN} + 3.67\text{ kN} = 0$$

$$\boxed{n_A = 11.0\text{ kN}}$$



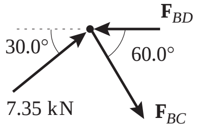
At Pin *A*:

$$\sum F_y = -7.35\text{ kN} + 11.0\text{ kN} - F_{AB}\sin 30.0° = 0$$

$$\boxed{F_{AB} = 7.35\text{ kN (compression)}}$$

$$\sum F_x = F_{AC} - (7.35\text{ kN})\cos 30.0° = 0$$

$$\boxed{F_{AC} = 6.37\text{ kN (tension)}}$$



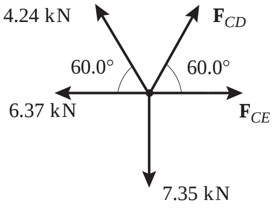
At Pin *B*:

$$\sum F_y = -(7.35\text{ kN})\sin 30.0° - F_{BC}\sin 60.0° = 0$$

$$\boxed{F_{BC} = 4.24\text{ kN (tension)}}$$

$$\sum F_x = (7.35\text{ kN})\cos 30.0° + (4.24\text{ kN})\cos 60.0° - F_{BD} = 0$$

$$\boxed{F_{BD} = 8.49\text{ kN (compression)}}$$



At Pin *C*:

$$\sum F_y = (4.24\text{ kN})\sin 60.0° + F_{CD}\sin 60.0° - 7.35\text{ kN} = 0$$

$$\boxed{F_{CD} = 4.24\text{ kN (tension)}}$$

$$\sum F_x = -6.37\text{ kN} - (4.24\text{ kN})\cos 60.0° + (4.24\text{ kN})\cos 60.0° + F_{CE} = 0$$

$$\boxed{F_{CE} = 6.37\text{ kN (tension)}}$$

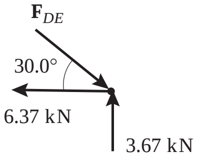


FIG. P12.69

At Pin *E*:

$$\sum F_y = -F_{DE}\sin 30.0° + 3.67\text{ kN} = 0$$

$$\boxed{F_{DE} = 7.35\text{ kN (compression)}}$$

or $\sum F_x = -6.37\text{ kN} - F_{DE}\cos 30.0° = 0$
which gives $F_{DE} = 7.35\text{ kN}$ as before.

**P12.70** (1) $ph = I\omega$

(2) $p = Mv_{\text{CM}}$

If the ball rolls without slipping, $R\omega = v_{\text{CM}}$

So, $h = \dfrac{I\omega}{p} = \dfrac{I\omega}{Mv_{\text{CM}}} = \dfrac{I}{MR} = \boxed{\dfrac{2}{5}R}$

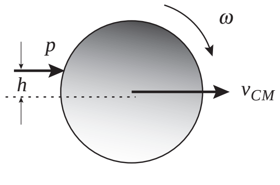


**FIG. P12.70**

**P12.71** (a) If the acceleration is $a$, we have $P_x = ma$ and $P_y + n - F_g = 0$. Taking the origin at the center of gravity, the torque equation gives

$$P_y(L-d) + P_x h - nd = 0.$$

Solving these equations, we find

$$P_y = \boxed{\frac{F_g}{L}\left(d - \frac{ah}{g}\right)}.$$

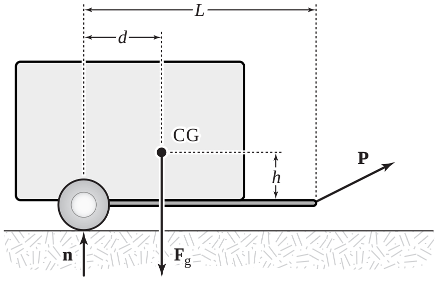


**FIG. P12.71**

(b) If $P_y = 0$, then $d = \dfrac{ah}{g} = \dfrac{(2.00\ \text{m/s}^2)(1.50\ \text{m})}{9.80\ \text{m/s}^2} = \boxed{0.306\ \text{m}}$.

(c) Using the given data, $P_x = -306\ \text{N}$ and $P_y = 553\ \text{N}$.

Thus, $\boxed{\mathbf{P} = \left(-306\hat{\mathbf{i}} + 553\hat{\mathbf{j}}\right)\ \text{N}}$.

***P12.72** When the cyclist is on the point of tipping over forward, the normal force on the rear wheel is zero. Parallel to the plane we have $f_1 - mg\sin\theta = ma$. Perpendicular to the plane, $n_1 - mg\cos\theta = 0$. Torque about the center of mass:

$$mg(0) - f_1(1.05\ \text{m}) + n_1(0.65\ \text{m}) = 0.$$

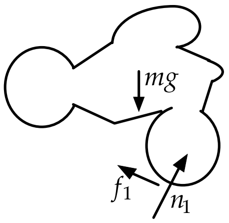


**FIG. P12.72**

Combining by substitution,

$$ma = f_1 - mg\sin\theta = \frac{n_1 0.65\ \text{m}}{1.05\ \text{m}} - mg\sin\theta = mg\cos\theta\frac{0.65\ \text{m}}{1.05\ \text{m}} - mg\sin\theta$$

$$a = g\left(\cos 20°\frac{0.65}{1.05} - \sin 20°\right) = \boxed{2.35\ \text{m/s}^2}$$

***P12.73** When the car is on the point of rolling over, the normal force on its inside wheels is zero.

$\sum F_y = ma_y$: $n - mg = 0$

$\sum F_x = ma_x$: $f = \dfrac{mv^2}{R}$

Take torque about the center of mass: $fh - n\dfrac{d}{2} = 0$.

Then by substitution $\dfrac{mv_{\max}^2}{R}h - \dfrac{mgd}{2} = 0$ $v_{\max} = \sqrt{\dfrac{gdR}{2h}}$

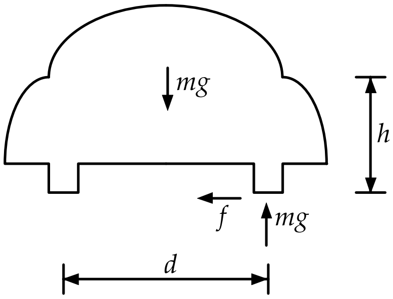


**FIG. P12.73**

A wider wheelbase (larger $d$) and a lower center of mass (smaller $h$) will reduce the risk of rollover.

# ANSWERS TO EVEN PROBLEMS

**P12.2** $F_y + R_y - F_g = 0$; $F_x - R_x = 0$;
$F_y \ell \cos\theta - F_g\left(\frac{\ell}{2}\right)\cos\theta - F_x \ell \sin\theta = 0$

**P12.4** see the solution

**P12.6** 0.750 m

**P12.8** $(2.54 \text{ m},\ 4.75 \text{ m})$

**P12.10** (a) 9.00 g; (b) 52.5 g; (c) 49.0 g

**P12.12** (a) 392 N; (b) $(339\hat{\mathbf{i}} + 0\hat{\mathbf{j}})$ N

**P12.14** (a) $f = \left[\frac{m_1 g}{2} + \frac{m_2 g x}{L}\right]\cot\theta$;
$n_g = (m_1 + m_2)g$; (b) $\mu = \frac{\left(\frac{m_1}{2} + \frac{m_2 d}{L}\right)\cot\theta}{m_1 + m_2}$

**P12.16** see the solution; 0.643 m

**P12.18** 36.7 N to the left; 31.2 N to the right

**P12.20** (a) 35.5 kN; (b) 11.5 kN to the right;
(c) 4.19 kN down

**P12.22** (a) 859 N; (b) 104 kN at 36.9° above the horizontal to the left

**P12.24** $\frac{3L}{4}$

**P12.26** (a) see the solution; (b) $\theta$ decreases;
(c) $R$ decreases

**P12.28** (a) 73.6 kN; (b) 2.50 mm

**P12.30** $\sim 1$ cm

**P12.32** $9.85 \times 10^{-5}$

**P12.34** 0.029 3 mm

**P12.36** (a) $-0.0538 \text{ m}^3$; (b) $1.09 \times 10^3 \text{ kg/m}^3$;
(c) Yes, in most practical circumstances

**P12.38** (a) 53.1°; (b) 1.04 kN; (c) 0.126 m, 51.2°;
(d) 1.07 kN; (e) 0.129 m, 51.1°; (f) 51.1°

**P12.40** (a) 0.400 mm; (b) 40.0 kN; (c) 2.00 mm;
(d) 2.40 mm; (e) 48.0 kN

**P12.42** at $A$: $Mg\frac{\sin\beta}{\sin(\alpha+\beta)}$; at $B$: $Mg\frac{\sin\alpha}{\sin(\alpha+\beta)}$

**P12.44** (a) 160 N to the right;
(b) 13.2 N to the right; (c) 292 N up;
(d) 192 N

**P12.46** 1.46 kN; $(1.33\hat{\mathbf{i}} + 2.58\hat{\mathbf{j}})$ kN

**P12.48** 0.789

**P12.50** $T = 1.68$ kN; $R = 2.34$ kN; $\theta = 21.2°$

**P12.52** (a) see the solution; (b) 60.0°

**P12.54** (a) 120 N; (b) 0.300; (c) 103 N at 31.0° above the horizontal to the right

**P12.56** (a), (b) see the solution;
(c) $C_{AB} = 732$ N; $T_{AC} = 634$ N; $C_{BC} = 897$ N

**P12.58** (a) $(9.09 \text{ m},\ 10.9 \text{ m})$; (b) $(10.0 \text{ m},\ 10.9 \text{ m})$;
(c) $0.114 \text{ m/s}$ to the right

**P12.60** $\frac{3}{8}F_g$

**P12.62** (a) $P_1 = 1.67$ N; $P_2 = 3.33$ N; $P_3 = 1.67$ N;
(b) 2.36 N

**P12.64** 4.90 cm

**P12.66** 16.7 cm above the center of the bottom

**P12.68** $C_{AB} = 10.4$ kN; $T_{AC} = 7.94$ kN;
$T_{BC} = 10.4$ kN; $C_{BD} = 15.9$ kN;
$C_{DE} = 10.4$ kN; $T_{DC} = 10.4$ kN;
$T_{EC} = 7.94$ kN

**P12.70** $\frac{2}{5}R$

**P12.72** $2.35 \text{ m/s}^2$

# 13

# Universal Gravitation

## CHAPTER OUTLINE

13.1 Newton's Law of Universal Gravitation
13.2 Measuring the Gravitational Constant
13.3 Free-Fall Acceleration and the Gravitational Force
13.4 Kepler's Laws and the Motion of Planets
13.5 The Gravitational Field
13.6 Gravitational Potential Energy
13.7 Energy Considerations in Planetary and Satellite Motion

## ANSWERS TO QUESTIONS

**Q13.1** Because $g$ is the same for all objects near the Earth's surface. The larger mass needs a larger force to give it just the same acceleration.

**Q13.2** To a good first approximation, your bathroom scale reading is unaffected because you, the Earth, and the scale are all in free fall in the Sun's gravitational field, in orbit around the Sun. To a precise second approximation, you weigh slightly less at noon and at midnight than you do at sunrise or sunset. The Sun's gravitational field is a little weaker at the center of the Earth than at the surface subsolar point, and a little weaker still on the far side of the planet. When the Sun is high in your sky, its gravity pulls up on you a little more strongly than on the Earth as a whole. At midnight the Sun pulls down on you a little less strongly than it does on the Earth below you. So you can have another doughnut with lunch, and your bedsprings will still last a little longer.

**Q13.3** Kepler's second law states that the angular momentum of the Earth is constant as the Earth orbits the sun. Since $L = m\omega r$, as the orbital radius decreases from June to December, then the orbital speed must increase accordingly.

**Q13.4** Because both the Earth and Moon are moving in orbit about the Sun. As described by $F_{\text{gravitational}} = ma_{\text{centripetal}}$, the gravitational force of the Sun merely keeps the Moon (and Earth) in a nearly circular orbit of radius 150 million kilometers. Because of its velocity, the Moon is kept in its orbit about the Earth by the gravitational force of the Earth. There is no imbalance of these forces, at new moon or full moon.

**Q13.5** Air resistance causes a decrease in the energy of the satellite-Earth system. This reduces the diameter of the orbit, bringing the satellite closer to the surface of the Earth. A satellite in a smaller orbit, however, must travel faster. Thus, the effect of air resistance is to speed up the satellite!

**Q13.6** Kepler's third law, which applies to all planets, tells us that the period of a planet is proportional to $r^{3/2}$. Because Saturn and Jupiter are farther from the Sun than Earth, they have longer periods. The Sun's gravitational field is much weaker at a distant Jovian planet. Thus, an outer planet experiences much smaller centripetal acceleration than Earth and has a correspondingly longer period.

**Q13.7** Ten terms are needed in the potential energy:

$$U = U_{12} + U_{13} + U_{14} + U_{15} + U_{23} + U_{24} + U_{25} + U_{34} + U_{35} + U_{45}.$$

With $N$ particles, you need $\sum_{i=1}^{N}(i-1) = \frac{N^2 - N}{2}$ terms.

**Q13.8** No, the escape speed does not depend on the mass of the rocket. If a rocket is launched at escape speed, then the total energy of the rocket-Earth system will be zero. When the separation distance becomes infinite $(U = 0)$ the rocket will stop $(K = 0)$. In the expression $\frac{1}{2}mv^2 - \frac{GM_E m}{r} = 0$, the mass $m$ of the rocket divides out.

**Q13.9** It takes 100 times more energy for the $10^5$ kg spacecraft to reach the moon than the $10^3$ kg spacecraft. Ideally, each spacecraft can reach the moon with zero velocity, so the only term that need be analyzed is the change in gravitational potential energy. $U$ is proportional to the mass of the spacecraft.

**Q13.10** The escape speed from the Earth is 11.2 km/s and that from the Moon is 2.3 km/s, smaller by a factor of 5. The energy required—and fuel—would be proportional to $v^2$, or 25 times more fuel is required to leave the Earth versus leaving the Moon.

**Q13.11** The satellites used for TV broadcast are in geosynchronous orbits. The centers of their orbits are the center of the Earth, and their orbital planes are the Earth's equatorial plane extended. This is the plane of the celestial equator. The communication satellites are so far away that they appear quite close to the celestial equator, from any location on the Earth's surface.

**Q13.12** For a satellite in orbit, one focus of an elliptical orbit, or the center of a circular orbit, must be located at the center of the Earth. If the satellite is over the northern hemisphere for half of its orbit, it must be over the southern hemisphere for the other half. We could share with Easter Island a satellite that would look straight down on Arizona each morning and vertically down on Easter Island each evening.

**Q13.13** The absolute value of the gravitational potential energy of the Earth-Moon system is twice the kinetic energy of the moon relative to the Earth.

**Q13.14** In a circular orbit each increment of displacement is perpendicular to the force applied. The dot product of force and displacement is zero. The work done by the gravitational force on a planet in an elliptical orbit speeds up the planet at closest approach, but negative work is done by gravity and the planet slows as it sweeps out to its farthest distance from the Sun. Therefore, net work in one complete orbit is zero.

**Q13.15** Every point $q$ on the sphere that does not lie along the axis connecting the center of the sphere and the particle will have companion point $q'$ for which the components of the gravitational force perpendicular to the axis will cancel. Point $q'$ can be found by rotating the sphere through 180° about the axis. The forces will not necessarily cancel if the mass is not uniformly distributed, unless the center of mass of the non-uniform sphere still lies along the axis.

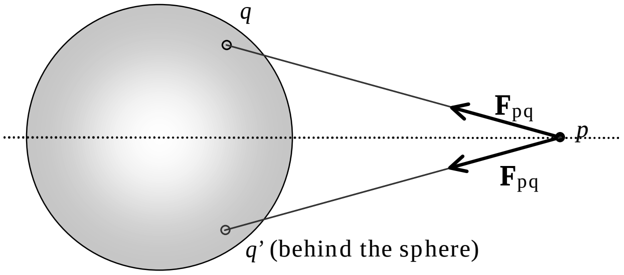


FIG. Q13.15

**Q13.16** Speed is maximum at closest approach. Speed is minimum at farthest distance.

**Q13.17** Set the universal description of the gravitational force, $F_g = \frac{GM_X m}{R_X^2}$, equal to the local description, $F_g = ma_{\text{gravitational}}$, where $M_X$ and $R_X$ are the mass and radius of planet *X*, respectively, and *m* is the mass of a "test particle." Divide both sides by *m*.

**Q13.18** The gravitational force of the Earth on an extra particle at its center must be zero, not infinite as one interpretation of Equation 13.1 would suggest. All the bits of matter that make up the Earth will pull in different outward directions on the extra particle.

**Q13.19** Cavendish determined *G*. Then from $g = \frac{GM}{R^2}$, one may determine the mass of the Earth.

**Q13.20** The gravitational force is conservative. An encounter with a stationary mass cannot permanently speed up a spacecraft. Jupiter is moving. A spacecraft flying across its orbit just behind the planet will gain kinetic energy as the planet's gravity does net positive work on it.

**Q13.21** **Method one:** Take measurements from an old kinescope of *Apollo* astronauts on the moon. From the motion of a freely falling object or from the period of a swinging pendulum you can find the acceleration of gravity on the moon's surface and calculate its mass. **Method two:** One could determine the approximate mass of the moon using an object hanging from an extremely sensitive balance, with knowledge of the position and distance of the moon and the radius of the Earth. First weigh the object when the moon is directly overhead. Then weigh of the object when the moon is just rising or setting. The slight difference between the measured weights reveals the cause of tides in the Earth's oceans, which is a difference in the strength of the moon's gravity between different points on the Earth. **Method three:** Much more precisely, from the motion of a spacecraft in orbit around the moon, its mass can be determined from Kepler's third law.

**Q13.22** The spacecraft did not have enough fuel to stop dead in its high-speed course for the Moon.

## SOLUTIONS TO PROBLEMS

### Section 13.1 **Newton's Law of Universal Gravitation**

**P13.1** For two 70-kg persons, modeled as spheres,

$$F_g = \frac{Gm_1m_2}{r^2} = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)(70\ \text{kg})(70\ \text{kg})}{(2\ \text{m})^2}\ \boxed{\sim 10^{-7}\ \text{N}}.$$

**P13.2** $F = m_1 g = \frac{Gm_1m_2}{r^2}$

$$g = \frac{Gm_2}{r^2} = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(4.00\times10^4\times10^3\ \text{kg}\right)}{(100\ \text{m})^2} = \boxed{2.67\times10^{-7}\ \text{m/s}^2}$$

**P13.3** (a) At the midpoint between the two objects, the forces exerted by the 200-kg and 500-kg objects are oppositely directed,

and from $F_g = \dfrac{Gm_1m_2}{r^2}$

we have $\sum F = \dfrac{G(50.0\text{ kg})(500\text{ kg}-200\text{ kg})}{(0.200\text{ m})^2} = \boxed{2.50\times10^{-5}\text{ N}}$ toward the 500-kg object.

(b) At a point between the two objects at a distance $d$ from the 500-kg objects, the net force on the 50.0-kg object will be zero when

$$\frac{G(50.0\text{ kg})(200\text{ kg})}{(0.400\text{ m}-d)^2} = \frac{G(50.0\text{ kg})(500\text{ kg})}{d^2}$$

or $d = \boxed{0.245\text{ m}}$

**P13.4** $m_1 + m_2 = 5.00\text{ kg}$ $\qquad m_2 = 5.00\text{ kg} - m_1$

$$F = G\frac{m_1m_2}{r^2} \Rightarrow 1.00\times10^{-8}\text{ N} = \left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\frac{m_1(5.00\text{ kg}-m_1)}{(0.200\text{ m})^2}$$

$$(5.00\text{ kg})m_1 - m_1^2 = \frac{\left(1.00\times10^{-8}\text{ N}\right)\left(0.040\,0\text{ m}^2\right)}{6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2} = 6.00\text{ kg}^2$$

Thus, $m_1^2 - (5.00\text{ kg})m_1 + 6.00\text{ kg} = 0$

or $\quad (m_1 - 3.00\text{ kg})(m_1 - 2.00\text{ kg}) = 0$

giving $\boxed{m_1 = 3.00\text{ kg, so } m_2 = 2.00\text{ kg}}$. The answer $m_1 = 2.00\text{ kg}$ and $m_2 = 3.00\text{ kg}$ is physically equivalent.

**P13.5** The force exerted on the 4.00-kg mass by the 2.00-kg mass is directed upward and given by

$$\mathbf{F}_{24} = G\frac{m_4m_2}{r_{24}^2}\hat{\mathbf{j}} = \left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\frac{(4.00\text{ kg})(2.00\text{ kg})}{(3.00\text{ m})^2}\hat{\mathbf{j}}$$
$$= 5.93\times10^{-11}\,\hat{\mathbf{j}}\text{N}$$

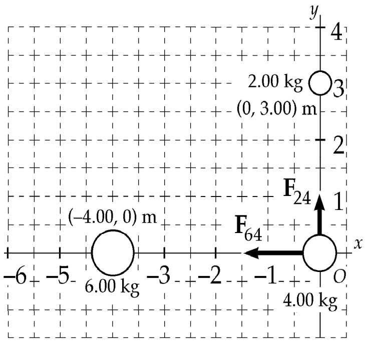


**FIG. P13.5**

The force exerted on the 4.00-kg mass by the 6.00-kg mass is directed to the left

$$\mathbf{F}_{64} = G\frac{m_4m_6}{r_{64}^2}\left(-\hat{\mathbf{i}}\right) = \left(-6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\frac{(4.00\text{ kg})(6.00\text{ kg})}{(4.00\text{ m})^2}\hat{\mathbf{i}}$$
$$= -10.0\times10^{-11}\,\hat{\mathbf{i}}\text{N}$$

Therefore, the resultant force on the 4.00-kg mass is $\mathbf{F}_4 = \mathbf{F}_{24} + \mathbf{F}_{64} = \boxed{\left(-10.0\hat{\mathbf{i}} + 5.93\hat{\mathbf{j}}\right)\times10^{-11}\text{ N}}$.

**P13.6** (a) The Sun-Earth distance is $1.496 \times 10^{11}$ m and the Earth-Moon distance is $3.84 \times 10^{8}$ m, so the distance from the Sun to the Moon during a solar eclipse is

$$1.496 \times 10^{11}\ \text{m} - 3.84 \times 10^{8}\ \text{m} = 1.492 \times 10^{11}\ \text{m}$$

The mass of the Sun, Earth, and Moon are $M_S = 1.99 \times 10^{30}$ kg

$M_E = 5.98 \times 10^{24}$ kg

and $M_M = 7.36 \times 10^{22}$ kg

We have $F_{SM} = \dfrac{Gm_1m_2}{r^2} = \dfrac{\left(6.67 \times 10^{-11}\right)\left(1.99 \times 10^{30}\right)\left(7.36 \times 10^{22}\right)}{\left(1.492 \times 10^{11}\right)^2} = \boxed{4.39 \times 10^{20}\ \text{N}}$

(b)
$$F_{EM} = \frac{\left(6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2\right)\left(5.98 \times 10^{24}\right)\left(7.36 \times 10^{22}\right)}{\left(3.84 \times 10^{8}\right)^2} = \boxed{1.99 \times 10^{20}\ \text{N}}$$

(c)
$$F_{SE} = \frac{\left(6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2\right)\left(1.99 \times 10^{30}\right)\left(5.98 \times 10^{24}\right)}{\left(1.496 \times 10^{11}\right)^2} = \boxed{3.55 \times 10^{22}\ \text{N}}$$

Note that the force exerted by the Sun on the Moon is much stronger than the force of the Earth on the Moon. In a sense, the Moon orbits the Sun more than it orbits the Earth. The Moon's path is everywhere concave toward the Sun. Only by subtracting out the solar orbital motion of the Earth-Moon system do we see the Moon orbiting the center of mass of this system.

## Section 13.2 **Measuring the Gravitational Constant**

**P13.7**
$$F = \frac{GMm}{r^2} = \left(6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2\right)\frac{(1.50\ \text{kg})\left(15.0 \times 10^{-3}\ \text{kg}\right)}{\left(4.50 \times 10^{-2}\ \text{m}\right)^2} = \boxed{7.41 \times 10^{-10}\ \text{N}}$$

**P13.8** Let $\theta$ represent the angle each cable makes with the vertical, $L$ the cable length, $x$ the distance each ball scrunches in, and $d = 1\text{ m}$ the original distance between them. Then $r = d - 2x$ is the separation of the balls. We have

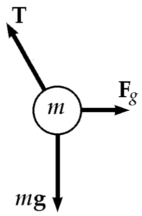


**FIG. P13.8**

$$\sum F_y = 0: \qquad T\cos\theta - mg = 0$$

$$\sum F_x = 0: \qquad T\sin\theta - \frac{Gmm}{r^2} = 0$$

Then $\quad \tan\theta = \dfrac{Gmm}{r^2 mg} \qquad \dfrac{x}{\sqrt{L^2 - x^2}} = \dfrac{Gm}{g(d-2x)^2} \qquad x(d-2x)^2 = \dfrac{Gm}{g}\sqrt{L^2 - x^2}$.

The factor $\dfrac{Gm}{g}$ is numerically small. There are two possibilities: either $x$ is small or else $d - 2x$ is small.

**Possibility one**: We can ignore $x$ in comparison to $d$ and $L$, obtaining

$$x(1\text{ m})^2 = \frac{\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)(100\text{ kg})}{\left(9.8\text{ m/s}^2\right)}45\text{ m} \qquad x = 3.06\times10^{-8}\text{ m}.$$

The separation distance is $r = 1\text{ m} - 2\left(3.06\times10^{-8}\text{ m}\right) = \boxed{1.000\text{ m} - 61.3\text{ nm}}$.

**Possibility two**: If $d - 2x$ is small, $x \approx 0.5\text{ m}$ and the equation becomes

$$(0.5\text{ m})r^2 = \frac{\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)(100\text{ kg})}{(9.8\text{ N/kg})}\sqrt{(45\text{ m})^2 - (0.5\text{ m})^2} \qquad r = \boxed{2.74\times10^{-4}\text{ m}}.$$

For this answer to apply, the spheres would have to be compressed to a density like that of the nucleus of atom.

## Section 13.3 **Free-Fall Acceleration and the Gravitational Force**

**P13.9** $a = \dfrac{MG}{(4R_E)^2} = \dfrac{9.80\text{ m/s}^2}{16.0} = \boxed{0.613\text{ m/s}^2}$ toward the Earth.

**P13.10** $$g = \frac{GM}{R^2} = \frac{G\rho\left(\frac{4\pi R^3}{3}\right)}{R^2} = \frac{4}{3}\pi G\rho R$$

If $\quad \dfrac{g_M}{g_E} = \dfrac{1}{6} = \dfrac{\frac{4\pi G\rho_M R_M}{3}}{\frac{4\pi G\rho_E R_E}{3}}$

then $\quad \dfrac{\rho_M}{\rho_E} = \left(\dfrac{g_M}{g_E}\right)\left(\dfrac{R_E}{R_M}\right) = \left(\dfrac{1}{6}\right)(4) = \boxed{\dfrac{2}{3}}$.

**P13.11** (a) At the zero-total field point, $\frac{GmM_E}{r_E^2}=\frac{GmM_M}{r_M^2}$

so $r_M=r_E\sqrt{\frac{M_M}{M_E}}=r_E\sqrt{\frac{7.36\times10^{22}}{5.98\times10^{24}}}=\frac{r_E}{9.01}$

$$r_E+r_M=3.84\times10^8\text{ m}=r_E+\frac{r_E}{9.01}$$

$$r_E=\frac{3.84\times10^8\text{ m}}{1.11}=\boxed{3.46\times10^8\text{ m}}$$

(b) At this distance the acceleration due to the Earth's gravity is

$$g_E=\frac{GM_E}{r_E^2}=\frac{\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\text{ kg}\right)}{\left(3.46\times10^8\text{ m}\right)^2}$$

$$g_E=\boxed{3.34\times10^{-3}\text{ m/s}^2\text{ directed toward the Earth}}$$

Section 13.4 **Kepler's Laws and the Motion of Planets**

**P13.12** (a) $v=\frac{2\pi r}{T}=\frac{2\pi(384\,400)\times10^3\text{ m}}{27.3\times(86\,400\text{ s})}=\boxed{1.02\times10^3\text{ m/s}}$.

(b) In one second, the Moon falls a distance

$$x=\frac{1}{2}at^2=\frac{1}{2}\frac{v^2}{r}t^2=\frac{1}{2}\frac{\left(1.02\times10^3\right)^2}{\left(3.844\times10^8\right)}\times(1.00)^2=1.35\times10^{-3}\text{ m}=\boxed{1.35\text{ mm}}.$$

The Moon only moves inward 1.35 mm for every 1020 meters it moves along a straight-line path.

**P13.13** Applying Newton's 2nd Law, $\sum F=ma$ yields $F_g=ma_c$ for each star:

$$\frac{GMM}{(2r)^2}=\frac{Mv^2}{r}\qquad\text{or}\qquad M=\frac{4v^2r}{G}.$$

We can write $r$ in terms of the period, $T$, by considering the time and distance of one complete cycle. The distance traveled in one orbit is the circumference of the stars' common orbit, so $2\pi r=vT$. Therefore

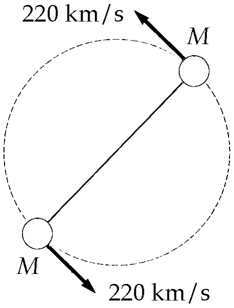


**FIG. P13.13**

$$M=\frac{4v^2r}{G}=\frac{4v^2}{G}\left(\frac{vT}{2\pi}\right)$$

so, $M=\frac{2v^3T}{\pi G}=\frac{2\left(220\times10^3\text{ m/s}\right)^3(14.4\text{ d})(86\,400\text{ s/d})}{\pi\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)}=\boxed{1.26\times10^{32}\text{ kg}=63.3\text{ solar masses}}$

**P13.14** Since speed is constant, the distance traveled between $t_1$ and $t_2$ is equal to the distance traveled between $t_3$ and $t_4$. The area of a triangle is equal to one-half its (base) width across one side times its (height) dimension perpendicular to that side.

So $\frac{1}{2}bv(t_2 - t_1) = \frac{1}{2}bv(t_4 - t_3)$

states that the particle's radius vector sweeps out equal areas in equal times.

**P13.15** $T^2 = \frac{4\pi^2 a^3}{GM}$ (Kepler's third law with $m << M$)

$$M = \frac{4\pi^2 a^3}{GT^2} = \frac{4\pi^2\left(4.22\times 10^8 \text{ m}\right)^3}{\left(6.67\times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.77\times 86\,400 \text{ s}\right)^2} = \boxed{1.90\times 10^{27} \text{ kg}}$$

(Approximately 316 Earth masses)

**P13.16** By conservation of angular momentum for the satellite,

$$r_p v_p = r_a v_a \qquad \frac{v_p}{v_a} = \frac{r_a}{r_p} = \frac{2\,289 \text{ km} + 6.37\times 10^3 \text{ km}}{459 \text{ km} + 6.37\times 10^3 \text{ km}} = \frac{8\,659 \text{ km}}{6\,829 \text{ km}} = \boxed{1.27}.$$

We do not need to know the period.

**P13.17** By Kepler's Third Law, $T^2 = ka^3$ ($a =$ semi-major axis)

For any object orbiting the Sun, with $T$ in years and $a$ in A.U., $k = 1.00$. Therefore, for Comet Halley

$$(75.6)^2 = (1.00)\left(\frac{0.570 + y}{2}\right)^3$$

The farthest distance the comet gets from the Sun is

$y = 2(75.6)^{2/3} - 0.570 = \boxed{35.2 \text{ A.U.}}$ (out around the orbit of Pluto)

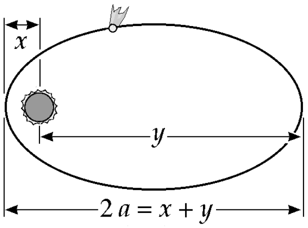


FIG. P13.17

**P13.18** $\sum F = ma$: $\quad \frac{Gm_{\text{planet}}M_{\text{star}}}{r^2} = \frac{m_{\text{planet}}v^2}{r}$

$$\frac{GM_{\text{star}}}{r} = v^2 = r^2\omega^2$$

$$GM_{\text{star}} = r^3\omega^3 = r_x^3\omega_x^2 = r_y^3\omega_y^2$$

$$\omega_y = \omega_x\left(\frac{r_x}{r_y}\right)^{3/2} \qquad \omega_y = \left(\frac{90.0°}{5.00 \text{ yr}}\right)3^{3/2} = \frac{468°}{5.00 \text{ yr}}$$

So $\boxed{\text{planet } Y \text{ has turned through 1.30 revolutions}}$.

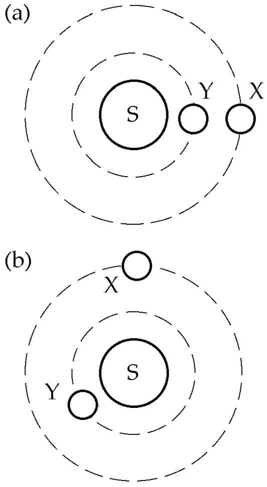


FIG. P13.18

**P13.19** $$\frac{GM_J}{(R_J+d)^2}=\frac{4\pi^2(R_J+d)}{T^2}$$

$$GM_JT^2=4\pi^2(R_J+d)^3$$

$$\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.90\times10^{27}\ \text{kg}\right)(9.84\times3\,600)^2=4\pi^2\left(6.99\times10^7+d\right)^3$$

$d=\boxed{8.92\times10^7\ \text{m}}=\boxed{89\,200\ \text{km}}$ above the planet

**P13.20** The gravitational force on a small parcel of material at the star's equator supplies the necessary centripetal force:

$$\frac{GM_sm}{R_s^2}=\frac{mv^2}{R_s}=mR_s\omega^2$$

so $$\omega=\sqrt{\frac{GM_s}{R_s^3}}=\sqrt{\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left[2\left(1.99\times10^{30}\ \text{kg}\right)\right]}{\left(10.0\times10^3\ \text{m}\right)^3}}$$

$$\omega=\boxed{1.63\times10^4\ \text{rad/s}}$$

***P13.21** The speed of a planet in a circular orbit is given by

$$\sum F=ma:\qquad \frac{GM_{\text{sun}}m}{r^2}=\frac{mv^2}{r}\qquad v=\sqrt{\frac{GM_{\text{sun}}}{r}}.$$

For Mercury the speed is $$v_M=\sqrt{\frac{\left(6.67\times10^{-11}\right)\left(1.99\times10^{30}\right)\text{m}^2}{\left(5.79\times10^{10}\right)\text{s}^2}}=4.79\times10^4\ \text{m/s}$$

and for Pluto, $$v_P=\sqrt{\frac{\left(6.67\times10^{-11}\right)\left(1.99\times10^{30}\right)\text{m}^2}{\left(5.91\times10^{12}\right)\text{s}^2}}=4.74\times10^3\ \text{m/s}.$$

With greater speed, Mercury will eventually move farther from the Sun than Pluto. With original distances $r_P$ and $r_M$ perpendicular to their lines of motion, they will be equally far from the Sun after time $t$ where

$$\sqrt{r_P^2+v_P^2t^2}=\sqrt{r_M^2+v_M^2t^2}$$

$$r_P^2-r_M^2=\left(v_M^2-v_P^2\right)t^2$$

$$t=\sqrt{\frac{\left(5.91\times10^{12}\ \text{m}\right)^2-\left(5.79\times10^{10}\ \text{m}\right)^2}{\left(4.79\times10^4\ \text{m/s}\right)^2-\left(4.74\times10^3\ \text{m/s}\right)^2}}=\sqrt{\frac{3.49\times10^{25}\ \text{m}^2}{2.27\times10^9\ \text{m}^2/\text{s}^2}}=1.24\times10^8\ \text{s}=\boxed{393\ \text{yr}}.$$

***P13.22** For the Earth, $\sum F = ma$: $\dfrac{GM_s m}{r^2} = \dfrac{mv^2}{r} = \dfrac{m}{r}\left(\dfrac{2\pi r}{T}\right)^2$.

Then $GM_s T^2 = 4\pi^2 r^3$.

Also the angular momentum $L = mvr = m\dfrac{2\pi r}{T}r$ is a constant for the Earth.

We eliminate $r = \sqrt{\dfrac{LT}{2\pi m}}$ between the equations:

$$GM_s T^2 = 4\pi^2\left(\frac{LT}{2\pi m}\right)^{3/2} \qquad GM_s T^{1/2} = 4\pi^2\left(\frac{L}{2\pi m}\right)^{3/2}.$$

Now the rate of change is described by

$$GM_s\left(\frac{1}{2}T^{-1/2}\frac{dT}{dt}\right) + G\left(1\frac{dM_s}{dt}T^{1/2}\right) = 0 \qquad \frac{dT}{dt} = -\frac{dM_s}{dt}\left(2\frac{T}{M_s}\right) \approx \frac{\Delta T}{T}$$

$$\Delta T \approx -\Delta t\frac{dM_s}{dt}\left(2\frac{T}{M_s}\right) = -5\,000\text{ yr}\left(\frac{3.16\times10^7\text{ s}}{1\text{ yr}}\right)\left(-3.64\times10^9\text{ kg/s}\right)\left(2\frac{1\text{ yr}}{1.991\times10^{30}\text{ kg}}\right)$$

$$\Delta T = \boxed{1.82\times10^{-2}\text{ s}}$$

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Section 13.5 **The Gravitational Field**

**P13.23** $\mathbf{g} = \dfrac{Gm}{l^2}\hat{\mathbf{i}} + \dfrac{Gm}{l^2}\hat{\mathbf{j}} + \dfrac{Gm}{2l^2}\left(\cos 45.0°\,\hat{\mathbf{i}} + \sin 45.0\hat{\mathbf{j}}\right)$

so $\mathbf{g} = \dfrac{GM}{l^2}\left(1 + \dfrac{1}{2\sqrt{2}}\right)\left(\hat{\mathbf{i}} + \hat{\mathbf{j}}\right)$ or

$$\mathbf{g} = \boxed{\frac{Gm}{l^2}\left(\sqrt{2} + \frac{1}{2}\right)\text{ toward the opposite corner}}$$

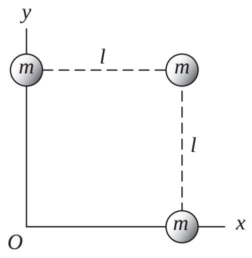

**FIG. P13.23**

**P13.24** (a) $F = \dfrac{GMm}{r^2} = \dfrac{\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left[100\left(1.99\times10^{30}\text{ kg}\right)\left(10^3\text{ kg}\right)\right]}{\left(1.00\times10^4\text{ m} + 50.0\text{ m}\right)^2} = \boxed{1.31\times10^{17}\text{ N}}$

(b) $\Delta F = \dfrac{GMm}{r_{\text{front}}^2} - \dfrac{GMm}{r_{\text{back}}^2}$

$$\Delta g = \frac{\Delta F}{m} = \frac{GM\left(r_{\text{back}}^2 - r_{\text{front}}^2\right)}{r_{\text{front}}^2 r_{\text{back}}^2}$$

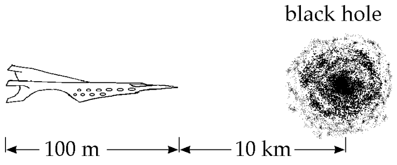

**FIG. P13.24**

$$\Delta g = \frac{\left(6.67\times10^{-11}\right)\left[100\left(1.99\times10^{30}\right)\right]\left[\left(1.01\times10^4\text{ m}\right)^2 - \left(1.00\times10^4\text{ m}\right)^2\right]}{\left(1.00\times10^4\text{ m}\right)^2\left(1.01\times10^4\text{ m}\right)^2}$$

$$\Delta g = \boxed{2.62\times10^{12}\text{ N/kg}}$$

**P13.25** $g_1 = g_2 = \dfrac{MG}{r^2 + a^2}$

$g_{1y} = -g_{2y}$ $\qquad g_y = g_{1y} + g_{2y}$

$g_{1x} = g_{2x} = g_2 \cos\theta$ $\qquad \cos\theta = \dfrac{r}{\left(a^2 + r^2\right)^{1/2}}$

$$\mathbf{g} = 2g_{2x}\left(-\hat{\mathbf{i}}\right)$$

or $\mathbf{g} = \boxed{\dfrac{2MGr}{\left(r^2 + a^2\right)^{3/2}} \text{ toward the center of mass}}$

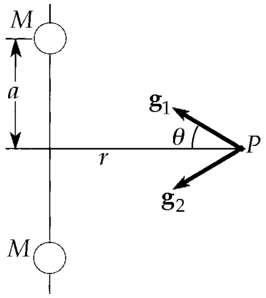

**FIG. P13.25**

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Section 13.6 **Gravitational Potential Energy**

**P13.26** (a) $U = -\dfrac{GM_E m}{r} = -\dfrac{\left(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98 \times 10^{24} \text{ kg}\right)(100 \text{ kg})}{(6.37 + 2.00) \times 10^6 \text{ m}} = \boxed{-4.77 \times 10^9 \text{ J}}.$

(b), (c) Planet and satellite exert forces of equal magnitude on each other, directed downward on the satellite and upward on the planet.

$$F = \frac{GM_E m}{r^2} = \frac{\left(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98 \times 10^{24} \text{ kg}\right)(100 \text{ kg})}{\left(8.37 \times 10^6 \text{ m}\right)^2} = \boxed{569 \text{ N}}$$

**P13.27** $U = -G\dfrac{Mm}{r}$ and $g = \dfrac{GM_E}{R_E^2}$

so that $\Delta U = -GMm\left(\dfrac{1}{3R_E} - \dfrac{1}{R_E}\right) = \dfrac{2}{3}mgR_E$

$$\Delta U = \frac{2}{3}(1\,000 \text{ kg})\left(9.80 \text{ m/s}^2\right)\left(6.37 \times 10^6 \text{ m}\right) = \boxed{4.17 \times 10^{10} \text{ J}}.$$

**P13.28** The height attained is not small compared to the radius of the Earth, so $U = mgy$ does not apply; $U = -\dfrac{GM_1M_2}{r}$ does. From launch to apogee at height $h$,

$K_i + U_i + \Delta E_{\text{mch}} = K_f + U_f$: $\quad \dfrac{1}{2}M_p v_i^2 - \dfrac{GM_E M_p}{R_E} + 0 = 0 - \dfrac{GM_E M_p}{R_E + h}$

$$\frac{1}{2}\left(10.0 \times 10^3 \text{ m/s}\right)^2 - \left(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(\frac{5.98 \times 10^{24} \text{ kg}}{6.37 \times 10^6 \text{ m}}\right)$$

$$= -\left(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(\frac{5.98 \times 10^{24} \text{ kg}}{6.37 \times 10^6 \text{ m} + h}\right)$$

$$\left(5.00 \times 10^7 \text{ m}^2/\text{s}^2\right) - \left(6.26 \times 10^7 \text{ m}^2/\text{s}^2\right) = \frac{-3.99 \times 10^{14} \text{ m}^3/\text{s}^2}{6.37 \times 10^6 \text{ m} + h}$$

$$6.37 \times 10^6 \text{ m} + h = \frac{3.99 \times 10^{14} \text{ m}^3/\text{s}^2}{1.26 \times 10^7 \text{ m}^2/\text{s}^2} = 3.16 \times 10^7 \text{ m}$$

$$\boxed{h = 2.52 \times 10^7 \text{ m}}$$

**P13.29** (a)

$$\rho = \frac{M_S}{\frac{4}{3}\pi r_E^2} = \frac{3\left(1.99\times10^{30}\text{ kg}\right)}{4\pi\left(6.37\times10^{6}\text{ m}\right)^3} = \boxed{1.84\times10^{9}\text{ kg/m}^3}$$

(b)

$$g = \frac{GM_S}{r_E^2} = \frac{\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.99\times10^{30}\text{ kg}\right)}{\left(6.37\times10^{6}\text{ m}\right)^2} = \boxed{3.27\times10^{6}\text{ m/s}^2}$$

(c)

$$U_g = -\frac{GM_S m}{r_E} = -\frac{\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.99\times10^{30}\text{ kg}\right)(1.00\text{ kg})}{6.37\times10^{6}\text{ m}} = \boxed{-2.08\times10^{13}\text{ J}}$$

**P13.30**

$$W = -\Delta U = -\left(\frac{-Gm_1m_2}{r} - 0\right)$$

$$W = \frac{\left(+6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(7.36\times10^{22}\text{ kg}\right)\left(1.00\times10^{3}\text{ kg}\right)}{1.74\times10^{6}\text{ m}} = \boxed{2.82\times10^{9}\text{ J}}$$

**P13.31** (a)

$$U_{\text{Tot}} = U_{12} + U_{13} + U_{23} = 3U_{12} = 3\left(-\frac{Gm_1m_2}{r_{12}}\right)$$

$$U_{\text{Tot}} = -\frac{3\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.00\times10^{-3}\text{ kg}\right)^2}{0.300\text{ m}} = \boxed{-1.67\times10^{-14}\text{ J}}$$

(b) $\boxed{\text{At the center}}$ of the equilateral triangle

***P13.32** (a) Energy conservation of the object-Earth system from release to radius *r*:

$$\left(K+U_g\right)_{\text{altitude } h} = \left(K+U_g\right)_{\text{radius } r}$$

$$0 - \frac{GM_E m}{R_E + h} = \frac{1}{2}mv^2 - \frac{GM_E m}{r}$$

$$v = \left(2GM_E\left(\frac{1}{r} - \frac{1}{R_E+h}\right)\right)^{1/2} = -\frac{dr}{dt}$$

(b) $\int_i^f dt = \int_i^f -\frac{dr}{v} = \int_f^i \frac{dr}{v}$. The time of fall is

$$\Delta t = \int_{R_E}^{R_E+h}\left(2GM_E\left(\frac{1}{r} - \frac{1}{R_E+h}\right)\right)^{-1/2} dr$$

$$\Delta t = \int_{6.37\times10^6\text{ m}}^{6.87\times10^6\text{ m}}\left[2\times6.67\times10^{-11}\times5.98\times10^{24}\left(\frac{1}{r} - \frac{1}{6.87\times10^{6}\text{ m}}\right)\right]^{-1/2} dr$$

We can enter this expression directly into a mathematical calculation program. Alternatively, to save typing we can change variables to $u = \frac{r}{10^6}$. Then

$$\Delta t = \left(7.977\times10^{14}\right)^{-1/2}\int_{6.37}^{6.87}\left(\frac{1}{10^6 u} - \frac{1}{6.87\times10^6}\right)^{-1/2} 10^6\,du = 3.541\times10^{-8}\frac{10^6}{\left(10^6\right)^{-1/2}}\int_{6.37}^{6.87}\left(\frac{1}{u} - \frac{1}{6.87}\right)^{-1/2} du$$

A mathematics program returns the value 9.596 for this integral, giving for the time of fall $\Delta t = 3.541\times10^{-8}\times10^{9}\times9.596 = 339.8 = \boxed{340\text{ s}}$.

## Section 13.7 **Energy Considerations in Planetary and Satellite Motion**

**P13.33** $$\frac{1}{2}mv_i^2+GM_Em\left(\frac{1}{r_f}-\frac{1}{r_i}\right)=\frac{1}{2}mv_f^2 \qquad \frac{1}{2}v_i^2+GM_E\left(0-\frac{1}{R_E}\right)=\frac{1}{2}v_f^2$$

or $$v_f^2=v_1^2-\frac{2GM_E}{R_E}$$

and $$v_f=\left(v_1^2-\frac{2GM_E}{R_E}\right)^{1/2}$$

$$v_f=\left[\left(2.00\times10^4\right)^2-1.25\times10^8\right]^{1/2}=\boxed{1.66\times10^4\ \text{m/s}}$$

**P13.34** (a) $$v_{\text{solar escape}}=\sqrt{\frac{2M_{\text{Sun}}G}{R_{E\text{-Sun}}}}=\boxed{42.1\ \text{km/s}}$$

(b) Let $r=R_{E\text{-}S}x$ represent variable distance from the Sun, with $x$ in astronomical units.

$$v=\sqrt{\frac{2M_{\text{Sun}}G}{R_{E\text{-}S}x}}=\frac{42.1}{\sqrt{x}}$$

If $v=\dfrac{125\,000\ \text{km}}{3\,600\ \text{s}}$, then $x=1.47\ \text{A.U.}=\boxed{2.20\times10^{11}\ \text{m}}$

(at or beyond the orbit of Mars, 125 000 km/h is sufficient for escape).

**P13.35** To obtain the orbital velocity, we use $$\sum F=\frac{mMG}{R^2}=\frac{mv^2}{R}$$

or $$v=\sqrt{\frac{MG}{R}}$$

We can obtain the escape velocity from $$\frac{1}{2}mv_{\text{esc}}^2=\frac{mMG}{R}$$

or $$v_{\text{esc}}=\sqrt{\frac{2MG}{R}}=\boxed{\sqrt{2}v}$$

**P13.36** $$\frac{v_i^2}{R_E+h}=\frac{GM_E}{(R_E+h)^2}$$

$$K_i=\frac{1}{2}mv_i^2=\frac{1}{2}\left(\frac{GM_Em}{R_E+h}\right)=\frac{1}{2}\left[\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)(500\ \text{kg})}{\left(6.37\times10^6\ \text{m}\right)+\left(0.500\times10^6\ \text{m}\right)}\right]=1.45\times10^{10}\ \text{J}$$

The change in gravitational potential energy of the satellite-Earth system is

$$\Delta U=\frac{GM_Em}{R_i}-\frac{GM_Em}{R_f}=GM_Em\left(\frac{1}{R_i}-\frac{1}{R_f}\right)$$
$$=\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)(500\ \text{kg})\left(-1.14\times10^{-8}\ \text{m}^{-1}\right)=-2.27\times10^9\ \text{J}$$

Also, $K_f=\dfrac{1}{2}mv_f^2=\dfrac{1}{2}(500\ \text{kg})\left(2.00\times10^3\ \text{m/s}\right)^2=1.00\times10^9\ \text{J}.$

The energy transformed due to friction is

$$\Delta E_{\text{int}}=K_i-K_f-\Delta U=(14.5-1.00+2.27)\times10^9\ \text{J}=\boxed{1.58\times10^{10}\ \text{J}}.$$

**P13.37** $F_c = F_G$ gives $\dfrac{mv^2}{r} = \dfrac{GmM_E}{r^2}$

which reduces to $v = \sqrt{\dfrac{GM_E}{r}}$

and period $= \dfrac{2\pi r}{v} = 2\pi r\sqrt{\dfrac{r}{GM_E}}$.

(a) $r = R_E + 200\text{ km} = 6\,370\text{ km} + 200\text{ km} = 6\,570\text{ km}$

Thus,

$$\text{period} = 2\pi\left(6.57\times10^{6}\text{ m}\right)\sqrt{\frac{\left(6.57\times10^{6}\text{ m}\right)}{\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\text{ kg}\right)}}$$

$$T = 5.30\times10^{3}\text{ s} = 88.3\text{ min} = \boxed{1.47\text{ h}}$$

(b) $v = \sqrt{\dfrac{GM_E}{r}} = \sqrt{\dfrac{\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\text{ kg}\right)}{\left(6.57\times10^{6}\text{ m}\right)}} = \boxed{7.79\text{ km/s}}$

(c) $K_f + U_f = K_i + U_i +$ energy input, gives

$$\text{input} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + \left(\frac{-GM_E m}{r_f}\right) - \left(\frac{-GM_E m}{r_i}\right) \qquad (1)$$

$$r_i = R_E = 6.37\times10^{6}\text{ m}$$

$$v_i = \frac{2\pi R_E}{86\,400\text{ s}} = 4.63\times10^{2}\text{ m/s}$$

Substituting the appropriate values into (1) yields the

$$\text{minimum energy input} = \boxed{6.43\times10^{9}\text{ J}}$$

**P13.38** The gravitational force supplies the needed centripetal acceleration.

Thus, $\frac{GM_E m}{(R_E+h)^2} = \frac{mv^2}{(R_E+h)}$ or $v^2 = \frac{GM_E}{R_E+h}$

(a) $T = \frac{2\pi r}{v} = \frac{2\pi(R_E+h)}{\sqrt{\frac{GM_E}{(R_E+h)}}}$ $T = \boxed{2\pi\sqrt{\frac{(R_E+h)^3}{GM_E}}}$

(b) $v = \boxed{\sqrt{\frac{GM_E}{R_E+h}}}$

(c) Minimum energy input is $\Delta E_{\min} = \left(K_f + U_{gf}\right) - \left(K_i - U_{gi}\right)$.

It is simplest to

$\boxed{\text{launch the satellite from a location on the equator, and launch it toward the east.}}$

This choice has the object starting with energy $K_i = \frac{1}{2}mv_i^2$

with $v_i = \frac{2\pi R_E}{1.00 \text{ day}} = \frac{2\pi R_E}{86\,400 \text{ s}}$ and $U_{gi} = -\frac{GM_E m}{R_E}$.

Thus, $\Delta E_{\min} = \frac{1}{2}m\left(\frac{GM_E}{R_E+h}\right) - \frac{GM_E m}{R_E+h} - \frac{1}{2}m\left[\frac{4\pi^2 R_E^2}{(86\,400 \text{ s})^2}\right] + \frac{GM_E m}{R_E}$

or $\Delta E_{\min} = \boxed{GM_E m\left[\frac{R_E+2h}{2R_E(R_E+h)}\right] - \frac{2\pi^2 R_E^2 m}{(86\,400 \text{ s})^2}}$

**P13.39** $E_{\text{tot}} = -\frac{GMm}{2r}$

$$\Delta E = \frac{GMm}{2}\left(\frac{1}{r_i} - \frac{1}{r_f}\right) = \frac{\left(6.67\times10^{-11}\right)\left(5.98\times10^{24}\right)}{2}\frac{10^3 \text{ kg}}{10^3 \text{ m}}\left(\frac{1}{6\,370+100} - \frac{1}{6\,370+200}\right)$$

$\Delta E = 4.69\times10^8 \text{ J} = \boxed{469 \text{ MJ}}$

**P13.40** $g_E = \frac{Gm_E}{r_E^2}$ $g_U = \frac{Gm_U}{r_U^2}$

(a) $\frac{g_U}{g_E} = \frac{m_U r_E^2}{m_E r_U^2} = 14.0\left(\frac{1}{3.70}\right)^2 = 1.02$ $g_U = (1.02)\left(9.80 \text{ m/s}^2\right) = \boxed{10.0 \text{ m/s}^2}$

(b) $v_{esc,E} = \sqrt{\frac{2Gm_E}{r_E}}$; $v_{esc,U} = \sqrt{\frac{2Gm_U}{r_U}}$: $\frac{v_{esc,E}}{v_{esc,U}} = \sqrt{\frac{m_U r_E}{m_E r_U}} = \sqrt{\frac{14.0}{3.70}} = 1.95$

For the Earth, from the text's table of escape speeds, $v_{esc,E} = 11.2 \text{ km/s}$

$\therefore v_{esc,U} = (1.95)(11.2 \text{ km/s}) = \boxed{21.8 \text{ km/s}}$

**P13.41** The rocket is in a potential well at Ganymede's surface with energy

$$U_1 = -\frac{Gm_1m_2}{r} = -\frac{6.67\times10^{-11}\ \mathrm{N\cdot m^2}m_2\left(1.495\times10^{23}\ \mathrm{kg}\right)}{\mathrm{kg}^2\left(2.64\times10^{6}\ \mathrm{m}\right)}$$

$$U_1 = -3.78\times10^{6}m_2\ \mathrm{m^2/s^2}$$

The potential well from Jupiter at the distance of Ganymede is

$$U_2 = -\frac{Gm_1m_2}{r} = -\frac{6.67\times10^{-11}\ \mathrm{N\cdot m^2}m_2\left(1.90\times10^{27}\ \mathrm{kg}\right)}{\mathrm{kg}^2\left(1.071\times10^{9}\ \mathrm{m}\right)}$$

$$U_2 = -1.18\times10^{8}m_2\ \mathrm{m^2/s^2}$$

To escape from both requires

$$\frac{1}{2}m_2v_{\mathrm{esc}}^2 = +\left(3.78\times10^{6}+1.18\times10^{8}\right)m_2\ \mathrm{m^2/s^2}$$

$$v_{\mathrm{esc}} = \sqrt{2\times1.22\times10^{8}\ \mathrm{m^2/s^2}} = \boxed{15.6\ \mathrm{km/s}}$$

**P13.42** We interpret "lunar escape speed" to be the escape speed from the surface of a stationary moon alone in the Universe:

$$\frac{1}{2}mv_{\mathrm{esc}}^2 = \frac{GM_mm}{R_m}$$

$$v_{\mathrm{esc}} = \sqrt{\frac{2GM_m}{R_m}}$$

$$v_{\mathrm{launch}} = 2\sqrt{\frac{2GM_m}{R_m}}$$

Now for the flight from moon to Earth

$$(K+U)_i = (K+U)_f$$

$$\frac{1}{2}mv_{\mathrm{launch}}^2 - \frac{GmM_m}{R_m} - \frac{GmM_E}{r_{\mathrm{el}}} = \frac{1}{2}mv_{\mathrm{impact}}^2 - \frac{GmM_m}{r_{m_2}} - \frac{GmM_E}{R_E}$$

$$\frac{4GM_m}{R_m} - \frac{GM_m}{R_m} - \frac{GM_E}{r_{\mathrm{el}}} = \frac{1}{2}v_{\mathrm{impact}}^2 - \frac{GM_m}{r_{m_2}} - \frac{GM_E}{R_E}$$

$$v_{\mathrm{impact}} = \left[2G\left(\frac{3M_m}{R_m} + \frac{M_m}{r_{m_2}} + \frac{M_E}{R_E} - \frac{M_E}{r_{\mathrm{el}}}\right)\right]^{1/2}$$

$$= \left[2G\left(\frac{3\times7.36\times10^{22}\ \mathrm{kg}}{1.74\times10^{6}\ \mathrm{m}} + \frac{7.36\times10^{22}\ \mathrm{kg}}{3.84\times10^{8}\ \mathrm{m}} + \frac{5.98\times10^{24}\ \mathrm{kg}}{6.37\times10^{6}\ \mathrm{m}} - \frac{5.98\times10^{24}\ \mathrm{kg}}{3.84\times10^{8}\ \mathrm{m}}\right)\right]^{1/2}$$

$$= \left[2G\left(1.27\times10^{17}+1.92\times10^{14}+9.39\times10^{17}-1.56\times10^{16}\right)\ \mathrm{kg/m}\right]^{1/2}$$

$$= \left[2\left(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}\right)10.5\times10^{17}\ \mathrm{kg/m}\right]^{1/2} = \boxed{11.8\ \mathrm{km/s}}$$

***P13.43** (a) Energy conservation for the object-Earth system from firing to apex:

$$\left(K+U_g\right)_i=\left(K+U_g\right)_f$$
$$\frac{1}{2}mv_i^2-\frac{GmM_E}{R_E}=0-\frac{GmM_E}{R_E+h}$$

where $\frac{1}{2}mv_{\text{esc}}^2=\frac{GmM_E}{R_E}$. Then

$$\frac{1}{2}v_i^2-\frac{1}{2}v_{\text{esc}}^2=-\frac{1}{2}v_{\text{esc}}^2\frac{R_E}{R_E+h}$$
$$v_{\text{esc}}^2-v_i^2=\frac{v_{\text{esc}}^2R_E}{R_E+h}$$
$$\frac{1}{v_{\text{esc}}^2-v_i^2}=\frac{R_E+h}{v_{\text{esc}}^2R_E}$$
$$h=\frac{v_{\text{esc}}^2R_E}{v_{\text{esc}}^2-v_i^2}-R_E=\frac{v_{\text{esc}}^2R_E-v_{\text{esc}}^2R_E+v_i^2R_E}{v_{\text{esc}}^2-v_i^2}$$
$$h=\frac{R_Ev_i^2}{v_{\text{esc}}^2-v_i^2}$$

(b) $$h=\frac{6.37\times10^6\text{ m }(8.76)^2}{(11.2)^2-(8.76)^2}=\boxed{1.00\times10^7\text{ m}}$$

(c) The fall of the meteorite is the time-reversal of the upward flight of the projectile, so it is described by the same energy equation

$$v_i^2=v_{\text{esc}}^2\left(1-\frac{R_E}{R_E+h}\right)=v_{\text{esc}}^2\left(\frac{h}{R_E+h}\right)=\left(11.2\times10^3\text{ m/s}\right)^2\left(\frac{2.51\times10^7\text{ m}}{6.37\times10^6\text{ m}+2.51\times10^7\text{ m}}\right)$$
$$=1.00\times10^8\text{ m}^2/\text{s}^2$$
$$v_i=\boxed{1.00\times10^4\text{ m/s}}$$

(d) With $v_i<<v_{\text{esc}}$, $h\approx\frac{R_Ev_i^2}{v_{\text{esc}}^2}=\frac{R_Ev_i^2R_E}{2GM_E}$. But $g=\frac{GM_E}{R_E^2}$, so $h=\frac{v_i^2}{2g}$, in agreement with $0^2=v_i^2+2(-g)(h-0)$.

**P13.44** For a satellite in an orbit of radius $r$ around the Earth, the total energy of the satellite-Earth system is $E=-\frac{GM_E}{2r}$. Thus, in changing from a circular orbit of radius $r=2R_E$ to one of radius $r=3R_E$, the required work is

$$W=\Delta E=-\frac{GM_Em}{2r_f}+\frac{GM_Em}{2r_i}=GM_Em\left[\frac{1}{4R_E}-\frac{1}{6R_E}\right]=\boxed{\frac{GM_Em}{12R_E}}.$$

***P13.45** (a) The major axis of the orbit is $2a = 50.5\text{ AU}$ so $a = 25.25\text{ AU}$

Further, in Figure 13.5, $a + c = 50\text{ AU}$ so $c = 24.75\text{ AU}$

Then $e = \frac{c}{a} = \frac{24.75}{25.25} = \boxed{0.980}$

(b) In $T^2 = K_s a^3$ for objects in solar orbit, the Earth gives us

$$(1\text{ yr})^2 = K_s(1\text{ AU})^3 \qquad K_s = \frac{(1\text{ yr})^2}{(1\text{ AU})^3}$$

Then $T^2 = \frac{(1\text{ yr})^2}{(1\text{ AU})^3}(25.25\text{ AU})^3 \qquad T = \boxed{127\text{ yr}}$

(c) $$U = -\frac{GMm}{r} = -\frac{\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.991\times10^{30}\text{ kg}\right)\left(1.2\times10^{10}\text{ kg}\right)}{50\left(1.496\times10^{11}\text{ m}\right)} = \boxed{-2.13\times10^{17}\text{ J}}$$

***P13.46** (a) For the satellite $\sum F = ma \qquad \frac{GmM_E}{r^2} = \frac{mv_0^2}{r}$

$$\boxed{v_0 = \left(\frac{GM_E}{r}\right)^{1/2}}$$

(b) Conservation of momentum in the forward direction for the exploding satellite:

$$\left(\sum mv\right)_i = \left(\sum mv\right)_f$$
$$5mv_0 = 4mv_i + m0$$
$$v_i = \frac{5}{4}v_0 = \boxed{\frac{5}{4}\left(\frac{GM_E}{r}\right)^{1/2}}$$

(c) With velocity perpendicular to radius, the orbiting fragment is at perigee. Its apogee distance and speed are related to $r$ and $v_i$ by $4mrv_i = 4mr_f v_f$ and $\frac{1}{2}4mv_i^2 - \frac{GM_E 4m}{r} = \frac{1}{2}4mv_f^2 - \frac{GM_E 4m}{r_f}$. Substituting $v_f = \frac{v_i r}{r_f}$ we have $\frac{1}{2}v_i^2 - \frac{GM_E}{r} = \frac{1}{2}\frac{v_i^2 r^2}{r_f^2} - \frac{GM_E}{r_f}$. Further, substituting $v_i^2 = \frac{25}{16}\frac{GM_E}{r}$ gives

$$\frac{25}{32}\frac{GM_E}{r} - \frac{GM_E}{r} = \frac{25}{32}\frac{GM_E r}{r_f^2} - \frac{GM_E}{r_f}$$
$$\frac{-7}{32r} = \frac{25r}{32r_f^2} - \frac{1}{r_f}$$

Clearing of fractions, $-7r_f^2 = 25r^2 - 32rr_f$ or $7\left(\frac{r_f}{r}\right)^2 - 32\left(\frac{r_f}{r}\right) + 25 = 0$ giving

$\frac{r_f}{r} = \frac{+32 \pm \sqrt{32^2 - 4(7)(25)}}{14} = \frac{50}{14}$ or $\frac{14}{14}$. The latter root describes the starting point. The outer end of the orbit has $\frac{r_f}{r} = \frac{25}{7}$; $\boxed{r_f = \frac{25r}{7}}$.

## Additional Problems

**P13.47** Let $m$ represent the mass of the spacecraft, $r_E$ the radius of the Earth's orbit, and $x$ the distance from Earth to the spacecraft.

The Sun exerts on the spacecraft a radial inward force of $F_s = \dfrac{GM_s m}{(r_E - x)^2}$

while the Earth exerts on it a radial outward force of $F_E = \dfrac{GM_E m}{x^2}$

The net force on the spacecraft must produce the correct centripetal acceleration for it to have an orbital period of 1.000 year.

Thus,
$$F_S - F_E = \frac{GM_S m}{(r_E - x)^2} - \frac{GM_E m}{x^2} = \frac{mv^2}{(r_E - x)} = \frac{m}{(r_E - x)}\left[\frac{2\pi(r_E - x)}{T}\right]^2$$

which reduces to
$$\frac{GM_S}{(r_E - x)^2} - \frac{GM_E}{x^2} = \frac{4\pi^2(r_E - x)}{T^2}. \qquad (1)$$

Cleared of fractions, this equation would contain powers of $x$ ranging from the fifth to the zeroth. We do not solve it algebraically. We may test the assertion that $x$ is between $1.47 \times 10^9$ m and $1.48 \times 10^9$ m by substituting both of these as trial solutions, along with the following data: $M_S = 1.991 \times 10^{30}$ kg, $M_E = 5.983 \times 10^{24}$ kg, $r_E = 1.496 \times 10^{11}$ m, and $T = 1.000 \text{ yr} = 3.156 \times 10^7$ s.

With $x = 1.47 \times 10^9$ m substituted into equation (1), we obtain

$$6.052 \times 10^{-3} \text{ m/s}^2 - 1.85 \times 10^{-3} \text{ m/s}^2 \approx 5.871 \times 10^{-3} \text{ m/s}^2$$

or $$5.868 \times 10^{-3} \text{ m/s}^2 \approx 5.871 \times 10^{-3} \text{ m/s}^2$$

With $x = 1.48 \times 10^9$ m substituted into the same equation, the result is

$$6.053 \times 10^{-3} \text{ m/s}^2 - 1.82 \times 10^{-3} \text{ m/s}^2 \approx 5.870\,8 \times 10^{-3} \text{ m/s}^2$$

or $$5.870\,9 \times 10^{-3} \text{ m/s}^2 \approx 5.870\,8 \times 10^{-3} \text{ m/s}^2 .$$

Since the first trial solution makes the left-hand side of equation (1) slightly less than the right hand side, and the second trial solution does the opposite, the true solution is determined as between the trial values. To three-digit precision, it is $1.48 \times 10^9$ m.

As an equation of fifth degree, equation (1) has five roots. The Sun-Earth system has five Lagrange points, all revolving around the Sun synchronously with the Earth. The SOHO and ACE satellites are at one. Another is beyond the far side of the Sun. Another is beyond the night side of the Earth. Two more are on the Earth's orbit, ahead of the planet and behind it by 60°. Plans are under way to gain perspective on the Sun by placing a spacecraft at one of these two co-orbital Lagrange points. The Greek and Trojan asteroids are at the co-orbital Lagrange points of the Jupiter-Sun system.

**P13.48** The acceleration of an object at the center of the Earth due to the gravitational force of the Moon is given by

$$a = G\frac{M_{\text{Moon}}}{d^2}$$

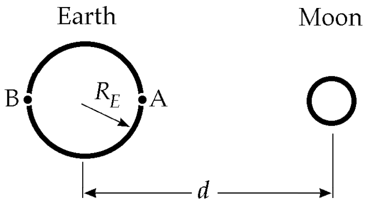

**FIG. P13.48**

At the point A nearest the Moon, $a_+ = G\frac{M_M}{(d-r)^2}$

At the point B farthest from the Moon, $a_- = G\frac{M_M}{(d+r)^2}$

$$\Delta a = a_+ - a = GM_M\left[\frac{1}{(d-r)^2} - \frac{1}{d^2}\right]$$

For $d >> r$, $\Delta a = \frac{2GM_M r}{d^3} = 1.11 \times 10^{-6}\ \text{m/s}^2$

Across the planet, $\frac{\Delta g}{g} = \frac{2\Delta a}{g} = \frac{2.22 \times 10^{-6}\ \text{m/s}^2}{9.80\ \text{m/s}^2} = \boxed{2.26 \times 10^{-7}}$

***P13.49** Energy conservation for the two-sphere system from release to contact:

$$-\frac{Gmm}{R} = -\frac{Gmm}{2r} + \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

$$Gm\left(\frac{1}{2r} - \frac{1}{R}\right) = v^2 \qquad v = \left(Gm\left[\frac{1}{2r} - \frac{1}{R}\right]\right)^{1/2}$$

(a) The injected impulse is the final momentum of each sphere,

$$mv = m^{2/2}\left(Gm\left[\frac{1}{2r} - \frac{1}{R}\right]\right)^{1/2} = \left[Gm^3\left(\frac{1}{2r} - \frac{1}{R}\right)\right]^{1/2}.$$

(b) If they now collide elastically each sphere reverses its velocity to receive impulse

$$mv - (-mv) = 2mv = \boxed{2\left[Gm^3\left(\frac{1}{2r} - \frac{1}{R}\right)\right]^{1/2}}$$

**P13.50** Momentum is conserved:

$$m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = m_1\mathbf{v}_{1f} + m_2\mathbf{v}_{2f}$$

$$0 = M\mathbf{v}_{1f} + 2M\mathbf{v}_{2f}$$

$$\mathbf{v}_{2f} = -\frac{1}{2}\mathbf{v}_{1f}$$

Energy is conserved:

$$(K+U)_i + \Delta E = (K+U)_f$$

$$0 - \frac{Gm_1m_2}{r_i} + 0 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2 - \frac{Gm_1m_2}{r_f}$$

$$-\frac{GM(2M)}{12R} = \frac{1}{2}Mv_{1f}^2 + \frac{1}{2}(2M)\left(\frac{1}{2}v_{1f}\right)^2 - \frac{GM(2M)}{4R}$$

$$v_{1f} = \boxed{\frac{2}{3}\sqrt{\frac{GM}{R}}} \qquad v_{2f} = \frac{1}{2}v_{1f} = \boxed{\frac{1}{3}\sqrt{\frac{GM}{R}}}$$

**P13.51** (a) $a_c = \frac{v^2}{r}$ $\qquad a_c = \frac{\left(1.25\times10^{6}\text{ m/s}\right)^2}{1.53\times10^{11}\text{ m}} = \boxed{10.2\text{ m/s}^2}$

(b) $\text{diff} = 10.2 - 9.90 = 0.312\text{ m/s}^2 = \frac{GM}{r^2}$

$$M = \frac{\left(0.312\text{ m/s}^2\right)\left(1.53\times10^{11}\text{ m}\right)^2}{6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2} = \boxed{1.10\times10^{32}\text{ kg}}$$

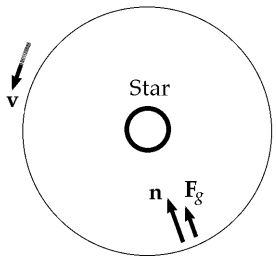

FIG. P13.51

**P13.52** (a) The free-fall acceleration produced by the Earth is $g = \frac{GM_E}{r^2} = GM_E r^{-2}$ (directed downward)

Its rate of change is $\qquad \frac{dg}{dr} = GM_E(-2)r^{-3} = -2GM_E r^{-3}$.

The minus sign indicates that $g$ decreases with increasing height.

At the Earth's surface, $\qquad \boxed{\frac{dg}{dr} = -\frac{2GM_E}{R_E^3}}$.

(b) For small differences,

$\frac{|\Delta g|}{\Delta r} = \frac{|\Delta g|}{h} = \frac{2GM_E}{R_E^3}$ $\qquad$ Thus, $\qquad \boxed{|\Delta g| = \frac{2GM_E h}{R_E^3}}$

(c) $$|\Delta g| = \frac{2\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\text{ kg}\right)(6.00\text{ m})}{\left(6.37\times10^{6}\text{ m}\right)^3} = \boxed{1.85\times10^{-5}\text{ m/s}^2}$$

***P13.53** (a) Each bit of mass $dm$ in the ring is at the same distance from the object at A. The separate contributions $-\frac{Gmdm}{r}$ to the system energy add up to $-\frac{GmM_{\text{ring}}}{r}$. When the object is at A, this is

$$\frac{-6.67\times10^{-11}\text{ N}\cdot\text{m}^2 1\,000\text{ kg } 2.36\times10^{20}\text{ kg}}{\text{kg}^2\sqrt{\left(1\times10^{8}\text{ m}\right)^2 + \left(2\times10^{8}\text{ m}\right)^2}} = \boxed{-7.04\times10^{4}\text{ J}}.$$

(b) When the object is at the center of the ring, the potential energy is

$$-\frac{6.67\times10^{-11}\text{ N}\cdot\text{m}^2\ 1\,000\text{ kg } 2.36\times10^{20}\text{ kg}}{\text{kg}^2 1\times10^{8}\text{ m}} = \boxed{-1.57\times10^{5}\text{ J}}.$$

(c) Total energy of the object-ring system is conserved:

$$\left(K + U_g\right)_A = \left(K + U_g\right)_B$$

$$0 - 7.04\times10^{4}\text{ J} = \frac{1}{2}1\,000\text{ kg}v_B^2 - 1.57\times10^{5}\text{ J}$$

$$v_B = \left(\frac{2\times8.70\times10^{4}\text{ J}}{1\,000\text{ kg}}\right)^{1/2} = \boxed{13.2\text{ m/s}}$$

**P13.54** To approximate the height of the sulfur, set

$$\frac{mv^2}{2} = mg_{Io}h \qquad h = 70\,000\text{ m} \qquad g_{Io} = \frac{GM}{r^2} = 1.79\text{ m/s}^2$$

$$v = \sqrt{2g_{Io}h} \qquad v = \sqrt{2(1.79)(70\,000)} \approx 500\text{ m/s (over } 1\,000\text{ mi/h)}$$

A more precise answer is given by

$$\frac{1}{2}mv^2 - \frac{GMm}{r_1} = -\frac{GMm}{r_2}$$

$$\frac{1}{2}v^2 = \left(6.67\times10^{-11}\right)\left(8.90\times10^{22}\right)\left(\frac{1}{1.82\times10^6} - \frac{1}{1.89\times10^6}\right) \qquad v = \boxed{492\text{ m/s}}$$

**P13.55** From the walk, $2\pi r = 25\,000\text{ m}$. Thus, the radius of the planet is $r = \frac{25\,000\text{ m}}{2\pi} = 3.98\times10^3\text{ m}$

From the drop: $\Delta y = \frac{1}{2}gt^2 = \frac{1}{2}g(29.2\text{ s})^2 = 1.40\text{ m}$

so, $g = \frac{2(1.40\text{ m})}{(29.2\text{ s})^2} = 3.28\times10^{-3}\text{ m/s}^2 = \frac{MG}{r^2}$ $\qquad \therefore M = \boxed{7.79\times10^{14}\text{ kg}}$

***P13.56** The distance between the orbiting stars is $d = 2r\cos 30° = \sqrt{3}r$ since $\cos 30° = \frac{\sqrt{3}}{2}$. The net inward force on one orbiting star is

$$\frac{Gmm}{d^2}\cos 30° + \frac{GMm}{r^2} + \frac{Gmm}{d^2}\cos 30° = \frac{mv^2}{r}$$

$$\frac{Gm2\cos 30°}{3r^2} + \frac{GM}{r^2} = \frac{4\pi^2r^2}{rT^2}$$

$$G\left(\frac{m}{\sqrt{3}} + M\right) = \frac{4\pi^2r^3}{T^2}$$

$$T^2 = \frac{4\pi^2r^3}{G\left(M + \frac{m}{\sqrt{3}}\right)}$$

$$T = 2\pi\left(\frac{r^3}{G\left(M + \frac{m}{\sqrt{3}}\right)}\right)^{1/2}$$

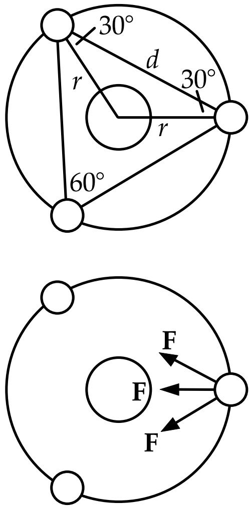

**FIG. P13.56**

**P13.57** For a 6.00 km diameter cylinder, $r = 3\,000\text{ m}$ and to simulate $1g = 9.80\text{ m/s}^2$

$$g = \frac{v^2}{r} = \omega^2 r$$

$$\omega = \sqrt{\frac{g}{r}} = \boxed{0.057\,2\text{ rad/s}}$$

The required rotation rate of the cylinder is $\boxed{\frac{1\text{ rev}}{110\text{ s}}}$

(For a description of proposed cities in space, see Gerard K. O'Neill in *Physics Today*, Sept. 1974.)

**P13.58** (a) $G$ has units $\frac{\mathrm{N\cdot m^2}}{\mathrm{kg^2}}=\frac{\mathrm{kg\cdot m\cdot m^2}}{\mathrm{s^2\cdot kg^2}}=\frac{\mathrm{m^3}}{\mathrm{s^2\cdot kg}}$

and dimensions $[\mathrm{G}]=\frac{\mathrm{L^3}}{\mathrm{T^2\cdot M}}$.

The speed of light has dimensions of $[c]=\frac{\mathrm{L}}{\mathrm{T}}$, and Planck's constant has the same dimensions as angular momentum or $[h]=\frac{\mathrm{M\cdot L^2}}{\mathrm{T}}$.

We require $\left[G^p c^q h^r\right]=\mathrm{L}$, or $\mathrm{L}^{3p}\mathrm{T}^{-2p}\mathrm{M}^{-p}\mathrm{L}^q\mathrm{T}^{-q}\mathrm{M}^r\mathrm{L}^{2r}\mathrm{T}^{-r}=\mathrm{L}^1\mathrm{M}^0\mathrm{T}^0$.

Thus, $3p+q+2r=1$

$-2p-q-r=0$

$-p+r=0$

which reduces (using $r=p$) to $3p+q+2p=1$

$-2p-q-p=0$

These equations simplify to $5p+q=1$ and $q=-3p$.

Then, $5p-3p=1$, yielding $p=\frac{1}{2}$, $q=-\frac{3}{2}$, and $r=\frac{1}{2}$.

Therefore, Planck length $=\boxed{G^{1/2}c^{-3/2}h^{1/2}}$.

(b) $\left(6.67\times10^{-11}\right)^{1/2}\left(3\times10^{8}\right)^{-3/2}\left(6.63\times10^{-34}\right)^{1/2}=\left(1.64\times10^{-69}\right)^{1/2}=4.05\times10^{-35}\text{ m}\boxed{\sim10^{-34}\text{ m}}$

**P13.59** $\frac{1}{2}m_0v_{\mathrm{esc}}^2=\frac{Gm_pm_0}{R}$

$v_{\mathrm{esc}}=\sqrt{\frac{2Gm_p}{R}}$

With $m_p=\rho\frac{4}{3}\pi R^3$, we have

$$v_{\mathrm{esc}}=\sqrt{\frac{2G\rho\frac{4}{3}\pi R^3}{R}}$$

$$=\boxed{\sqrt{\frac{8\pi G\rho}{3}}R}$$

So, $v_{\mathrm{esc}}\propto R$.

***P13.60** For both circular orbits,

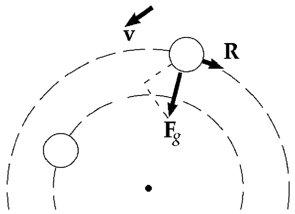

FIG. P13.60

$$\sum F = ma: \qquad \frac{GM_E m}{r^2} = \frac{mv^2}{r}$$

$$v = \sqrt{\frac{GM_E}{r}}$$

(a) The original speed is $v_i = \sqrt{\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)}{\left(6.37\times10^6\ \text{m}+2\times10^5\ \text{m}\right)}} = \boxed{7.79\times10^3\ \text{m/s}}$.

(b) The final speed is $v_i = \sqrt{\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)}{\left(6.47\times10^6\ \text{m}\right)}} = \boxed{7.85\times10^3\ \text{m/s}}$.

The energy of the satellite-Earth system is

$$K + U_g = \frac{1}{2}mv^2 - \frac{GM_E m}{r} = \frac{1}{2}m\frac{GM_E}{r} - \frac{GM_E}{r} = -\frac{GM_E m}{2r}$$

(c) Originally $E_i = -\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)(100\ \text{kg})}{2\left(6.57\times10^6\ \text{m}\right)} = \boxed{-3.04\times10^9\ \text{J}}$.

(d) Finally $E_f = -\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)(100\ \text{kg})}{2\left(6.47\times10^6\ \text{m}\right)} = \boxed{-3.08\times10^9\ \text{J}}$.

(e) Thus the object speeds up as it spirals down to the planet. The loss of gravitational energy is so large that the total energy decreases by

$$E_i - E_f = -3.04\times10^9\ \text{J} - \left(-3.08\times10^9\ \text{J}\right) = \boxed{4.69\times10^7\ \text{J}}.$$

(f) The only forces on the object are the backward force of air resistance $R$, comparatively very small in magnitude, and the force of gravity. Because the spiral path of the satellite is not perpendicular to the gravitational force, $\boxed{\text{one component of the gravitational force}}$ pulls forward on the satellite to do positive work and make its speed increase.

**P13.61** (a) At infinite separation $U = 0$ and at rest $K = 0$. Since energy of the two-planet system is conserved we have,

$$0 = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 - \frac{Gm_1 m_2}{d} \qquad (1)$$

The initial momentum of the system is zero and momentum is conserved.

Therefore, $$0 = m_1 v_1 - m_2 v_2 \qquad (2)$$

Combine equations (1) and (2): $\boxed{v_1 = m_2\sqrt{\frac{2G}{d(m_1+m_2)}}}$ and $\boxed{v_2 = m_1\sqrt{\frac{2G}{d(m_1+m_2)}}}$

Relative velocity $v_r = v_1 - (-v_2) = \boxed{\sqrt{\frac{2G(m_1+m_2)}{d}}}$

(b) Substitute given numerical values into the equation found for $v_1$ and $v_2$ in part (a) to find

$v_1 = 1.03 \times 10^4 \text{ m/s}$ and $v_2 = 2.58 \times 10^3 \text{ m/s}$

Therefore, $K_1 = \frac{1}{2}m_1 v_1^2 = \boxed{1.07 \times 10^{32}\text{ J}}$ and $K_2 = \frac{1}{2}m_2 v_2^2 = \boxed{2.67 \times 10^{31}\text{ J}}$

**P13.62** (a) The net torque exerted on the Earth is zero. Therefore, the angular momentum of the Earth is conserved;

$$mr_a v_a = mr_p v_p \text{ and } v_a = v_p\left(\frac{r_p}{r_a}\right) = \left(3.027 \times 10^4 \text{ m/s}\right)\left(\frac{1.471}{1.521}\right) = \boxed{2.93 \times 10^4 \text{ m/s}}$$

(b) $$K_p = \frac{1}{2}mv_p^2 = \frac{1}{2}\left(5.98 \times 10^{24}\right)\left(3.027 \times 10^4\right)^2 = \boxed{2.74 \times 10^{33}\text{ J}}$$

$$U_p = -\frac{GmM}{r_p} = -\frac{\left(6.673 \times 10^{-11}\right)\left(5.98 \times 10^{24}\right)\left(1.99 \times 10^{30}\right)}{1.471 \times 10^{11}} = \boxed{-5.40 \times 10^{33}\text{ J}}$$

(c) Using the same form as in part (b), $K_a = \boxed{2.57 \times 10^{33}\text{ J}}$ and $U_a = \boxed{-5.22 \times 10^{33}\text{ J}}$.

Compare to find that $K_p + U_p = \boxed{-2.66 \times 10^{33}\text{ J}}$ and $K_a + U_a = \boxed{-2.65 \times 10^{33}\text{ J}}$. They agree.

**P13.63** (a) The work must provide the increase in gravitational energy

$$\begin{aligned}W &= \Delta U_g = U_{gf} - U_{gi}\\ &= -\frac{GM_EM_p}{r_f} + \frac{GM_EM_p}{r_i}\\ &= -\frac{GM_EM_p}{R_E + y} + \frac{GM_EM_p}{R_E}\\ &= GM_EM_p\left(\frac{1}{R_E} - \frac{1}{R_E + y}\right)\\ &= \left(\frac{6.67\times10^{-11}\ \mathrm{N\cdot m^2}}{\mathrm{kg}^2}\right)\left(5.98\times10^{24}\ \mathrm{kg}\right)(100\ \mathrm{kg})\left(\frac{1}{6.37\times10^6\ \mathrm{m}} - \frac{1}{7.37\times10^6\ \mathrm{m}}\right)\\ W &= \boxed{850\ \mathrm{MJ}}\end{aligned}$$

(b) In a circular orbit, gravity supplies the centripetal force:

$$\frac{GM_EM_p}{(R_E + y)^2} = \frac{M_pv^2}{(R_E + y)}$$

Then, $\frac{1}{2}M_pv^2 = \frac{1}{2}\frac{GM_EM_p}{(R_E + y)}$

So, additional work = kinetic energy required

$$= \frac{1}{2}\frac{\left(6.67\times10^{-11}\ \mathrm{N\cdot m^2}\right)\left(5.98\times10^{24}\ \mathrm{kg}\right)(100\ \mathrm{kg})}{\left(\mathrm{kg}^2\right)\left(7.37\times10^6\ \mathrm{m}\right)}$$

$$\Delta W = \boxed{2.71\times10^9\ \mathrm{J}}$$

**P13.64** Centripetal acceleration comes from gravitational acceleration.

$$\frac{v^2}{r} = \frac{M_cG}{r^2} = \frac{4\pi^2r^2}{T^2r}$$

$$GM_cT^2 = 4\pi^2r^3$$

$$\left(6.67\times10^{-11}\right)(20)\left(1.99\times10^{30}\right)\left(5.00\times10^{-3}\right)^2 = 4\pi^2r^3$$

$$r_{\mathrm{orbit}} = \boxed{119\ \mathrm{km}}$$

**P13.65** (a) $T = \frac{2\pi r}{v} = \frac{2\pi\left(30\,000\times9.46\times10^{15}\ \mathrm{m}\right)}{2.50\times10^5\ \mathrm{m/s}} = 7\times10^{15}\ \mathrm{s} = \boxed{2\times10^8\ \mathrm{yr}}$

(b) $M = \frac{4\pi^2a^3}{GT^2} = \frac{4\pi^2\left(30\,000\times9.46\times10^{15}\ \mathrm{m}\right)^3}{\left(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}\right)\left(7.13\times10^{15}\ \mathrm{s}\right)^2} = 2.66\times10^{41}\ \mathrm{kg}$

$M = 1.34\times10^{11}$ solar masses $\boxed{\sim10^{11}\ \text{solar masses}}$

The number of stars is $\boxed{\text{on the order of } 10^{11}}$.

**P13.66** (a) From the data about perigee, the energy of the satellite-Earth system is

$$E = \frac{1}{2}mv_p^2 - \frac{GM_E m}{r_p} = \frac{1}{2}(1.60)\left(8.23\times10^3\right)^2 - \frac{\left(6.67\times10^{-11}\right)\left(5.98\times10^{24}\right)(1.60)}{7.02\times10^6}$$

or $E = \boxed{-3.67\times10^7\ \text{J}}$

(b) $L = mvr\sin\theta = mv_p r_p \sin 90.0° = (1.60\ \text{kg})\left(8.23\times10^3\ \text{m/s}\right)\left(7.02\times10^6\ \text{m}\right)$

$= \boxed{9.24\times10^{10}\ \text{kg}\cdot\text{m}^2/\text{s}}$

(c) Since both the energy of the satellite-Earth system and the angular momentum of the Earth are conserved,

at apogee we must have $\frac{1}{2}mv_a^2 - \frac{GMm}{r_a} = E$

and $mv_a r_a \sin 90.0° = L$.

Thus, $\frac{1}{2}(1.60)v_a^2 - \frac{\left(6.67\times10^{-11}\right)\left(5.98\times10^{24}\right)(1.60)}{r_s} = -3.67\times10^7\ \text{J}$

and $(1.60\ \text{kg})v_a r_a = 9.24\times10^{10}\ \text{kg}\cdot\text{m}^2/\text{s}$.

Solving simultaneously, $\frac{1}{2}(1.60)v_a^2 - \frac{\left(6.67\times10^{-11}\right)\left(5.98\times10^{24}\right)(1.60)(1.60)v_a}{9.24\times10^{10}} = -3.67\times10^7$

which reduces to $0.800v_a^2 - 11\,046v_a + 3.672\,3\times10^7 = 0$

so $v_a = \frac{11\,046 \pm \sqrt{(11\,046)^2 - 4(0.800)\left(3.672\,3\times10^7\right)}}{2(0.800)}$.

This gives $v_a = 8\,230\ \text{m/s}$ or $\boxed{5\,580\ \text{m/s}}$. The smaller answer refers to the velocity at the apogee while the larger refers to perigee.

Thus, $r_a = \frac{L}{mv_a} = \frac{9.24\times10^{10}\ \text{kg}\cdot\text{m}^2/\text{s}}{(1.60\ \text{kg})\left(5.58\times10^3\ \text{m/s}\right)} = \boxed{1.04\times10^7\ \text{m}}$.

(d) The major axis is $2a = r_p + r_a$, so the semi-major axis is

$$a = \frac{1}{2}\left(7.02\times10^6\ \text{m} + 1.04\times10^7\ \text{m}\right) = \boxed{8.69\times10^6\ \text{m}}$$

(e) $T = \sqrt{\frac{4\pi^2 a^3}{GM_E}} = \sqrt{\frac{4\pi^2\left(8.69\times10^6\ \text{m}\right)^3}{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)}}$

$T = 8\,060\ \text{s} = \boxed{134\ \text{min}}$

***P13.67** Let $m$ represent the mass of the meteoroid and $v_i$ its speed when far away. No torque acts on the meteoroid, so its angular momentum is conserved as it moves between the distant point and the point where it grazes the Earth, moving perpendicular to the radius:

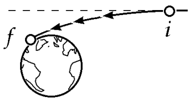

**FIG. P13.67**

$$L_i = L_f: \qquad m\mathbf{r}_i \times \mathbf{v}_i = m\mathbf{r}_f \times \mathbf{v}_f$$

$$m(3R_E v_i) = mR_E v_f$$

$$v_f = 3v_i$$

Now energy of the meteoroid-Earth system is also conserved:

$$\left(K+U_g\right)_i = \left(K+U_g\right)_f: \qquad \frac{1}{2}mv_i^2 + 0 = \frac{1}{2}mv_f^2 - \frac{GM_E m}{R_E}$$

$$\frac{1}{2}v_i^2 = \frac{1}{2}\left(9v_i^2\right) - \frac{GM_E}{R_E}$$

$$\frac{GM_E}{R_E} = 4v_i^2: \qquad \boxed{v_i = \sqrt{\frac{GM_E}{4R_E}}}$$

***P13.68** From Kepler's third law, minimum period means minimum orbit size. The "treetop satellite" in Figure P13.35 has minimum period. The radius of the satellite's circular orbit is essentially equal to the radius $R$ of the planet.

$$\sum F = ma: \qquad \frac{GMm}{R^2} = \frac{mv^2}{R} = \frac{m}{R}\left(\frac{2\pi R}{T}\right)^2$$

$$G\rho V = \frac{R^2\left(4\pi^2 R^2\right)}{RT^2}$$

$$G\rho\left(\frac{4}{3}\pi R^3\right) = \frac{4\pi^2 R^3}{T^2}$$

The radius divides out: $T^2 G\rho = 3\pi$ $\qquad \boxed{T = \sqrt{\frac{3\pi}{G\rho}}}$

**P13.69** If we choose the coordinate of the center of mass at the origin, then

$$0 = \frac{(Mr_2 - mr_1)}{M+m} \qquad \text{and} \qquad Mr_2 = mr_1$$

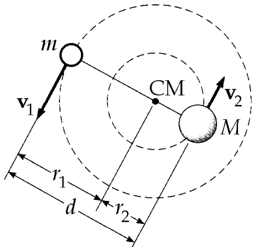

**FIG. P13.69**

(Note: this is equivalent to saying that the net torque must be zero and the two experience no angular acceleration.) For each mass $F = ma$ so

$$mr_1\omega_1^2 = \frac{MGm}{d^2} \qquad \text{and} \qquad Mr_2\omega_2^2 = \frac{MGm}{d^2}$$

Combining these two equations and using $d = r_1 + r_2$ gives $(r_1 + r_2)\omega^2 = \frac{(M+m)G}{d^2}$

with $\omega_1 = \omega_2 = \omega$

and $T = \frac{2\pi}{\omega}$

we find $\boxed{T^2 = \frac{4\pi^2 d^3}{G(M+m)}}$.

**P13.70** (a) The gravitational force exerted on $m_2$ by the Earth (mass $m_1$) accelerates $m_2$ according to: $m_2 g_2 = \frac{G m_1 m_2}{r^2}$. The equal magnitude force exerted on the Earth by $m_2$ produces negligible acceleration of the Earth. The acceleration of relative approach is then

$$g_2 = \frac{G m_1}{r^2} = \frac{\left(6.67\times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times 10^{24}\ \text{kg}\right)}{\left(1.20\times 10^{7}\ \text{m}\right)^2} = \boxed{2.77\ \text{m/s}^2}.$$

(b) Again, $m_2$ accelerates toward the center of mass with $g_2 = 2.77\ \text{m/s}^2$. Now the Earth accelerates toward $m_2$ with an acceleration given as

$$m_1 g_1 = \frac{G m_1 m_2}{r^2}$$

$$g_1 = \frac{G m_2}{r^2} = \frac{\left(6.67\times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(2.00\times 10^{24}\ \text{kg}\right)}{\left(1.20\times 10^{7}\ \text{m}\right)^2} = 0.926\ \text{m/s}^2$$

The distance between the masses closes with relative acceleration of

$$g_{\text{rel}} = g_1 + g_2 = 0.926\ \text{m/s}^2 + 2.77\ \text{m/s}^2 = \boxed{3.70\ \text{m/s}^2}.$$

**P13.71** *Initial Conditions and Constants:*

| | |
|---|---|
| Mass of planet: | $5.98\times 10^{24}$ kg |
| Radius of planet: | $6.37\times 10^{6}$ m |
| Initial $x$: | 0.0 planet radii |
| Initial $y$: | 2.0 planet radii |
| Initial $v_x$: | +5 000 m/s |
| Initial $v_y$: | 0.0 m/s |
| Time interval: | 10.9 s |

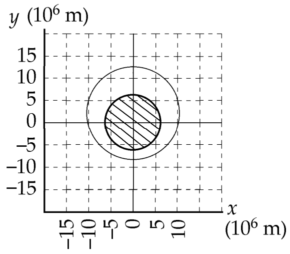

**FIG. P13.71**

| $t$ (s) | $x$ (m) | $y$ (m) | $r$ (m) | $v_x$ (m/s) | $v_y$ (m/s) | $a_x$ $\left(\text{m/s}^2\right)$ | $a_y$ $\left(\text{m/s}^2\right)$ |
|---:|---:|---:|---:|---:|---:|---:|---:|
| 0.0 | 0.0 | 12 740 000.0 | 12 740 000.0 | 5 000.0 | 0.0 | 0.000 0 | –2.457 5 |
| 10.9 | 54 315.3 | 12 740 000.0 | 12 740 115.8 | 4 999.9 | –26.7 | –0.010 0 | –2.457 4 |
| 21.7 | 108 629.4 | 12 739 710.0 | 12 740 173.1 | 4 999.7 | –53.4 | –0.021 0 | –2.457 3 |
| 32.6 | 162 941.1 | 12 739 130.0 | 12 740 172.1 | 4 999.3 | –80.1 | –0.031 0 | –2.457 2 |
| … | | | | | | | |
| 5 431.6 | 112 843.8 | –8 466 816.0 | 8 467 567.9 | –7 523.0 | –39.9 | –0.074 0 | 5.562 5 |
| 5 442.4 | 31 121.4 | –8 467 249.7 | 8 467 306.9 | –7 523.2 | 20.5 | –0.020 0 | 5.563 3 |
| 5 453.3 | –50 603.4 | –8 467 026.9 | 8 467 178.2 | –7 522.8 | 80.9 | 0.033 0 | 5.563 4 |
| 5 464.1 | –132 324.3 | –8 466 147.7 | 8 467 181.7 | –7 521.9 | 141.4 | 0.087 0 | 5.562 8 |
| … | | | | | | | |
| 10 841.3 | –108 629.0 | 12 739 134.4 | 12 739 597.5 | 4 999.9 | 53.3 | 0.021 0 | –2.457 5 |
| 10 852.2 | –54 314.9 | 12 739 713.4 | 12 739 829.2 | 5 000.0 | 26.6 | 0.010 0 | –2.457 5 |
| 10 863.1 | 0.4 | 12 740 002.4 | 12 740 002.4 | 5 000.0 | –0.1 | 0.000 0 | –2.457 5 |

The $\boxed{\text{object does not hit the Earth}}$; its minimum radius is $\boxed{1.33 R_E}$.

Its period is $\boxed{1.09\times 10^{4}\ \text{s}}$. A circular orbit would require a speed of $\boxed{5.60\ \text{km/s}}$.

## ANSWERS TO EVEN PROBLEMS

| | |
|---|---|
| **P13.2** | $2.67\times10^{-7}\ \text{m/s}^2$ |
| **P13.4** | 3.00 kg and 2.00 kg |
| **P13.6** | (a) $4.39\times10^{20}$ N toward the Sun;<br>(b) $1.99\times10^{20}$ N toward the Earth;<br>(c) $3.55\times10^{22}$ N toward the Sun |
| **P13.8** | see the solution; either $1\ \text{m}-61.3\ \text{nm}$ or $2.74\times10^{-4}$ m |
| **P13.10** | $\dfrac{2}{3}$ |
| **P13.12** | (a) 1.02 km/s; (b) 1.35 mm |
| **P13.14** | see the solution |
| **P13.16** | 1.27 |
| **P13.18** | Planet *Y* has turned through 1.30 revolutions |
| **P13.20** | $1.63\times10^{4}$ rad/s |
| **P13.22** | 18.2 ms |
| **P13.24** | (a) $1.31\times10^{17}$ N toward the center;<br>(b) $2.62\times10^{12}$ N/kg |
| **P13.26** | (a) $-4.77\times10^{9}$ J; (b) 569 N down;<br>(c) 569 N up |
| **P13.28** | $2.52\times10^{7}$ m |
| **P13.30** | $2.82\times10^{9}$ J |
| **P13.32** | (a) see the solution; (b) 340 s |
| **P13.34** | (a) 42.1 km/s; (b) $2.20\times10^{11}$ m |
| **P13.36** | $1.58\times10^{10}$ J |
| **P13.38** | (a) $2\pi(R_E+h)^{3/2}(GM_E)^{-1/2}$;<br>(b) $(GM_E)^{1/2}(R_E+h)^{-1/2}$;<br>(c) $GM_Em\left[\dfrac{R_E+2h}{2R_E(R_E+h)}\right]-\dfrac{2\pi^2R_E^2m}{(86\,400\ \text{s})^2}$<br>The satellite should be launched from the Earth's equator toward the east. |
| **P13.40** | (a) $10.0\ \text{m/s}^2$; (b) 21.8 km/s |
| **P13.42** | 11.8 km/s |
| **P13.44** | $\dfrac{GM_Em}{12R_E}$ |
| **P13.46** | (a) $v_0=\left(\dfrac{GM_E}{r}\right)^{1/2}$; (b) $v_i=\dfrac{5\left(\frac{GM_E}{r}\right)^{1/2}}{4}$;<br>(c) $r_f=\dfrac{25r}{7}$ |
| **P13.48** | $2.26\times10^{-7}$ |
| **P13.50** | $\dfrac{2}{3}\sqrt{\dfrac{GM}{R}}$; $\dfrac{1}{3}\sqrt{\dfrac{GM}{R}}$ |
| **P13.52** | (a), (b) see the solution;<br>(c) $1.85\times10^{-5}\ \text{m/s}^2$ |
| **P13.54** | 492 m/s |
| **P13.56** | see the solution |
| **P13.58** | (a) $G^{1/2}c^{-3/2}h^{1/2}$; (b) $\sim10^{-34}$ m |
| **P13.60** | (a) 7.79 km/s; (b) 7.85 km/s;(c) $-3.04$ GJ;<br>(d) $-3.08$ GJ; (e) loss = 46.9 MJ;<br>(f) A component of the Earth's gravity pulls forward on the satellite in its downward banking trajectory. |
| **P13.62** | (a) 29.3 km/s; (b) $K_p=2.74\times10^{33}$ J;<br>$U_p=-5.40\times10^{33}$ J;(c) $K_a=2.57\times10^{33}$ J;<br>$U_a=-5.22\times10^{33}$ J; yes |
| **P13.64** | 119 km |
| **P13.66** | (a) $-36.7$ MJ; (b) $9.24\times10^{10}\ \text{kg}\cdot\text{m}^2/\text{s}$;<br>(c) 5.58 km/s; 10.4 Mm; (d) 8.69 Mm;<br>(e) 134 min |
| **P13.68** | see the solution |
| **P13.70** | (a) $2.77\ \text{m/s}^2$; (b) $3.70\ \text{m/s}^2$ |

# 14

# Fluid Mechanics

## CHAPTER OUTLINE

14.1 Pressure
14.2 Variation of Pressure with Depth
14.3 Pressure Measurements
14.4 Buoyant Forces and Archimede's Principle
14.5 Fluid Dynamics
14.6 Bernoulli's Equation
14.7 Other Applications of Fluid Dynamics

## ANSWERS TO QUESTIONS

**Q14.1** The weight depends upon the total volume of glass. The pressure depends only on the depth.

**Q14.2** Both must be built the same. The force on the back of each dam is the average pressure of the water times the area of the dam. If both reservoirs are equally deep, the force is the same.

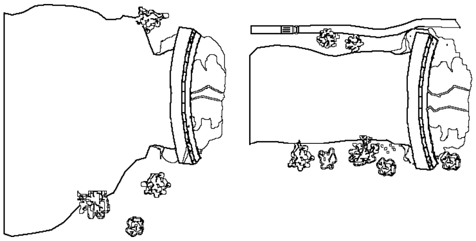

**FIG. Q14.2**

**Q14.3** If the tube were to fill up to the height of several stories of the building, the pressure at the bottom of the depth of the tube of fluid would be very large according to Equation 14.4. This pressure is much larger than that originally exerted by inward elastic forces of the rubber on the water. As a result, water is pushed into the bottle from the tube. As more water is added to the tube, more water continues to enter the bottle, stretching it thin. For a typical bottle, the pressure at the bottom of the tube can become greater than the pressure at which the rubber material will rupture, so the bottle will simply fill with water and expand until it bursts. Blaise Pascal splintered strong barrels by this method.

**Q14.4** About 1 000 N: that's about 250 pounds.

**Q14.5** The submarine would stop if the density of the surrounding water became the same as the average density of the submarine. Unfortunately, because the water is almost incompressible, this will be much deeper than the crush depth of the submarine.

**Q14.6** Yes. The propulsive force of the fish on the water causes the scale reading to fluctuate. Its average value will still be equal to the total weight of bucket, water, and fish.

**Q14.7** The boat floats higher in the ocean than in the inland lake. According to Archimedes's principle, the magnitude of buoyant force on the ship is equal to the weight of the water displaced by the ship. Because the density of salty ocean water is greater than fresh lake water, less ocean water needs to be displaced to enable the ship to float.

**Q14.8** In the ocean, the ship floats due to the buoyant force from *salt water*. Salt water is denser than fresh water. As the ship is pulled up the river, the buoyant force from the fresh water in the river is not sufficient to support the weight of the ship, and it sinks.

**Q14.9** Exactly the same. Buoyancy equals density of water times volume displaced.

**Q14.10** At lower elevation the water pressure is greater because pressure increases with increasing depth below the water surface in the reservoir (or water tower). The penthouse apartment is not so far below the water surface. The pressure behind a closed faucet is weaker there and the flow weaker from an open faucet. Your fire department likely has a record of the precise elevation of every fire hydrant.

**Q14.11** As the wind blows over the chimney, it creates a lower pressure at the top of the chimney. The smoke flows from the relatively higher pressure in front of the fireplace to the low pressure outside. Science doesn't suck; the smoke is pushed from below.

**Q14.12** The rapidly moving air above the ball exerts less pressure than the atmospheric pressure below the ball. This can give substantial lift to balance the weight of the ball.

**Q14.13** The ski–jumper gives her body the shape of an airfoil. She deflects downward the air stream as it rushes past and it deflects her upward by Newton's third law. The air exerts on her a lift force, giving her a higher and longer trajectory. To say it in different words, the pressure on her back is less than the pressure on her front.

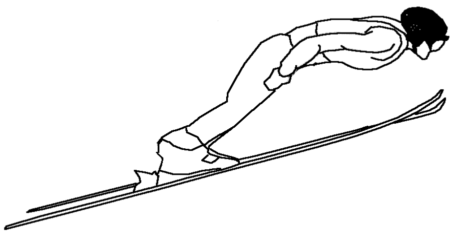

**FIG. Q14.13**

**Q14.14** The horizontal force exerted by the outside fluid, on an area element of the object's side wall, has equal magnitude and opposite direction to the horizontal force the fluid exerts on another element diametrically opposite the first.

**Q14.15** The glass may have higher density than the liquid, but the air inside has lower density. The total weight of the bottle can be less than the weight of an equal volume of the liquid.

**Q14.16** Breathing in makes your volume greater and increases the buoyant force on you. You instinctively take a deep breath if you fall into the lake.

**Q14.17** No. The somewhat lighter barge will float higher in the water.

**Q14.18** The level of the pond falls. This is because the anchor displaces more water while in the boat. A floating object displaces a volume of water whose weight is equal to the weight of the object. A submerged object displaces a volume of water equal to the volume of the object. Because the density of the anchor is greater than that of water, a volume of water that weighs the same as the anchor will be greater than the volume of the anchor.

**Q14.19** The metal is more dense than water. If the metal is sufficiently thin, it can float like a ship, with the lip of the dish above the water line. Most of the volume below the water line is filled with air. The mass of the dish divided by the volume of the part below the water line is just equal to the density of water. Placing a bar of soap into this space to replace the air raises the average density of the compound object and the density can become greater than that of water. The dish sinks with its cargo.

**Q14.20** The excess pressure is transmitted undiminished throughout the container. It will compress air inside the wood. The water driven into the wood raises its average density and makes if float lower in the water. Add some thumbtacks to reach neutral buoyancy and you can make the wood sink or rise at will by subtly squeezing a large clear–plastic soft–drink bottle. Bored with graph paper and proving his own existence, René Descartes invented this toy or trick.

**Q14.21** The plate must be horizontal. Since the pressure of a fluid increases with increasing depth, other orientations of the plate will give a non-uniform pressure on the flat faces.

**Q14.22** The air in your lungs, the blood in your arteries and veins, and the protoplasm in each cell exert nearly the same pressure, so that the wall of your chest can be in equilibrium.

**Q14.23** Use a balance to determine its mass. Then partially fill a graduated cylinder with water. Immerse the rock in the water and determine the volume of water displaced. Divide the mass by the volume and you have the density.

**Q14.24** When taking off into the wind, the increased airspeed over the wings gives a larger lifting force, enabling the pilot to take off in a shorter length of runway.

**Q14.25** Like the ball, the balloon will remain in front of you. It will not bob up to the ceiling. Air pressure will be no higher at the floor of the sealed car than at the ceiling. The balloon will experience no buoyant force. You might equally well switch off gravity.

**Q14.26** Styrofoam is a little more dense than air, so the first ship floats lower in the water.

**Q14.27** We suppose the compound object floats. In both orientations it displaces its own weight of water, so it displaces equal volumes of water. The water level in the tub will be unchanged when the object is turned over. Now the steel is underwater and the water exerts on the steel a buoyant force that was not present when the steel was on top surrounded by air. Thus, slightly less wood will be below the water line on the block. It will appear to float higher.

**Q14.28** A breeze from any direction speeds up to go over the mound and the air pressure drops. Air then flows through the burrow from the lower entrance to the upper entrance.

**Q14.29** Regular cola contains a considerable mass of dissolved sugar. Its density is higher than that of water. Diet cola contains a very small mass of artificial sweetener and has nearly the same density as water. The low–density air in the can has a bigger effect than the thin aluminum shell, so the can of diet cola floats.

**Q14.30** (a) Lowest density: oil; highest density: mercury

(b) The density must increase from top to bottom.

**Q14.31** (a) Since the velocity of the air in the right-hand section of the pipe is lower than that in the middle, the pressure is higher.

(b) The equation that predicts the same pressure in the far right and left-hand sections of the tube assumes laminar flow without viscosity. Internal friction will cause some loss of mechanical energy and turbulence will also progressively reduce the pressure. If the pressure at the left were not higher than at the right, the flow would stop.

**Q14.32** Clap your shoe or wallet over the hole, or a seat cushion, or your hand. Anything that can sustain a force on the order of 100 N is strong enough to cover the hole and greatly slow down the escape of the cabin air. You need not worry about the air rushing out instantly, or about your body being "sucked" through the hole, or about your blood boiling or your body exploding. If the cabin pressure drops a lot, your ears will pop and the saliva in your mouth may boil—at body temperature—but you will still have a couple of minutes to plug the hole and put on your emergency oxygen mask. Passengers who have been drinking carbonated beverages may find that the carbon dioxide suddenly comes out of solution in their stomachs, distending their vests, making them belch, and all but frothing from their ears; so you might warn them of this effect.

## SOLUTIONS TO PROBLEMS

### Section 14.1 **Pressure**

**P14.1** $M = \rho_{\text{iron}} V = \left(7\,860 \text{ kg/m}^3\right)\left[\frac{4}{3}\pi(0.015\,0 \text{ m})^3\right]$

$M = \boxed{0.111 \text{ kg}}$

**P14.2** The density of the nucleus is of the same order of magnitude as that of one proton, according to the assumption of close packing:

$$\rho = \frac{m}{V} \sim \frac{1.67 \times 10^{-27} \text{ kg}}{\frac{4}{3}\pi\left(10^{-15} \text{ m}\right)^3} \boxed{\sim 10^{18} \text{ kg/m}^3}.$$

With vastly smaller average density, a macroscopic chunk of matter or an atom must be mostly empty space.

**P14.3** $P = \frac{F}{A} = \frac{50.0(9.80)}{\pi\left(0.500 \times 10^{-2}\right)^2} = \boxed{6.24 \times 10^6 \text{ N/m}^2}$

**P14.4** Let $F_g$ be its weight. Then each tire supports $\frac{F_g}{4}$,

so $P = \frac{F}{A} = \frac{F_g}{4A}$

yielding $F_g = 4AP = 4\left(0.024\,0 \text{ m}^2\right)\left(200 \times 10^3 \text{ N/m}^2\right) = \boxed{1.92 \times 10^4 \text{ N}}$

**P14.5** The Earth's surface area is $4\pi R^2$. The force pushing inward over this area amounts to

$$F = P_0 A = P_0\left(4\pi R^2\right).$$

This force is the weight of the air:

$$F_g = mg = P_0\left(4\pi R^2\right)$$

so the mass of the air is

$$m = \frac{P_0\left(4\pi R^2\right)}{g} = \frac{\left(1.013 \times 10^5 \text{ N/m}^2\right)\left[4\pi\left(6.37 \times 10^6 \text{ m}\right)^2\right]}{9.80 \text{ m/s}^2} = \boxed{5.27 \times 10^{18} \text{ kg}}.$$

## Section 14.2 Variation of Pressure with Depth

**P14.6** (a) $P = P_0 + \rho g h = 1.013\times10^5\ \text{Pa} + \left(1\,024\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)\left(1\,000\ \text{m}\right)$

$P = \boxed{1.01\times10^7\ \text{Pa}}$

(b) The gauge pressure is the difference in pressure between the water outside and the air inside the submarine, which we suppose is at 1.00 atmosphere.

$$P_{\text{gauge}} = P - P_0 = \rho g h = 1.00\times10^7\ \text{Pa}$$

The resultant inward force on the porthole is then

$$F = P_{\text{gauge}}A = 1.00\times10^7\ \text{Pa}\left[\pi(0.150\ \text{m})^2\right] = \boxed{7.09\times10^5\ \text{N}}.$$

**P14.7** $F_{el} = F_{\text{fluid}}$ or $kx = \rho g h A$

and $h = \dfrac{kx}{\rho g A}$

$$h = \frac{\left(1\,000\ \text{N/m}^2\right)\left(5.00\times10^{-3}\ \text{m}\right)}{\left(10^3\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)\left[\pi\left(1.00\times10^{-2}\ \text{m}\right)^2\right]} = \boxed{1.62\ \text{m}}$$

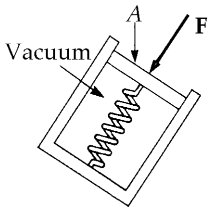

FIG. P14.7

**P14.8** Since the pressure is the same on both sides, $\dfrac{F_1}{A_1} = \dfrac{F_2}{A_2}$

In this case, $\dfrac{15\,000}{200} = \dfrac{F_2}{3.00}$ or $F_2 = \boxed{225\ \text{N}}$

**P14.9** $F_g = 80.0\ \text{kg}\left(9.80\ \text{m/s}^2\right) = 784\ \text{N}$

When the cup barely supports the student, the normal force of the ceiling is zero and the cup is in equilibrium.

$$F_g = F = PA = \left(1.013\times10^5\ \text{Pa}\right)A$$

$$A = \frac{F_g}{P} = \frac{784}{1.013\times10^5} = \boxed{7.74\times10^{-3}\ \text{m}^2}$$

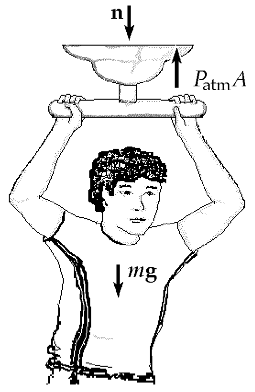

FIG. P14.9

**P14.10** (a) Suppose the "vacuum cleaner" functions as a high–vacuum pump. The air below the brick will exert on it a lifting force

$$F = PA = 1.013\times10^5\ \text{Pa}\left[\pi\left(1.43\times10^{-2}\ \text{m}\right)^2\right] = \boxed{65.1\ \text{N}}.$$

(b) The octopus can pull the bottom away from the top shell with a force that could be no larger than

$$F = PA = (P_0 + \rho g h)A = \left[1.013\times10^5\ \text{Pa} + \left(1\,030\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)(32.3\ \text{m})\right]\left[\pi\left(1.43\times10^{-2}\ \text{m}\right)^2\right]$$

$F = \boxed{275\ \text{N}}$

**P14.11** The excess water pressure (over air pressure) halfway down is

$$P_{\text{gauge}} = \rho g h = \left(1\,000\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)(1.20\ \text{m}) = 1.18 \times 10^4\ \text{Pa}.$$

The force on the wall due to the water is

$$F = P_{\text{gauge}} A = \left(1.18 \times 10^4\ \text{Pa}\right)(2.40\ \text{m})(9.60\ \text{m}) = \boxed{2.71 \times 10^5\ \text{N}}$$

horizontally toward the back of the hole.

**P14.12** The pressure on the bottom due to the water is $P_b = \rho g z = 1.96 \times 10^4\ \text{Pa}$

So, $F_b = P_b A = \boxed{5.88 \times 10^6\ \text{N}}$

On each end, $F = \overline{P} A = 9.80 \times 10^3\ \text{Pa}\left(20.0\ \text{m}^2\right) = \boxed{196\ \text{kN}}$

On the side, $F = \overline{P} A = 9.80 \times 10^3\ \text{Pa}\left(60.0\ \text{m}^2\right) = \boxed{588\ \text{kN}}$

**P14.13** In the reference frame of the fluid, the cart's acceleration causes a fictitious force to act backward, as if the acceleration of gravity were $\sqrt{g^2 + a^2}$ directed downward and backward at $\theta = \tan^{-1}\left(\frac{a}{g}\right)$ from the vertical. The center of the spherical shell is at depth $\frac{d}{2}$ below the air bubble and the pressure there is

$$P = P_0 + \rho g_{\text{eff}} h = \boxed{P_0 + \frac{1}{2}\rho d \sqrt{g^2 + a^2}}.$$

**P14.14** The air outside and water inside both exert atmospheric pressure, so only the excess water pressure $\rho g h$ counts for the net force. Take a strip of hatch between depth $h$ and $h + dh$. It feels force

$$dF = P\,dA = \rho g h (2.00\ \text{m})\,dh.$$

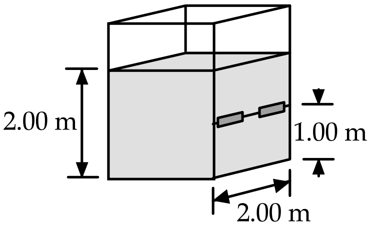

FIG. P14.14

(a) The total force is

$$F = \int dF = \int_{h=1.00\ \text{m}}^{2.00\ \text{m}} \rho g h (2.00\ \text{m})\,dh$$

$$F = \rho g (2.00\ \text{m}) \frac{h^2}{2}\Bigg|_{1.00\ \text{m}}^{2.00\ \text{m}} = \left(1\,000\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)\frac{(2.00\ \text{m})}{2}\left[(2.00\ \text{m})^2 - (1.00\ \text{m})^2\right]$$

$$F = \boxed{29.4\ \text{kN (to the right)}}$$

(b) The lever arm of $dF$ is the distance $(h - 1.00\ \text{m})$ from hinge to strip:

$$\tau = \int d\tau = \int_{h=1.00\ \text{m}}^{2.00\ \text{m}} \rho g h (2.00\ \text{m})(h - 1.00\ \text{m})\,dh$$

$$\tau = \rho g (2.00\ \text{m})\left[\frac{h^3}{3} - (1.00\ \text{m})\frac{h^2}{2}\right]_{1.00\ \text{m}}^{2.00\ \text{m}}$$

$$\tau = \left(1\,000\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)(2.00\ \text{m})\left(\frac{7.00\ \text{m}^3}{3} - \frac{3.00\ \text{m}^3}{2}\right)$$

$$\tau = \boxed{16.3\ \text{kN}\cdot\text{m counterclockwise}}$$

**P14.15** The bell is uniformly compressed, so we can model it with any shape. We choose a sphere of diameter 3.00 m.
The pressure on the ball is given by: $P = P_{\text{atm}} + \rho_w gh$ so the change in pressure on the ball from when it is on the surface of the ocean to when it is at the bottom of the ocean is $\Delta P = \rho_w gh$.
In addition:

$$\Delta V = \frac{-V\Delta P}{B} = -\frac{\rho_w ghV}{B} = -\frac{4\pi\rho_w ghr^3}{3B}\text{, where } B \text{ is the Bulk Modulus.}$$

$$\Delta V = -\frac{4\pi\left(1\,030\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)(10\,000\ \text{m})(1.50\ \text{m})^3}{(3)\left(14.0\times10^{10}\ \text{Pa}\right)} = -0.010\,2\ \text{m}^3$$

Therefore, the volume of the ball at the bottom of the ocean is

$$V - \Delta V = \frac{4}{3}\pi(1.50\ \text{m})^3 - 0.010\,2\ \text{m}^3 = 14.137\ \text{m}^3 - 0.010\,2\ \text{m}^3 = 14.127\ \text{m}^3.$$

This gives a radius of 1.499 64 m and a new diameter of 2.999 3 m. Therefore the diameter decreases by $\boxed{0.722\ \text{mm}}$.

## Section 14.3 **Pressure Measurements**

**P14.16** (a) We imagine the superhero to produce a perfect vacuum in the straw. Take point 1 at the water surface in the basin and point 2 at the water surface in the straw:

$$P_1 + \rho gy_1 = P_2 + \rho gy_2$$

$$1.013\times10^5\ \text{N/m}^2 + 0 = 0 + \left(1\,000\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)y_2 \qquad y_2 = \boxed{10.3\ \text{m}}$$

(b) No atmosphere can lift the water in the straw through $\boxed{\text{zero}}$ height difference.

**P14.17** $P_0 = \rho gh$

$$h = \frac{P_0}{\rho g} = \frac{10.13\times10^5\ \text{Pa}}{\left(0.984\times10^3\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)} = \boxed{10.5\ \text{m}}$$

$\boxed{\text{No. Some alcohol and water will evaporate.}}$ The equilibrium vapor pressures of alcohol and water are higher than the vapor pressure of mercury.

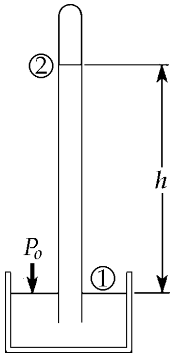

**FIG. P14.17**

**P14.18** (a) Using the definition of density, we have

$$h_w = \frac{m_{\text{water}}}{A_2\rho_{\text{water}}} = \frac{100\text{ g}}{5.00\text{ cm}^2\left(1.00\text{ g/cm}^3\right)} = \boxed{20.0\text{ cm}}$$

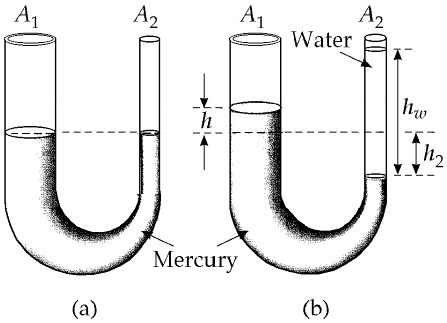

**FIG. P14.18**

(b) Sketch (b) at the right represents the situation after the water is added. A volume $(A_2h_2)$ of mercury has been displaced by water in the right tube. The additional volume of mercury now in the left tube is $A_1h$. Since the total volume of mercury has not changed,

$$A_2h_2 = A_1h \qquad \text{or} \qquad h_2 = \frac{A_1}{A_2}h \qquad (1)$$

At the level of the mercury–water interface in the right tube, we may write the absolute pressure as:

$$P = P_0 + \rho_{\text{water}}gh_w$$

The pressure at this same level in the left tube is given by

$$P = P_0 + \rho_{\text{Hg}}g(h+h_2) = P_0 + \rho_{\text{water}}gh_w$$

which, using equation (1) above, reduces to

$$\rho_{\text{Hg}}h\left[1+\frac{A_1}{A_2}\right] = \rho_{\text{water}}h_w$$

or $h = \dfrac{\rho_{\text{water}}h_w}{\rho_{\text{Hg}}\left(1+\frac{A_1}{A_2}\right)}$.

Thus, the level of mercury has risen a distance of $h = \dfrac{\left(1.00\text{ g/cm}^3\right)(20.0\text{ cm})}{\left(13.6\text{ g/cm}^3\right)\left(1+\frac{10.0}{5.00}\right)} = \boxed{0.490\text{ cm}}$

above the original level.

**P14.19** $\Delta P_0 = \rho g\Delta h = -2.66\times 10^3\text{ Pa}: \quad P = P_0 + \Delta P_0 = (1.013 - 0.026\,6)\times 10^5\text{ Pa} = \boxed{0.986\times 10^5\text{ Pa}}$

**P14.20** Let *h* be the height of the water column added to the right side of the U–tube. Then when equilibrium is reached, the situation is as shown in the sketch at right. Now consider two points, *A* and *B* shown in the sketch, at the level of the water–mercury interface. By Pascal's Principle, the absolute pressure at *B* is the same as that at *A*. But,

$$P_A = P_0 + \rho_w gh + \rho_{\text{Hg}}gh_2 \text{ and}$$
$$P_B = P_0 + \rho_w g(h_1 + h + h_2).$$

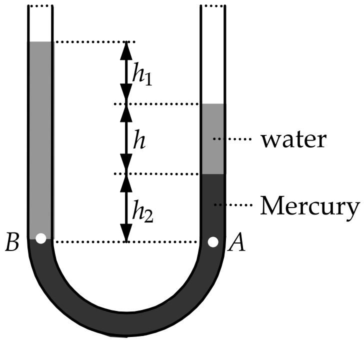

**FIG. P14.20**

Thus, from $P_A = P_B$, $\rho_w h_1 + \rho_w h + \rho_w h_2 = \rho_w h + \rho_{\text{Hg}}h_2$, or

$$h_1 = \left[\frac{\rho_{\text{Hg}}}{\rho_w} - 1\right]h_2 = (13.6-1)(1.00\text{ cm}) = \boxed{12.6\text{ cm}}.$$

***P14.21** (a) $P = P_0 + \rho g h$

The gauge pressure is

$$P - P_0 = \rho g h = 1\,000\text{ kg}\left(9.8\text{ m/s}^2\right)(0.160\text{ m}) = \boxed{1.57\text{ kPa}} = 1.57\times10^3\text{ Pa}\left(\frac{1\text{ atm}}{1.013\times10^5\text{ Pa}}\right)$$

$$= \boxed{0.015\,5\text{ atm}}.$$

It would lift a mercury column to height

$$h = \frac{P - P_0}{\rho g} = \frac{1\,568\text{ Pa}}{\left(13\,600\text{ kg/m}^3\right)\left(9.8\text{ m/s}^2\right)} = \boxed{11.8\text{ mm}}.$$

(b) $\boxed{\text{Increased pressure of the cerebrospinal fluid will raise the level of the fluid in the spinal tap.}}$

(c) $\boxed{\text{Blockage of the fluid within the spinal column or between the skull and the spinal column would prevent the fluid level from rising.}}$

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## Section 14.4 Buoyant Forces and Archimede's Principle

**P14.22** (a) The balloon is nearly in equilibrium:

$$\sum F_y = m a_y \Rightarrow B - \left(F_g\right)_{\text{helium}} - \left(F_g\right)_{\text{payload}} = 0$$

or $\rho_{\text{air}} g V - \rho_{\text{helium}} g V - m_{\text{payload}} g = 0$

This reduces to

$$m_{\text{payload}} = \left(\rho_{\text{air}} - \rho_{\text{helium}}\right)V = \left(1.29\text{ kg/m}^3 - 0.179\text{ kg/m}^3\right)\left(400\text{ m}^3\right)$$

$$m_{\text{payload}} = \boxed{444\text{ kg}}$$

(b) Similarly,

$$m_{\text{payload}} = \left(\rho_{\text{air}} - \rho_{\text{hydrogen}}\right)V = \left(1.29\text{ kg/m}^3 - 0.089\,9\text{ kg/m}^3\right)\left(400\text{ m}^3\right)$$

$$m_{\text{payload}} = \boxed{480\text{ kg}}$$

The air does the lifting, nearly the same for the two balloons.

**P14.23** At equilibrium $\sum F = 0$ or $F_{app} + mg = B$

where $B$ is the buoyant force.

The applied force, $F_{app} = B - mg$

where $B = \text{Vol}\left(\rho_{\text{water}}\right)g$

and $m = (\text{Vol})\rho_{\text{ball}}$.

So, $F_{app} = (\text{Vol})g\left(\rho_{\text{water}} - \rho_{\text{ball}}\right) = \frac{4}{3}\pi r^3 g\left(\rho_{\text{water}} - \rho_{\text{ball}}\right)$

$$F_{app} = \frac{4}{3}\pi\left(1.90\times10^{-2}\text{ m}\right)^3\left(9.80\text{ m/s}^2\right)\left(10^3\text{ kg/m}^3 - 84.0\text{ kg/m}^3\right) = \boxed{0.258\text{ N}}$$

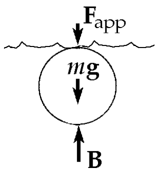

FIG. P14.23

**P14.24** $F_g = \left(m + \rho_s V\right)g$ must be equal to $F_b = \rho_w V g$

Since $V = Ah$, $m + \rho_s Ah = \rho_w Ah$

and $A = \boxed{\dfrac{m}{\left(\rho_w - \rho_s\right)h}}$

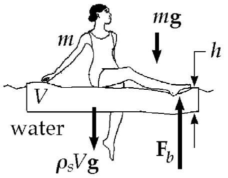

FIG. P14.24

**P14.25** (a) Before the metal is immersed:

$$\sum F_y = T_1 - Mg = 0 \text{ or}$$

$$T_1 = Mg = (1.00\ \text{kg})(9.80\ \text{m/s}^2) = \boxed{9.80\ \text{N}}$$

(b) After the metal is immersed:

$$\sum F_y = T_2 + B - Mg = 0 \text{ or}$$

$$T_2 = Mg - B = Mg - (\rho_w V)g$$

$$V = \frac{M}{\rho} = \frac{1.00\ \text{kg}}{2\,700\ \text{kg/m}^3}$$

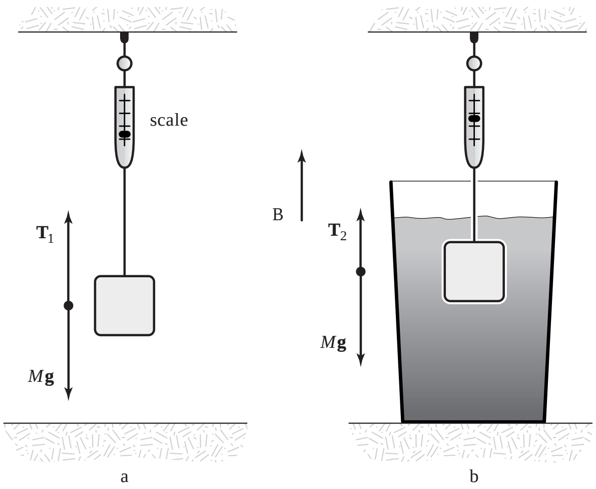

**FIG. P14.25**

Thus,

$$T_2 = Mg - B = 9.80\ \text{N} - (1\,000\ \text{kg/m}^3)\left(\frac{1.00\ \text{kg}}{2\,700\ \text{kg/m}^3}\right)(9.80\ \text{m/s}^2) = \boxed{6.17\ \text{N}}.$$

***P14.26** (a)

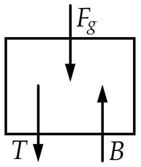

**FIG. P14.26(a)**

(b) $\sum F_y = 0:\quad -15\ \text{N} - 10\ \text{N} + B = 0$

$$\boxed{B = 25.0\ \text{N}}$$

(c) The oil pushes $\boxed{\text{horizontally inward}}$ on each side of the block.

(d) $\boxed{\text{String tension increases}}$. The oil causes the water below to be under greater pressure, and the water pushes up more strongly on the bottom of the block.

(e) Consider the equilibrium just before the string breaks:

$$-15\ \text{N} - 60\ \text{N} + 25\ \text{N} + B_{\text{oil}} = 0$$

$$B_{\text{oil}} = 50\ \text{N}$$

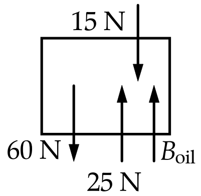

**FIG. P14.26(e)**

For the buoyant force of the water we have

$$B = \rho V g \qquad 25\ \text{N} = (1\,000\ \text{kg/m}^3)(0.25V_{\text{block}})9.8\ \text{m/s}^2$$

$$V_{\text{block}} = 1.02 \times 10^{-2}\ \text{m}^3$$

For the buoyant force of the oil

$$50\ \text{N} = (800\ \text{kg/m}^3) f_e (1.02 \times 10^{-2}\ \text{m}^3) 9.8\ \text{m/s}^2$$

$$f_e = 0.625 = \boxed{62.5\%}$$

(f) $-15\ \text{N} + (800\ \text{kg/m}^3) f_f (1.02 \times 10^{-2}\ \text{m}^3) 9.8\ \text{m/s}^2 = 0$

$$f_f = 0.187 = \boxed{18.7\%}$$

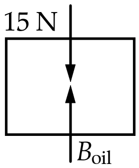

**FIG. P14.26(f)**

**P14.27** (a) $P = P_0 + \rho g h$

Taking $P_0 = 1.013 \times 10^5 \text{ N/m}^2$ and $h = 5.00 \text{ cm}$

we find $P_{\text{top}} = 1.0179 \times 10^5 \text{ N/m}^2$

For $h = 17.0 \text{ cm}$, we get $P_{\text{bot}} = 1.0297 \times 10^5 \text{ N/m}^2$

Since the areas of the top and bottom are $A = (0.100 \text{ m})^2 = 10^{-2} \text{ m}^2$

we find $F_{\text{top}} = P_{\text{top}} A = \boxed{1.0179 \times 10^3 \text{ N}}$

and $F_{\text{bot}} = \boxed{1.0297 \times 10^3 \text{ N}}$

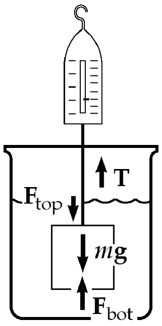

**FIG. P14.27**

(b) $T + B - Mg = 0$

where $B = \rho_w V g = \left(10^3 \text{ kg/m}^3\right)\left(1.20 \times 10^{-3} \text{ m}^3\right)\left(9.80 \text{ m/s}^2\right) = 11.8 \text{ N}$

and $Mg = 10.0(9.80) = 98.0 \text{ N}$

Therefore, $T = Mg - B = 98.0 - 11.8 = \boxed{86.2 \text{ N}}$

(c) $F_{\text{bot}} - F_{\text{top}} = (1.0297 - 1.0179) \times 10^3 \text{ N} = \boxed{11.8 \text{ N}}$

which is equal to $B$ found in part (b).

**P14.28** Consider spherical balloons of radius 12.5 cm containing helium at STP and immersed in air at 0°C and 1 atm. If the rubber envelope has mass 5.00 g, the upward force on each is

$$B - F_{g,\text{He}} - F_{g,env} = \rho_{\text{air}} V g - \rho_{\text{He}} V g - m_{env} g$$

$$F_{up} = (\rho_{\text{air}} - \rho_{\text{He}})\left(\frac{4}{3}\pi r^3\right) g - m_{env} g$$

$$F_{up} = \left[(1.29 - 0.179) \text{ kg/m}^3\right]\left[\frac{4}{3}\pi(0.125 \text{ m})^3\right]\left(9.80 \text{ m/s}^2\right) - 5.00 \times 10^{-3} \text{ kg}\left(9.80 \text{ m/s}^2\right) = 0.0401 \text{ N}$$

If your weight (including harness, strings, and submarine sandwich) is

$$70.0 \text{ kg}\left(9.80 \text{ m/s}^2\right) = 686 \text{ N}$$

you need this many balloons: $\dfrac{686 \text{ N}}{0.0401 \text{ N}} = 17\,000 \boxed{\sim 10^4}$.

**P14.29** (a) According to Archimedes, $B = \rho_{\text{water}} V_{\text{water}} g = \left(1.00 \text{ g/cm}^3\right)\left[20.0 \times 20.0 \times (20.0 - h)\right] g$

But $B = \text{Weight of block} = mg = \rho_{\text{wood}} V_{\text{wood}} g = \left(0.650 \text{ g/cm}^3\right)(20.0 \text{ cm})^3 g$

$0.650(20.0)^3 g = 1.00(20.0)(20.0)(20.0 - h) g$

$20.0 - h = 20.0(0.650)$ so $h = 20.0(1 - 0.650) = \boxed{7.00 \text{ cm}}$

(b) $B = F_g + Mg$ where $M = \text{mass of lead}$

$1.00(20.0)^3 g = 0.650(20.0)^3 g + Mg$

$M = (1.00 - 0.650)(20.0)^3 = 0.350(20.0)^3 = 2\,800 \text{ g} = \boxed{2.80 \text{ kg}}$

***P14.30** (a) The weight of the ball must be equal to the buoyant force of the water:

$$1.26\text{ kg}g = \rho_{\text{water}}\frac{4}{3}\pi r_{\text{outer}}^3 g$$

$$r_{\text{outer}} = \left(\frac{3\times 1.26\text{ kg}}{4\pi 1\,000\text{ kg/m}^3}\right)^{1/3} = \boxed{6.70\text{ cm}}$$

(b) The mass of the ball is determined by the density of aluminum:

$$m = \rho_{\text{Al}}V = \rho_{\text{Al}}\left(\frac{4}{3}\pi r_0^3 - \frac{4}{3}\pi r_i^3\right)$$

$$1.26\text{ kg} = 2\,700\text{ kg/m}^3\left(\frac{4}{3}\pi\right)\left((0.067\text{ m})^3 - r_i^3\right)$$

$$1.11\times 10^{-4}\text{ m}^3 = 3.01\times 10^{-4}\text{ m}^3 - r_i^3$$

$$r_i = \left(1.89\times 10^{-4}\text{ m}^3\right)^{1/3} = \boxed{5.74\text{ cm}}$$

***P14.31** Let $A$ represent the horizontal cross-sectional area of the rod, which we presume to be constant. The rod is in equilibrium:

$$\sum F_y = 0: \qquad -mg + B = 0 = -\rho_0 V_{\text{whole rod}}g + \rho_{\text{fluid}}V_{\text{immersed}}g$$

$$\rho_0 ALg = \rho A(L-h)g$$

The density of the liquid is $\rho = \dfrac{\rho_0 L}{L-h}$.

***P14.32** We use the result of Problem 14.31. For the rod floating in a liquid of density $0.98\text{ g/cm}^3$,

$$\rho = \rho_0\frac{L}{L-h}$$

$$0.98\text{ g/cm}^3 = \frac{\rho_0 L}{(L-0.2\text{ cm})}$$

$$0.98\text{ g/cm}^3\,L - \left(0.98\text{ g/cm}^3\right)0.2\text{ cm} = \rho_0 L$$

For floating in the dense liquid,

$$1.14\text{ g/cm}^3 = \frac{\rho_0 L}{(L-1.8\text{ cm})}$$

$$1.14\text{ g/cm}^3 - \left(1.14\text{ g/cm}^3\right)1.8\text{ cm} = \rho_0 L$$

(a) By substitution,

$$1.14L - 1.14(1.8\text{ cm}) = 0.98L - 0.2(0.98)$$

$$0.16L = 1.856\text{ cm}$$

$$L = \boxed{11.6\text{ cm}}$$

(b) Substituting back,

$$0.98\text{ g/cm}^3(11.6\text{ cm} - 0.2\text{ cm}) = \rho_0 11.6\text{ cm}$$

$$\rho_0 = \boxed{0.963\text{ g/cm}^3}$$

(c) The marks are not equally spaced. Because $\rho = \dfrac{\rho_0 L}{L-h}$ is not of the form $\rho = a + bh$, equal-size steps of $\rho$ do not correspond to equal-size steps of $h$.

**P14.33** The balloon stops rising when $(\rho_{\text{air}} - \rho_{\text{He}})gV = Mg$ and $(\rho_{\text{air}} - \rho_{\text{He}})V = M$,

Therefore, $V = \dfrac{M}{\rho_{\text{air}} - \rho_{\text{He}}} = \dfrac{400}{1.25e^{-1} - 0.180}$ $V = \boxed{1\,430 \text{ m}^3}$

**P14.34** Since the frog floats, the buoyant force = the weight of the frog. Also, the weight of the displaced water = weight of the frog, so

$$\rho_{\text{ooze}} Vg = m_{\text{frog}} g$$

or $m_{\text{frog}} = \rho_{\text{ooze}} V = \rho_{\text{ooze}} \dfrac{1}{2}\left(\dfrac{4}{3}\pi r^3\right) = \left(1.35 \times 10^3 \text{ kg/m}^3\right)\dfrac{2\pi}{3}\left(6.00 \times 10^{-2} \text{ m}\right)^3$

Hence, $m_{\text{frog}} = \boxed{0.611 \text{ kg}}$.

**P14.35** $B = F_g$

$$\rho_{\text{H}_2\text{O}} g \frac{V}{2} = \rho_{\text{sphere}} gV$$

$$\rho_{\text{sphere}} = \frac{1}{2}\rho_{\text{H}_2\text{O}} = \boxed{500 \text{ kg/m}^3}$$

$$\rho_{\text{glycerin}} g\left(\frac{4}{10}V\right) - \rho_{\text{sphere}} gV = 0$$

$$\rho_{\text{glycerin}} = \frac{10}{4}\left(500 \text{ kg/m}^3\right) = \boxed{1\,250 \text{ kg/m}^3}$$

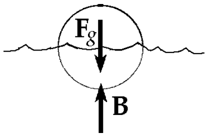

**FIG. P14.35**

**P14.36** Constant velocity implies zero acceleration, which means that the submersible is in equilibrium under the gravitational force, the upward buoyant force, and the upward resistance force:

$$\sum F_y = ma_y = 0 \qquad -\left(1.20 \times 10^4 \text{ kg} + m\right)g + \rho_w gV + 1\,100 \text{ N} = 0$$

where $m$ is the mass of the added water and $V$ is the sphere's volume.

$$1.20 \times 10^4 \text{ kg} + m = 1.03 \times 10^3\left[\frac{4}{3}\pi(1.50)^3\right] + \frac{1\,100 \text{ N}}{9.8 \text{ m/s}^2}$$

so $m = \boxed{2.67 \times 10^3 \text{ kg}}$

**P14.37** By Archimedes's principle, the weight of the fifty planes is equal to the weight of a horizontal slice of water 11.0 cm thick and circumscribed by the water line:

$$\Delta B = \rho_{\text{water}} g(\Delta V)$$

$$50\left(2.90 \times 10^4 \text{ kg}\right)g = \left(1\,030 \text{ kg/m}^3\right)g(0.110 \text{ m})A$$

giving $A = \boxed{1.28 \times 10^4 \text{ m}^2}$. The acceleration of gravity does not affect the answer.

## Section 14.5 **Fluid Dynamics**

## Section 14.6 **Bernoulli's Equation**

**P14.38** By Bernoulli's equation,

$$8.00\times10^4\ \text{N/m}^2+\frac{1}{2}(1\,000)v^2=6.00\times10^4\ \text{N/m}^2+\frac{1}{2}(1\,000)16v^2$$

$$2.00\times10^4\ \text{N/m}^2=\frac{1}{2}(1\,000)15v^2$$

$$v=1.63\ \text{m/s}$$

$$\frac{dm}{dt}=\rho Av=1\,000\pi\left(5.00\times10^{-2}\right)^2(1.63\ \text{m/s})=\boxed{12.8\ \text{kg/s}}$$

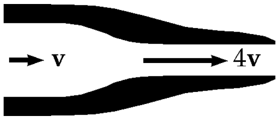

FIG. P14.38

**P14.39** Assuming the top is open to the atmosphere, then

$P_1=P_0$.

Note $P_2=P_0$.

$$\text{Flow rate}=2.50\times10^{-3}\ \text{m}^3/\text{min}=4.17\times10^{-5}\ \text{m}^3/\text{s}.$$

(a) $A_1 >> A_2$ so $v_1 << v_2$

Assuming $v_1=0$,

$$P_1+\frac{\rho v_1^2}{2}+\rho gy_1=P_2+\frac{\rho v_2^2}{2}+\rho gy_2$$

$$v_2=(2gy_1)^{1/2}=[2(9.80)(16.0)]^{1/2}=\boxed{17.7\ \text{m/s}}$$

(b) $$\text{Flow rate}=A_2v_2=\left(\frac{\pi d^2}{4}\right)(17.7)=4.17\times10^{-5}\ \text{m}^3/\text{s}$$

$$d=\boxed{1.73\times10^{-3}\ \text{m}}=1.73\ \text{mm}$$

***P14.40** Take point ① at the free surface of the water in the tank and ② inside the nozzle.

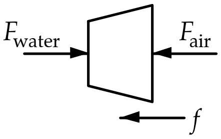

FIG. P14.40

(a) With the cork in place $P_1+\rho gy_1+\frac{1}{2}\rho v_1^2=P_2+\rho gy_2+\frac{1}{2}\rho v_2^2$ becomes

$P_0+1\,000\ \text{kg/m}^3\ 9.8\ \text{m/s}^2\ 7.5\ \text{m}+0=P_2+0+0$; $P_2-P_0=7.35\times10^4\ \text{Pa}$.

For the stopper $\sum F_x=0$

$$F_{\text{water}}-F_{\text{air}}-f=0$$

$$P_2A-P_0A=f$$

$$f=7.35\times10^4\ \text{Pa}\,\pi(0.011\ \text{m})^2=\boxed{27.9\ \text{N}}$$

(b) Now Bernoulli's equation gives

$$P_0+7.35\times10^4\ \text{Pa}+0=P_0+0+\frac{1}{2}\left(1\,000\ \text{kg/m}^3\right)v_2^2$$

$$v_2=12.1\ \text{m/s}$$

The quantity leaving the nozzle in 2 h is

$$\rho V=\rho Av_2t=\left(1\,000\ \text{kg/m}^3\right)\pi(0.011\ \text{m})^2(12.1\ \text{m/s})7\,200\ \text{s}=\boxed{3.32\times10^4\ \text{kg}}.$$

*continued on next page*

(c) Take point 1 in the wide hose and 2 just outside the nozzle. Continuity:

$$A_1v_1 = A_2v_2$$
$$\pi\left(\frac{6.6\text{ cm}}{2}\right)^2 v_1 = \pi\left(\frac{2.2\text{ cm}}{2}\right)^2 12.1\text{ m/s}$$
$$v_1 = \frac{12.1\text{ m/s}}{9} = 1.35\text{ m/s}$$
$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$
$$P_1 + 0 + \frac{1}{2}\left(1\,000\text{ kg/m}^3\right)(1.35\text{ m/s})^2 = P_0 + 0 + \frac{1}{2}\left(1\,000\text{ kg/m}^3\right)(12.1\text{ m/s})^2$$
$$P_1 - P_0 = 7.35\times10^4\text{ Pa} - 9.07\times10^2\text{ Pa} = \boxed{7.26\times10^4\text{ Pa}}$$

**P14.41** Flow rate $Q = 0.012\,0\text{ m}^3/\text{s} = v_1A_1 = v_2A_2$

$$v_2 = \frac{Q}{A_2} = \frac{0.012\,0}{A_2} = \boxed{31.6\text{ m/s}}$$

***P14.42** (a) $\mathcal{P} = \frac{\Delta E}{\Delta t} = \frac{\Delta mgh}{\Delta t} = \left(\frac{\Delta m}{\Delta t}\right)gh = Rgh$

(b) $\mathcal{P}_{\text{EL}} = 0.85\left(8.5\times10^5\right)(9.8)(87) = \boxed{616\text{ MW}}$

***P14.43** The volume flow rate is

$$\frac{125\text{ cm}^3}{16.3\text{ s}} = Av_1 = \pi\left(\frac{0.96\text{ cm}}{2}\right)^2 v_1.$$

The speed at the top of the falling column is

$$v_1 = \frac{7.67\text{ cm}^3/\text{s}}{0.724\text{ cm}^2} = 10.6\text{ cm/s}.$$

Take point 2 at 13 cm below:

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$
$$P_0 + \left(1\,000\text{ kg/m}^3\right)\left(9.8\text{ m/s}^2\right)0.13\text{ m} + \frac{1}{2}\left(1\,000\text{ kg/m}^3\right)(0.106\text{ m/s})^2$$
$$= P_0 + 0 + \frac{1}{2}\left(1\,000\text{ kg/m}^3\right)v_2^2$$
$$v_2 = \sqrt{2\left(9.8\text{ m/s}^2\right)0.13\text{ m} + (0.106\text{ m/s})^2} = 1.60\text{ m/s}$$

The volume flow rate is constant:

$$7.67\text{ cm}^3/\text{s} = \pi\left(\frac{d}{2}\right)^2 160\text{ cm/s}$$
$$d = \boxed{0.247\text{ cm}}$$

***P14.44** (a) Between sea surface and clogged hole: $P_1+\frac{1}{2}\rho v_1^2+\rho g y_1=P_2+\frac{1}{2}\rho v_2^2+\rho g y_2$

$$1\text{ atm}+0+\left(1\,030\text{ kg/m}^3\right)\left(9.8\text{ m/s}^2\right)(2\text{ m})=P_2+0+0 \qquad P_2=1\text{ atm}+20.2\text{ kPa}$$

The air on the back of his hand pushes opposite the water, so the net force on his hand is

$$F=PA=\left(20.2\times10^3\text{ N/m}^2\right)\left(\frac{\pi}{4}\right)\left(1.2\times10^{-2}\text{ m}\right)^2 \qquad F=\boxed{2.28\text{ N}}$$

(b) Now, Bernoulli's theorem is

$$1\text{ atm}+0+20.2\text{ kPa}=1\text{ atm}+\frac{1}{2}\left(1\,030\text{ kg/m}^3\right)v_2^2+0 \qquad v_2=6.26\text{ m/s}$$

The volume rate of flow is $A_2v_2=\frac{\pi}{4}\left(1.2\times10^{-2}\text{ m}\right)^2(6.26\text{ m/s})=7.08\times10^{-4}\text{ m}^3/\text{s}$

One acre–foot is $4\,047\text{ m}^2\times0.304\,8\text{ m}=1\,234\text{ m}^3$

Requiring $\dfrac{1\,234\text{ m}^3}{7.08\times10^{-4}\text{ m}^3/\text{s}}=\boxed{1.74\times10^6\text{ s}}=20.2\text{ days}$

**P14.45** (a) Suppose the flow is very slow: $\left(P+\frac{1}{2}\rho v^2+\rho g y\right)_{\text{river}}=\left(P+\frac{1}{2}\rho v^2+\rho g y\right)_{\text{rim}}$

$$P+0+\rho g(564\text{ m})=1\text{ atm}+0+\rho g(2\,096\text{ m})$$
$$P=1\text{ atm}+\left(1\,000\text{ kg/m}^3\right)\left(9.8\text{ m/s}^2\right)(1\,532\text{ m})=\boxed{1\text{ atm}+15.0\text{ MPa}}$$

(b) The volume flow rate is $4\,500\text{ m}^3/\text{d}=Av=\dfrac{\pi d^2v}{4}$

$$v=\left(4\,500\text{ m}^3/\text{d}\right)\left(\frac{1\text{ d}}{86\,400\text{ s}}\right)\left(\frac{4}{\pi(0.150\text{ m})^2}\right)=\boxed{2.95\text{ m/s}}$$

(c) Imagine the pressure as applied to stationary water at the bottom of the pipe:

$$\left(P+\frac{1}{2}\rho v^2+\rho g y\right)_{\text{bottom}}=\left(P+\frac{1}{2}\rho v^2+\rho g y\right)_{\text{top}}$$
$$P+0=1\text{ atm}+\frac{1}{2}\left(1\,000\text{ kg/m}^3\right)(2.95\text{ m/s})^2+1\,000\text{ kg}\left(9.8\text{ m/s}^2\right)(1\,532\text{ m})$$
$$P=1\text{ atm}+15.0\text{ MPa}+4.34\text{ kPa}$$

The additional pressure is $\boxed{4.34\text{ kPa}}$.

**P14.46** (a) For upward flight of a water-drop projectile from geyser vent to fountain–top, $v_{yf}^2 = v_{yi}^2 + 2a_y\Delta y$

Then $0 = v_i^2 + 2\left(-9.80 \text{ m/s}^2\right)(+40.0 \text{ m})$ and $v_i = \boxed{28.0 \text{ m/s}}$

(b) Between geyser vent and fountain–top: $P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$

Air is so low in density that very nearly $P_1 = P_2 = 1 \text{ atm}$

Then, $\frac{1}{2}v_i^2 + 0 = 0 + \left(9.80 \text{ m/s}^2\right)(40.0 \text{ m})$

$v_1 = \boxed{28.0 \text{ m/s}}$

(c) Between the chamber and the fountain-top: $P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$

$$P_1 + 0 + \left(1\,000 \text{ kg/m}^3\right)\left(9.80 \text{ m/s}^2\right)(-175 \text{ m}) = P_0 + 0 + \left(1\,000 \text{ kg/m}^3\right)\left(9.80 \text{ m/s}^2\right)(+40.0 \text{ m})$$

$$P_1 - P_0 = \left(1\,000 \text{ kg/m}^3\right)\left(9.80 \text{ m/s}^2\right)(215 \text{ m}) = \boxed{2.11 \text{ MPa}}$$

**P14.47** $P_1 + \frac{\rho v_1^2}{2} = P_2 + \frac{\rho_2^2}{2}$ (Bernoulli equation), $v_1 A_1 = v_2 A_2$ where $\frac{A_1}{A_2} = 4$

$$\Delta P = P_1 - P_2 = \frac{\rho}{2}\left(v_2^2 - v_1^2\right) = \frac{\rho}{2}v_1^2\left(\frac{A_1^2}{A_2^2} - 1\right) \text{ and } \Delta P = \frac{\rho v_1^2}{2}15 = 21\,000 \text{ Pa}$$

$v_1 = 2.00 \text{ m/s}$; $v_2 = 4v_1 = 8.00 \text{ m/s}$:

The volume flow rate is $v_1 A_1 = \boxed{2.51 \times 10^{-3} \text{ m}^3\text{/s}}$

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## Section 14.7 Other Applications of Fluid Dynamics

**P14.48** $Mg = \left(P_1 - P_2\right)A$ for a balanced condition $\frac{16\,000(9.80)}{A} = 7.00 \times 10^4 - P_2$

where $A = 80.0 \text{ m}^2$ $\therefore P_2 = 7.0 \times 10^4 - 0.196 \times 10^4 = \boxed{6.80 \times 10^4 \text{ Pa}}$

**P14.49** $\rho_{\text{air}}\frac{v^2}{2} = \Delta P = \rho_{\text{Hg}} g \Delta h$

$$v = \sqrt{\frac{2\rho_{\text{Hg}} g \Delta h}{\rho_{\text{air}}}} = \boxed{103 \text{ m/s}}$$

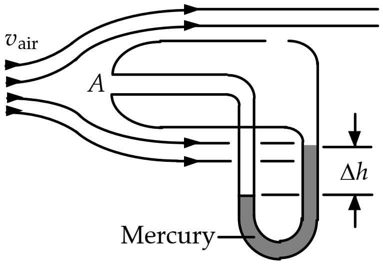

FIG. P14.49

**P14.50** The assumption of incompressibility is surely unrealistic, but allows an estimate of the speed:

$$P_1+\rho g y_1+\frac{1}{2}\rho v_1^2=P_2+\rho g y_2+\frac{1}{2}\rho v_2^2$$

$$1.00\text{ atm}+0+0=0.287\text{ atm}+0+\frac{1}{2}\left(1.20\text{ kg/m}^3\right)v_2^2$$

$$v_2=\sqrt{\frac{2(1.00-0.287)\left(1.013\times10^5\text{ N/m}^2\right)}{1.20\text{ kg/m}^3}}=\boxed{347\text{ m/s}}$$

**P14.51** (a) $P_0+\rho g h+0=P_0+0+\frac{1}{2}\rho v_3^2$ $\qquad v_3=\sqrt{2gh}$

If $h=1.00$ m, $\qquad v_3=\boxed{4.43\text{ m/s}}$

(b) $P+\rho g y+\frac{1}{2}\rho v_2^2=P_0+0+\frac{1}{2}\rho v_3^2$

Since $v_2=v_3$, $\qquad P=P_0-\rho g y$

Since $P\geq 0$ $\qquad y\leq\frac{P_0}{\rho g}=\frac{1.013\times10^5\text{ Pa}}{\left(10^3\text{ kg/m}^3\right)\left(9.8\text{ m/s}^2\right)}=\boxed{10.3\text{ m}}$

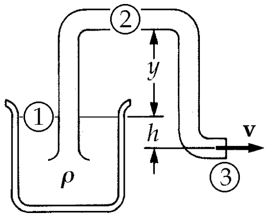

**FIG. P14.51**

***P14.52** Take points 1 and 2 in the air just inside and outside the window pane.

$$P_1+\frac{1}{2}\rho v_1^2+\rho g y_1=P_2+\frac{1}{2}\rho v_2^2+\rho g y_2$$

$$P_0+0=P_2+\frac{1}{2}\left(1.30\text{ kg/m}^3\right)(11.2\text{ m/s})^2 \qquad P_2=P_0-81.5\text{ Pa}$$

(a) The total force exerted by the air is outward,

$$P_1A-P_2A=P_0A-P_0A+\left(81.5\text{ N/m}^2\right)(4\text{ m})(1.5\text{ m})=\boxed{489\text{ N outward}}$$

(b) $$P_1A-P_2A=\frac{1}{2}\rho v_2^2A=\frac{1}{2}\left(1.30\text{ kg/m}^3\right)(22.4\text{ m/s})^2(4\text{ m})(1.5\text{ m})=\boxed{1.96\text{ kN outward}}$$

**P14.53** In the reservoir, the gauge pressure is $\qquad \Delta P=\frac{2.00\text{ N}}{2.50\times10^{-5}\text{ m}^2}=8.00\times10^4\text{ Pa}$

From the equation of continuity: $\qquad A_1v_1=A_2v_2$

$\left(2.50\times10^{-5}\text{ m}^2\right)v_1=\left(1.00\times10^{-8}\text{ m}^2\right)v_2 \qquad v_1=\left(4.00\times10^{-4}\right)v_2$

Thus, $v_1^2$ is negligible in comparison to $v_2^2$.

Then, from Bernoulli's equation: $\qquad (P_1-P_2)+\frac{1}{2}\rho v_1^2+\rho g y_1=\frac{1}{2}\rho v_2^2+\rho g y_2$

$$8.00\times10^4\text{ Pa}+0+0=0+\frac{1}{2}\left(1\,000\text{ kg/m}^3\right)v_2^2$$

$$v_2=\sqrt{\frac{2\left(8.00\times10^4\text{ Pa}\right)}{1\,000\text{ kg/m}^3}}=\boxed{12.6\text{ m/s}}$$

## Additional Problems

**P14.54** Consider the diagram and apply Bernoulli's equation to points A and B, taking $y = 0$ at the level of point B, and recognizing that $v_A$ is approximately zero. This gives:

$$P_A + \frac{1}{2}\rho_w(0)^2 + \rho_w g(h - L\sin\theta) = P_B + \frac{1}{2}\rho_w v_B^2 + \rho_w g(0)$$

Now, recognize that $P_A = P_B = P_{\text{atmosphere}}$ since both points are open to the atmosphere (neglecting variation of atmospheric pressure with altitude). Thus, we obtain

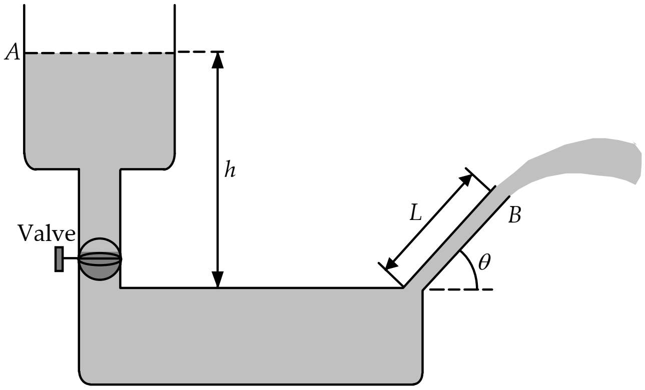

**FIG. P14.54**

$$v_B = \sqrt{2g(h - L\sin\theta)} = \sqrt{2\left(9.80\ \text{m/s}^2\right)\left[10.0\ \text{m} - (2.00\ \text{m})\sin 30.0°\right]}$$
$$v_B = 13.3\ \text{m/s}$$

Now the problem reduces to one of projectile motion with $v_{yi} = v_B \sin 30.0° = 6.64\ \text{m/s}$. Then, $v_{yf}^2 = v_{yi}^2 + 2a(\Delta y)$ gives at the top of the arc (where $y = y_{\max}$ and $v_{yf} = 0$)

$$0 = (6.64\ \text{m/s})^2 + 2\left(-9.80\ \text{m/s}^2\right)(y_{\max} - 0)$$

or $y_{\max} = \boxed{2.25\ \text{m (above the level where the water emerges)}}$.

**P14.55** When the balloon comes into equilibrium, we must have

$$\sum F_y = B - F_{g,\ \text{balloon}} - F_{g,\ \text{He}} - F_{g,\ \text{string}} = 0$$

$F_{g,\ \text{string}}$ is the weight of the string above the ground, and $B$ is the buoyant force. Now

$$F_{g,\ \text{balloon}} = m_{\text{balloon}} g$$
$$F_{g,\ \text{He}} = \rho_{\text{He}} V g$$
$$B = \rho_{\text{air}} V g$$

and $F_{g,\ \text{string}} = m_{\text{string}} \dfrac{h}{L} g$

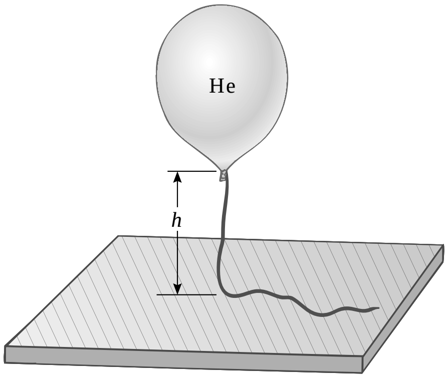

**FIG. P14.55**

Therefore, we have

$$\rho_{\text{air}} V g - m_{\text{balloon}} g - \rho_{\text{He}} V g - m_{\text{string}} \frac{h}{L} g = 0$$

or $h = \dfrac{(\rho_{\text{air}} - \rho_{\text{He}})V - m_{\text{balloon}}}{m_{\text{string}}} L$

giving,

$$h = \frac{(1.29 - 0.179)\left(\text{kg/m}^3\right)\left(\frac{4\pi(0.400\ \text{m})^3}{3}\right) - 0.250\ \text{kg}}{0.050\,0\ \text{kg}}(2.00\ \text{m}) = \boxed{1.91\ \text{m}}.$$

**P14.56** Assume $v_{\text{inside}} \approx 0$

$$P + 0 + 0 = 1\text{ atm} + \frac{1}{2}(1\,000)(30.0)^2 + 1\,000(9.80)(0.500)$$

$$P_{\text{gauge}} = P - 1\text{ atm} = 4.50\times10^5 + 4.90\times10^3 = \boxed{455\text{ kPa}}$$

**P14.57** The "balanced" condition is one in which the apparent weight of the body equals the apparent weight of the weights. This condition can be written as:

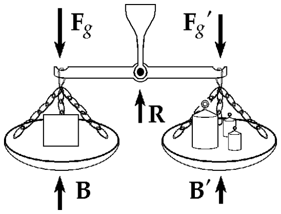

**FIG. P14.57**

$$F_g - B = F_g' - B'$$

where $B$ and $B'$ are the buoyant forces on the body and weights respectively. The buoyant force experienced by an object of volume $V$ in air equals:

$$\text{Buoyant force} = (\text{Volume of object})\rho_{\text{air}}g$$

so we have $B = V\rho_{\text{air}}g$ and $B' = \left(\dfrac{F_g'}{\rho g}\right)\rho_{\text{air}}g$.

Therefore, $F_g = F_g' + \left(V - \dfrac{F_g'}{\rho g}\right)\rho_{\text{air}}g$.

**P14.58** The cross–sectional area above water is

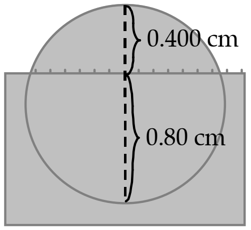

**FIG. P14.58**

$$\frac{2.46\text{ rad}}{2\pi}\pi(0.600\text{ cm})^2 - (0.200\text{ cm})(0.566\text{ cm}) = 0.330\text{ cm}^2$$

$$A_{\text{all}} = \pi(0.600)^2 = 1.13\text{ cm}^2$$

$$\rho_{\text{water}}gA_{\text{under}} = \rho_{\text{wood}}A_{\text{all}}g$$

$$\rho_{\text{wood}} = \frac{1.13 - 0.330}{1.13} = 0.709\text{ g/cm}^3 = \boxed{709\text{ kg/m}^3}$$

**P14.59** At equilibrium, $\sum F_y = 0$: $B - F_{\text{spring}} - F_{g,\text{ He}} - F_{g,\text{ balloon}} = 0$

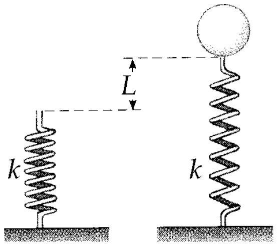

**FIG. P14.59**

giving $F_{\text{spring}} = kL = B - (m_{\text{He}} + m_{\text{balloon}})g$.

But $B = \text{weight of displaced air} = \rho_{\text{air}}Vg$

and $m_{\text{He}} = \rho_{\text{He}}V$.

Therefore, we have: $kL = \rho_{\text{air}}Vg - \rho_{\text{He}}Vg - m_{\text{balloon}}g$

or $L = \dfrac{(\rho_{\text{air}} - \rho_{\text{He}})V - m_{\text{balloon}}}{k}g$.

From the data given, $L = \dfrac{(1.29\text{ kg/m}^3 - 0.180\text{ kg/m}^3)5.00\text{ m}^3 - 2.00\times10^{-3}\text{ kg}}{90.0\text{ N/m}}(9.80\text{ m/s}^2)$.

Thus, this gives $L = \boxed{0.604\text{ m}}$.

**P14.60** $P = \rho gh$ $\quad 1.013\times10^5 = 1.29(9.80)h$

$h = \boxed{8.01\text{ km}}$ $\quad$ For Mt. Everest, $\quad 29\,300\text{ ft} = 8.88\text{ km}$ $\quad \boxed{\text{Yes}}$

**P14.61** The torque is $\tau = \int d\tau = \int r dF$

From the figure $\tau = \int_0^H y\left[\rho g(H-y)wdy\right] = \boxed{\frac{1}{6}\rho g w H^3}$

The total force is given as $\frac{1}{2}\rho g w H^2$

If this were applied at a height $y_{eff}$ such that the torque remains unchanged, we have

$$\frac{1}{6}\rho g w H^3 = y_{eff}\left[\frac{1}{2}\rho g w H^2\right] \quad \text{and} \quad y_{eff} = \boxed{\frac{1}{3}H}.$$

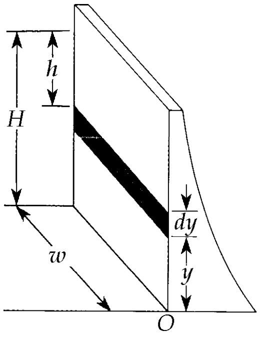

FIG. P14.61

**P14.62** (a) The pressure on the surface of the two hemispheres is constant at all points, and the force on each element of surface area is directed along the radius of the hemispheres. The applied force along the axis must balance the force on the "effective" area, which is the projection of the actual surface onto a plane perpendicular to the $x$ axis,

$$A = \pi R^2$$

Therefore, $F = \boxed{(P_0 - P)\pi R^2}$

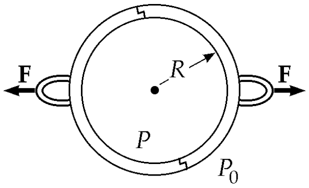

FIG. P14.62

(b) For the values given $F = (P_0 - 0.100P_0)\left[\pi(0.300 \text{ m})^2\right] = 0.254P_0 = \boxed{2.58 \times 10^4 \text{ N}}$

**P14.63** Looking first at the top scale and the iron block, we have:

$$T_1 + B = F_{g,\text{ iron}}$$

where $T_1$ is the tension in the spring scale, $B$ is the buoyant force, and $F_{g,\text{ iron}}$ is the weight of the iron block. Now if $m_{\text{iron}}$ is the mass of the iron block, we have

$$m_{\text{iron}} = \rho_{\text{iron}}V \qquad \text{so} \qquad V = \frac{m_{\text{iron}}}{\rho_{\text{iron}}} = V_{\text{displaced oil}}$$

Then, $B = \rho_{\text{oil}}V_{\text{iron}}g$

Therefore, $T_1 = F_{g,\text{ iron}} - \rho_{\text{oil}}V_{\text{iron}}g = m_{\text{iron}}g - \rho_{\text{oil}}\frac{m_{\text{iron}}}{\rho_{\text{iron}}}g$

or $T_1 = \left(1 - \frac{\rho_{\text{oil}}}{\rho_{\text{iron}}}\right)m_{\text{iron}}g = \left(1 - \frac{916}{7\,860}\right)(2.00)(9.80) = \boxed{17.3 \text{ N}}$

Next, we look at the bottom scale which reads $T_2$ (i.e., exerts an upward force $T_2$ on the system). Consider the external vertical forces acting on the beaker–oil–iron combination.

$\sum F_y = 0$ gives

$$T_1 + T_2 - F_{g,\text{ beaker}} - F_{g,\text{ oil}} - F_{g,\text{ iron}} = 0$$

or $T_2 = (m_{\text{beaker}} + m_{\text{oil}} + m_{\text{iron}})g - T_1 = (5.00 \text{ kg})(9.80 \text{ m/s}^2) - 17.3 \text{ N}$

Thus, $T_2 = \boxed{31.7 \text{ N}}$ is the lower scale reading.

**P14.64** Looking at the top scale and the iron block:

$$T_1 + B = F_{g,\,\mathrm{Fe}} \qquad \text{where} \qquad B = \rho_0 V_{\mathrm{Fe}} g = \rho_0 \left(\frac{m_{\mathrm{Fe}}}{\rho_{\mathrm{Fe}}}\right) g$$

is the buoyant force exerted on the iron block by the oil.

Thus, $$T_1 = F_{g,\,\mathrm{Fe}} - B = m_{\mathrm{Fe}} g - \rho_0 \left(\frac{m_{\mathrm{Fe}}}{\rho_{\mathrm{Fe}}}\right) g$$

or $$T_1 = \boxed{\left(1 - \frac{\rho_0}{\rho_{\mathrm{Fe}}}\right) m_{\mathrm{Fe}} g} \text{ is the reading on the top scale.}$$

Now, consider the bottom scale, which exerts an upward force of $T_2$ on the beaker–oil–iron combination.

$$\sum F_y = 0: \qquad T_1 + T_2 - F_{g,\,\text{beaker}} - F_{g,\,\text{oil}} - F_{g,\,\mathrm{Fe}} = 0$$

$$T_2 = F_{g,\,\text{beaker}} + F_{g,\,\text{oil}} + F_{g,\,\mathrm{Fe}} - T_1 = \left(m_b + m_0 + m_{\mathrm{Fe}}\right) g - \left(1 - \frac{\rho_0}{\rho_{\mathrm{Fe}}}\right) m_{\mathrm{Fe}} g$$

or $$T_2 = \boxed{\left[m_b + m_0 + \left(\frac{\rho_0}{\rho_{\mathrm{Fe}}}\right) m_{\mathrm{Fe}}\right] g} \text{ is the reading on the bottom scale.}$$

**P14.65** $\rho_{\mathrm{Cu}} V = 3.083 \text{ g}$

$\rho_{\mathrm{Zn}}(xV) + \rho_{\mathrm{Cu}}(1 - x)V = 2.517 \text{ g}$

$$\rho_{\mathrm{Zn}} \left(\frac{3.083}{\rho_{\mathrm{Cu}}}\right) x + 3.083(1 - x) = 2.517$$

$$\left(1 - \frac{7.133}{8.960}\right) x = \left(1 - \frac{2.517}{3.083}\right)$$

$x = 0.900\,4$

$\%\mathrm{Zn} = \boxed{90.04\%}$

**P14.66** (a) From $\sum F = ma$

$$B - m_{\text{shell}} g - m_{\mathrm{He}} g = m_{\text{total}} a = \left(m_{\text{shell}} + m_{\mathrm{He}}\right) a \qquad (1)$$

Where $B = \rho_{\text{water}} V g$ and $m_{\mathrm{He}} = \rho_{\mathrm{He}} V$

Also, $V = \dfrac{4}{3}\pi r^3 = \dfrac{\pi d^3}{6}$

Putting these into equation (1) above,

$$\left(m_{\text{shell}} + \rho_{\mathrm{He}} \frac{\pi d^3}{6}\right) a = \left(\rho_{\text{water}} \frac{\pi d^3}{6} - m_{\text{shell}} - \rho_{\mathrm{He}} \frac{\pi d^3}{6}\right) g$$

which gives

$$a = \frac{\left(\rho_{\text{water}} - \rho_{\mathrm{He}}\right) \frac{\pi d^3}{6} - m_{\text{shell}}}{m_{\text{shell}} + \rho_{\mathrm{He}} \frac{\pi d^3}{6}} g$$

or $$a = \frac{(1\,000 - 0.180)\left(\mathrm{kg/m^3}\right) \frac{\pi (0.200 \text{ m})^3}{6} - 4.00 \text{ kg}}{4.00 \text{ kg} + \left(0.180 \text{ kg/m}^3\right) \frac{\pi (0.200 \text{ m})^3}{6}} 9.80 \text{ m/s}^2 = \boxed{0.461 \text{ m/s}^2}$$

(b) $$t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(h - d)}{a}} = \sqrt{\frac{2(4.00 \text{ m} - 0.200 \text{ m})}{0.461 \text{ m/s}^2}} = \boxed{4.06 \text{ s}}$$

**P14.67** Inertia of the disk: $I = \frac{1}{2}MR^2 = \frac{1}{2}(10.0\text{ kg})(0.250\text{ m})^2 = 0.312\text{ kg}\cdot\text{m}^2$

Angular acceleration: $\omega_f = \omega_i + \alpha t$

$$\alpha = \left(\frac{0 - 300\text{ rev/min}}{60.0\text{ s}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60.0\text{ s}}\right) = -0.524\text{ rad/s}^2$$

Braking torque: $\sum \tau = I\alpha \Rightarrow -fd = I\alpha$, so $f = \frac{-I\alpha}{d}$

Friction force: $f = \frac{(0.312\text{ kg}\cdot\text{m}^2)(0.524\text{ rad/s}^2)}{0.220\text{ m}} = 0.744\text{ N}$

Normal force: $f = \mu_k n \Rightarrow n = \frac{f}{\mu_k} = \frac{0.744\text{ N}}{0.500} = 1.49\text{ N}$

gauge pressure: $P = \frac{n}{A} = \frac{1.49\text{ N}}{\pi(2.50\times 10^{-2}\text{ m})^2} = \boxed{758\text{ Pa}}$

**P14.68** The incremental version of $P - P_0 = \rho g y$ is $dP = -\rho g dy$

We assume that the density of air is proportional to pressure, or $\frac{P}{\rho} = \frac{P_0}{\rho_0}$

Combining these two equations we have $dP = -P\frac{\rho_0}{P_0}g dy$

$$\int_{P_0}^{P}\frac{dP}{P} = -g\frac{\rho_0}{P_0}\int_0^h dy$$

and integrating gives $\ln\left(\frac{P}{P_0}\right) = -\frac{\rho_0 g h}{P_0}$

so where $\alpha = \frac{\rho_0 g}{P_0}$, $P = P_0 e^{-\alpha h}$

**P14.69** Energy for the fluid-Earth system is conserved.

$(K+U)_i + \Delta E_{\text{mech}} = (K+U)_f$: $0 + \frac{mgL}{2} + 0 = \frac{1}{2}mv^2 + 0$

$$v = \sqrt{gL} = \sqrt{2.00\text{ m}(9.8\text{ m/s}^2)} = \boxed{4.43\text{ m/s}}$$

**P14.70** Let $s$ stand for the edge of the cube, $h$ for the depth of immersion, $\rho_{\text{ice}}$ stand for the density of the ice, $\rho_w$ stand for density of water, and $\rho_a$ stand for density of the alcohol.

(a) According to Archimedes's principle, at equilibrium we have

$$\rho_{\text{ice}} g s^3 = \rho_w g h s^2 \Rightarrow h = s\frac{\rho_{\text{ice}}}{\rho_w}$$

With $\rho_{\text{ice}} = 0.917\times 10^3 \text{ kg/m}^3$

$\rho_w = 1.00\times 10^3 \text{ kg/m}^3$

and $s = 20.0 \text{ mm}$

we get $h = 20.0(0.917) = 18.34 \text{ mm} \approx \boxed{18.3 \text{ mm}}$

(b) We assume that the top of the cube is still above the alcohol surface. Letting $h_a$ stand for the thickness of the alcohol layer, we have

$$\rho_a g s^2 h_a + \rho_w g s^2 h_w = \rho_{\text{ice}} g s^3 \qquad \text{so} \qquad h_w = \left(\frac{\rho_{\text{ice}}}{\rho_w}\right)s - \left(\frac{\rho_a}{\rho_w}\right)h_a$$

With $\rho_a = 0.806\times 10^3 \text{ kg/m}^3$

and $h_a = 5.00 \text{ mm}$

we obtain $h_w = 18.34 - 0.806(5.00) = 14.31 \text{ mm} \approx \boxed{14.3 \text{ mm}}$

(c) Here $h'_w = s - h'_a$, so Archimedes's principle gives

$$\rho_a g s^2 h'_a + \rho_w g s^2 (s - h'_a) = \rho_{\text{ice}} g s^3 \Rightarrow \rho_a h'_a + \rho_w (s - h'_a) = \rho_{\text{ice}} s$$

$$h'_a = s\frac{(\rho_w - \rho_{\text{ice}})}{(\rho_w - \rho_a)} = 20.0\frac{(1.000 - 0.917)}{(1.000 - 0.806)} = 8.557 \approx \boxed{8.56 \text{ mm}}$$

**P14.71** **Note**: Variation of atmospheric pressure with altitude is included in this solution. Because of the small distances involved, this effect is unimportant in the final answers.

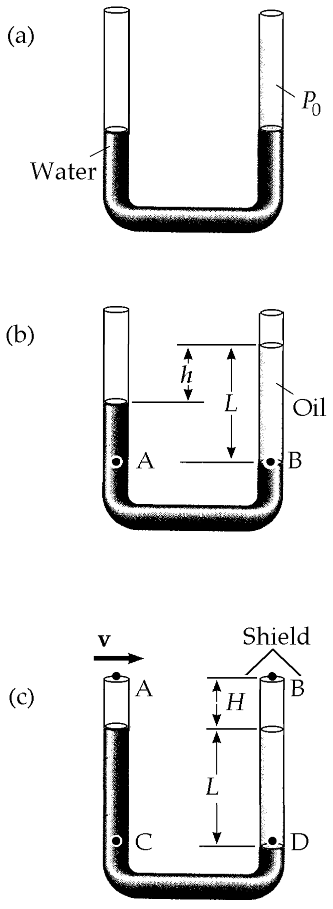

**FIG. P14.71**

(a) Consider the pressure at points A and B in part (b) of the figure:

Using the left tube: $P_A = P_{\text{atm}} + \rho_a gh + \rho_w g(L-h)$ where the second term is due to the variation of air pressure with altitude.

Using the right tube: $P_B = P_{\text{atm}} + \rho_0 gL$

But Pascal's principle says that $P_A = P_B$.

Therefore, $P_{\text{atm}} + \rho_0 gL = P_{\text{atm}} + \rho_a gh + \rho_w g(L-h)$

or $(\rho_w - \rho_a)h = (\rho_w - \rho_0)L$, giving

$$h = \left(\frac{\rho_w - \rho_0}{\rho_w - \rho_a}\right)L = \left(\frac{1\,000 - 750}{1\,000 - 1.29}\right)5.00\text{ cm} = \boxed{1.25\text{ cm}}$$

(b) Consider part (c) of the diagram showing the situation when the air flow over the left tube equalizes the fluid levels in the two tubes. First, apply Bernoulli's equation to points A and B $(y_A = y_B,\ v_A = v,\ \text{and } v_B = 0)$

This gives: $P_A + \frac{1}{2}\rho_a v^2 + \rho_a g y_A = P_B + \frac{1}{2}\rho_a (0)^2 + \rho_a g y_B$

and since $y_A = y_B$, this reduces to: $P_B - P_A = \frac{1}{2}\rho_a v^2$ (1)

Now consider points C and D, both at the level of the oil–water interface in the right tube. Using the variation of pressure with depth in static fluids, we have:

$P_C = P_A + \rho_a gH + \rho_w gL$ and $P_D = P_B + \rho_a gH + \rho_0 gL$

But Pascal's principle says that $P_C = P_D$. Equating these two gives:

$P_B + \rho_a gH + \rho_0 gL = P_A + \rho_a gH + \rho_w gL$ or $P_B - P_A = (\rho_w - \rho_0)gL$ (2)

Substitute equation (1) for $P_B - P_A$ into (2) to obtain $\frac{1}{2}\rho_a v^2 = (\rho_w - \rho_0)gL$

or $$v = \sqrt{\frac{2gL(\rho_w - \rho_0)}{\rho_a}} = \sqrt{2\left(9.80\text{ m/s}^2\right)(0.050\,0\text{ m})\left(\frac{1\,000 - 750}{1.29}\right)}$$

$$v = \boxed{13.8\text{ m/s}}$$

**P14.72** (a) The flow rate, $Av$, as given may be expressed as follows:

$$\frac{25.0\text{ liters}}{30.0\text{ s}} = 0.833\text{ liters/s} = 833\text{ cm}^3/\text{s}.$$

The area of the faucet tap is $\pi\,\text{cm}^2$, so we can find the velocity as

$$v = \frac{\text{flow rate}}{A} = \frac{833\text{ cm}^3/\text{s}}{\pi\,\text{cm}^2} = 265\text{ cm/s} = \boxed{2.65\text{ m/s}}.$$

(b) We choose point 1 to be in the entrance pipe and point 2 to be at the faucet tap. $A_1v_1 = A_2v_2$ gives $v_1 = 0.295\text{ m/s}$. Bernoulli's equation is:

$$P_1 - P_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) + \rho g\left(y_2 - y_1\right)$$

and gives

$$P_1 - P_2 = \frac{1}{2}\left(10^3\text{ kg/m}^3\right)\left[(2.65\text{ m/s})^2 - (0.295\text{ m/s})^2\right] + \left(10^3\text{ kg/m}^3\right)\left(9.80\text{ m/s}^2\right)(2.00\text{ m})$$

or $P_{\text{gauge}} = P_1 - P_2 = \boxed{2.31\times 10^4\text{ Pa}}$.

**P14.73** (a) Since the upward buoyant force is balanced by the weight of the sphere,

$$m_1 g = \rho V g = \rho\left(\frac{4}{3}\pi R^3\right)g.$$

In this problem, $\rho = 0.789\,45\text{ g/cm}^3$ at 20.0°C, and $R = 1.00\text{ cm}$ so we find:

$$m_1 = \rho\left(\frac{4}{3}\pi R^3\right) = \left(0.789\,45\text{ g/cm}^3\right)\left[\frac{4}{3}\pi(1.00\text{ cm})^3\right] = \boxed{3.307\text{ g}}.$$

(b) Following the same procedure as in part (a), with $\rho' = 0.780\,97\text{ g/cm}^3$ at 30.0°C, we find:

$$m_2 = \rho'\left(\frac{4}{3}\pi R^3\right) = \left(0.780\,97\text{ g/cm}^3\right)\left[\frac{4}{3}\pi(1.00\text{ cm})^3\right] = \boxed{3.271\text{ g}}.$$

(c) When the first sphere is resting on the bottom of the tube,

$n + B = F_{g1} = m_1 g$, where $n$ is the normal force.

Since $B = \rho' V g$

$$n = m_1 g - \rho' V g = \left[3.307\text{ g} - \left(0.780\,97\text{ g/cm}^3\right)(1.00\text{ cm})^3\right]980\text{ cm/s}^2$$

$$n = 34.8\text{ g}\cdot\text{cm/s}^2 = \boxed{3.48\times 10^{-4}\text{ N}}$$

***P14.74** (a) Take point ① at the free water surface in the tank and point ② at the bottom end of the tube:

$$P_1+\rho g y_1+\frac{1}{2}\rho v_1^2=P_2+\rho g y_2+\frac{1}{2}\rho v_2^2$$

$$P_0+\rho g d+0=P_0+0+\frac{1}{2}\rho v_2^2$$

$$v_2=\sqrt{2gd}$$

The volume flow rate is $\frac{V}{t}=\frac{Ah}{t}=v_2A'$. Then $t=\frac{Ah}{v_2A'}=\frac{Ah}{A'\sqrt{2gd}}$.

(b) $t=\frac{(0.5\text{ m})^2\,0.5\text{ m}}{2\times10^{-4}\text{ m}^2\sqrt{2\left(9.8\text{ m/s}^2\right)10\text{ m}}}=\boxed{44.6\text{ s}}$

***P14.75** (a) For diverging stream lines that pass just above and just below the hydrofoil we have

$$P_t+\rho g y_t+\frac{1}{2}\rho v_t^2=P_b+\rho g y_b+\frac{1}{2}\rho v_b^2.$$

Ignoring the buoyant force means taking $y_t\approx y_b$

$$P_t+\frac{1}{2}\rho(nv_b)^2=P_b+\frac{1}{2}\rho v_b^2$$

$$P_b-P_t=\frac{1}{2}\rho v_b^2\left(n^2-1\right)$$

The lift force is $(P_b-P_t)A=\frac{1}{2}\rho v_b^2\left(n^2-1\right)A$.

(b) For liftoff,

$$\frac{1}{2}\rho v_b^2\left(n^2-1\right)A=Mg$$

$$v_b=\left(\frac{2Mg}{\rho\left(n^2-1\right)A}\right)^{1/2}$$

The speed of the boat relative to the shore must be nearly equal to this speed of the water below the hydrofoil relative to the boat.

(c) $v^2\left(n^2-1\right)A\rho=2Mg$

$$A=\frac{2(800\text{ kg})9.8\text{ m/s}^2}{(9.5\text{ m/s})^2\left(1.05^2-1\right)1\,000\text{ kg/m}^3}=\boxed{1.70\text{ m}^2}$$

## ANSWERS TO EVEN PROBLEMS

**P14.2** $\sim 10^{18}\ \text{kg/m}^3$; matter is mostly empty space

**P14.4** $1.92 \times 10^4\ \text{N}$

**P14.6** (a) $1.01 \times 10^7\ \text{Pa}$; (b)$7.09 \times 10^5\ \text{N}$ outward

**P14.8** 255 N

**P14.10** (a) 65.1 N; (b) 275 N

**P14.12** $5.88 \times 10^6\ \text{N}$ down; 196 kN outward; 588 kN outward

**P14.14** (a) 29.4 kN to the right; (b) 16.3 $\text{kN} \cdot \text{m}$ counterclockwise

**P14.16** (a) 10.3 m; (b) zero

**P14.18** (a) 20.0 cm; (b) 0.490 cm

**P14.20** 12.6 cm

**P14.22** (a) 444 kg; (b) 480 kg

**P14.24** $\dfrac{m}{(\rho_w - \rho_s)h}$

**P14.26** (a) see the solution; (b) 25.0 N up; (c) horizontally inward; (d) tension increases; see the solution; (e) 62.5%; (f) 18.7%

**P14.28** $\sim 10^4$ balloons of 25-cm diameter

**P14.30** (a) 6.70 cm; (b) 5.74 cm

**P14.32** (a) 11.6 cm; (b) $0.963\ \text{g/cm}^3$; (c) no; see the solution

**P14.34** 0.611 kg

**P14.36** $2.67 \times 10^3\ \text{kg}$

**P14.38** $12.8\ \text{kg/s}$

**P14.40** (a) 27.9 N; (b) $3.32 \times 10^4\ \text{kg}$; (c) $7.26 \times 10^4\ \text{Pa}$

**P14.42** (a) see the solution; (b) 616 MW

**P14.44** (a) 2.28 N toward Holland; (b) $1.74 \times 10^6\ \text{s}$

**P14.46** (a), (b) $28.0\ \text{m/s}$; (c) 2.11 MPa

**P14.48** $6.80 \times 10^4\ \text{Pa}$

**P14.50** $347\ \text{m/s}$

**P14.52** (a) 489 N outward; (b) 1.96 kN outward

**P14.54** 2.25 m above the level where the water emerges

**P14.56** 455 kPa

**P14.58** $709\ \text{kg/m}^3$

**P14.60** 8.01 km; yes

**P14.62** (a) see the solution; (b) $2.58 \times 10^4\ \text{N}$

**P14.64** top scale: $\left(1 - \dfrac{\rho_0}{\rho_{\text{Fe}}}\right) m_{\text{Fe}} g$;

bottom scale: $\left(m_b + m_0 + \dfrac{\rho_0 m_{\text{Fe}}}{\rho_{\text{Fe}}}\right) g$

**P14.66** (a) $0.461\ \text{m/s}^2$; (b) 4.06 s

**P14.68** see the solution

**P14.70** (a) 18.3 mm; (b) 14.3 mm; (c) 8.56 mm

**P14.72** (a) $2.65\ \text{m/s}$; (b) $2.31 \times 10^4\ \text{Pa}$

**P14.74** (a) see the solution; (b) 44.6 s

# 15

# Oscillatory Motion

CHAPTER OUTLINE

15.1 Motion of an Object Attached to a Spring
15.2 Mathematical Representation of Simple Harmonic Motion
15.3 Energy of the Simple Harmonic Oscillator
15.4 Comparing Simple Harmonic Motion with Uniform Circular Motion
15.5 The Pendulum
15.6 Damped Oscillations
15.7 Forced Oscillations

## ANSWERS TO QUESTIONS

**Q15.1** Neither are examples of simple harmonic motion, although they are both periodic motion. In neither case is the acceleration proportional to the position. Neither motion is so smooth as SHM. The ball's acceleration is very large when it is in contact with the floor, and the student's when the dismissal bell rings.

**Q15.2** You can take $\phi = \pi$, or equally well, $\phi = -\pi$. At $t = 0$, the particle is at its turning point on the negative side of equilibrium, at $x = -A$.

**Q15.3** The two will be equal if and only if the position of the particle at time zero is its equilibrium position, which we choose as the origin of coordinates.

**Q15.4** (a) In simple harmonic motion, one-half of the time, the velocity is in the same direction as the displacement away from equilibrium.

(b) Velocity and acceleration are in the same direction half the time.

(c) Acceleration is always opposite to the position vector, and never in the same direction.

**Q15.5** No. It is necessary to know both the position and velocity at time zero.

**Q15.6** The motion will still be simple harmonic motion, but the period of oscillation will be a bit larger. The effective mass of the system in $\omega = \left(\dfrac{k}{m_{eff}}\right)^{1/2}$ will need to include a certain fraction of the mass of the spring.

**Q15.7** We assume that the coils of the spring do not hit one another. The frequency will be higher than $f$ by the factor $\sqrt{2}$. When the spring with two blocks is set into oscillation in space, the coil in the center of the spring does not move. We can imagine clamping the center coil in place without affecting the motion. We can effectively duplicate the motion of each individual block in space by hanging a single block on a half-spring here on Earth. The half-spring with its center coil clamped—or its other half cut off—has twice the spring constant as the original uncut spring, because an applied force of the same size would produce only one-half the extension distance. Thus the oscillation frequency in space is $\left(\frac{1}{2\pi}\right)\left(\frac{2k}{m}\right)^{1/2} = \sqrt{2}f$. The absence of a force required to support the vibrating system in orbital free fall has no effect on the frequency of its vibration.

**Q15.8** No; Kinetic, Yes; Potential, No. For constant amplitude, the total energy $\frac{1}{2}kA^2$ stays constant. The kinetic energy $\frac{1}{2}mv^2$ would increase for larger mass if the speed were constant, but here the greater mass causes a decrease in frequency and in the average and maximum speed, so that the kinetic and potential energies at every point are unchanged.

**Q15.9** Since the acceleration is not constant in simple harmonic motion, none of the equations in Table 2.2 are valid.

| *Equation* | *Information given by equation* |
|---|---|
| $x(t) = A\cos(\omega t + \phi)$ | position as a function of time |
| $v(t) = -\omega A\sin(\omega t + \phi)$ | velocity as a function of time |
| $v(x) = \pm\omega\left(A^2 - x^2\right)^{1/2}$ | velocity as a function of position |
| $a(t) = -\omega^2 A\cos(\omega t + \phi)$ | acceleration as a function of time |
| $a(t) = -\omega^2 x(t)$ | acceleration as a function of position |

The angular frequency $\omega$ appears in every equation. It is a good idea to figure out the value of angular frequency early in the solution to a problem about vibration, and to store it in calculator memory.

**Q15.10** We have $T_i = \sqrt{\frac{L_i}{g}}$ and $T_f = \sqrt{\frac{L_f}{g}} = \sqrt{\frac{2L_i}{g}} = \sqrt{2}T_i$. The period gets larger by $\sqrt{2}$ times. Changing the mass has no effect on the period of a simple pendulum.

**Q15.11** (a) Period decreases. (b) Period increases. (c) No change.

**Q15.12** No, the equilibrium position of the pendulum will be shifted (angularly) towards the back of the car. The period of oscillation will increase slightly, since the restoring force (in the reference frame of the accelerating car) is reduced.

**Q15.13** The motion will be periodic—that is, it will repeat. The period is nearly constant as the angular amplitude increases through small values; then the period becomes noticeably larger as $\theta$ increases farther.

**Q15.14** Shorten the pendulum to decrease the period between ticks.

**Q15.15** No. If the resistive force is greater than the restoring force of the spring (in particular, if $b^2 > 4mk$), the system will be overdamped and will not oscillate.

**Q15.16** Yes. An oscillator with damping can vibrate at resonance with amplitude that remains constant in time. Without damping, the amplitude would increase without limit at resonance.

**Q15.17** The phase constant must be $\pi$ rad.

**Q15.18** Higher frequency. When it supports your weight, the center of the diving board flexes down less than the end does when it supports your weight. Thus the stiffness constant describing the center of the board is greater than the stiffness constant describing the end. And then $f = \left(\frac{1}{2\pi}\right)\sqrt{\frac{k}{m}}$ is greater for you bouncing on the center of the board.

**Q15.19** The release of air from one side of the parachute can make the parachute turn in the opposite direction, causing it to release air from the opposite side. This behavior will result in a periodic driving force that can set the parachute into side-to-side oscillation. If the amplitude becomes large enough, the parachute will not supply the needed air resistance to slow the fall of the unfortunate skydiver.

**Q15.20** An imperceptibly slight breeze may be blowing past the leaves in tiny puffs. As a leaf twists in the wind, the fibers in its stem provide a restoring torque. If the frequency of the breeze matches the natural frequency of vibration of one particular leaf as a torsional pendulum, that leaf can be driven into a large-amplitude resonance vibration. Note that it is not the *size* of the driving force that sets the leaf into resonance, but the *frequency* of the driving force. If the frequency changes, another leaf will be set into resonant oscillation.

**Q15.21** We assume the diameter of the bob is not very small compared to the length of the cord supporting it. As the water leaks out, the center of mass of the bob moves down, increasing the effective length of the pendulum and slightly lowering its frequency. As the last drops of water dribble out, the center of mass of the bob hops back up to the center of the sphere, and the pendulum frequency quickly increases to its original value.

## SOLUTIONS TO PROBLEMS

### Section 15.1 **Motion of an Object Attached to a Spring**

**P15.1** (a) Since the collision is perfectly elastic, the ball will rebound to the height of 4.00 m and then repeat the motion over and over again. Thus, the $\boxed{\text{motion is periodic}}$.

(b) To determine the period, we use: $x = \frac{1}{2}gt^2$.

The time for the ball to hit the ground is $t = \sqrt{\frac{2x}{g}} = \sqrt{\frac{2(4.00\text{ m})}{9.80\text{ m/s}^2}} = 0.909\text{ s}$

This equals one-half the period, so $T = 2(0.909\text{ s}) = \boxed{1.82\text{ s}}$.

(c) $\boxed{\text{No}}$. The net force acting on the ball is a constant given by $F = -mg$ (except when it is in contact with the ground), which is not in the form of Hooke's law.

## Section 15.2 **Mathematical Representation of Simple Harmonic Motion**

**P15.2** (a) $x=(5.00\text{ cm})\cos\left(2t+\dfrac{\pi}{6}\right)$ At $t=0$, $x=(5.00\text{ cm})\cos\left(\dfrac{\pi}{6}\right)=\boxed{4.33\text{ cm}}$

(b) $v=\dfrac{dx}{dt}=-(10.0\text{ cm/s})\sin\left(2t+\dfrac{\pi}{6}\right)$ At $t=0$, $v=\boxed{-5.00\text{ cm/s}}$

(c) $a=\dfrac{dv}{dt}=-\left(20.0\text{ cm/s}^2\right)\cos\left(2t+\dfrac{\pi}{6}\right)$ At $t=0$, $a=\boxed{-17.3\text{ cm/s}^2}$

(d) $A=\boxed{5.00\text{ cm}}$ and $T=\dfrac{2\pi}{\omega}=\dfrac{2\pi}{2}=\boxed{3.14\text{ s}}$

**P15.3** $x=(4.00\text{ m})\cos(3.00\pi t+\pi)$ Compare this with $x=A\cos(\omega t+\phi)$ to find

(a) $\omega=2\pi f=3.00\pi$

or $\boxed{f=1.50\text{ Hz}}$ $T=\dfrac{1}{f}=\boxed{0.667\text{ s}}$

(b) $A=\boxed{4.00\text{ m}}$

(c) $\phi=\boxed{\pi\text{ rad}}$

(d) $x(t=0.250\text{ s})=(4.00\text{ m})\cos(1.75\pi)=\boxed{2.83\text{ m}}$

***P15.4** (a) The spring constant of this spring is

$$k=\frac{F}{x}=\frac{0.45\text{ kg }9.8\text{ m/s}^2}{0.35\text{ m}}=12.6\text{ N/m}$$

we take the $x$-axis pointing downward, so $\phi=0$

$$x=A\cos\omega t=18.0\text{ cm}\cos\sqrt{\frac{12.6\text{ kg}}{0.45\text{ kg}\cdot\text{s}^2}}84.4\text{ s}=18.0\text{ cm}\cos 446.6\text{ rad}=\boxed{15.8\text{ cm}}$$

(d) Now $446.6\text{ rad}=71\times 2\pi+0.497\text{ rad}$. In each cycle the object moves $4(18)=72\text{ cm}$, so it has moved $71(72\text{ cm})+(18-15.8)\text{ cm}=\boxed{51.1\text{ m}}$.

(b) By the same steps, $k=\dfrac{0.44\text{ kg }9.8\text{ m/s}^2}{0.355\text{ m}}=12.1\text{ N/m}$

$$x=A\cos\sqrt{\frac{k}{m}}t=18.0\text{ cm}\cos\sqrt{\frac{12.1}{0.44}}84.4=18.0\text{ cm }\cos 443.5\text{ rad}=\boxed{-15.9\text{ cm}}$$

(e) $443.5\text{ rad}=70(2\pi)+3.62\text{ rad}$

$$\text{Distance moved }=70(72\text{ cm})+18+15.9\text{ cm}=\boxed{50.7\text{ m}}$$

(c) The answers to (d) and (e) are not very different given the difference in the data about the two vibrating systems. But when we ask about details of the future, the imprecision in our knowledge about the present makes it impossible to make precise predictions. The two oscillations start out in phase but get completely out of phase.

**P15.5** (a) At $t = 0$, $x = 0$ and $v$ is positive (to the right). Therefore, this situation corresponds to $x = A\sin\omega t$

and $v = v_i\cos\omega t$

Since $f = 1.50\text{ Hz}$, $\omega = 2\pi f = 3.00\pi$

Also, $A = 2.00\text{ cm}$, so that $\boxed{x = (2.00\text{ cm})\sin 3.00\pi t}$

(b) $v_{\max} = v_i = A\omega = 2.00(3.00\pi) = 6.00\pi\text{ cm/s} = \boxed{18.8\text{ cm/s}}$

The particle has this speed at $t = 0$ and next at $t = \frac{T}{2} = \boxed{\frac{1}{3}\text{ s}}$

(c) $a_{\max} = A\omega^2 = 2.00(3.00\pi)^2 = 18.0\pi^2\text{ cm/s}^2 = \boxed{178\text{ cm/s}^2}$

This positive value of acceleration first occurs at $t = \frac{3}{4}T = \boxed{0.500\text{ s}}$

(d) Since $T = \frac{2}{3}\text{ s}$ and $A = 2.00\text{ cm}$, the particle will travel 8.00 cm in this time.

Hence, in 1.00 s$\left(= \frac{3}{2}T\right)$, the particle will travel $8.00\text{ cm} + 4.00\text{ cm} = \boxed{12.0\text{ cm}}$.

**P15.6** The proposed solution $x(t) = x_i\cos\omega t + \left(\frac{v_i}{\omega}\right)\sin\omega t$

implies velocity $v = \frac{dx}{dt} = -x_i\omega\sin\omega t + v_i\cos\omega t$

and acceleration $a = \frac{dv}{dt} = -x_i\omega^2\cos\omega t - v_i\omega\sin\omega t = -\omega^2\left(x_i\cos\omega t + \left(\frac{v_i}{\omega}\right)\sin\omega t\right) = -\omega^2 x$

(a) The acceleration being a negative constant times position means we do have SHM, and its angular frequency is $\omega$. At $t = 0$ the equations reduce to $x = x_i$ and $v = v_i$ so they satisfy all the requirements.

(b)

$$v^2 - ax = \left(-x_i\omega\sin\omega t + v_i\cos\omega t\right)^2 - \left(-x_i\omega^2\cos\omega t - v_i\sin\omega t\right)\left(x_i\cos\omega t + \left(\frac{v_i}{\omega}\right)\sin\omega t\right)$$

$$v^2 - ax = x_i^2\omega^2\sin^2\omega t - 2x_iv_i\omega\sin\omega t\cos\omega t + v_i^2\cos^2\omega t$$

$$+x_i^2\omega^2\cos^2\omega t + x_iv_i\omega\cos\omega t\sin\omega t + x_iv_i\omega\sin\omega t\cos\omega t + v_i^2\sin^2\omega t = x_i^2\omega^2 + v_i^2$$

So this expression is constant in time. On one hand, it must keep its original value $v_i^2 - a_ix_i$. On the other hand, if we evaluate it at a turning point where $v = 0$ and $x = A$, it is $A^2\omega^2 + 0^2 = A^2\omega^2$. Thus it is proved.

**P15.7** (a) $T = \frac{12.0\text{ s}}{5} = \boxed{2.40\text{ s}}$

(b) $f = \frac{1}{T} = \frac{1}{2.40} = \boxed{0.417\text{ Hz}}$

(c) $\omega = 2\pi f = 2\pi(0.417) = \boxed{2.62\text{ rad/s}}$

***P15.8** The mass of the cube is

$$m=\rho V=\left(2.7\times10^{3}\ \mathrm{kg/m^{3}}\right)(0.015\ \mathrm{m})^{3}=9.11\times10^{-3}\ \mathrm{kg}$$

The spring constant of the strip of steel is

$$k=\frac{F}{x}=\frac{14.3\ \mathrm{N}}{0.027\,5\ \mathrm{m}}=52.0\ \mathrm{N/m}$$

$$f=\frac{\omega}{2\pi}=\frac{1}{2\pi}\sqrt{\frac{k}{m}}=\frac{1}{2\pi}\sqrt{\frac{52\ \mathrm{kg}}{\mathrm{s}^{2}9.11\times10^{-3}\ \mathrm{kg}}}=\boxed{12.0\ \mathrm{Hz}}$$

**P15.9** $f=\frac{\omega}{2\pi}=\frac{1}{2\pi}\sqrt{\frac{k}{m}}$ or $T=\frac{1}{f}=2\pi\sqrt{\frac{m}{k}}$

Solving for $k$, $k=\frac{4\pi^{2}m}{T^{2}}=\frac{4\pi^{2}(7.00\ \mathrm{kg})}{(2.60\ \mathrm{s})^{2}}=\boxed{40.9\ \mathrm{N/m}}$.

***P15.10** $x=A\cos\omega t$ $\quad A=0.05\ \mathrm{m}$ $\quad v=-A\omega\sin\omega t$ $\quad a=-A\omega^{2}\cos\omega t$

If $f=3\,600\ \mathrm{rev/min}=60\ \mathrm{Hz}$, then $\omega=120\pi\,\mathrm{s}^{-1}$

$v_{\max}=0.05(120\pi)\ \mathrm{m/s}=\boxed{18.8\ \mathrm{m/s}}$ $\qquad a_{\max}=0.05(120\pi)^{2}\ \mathrm{m/s^{2}}=\boxed{7.11\ \mathrm{km/s^{2}}}$

**P15.11** (a) $\omega=\sqrt{\frac{k}{m}}=\sqrt{\frac{8.00\ \mathrm{N/m}}{0.500\ \mathrm{kg}}}=4.00\ \mathrm{s}^{-1}$ so position is given by $x=10.0\sin(4.00t)\ \mathrm{cm}$.

From this we find that $v=40.0\cos(4.00t)\ \mathrm{cm/s}$ $\quad v_{\max}=\boxed{40.0\ \mathrm{cm/s}}$

$a=-160\sin(4.00t)\ \mathrm{cm/s^{2}}$ $\quad a_{\max}=\boxed{160\ \mathrm{cm/s^{2}}}$.

(b) $t=\left(\frac{1}{4.00}\right)\sin^{-1}\left(\frac{x}{10.0}\right)$ and when $x=6.00\ \mathrm{cm}$, $t=0.161\ \mathrm{s}$.

We find $v=40.0\cos\left[4.00(0.161)\right]=\boxed{32.0\ \mathrm{cm/s}}$

$a=-160\sin\left[4.00(0.161)\right]=\boxed{-96.0\ \mathrm{cm/s^{2}}}$.

(c) Using $t=\left(\frac{1}{4.00}\right)\sin^{-1}\left(\frac{x}{10.0}\right)$

when $x=0$, $t=0$ and when $x=8.00\ \mathrm{cm}$, $t=0.232\ \mathrm{s}$.

Therefore, $\Delta t=\boxed{0.232\ \mathrm{s}}$.

**P15.12** $m = 1.00\text{ kg}$, $k = 25.0\text{ N/m}$, and $A = 3.00\text{ cm}$. At $t = 0$, $x = -3.00\text{ cm}$

(a) $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{25.0}{1.00}} = 5.00\text{ rad/s}$

so that, $T = \frac{2\pi}{\omega} = \frac{2\pi}{5.00} = \boxed{1.26\text{ s}}$

(b) $v_{\max} = A\omega = 3.00 \times 10^{-2}\text{ m}(5.00\text{ rad/s}) = \boxed{0.150\text{ m/s}}$

$a_{\max} = A\omega^2 = 3.00 \times 10^{-2}\text{ m}(5.00\text{ rad/s})^2 = \boxed{0.750\text{ m/s}^2}$

(c) Because $x = -3.00\text{ cm}$ and $v = 0$ at $t = 0$, the required solution is $x = -A\cos\omega t$

or $\boxed{x = -3.00\cos(5.00t)\text{ cm}}$

$$v = \frac{dx}{dt} = \boxed{15.0\sin(5.00t)\text{ cm/s}}$$

$$a = \frac{dv}{dt} = \boxed{75.0\cos(5.00t)\text{ cm/s}^2}$$

**P15.13** The 0.500 s must elapse between one turning point and the other. Thus the period is 1.00 s.

$$\omega = \frac{2\pi}{T} = 6.28/\text{s}$$

and $v_{\max} = \omega A = (6.28/\text{s})(0.100\text{ m}) = \boxed{0.628\text{ m/s}}$.

**P15.14** (a) $v_{\max} = \omega A$

$$A = \frac{v_{\max}}{\omega} = \boxed{\frac{v}{\omega}}$$

(b) $x = -A\sin\omega t = \boxed{-\left(\frac{v}{\omega}\right)\sin\omega t}$

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Section 15.3 **Energy of the Simple Harmonic Oscillator**

**P15.15** (a) Energy is conserved for the block-spring system between the maximum-displacement and the half-maximum points:

$$(K+U)_i = (K+U)_f \qquad 0 + \frac{1}{2}kA^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$\frac{1}{2}(6.50\text{ N/m})(0.100\text{ m})^2 = \frac{1}{2}m(0.300\text{ m/s})^2 + \frac{1}{2}(6.50\text{ N/m})\left(5.00 \times 10^{-2}\text{ m}\right)^2$$

$$32.5\text{ mJ} = \frac{1}{2}m(0.300\text{ m/s})^2 + 8.12\text{ mJ} \qquad m = \frac{2(24.4\text{ mJ})}{9.0 \times 10^{-2}\text{ m}^2/\text{s}^2} = \boxed{0.542\text{ kg}}$$

(b) $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{6.50\text{ N/m}}{0.542\text{ kg}}} = 3.46\text{ rad/s}$ $\qquad \therefore T = \frac{2\pi}{\omega} = \frac{2\pi\text{ rad}}{3.46\text{ rad/s}} = \boxed{1.81\text{ s}}$

(c) $a_{\max} = A\omega^2 = 0.100\text{ m}(3.46\text{ rad/s})^2 = \boxed{1.20\text{ m/s}^2}$

**P15.16** $m = 200\ \text{g},\ T = 0.250\ \text{s},\ E = 2.00\ \text{J};\ \omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{0.250} = 25.1\ \text{rad/s}$

(a) $k = m\omega^2 = 0.200\ \text{kg}(25.1\ \text{rad/s})^2 = \boxed{126\ \text{N/m}}$

(b) $E = \dfrac{kA^2}{2} \Rightarrow A = \sqrt{\dfrac{2E}{k}} = \sqrt{\dfrac{2(2.00)}{126}} = \boxed{0.178\ \text{m}}$

**P15.17** Choose the car with its shock-absorbing bumper as the system; by conservation of energy,

$\dfrac{1}{2}mv^2 = \dfrac{1}{2}kx^2$: $\quad v = x\sqrt{\dfrac{k}{m}} = \left(3.16\times10^{-2}\ \text{m}\right)\sqrt{\dfrac{5.00\times10^6}{10^3}} = \boxed{2.23\ \text{m/s}}$

**P15.18** (a) $E = \dfrac{kA^2}{2} = \dfrac{250\ \text{N/m}\left(3.50\times10^{-2}\ \text{m}\right)^2}{2} = \boxed{0.153\ \text{J}}$

(b) $v_{\max} = A\omega$ where $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{250}{0.500}} = 22.4\ \text{s}^{-1}$ $\quad v_{\max} = \boxed{0.784\ \text{m/s}}$

(c) $a_{\max} = A\omega^2 = 3.50\times10^{-2}\ \text{m}\left(22.4\ \text{s}^{-1}\right)^2 = \boxed{17.5\ \text{m/s}^2}$

**P15.19** (a) $E = \dfrac{1}{2}kA^2 = \dfrac{1}{2}(35.0\ \text{N/m})\left(4.00\times10^{-2}\ \text{m}\right)^2 = \boxed{28.0\ \text{mJ}}$

(b) $|v| = \omega\sqrt{A^2 - x^2} = \sqrt{\dfrac{k}{m}}\sqrt{A^2 - x^2}$

$|v| = \sqrt{\dfrac{35.0}{50.0\times10^{-3}}}\sqrt{\left(4.00\times10^{-2}\right)^2 - \left(1.00\times10^{-2}\right)^2} = \boxed{1.02\ \text{m/s}}$

(c) $\dfrac{1}{2}mv^2 = \dfrac{1}{2}kA^2 - \dfrac{1}{2}kx^2 = \dfrac{1}{2}(35.0)\left[\left(4.00\times10^{-2}\right)^2 - \left(3.00\times10^{-2}\right)^2\right] = \boxed{12.2\ \text{mJ}}$

(d) $\dfrac{1}{2}kx^2 = E - \dfrac{1}{2}mv^2 = \boxed{15.8\ \text{mJ}}$

**P15.20** (a) $k = \dfrac{|F|}{x} = \dfrac{20.0\ \text{N}}{0.200\ \text{m}} = \boxed{100\ \text{N/m}}$

(b) $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{50.0}\ \text{rad/s}$ so $f = \dfrac{\omega}{2\pi} = \boxed{1.13\ \text{Hz}}$

(c) $v_{\max} = \omega A = \sqrt{50.0}(0.200) = \boxed{1.41\ \text{m/s}}$ at $x = 0$

(d) $a_{\max} = \omega^2 A = 50.0(0.200) = \boxed{10.0\ \text{m/s}^2}$ at $x = \pm A$

(e) $E = \dfrac{1}{2}kA^2 = \dfrac{1}{2}(100)(0.200)^2 = \boxed{2.00\ \text{J}}$

(f) $|v| = \omega\sqrt{A^2 - x^2} = \sqrt{50.0}\sqrt{\dfrac{8}{9}(0.200)^2} = \boxed{1.33\ \text{m/s}}$

(g) $|a| = \omega^2 x = 50.0\left(\dfrac{0.200}{3}\right) = \boxed{3.33\ \text{m/s}^2}$

**P15.21** (a) $E=\frac{1}{2}kA^2$, so if $A'=2A$, $E'=\frac{1}{2}k(A')^2=\frac{1}{2}k(2A)^2=4E$

Therefore $\boxed{E \text{ increases by factor of 4}}$.

(b) $v_{\max}=\sqrt{\frac{k}{m}}A$, so if $A$ is doubled, $\boxed{v_{\max} \text{ is doubled}}$.

(c) $a_{\max}=\frac{k}{m}A$, so if $A$ is doubled, $\boxed{a_{\max} \text{ also doubles}}$.

(d) $T=2\pi\sqrt{\frac{m}{k}}$ is independent of $A$, so $\boxed{\text{the period is unchanged}}$.

***P15.22** (a)

$$y_f=y_i+v_{yi}t+\frac{1}{2}a_yt^2$$
$$-11\text{ m}=0+0+\frac{1}{2}\left(-9.8\text{ m/s}^2\right)t^2$$
$$t=\sqrt{\frac{22\text{ m}\cdot\text{s}^2}{9.8\text{ m}}}=\boxed{1.50\text{ s}}$$

(b) Take the initial point where she steps off the bridge and the final point at the bottom of her motion.

$$\left(K+U_g+U_s\right)_i=\left(K+U_g+U_s\right)_f$$
$$0+mgy+0=0+0+\frac{1}{2}kx^2$$
$$65\text{ kg }9.8\text{ m/s}^2\ 36\text{ m}=\frac{1}{2}k(25\text{ m})^2$$
$$k=\boxed{73.4\text{ N/m}}$$

(c) The spring extension at equilibrium is $x=\frac{F}{k}=\frac{65\text{ kg }9.8\text{ m/s}^2}{73.4\text{ N/m}}=8.68\text{ m}$, so this point is $11+8.68\text{ m}=\boxed{19.7\text{ m below the bridge}}$ and the amplitude of her oscillation is $36-19.7=16.3\text{ m}$.

(d) $\omega=\sqrt{\frac{k}{m}}=\sqrt{\frac{73.4\text{ N/m}}{65\text{ kg}}}=\boxed{1.06\text{ rad/s}}$

(e) Take the phase as zero at maximum downward extension. We find what the phase was 25 m higher when $x=-8.68$ m:

In $x=A\cos\omega t$, $16.3\text{ m}=16.3\text{ m}\cos 0$

$-8.68\text{ m}=16.3\text{ m}\cos\left(1.06\ \frac{t}{\text{s}}\right)$ $1.06\frac{t}{\text{s}}=-122°=-2.13\text{ rad}$

$t=-2.01\text{ s}$

Then $\boxed{+2.01\text{ s}}$ is the time over which the spring stretches.

(f) total time $=1.50\text{ s}+2.01\text{ s}=\boxed{3.50\text{ s}}$

**P15.23** Model the oscillator as a block-spring system.

From energy considerations, $v^2+\omega^2x^2=\omega^2A^2$

$v_{\max}=\omega A$ and $v=\dfrac{\omega A}{2}$ so $\left(\dfrac{\omega A}{2}\right)^2+\omega^2x^2=\omega^2A^2$

From this we find $x^2=\dfrac{3}{4}A^2$ and $x=\dfrac{\sqrt{3}}{2}A=\boxed{\pm 2.60\text{ cm}}$ where $A=3.00\text{ cm}$

**P15.24** The potential energy is

$$U_s=\frac{1}{2}kx^2=\frac{1}{2}kA^2\cos^2(\omega t).$$

The rate of change of potential energy is

$$\frac{dU_s}{dt}=\frac{1}{2}kA^2\,2\cos(\omega t)\big[-\omega\sin(\omega t)\big]=-\frac{1}{2}kA^2\omega\sin 2\omega t.$$

(a) This rate of change is maximal and negative at

$$2\omega t=\frac{\pi}{2},\ 2\omega t=2\pi+\frac{\pi}{2},\text{ or in general, }2\omega t=2n\pi+\frac{\pi}{2}\text{ for integer }n.$$

Then, $t=\dfrac{\pi}{4\omega}(4n+1)=\dfrac{\pi(4n+1)}{4\left(3.60\text{ s}^{-1}\right)}$

For $n=0$, this gives $t=\boxed{0.218\text{ s}}$ while $n=1$ gives $t=\boxed{1.09\text{ s}}$.

All other values of $n$ yield times outside the specified range.

(b) $$\left|\frac{dU_s}{dt}\right|_{\max}=\frac{1}{2}kA^2\omega=\frac{1}{2}(3.24\text{ N/m})\left(5.00\times10^{-2}\text{ m}\right)^2\left(3.60\text{ s}^{-1}\right)=\boxed{14.6\text{ mW}}$$

## Section 15.4 **Comparing Simple Harmonic Motion with Uniform Circular Motion**

**P15.25** (a) The motion is simple harmonic because the tire is rotating with constant velocity and you are looking at the motion of the bump projected in a plane perpendicular to the tire.

(b) Since the car is moving with speed $v=3.00\text{ m/s}$, and its radius is 0.300 m, we have:

$$\omega=\frac{3.00\text{ m/s}}{0.300\text{ m}}=10.0\text{ rad/s}.$$

Therefore, the period of the motion is:

$$T=\frac{2\pi}{\omega}=\frac{2\pi}{(10.0\text{ rad/s})}=\boxed{0.628\text{ s}}.$$

**P15.26** The angle of the crank pin is $\theta = \omega t$. Its $x$-coordinate is

$$\boxed{x = A\cos\theta = A\cos\omega t}$$

where $A$ is the distance from the center of the wheel to the crank pin. This is of the form $x = A\cos(\omega t + \phi)$, so the yoke and piston rod move with simple harmonic motion.

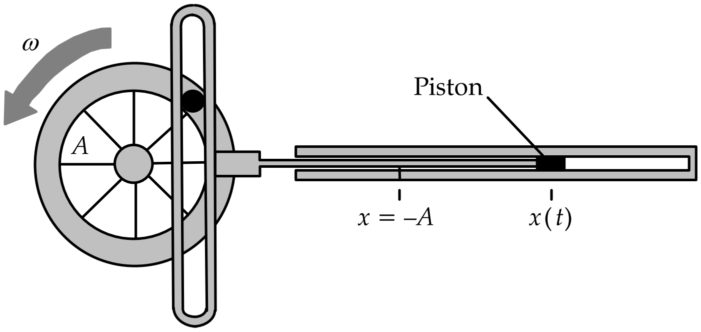


**FIG. P15.26**

## Section 15.5 The Pendulum

**P15.27** (a) $T = 2\pi\sqrt{\dfrac{L}{g}}$

$$L = \frac{gT^2}{4\pi^2} = \frac{(9.80\ \text{m/s}^2)(12.0\ \text{s})^2}{4\pi^2} = \boxed{35.7\ \text{m}}$$

(b) $T_{\text{moon}} = 2\pi\sqrt{\dfrac{L}{g_{\text{moon}}}} = 2\pi\sqrt{\dfrac{35.7\ \text{m}}{1.67\ \text{m/s}^2}} = \boxed{29.1\ \text{s}}$

**P15.28** The period in Tokyo is $T_T = 2\pi\sqrt{\dfrac{L_T}{g_T}}$

and the period in Cambridge is $T_C = 2\pi\sqrt{\dfrac{L_C}{g_C}}$

We know $T_T = T_C = 2.00\ \text{s}$

For which, we see $\dfrac{L_T}{g_T} = \dfrac{L_C}{g_C}$

or $\dfrac{g_C}{g_T} = \dfrac{L_C}{L_T} = \dfrac{0.994\,2}{0.992\,7} = \boxed{1.001\,5}$

**P15.29** The swinging box is a physical pendulum with period $T = 2\pi\sqrt{\dfrac{I}{mgd}}$.

The moment of inertia is given approximately by

$$I = \frac{1}{3}mL^2 \text{ (treating the box as a rod suspended from one end).}$$

Then, with $L \approx 1.0\ \text{m}$ and $d \approx \dfrac{L}{2}$,

$$T \approx 2\pi\sqrt{\frac{\frac{1}{3}mL^2}{mg\left(\frac{L}{2}\right)}} = 2\pi\sqrt{\frac{2L}{3g}} = 2\pi\sqrt{\frac{2(1.0\ \text{m})}{3(9.8\ \text{m/s}^2)}} = 1.6\ \text{s or } T \sim \boxed{10^0\ \text{s}}.$$

**P15.30** $\omega = \frac{2\pi}{T}$: $\quad T = \frac{2\pi}{\omega} = \frac{2\pi}{4.43} = \boxed{1.42 \text{ s}}$

$\omega = \sqrt{\frac{g}{L}}$: $\quad L = \frac{g}{\omega^2} = \frac{9.80}{(4.43)^2} = \boxed{0.499 \text{ m}}$

**P15.31** Using the simple harmonic motion model:

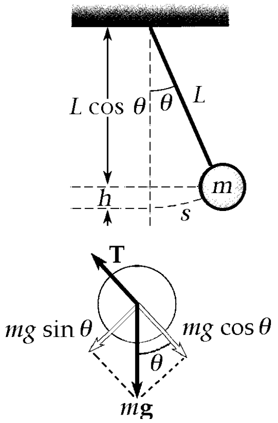


**FIG. P15.31**

$$A = r\theta = 1 \text{ m } 15° \frac{\pi}{180°} = 0.262 \text{ m}$$

$$\omega = \sqrt{\frac{g}{L}} = \sqrt{\frac{9.8 \text{ m/s}^2}{1 \text{ m}}} = 3.13 \text{ rad/s}$$

(a) $v_{\text{max}} = A\omega = 0.262 \text{ m } 3.13/\text{s} = \boxed{0.820 \text{ m/s}}$

(b) $a_{\text{max}} = A\omega^2 = 0.262 \text{ m}(3.13/\text{s})^2 = 2.57 \text{ m/s}^2$

$a_{\tan} = r\alpha \qquad \alpha = \frac{a_{\tan}}{r} = \frac{2.57 \text{ m/s}^2}{1 \text{ m}} = \boxed{2.57 \text{ rad/s}^2}$

(c) $F = ma = 0.25 \text{ kg } 2.57 \text{ m/s}^2 = \boxed{0.641 \text{ N}}$

More precisely,

(a) $mgh = \frac{1}{2}mv^2$ and $h = L(1-\cos\theta)$

$\therefore v_{\text{max}} = \sqrt{2gL(1-\cos\theta)} = \boxed{0.817 \text{ m/s}}$

(b) $I\alpha = mgL\sin\theta$

$\alpha_{\text{max}} = \frac{mgL\sin\theta}{mL^2} = \frac{g}{L}\sin\theta_i = \boxed{2.54 \text{ rad/s}^2}$

(c) $F_{\text{max}} = mg\sin\theta_i = 0.250(9.80)(\sin 15.0°) = \boxed{0.634 \text{ N}}$

**P15.32** (a) The string tension must support the weight of the bob, accelerate it upward, and also provide the restoring force, just as if the elevator were at rest in a gravity field $(9.80+5.00) \text{ m/s}^2$

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{5.00 \text{ m}}{14.8 \text{ m/s}^2}}$$

$$T = \boxed{3.65 \text{ s}}$$

(b) $T = 2\pi\sqrt{\frac{5.00 \text{ m}}{\left(9.80 \text{ m/s}^2 - 5.00 \text{ m/s}^2\right)}} = \boxed{6.41 \text{ s}}$

(c) $g_{eff} = \sqrt{\left(9.80 \text{ m/s}^2\right)^2 + \left(5.00 \text{ m/s}^2\right)^2} = 11.0 \text{ m/s}^2$

$T = 2\pi = \sqrt{\frac{5.00 \text{ m}}{11.0 \text{ m/s}^2}} = \boxed{4.24 \text{ s}}$

**P15.33** Referring to the sketch we have

$F = -mg\sin\theta$ and $\tan\theta = \dfrac{x}{R}$

For small displacements, $\tan\theta \approx \sin\theta$

and $F = -\dfrac{mg}{R}x = -kx$

Since the restoring force is proportional to the displacement from equilibrium, the motion is simple harmonic motion.

Comparing to $F = -m\omega^2 x$ shows $\boxed{\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{g}{R}}}$.

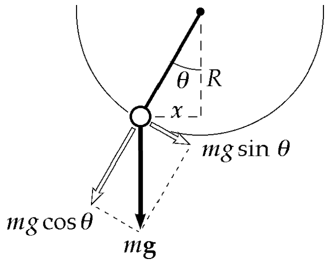


FIG. P15.33

**P15.34** (a) $T = \dfrac{\text{total measured time}}{50}$

The measured periods are:

| Length, $L$ (m) | 1.000 | 0.750 | 0.500 |
|---|---|---|---|
| Period, $T$ (s) | 1.996 | 1.732 | 1.422 |

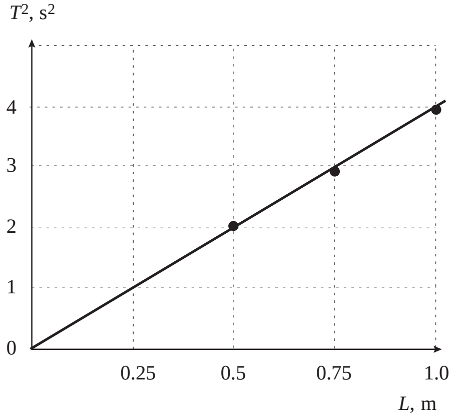


FIG. P15.34

(b) $T = 2\pi\sqrt{\dfrac{L}{g}}$ so $g = \dfrac{4\pi^2 L}{T^2}$

The calculated values for $g$ are:

| Period, $T$ (s) | 1.996 | 1.732 | 1.422 |
|---|---|---|---|
| $g\left(\text{m/s}^2\right)$ | 9.91 | 9.87 | 9.76 |

Thus, $g_{\text{ave}} = \boxed{9.85\ \text{m/s}^2}$ this agrees with the accepted value of $g = 9.80\ \text{m/s}^2$ within 0.5%.

(c) From $T^2 = \left(\dfrac{4\pi^2}{g}\right)L$, the slope of $T^2$ versus $L$ graph $= \dfrac{4\pi^2}{g} = 4.01\ \text{s}^2/\text{m}$.

Thus, $g = \dfrac{4\pi^2}{\text{slope}} = \boxed{9.85\ \text{m/s}^2}$. This is the same as the value in (b).

**P15.35** $f = 0.450\ \text{Hz}$, $d = 0.350\ \text{m}$, and $m = 2.20\ \text{kg}$

$$T = \frac{1}{f};$$

$$T = 2\pi\sqrt{\frac{I}{mgd}}; \quad T^2 = \frac{4\pi^2 I}{mgd}$$

$$I = T^2\frac{mgd}{4\pi^2} = \left(\frac{1}{f}\right)^2\frac{mgd}{4\pi^2} = \frac{2.20(9.80)(0.350)}{4\pi^2\left(0.450\ \text{s}^{-1}\right)^2} = \boxed{0.944\ \text{kg}\cdot\text{m}^2}$$

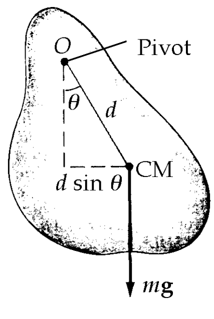


FIG. P15.35

**P15.36** (a) The parallel-axis theorem:

$$I = I_{\text{CM}} + Md^2 = \frac{1}{12}ML^2 + Md^2 = \frac{1}{12}M(1.00\text{ m})^2 + M(1.00\text{ m})^2$$

$$= M\left(\frac{13}{12}\text{ m}^2\right)$$

$$T = 2\pi\sqrt{\frac{I}{Mgd}} = 2\pi\sqrt{\frac{M\left(13\text{ m}^2\right)}{12Mg(1.00\text{ m})}} = 2\pi\sqrt{\frac{13\text{ m}}{12\left(9.80\text{ m/s}^2\right)}} = \boxed{2.09\text{ s}}$$

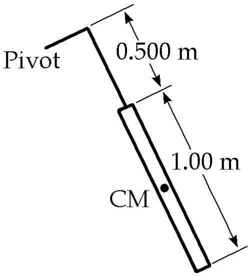


**FIG. P15.36**

(b) For the simple pendulum

$$T = 2\pi\sqrt{\frac{1.00\text{ m}}{9.80\text{ m/s}^2}} = 2.01\text{ s} \qquad \text{difference} = \frac{2.09\text{ s} - 2.01\text{ s}}{2.01\text{ s}} = \boxed{4.08\%}$$

**P15.37** (a) The parallel axis theorem says directly $I = I_{\text{CM}} + md^2$

$$\text{so} \qquad T = 2\pi\sqrt{\frac{I}{mgd}} = \boxed{2\pi\sqrt{\frac{\left(I_{\text{CM}} + md^2\right)}{mgd}}}$$

(b) When $d$ is very large $T \to 2\pi\sqrt{\frac{d}{g}}$ gets large.

When $d$ is very small $T \to 2\pi\sqrt{\frac{I_{\text{CM}}}{mgd}}$ gets large.

So there must be a minimum, found by

$$\frac{dT}{dd} = 0 = \frac{d}{dd}2\pi\left(I_{\text{CM}} + md^2\right)^{1/2}(mgd)^{-1/2}$$

$$= 2\pi\left(I_{\text{CM}} + md^2\right)^{1/2}\left(-\frac{1}{2}\right)(mgd)^{-3/2}mg + 2\pi(mgd)^{-1/2}\left(\frac{1}{2}\right)\left(I_{\text{CM}} + md^2\right)^{-1/2}2md$$

$$= \frac{-\pi\left(I_{\text{CM}} + md^2\right)mg}{\left(I_{\text{CM}} + md^2\right)^{1/2}(mgd)^{3/2}} + \frac{2\pi\, md\, mgd}{\left(I_{\text{CM}} + md^2\right)^{1/2}(mgd)^{3/2}} = 0$$

This requires

$$-I_{\text{CM}} - md^2 + 2md^2 = 0$$

or $\boxed{I_{\text{CM}} = md^2}$.

**P15.38** We suppose the stick moves in a horizontal plane. Then,

$$I = \frac{1}{12}mL^2 = \frac{1}{12}(2.00\text{ kg})(1.00\text{ m})^2 = 0.167\text{ kg}\cdot\text{m}^2$$

$$T = 2\pi\sqrt{\frac{I}{\kappa}}$$

$$\kappa = \frac{4\pi^2 I}{T^2} = \frac{4\pi^2\left(0.167\text{ kg}\cdot\text{m}^2\right)}{(180\text{ s})^2} = \boxed{203\ \mu\text{N}\cdot\text{m}}$$

**P15.39** $T = 0.250\text{ s}$, $I = mr^2 = \left(20.0\times10^{-3}\text{ kg}\right)\left(5.00\times10^{-3}\text{ m}\right)^2$

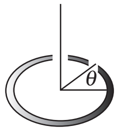


**FIG. P15.39**

(a) $I = \boxed{5.00\times10^{-7}\text{ kg}\cdot\text{m}^2}$

(b) $I\dfrac{d^2\theta}{dt^2} = -\kappa\theta;\ \sqrt{\dfrac{\kappa}{I}} = \omega = \dfrac{2\pi}{T}$

$$\kappa = I\omega^2 = \left(5.00\times10^{-7}\right)\left(\frac{2\pi}{0.250}\right)^2 = \boxed{3.16\times10^{-4}\ \frac{\text{N}\cdot\text{m}}{\text{rad}}}$$

## Section 15.6 **Damped Oscillations**

**P15.40** The total energy is $E = \dfrac{1}{2}mv^2 + \dfrac{1}{2}kx^2$

Taking the time-derivative, $\dfrac{dE}{dt} = mv\dfrac{d^2x}{dt^2} + kxv$

Use Equation 15.31: $\dfrac{md^2x}{dt^2} = -kx - bv$

$$\frac{dE}{dt} = v(-kx - bv) + kvx$$

Thus, $\boxed{\dfrac{dE}{dt} = -bv^2 < 0}$

**P15.41** $\theta_i = 15.0°$ $\qquad \theta(t = 1\,000) = 5.50°$

$x = Ae^{-bt/2m}$ $\qquad \dfrac{x_{1\,000}}{x_i} = \dfrac{Ae^{-bt/2m}}{A} = \dfrac{5.50}{15.0} = e^{-b(1\,000)/2m}$

$$\ln\left(\frac{5.50}{15.0}\right) = -1.00 = \frac{-b(1\,000)}{2m}$$

$$\therefore \frac{b}{2m} = \boxed{1.00\times10^{-3}\text{ s}^{-1}}$$

**P15.42** Show that $x = Ae^{-bt/2m}\cos(\omega t + \phi)$

is a solution of $-kx - b\dfrac{dx}{dt} = m\dfrac{d^2x}{dt^2}$ (1)

where $\omega = \sqrt{\dfrac{k}{m} - \left(\dfrac{b}{2m}\right)^2}$. (2)

$$x = Ae^{-bt/2m}\cos(\omega t + \phi) \tag{3}$$

$$\frac{dx}{dt} = Ae^{-bt/2m}\left(-\frac{b}{2m}\right)\cos(\omega t + \phi) - Ae^{-bt/2m}\omega\sin(\omega t + \phi) \tag{4}$$

$$\frac{d^2x}{dt^2} = -\frac{b}{2m}\left[Ae^{-bt/2m}\left(-\frac{b}{2m}\right)\cos(\omega t + \phi) - Ae^{-bt/2m}\omega\sin(\omega t + \phi)\right] - \left[Ae^{-bt/2m}\left(-\frac{b}{2m}\right)\omega\sin(\omega t + \phi) + Ae^{-bt/2m}\omega^2\cos(\omega t + \phi)\right] \tag{5}$$

*continued on next page*

Substitute (3), (4) into the left side of (1) and (5) into the right side of (1);

$$-kAe^{-bt/2m}\cos(\omega t+\phi)+\frac{b^2}{2m}Ae^{-bt/2m}\cos(\omega t+\phi)+b\omega Ae^{-bt/2m}\sin(\omega t+\phi)$$
$$=-\frac{b}{2}\left[Ae^{-bt/2m}\left(-\frac{b}{2m}\right)\cos(\omega t+\phi)-Ae^{-bt/2m}\omega\sin(\omega t+\phi)\right]$$
$$+\frac{b}{2}Ae^{-bt/2m}\omega\sin(\omega t+\phi)-m\omega^2Ae^{-bt/2m}\cos(\omega t+\phi)$$

Compare the coefficients of $Ae^{-bt/2m}\cos(\omega t+\phi)$ and $Ae^{-bt/2m}\sin(\omega t+\phi)$:

cosine-term: $-k+\frac{b^2}{2m}=-\frac{b}{2}\left(-\frac{b}{2m}\right)-m\omega^2=\frac{b^2}{4m}-m\left(\frac{k}{m}-\frac{b^2}{4m^2}\right)=-k+\frac{b^2}{2m}$

sine-term: $b\omega=+\frac{b}{2}(\omega)+\frac{b}{2}(\omega)=b\omega$

Since the coefficients are equal, $x=Ae^{-bt/2m}\cos(\omega t+\phi)$ is a solution of the equation.

***P15.43** The frequency if undamped would be $\omega_0=\sqrt{\frac{k}{m}}=\sqrt{\frac{2.05\times10^4\ \text{N/m}}{10.6\ \text{kg}}}=44.0/\text{s}$.

(a) With damping

$$\omega=\sqrt{\omega_0^2-\left(\frac{b}{2m}\right)^2}=\sqrt{\left(44\ \frac{1}{\text{s}}\right)^2-\left(\frac{3\ \text{kg}}{\text{s}\ 2\ 10.6\ \text{kg}}\right)^2}$$
$$=\sqrt{1\,933.96-0.02}=44.0\ \frac{1}{\text{s}}$$
$$f=\frac{\omega}{2\pi}=\frac{44.0}{2\pi\,\text{s}}=\boxed{7.00\ \text{Hz}}$$

(b) In $x=A_0e^{-bt/2m}\cos(\omega t+\phi)$ over one cycle, a time $T=\frac{2\pi}{\omega}$, the amplitude changes from $A_0$ to $A_0e^{-b2\pi/2m\omega}$ for a fractional decrease of

$$\frac{A_0-A_0e^{-\pi b/m\omega}}{A_0}=1-e^{-\pi3/(10.6\cdot44.0)}=1-e^{-0.020\,2}=1-0.979\,98=0.020\,0=\boxed{2.00\%}.$$

(c) The energy is proportional to the square of the amplitude, so its fractional rate of decrease is twice as fast:

$$E=\frac{1}{2}kA^2=\frac{1}{2}kA_0^2e^{-2bt/2m}=E_0e^{-bt/m}.$$

We specify

$$0.05E_0=E_0e^{-3t/10.6}$$
$$0.05=e^{-3t/10.6}$$
$$e^{+3t/10.6}=20$$
$$\frac{3t}{10.6}=\ln20=3.00$$
$$t=\boxed{10.6\ \text{s}}$$

## Section 15.7 **Forced Oscillations**

**P15.44** (a) For resonance, her frequency must match

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{4.30\times 10^3 \text{ N/m}}{12.5 \text{ kg}}} = \boxed{2.95 \text{ Hz}}.$$

(b) From $x = A\cos\omega t$, $v = \frac{dx}{dt} = -A\omega\sin\omega t$, and $a = \frac{dv}{dt} = -A\omega^2\cos\omega t$, the maximum acceleration is $A\omega^2$. When this becomes equal to the acceleration due to gravity, the normal force exerted on her by the mattress will drop to zero at one point in the cycle:

$$A\omega^2 = g \qquad \text{or} \qquad A = \frac{g}{\omega^2} = \frac{g}{\frac{k}{m}} = \frac{gm}{k} \qquad A = \frac{\left(9.80 \text{ m/s}^2\right)(12.5 \text{ kg})}{4.30\times 10^3 \text{ N/m}} = \boxed{2.85 \text{ cm}}$$

**P15.45** $F = 3.00\cos(2\pi t)$ N and $k = 20.0$ N/m

(a) $\omega = \frac{2\pi}{T} = 2\pi$ rad/s so $T = \boxed{1.00 \text{ s}}$

(b) In this case, $\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{20.0}{2.00}} = 3.16$ rad/s

The equation for the amplitude of a driven oscillator,

with $b = 0$, gives $A = \left(\frac{F_0}{m}\right)\left(\omega^2 - \omega_0^2\right)^{-1} = \frac{3}{2}\left[4\pi^2 - (3.16)^2\right]^{-1}$

Thus $A = 0.050\,9 \text{ m} = \boxed{5.09 \text{ cm}}$.

**P15.46** $F_0\cos\omega t - kx = m\frac{d^2x}{dt^2} \qquad \omega_0 = \sqrt{\frac{k}{m}}$ (1)

$x = A\cos(\omega t + \phi)$ (2)

$\frac{dx}{dt} = -A\omega\sin(\omega t + \phi)$ (3)

$\frac{d^2x}{dt^2} = -A\omega^2\cos(\omega t + \phi)$ (4)

Substitute (2) and (4) into (1): $F_0\cos\omega t - kA\cos(\omega t + \phi) = m\left(-A\omega^2\right)\cos(\omega t + \phi)$

Solve for the amplitude: $\left(kA - mA\omega^2\right)\cos(\omega t + \phi) = F_0\cos\omega t$

These will be equal, provided only that $\phi$ must be zero and $kA - mA\omega^2 = F_0$

Thus, $A = \frac{\frac{F_0}{m}}{\left(\frac{k}{m}\right) - \omega^2}$

**P15.47** From the equation for the amplitude of a driven oscillator with no damping,

$$A=\frac{F_0/m}{\sqrt{\left(\omega^2-\omega_0^2\right)^2}}$$

$$\omega=2\pi f=\left(20.0\pi\,\text{s}^{-1}\right) \qquad \omega_0^2=\frac{k}{m}=\frac{200}{\left(\frac{40.0}{9.80}\right)}=49.0\ \text{s}^{-2}$$

$$F_0=mA\left(\omega^2-\omega_0^2\right)$$

$$F_0=\left(\frac{40.0}{9.80}\right)\left(2.00\times10^{-2}\right)(3\,950-49.0)=\boxed{318\ \text{N}}$$

**P15.48** $A=\dfrac{F_{\text{ext}}/m}{\sqrt{\left(\omega^2-\omega_0^2\right)^2+(b\omega/m)^2}}$

With $b=0$, $\quad A=\dfrac{F_{\text{ext}}/m}{\sqrt{\left(\omega^2-\omega_0^2\right)^2}}=\dfrac{F_{\text{ext}}/m}{\pm\left(\omega^2-\omega_0^2\right)}=\pm\dfrac{F_{\text{ext}}/m}{\omega^2-\omega_0^2}$

Thus, $\quad \omega^2=\omega_0^2\pm\dfrac{F_{\text{ext}}/m}{A}=\dfrac{k}{m}\pm\dfrac{F_{\text{ext}}}{mA}=\dfrac{6.30\ \text{N/m}}{0.150\ \text{kg}}\pm\dfrac{1.70\ \text{N}}{(0.150\ \text{kg})(0.440\ \text{m})}$

This yields $\quad \omega=8.23\ \text{rad/s}$ or $\omega=4.03\ \text{rad/s}$

Then, $\quad f=\dfrac{\omega}{2\pi}$ gives either $f=\boxed{1.31\ \text{Hz}}$ or $f=\boxed{0.641\ \text{Hz}}$

**P15.49** The beeper must resonate at the frequency of a simple pendulum of length 8.21 cm:

$$f=\frac{1}{2\pi}\sqrt{\frac{g}{L}}=\frac{1}{2\pi}\sqrt{\frac{9.80\ \text{m/s}^2}{0.082\,1\ \text{m}}}=\boxed{1.74\ \text{Hz}}.$$

***P15.50** For the resonance vibration with the occupants in the car, we have for the spring constant of the suspension

$$f=\frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad k=4\pi^2f^2m=4\pi^2\left(1.8\ \text{s}^{-1}\right)^2\left(1\,130\ \text{kg}+4(72.4\ \text{kg})\right)=1.82\times10^5\ \text{kg/s}^2$$

Now as the occupants exit $\quad x=\dfrac{F}{k}=\dfrac{4(72.4\ \text{kg})\left(9.8\ \text{m/s}^2\right)}{1.82\times10^5\ \text{kg/s}^2}=\boxed{1.56\times10^{-2}\ \text{m}}$

## Additional Problems

**P15.51** Let $F$ represent the tension in the rod.

(a) At the pivot, $F = Mg + Mg = \boxed{2Mg}$

A fraction of the rod's weight $Mg\left(\frac{y}{L}\right)$ as well as the weight of the ball pulls down on point $P$. Thus, the tension in the rod at point $P$ is

$$F = Mg\left(\frac{y}{L}\right) + Mg = \boxed{Mg\left(1+\frac{y}{L}\right)}.$$

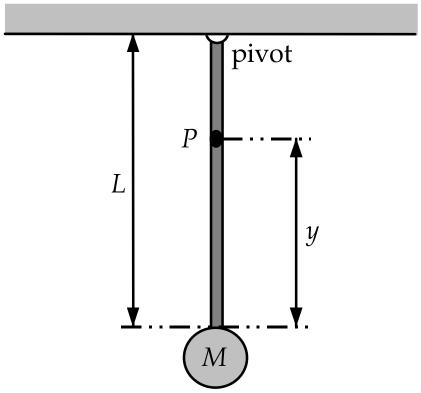


**FIG. P15.51**

(b) Relative to the pivot, $I = I_{\text{rod}} + I_{\text{ball}} = \frac{1}{3}ML^2 + ML^2 = \frac{4}{3}ML^2$

For the physical pendulum, $T = 2\pi\sqrt{\frac{I}{mgd}}$ where $m = 2M$ and $d$ is the distance from the pivot to the center of mass of the rod and ball combination. Therefore,

$$d = \frac{M\left(\frac{L}{2}\right) + ML}{M+M} = \frac{3L}{4} \text{ and } T = 2\pi\sqrt{\frac{\frac{4}{3}ML^2}{(2M)g\left(\frac{3L}{4}\right)}} = \boxed{\frac{4\pi}{3}\sqrt{\frac{2L}{g}}}.$$

For $L = 2.00\text{ m}$, $T = \frac{4\pi}{3}\sqrt{\frac{2(2.00\text{ m})}{9.80\text{ m/s}^2}} = \boxed{2.68\text{ s}}$.

**P15.52** (a) Total energy $= \frac{1}{2}kA^2 = \frac{1}{2}(100\text{ N/m})(0.200\text{ m})^2 = 2.00\text{ J}$

At equilibrium, the total energy is:

$$\frac{1}{2}(m_1+m_2)v^2 = \frac{1}{2}(16.0\text{ kg})v^2 = (8.00\text{ kg})v^2.$$

Therefore,

$$(8.00\text{ kg})v^2 = 2.00\text{ J}, \text{ and } v = \boxed{0.500\text{ m/s}}.$$

This is the speed of $m_1$ and $m_2$ at the equilibrium point. Beyond this point, the mass $m_2$ moves with the constant speed of 0.500 m/s while mass $m_1$ starts to slow down due to the restoring force of the spring.

*continued on next page*

(b) The energy of the $m_1$-spring system at equilibrium is:

$$\frac{1}{2}m_1 v^2 = \frac{1}{2}(9.00\text{ kg})(0.500\text{ m/s})^2 = 1.125\text{ J}.$$

This is also equal to $\frac{1}{2}k(A')^2$, where $A'$ is the amplitude of the $m_1$-spring system. Therefore,

$$\frac{1}{2}(100)(A')^2 = 1.125 \text{ or } A' = 0.150\text{ m}.$$

The period of the $m_1$-spring system is $T = 2\pi\sqrt{\frac{m_1}{k}} = 1.885\text{ s}$

and it takes $\frac{1}{4}T = 0.471\text{ s}$ after it passes the equilibrium point for the spring to become fully stretched the first time. The distance separating $m_1$ and $m_2$ at this time is:

$$D = v\left(\frac{T}{4}\right) - A' = 0.500\text{ m/s}(0.471\text{ s}) - 0.150\text{ m} = 0.0856 = \boxed{8.56\text{ cm}}.$$

**P15.53** $\left(\frac{d^2x}{dt^2}\right)_{\max} = A\omega^2$

$f_{\max} = \mu_s n = \mu_s mg = mA\omega^2$

$A = \frac{\mu_s g}{\omega^2} = \boxed{6.62\text{ cm}}$

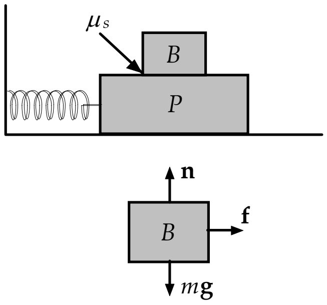


**FIG. P15.53**

**P15.54** The maximum acceleration of the oscillating system is $a_{\max} = A\omega^2 = 4\pi^2 Af^2$. The friction force exerted between the two blocks must be capable of accelerating block B at this rate. Thus, if Block *B* is about to slip,

$$f = f_{\max} = \mu_s n = \mu_s mg = m\left(4\pi^2 Af^2\right) \qquad \text{or} \qquad A = \boxed{\frac{\mu_s g}{4\pi^2 f^2}}.$$

**P15.55** Deuterium is the isotope of the element hydrogen with atoms having nuclei consisting of one proton and one neutron. For brevity we refer to the molecule formed by two deuterium atoms as *D* and to the diatomic molecule of hydrogen-1 as *H*.

$$M_D = 2M_H \qquad \frac{\omega_D}{\omega_H} = \frac{\sqrt{\frac{k}{M_D}}}{\sqrt{\frac{k}{M_H}}} = \sqrt{\frac{M_H}{M_D}} = \sqrt{\frac{1}{2}} \qquad f_D = \frac{f_H}{\sqrt{2}} = \boxed{0.919 \times 10^{14}\text{ Hz}}$$

**P15.56** The kinetic energy of the ball is $K=\frac{1}{2}mv^2+\frac{1}{2}I\Omega^2$, where $\Omega$ is the rotation rate of the ball about its center of mass. Since the center of the ball moves along a circle of radius $4R$, its displacement from equilibrium is $s=(4R)\theta$ and its speed is $v=\frac{ds}{dt}=4R\left(\frac{d\theta}{dt}\right)$. Also, since the ball rolls without slipping,

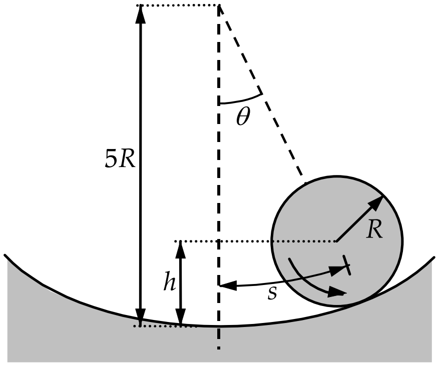


**FIG. P15.56**

$$v=\frac{ds}{dt}=R\Omega \qquad \text{so} \qquad \Omega=\frac{v}{R}=4\left(\frac{d\theta}{dt}\right)$$

The kinetic energy is then

$$\begin{aligned}K&=\frac{1}{2}m\left(4R\frac{d\theta}{dt}\right)^2+\frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(4\frac{d\theta}{dt}\right)^2\\&=\frac{112mR^2}{10}\left(\frac{d\theta}{dt}\right)^2\end{aligned}$$

When the ball has an angular displacement $\theta$, its center is distance $h=4R(1-\cos\theta)$ higher than when at the equilibrium position. Thus, the potential energy is $U_g=mgh=4mgR(1-\cos\theta)$. For small angles, $(1-\cos\theta)\approx\frac{\theta^2}{2}$ (see Appendix B). Hence, $U_g\approx 2mgR\theta^2$, and the total energy is

$$E=K+U_g=\frac{112mR^2}{10}\left(\frac{d\theta}{dt}\right)^2+2mgR\theta^2.$$

Since $E=$ constant in time, $\frac{dE}{dt}=0=\frac{112mR^2}{5}\left(\frac{d\theta}{dt}\right)\frac{d^2\theta}{dt^2}+4mgR\theta\left(\frac{d\theta}{dt}\right)$.

This reduces to $\frac{28R}{5}\frac{d^2\theta}{dt^2}+g\theta=0$, or $\frac{d^2\theta}{dt^2}=-\left(\frac{5g}{28R}\right)\theta$.

With the angular acceleration equal to a negative constant times the angular position, this is in the defining form of a simple harmonic motion equation with $\omega=\sqrt{\frac{5g}{28R}}$.

The period of the simple harmonic motion is then $T=\frac{2\pi}{\omega}=\boxed{2\pi\sqrt{\frac{28R}{5g}}}$.

**P15.57** (a)

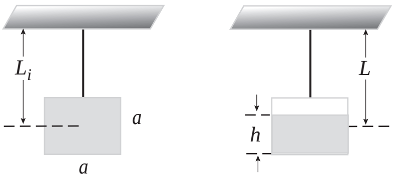


**FIG. P15.57(a)**

(b) $T = 2\pi\sqrt{\frac{L}{g}}$ $$\frac{dT}{dt} = \frac{\pi}{\sqrt{g}}\frac{1}{\sqrt{L}}\frac{dL}{dt} \qquad (1)$$

We need to find $L(t)$ and $\frac{dL}{dt}$. From the diagram in (a),

$$L = L_i + \frac{a}{2} - \frac{h}{2};\ \frac{dL}{dt} = -\left(\frac{1}{2}\right)\frac{dh}{dt}.$$

But $\frac{dM}{dt} = \rho\frac{dV}{dt} = -\rho A\frac{dh}{dt}$. Therefore,

$$\frac{dh}{dt} = -\frac{1}{\rho A}\frac{dM}{dt};\ \frac{dL}{dt} = \left(\frac{1}{2\rho A}\right)\frac{dM}{dt} \qquad (2)$$

Also, $$\int_{L_i}^{L} dL = \left(\frac{1}{2\rho A}\right)\left(\frac{dM}{dt}\right)t = L - L_i \qquad (3)$$

Substituting Equation (2) and Equation (3) into Equation (1):

$$\frac{dT}{dt} = \boxed{\frac{\pi}{\sqrt{g}}\left(\frac{1}{2\rho a^2}\right)\left(\frac{dM}{dt}\right)\frac{1}{\sqrt{L_i + \frac{1}{2\rho a^2}\left(\frac{dM}{dt}\right)t}}}.$$

(c) Substitute Equation (3) into the equation for the period.

$$T = \boxed{\frac{2\pi}{\sqrt{g}}\sqrt{L_i + \frac{1}{2\rho a^2}\left(\frac{dM}{dt}\right)t}}$$

Or one can obtain $T$ by integrating (b):

$$\int_{T}^{T_i} dT = \frac{\pi}{\sqrt{g}}\left(\frac{1}{2\rho a^2}\right)\left(\frac{dM}{dt}\right)\int_0^t \frac{dt}{\sqrt{L_i + \frac{1}{2\rho a^2}\left(\frac{dM}{dt}\right)t}}$$

$$T - T_i = \frac{\pi}{\sqrt{g}}\left(\frac{1}{2\rho a^2}\right)\left(\frac{dM}{dt}\right)\left[\frac{2}{\frac{1}{2\rho a^2}\left(\frac{dM}{dt}\right)}\right]\left[\sqrt{L_i + \frac{1}{2\rho a^2}\left(\frac{dM}{dt}\right)t} - \sqrt{L_i}\right]$$

But $T_i = 2\pi\sqrt{\frac{L_i}{g}}$, so $T = \frac{2\pi}{\sqrt{g}}\sqrt{L_i + \frac{1}{2\rho a^2}\left(\frac{dM}{dt}\right)t}$.

**P15.58** $\omega = \sqrt{\dfrac{k}{m}} = \dfrac{2\pi}{T}$

(a) $k = \omega^2 m = \boxed{\dfrac{4\pi^2 m}{T^2}}$  (b) $m' = \dfrac{k(T')^2}{4\pi^2} = \boxed{m\left(\dfrac{T'}{T}\right)^2}$

**P15.59** We draw a free-body diagram of the pendulum. The force **H** exerted by the hinge causes no torque about the axis of rotation.

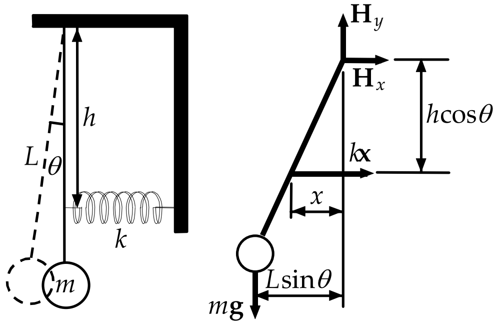


**FIG. P15.59**

$\tau = I\alpha$ and $\dfrac{d^2\theta}{dt^2} = -\alpha$

$$\tau = MgL\sin\theta + kxh\cos\theta = -I\frac{d^2\theta}{dt^2}$$

For small amplitude vibrations, use the approximations: $\sin\theta \approx \theta$, $\cos\theta \approx 1$, and $x \approx s = h\theta$.

Therefore, $\dfrac{d^2\theta}{dt^2} = -\left(\dfrac{MgL + kh^2}{I}\right)\theta = -\omega^2\theta$  $\omega = \sqrt{\dfrac{MgL + kh^2}{ML^2}} = 2\pi f$

$$f = \boxed{\frac{1}{2\pi}\sqrt{\frac{MgL + kh^2}{ML^2}}}$$

***P15.60** (a) In $x = A\cos(\omega t + \phi)$, $v = -\omega A\sin(\omega t + \phi)$

we have at $t = 0$ $v = -\omega A\sin\phi = -v_{\max}$

This requires $\phi = 90°$, so $x = A\cos(\omega t + 90°)$

And this is equivalent to $x = -A\sin\omega t$

Numerically we have $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{50\text{ N/m}}{0.5\text{ kg}}} = 10\text{ s}^{-1}$

and $v_{\max} = \omega A$  $20\text{ m/s} = \left(10\text{ s}^{-1}\right)A$  $A = 2\text{ m}$

So $\boxed{x = (-2\text{ m})\sin\left[\left(10\text{ s}^{-1}\right)t\right]}$

(b) In $\dfrac{1}{2}mv^2 + \dfrac{1}{2}kx^2 = \dfrac{1}{2}kA^2$,  $\dfrac{1}{2}kx^2 = 3\left(\dfrac{1}{2}mv^2\right)$

implies $\dfrac{1}{3}\dfrac{1}{2}kx^2 + \dfrac{1}{2}kx^2 = \dfrac{1}{2}kA^2$  $\dfrac{4}{3}x^2 = A^2$

$$x = \pm\sqrt{\frac{3}{4}}A = \pm 0.866A = \boxed{\pm 1.73\text{ m}}$$

*continued on next page*

(c) $\omega = \sqrt{\dfrac{g}{L}}$ $\qquad L = \dfrac{g}{\omega^2} = \dfrac{9.8\ \text{m/s}^2}{\left(10\ \text{s}^{-1}\right)^2} = \boxed{0.098\,0\ \text{m}}$

(d) In $\qquad x = (-2\ \text{m})\sin\left[\left(10\ \text{s}^{-1}\right)t\right]$

the particle is at $x = 0$ at $t = 0$, at $10t = \pi\,\text{s}$, and so on.

The particle is at $\qquad x = 1\ \text{m}$

when $\qquad -\dfrac{1}{2} = \sin\left[\left(10\ \text{s}^{-1}\right)t\right]$

with solutions $\qquad \left(10\ \text{s}^{-1}\right)t = -\dfrac{\pi}{6}$

$\left(10\ \text{s}^{-1}\right)t = \pi + \dfrac{\pi}{6}$, and so on.

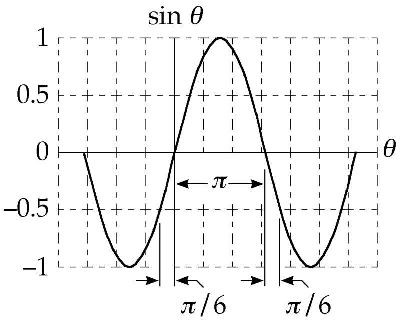


**FIG. P15.60(d)**

The minimum time for the motion is $\Delta t$ in $10\Delta t = \left(\dfrac{\pi}{6}\right)$ s

$$\Delta t = \left(\frac{\pi}{60}\right)\text{s} = \boxed{0.052\,4\ \text{s}}$$

**P15.61** (a) At equilibrium, we have

$$\sum \tau = 0 - mg\left(\frac{L}{2}\right) + kx_0 L$$

where $x_0$ is the equilibrium compression.

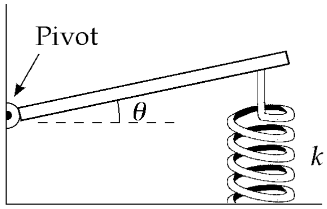


**FIG. P15.61**

After displacement by a small angle,

$$\sum \tau = -mg\left(\frac{L}{2}\right) + kxL = -mg\left(\frac{L}{2}\right) + k\left(x_0 - L\theta\right)L = -k\theta L^2$$

But, $\displaystyle\sum \tau = I\alpha = \frac{1}{3}mL^2\frac{d^2\theta}{dt^2}$. $\qquad$ So $\dfrac{d^2\theta}{dt^2} = -\dfrac{3k}{m}\theta$.

The angular acceleration is opposite in direction and proportional to the displacement, so we have simple harmonic motion with $\boxed{\omega^2 = \dfrac{3k}{m}}$.

(b) $$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{3k}{m}} = \frac{1}{2\pi}\sqrt{\frac{3(100\ \text{N/m})}{5.00\ \text{kg}}} = \boxed{1.23\ \text{Hz}}$$

***P15.62** As it passes through equilibrium, the 4-kg object has speed

$$v_{\max} = \omega A = \sqrt{\frac{k}{m}}A = \sqrt{\frac{100 \text{ N/m}}{4 \text{ kg}}}2 \text{ m} = 10.0 \text{ m/s}.$$

In the completely inelastic collision momentum of the two-object system is conserved. So the new 10-kg object starts its oscillation with speed given by

$$4 \text{ kg}(10 \text{ m/s}) + (6 \text{ kg})0 = (10 \text{ kg})v_{\max}$$
$$v_{\max} = 4.00 \text{ m/s}$$

(a) The new amplitude is given by $\frac{1}{2}mv_{\max}^2 = \frac{1}{2}kA^2$

$$10 \text{ kg}(4 \text{ m/s})^2 = (100 \text{ N/m})A^2$$
$$A = 1.26 \text{ m}$$

Thus the amplitude has $\boxed{\text{decreased by}}$ $2.00 \text{ m} - 1.26 \text{ m} = \boxed{0.735 \text{ m}}$

(b) The old period was $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{4 \text{ kg}}{100 \text{ N/m}}} = 1.26 \text{ s}$

The new period is $T = 2\pi\sqrt{\frac{10}{100} \text{ s}^2} = 1.99 \text{ s}$

The period has $\boxed{\text{increased by}}$ $1.99 \text{ m} - 1.26 \text{ m} = \boxed{0.730 \text{ s}}$

(c) The old energy was $\frac{1}{2}mv_{\max}^2 = \frac{1}{2}(4 \text{ kg})(10 \text{ m/s})^2 = 200 \text{ J}$

The new mechanical energy is $\frac{1}{2}(10 \text{ kg})(4 \text{ m/s})^2 = 80 \text{ J}$

The energy has $\boxed{\text{decreased by 120 J}}$.

(d) The missing mechanical energy has turned into internal energy in the completely inelastic collision.

**P15.63** (a) $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}} = \boxed{3.00 \text{ s}}$

(b) $E = \frac{1}{2}mv^2 = \frac{1}{2}(6.74)(2.06)^2 = \boxed{14.3 \text{ J}}$

(c) At maximum angular displacement $mgh = \frac{1}{2}mv^2$ $h = \frac{v^2}{2g} = 0.217 \text{ m}$

$h = L - L\cos\theta = L(1 - \cos\theta)$ $\cos\theta = 1 - \frac{h}{L}$ $\boxed{\theta = 25.5°}$

**P15.64** One can write the following equations of motion:

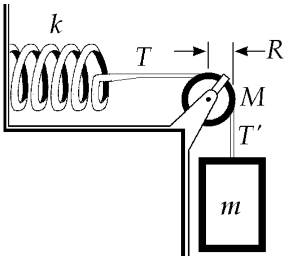


FIG. P15.64

$T - kx = 0$ (describes the spring)

$mg - T' = ma = m\dfrac{d^2x}{dt^2}$ (for the hanging object)

$R(T' - T) = I\dfrac{d^2\theta}{dt^2} = \dfrac{I}{R}\dfrac{d^2x}{dt^2}$ (for the pulley)

$$\text{with } I = \frac{1}{2}MR^2$$

Combining these equations gives the equation of motion

$$\left(m + \frac{1}{2}M\right)\frac{d^2x}{dt^2} + kx = mg.$$

The solution is $x(t) = A\sin\omega t + \dfrac{mg}{k}$ (where $\dfrac{mg}{k}$ arises because of the extension of the spring due to the weight of the hanging object), with frequency

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m + \frac{1}{2}M}} = \frac{1}{2\pi}\sqrt{\frac{100\ \text{N/m}}{0.200\ \text{kg} + \frac{1}{2}M}}.$$

(a) For $M = 0$ $f = \boxed{3.56\ \text{Hz}}$

(b) For $M = 0.250$ kg $f = \boxed{2.79\ \text{Hz}}$

(c) For $M = 0.750$ kg $f = \boxed{2.10\ \text{Hz}}$

**P15.65** Suppose a 100-kg biker compresses the suspension 2.00 cm.

Then, $k = \dfrac{F}{x} = \dfrac{980\ \text{N}}{2.00 \times 10^{-2}\ \text{m}} = 4.90 \times 10^4\ \text{N/m}$

If total mass of motorcycle and biker is 500 kg, the frequency of free vibration is

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{4.90 \times 10^4\ \text{N/m}}{500\ \text{kg}}} = 1.58\ \text{Hz}$$

If he encounters washboard bumps at the same frequency, resonance will make the motorcycle bounce a lot. Assuming a speed of 20.0 m/s, we find these ridges are separated by

$$\frac{20.0\ \text{m/s}}{1.58\ \text{s}^{-1}} = 12.7\ \text{m}\ \boxed{\sim 10^1\ \text{m}}.$$

In addition to this vibration mode of bouncing up and down as one unit, the motorcycle can also vibrate at higher frequencies by rocking back and forth between front and rear wheels, by having just the front wheel bounce inside its fork, or by doing other things. Other spacing of bumps will excite all of these other resonances.

**P15.66** (a) For each segment of the spring

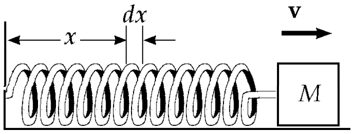


**FIG. P15.66**

$$dK = \frac{1}{2}(dm)v_x^2.$$

Also, $v_x = \frac{x}{\ell}v$ and $dm = \frac{m}{\ell}dx$.

Therefore, the total kinetic energy of the block-spring system is

$$K = \frac{1}{2}Mv^2 + \frac{1}{2}\int_0^{\ell}\left(\frac{x^2v^2}{\ell^2}\right)\frac{m}{\ell}dx = \boxed{\frac{1}{2}\left(M + \frac{m}{3}\right)v^2}.$$

(b) $\omega = \sqrt{\frac{k}{m_{eff}}}$ and $\frac{1}{2}m_{eff}v^2 = \frac{1}{2}\left(M + \frac{m}{3}\right)v^2$

Therefore, $T = \frac{2\pi}{\omega} = \boxed{2\pi\sqrt{\frac{M + \frac{m}{3}}{k}}}$.

**P15.67** (a) $\sum \mathbf{F} = -2T\sin\theta\,\hat{\mathbf{j}}$ where $\theta = \tan^{-1}\left(\frac{y}{L}\right)$

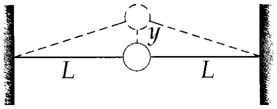


**FIG. P15.67**

Therefore, for a small displacement

$\sin\theta \approx \tan\theta = \frac{y}{L}$ and $\boxed{\sum \mathbf{F} = \frac{-2Ty}{L}\hat{\mathbf{j}}}$

(b) The total force exerted on the ball is opposite in direction and proportional to its displacement from equilibrium, so the ball moves with simple harmonic motion. For a spring system,

$$\sum \mathbf{F} = -k\mathbf{x} \quad \text{becomes here} \quad \sum \mathbf{F} = -\frac{2T}{L}\mathbf{y}.$$

Therefore, the effective spring constant is $\frac{2T}{L}$ and $\boxed{\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{2T}{mL}}}$.

**P15.68** (a) Assuming a Hooke's Law type spring,

$$F = Mg = kx$$

and empirically

$$Mg = 1.74x - 0.113$$

so $k = \boxed{1.74\ \text{N/m} \pm 6\%}$.

| $M$, kg | $x$, m | $Mg$, N |
|---|---|---|
| 0.020 0 | 0.17 | 0.196 |
| 0.040 0 | 0.293 | 0.392 |
| 0.050 0 | 0.353 | 0.49 |
| 0.060 0 | 0.413 | 0.588 |
| 0.070 0 | 0.471 | 0.686 |
| 0.080 0 | 0.493 | 0.784 |

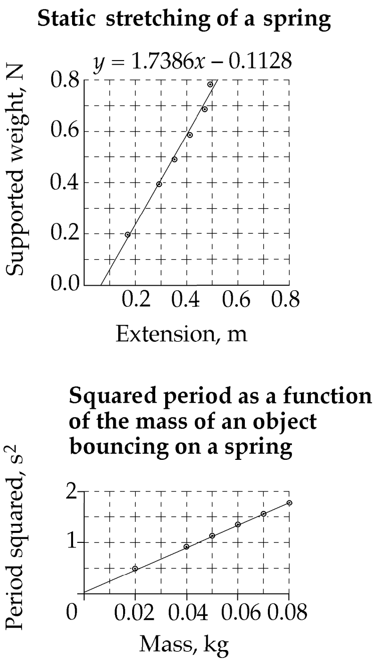


FIG. P15.68

(b) We may write the equation as theoretically

$$T^2 = \frac{4\pi^2}{k}M + \frac{4\pi^2}{3k}m_s$$

and empirically

$$T^2 = 21.7M + 0.058\,9$$

so $k = \frac{4\pi^2}{21.7} = \boxed{1.82\ \text{N/m} \pm 3\%}$

| Time, s | $T$, s | $M$, kg | $T^2$, $s^2$ |
|---|---|---|---|
| 7.03 | 0.703 | 0.020 0 | 0.494 |
| 9.62 | 0.962 | 0.040 0 | 0.925 |
| 10.67 | 1.067 | 0.050 0 | 1.138 |
| 11.67 | 1.167 | 0.060 0 | 1.362 |
| 12.52 | 1.252 | 0.070 0 | 1.568 |
| 13.41 | 1.341 | 0.080 0 | 1.798 |

The $k$ values 1.74 N/m ± 6%

and 1.82 N/m ± 3% differ by 4%

so $\boxed{\text{they agree.}}$

(c) Utilizing the axis-crossing point, $m_s = 3\left(\frac{0.058\,9}{21.7}\right)\text{kg} = \boxed{8\ \text{grams} \pm 12\%}$

$\boxed{\text{in agreement}}$ with 7.4 grams.

**P15.69** (a) $\Delta K + \Delta U = 0$

Thus, $K_{\text{top}} + U_{\text{top}} = K_{\text{bot}} + U_{\text{bot}}$

where $K_{\text{top}} = U_{\text{bot}} = 0$

Therefore, $mgh = \frac{1}{2}I\omega^2$, but

$$h = R - R\cos\theta = R(1-\cos\theta)$$

$$\omega = \frac{v}{R}$$

and $I = \frac{MR^2}{2} + \frac{mr^2}{2} + mR^2$

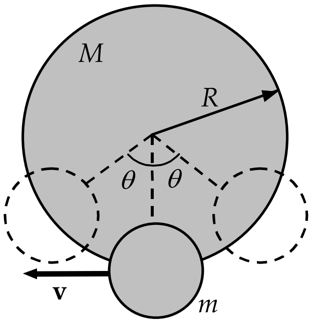


FIG. P15.69

Substituting we find

$$mgR(1-\cos\theta) = \frac{1}{2}\left(\frac{MR^2}{2} + \frac{mr^2}{2} + mR^2\right)\frac{v^2}{R^2}$$

$$mgR(1-\cos\theta) = \left[\frac{M}{4} + \frac{mr^2}{4R^2} + \frac{m}{2}\right]v^2$$

and $v^2 = 4gR\frac{(1-\cos\theta)}{\left(\frac{M}{m} + \frac{r^2}{R^2} + 2\right)}$

so $\boxed{v = 2\sqrt{\frac{Rg(1-\cos\theta)}{\frac{M}{m} + \frac{r^2}{R^2} + 2}}}$

(b) $T = 2\pi\sqrt{\frac{I}{m_T g d_{\text{CM}}}}$

$m_T = m + M$ $\qquad$ $d_{\text{CM}} = \frac{mR + M(0)}{m+M}$

$T = \boxed{2\pi\sqrt{\frac{\frac{1}{2}MR^2 + \frac{1}{2}mr^2 + mR^2}{mgR}}}$

**P15.70** (a) We require $Ae^{-bt/2m} = \frac{A}{2}$ $\qquad$ $e^{+bt/2m} = 2$

or $\frac{bt}{2m} = \ln 2$ $\qquad$ or $\frac{0.100\text{ kg/s}}{2(0.375\text{ kg})}t = 0.693$ $\qquad$ $\therefore t = \boxed{5.20\text{ s}}$

The spring constant is irrelevant.

(b) We can evaluate the energy at successive turning points, where $\cos(\omega t + \phi) = \pm 1$ and the energy is $\frac{1}{2}kx^2 = \frac{1}{2}kA^2e^{-bt/2m}$. We require $\frac{1}{2}kA^2e^{-bt/2m} = \frac{1}{2}\left(\frac{1}{2}kA^2\right)$

or $e^{+bt/m} = 2$ $\qquad$ $\therefore t = \frac{m\ln 2}{b} = \frac{0.375\text{ kg}(0.693)}{0.100\text{ kg/s}} = \boxed{2.60\text{ s}}$.

(c) From $E = \frac{1}{2}kA^2$, the fractional rate of change of energy over time is

$$\frac{\frac{dE}{dt}}{E} = \frac{\frac{d}{dt}\left(\frac{1}{2}kA^2\right)}{\frac{1}{2}kA^2} = \frac{\frac{1}{2}k(2A)\frac{dA}{dt}}{\frac{1}{2}kA^2} = 2\frac{\frac{dA}{dt}}{A}$$

two times faster than the fractional rate of change in amplitude.

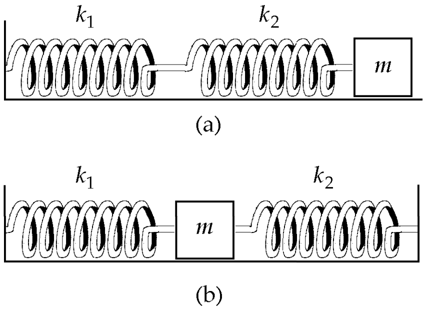


**FIG. P15.71**

**P15.71** (a) When the mass is displaced a distance $x$ from equilibrium, spring 1 is stretched a distance $x_1$ and spring 2 is stretched a distance $x_2$.

By Newton's third law, we expect

$$k_1x_1 = k_2x_2.$$

When this is combined with the requirement that

$$x = x_1 + x_2,$$

we find $x_1 = \left[\frac{k_2}{k_1+k_2}\right]x$

The force on either spring is given by $F_1 = \left[\frac{k_1k_2}{k_1+k_2}\right]x = ma$

where $a$ is the acceleration of the mass $m$.

This is in the form $F = k_{eff}x = ma$

and $T = 2\pi\sqrt{\frac{m}{k_{eff}}} = \boxed{2\pi\sqrt{\frac{m(k_1+k_2)}{k_1k_2}}}$

(b) In this case each spring is distorted by the distance $x$ which the mass is displaced. Therefore, the restoring force is

$$F = -(k_1+k_2)x \qquad \text{and} \qquad k_{eff} = k_1 + k_2$$

so that $T = \boxed{2\pi\sqrt{\frac{m}{(k_1+k_2)}}}$.

**P15.72** Let $\ell$ represent the length below water at equilibrium and $M$ the tube's mass:

$$\sum F_y = 0 \Rightarrow -Mg + \rho\pi r^2\ell g = 0.$$

Now with any excursion $x$ from equilibrium

$$-Mg + \rho\pi r^2(\ell - x)g = Ma.$$

Subtracting the equilibrium equation gives

$$-\rho\pi r^2 gx = Ma$$

$$a = -\left(\frac{\rho\pi r^2 g}{M}\right)x = -\omega^2 x$$

The opposite direction and direct proportionality of $a$ to $x$ imply SHM with angular frequency

$$\omega = \sqrt{\frac{\rho\pi r^2 g}{M}}$$

$$T = \frac{2\pi}{\omega} = \boxed{\left(\frac{2}{r}\right)\sqrt{\frac{\pi M}{\rho g}}}$$

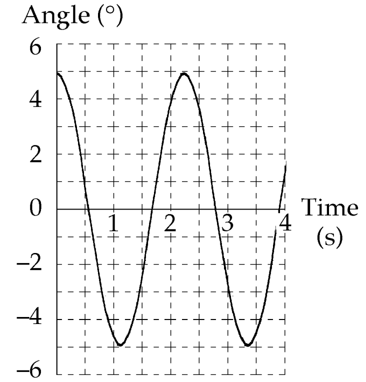
**P15.73** For $\theta_{\max} = 5.00°$, the motion calculated by the Euler method agrees quite precisely with the prediction of $\theta_{\max}\cos\omega t$. The period is $T = 2.20\text{ s}$.

| Time, $t$ (s) | Angle, $\theta$ (°) | Ang. speed (°/s) | Ang. Accel. $\left(°/\text{s}^2\right)$ | $\theta_{\max}\cos\omega t$ |
|---|---|---|---|---|
| 0.000 | 5.000 0 | 0.000 0 | –40.781 5 | 5.000 0 |
| 0.004 | 4.999 3 | –0.163 1 | –40.776 2 | 4.999 7 |
| 0.008 | 4.998 0 | –0.326 2 | –40.765 6 | 4.998 7 |
| … | | | | |
| 0.544 | 0.056 0 | –14.282 3 | –0.457 6 | 0.081 0 |
| 0.548 | –0.001 1 | –14.284 2 | 0.009 0 | 0.023 9 |
| 0.552 | –0.058 2 | –14.284 1 | 0.475 6 | –0.033 3 |
| … | | | | |
| 1.092 | –4.999 4 | –0.319 9 | 40.776 5 | –4.998 9 |
| 1.096 | –5.000 0 | –0.156 8 | 40.781 6 | –4.999 8 |
| 1.100 | –5.000 0 | 0.006 3 | 40.781 4 | –5.000 0 |
| 1.104 | –4.999 3 | 0.169 4 | 40.775 9 | –4.999 6 |
| … | | | | |
| 1.644 | –0.063 8 | 14.282 4 | 0.439 7 | –0.071 6 |
| 1.648 | 0.003 3 | 14.284 2 | –0.027 0 | –0.014 5 |
| 1.652 | 0.060 4 | 14.284 1 | –0.493 6 | 0.042 7 |
| … | | | | |
| 2.192 | 4.999 4 | 0.313 7 | –40.776 8 | 4.999 1 |
| 2.196 | 5.000 0 | 0.150 6 | –40.781 7 | 4.999 9 |
| 2.200 | 5.000 0 | –0.012 6 | –40.781 3 | 5.000 0 |
| 2.204 | 4.999 3 | –0.175 7 | –40.775 6 | 4.999 4 |

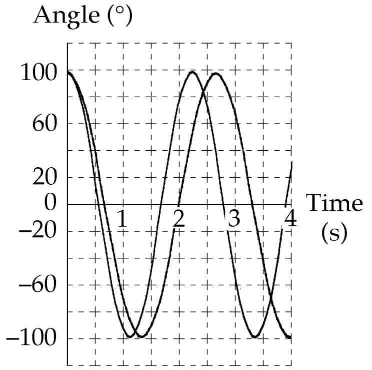
For $\theta_{\max} = 100°$, the simple harmonic motion approximation $\theta_{\max}\cos\omega t$ diverges greatly from the Euler calculation. The period is $T = 2.71\text{ s}$, larger than the small-angle period by 23%.

| Time, $t$ (s) | Angle, $\theta$ (°) | Ang. speed (°/s) | Ang. Accel. $\left(°/\text{s}^2\right)$ | $\theta_{\max}\cos\omega t$ |
|---|---|---|---|---|
| 0.000 | 100.000 0 | 0.000 0 | –460.606 6 | 100.000 0 |
| 0.004 | 99.992 6 | –1.843 2 | –460.817 3 | 99.993 5 |
| 0.008 | 99.977 6 | –3.686 5 | –460.838 2 | 99.973 9 |
| … | | | | |
| 1.096 | –84.744 9 | –120.191 0 | 465.948 8 | –99.995 4 |
| 1.100 | –85.218 2 | –118.327 2 | 466.286 9 | –99.999 8 |
| 1.104 | –85.684 0 | –116.462 0 | 466.588 6 | –99.991 1 |
| … | | | | |
| 1.348 | –99.996 0 | –3.053 3 | 460.812 5 | –75.797 9 |
| 1.352 | –100.000 8 | –1.210 0 | 460.805 7 | –75.047 4 |
| 1.356 | –99.998 3 | 0.633 2 | 460.809 3 | –74.287 0 |
| … | | | | |
| 2.196 | 40.150 9 | 224.867 7 | –301.713 2 | 99.997 1 |
| 2.200 | 41.045 5 | 223.660 9 | –307.260 7 | 99.999 3 |
| 2.204 | 41.935 3 | 222.431 8 | –312.703 5 | 99.988 5 |
| … | | | | |
| 2.704 | 99.998 5 | 2.420 0 | –460.809 0 | 12.642 2 |
| 2.708 | 100.000 8 | 0.576 8 | –460.805 7 | 11.507 5 |
| 2.712 | 99.995 7 | –1.266 4 | –460.812 9 | 10.371 2 |

**Motion of a Simple Pendulum**



**Small amplitude**



**Large amplitude**

**FIG. P15.73**

***P15.74** (a) The block moves with the board in what we take as the positive $x$ direction, stretching the spring until the spring force $-kx$ is equal in magnitude to the maximum force of static friction $\mu_s n = \mu_s mg$. This occurs at $x = \frac{\mu_s mg}{k}$.

(b) Since $v$ is small, the block is nearly at the rest at this break point. It starts almost immediately to move back to the left, the forces on it being $-kx$ and $+\mu_k mg$. While it is sliding the net force exerted on it can be written as

$$-kx + \mu_k mg = -kx + \frac{k\mu_k mg}{k} = -k\left(x - \frac{\mu_k mg}{k}\right) = -kx_{rel}$$

where $x_{rel}$ is the excursion of the block away from the point $\frac{\mu_k mg}{k}$.

Conclusion: the block goes into simple harmonic motion centered about the equilibrium position where the spring is stretched by $\frac{\mu_k mg}{k}$.

(d) The amplitude of its motion is its original displacement, $A = \frac{\mu_s mg}{k} - \frac{\mu_k mg}{k}$. It first comes to rest at spring extension $\frac{\mu_k mg}{k} - A = \frac{(2\mu_k - \mu_s)mg}{k}$. Almost immediately at this point it latches onto the slowly-moving board to move with the board. The board exerts a force of static friction on the block, and the cycle continues.

(c) The graph of the motion looks like this:

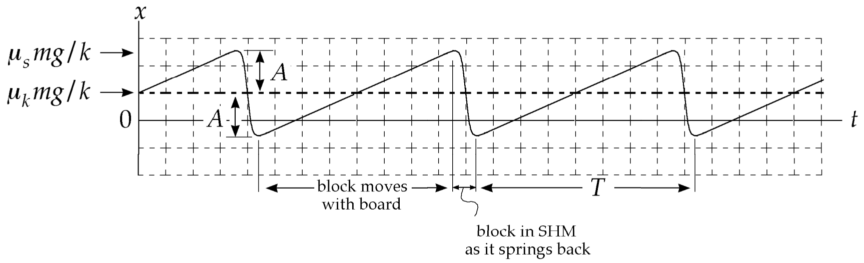


FIG. P15.74(c)

(e) The time during each cycle when the block is moving with the board is $\frac{2A}{v} = \frac{2(\mu_s - \mu_k)mg}{kv}$. The time for which the block is springing back is one half a cycle of simple harmonic motion, $\frac{1}{2}\left(2\pi\sqrt{\frac{m}{k}}\right) = \pi\sqrt{\frac{m}{k}}$. We ignore the times at the end points of the motion when the speed of the block changes from $v$ to 0 and from 0 to $v$. Since $v$ is small compared to $\frac{2A}{\pi\sqrt{\frac{m}{k}}}$, these times are negligible. Then the period is

$$T = \frac{2(\mu_s - \mu_k)mg}{kv} + \pi\sqrt{\frac{m}{k}}.$$

*continued on next page*

(f) $$T = \frac{2(0.4-0.25)(0.3\text{ kg})\left(9.8\text{ m/s}^2\right)}{(0.024\text{ m/s})(12\text{ N/m})} + \pi\sqrt{\frac{0.3\text{ kg}}{12\text{ N/m}}} = 3.06\text{ s} + 0.497\text{ s} = 3.56\text{ s}$$

Then $f = \frac{1}{T} = \boxed{0.281\text{ Hz}}$.

(g) $T = \frac{2(\mu_s - \mu_k)mg}{kv} + \pi\sqrt{\frac{m}{k}}$ increases as $m$ increases, so the frequency $\boxed{\text{decreases}}$.

(h) As $k$ increases, $T$ decreases and $f$ $\boxed{\text{increases}}$.

(i) As $v$ increases, $T$ decreases and $f$ $\boxed{\text{increases}}$.

(j) As $(\mu_s - \mu_k)$ increases, $T$ increases and $f$ $\boxed{\text{decreases}}$.

***P15.75** (a) Newton's law of universal gravitation is $F = -\frac{GMm}{r^2} = -\frac{Gm}{r^2}\left(\frac{4}{3}\pi r^3\right)\rho$

Thus, $F = -\left(\frac{4}{3}\pi\rho Gm\right)r$

Which is of Hooke's law form with $k = \frac{4}{3}\pi\rho Gm$

(b) The sack of mail moves without friction according to $-\left(\frac{4}{3}\right)\pi\rho Gmr = ma$

$$a = -\left(\frac{4}{3}\right)\pi\rho Gr = -\omega^2 r$$

Since acceleration is a negative constant times excursion from equilibrium, it executes SHM with

$\omega = \sqrt{\frac{4\pi\rho G}{3}}$ and period $T = \frac{2\pi}{\omega} = \sqrt{\frac{3\pi}{\rho G}}$

The time for a one-way trip through the earth is $\frac{T}{2} = \sqrt{\frac{3\pi}{4\rho G}}$

We have also $g = \frac{GM_e}{R_e^2} = \frac{G4\pi R_e^3\rho}{3R_e^2} = \frac{4}{3}\pi\rho GR_e$

so $\frac{4\rho G}{3} = \frac{g}{(\pi R_e)}$ and $\frac{T}{2} = \pi\sqrt{\frac{R_e}{g}} = \pi\sqrt{\frac{6.37\times10^6\text{ m}}{9.8\text{ m/s}^2}} = 2.53\times10^3\text{ s} = \boxed{42.2\text{ min}}$.

## ANSWERS TO EVEN PROBLEMS

**P15.2** (a) 4.33 cm; (b) $-5.00$ cm/s; (c) $-17.3\text{ cm/s}^2$; (d) 3.14 s; 5.00 cm

**P15.4** (a) 15.8 cm; (b) $-15.9$ cm; (c) see the solution; (d) 51.1 m; (e) 50.7 m

**P15.6** see the solution

**P15.8** 12.0 Hz

**P15.10** 18.8 m/s; $7.11\text{ km/s}^2$

| | |
|---|---|
| **P15.12** | (a) 1.26 s; (b) 0.150 $\text{m/s}$; 0.750 $\text{m/s}^2$;<br>(c) $x = -3\text{ cm}\cos 5t$; $v = \left(\frac{15\text{ cm}}{\text{s}}\right)\sin 5t$;<br>$a = \left(\frac{75\text{ cm}}{\text{s}^2}\right)\cos 5t$ |
| **P15.14** | (a) $\frac{v}{\omega}$; (b) $x = -\left(\frac{v}{\omega}\right)\sin\omega t$ |
| **P15.16** | (a) 126 $\text{N/m}$; (b) 0.178 m |
| **P15.18** | (a) 0.153 J; (b) 0.784 $\text{m/s}$; (c) 17.5 $\text{m/s}^2$ |
| **P15.20** | (a) 100 $\text{N/m}$; (b) 1.13 Hz;<br>(c) 1.41 $\text{m/s}$ at $x = 0$;<br>(d) 10.0 $\text{m/s}^2$ at $x = \pm A$; (e) 2.00 J;<br>(f) 1.33 $\text{m/s}$; (g) 3.33 $\text{m/s}^2$ |
| **P15.22** | (a) 1.50 s; (b) 73.4 $\text{N/m}$;<br>(c) 19.7 m below the bridge; (d) 1.06 $\text{rad/s}$;<br>(e) 2.01 s; (f) 3.50 s |
| **P15.24** | (a) 0.218 s and 1.09 s; (b) 14.6 mW |
| **P15.26** | The position of the piston is given by<br>$x = A\cos\omega t$. |
| **P15.28** | $\frac{g_C}{g_T} = 1.001\,5$ |
| **P15.30** | 1.42 s; 0.499 m |
| **P15.32** | (a) 3.65 s; (b) 6.41 s; (c) 4.24 s |
| **P15.34** | (a) see the solution;<br>(b), (c) 9.85 $\text{m/s}^2$; agreeing with the<br>accepted value within 0.5% |
| **P15.36** | (a) 2.09 s; (b) 4.08% |
| **P15.38** | 203 $\mu\text{N}\cdot\text{m}$ |
| **P15.40** | see the solution |
| **P15.42** | see the solution |
| **P15.44** | (a) 2.95 Hz; (b) 2.85 cm |
| **P15.46** | see the solution |
| **P15.48** | either 1.31 Hz or 0.641 Hz |
| **P15.50** | 1.56 cm |
| **P15.52** | (a) 0.500 $\text{m/s}$; (b) 8.56 cm |
| **P15.54** | $A = \frac{\mu_s g}{4\pi^2 f^2}$ |
| **P15.56** | see the solution |
| **P15.58** | (a) $k = \frac{4\pi^2 m}{T^2}$; (b) $m' = m\left(\frac{T'}{T}\right)^2$ |
| **P15.60** | (a) $x = (-2\text{ m})\sin(10t)$; (b) at $x \pm 1.73$ m;<br>(c) 98.0 mm; (d) 52.4 ms |
| **P15.62** | (a) decreased by 0.735 m;<br>(b) increased by 0.730 s;<br>(c) decreased by 120 J; (d) see the solution |
| **P15.64** | (a) 3.56 Hz; (b) 2.79 Hz; (c) 2.10 Hz |
| **P15.66** | (a) $\frac{1}{2}\left(M + \frac{m}{3}\right)v^2$; (b) $T = 2\pi\sqrt{\frac{M + \frac{m}{3}}{k}}$ |
| **P15.68** | see the solution; (a) $k = 1.74\text{ N/m} \pm 6\%$;<br>(b) $1.82\text{ N/m} \pm 3\%$; they agree;<br>(c) $8\text{ g} \pm 12\%$; it agrees |
| **P15.70** | (a) 5.20 s; (b) 2.60 s; (c) see the solution |
| **P15.72** | see the solution; $T = \left(\frac{2}{r}\right)\sqrt{\frac{\pi M}{\rho g}}$ |
| **P15.74** | see the solution; (f) 0.281 Hz;<br>(g) decreases; (h) increases; (i) increases;<br>(j) decreases |

# 16

# Wave Motion

### CHAPTER OUTLINE

16.1 Propagation of a Disturbance
16.2 Sinusoidal Waves
16.3 The Speed of Waves on Strings
16.4 Reflection and Transmission
16.5 Rate of Energy Transfer by Sinusoidal Waves on Strings
16.6 The Linear Wave Equation

## ANSWERS TO QUESTIONS

**Q16.1** As the pulse moves down the string, the particles of the string itself move side to side. Since the medium—here, the string—moves perpendicular to the direction of wave propagation, the wave is transverse by definition.

**Q16.2** To use a slinky to create a longitudinal wave, pull a few coils back and release. For a transverse wave, jostle the end coil side to side.

**Q16.3** From $v=\sqrt{\frac{T}{\mu}}$, we must increase the tension by a factor of 4.

**Q16.4** It depends on from what the wave reflects. If reflecting from a less dense string, the reflected part of the wave will be right side up.

**Q16.5** Yes, among other things it depends on. $v_{\max}=\omega A=2\pi fA=\frac{2\pi vA}{\lambda}$. Here $v$ is the speed of the wave.

**Q16.6** Since the frequency is 3 cycles per second, the period is $\frac{1}{3}$ second = 333 ms.

**Q16.7** Amplitude is increased by a factor of $\sqrt{2}$. The wave speed does not change.

**Q16.8** The section of rope moves up and down in SHM. Its speed is always changing. The wave continues on with constant speed in one direction, setting further sections of the rope into up-and-down motion.

**Q16.9** Each element of the rope must support the weight of the rope below it. The tension increases with height. (It increases linearly, if the rope does not stretch.) Then the wave speed $v=\sqrt{\frac{T}{\mu}}$ increases with height.

**Q16.10** The difference is in the direction of motion of the elements of the medium. In longitudinal waves, the medium moves back and forth parallel to the direction of wave motion. In transverse waves, the medium moves perpendicular to the direction of wave motion.

**Q16.11** Slower. Wave speed is inversely proportional to the square root of linear density.

**Q16.12** As the wave passes from the massive string to the less massive string, the wave speed will increase according to $v=\sqrt{\frac{T}{\mu}}$. The frequency will remain unchanged. Since $v=f\lambda$, the wavelength must increase.

**Q16.13** Higher tension makes wave speed higher. Greater linear density makes the wave move more slowly.

**Q16.14** The wave speed is independent of the maximum particle speed. The source determines the maximum particle speed, through its frequency and amplitude. The wave speed depends instead on properties of the medium.

**Q16.15** Longitudinal waves depend on the compressibility of the fluid for their propagation. Transverse waves require a restoring force in response to sheer strain. Fluids do not have the underlying structure to supply such a force. A fluid cannot support static sheer. A viscous fluid can temporarily be put under sheer, but the higher its viscosity the more quickly it converts input work into internal energy. A local vibration imposed on it is strongly damped, and not a source of wave propagation.

**Q16.16** Let $\Delta t=t_s-t_p$ represent the difference in arrival times of the two waves at a station at distance $d=v_st_s=v_pt_p$ from the hypocenter. Then $d=\Delta t\left(\frac{1}{v_s}-\frac{1}{v_p}\right)^{-1}$. Knowing the distance from the first station places the hypocenter on a sphere around it. A measurement from a second station limits it to another sphere, which intersects with the first in a circle. Data from a third non-collinear station will generally limit the possibilities to a point.

**Q16.17** The speed of a wave on a "massless" string would be infinite!

## SOLUTIONS TO PROBLEMS

### Section 16.1 **Propagation of a Disturbance**

**P16.1** Replace *x* by $x-vt=x-4.5t$

to get $\boxed{y=\frac{6}{\left[(x-4.5t)^2+3\right]}}$

**P16.2**

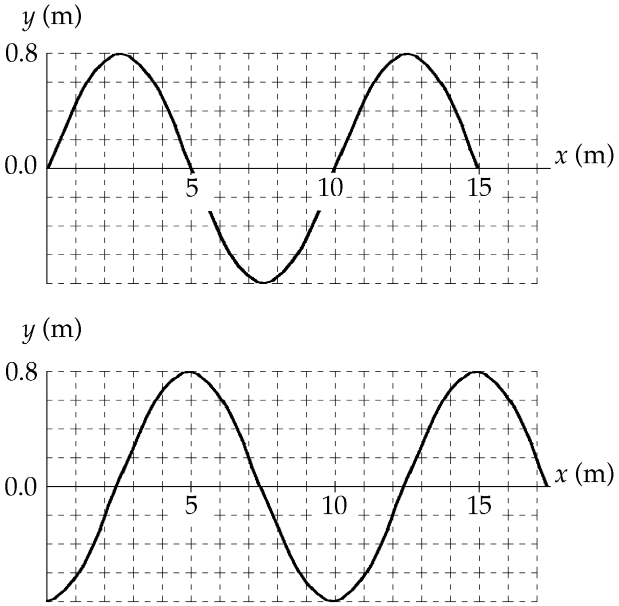


**FIG. P16.2**

**P16.3** $5.00e^{-(x+5t)^2}$ is of the form $f(x+vt)$

so it describes a wave moving to the $\boxed{\text{left}}$ at $v = \boxed{5.00\text{ m/s}}$.

**P16.4** (a) The $\boxed{\text{longitudinal}}$ wave travels a shorter distance and is moving faster, so it will arrive at point $B$ first.

(b) The wave that travels through the Earth must travel

a distance of $2R\sin 30.0° = 2(6.37\times 10^6\text{ m})\sin 30.0° = 6.37\times 10^6\text{ m}$

at a speed of $7\,800$ m/s

Therefore, it takes $\dfrac{6.37\times 10^6\text{ m}}{7\,800\text{ m/s}} = 817\text{ s}$

The wave that travels along the Earth's surface must travel

a distance of $s = R\theta = R\left(\dfrac{\pi}{3}\text{ rad}\right) = 6.67\times 10^6\text{ m}$

at a speed of $4\,500$ m/s

Therefore, it takes $\dfrac{6.67\times 10^6}{4\,500} = 1\,482\text{ s}$

The time difference is $\boxed{665\text{ s}} = 11.1\text{ min}$

**P16.5** The distance the waves have traveled is $d = (7.80 \text{ km/s})t = (4.50 \text{ km/s})(t + 17.3 \text{ s})$

where $t$ is the travel time for the faster wave.

Then, $(7.80 - 4.50)(\text{km/s})t = (4.50 \text{ km/s})(17.3 \text{ s})$

or $t = \dfrac{(4.50 \text{ km/s})(17.3 \text{ s})}{(7.80 - 4.50) \text{ km/s}} = 23.6 \text{ s}$

and the distance is $d = (7.80 \text{ km/s})(23.6 \text{ s}) = \boxed{184 \text{ km}}$.

## Section 16.2 **Sinusoidal Waves**

**P16.6** Using data from the observations, we have $\lambda = 1.20 \text{ m}$

and $f = \dfrac{8.00}{12.0 \text{ s}}$

Therefore, $v = \lambda f = (1.20 \text{ m})\left(\dfrac{8.00}{12.0 \text{ s}}\right) = \boxed{0.800 \text{ m/s}}$

**P16.7** $f = \dfrac{40.0 \text{ vibrations}}{30.0 \text{ s}} = \dfrac{4}{3} \text{ Hz}$ $v = \dfrac{425 \text{ cm}}{10.0 \text{ s}} = 42.5 \text{ cm/s}$

$\lambda = \dfrac{v}{f} = \dfrac{42.5 \text{ cm/s}}{\frac{4}{3} \text{ Hz}} = 31.9 \text{ cm} = \boxed{0.319 \text{ m}}$

**P16.8** $v = f\lambda = (4.00 \text{ Hz})(60.0 \text{ cm}) = 240 \text{ cm/s} = \boxed{2.40 \text{ m/s}}$

**P16.9** $y = (0.020\,0 \text{ m})\sin(2.11x - 3.62t)$ in SI units $A = \boxed{2.00 \text{ cm}}$

$k = 2.11 \text{ rad/m}$ $\lambda = \dfrac{2\pi}{k} = \boxed{2.98 \text{ m}}$

$\omega = 3.62 \text{ rad/s}$ $f = \dfrac{\omega}{2\pi} = \boxed{0.576 \text{ Hz}}$

$v = f\lambda = \dfrac{\omega}{2\pi}\dfrac{2\pi}{k} = \dfrac{3.62}{2.11} = \boxed{1.72 \text{ m/s}}$

**P16.10** $y = (0.005\,1 \text{ m})\sin(310x - 9.30t)$ SI units

$v = \dfrac{\omega}{k} = \dfrac{9.30}{310} = 0.030\,0 \text{ m/s}$

$s = vt = \boxed{0.300 \text{ m in positive } x\text{-direction}}$

***P16.11** From $y=(12.0\text{ cm})\sin\left((1.57\text{ rad/m})x-(31.4\text{ rad/s})t\right)$

(a) The transverse velocity is $\frac{\partial y}{\partial t}=-A\omega\cos(kx-\omega t)$

Its maximum magnitude is $A\omega=12\text{ cm}(31.4\text{ rad/s})=\boxed{3.77\text{ m/s}}$

(b) $$a_y=\frac{\partial v_y}{\partial t}=\frac{\partial}{\partial t}\left(-A\omega\cos(kx-\omega t)\right)=-A\omega^2\sin(kx-\omega t)$$

The maximum value is $A\omega^2=(0.12\text{ m})\left(31.4\text{ s}^{-1}\right)^2=\boxed{118\text{ m/s}^2}$

**P16.12** At time $t$, the phase of $y=(15.0\text{ cm})\cos(0.157x-50.3t)$ at coordinate $x$ is $\phi=(0.157\text{ rad/cm})x-(50.3\text{ rad/s})t$. Since $60.0°=\frac{\pi}{3}\text{ rad}$, the requirement for point $B$ is that $\phi_B=\phi_A\pm\frac{\pi}{3}\text{ rad}$, or (since $x_A=0$),

$$(0.157\text{ rad/cm})x_B-(50.3\text{ rad/s})t=0-(50.3\text{ rad/s})t\pm\frac{\pi}{3}\text{ rad}.$$

This reduces to $x_B=\frac{\pm\pi\text{ rad}}{3(0.157\text{ rad/cm})}=\boxed{\pm 6.67\text{ cm}}$.

**P16.13** $y=0.250\sin(0.300x-40.0t)\text{ m}$

Compare this with the general expression $y=A\sin(kx-\omega t)$

(a) $A=\boxed{0.250\text{ m}}$

(b) $\omega=\boxed{40.0\text{ rad/s}}$

(c) $k=\boxed{0.300\text{ rad/m}}$

(d) $$\lambda=\frac{2\pi}{k}=\frac{2\pi}{0.300\text{ rad/m}}=\boxed{20.9\text{ m}}$$

(e) $$v=f\lambda=\left(\frac{\omega}{2\pi}\right)\lambda=\left(\frac{40.0\text{ rad/s}}{2\pi}\right)(20.9\text{ m})=\boxed{133\text{ m/s}}$$

(f) The wave moves to the right, $\boxed{\text{in }+x\text{ direction}}$.

**P16.14** (a) See figure at right.

(b) $T = \frac{2\pi}{\omega} = \frac{2\pi}{50.3} = \boxed{0.125\text{ s}}$

This agrees with the period found in the example in the text.

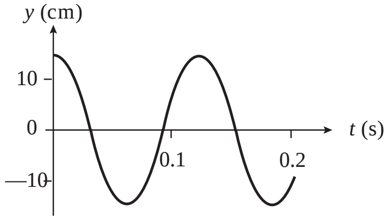


**FIG. P16.14**

**P16.15** (a) $A = y_{\max} = 8.00\text{ cm} = 0.080\,0\text{ m}$ $\quad k = \frac{2\pi}{\lambda} = \frac{2\pi}{(0.800\text{ m})} = 7.85\text{ m}^{-1}$

$\omega = 2\pi f = 2\pi(3.00) = 6.00\pi\text{ rad/s}$

Therefore, $y = A\sin(kx + \omega t)$

Or (where $y(0,\ t) = 0$ at $t = 0$) $\quad \boxed{y = (0.080\,0)\sin(7.85x + 6\pi t)\text{ m}}$

(b) In general, $y = 0.080\,0\sin(7.85x + 6\pi t + \phi)$

Assuming $y(x,\ 0) = 0$ at $x = 0.100\text{ m}$

then we require that $0 = 0.080\,0\sin(0.785 + \phi)$

or $\phi = -0.785$

Therefore, $\boxed{y = 0.080\,0\sin(7.85x + 6\pi t - 0.785)\text{ m}}$

**P16.16** (a)

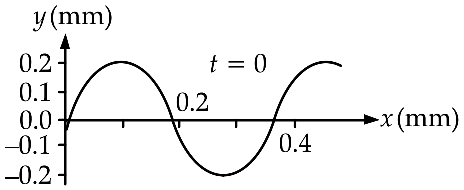


**FIG. P16.16(a)**

(b) $k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.350\text{ m}} = \boxed{18.0\ \text{rad/m}}$

$T = \frac{1}{f} = \frac{1}{12.0/\text{s}} = \boxed{0.083\,3\text{ s}}$

$\omega = 2\pi f = 2\pi 12.0/\text{s} = \boxed{75.4\ \text{rad/s}}$

$|v| = f\lambda = (12.0/\text{s})(0.350\text{ m}) = \boxed{4.20\ \text{m/s}}$

(c) $y = A\sin(kx + \omega t + \phi)$ specializes to

$y = 0.200\text{ m}\sin(18.0\,x/\text{m} + 75.4t/\text{s} + \phi)$

at $x = 0$, $t = 0$ we require

$-3.00 \times 10^{-2}\text{ m} = 0.200\text{ m}\sin(+\phi)$

$\phi = -8.63° = -0.151\text{ rad}$

so $y(x,\ t) = \boxed{(0.200\text{ m})\sin(18.0x/\text{m} + 75.4t/\text{s} - 0.151\text{ rad})}$

**P16.17** $y = (0.120\text{ m})\sin\left(\dfrac{\pi}{8}x + 4\pi t\right)$

(a) $v = \dfrac{dy}{dt}$: $\quad x = (0.120)(4\pi)\cos\left(\dfrac{\pi}{8}x + 4\pi t\right)$

$$v(0.200\text{ s},\ 1.60\text{ m}) = \boxed{-1.51\text{ m/s}}$$

$a = \dfrac{dv}{dt}$: $\quad a = (-0.120\text{ m})(4\pi)^2\sin\left(\dfrac{\pi}{8}x + 4\pi t\right)$

$$a(0.200\text{ s},\ 1.60\text{ m}) = \boxed{0}$$

(b) $k = \dfrac{\pi}{8} = \dfrac{2\pi}{\lambda}$: $\quad \lambda = \boxed{16.0\text{ m}}$

$\omega = 4\pi = \dfrac{2\pi}{T}$: $\quad T = \boxed{0.500\text{ s}}$

$$v = \frac{\lambda}{T} = \frac{16.0\text{ m}}{0.500\text{ s}} = \boxed{32.0\text{ m/s}}$$

**P16.18** (a) Let us write the wave function as $\quad y(x,\ t) = A\sin(kx + \omega t + \phi)$

$$y(0,\ 0) = A\sin\phi = 0.020\,0\text{ m}$$

$$\left.\frac{dy}{dt}\right|_{0,\ 0} = A\omega\cos\phi = -2.00\text{ m/s}$$

Also, $\quad \omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{0.025\,0\text{ s}} = 80.0\,\pi/\text{s}$

$$A^2 = x_i^2 + \left(\frac{v_i}{\omega}\right)^2 = (0.020\,0\text{ m})^2 + \left(\frac{2.00\text{ m/s}}{80.0\,\pi/\text{s}}\right)^2$$

$$A = \boxed{0.021\,5\text{ m}}$$

(b) $\dfrac{A\sin\phi}{A\cos\phi} = \dfrac{0.020\,0}{\frac{-2}{80.0\pi}} = -2.51 = \tan\phi$

Your calculator's answer $\tan^{-1}(-2.51) = -1.19\text{ rad}$ has a negative sine and positive cosine, just the reverse of what is required. You must look beyond your calculator to find

$\phi = \pi - 1.19\text{ rad} = \boxed{1.95\text{ rad}}$

(c) $v_{y,\,\max} = A\omega = 0.021\,5\text{ m}(80.0\pi/\text{s}) = \boxed{5.41\text{ m/s}}$

(d) $\lambda = v_x T = (30.0\text{ m/s})(0.025\,0\text{ s}) = 0.750\text{ m}$

$k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{0.750\text{ m}} = 8.38/\text{m}$ $\qquad \omega = 80.0\pi/\text{s}$

$$\boxed{y(x,\ t) = (0.021\,5\text{ m})\sin(8.38x\text{ rad/m} + 80.0\pi t\text{ rad/s} + 1.95\text{ rad})}$$

**P16.19** (a) $f = \frac{v}{\lambda} = \frac{(1.00\ \text{m/s})}{2.00\ \text{m}} = \boxed{0.500\ \text{Hz}}$

$\omega = 2\pi f = 2\pi(0.500/\text{s}) = \boxed{3.14\ \text{rad/s}}$

(b) $k = \frac{2\pi}{\lambda} = \frac{2\pi}{2.00\ \text{m}} = \boxed{3.14\ \text{rad/m}}$

(c) $y = A\sin(kx - \omega t + \phi)$ becomes

$y = \boxed{(0.100\ \text{m})\sin(3.14x/\text{m} - 3.14t/\text{s} + 0)}$

(d) For $x = 0$ the wave function requires

$\boxed{y = (0.100\ \text{m})\sin(-3.14t/\text{s})}$

(e) $\boxed{y = (0.100\ \text{m})\sin(4.71\ \text{rad} - 3.14t/\text{s})}$

(f) $v_y = \frac{\partial y}{\partial t} = 0.100\ \text{m}(-3.14/\text{s})\cos(3.14x/\text{m} - 3.14t/\text{s})$

The cosine varies between +1 and –1, so

$v_y \le (0.314\ \text{m/s})$

**P16.20** (a) at $x = 2.00\ \text{m}$, $y = \boxed{(0.100\ \text{m})\sin(1.00\ \text{rad} - 20.0t)}$

(b) $y = (0.100\ \text{m})\sin(0.500x - 20.0t) = A\sin(kx - \omega t)$

so $\omega = 20.0\ \text{rad/s}$ and $f = \frac{\omega}{2\pi} = \boxed{3.18\ \text{Hz}}$

## Section 16.3 **The Speed of Waves on Strings**

**P16.21** The down and back distance is $4.00\ \text{m} + 4.00\ \text{m} = 8.00\ \text{m}$.

The speed is then $v = \frac{d_{\text{total}}}{t} = \frac{4(8.00\ \text{m})}{0.800\ \text{s}} = 40.0\ \text{m/s} = \sqrt{\frac{T}{\mu}}$

Now, $\mu = \frac{0.200\ \text{kg}}{4.00\ \text{m}} = 5.00 \times 10^{-2}\ \text{kg/m}$

So $T = \mu v^2 = (5.00 \times 10^{-2}\ \text{kg/m})(40.0\ \text{m/s})^2 = \boxed{80.0\ \text{N}}$

**P16.22** The mass per unit length is: $\mu = \frac{0.060\,0\ \text{kg}}{5.00\ \text{m}} = 1.20 \times 10^{-2}\ \text{kg/m}$.

The required tension is: $T = \mu v^2 = (0.012\,0\ \text{kg/m})(50.0\ \text{m/s})^2 = \boxed{30.0\ \text{N}}$.

**P16.23** $v=\sqrt{\dfrac{T}{\mu}}=\sqrt{\dfrac{1\,350\ \text{kg}\cdot\text{m/s}^2}{5.00\times10^{-3}\ \text{kg/m}}}=\boxed{520\ \text{m/s}}$

**P16.24** (a) $\omega=2\pi f=2\pi(500)=3\,140\ \text{rad/s}$, $k=\dfrac{\omega}{v}=\dfrac{3\,140}{196}=16.0\ \text{rad/m}$

$$\boxed{y=\left(2.00\times10^{-4}\ \text{m}\right)\sin(16.0x-3\,140t)}$$

(b) $v=196\ \text{m/s}=\sqrt{\dfrac{T}{4.10\times10^{-3}\ \text{kg/m}}}$

$T=\boxed{158\ \text{N}}$

**P16.25** $T=Mg$ is the tension; $v=\sqrt{\dfrac{T}{\mu}}=\sqrt{\dfrac{Mg}{\frac{m}{L}}}=\sqrt{\dfrac{MgL}{m}}=\dfrac{L}{t}$ is the wave speed.

Then, $\dfrac{MgL}{m}=\dfrac{L^2}{t^2}$

and $g=\dfrac{Lm}{Mt^2}=\dfrac{1.60\ \text{m}\left(4.00\times10^{-3}\ \text{kg}\right)}{3.00\ \text{kg}\left(3.61\times10^{-3}\ \text{s}\right)^2}=\boxed{1.64\ \text{m/s}^2}$

**P16.26** $v=\sqrt{\dfrac{T}{\mu}}$

$T=\mu v^2=\rho A v^2=\rho\pi r^2v^2$

$T=\left(8\,920\ \text{kg/m}^3\right)(\pi)\left(7.50\times10^{-4}\ \text{m}\right)^2(200\ \text{m/s})^2$

$T=\boxed{631\ \text{N}}$

**P16.27** Since $\mu$ is constant, $\mu=\dfrac{T_2}{v_2^2}=\dfrac{T_1}{v_1^2}$ and

$$T_2=\left(\frac{v_2}{v_1}\right)^2T_1=\left(\frac{30.0\ \text{m/s}}{20.0\ \text{m/s}}\right)^2(6.00\ \text{N})=\boxed{13.5\ \text{N}}.$$

**P16.28** The period of the pendulum is $T=2\pi\sqrt{\dfrac{L}{g}}$

Let $F$ represent the tension in the string (to avoid confusion with the period) when the pendulum is vertical and stationary. The speed of waves in the string is then:

$$v=\sqrt{\frac{F}{\mu}}=\sqrt{\frac{Mg}{\frac{m}{L}}}=\sqrt{\frac{MgL}{m}}$$

Since it might be difficult to measure $L$ precisely, we eliminate $\sqrt{L}=\dfrac{T\sqrt{g}}{2\pi}$

so $v=\sqrt{\dfrac{Mg}{m}}\dfrac{T\sqrt{g}}{2\pi}=\boxed{\dfrac{Tg}{2\pi}\sqrt{\dfrac{M}{m}}}$.

**P16.29** If the tension in the wire is $T$, the tensile stress is

$$\text{Stress} = \frac{T}{A} \qquad \text{so} \qquad T = A(\text{stress}).$$

The speed of transverse waves in the wire is

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{A(\text{Stress})}{\frac{m}{L}}} = \sqrt{\frac{\text{Stress}}{\frac{m}{AL}}} = \sqrt{\frac{\text{Stress}}{\frac{m}{\text{Volume}}}} = \sqrt{\frac{\text{Stress}}{\rho}}$$

where $\rho$ is the density. The maximum velocity occurs when the stress is a maximum:

$$v_{\max} = \sqrt{\frac{2.70 \times 10^8 \text{ Pa}}{7\,860 \text{ kg/m}^3}} = \boxed{185 \text{ m/s}}.$$

**P16.30** From the free-body diagram $\quad mg = 2T\sin\theta$

$$T = \frac{mg}{2\sin\theta}$$

The angle $\theta$ is found from $\quad \cos\theta = \frac{\frac{3L}{8}}{\frac{L}{2}} = \frac{3}{4}$

$$\therefore \theta = 41.4°$$

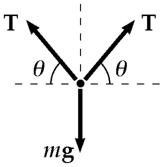


**FIG. P16.30**

(a) $\quad v = \sqrt{\frac{T}{\mu}}$

$$v = \sqrt{\frac{mg}{2\mu\sin 41.4°}} = \left(\sqrt{\frac{9.80 \text{ m/s}^2}{2\left(8.00 \times 10^{-3} \text{ kg/m}\right)\sin 41.4°}}\right)\sqrt{m}$$

or $\quad v = \boxed{\left(30.4 \frac{\text{m/s}}{\sqrt{\text{kg}}}\right)\sqrt{m}}$

(b) $\quad v = 60.0 = 30.4\sqrt{m}$ and $\boxed{m = 3.89 \text{ kg}}$

**P16.31** The total time is the sum of the two times.

In each wire $\quad t = \frac{L}{v} = L\sqrt{\frac{\mu}{T}}$

Let $A$ represent the cross-sectional area of one wire. The mass of one wire can be written both as $m = \rho V = \rho AL$ and also as $m = \mu L$.

Then we have $\quad \mu = \rho A = \frac{\pi\rho d^2}{4}$

Thus, $\quad t = L\left(\frac{\pi\rho d^2}{4T}\right)^{1/2}$

For copper, $\quad t = (20.0)\left[\frac{(\pi)(8\,920)\left(1.00 \times 10^{-3}\right)^2}{(4)(150)}\right]^{1/2} = 0.137 \text{ s}$

For steel, $\quad t = (30.0)\left[\frac{(\pi)(7\,860)\left(1.00 \times 10^{-3}\right)^2}{(4)(150)}\right]^{1/2} = 0.192 \text{ s}$

The total time is $\quad 0.137 + 0.192 = \boxed{0.329 \text{ s}}$

**P16.32** Refer to the diagrams. From the free-body diagram of point $A$:

$$\sum F_y = 0 \Rightarrow T_1 \sin\theta = Mg \qquad \text{and} \qquad \sum F_x = 0 \Rightarrow T_1 \cos\theta = T$$

Combining these equations to eliminate $T_1$ gives the tension in the string connecting points $A$ and $B$ as: $T = \dfrac{Mg}{\tan\theta}$.

The speed of transverse waves in this segment of string is then

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{\frac{Mg}{\tan\theta}}{\frac{m}{L}}} = \sqrt{\frac{MgL}{m\tan\theta}}$$

and the time for a pulse to travel from $A$ to $B$ is

$$t = \frac{\frac{L}{2}}{v} = \boxed{\sqrt{\frac{mL\tan\theta}{4Mg}}}.$$

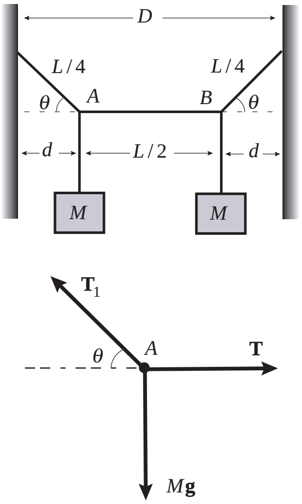


**FIG. P16.32**

***P16.33** (a) $f$ has units $\text{Hz} = 1/\text{s}$, so $T = \dfrac{1}{f}$ has units of seconds, $\boxed{\text{s}}$. For the other $T$ we have $T = \mu v^2$, with units $\dfrac{\text{kg}}{\text{m}}\dfrac{\text{m}^2}{\text{s}^2} = \dfrac{\text{kg}\cdot\text{m}}{\text{s}^2} = \boxed{\text{N}}$.

(b) The first $T$ is $\boxed{\text{period}}$ of time; the second is $\boxed{\text{force}}$ of tension.

## Section 16.4 **Reflection and Transmission**

Problem 7 in Chapter 18 can be assigned with this section.

## Section 16.5 **Rate of Energy Transfer by Sinusoidal Waves on Strings**

**P16.34** $f = \dfrac{v}{\lambda} = \dfrac{30.0}{0.500} = 60.0\text{ Hz}$ $\qquad \omega = 2\pi f = 120\pi\text{ rad/s}$

$$\mathcal{P} = \frac{1}{2}\mu\omega^2 A^2 v = \frac{1}{2}\left(\frac{0.180}{3.60}\right)(120\pi)^2(0.100)^2(30.0) = \boxed{1.07\text{ kW}}$$

**P16.35** Suppose that no energy is absorbed or carried down into the water. Then a fixed amount of power is spread thinner farther away from the source, spread over the circumference $2\pi r$ of an expanding circle. The power-per-width across the wave front

$$\frac{\mathcal{P}}{2\pi r}$$

is proportional to amplitude squared so amplitude is proportional to

$$\sqrt{\frac{\mathcal{P}}{2\pi r}}.$$

**P16.36** $T=$ constant; $v=\sqrt{\dfrac{T}{\mu}}$; $\mathscr{P}=\dfrac{1}{2}\mu\omega^2A^2v$

(a) If $L$ is doubled, $v$ remains constant and $\boxed{\mathscr{P}\text{ is constant}}$.

(b) If $A$ is doubled and $\omega$ is halved, $\mathscr{P}\propto\omega^2A^2$ $\boxed{\text{remains constant}}$.

(c) If $\lambda$ and $A$ are doubled, the product $\omega^2A^2\propto\dfrac{A^2}{\lambda^2}$ remains constant, so $\boxed{\mathscr{P}\text{ remains constant}}$.

(d) If $L$ and $\lambda$ are halved, then $\omega^2\propto\dfrac{1}{\lambda^2}$ is quadrupled, so $\boxed{\mathscr{P}\text{ is quadrupled}}$. (Changing $L$ doesn't affect $\mathscr{P}$).

**P16.37** $A=5.00\times10^{-2}$ m $\quad\mu=4.00\times10^{-2}$ kg/m $\quad\mathscr{P}=300$ W $\quad T=100$ N

Therefore, $v=\sqrt{\dfrac{T}{\mu}}=50.0$ m/s

$$\mathscr{P}=\frac{1}{2}\mu\omega^2A^2v:\quad \omega^2=\frac{2\mathscr{P}}{\mu A^2v}=\frac{2(300)}{(4.00\times10^{-2})(5.00\times10^{-2})^2(50.0)}$$

$$\omega=346\text{ rad/s}$$

$$f=\frac{\omega}{2\pi}=\boxed{55.1\text{ Hz}}$$

**P16.38** $\mu=30.0\text{ g/m}=30.0\times10^{-3}\text{ kg/m}$

$\lambda=1.50$ m

$f=50.0$ Hz: $\quad\omega=2\pi f=314\text{ s}^{-1}$

$2A=0.150$ m: $\quad A=7.50\times10^{-2}$ m

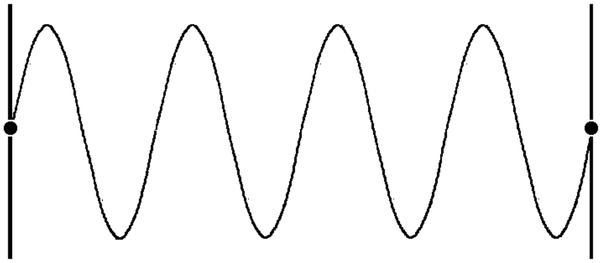


**FIG. P16.38**

(a) $y=A\sin\left(\dfrac{2\pi}{\lambda}x-\omega t\right)$

$$\boxed{y=(7.50\times10^{-2})\sin(4.19x-314t)}$$

(b) $\mathscr{P}=\dfrac{1}{2}\mu\omega^2A^2v=\dfrac{1}{2}(30.0\times10^{-3})(314)^2(7.50\times10^{-2})^2\left(\dfrac{314}{4.19}\right)$ W $\quad\boxed{\mathscr{P}=625\text{ W}}$

**P16.39** (a) $v=f\lambda=\dfrac{\omega}{2\pi}\dfrac{2\pi}{k}=\dfrac{\omega}{k}=\dfrac{50.0}{0.800}$ m/s $=\boxed{62.5\text{ m/s}}$

(b) $\lambda=\dfrac{2\pi}{k}=\dfrac{2\pi}{0.800}$ m $=\boxed{7.85\text{ m}}$

(c) $f=\dfrac{50.0}{2\pi}=\boxed{7.96\text{ Hz}}$

(d) $\mathscr{P}=\dfrac{1}{2}\mu\omega^2A^2v=\dfrac{1}{2}(12.0\times10^{-3})(50.0)^2(0.150)^2(62.5)$ W $=\boxed{21.1\text{ W}}$

***P16.40** Comparing $y = 0.35\sin\left(10\pi t - 3\pi x + \frac{\pi}{4}\right)$ with $y = A\sin(kx - \omega t + \phi) = A\sin(\omega t - kx - \phi + \pi)$ we have $k = \frac{3\pi}{m}$, $\omega = 10\pi/\text{s}$, $A = 0.35\text{ m}$. Then $v = f\lambda = 2\pi f\frac{\lambda}{2\pi} = \frac{\omega}{k} = \frac{10\pi/\text{s}}{3\pi/\text{m}} = 3.33\text{ m/s}$.

(a) The rate of energy transport is

$$\mathcal{P} = \frac{1}{2}\mu\omega^2A^2v = \frac{1}{2}\left(75\times10^{-3}\text{ kg/m}\right)(10\pi/\text{s})^2(0.35\text{ m})^2 3.33\text{ m/s} = \boxed{15.1\text{ W}}.$$

(b) The energy per cycle is

$$E_\lambda = \mathcal{P}T = \frac{1}{2}\mu\omega^2A^2\lambda = \frac{1}{2}\left(75\times10^{-3}\text{ kg/m}\right)(10\pi/\text{s})^2(0.35\text{ m})^2\frac{2\pi\text{ m}}{3\pi} = \boxed{3.02\text{ J}}.$$

**P16.41** Originally,

$$\mathcal{P}_0 = \frac{1}{2}\mu\omega^2A^2v$$
$$\mathcal{P}_0 = \frac{1}{2}\mu\omega^2A^2\sqrt{\frac{T}{\mu}}$$
$$\mathcal{P}_0 = \frac{1}{2}\omega^2A^2\sqrt{T\mu}$$

The doubled string will have doubled mass-per-length. Presuming that we hold tension constant, it can carry power larger by $\sqrt{2}$ times.

$$\boxed{\sqrt{2}\mathcal{P}_0} = \frac{1}{2}\omega^2A^2\sqrt{T2\mu}$$

***P16.42** As for a strong wave, the rate of energy transfer is proportional to the square of the amplitude and to the speed. We write $\mathcal{P} = FvA^2$ where $F$ is some constant. With no absorption of energy,

$$Fv_{\text{bedrock}}A^2_{\text{bedrock}} = Fv_{\text{mudfill}}A^2_{\text{mudfill}}$$
$$\sqrt{\frac{v_{\text{bedrock}}}{v_{\text{mudfill}}}} = \frac{A_{\text{mudfill}}}{A_{\text{bedrock}}} = \sqrt{\frac{25v_{\text{mudfill}}}{v_{\text{mudfill}}}} = 5$$

$\boxed{\text{The amplitude increases by 5.00 times.}}$

## Section 16.6 **The Linear Wave Equation**

**P16.43** (a) $A = (7.00 + 3.00)4.00$ yields $\boxed{A = 40.0}$

(b) In order for two vectors to be equal, they must have the same magnitude and the same direction in three-dimensional space. All of their components must be equal. Thus, $7.00\hat{\mathbf{i}} + 0\hat{\mathbf{j}} + 3.00\hat{\mathbf{k}} = A\hat{\mathbf{i}} + B\hat{\mathbf{j}} + C\hat{\mathbf{k}}$ requires $\boxed{A = 7.00,\ B = 0,\ \text{and } C = 3.00}$.

(c) In order for two functions to be identically equal, they must be equal for every value of every variable. They must have the same graphs. In

$$A + B\cos(Cx + Dt + E) = 0 + 7.00\ \text{mm}\cos(3.00x + 4.00t + 2.00),$$

the equality of average values requires that $\boxed{A = 0}$. The equality of maximum values requires $\boxed{B = 7.00\ \text{mm}}$. The equality for the wavelength or periodicity as a function of $x$ requires $\boxed{C = 3.00\ \text{rad/m}}$. The equality of period requires $\boxed{D = 4.00\ \text{rad/s}}$, and the equality of zero-crossings requires $\boxed{E = 2.00\ \text{rad}}$.

***P16.44** The linear wave equation is $\dfrac{\partial^2 y}{\partial x^2} = \dfrac{1}{v^2}\dfrac{\partial^2 y}{\partial t^2}$

If $y = e^{b(x-vt)}$

then $\dfrac{\partial y}{\partial t} = -bve^{b(x-vt)}$ and $\dfrac{\partial y}{\partial x} = be^{b(x-vt)}$

$$\frac{\partial^2 y}{\partial t^2} = b^2v^2e^{b(x-vt)} \text{ and } \frac{\partial^2 y}{\partial x^2} = b^2e^{b(x-vt)}$$

Therefore, $\dfrac{\partial^2 y}{\partial t^2} = v^2\dfrac{\partial^2 y}{\partial x^2}$, demonstrating that $e^{b(x-vt)}$ is a solution

**P16.45** The linear wave equation is $\dfrac{1}{v^2}\dfrac{\partial^2 y}{\partial t^2} = \dfrac{\partial^2 y}{\partial x^2}$

To show that $y = \ln[b(x - vt)]$ is a solution, we find its first and second derivatives with respect to $x$ and $t$ and substitute into the equation.

$$\frac{\partial y}{\partial t} = \frac{1}{b(x-vt)}(-bv) \qquad \frac{\partial^2 y}{\partial t^2} = \frac{-1(-bv)^2}{b^2(x-vt)^2} = -\frac{v^2}{(x-vt)^2}$$

$$\frac{\partial y}{\partial x} = [b(x-vt)]^{-1}b \qquad \frac{\partial^2 y}{\partial x^2} = -\frac{b}{b}(x-vt)^2 = -\frac{1}{(x-vt)^2}$$

Then $\dfrac{1}{v^2}\dfrac{\partial^2 y}{\partial t^2} = \dfrac{1}{v^2}\dfrac{(-v^2)}{(x-vt)^2} = -\dfrac{1}{(x-vt)^2} = \dfrac{\partial^2 y}{\partial x^2}$ so the given wave function is a solution.

**P16.46** (a) From $y = x^2 + v^2t^2$,

evaluate

$$\frac{\partial y}{\partial x} = 2x \qquad \frac{\partial^2 y}{\partial x^2} = 2$$

$$\frac{\partial y}{\partial t} = v^2 2t \qquad \frac{\partial^2 y}{\partial t^2} = 2v^2$$

Does $\frac{\partial^2 y}{\partial t^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$ ?

By substitution: $2 = \frac{1}{v^2} 2v^2$ and this is true, so the wave function does satisfy the wave equation.

(b) Note $\frac{1}{2}(x+vt)^2 + \frac{1}{2}(x-vt)^2 = \frac{1}{2}x^2 + xvt + \frac{1}{2}v^2t^2 + \frac{1}{2}x^2 - xvt + \frac{1}{2}v^2t^2$

$= x^2 + v^2t^2$ as required.

So $\boxed{f(x+vt) = \frac{1}{2}(x+vt)^2}$ and $\boxed{g(x-vt) = \frac{1}{2}(x-vt)^2}$.

(c) $y = \sin x \cos vt$ makes

$$\frac{\partial y}{\partial x} = \cos x \cos vt \qquad \frac{\partial^2 y}{\partial x^2} = -\sin x \cos vt$$

$$\frac{\partial y}{\partial t} = -v \sin x \sin vt \qquad \frac{\partial^2 y}{\partial t^2} = -v^2 \sin x \cos vt$$

Then $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$

becomes $-\sin x \cos vt = \frac{-1}{v^2} v^2 \sin x \cos vt$ which is true as required.

Note $\sin(x+vt) = \sin x \cos vt + \cos x \sin vt$

$\sin(x-vt) = \sin x \cos vt - \cos x \sin vt$.

So $\sin x \cos vt = f(x+vt) + g(x-vt)$ with

$$\boxed{f(x+vt) = \frac{1}{2}\sin(x+vt)} \quad \text{and} \quad \boxed{g(x-vt) = \frac{1}{2}\sin(x-vt)}.$$

## Additional Problems

**P16.47** Assume a typical distance between adjacent people $\sim 1$ m.

Then the wave speed is $v = \frac{\Delta x}{\Delta t} \sim \frac{1 \text{ m}}{0.1 \text{ s}} \sim 10 \text{ m/s}$

Model the stadium as a circle with a radius of order 100 m. Then, the time for one circuit around the stadium is

$$T = \frac{2\pi r}{v} \sim \frac{2\pi(10^2)}{10 \text{ m/s}} = 63 \text{ s} \boxed{\sim 1 \text{ min}}.$$

**P16.48** Compare the given wave function $y = 4.00\sin(2.00x - 3.00t)$ cm to the general form $y = A\sin(kx - \omega t)$ to find

(a) amplitude $A = 4.00\text{ cm} = \boxed{0.040\,0\text{ m}}$

(b) $k = \dfrac{2\pi}{\lambda} = 2.00\text{ cm}^{-1}$ and $\lambda = \pi\,\text{cm} = \boxed{0.031\,4\text{ m}}$

(c) $\omega = 2\pi f = 3.00\text{ s}^{-1}$ and $f = \boxed{0.477\text{ Hz}}$

(d) $T = \dfrac{1}{f} = \boxed{2.09\text{ s}}$

(e) The minus sign indicates that the wave is traveling in the $\boxed{\text{positive } x\text{-direction}}$.

**P16.49** (a) Let $u = 10\pi t - 3\pi x + \dfrac{\pi}{4}$   $\dfrac{du}{dt} = 10\pi - 3\pi\dfrac{dx}{dt} = 0$ at a point of constant phase

$$\frac{dx}{dt} = \frac{10}{3} = \boxed{3.33\text{ m/s}}$$

The velocity is in the $\boxed{\text{positive } x\text{-direction}}$.

(b) $y(0.100,\ 0) = (0.350\text{ m})\sin\left(-0.300\pi + \dfrac{\pi}{4}\right) = -0.054\,8\text{ m} = \boxed{-5.48\text{ cm}}$

(c) $k = \dfrac{2\pi}{\lambda} = 3\pi:\ \ \lambda = \boxed{0.667\text{ m}}$   $\omega = 2\pi f = 10\pi:\ f = \boxed{5.00\text{ Hz}}$

(d) $v_y = \dfrac{\partial y}{\partial t} = (0.350)(10\pi)\cos\left(10\pi t - 3\pi x + \dfrac{\pi}{4}\right)$   $v_{y,\,\max} = (10\pi)(0.350) = \boxed{11.0\text{ m/s}}$

***P16.50** (a) $0.175\text{ m} = (0.350\text{ m})\sin\left[(99.6\text{ rad/s})t\right]$

$\therefore \sin\left[(99.6\text{ rad/s})t\right] = 0.5$

The smallest two angles for which the sine function is 0.5 are 30° and 150°, i.e., 0.523 6 rad and 2.618 rad.

$(99.6\text{ rad/s})t_1 = 0.523\,6\text{ rad}$, thus $t_1 = 5.26\text{ ms}$

$(99.6\text{ rad/s})t_2 = 2.618\text{ rad}$, thus $t_2 = 26.3\text{ ms}$

$\Delta t \equiv t_2 - t_1 = 26.3\text{ ms} - 5.26\text{ ms} = \boxed{21.0\text{ ms}}$

(b) Distance traveled by the wave $= \left(\dfrac{\omega}{k}\right)\Delta t = \left(\dfrac{99.6\text{ rad/s}}{1.25\text{ rad/m}}\right)\left(21.0\times10^{-3}\text{ s}\right) = \boxed{1.68\text{ m}}$.

**P16.51** The equation $v = \lambda f$ is a special case of

speed = (cycle length)(repetition rate).

Thus, $v = \left(19.0\times10^{-3}\text{ m/frame}\right)\left(24.0\text{ frames/s}\right) = \boxed{0.456\text{ m/s}}$.

**P16.52** Assuming the incline to be frictionless and taking the positive $x$-direction to be up the incline:

$\sum F_x = T - Mg\sin\theta = 0$ or the tension in the string is $T = Mg\sin\theta$

The speed of transverse waves in the string is then $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{Mg\sin\theta}{\frac{m}{L}}} = \sqrt{\dfrac{MgL\sin\theta}{m}}$

The time interval for a pulse to travel the string's length is $\Delta t = \dfrac{L}{v} = L\sqrt{\dfrac{m}{MgL\sin\theta}} = \boxed{\sqrt{\dfrac{mL}{Mg\sin\theta}}}$

**P16.53** Energy is conserved as the block moves down distance $x$:

$$\left(K + U_g + U_s\right)_{\text{top}} + \Delta E = \left(K + U_g + U_s\right)_{\text{bottom}}$$

$$0 + Mgx + 0 + 0 = 0 + 0 + \frac{1}{2}kx^2$$

$$x = \frac{2Mg}{k}$$

(a) $T = kx = 2Mg = 2(2.00\text{ kg})\left(9.80\text{ m/s}^2\right) = \boxed{39.2\text{ N}}$

(b) $L = L_0 + x = L_0 + \dfrac{2Mg}{k}$

$L = 0.500\text{ m} + \dfrac{39.2\text{ N}}{100\text{ N/m}} = \boxed{0.892\text{ m}}$

(c) $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{TL}{m}}$

$v = \sqrt{\dfrac{39.2\text{ N} \times 0.892\text{ m}}{5.0 \times 10^{-3}\text{ kg}}}$

$v = \boxed{83.6\text{ m/s}}$

**P16.54** $Mgx = \dfrac{1}{2}kx^2$

(a) $T = kx = \boxed{2Mg}$

(b) $L = L_0 + x = \boxed{L_0 + \dfrac{2Mg}{k}}$

(c) $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{TL}{m}} = \boxed{\sqrt{\dfrac{2Mg}{m}\left(L_0 + \dfrac{2Mg}{k}\right)}}$

**P16.55** (a) $$v=\sqrt{\frac{T}{\mu}}=\sqrt{\frac{80.0\text{ N}}{\left(5.00\times10^{-3}\text{ kg}/2.00\text{ m}\right)}}=\boxed{179\text{ m/s}}$$

(b) From Equation 16.21, $\mathcal{P}=\frac{1}{2}\mu v\omega^2A^2$ and $\omega=2\pi\left(\frac{v}{\lambda}\right)$

$$\mathcal{P}=\frac{1}{2}\mu vA^2\left(\frac{2\pi v}{\lambda}\right)^2=\frac{2\pi^2\mu A^2v^3}{\lambda^2}$$

$$\mathcal{P}=\frac{2\pi^2\left(\frac{5.00\times10^{-3}\text{ kg}}{2.00\text{ m}}\right)(0.040\,0\text{ m})^2(179\text{ m/s})^3}{(0.160\text{ m})^2}$$

$$\mathcal{P}=1.77\times10^4\text{ W}=\boxed{17.7\text{ kW}}$$

**P16.56** $v=\sqrt{\frac{T}{\mu}}$ and in this case $T=mg$; therefore, $m=\frac{\mu v^2}{g}$.

Now $v=f\lambda$ implies $v=\frac{\omega}{k}$ so that

$$m=\frac{\mu}{g}\left(\frac{\omega}{k}\right)^2=\frac{0.250\text{ kg/m}}{9.80\text{ m/s}^2}\left[\frac{18\pi\text{ s}^{-1}}{0.750\pi\text{ m}^{-1}}\right]^2=\boxed{14.7\text{ kg}}.$$

***P16.57** Let $M=$ mass of block, $m=$ mass of string. For the block, $\sum F=ma$ implies $T=\frac{mv_b^2}{r}=m\omega^2r$. The speed of a wave on the string is then

$$v=\sqrt{\frac{T}{\mu}}=\sqrt{\frac{M\omega^2r}{\frac{m}{r}}}=r\omega\sqrt{\frac{M}{m}}$$

$$t=\frac{r}{v}=\frac{1}{\omega}\sqrt{\frac{m}{M}}$$

$$\theta=\omega t=\sqrt{\frac{m}{M}}=\sqrt{\frac{0.003\,2\text{ kg}}{0.450\text{ kg}}}=\boxed{0.084\,3\text{ rad}}$$

**P16.58** (a) $$\mu=\frac{dm}{dL}=\rho A\frac{dx}{dx}=\rho A$$

$$v=\sqrt{\frac{T}{\mu}}=\sqrt{\frac{T}{\rho A}}=\sqrt{\frac{T}{\left[\rho(ax+b)\right]}}=\sqrt{\frac{T}{\left[\rho\left(10^{-3}x+10^{-2}\right)\text{cm}^2\right]}}$$

With all SI units, $$\boxed{v=\sqrt{\frac{T}{\left[\rho\left(10^{-3}x+10^{-2}\right)10^{-4}\right]}}\text{ m/s}}$$

(b) $$v\big|_{x=0}=\sqrt{\frac{24.0}{\left[(2\,700)\left(0+10^{-2}\right)\left(10^{-4}\right)\right]}}=\boxed{94.3\text{ m/s}}$$

$$v\big|_{x=10.0}=\sqrt{\frac{24.0}{\left[(2\,700)\left(10^{-2}+10^{-2}\right)\left(10^{-4}\right)\right]}}=\boxed{66.7\text{ m/s}}$$

**P16.59** $v=\sqrt{\frac{T}{\mu}}$ where $T=\mu xg$, the weight of a length $x$, of rope.

Therefore, $v=\sqrt{gx}$

But $v=\frac{dx}{dt}$, so that $dt=\frac{dx}{\sqrt{gx}}$

and $t=\int_0^L \frac{dx}{\sqrt{gx}}=\frac{1}{\sqrt{g}}\left.\frac{\sqrt{x}}{\frac{1}{2}}\right|_0^L=\boxed{2\sqrt{\frac{L}{g}}}$

**P16.60** At distance $x$ from the bottom, the tension is $T=\left(\frac{mxg}{L}\right)+Mg$, so the wave speed is:

$$v=\sqrt{\frac{T}{\mu}}=\sqrt{\frac{TL}{m}}=\sqrt{xg+\left(\frac{MgL}{m}\right)}=\frac{dx}{dt}.$$

(a) Then $t=\int_0^t dt=\int_0^L\left[xg+\left(\frac{MgL}{m}\right)\right]^{-1/2}dx$ $\qquad t=\frac{1}{g}\left.\frac{\left[xg+(MgL/m)\right]^{1/2}}{\frac{1}{2}}\right|_{x=0}^{x=L}$

$$t=\frac{2}{g}\left[\left(Lg+\frac{MgL}{m}\right)^{1/2}-\left(\frac{MgL}{m}\right)^{1/2}\right] \qquad \boxed{t=2\sqrt{\frac{L}{g}}\left(\frac{\sqrt{m+M}-\sqrt{M}}{\sqrt{m}}\right)}$$

(b) When $M=0$, as in the previous problem, $t=2\sqrt{\frac{L}{g}}\left(\frac{\sqrt{m}-0}{\sqrt{m}}\right)=\boxed{2\sqrt{\frac{L}{g}}}$

(c) As $m\to 0$ we expand $\sqrt{m+M}=\sqrt{M}\left(1+\frac{m}{M}\right)^{1/2}=\sqrt{M}\left(1+\frac{1}{2}\frac{m}{M}-\frac{1}{8}\frac{m^2}{M^2}+\ldots\right)$

to obtain $t=2\sqrt{\frac{L}{g}}\left(\frac{\sqrt{M}+\frac{1}{2}\left(m/\sqrt{M}\right)-\frac{1}{8}\left(m^2/M^{3/2}\right)+\ldots-\sqrt{M}}{\sqrt{m}}\right)$

$$t\approx 2\sqrt{\frac{L}{g}}\left(\frac{1}{2}\sqrt{\frac{m}{M}}\right)=\boxed{\sqrt{\frac{mL}{Mg}}}$$

**P16.61** (a) The speed in the lower half of a rope of length $L$ is the same function of distance (from the bottom end) as the speed along the entire length of a rope of length $\left(\frac{L}{2}\right)$.

Thus, the time required $=2\sqrt{\frac{L'}{g}}$ with $L'=\frac{L}{2}$

and the time required $=2\sqrt{\frac{L}{2g}}=\boxed{0.707\left(2\sqrt{\frac{L}{g}}\right)}$.

It takes the pulse more that 70% of the total time to cover 50% of the distance.

(b) By the same reasoning applied in part (a), the distance climbed in $\tau$ is given by $d=\frac{g\tau^2}{4}$.

For $\tau=\frac{t}{2}=\sqrt{\frac{L}{g}}$, we find the *distance climbed* $=\boxed{\frac{L}{4}}$.

In half the total trip time, the pulse has climbed $\frac{1}{4}$ of the total length.

**P16.62** (a) $v = \frac{\omega}{k} = \frac{15.0}{3.00} = \boxed{5.00 \text{ m/s in positive } x\text{-direction}}$

(b) $v = \frac{15.0}{3.00} = \boxed{5.00 \text{ m/s in negative } x\text{-direction}}$

(c) $v = \frac{15.0}{2.00} = \boxed{7.50 \text{ m/s in negative } x\text{-direction}}$

(d) $v = \frac{12.0}{\frac{1}{2}} = \boxed{24.0 \text{ m/s in positive } x\text{-direction}}$

**P16.63** Young's modulus for the wire may be written as $Y = \frac{\frac{T}{A}}{\frac{\Delta L}{L}}$, where $T$ is the tension maintained in the wire and $\Delta L$ is the elongation produced by this tension. Also, the mass density of the wire may be expressed as $\rho = \frac{\mu}{A}$.

The speed of transverse waves in the wire is then

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{\frac{T}{A}}{\frac{\mu}{A}}} = \sqrt{\frac{Y\left(\frac{\Delta L}{L}\right)}{\rho}}$$

and the strain in the wire is $\frac{\Delta L}{L} = \frac{\rho v^2}{Y}$.

If the wire is aluminum and $v = 100$ m/s, the strain is

$$\frac{\Delta L}{L} = \frac{\left(2.70 \times 10^3 \text{ kg/m}^3\right)(100 \text{ m/s})^2}{7.00 \times 10^{10} \text{ N/m}^2} = \boxed{3.86 \times 10^{-4}}.$$

***P16.64** (a) Consider a short section of chain at the top of the loop. A free-body diagram is shown. Its length is $s = R(2\theta)$ and its mass is $\mu R 2\theta$. In the frame of reference of the center of the loop, Newton's second law is

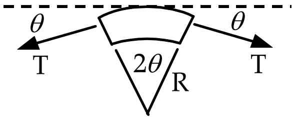


**FIG. P16.64(a)**

$$\sum F_y = ma_y \qquad 2T\sin\theta \text{ down} = \frac{mv_0^2}{R} \text{ down} = \frac{\mu R 2\theta v_0^2}{R}$$

For a very short section, $\sin\theta = \theta$ and $\boxed{T = \mu v_0^2}$.

(b) The wave speed is $v = \sqrt{\frac{T}{\mu}} = \boxed{v_0}$.

(c) In the frame of reference of the center of the loop, each pulse moves with equal speed clockwise and counterclockwise.

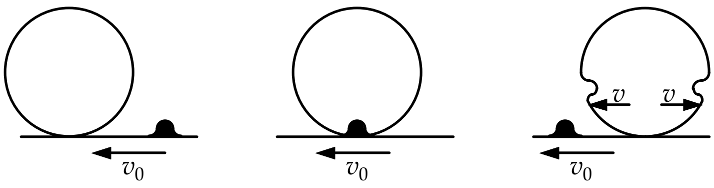


**FIG. P16.64(c-1)**

*continued on next page*

In the frame of reference of the ground, once pulse moves backward at speed $v_0 + v = 2v_0$ and the other forward at $v_0 - v = 0$. The one pulse makes two revolutions while the loop makes one revolution and the other pulse does not move around the loop. If it is generated at the six-o'clock position, it will stay at the six-o'clock position.

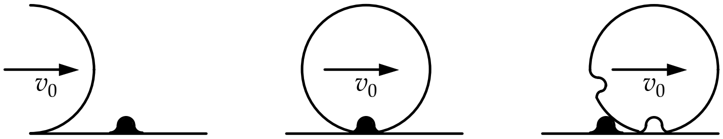


**FIG. P16.64(c-2)**

**P16.65** (a) Assume the spring is originally stationary throughout, extended to have a length *L* much greater than its equilibrium length. We start moving one end forward with the speed *v* at which a wave propagates on the spring. In this way we create a single pulse of compression that moves down the length of the spring. For an increment of spring with length *dx* and mass *dm*, just as the pulse swallows it up, $\sum F = ma$

becomes $kdx = adm$ or $\frac{k}{\frac{dm}{dx}} = a$.

But $\frac{dm}{dx} = \mu$ so $a = \frac{k}{\mu}$.

Also, $a = \frac{dv}{dt} = \frac{v}{t}$ when $v_i = 0$. But $L = vt$, so $a = \frac{v^2}{L}$.

Equating the two expressions for *a*, we have $\frac{k}{\mu} = \frac{v^2}{L}$ or $\boxed{v = \sqrt{\frac{kL}{\mu}}}$.

(b) Using the expression from part (a) $v = \sqrt{\frac{kL}{\mu}} = \sqrt{\frac{kL^2}{m}} = \sqrt{\frac{(100\ \text{N/m})(2.00\ \text{m})^2}{0.400\ \text{kg}}} = \boxed{31.6\ \text{m/s}}$.

**P16.66** (a) $v = \left(\frac{T}{\mu}\right)^{1/2} = \left(\frac{2T_0}{\mu_0}\right)^{1/2} = \boxed{v_0\sqrt{2}}$ where $v_0 \equiv \left(\frac{T_0}{\mu_0}\right)^{1/2}$

$v' = \left(\frac{T}{\mu'}\right)^{1/2} = \left(\frac{2T_0}{3\mu_0}\right)^{1/2} = \boxed{v_0\sqrt{\frac{2}{3}}}$

(b) $\Delta t_{\text{left}} = \frac{\frac{L}{2}}{v} = \frac{L}{2v_0\sqrt{2}} = \frac{\Delta t_0}{2\sqrt{2}} = 0.354\Delta t_0$ where $\Delta t_0 \equiv \frac{L}{v_0}$

$\Delta t_{\text{right}} = \frac{\frac{L}{2}}{v'} = \frac{L}{2v_0\sqrt{\frac{2}{3}}} = \frac{\Delta t_0}{2\sqrt{\frac{2}{3}}} = 0.612\Delta t_0$

$\Delta t_{\text{left}} + \Delta t_{\text{right}} = \boxed{0.966\Delta t_0}$

**P16.67** (a) $\mathscr{P}(x)=\frac{1}{2}\mu\omega^2A^2v=\frac{1}{2}\mu\omega^2A_0^2e^{-2bx}\left(\frac{\omega}{k}\right)=\boxed{\frac{\mu\omega^3}{2k}A_0^2e^{-2bx}}$

(b) $\mathscr{P}(0)=\boxed{\frac{\mu\omega^3}{2k}A_0^2}$

(c) $\frac{\mathscr{P}(x)}{\mathscr{P}(0)}=\boxed{e^{-2bx}}$

**P16.68** $v=\frac{4\,450\text{ km}}{9.50\text{ h}}=468\text{ km/h}=\boxed{130\text{ m/s}}$

$\bar{d}=\frac{v^2}{g}=\frac{(130\text{ m/s})^2}{\left(9.80\text{ m/s}^2\right)}=\boxed{1\,730\text{ m}}$

***P16.69** (a) $\mu(x)$ is a linear function, so it is of the form $\mu(x)=mx+b$

To have $\mu(0)=\mu_0$ we require $b=\mu_0$. Then $\mu(L)=\mu_L=mL+\mu_0$

so $m=\frac{\mu_L-\mu_0}{L}$

Then $\boxed{\mu(x)=\frac{(\mu_L-\mu_0)x}{L}+\mu_0}$

(b) From $v=\frac{dx}{dt}$, the time required to move from $x$ to $x+dx$ is $\frac{dx}{v}$. The time required to move from 0 to $L$ is

$$\Delta t=\int_0^L\frac{dx}{v}=\int_0^L\frac{dx}{\sqrt{\frac{T}{\mu}}}=\frac{1}{\sqrt{T}}\int_0^L\sqrt{\mu(x)}\,dx$$

$$\Delta t=\frac{1}{\sqrt{T}}\int_0^L\left(\frac{(\mu_L-\mu_0)x}{L}+\mu_0\right)^{1/2}\left(\frac{\mu_L-\mu_0}{L}\right)dx\left(\frac{L}{\mu_L-\mu_0}\right)$$

$$\Delta t=\frac{1}{\sqrt{T}}\left(\frac{L}{\mu_L-\mu_0}\right)\left(\frac{(\mu_L-\mu_0)x}{L}+\mu_0\right)^{3/2}\frac{1}{\frac{3}{2}}\Bigg|_0^L$$

$$\Delta t=\frac{2L}{3\sqrt{T}(\mu_L-\mu_0)}\left(\mu_L^{3/2}-\mu_0^{3/2}\right)$$

$$\Delta t=\frac{2L\left(\sqrt{\mu_L}-\sqrt{\mu_0}\right)\left(\mu_L+\sqrt{\mu_L\mu_0}+\mu_0\right)}{3\sqrt{T}\left(\sqrt{\mu_L}-\sqrt{\mu_0}\right)\left(\sqrt{\mu_L}+\sqrt{\mu_0}\right)}$$

$$\Delta t=\frac{2L}{3\sqrt{T}}\left(\frac{\mu_L+\sqrt{\mu_L\mu_0}+\mu_0}{\sqrt{\mu_L}+\sqrt{\mu_0}}\right)$$

## ANSWERS TO EVEN PROBLEMS

| | |
|---|---|
| **P16.2** | see the solution |
| **P16.4** | (a) the P wave; (b) 665 s |
| **P16.6** | $0.800\ \text{m/s}$ |
| **P16.8** | $2.40\ \text{m/s}$ |
| **P16.10** | 0.300 m in the positive $x$-direction |
| **P16.12** | ±6.67 cm |
| **P16.14** | (a) see the solution; (b) 0.125 s; in agreement with the example |
| **P16.16** | (a) see the solution; (b) $18.0/\text{m}$; $83.3\ \text{ms}$;<br>$75.4\ \text{rad/s}$; $4.20\ \text{m/s}$;<br>(c) $(0.2\ \text{m})\sin(18x+75.4t-0.151)$ |
| **P16.18** | (a) 0.021 5 m; (b) 1.95 rad; (c) $5.41\ \text{m/s}$;<br>(d) $y(x,\ t)=$<br>$(0.021\,5\ \text{m})\sin(8.38x+80.0\pi t+1.95)$ |
| **P16.20** | (a) see the solution; (b) 3.18 Hz |
| **P16.22** | 30.0 N |
| **P16.24** | (a) $y=(0.2\ \text{mm})\sin(16x-3\,140t)$;<br>(b) 158 N |
| **P16.26** | 631 N |
| **P16.28** | $v=\dfrac{Tg}{2\pi}\sqrt{\dfrac{M}{m}}$ |
| **P16.30** | (a) $v=\left(30.4\ \dfrac{\text{m}}{\text{s}\cdot\sqrt{\text{kg}}}\right)\sqrt{m}$; (b) 3.89 kg |
| **P16.32** | $\sqrt{\dfrac{mL\tan\theta}{4Mg}}$ |
| **P16.34** | 1.07 kW |
| **P16.36** | (a), (b), (c) $\mathscr{P}$ is constant;<br>(d) $\mathscr{P}$ is quadrupled |
| **P16.38** | (a) $y=(0.075\,0)\sin(4.19x-314t)$;<br>(b) 625 W |
| **P16.40** | (a) $15.1\ \text{W}$; (b) 3.02 J |
| **P16.42** | The amplitude increases by 5.00 times |
| **P16.44** | see the solution |
| **P16.46** | (a) see the solution;<br>(b) $\frac{1}{2}(x+vt)^2+\frac{1}{2}(x-vt)^2$;<br>(c) $\frac{1}{2}\sin(x+vt)+\frac{1}{2}\sin(x-vt)$ |
| **P16.48** | (a) 0.040 0 m; (b) $0.031\,4\ \text{m}$;<br>(c) $0.477\ \text{Hz}$; (d) $2.09\ \text{s}$;<br>(e) positive $x$-direction |
| **P16.50** | (a) $21.0\ \text{ms}$; (b) 1.68 m |
| **P16.52** | $\Delta t=\sqrt{\dfrac{mL}{Mg\sin\theta}}$ |
| **P16.54** | (a) $2Mg$; (b) $L_0+\dfrac{2Mg}{k}$;<br>(c) $\sqrt{\dfrac{2Mg}{m}\left(L_0+\dfrac{2Mg}{k}\right)}$ |
| **P16.56** | 14.7 kg |
| **P16.58** | (a) $v=\sqrt{\dfrac{T}{\rho\left(10^{-7}x+10^{-6}\right)}}$ in SI units;<br>(b) $94.3\ \text{m/s}$; $66.7\ \text{m/s}$ |
| **P16.60** | see the solution |
| **P16.62** | (a) $5.00\hat{\mathbf{i}}\ \text{m/s}$; (b) $-5.00\hat{\mathbf{i}}\ \text{m/s}$;<br>(c) $-7.50\hat{\mathbf{i}}\ \text{m/s}$; (d) $24.0\hat{\mathbf{i}}\ \text{m/s}$ |
| **P16.64** | (a) $\mu v_0^2$; (b) $v_0$;<br>(c) One travels 2 rev and the other does not move around the loop. |

**P16.66** (a) $v = \left(\frac{2T_0}{\mu_0}\right)^{1/2} = v_0\sqrt{2}$;

$v' = \left(\frac{2T_0}{3\mu_0}\right)^{1/2} = v_0\sqrt{\frac{2}{3}}$; (b) $0.966\Delta t_0$

**P16.68** 130 m/s; 1.73 km

# 17

# Sound Waves

**CHAPTER OUTLINE**

17.1 Speed of Sound Waves
17.2 Periodic Sound Waves
17.3 Intensity of Periodic Sound Waves
17.4 The Doppler Effect
17.5 Digital Sound Recording
17.6 Motion Picture Sound

## ANSWERS TO QUESTIONS

**Q17.1** Sound waves are longitudinal because elements of the medium—parcels of air—move parallel and antiparallel to the direction of wave motion.

**Q17.2** We assume that a perfect vacuum surrounds the clock. The sound waves require a medium for them to travel to your ear. The hammer on the alarm will strike the bell, and the vibration will spread as sound waves through the body of the clock. If a bone of your skull were in contact with the clock, you would hear the bell. However, in the absence of a surrounding medium like air or water, no sound can be radiated away. A larger-scale example of the same effect: Colossal storms raging on the Sun are deathly still for us.

What happens to the sound energy within the clock? Here is the answer: As the sound wave travels through the steel and plastic, traversing joints and going around corners, its energy is converted into additional internal energy, raising the temperature of the materials. After the sound has died away, the clock will glow very slightly brighter in the infrared portion of the electromagnetic spectrum.

**Q17.3** If an object is $\frac{1}{2}$ meter from the sonic ranger, then the sensor would have to measure how long it would take for a sound pulse to travel one meter. Since sound of any frequency moves at about $343\ \text{m/s}$, then the sonic ranger would have to be able to measure a time difference of under 0.003 seconds. This small time measurement is possible with modern electronics. But it would be more expensive to outfit sonic rangers with the more sensitive equipment than it is to print “do not use to measure distances less than $\frac{1}{2}$ meter” in the users’ manual.

**Q17.4** The speed of sound to two significant figures is $340\ \text{m/s}$. Let’s assume that you can measure time to $\frac{1}{10}$ second by using a stopwatch. To get a speed to two significant figures, you need to measure a time of at least 1.0 seconds. Since $d = vt$, the minimum distance is 340 meters.

**Q17.5** The frequency increases by a factor of 2 because the wave speed, which is dependent only on the medium through which the wave travels, remains constant.

**Q17.6** When listening, you are approximately the same distance from all of the members of the group. If different frequencies traveled at different speeds, then you might hear the higher pitched frequencies before you heard the lower ones produced at the same time. Although it might be interesting to think that each listener heard his or her own personal performance depending on where they were seated, a time lag like this could make a Beethoven sonata sound as if it were written by Charles Ives.

**Q17.7** Since air is a viscous fluid, some of the energy of sound vibration is turned into internal energy. At such great distances, the amplitude of the signal is so decreased by this effect you re unable to hear it.

**Q17.8** We suppose that a point source has no structure, and radiates sound equally in all directions (isotropically). The sound wavefronts are expanding spheres, so the area over which the sound energy spreads increases according to $A = 4\pi r^2$. Thus, if the distance is tripled, the area increases by a factor of nine, and the new intensity will be one-ninth of the old intensity. This answer according to the inverse-square law applies if the medium is uniform and unbounded.

For contrast, suppose that the sound is confined to move in a horizontal layer. (Thermal stratification in an ocean can have this effect on sonar "pings.") Then the area over which the sound energy is dispersed will only increase according to the circumference of an expanding circle: $A = 2\pi rh$, and so three times the distance will result in one third the intensity.

In the case of an entirely enclosed speaking tube (such as a ship's telephone), the area perpendicular to the energy flow stays the same, and increasing the distance will not change the intensity appreciably.

**Q17.9** He saw the first wave he encountered, light traveling at $3.00 \times 10^8$ m/s. At the same moment, infrared as well as visible light began warming his skin, but some time was required to raise the temperature of the outer skin layers before he noticed it. The meteor produced compressional waves in the air and in the ground. The wave in the ground, which can be called either sound or a seismic wave, traveled much faster than the wave in air, since the ground is much stiffer against compression. Our witness received it next and noticed it as a little earthquake. He was no doubt unable to distinguish the P and S waves. The first air-compression wave he received was a shock wave with an amplitude on the order of meters. It transported him off his doorstep. Then he could hear some additional direct sound, reflected sound, and perhaps the sound of the falling trees.

**Q17.10** A microwave pulse is reflected from a moving object. The waves that are reflected back are Doppler shifted in frequency according to the speed of the target. The receiver in the radar gun detects the reflected wave and compares its frequency to that of the emitted pulse. Using the frequency shift, the speed can be calculated to high precision. Be forewarned: this technique works if you are either traveling toward or away from your local law enforcement agent!

**Q17.11** As you move towards the canyon wall, the echo of your car horn would be shifted up in frequency; as you move away, the echo would be shifted down in frequency.

**Q17.12** Normal conversation has an intensity level of about 60 dB.

**Q17.13** A rock concert has an intensity level of about 120 dB.
A cheering crowd has an intensity level of about 90 dB.
Normal conversation has an intensity level of about 50–60 dB.
Turning a page in the textbook has an intensity level of about 10–20 dB.

**Q17.14** One would expect the spectra of the light to be Doppler shifted up in frequency (blue shift) as the star approaches us. As the star recedes in its orbit, the frequency spectrum would be shifted down (red shift). While the star is moving perpendicular to our line of sight, there will be no frequency shift at all. Overall, the spectra would oscillate with a period equal to that of the orbiting stars.

**Q17.15** For the sound from a source not to shift in frequency, the radial velocity of the source relative to the observer must be zero; that is, the source must not be moving toward or away from the observer. The source can be moving in a plane perpendicular to the line between it and the observer. Other possibilities: The source and observer might both have zero velocity. They might have equal velocities relative to the medium. The source might be moving around the observer on a sphere of constant radius. Even if the source speeds up on the sphere, slows down, or stops, the frequency heard will be equal to the frequency emitted by the source.

**Q17.16** Wind can change a Doppler shift but cannot cause one. Both $v_o$ and $v_s$ in our equations must be interpreted as speeds of observer and source relative to the air. If source and observer are moving relative to each other, the observer will hear one shifted frequency in still air and a different shifted frequency if wind is blowing. If the distance between source and observer is constant, there will never be a Doppler shift.

**Q17.17** If the object being tracked is moving away from the observer, then the sonic pulse would never reach the object, as the object is moving away faster than the wave speed. If the object being tracked is moving towards the observer, then the object itself would reach the detector before reflected pulse.

**Q17.18** New-fallen snow is a wonderful acoustic absorber as it reflects very little of the sound that reaches it. It is full of tiny intricate air channels and does not spring back when it is distorted. It acts very much like acoustic tile in buildings. So where does the absorbed energy go? It turns into internal energy—albeit a very small amount.

**Q17.19** As a sound wave moves away from the source, its intensity decreases. With an echo, the sound must move from the source to the reflector and then back to the observer, covering a significant distance.

**Q17.20** The observer would most likely hear the sonic boom of the plane itself and then beep, baap, boop. Since the plane is supersonic, the loudspeaker would pull ahead of the leading "boop" wavefront before emitting the "baap", and so forth.

"How are you?" would be heard as "?uoy era woH"

**Q17.21** This system would be seen as a star moving in an elliptical path. Just like the light from a star in a binary star system, described in the answer to question 14, the spectrum of light from the star would undergo a series of Doppler shifts depending on the star's speed and direction of motion relative to the observer. The repetition rate of the Doppler shift pattern is the period of the orbit. Information about the orbit size can be calculated from the size of the Doppler shifts.

## SOLUTIONS TO PROBLEMS

### Section 17.1 **Speed of Sound Waves**

**P17.1** Since $v_{\text{light}} >> v_{\text{sound}}$: $d \approx (343\ \text{m/s})(16.2\ \text{s}) = \boxed{5.56\ \text{km}}$

**P17.2** $v = \sqrt{\dfrac{B}{\rho}} = \sqrt{\dfrac{2.80 \times 10^{10}}{13.6 \times 10^{3}}} = \boxed{1.43\ \text{km/s}}$

**P17.3** Sound takes this time to reach the man: $\frac{(20.0\text{ m}-1.75\text{ m})}{343\text{ m/s}}=5.32\times10^{-2}\text{ s}$

so the warning should be shouted no later than before the pot strikes. $0.300\text{ s}+5.32\times10^{-2}\text{ s}=0.353\text{ s}$

Since the whole time of fall is given by $y=\frac{1}{2}gt^2$: $18.25\text{ m}=\frac{1}{2}\left(9.80\text{ m/s}^2\right)t^2$

$t=1.93\text{ s}$

the warning needs to come $1.93\text{ s}-0.353\text{ s}=1.58\text{ s}$

into the fall, when the pot has fallen $\frac{1}{2}\left(9.80\text{ m/s}^2\right)(1.58\text{ s})^2=12.2\text{ m}$

to be above the ground by $20.0\text{ m}-12.2\text{ m}=\boxed{7.82\text{ m}}$

**P17.4** (a) At 9 000 m, $\Delta T=\left(\frac{9\,000}{150}\right)(-1.00°\text{C})=-60.0°\text{C}$ so $T=-30.0°\text{C}$.

Using the chain rule:

$$\frac{dv}{dt}=\frac{dv}{dT}\frac{dT}{dx}\frac{dx}{dt}=v\frac{dv}{dT}\frac{dT}{dx}=v(0.607)\left(\frac{1}{150}\right)=\frac{v}{247}\text{, so } dt=(247\text{ s})\frac{dv}{v}$$

$$\int_0^t dt=(247\text{ s})\int_{v_i}^{v_f}\frac{dv}{v}$$

$$t=(247\text{ s})\ln\left(\frac{v_f}{v_i}\right)=(247\text{ s})\ln\left[\frac{331.5+0.607(30.0)}{331.5+0.607(-30.0)}\right]$$

$t=\boxed{27.2\text{ s}}$ for sound to reach ground.

(b) $$t=\frac{h}{v}=\frac{9\,000}{[331.5+0.607(30.0)]}=\boxed{25.7\text{ s}}$$

It takes longer when the air cools off than if it were at a uniform temperature.

***P17.5** Let $x_1$ represent the cowboy's distance from the nearer canyon wall and $x_2$ his distance from the farther cliff. The sound for the first echo travels distance $2x_1$. For the second, $2x_2$. For the third, $2x_1+2x_2$. For the fourth echo, $2x_1+2x_2+2x_1$. Then $\frac{2x_2-2x_1}{340\text{ m/s}}=1.92\text{ s}$ and $\frac{2x_1+2x_2-2x_2}{340\text{ m/s}}=1.47\text{ s}$. Thus $x_1=\frac{1}{2}340\text{ m/s }1.47\text{ s}=250\text{ m}$ and $\frac{2x_2}{340\text{ m/s}}=1.92\text{ s}+1.47\text{ s}$; $x_2=576\text{ m}$.

(a) So $x_1+x_2=\boxed{826\text{ m}}$

(b) $$\frac{2x_1+2x_2+2x_1-(2x_1+2x_2)}{340\text{ m/s}}=\boxed{1.47\text{ s}}$$

**P17.6** It is easiest to solve part (b) first:

(b) The distance the sound travels to the plane is $d_s = \sqrt{h^2 + \left(\frac{h}{2}\right)^2} = \frac{h\sqrt{5}}{2}$.

The sound travels this distance in 2.00 s, so

$$d_s = \frac{h\sqrt{5}}{2} = (343\ \text{m/s})(2.00\ \text{s}) = 686\ \text{m}$$

giving the altitude of the plane as $h = \frac{2(686\ \text{m})}{\sqrt{5}} = \boxed{614\ \text{m}}$.

(a) The distance the plane has traveled in 2.00 s is $v(2.00\ \text{s}) = \frac{h}{2} = 307\ \text{m}$.

Thus, the speed of the plane is: $v = \frac{307\ \text{m}}{2.00\ \text{s}} = \boxed{153\ \text{m/s}}$.

## Section 17.2 **Periodic Sound Waves**

**P17.7** $\lambda = \frac{v}{f} = \frac{340\ \text{m/s}}{60.0 \times 10^3\ \text{s}^{-1}} = \boxed{5.67\ \text{mm}}$

***P17.8** The sound speed is $v = 331\ \text{m/s}\sqrt{1 + \frac{26°\text{C}}{273°\text{C}}} = 346\ \text{m/s}$

(a) Let $t$ represent the time for the echo to return. Then

$$d = \frac{1}{2}vt = \frac{1}{2}346\ \text{m/s}\ 24 \times 10^{-3}\ \text{s} = \boxed{4.16\ \text{m}}.$$

(b) Let $\Delta t$ represent the duration of the pulse:

$$\Delta t = \frac{10\lambda}{v} = \frac{10\lambda}{f\lambda} = \frac{10}{f} = \frac{10}{22 \times 10^6\ 1/\text{s}} = \boxed{0.455\ \mu\text{s}}.$$

(c) $L = 10\lambda = \frac{10v}{f} = \frac{10(346\ \text{m/s})}{22 \times 10^6\ 1/\text{s}} = \boxed{0.157\ \text{mm}}$

***P17.9** If $f = 1$ MHz, $\lambda = \frac{v}{f} = \frac{1\,500\ \text{m/s}}{10^6/\text{s}} = \boxed{1.50\ \text{mm}}$

If $f = 20$ MHz, $\lambda = \frac{1\,500\ \text{m/s}}{2 \times 10^7/\text{s}} = \boxed{75.0\ \mu\text{m}}$

**P17.10** $\Delta P_{\max} = \rho v \omega s_{\max}$

$$s_{\max} = \frac{\Delta P_{\max}}{\rho v \omega} = \frac{\left(4.00 \times 10^{-3}\ \text{N/m}^2\right)}{\left(1.20\ \text{kg/m}^3\right)(343\ \text{m/s})(2\pi)\left(10.0 \times 10^3\ \text{s}^{-1}\right)} = \boxed{1.55 \times 10^{-10}\ \text{m}}$$

**P17.11** (a) $A = \boxed{2.00\ \mu\text{m}}$

$$\lambda = \frac{2\pi}{15.7} = 0.400\ \text{m} = \boxed{40.0\ \text{cm}}$$

$$v = \frac{\omega}{k} = \frac{858}{15.7} = \boxed{54.6\ \text{m/s}}$$

(b) $$s = 2.00\cos\left[(15.7)(0.050\,0) - (858)\left(3.00\times10^{-3}\right)\right] = \boxed{-0.433\ \mu\text{m}}$$

(c) $$v_{\max} = A\omega = (2.00\ \mu\text{m})\left(858\ \text{s}^{-1}\right) = \boxed{1.72\ \text{mm/s}}$$

**P17.12** (a) $$\Delta P = (1.27\ \text{Pa})\sin\left(\frac{\pi x}{\text{m}} - \frac{340\pi t}{\text{s}}\right)\ \text{(SI units)}$$

The pressure amplitude is: $\Delta P_{\max} = \boxed{1.27\ \text{Pa}}$.

(b) $\omega = 2\pi f = 340\pi/\text{s}$, so $f = \boxed{170\ \text{Hz}}$

(c) $k = \frac{2\pi}{\lambda} = \pi/\text{m}$, giving $\lambda = \boxed{2.00\ \text{m}}$

(d) $v = \lambda f = (2.00\ \text{m})(170\ \text{Hz}) = \boxed{340\ \text{m/s}}$

**P17.13** $$k = \frac{2\pi}{\lambda} = \frac{2\pi}{(0.100\ \text{m})} = 62.8\ \text{m}^{-1}$$

$$\omega = \frac{2\pi v}{\lambda} = \frac{2\pi(343\ \text{m/s})}{(0.100\ \text{m})} = 2.16\times10^{4}\ \text{s}^{-1}$$

Therefore, $\boxed{\Delta P = (0.200\ \text{Pa})\sin\left[62.8\,x/\text{m} - 2.16\times10^{4}\,t/\text{s}\right]}$.

**P17.14** $$\omega = 2\pi f = \frac{2\pi v}{\lambda} = \frac{2\pi(343\ \text{m/s})}{(0.100\ \text{m})} = 2.16\times10^{4}\ \text{rad/s}$$

$$s_{\max} = \frac{\Delta P_{\max}}{\rho v\omega} = \frac{(0.200\ \text{Pa})}{\left(1.20\ \text{kg/m}^3\right)(343\ \text{m/s})\left(2.16\times10^{4}\ \text{s}^{-1}\right)} = 2.25\times10^{-8}\ \text{m}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{(0.100\ \text{m})} = 62.8\ \text{m}^{-1}$$

Therefore, $\boxed{s = s_{\max}\cos(kx - \omega t) = \left(2.25\times10^{-8}\ \text{m}\right)\cos\left(62.8x/\text{m} - 2.16\times10^{4}\,t/\text{s}\right)}$.

**P17.15** $$\Delta P_{\max} = \rho v\omega s_{\max} = \rho v\left(\frac{2\pi v}{\lambda}\right)s_{\max}$$

$$\lambda = \frac{2\pi\rho v^2 s_{\max}}{\Delta P_{\max}} = \frac{2\pi(1.20)(343)^2\left(5.50\times10^{-6}\right)}{0.840} = \boxed{5.81\ \text{m}}$$

**P17.16** (a) The sound "pressure" is extra tensile stress for one-half of each cycle. When it becomes $(0.500\%)\left(13.0\times10^{10}\ \text{Pa}\right)=6.50\times10^{8}\ \text{Pa}$, the rod will break. Then, $\Delta P_{\max}=\rho v\omega s_{\max}$

$$s_{\max}=\frac{\Delta P_{\max}}{\rho v\omega}=\frac{6.50\times10^{8}\ \text{N/m}^2}{\left(8.92\times10^{3}\ \text{kg/m}^3\right)(5\,010\ \text{m/s})(2\pi 500/\text{s})}=\boxed{4.63\ \text{mm}}.$$

(b) From $s=s_{\max}\cos(kx-\omega t)$

$$v=\frac{\partial s}{\partial t}=-\omega s_{\max}\sin(kx-\omega t)$$

$$v_{\max}=\omega s_{\max}=(2\pi 500/\text{s})(4.63\ \text{mm})=\boxed{14.5\ \text{m/s}}$$

(c) $$I=\frac{1}{2}\rho v\left(\omega s_{\max}\right)^2=\frac{1}{2}\rho v v_{\max}^2=\frac{1}{2}\left(8.92\times10^{3}\ \text{kg/m}^3\right)(5\,010\ \text{m/s})(14.5\ \text{m/s})^2$$

$$=\boxed{4.73\times10^{9}\ \text{W/m}^2}$$

***P17.17** Let $P(x)$ represent absolute pressure as a function of $x$. The net force to the right on the chunk of air is $+P(x)A-P(x+\Delta x)A$. Atmospheric pressure subtracts out, leaving $\left[-\Delta P(x+\Delta x)+\Delta P(x)\right]A=-\frac{\partial \Delta P}{\partial x}\Delta x A$.

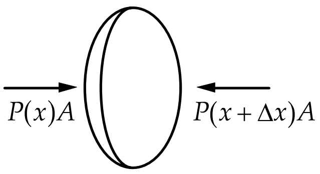


**FIG. P17.17**

The mass of the air is $\Delta m=\rho\Delta V=\rho A\Delta x$ and its acceleration is $\frac{\partial^2 s}{\partial t^2}$. So Newton's second law becomes

$$-\frac{\partial \Delta P}{\partial x}\Delta x A=\rho A\Delta x\frac{\partial^2 s}{\partial t^2}$$

$$-\frac{\partial}{\partial x}\left(-B\frac{\partial s}{\partial x}\right)=\rho\frac{\partial^2 s}{\partial t^2}$$

$$\frac{B}{\rho}\frac{\partial^2 s}{\partial x^2}=\frac{\partial^2 s}{\partial t^2}$$

Into this wave equation as a trial solution we substitute the wave function $s(x,\,t)=s_{\max}\cos(kx-\omega t)$ we find

$$\frac{\partial s}{\partial x}=-ks_{\max}\sin(kx-\omega t)$$

$$\frac{\partial^2 s}{\partial x^2}=-k^2 s_{\max}\cos(kx-\omega t)$$

$$\frac{\partial s}{\partial t}=+\omega s_{\max}\sin(kx-\omega t)$$

$$\frac{\partial^2 s}{\partial t^2}=-\omega^2 s_{\max}\cos(kx-\omega t)$$

$\frac{B}{\rho}\frac{\partial^2 s}{\partial x^2}=\frac{\partial^2 s}{\partial t^2}$ becomes $-\frac{B}{\rho}k^2 s_{\max}\cos(kx-\omega t)=-\omega^2 s_{\max}\cos(kx-\omega t)$

This is true provided $\frac{B}{\rho}\frac{4\pi^2}{\lambda^2}=4\pi^2 f^2$.

The sound wave can propagate provided it has $\lambda^2 f^2=v^2=\frac{B}{\rho}$; that is, provided it propagates with speed $v=\sqrt{\frac{B}{\rho}}$.

## Section 17.3 **Intensity of Periodic Sound Waves**

***P17.18** The sound power incident on the eardrum is $\wp = IA$ where $I$ is the intensity of the sound and $A = 5.0 \times 10^{-5}\ \text{m}^2$ is the area of the eardrum.

(a) At the threshold of hearing, $I = 1.0 \times 10^{-12}\ \text{W/m}^2$, and

$$\wp = \left(1.0 \times 10^{-12}\ \text{W/m}^2\right)\left(5.0 \times 10^{-5}\ \text{m}^2\right) = \boxed{5.00 \times 10^{-17}\ \text{W}}.$$

(b) At the threshold of pain, $I = 1.0\ \text{W/m}^2$, and

$$\wp = \left(1.0\ \text{W/m}^2\right)\left(5.0 \times 10^{-5}\ \text{m}^2\right) = \boxed{5.00 \times 10^{-5}\ \text{W}}.$$

**P17.19** $\beta = 10\log\left(\dfrac{I}{I_0}\right) = 10\log\left(\dfrac{4.00 \times 10^{-6}}{1.00 \times 10^{-12}}\right) = \boxed{66.0\ \text{dB}}$

**P17.20** (a) $70.0\ \text{dB} = 10\log\left(\dfrac{I}{1.00 \times 10^{-12}\ \text{W/m}^2}\right)$

Therefore, $I = \left(1.00 \times 10^{-12}\ \text{W/m}^2\right)10^{(70.0/10)} = \boxed{1.00 \times 10^{-5}\ \text{W/m}^2}$.

(b) $I = \dfrac{\Delta P_{\max}^2}{2\rho v}$, so

$$\Delta P_{\max} = \sqrt{2\rho v I} = \sqrt{2\left(1.20\ \text{kg/m}^3\right)(343\ \text{m/s})\left(1.00 \times 10^{-5}\ \text{W/m}^2\right)}$$

$$\Delta P_{\max} = \boxed{90.7\ \text{mPa}}$$

**P17.21** $I = \dfrac{1}{2}\rho\omega^2 s_{\max}^2 v$

(a) At $f = 2\,500\ \text{Hz}$, the frequency is increased by a factor of 2.50, so the intensity (at constant $s_{\max}$) increases by $(2.50)^2 = 6.25$.

Therefore, $6.25(0.600) = \boxed{3.75\ \text{W/m}^2}$.

(b) $\boxed{0.600\ \text{W/m}^2}$

**P17.22** The original intensity is $I_1 = \dfrac{1}{2}\rho\omega^2 s_{\max}^2 v = 2\pi^2\rho v f^2 s_{\max}^2$

(a) If the frequency is increased to $f'$ while a constant displacement amplitude is maintained, the new intensity is

$$I_2 = 2\pi^2\rho v (f')^2 s_{\max}^2 \text{ so } \frac{I_2}{I_1} = \frac{2\pi^2\rho v (f')s_{\max}^2}{2\pi^2\rho v f^2 s_{\max}^2} = \left(\frac{f'}{f}\right)^2 \text{ or } \boxed{I_2 = \left(\frac{f'}{f}\right)^2 I_1}.$$

*continued on next page*

(b) If the frequency is reduced to $f' = \frac{f}{2}$ while the displacement amplitude is doubled, the new intensity is

$$I_2 = 2\pi^2 \rho v \left(\frac{f}{2}\right)^2 (2s_{\max})^2 = 2\pi^2 \rho v f^2 s_{\max}^2 = I_1$$

or the $\boxed{\text{intensity is unchanged}}$.

***P17.23** (a) For the low note the wavelength is $\lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{146.8/\text{s}} = \boxed{2.34 \text{ m}}$.

For the high note $\lambda = \frac{343 \text{ m/s}}{880/\text{s}} = \boxed{0.390 \text{ m}}$.

We observe that the ratio of the frequencies of these two notes is $\frac{880 \text{ Hz}}{146.8 \text{ Hz}} = 5.99$ nearly equal to a small integer. This fact is associated with the consonance of the notes D and A.

(b) $\beta = 10 \text{ dB} \log\left(\frac{I}{10^{-12} \text{ W/m}^2}\right) = 75 \text{ dB}$ gives $I = 3.16 \times 10^{-5} \text{ W/m}^2$

$$I = \frac{\Delta P_{\max}^2}{2\rho v}$$

$$\Delta P_{\max} = \sqrt{3.16 \times 10^{-5} \text{ W/m}^2\, 2(1.20 \text{ kg/m}^3)(343 \text{ m/s})} = \boxed{0.161 \text{ Pa}}$$

for both low and high notes.

(c) $I = \frac{1}{2}\rho v (\omega s_{\max})^2 = \frac{1}{2}\rho v 4\pi^2 f^2 s_{\max}^2$

$$s_{\max} = \sqrt{\frac{I}{2\pi^2 \rho v f^2}}$$

for the low note,

$$s_{\max} = \sqrt{\frac{3.16 \times 10^{-5} \text{ W/m}^2}{2\pi^2 1.20 \text{ kg/m}^3\, 343 \text{ m/s}}}\frac{1}{146.8/\text{s}}$$

$$= \frac{6.24 \times 10^{-5}}{146.8} \text{ m} = \boxed{4.25 \times 10^{-7} \text{ m}}$$

for the high note,

$$s_{\max} = \frac{6.24 \times 10^{-5}}{880} \text{ m} = \boxed{7.09 \times 10^{-8} \text{ m}}$$

(d) With both frequencies lower (numerically smaller) by the factor $\frac{146.8}{134.3} = \frac{880}{804.9} = 1.093$, the wavelengths and displacement amplitudes are made 1.093 times larger, and the pressure amplitudes are unchanged.

***P17.24** The power necessarily supplied to the speaker is the power carried away by the sound wave:

$$P = \frac{1}{2}\rho A v (\omega s_{\max})^2 = 2\pi^2 \rho A v f^2 s_{\max}^2$$

$$= 2\pi^2 (1.20 \text{ kg/m}^3)\pi\left(\frac{0.08 \text{ m}}{2}\right)^2 (343 \text{ m/s})(600 \text{ 1/s})^2 (0.12 \times 10^{-2} \text{ m})^2 = \boxed{21.2 \text{ W}}$$

**P17.25** (a) $I_1 = \left(1.00\times10^{-12}\ \text{W/m}^2\right)10^{(\beta_1/10)} = \left(1.00\times10^{-12}\ \text{W/m}^2\right)10^{80.0/10}$

or $I_1 = 1.00\times10^{-4}\ \text{W/m}^2$

$I_2 = \left(1.00\times10^{-12}\ \text{W/m}^2\right)10^{(\beta_2/10)} = \left(1.00\times10^{-12}\ \text{W/m}^2\right)10^{75.0/10}$

or $I_2 = 1.00\times10^{-4.5}\ \text{W/m}^2 = 3.16\times10^{-5}\ \text{W/m}^2$

When both sounds are present, the total intensity is

$$I = I_1 + I_2 = 1.00\times10^{-4}\ \text{W/m}^2 + 3.16\times10^{-5}\ \text{W/m}^2 = \boxed{1.32\times10^{-4}\ \text{W/m}^2}.$$

(b) The decibel level for the combined sounds is

$$\beta = 10\log\left(\frac{1.32\times10^{-4}\ \text{W/m}^2}{1.00\times10^{-12}\ \text{W/m}^2}\right) = 10\log\left(1.32\times10^{8}\right) = \boxed{81.2\ \text{dB}}.$$

***P17.26** (a) We have $\lambda = \dfrac{v}{f}$ and $f$ is the same for all three waves. Since the speed is smallest in air, $\lambda$ is smallest in air. It is larger by $\dfrac{1\,493\ \text{m/s}}{331\ \text{m/s}} = \boxed{4.51\ \text{times}}$ in water and by $\dfrac{5\,950}{331} = \boxed{18.0\ \text{times in iron}}$.

(b) From $I = \dfrac{1}{2}\rho v\omega^2 s_{\max}^2$; $s_{\max} = \sqrt{\dfrac{2I_0}{\rho v\omega_0^2}}$, $s_{\max}$ is smallest in iron, larger in water by $\sqrt{\dfrac{\rho_{\text{iron}}v_{\text{iron}}}{\rho_{\text{water}}v_{\text{water}}}} = \sqrt{\dfrac{7\,860\cdot5\,950}{1\,000\cdot1\,493}} = \boxed{5.60\ \text{times}}$, and larger in air by $\sqrt{\dfrac{7\,860\cdot5\,950}{1.29\cdot331}} = \boxed{331\ \text{times}}$.

(c) From $I = \dfrac{\Delta P_{\max}^2}{2\rho v}$; $\Delta P_{\max} = \sqrt{2I\rho v}$, $\Delta P_{\max}$ is smallest in air, larger in water by $\sqrt{\dfrac{1\,000\cdot1\,493}{1.29\cdot331}} = \boxed{59.1\ \text{times}}$, and larger in iron by $\sqrt{\dfrac{7\,860\cdot5\,950}{1.29\cdot331}} = \boxed{331\ \text{times}}$.

(d) $\lambda = \dfrac{v}{f} = \dfrac{v2\pi}{\omega} = \dfrac{(331\ \text{m/s})2\pi}{2\,000\,\pi/\text{s}} = \boxed{0.331\ \text{m}}$ in air

$\lambda = \dfrac{1\,493\ \text{m/s}}{1\,000/\text{s}} = \boxed{1.49\ \text{m}}$ in water $\qquad \lambda = \dfrac{5\,950\ \text{m/s}}{1\,000/\text{s}} = \boxed{5.95\ \text{m}}$ in iron

$s_{\max} = \sqrt{\dfrac{2I_0}{\rho v\omega_0^2}} = \sqrt{\dfrac{2\times10^{-6}\ \text{W/m}^2}{\left(1.29\ \text{kg/m}^3\right)(331\ \text{m/s})(6\,283\ 1/\text{s})^2}} = \boxed{1.09\times10^{-8}\ \text{m}}$ in air

$s_{\max} = \sqrt{\dfrac{2\times10^{-6}}{1\,000(1\,493)}}\ \dfrac{1}{6\,283} = \boxed{1.84\times10^{-10}\ \text{m}}$ in water

$s_{\max} = \sqrt{\dfrac{2\times10^{-6}}{7\,860(5\,950)}}\ \dfrac{1}{6\,283} = \boxed{3.29\times10^{-11}\ \text{m}}$ in iron

$\Delta P_{\max} = \sqrt{2I\rho v} = \sqrt{2\left(10^{-6}\ \text{W/m}^2\right)\left(1.29\ \text{kg/m}^3\right)331\ \text{m/s}} = \boxed{0.029\,2\ \text{Pa}}$ in air

$\Delta P_{\max} = \sqrt{2\times10^{-6}(1\,000)1\,493} = \boxed{1.73\ \text{Pa}}$ in water

$\Delta P_{\max} = \sqrt{2\times10^{-6}(7\,860)(5\,950)} = \boxed{9.67\ \text{Pa}}$ in iron

**P17.27** (a)

$$120\text{ dB}=10\text{ dB}\log\left[\frac{I}{10^{-12}\text{ W/m}^2}\right]$$

$$I=1.00\text{ W/m}^2=\frac{\wp}{4\pi r^2}$$

$$r=\sqrt{\frac{\wp}{4\pi I}}=\sqrt{\frac{6.00\text{ W}}{4\pi\left(1.00\text{ W/m}^2\right)}}=\boxed{0.691\text{ m}}$$

We have assumed the speaker is an isotropic point source.

(b)

$$0\text{ dB}=10\text{ dB}\log\left(\frac{I}{10^{-12}\text{ W/m}^2}\right)$$

$$I=1.00\times10^{-12}\text{ W/m}^2$$

$$r=\sqrt{\frac{\wp}{4\pi I}}=\sqrt{\frac{6.00\text{ W}}{4\pi\left(1.00\times10^{-12}\text{ W/m}^2\right)}}=\boxed{691\text{ km}}$$

We have assumed a uniform medium that absorbs no energy.

**P17.28** We begin with $\beta_2=10\log\left(\frac{I_2}{I_0}\right)$, and $\beta_1=10\log\left(\frac{I_1}{I_0}\right)$, so

$$\beta_2-\beta_1=10\log\left(\frac{I_2}{I_1}\right).$$

Also, $I_2=\frac{\wp}{4\pi r_2^2}$, and $I_1=\frac{\wp}{4\pi r_1^2}$, giving $\frac{I_2}{I_1}=\left(\frac{r_1}{r_2}\right)^2$.

Then, $\beta_2-\beta_1=10\log\left(\frac{r_1}{r_2}\right)^2=\boxed{20\log\left(\frac{r_1}{r_2}\right)}$.

**P17.29** Since intensity is inversely proportional to the square of the distance,

$$I_4=\frac{1}{100}I_{0.4}\text{ and }I_{0.4}=\frac{\Delta P_{\max}^2}{2\rho v}=\frac{(10.0)^2}{2(1.20)(343)}=0.121\text{ W/m}^2.$$

The difference in sound intensity level is

$$\Delta\beta=10\log\left(\frac{I_{4\text{ km}}}{I_{0.4\text{ km}}}\right)=10(-2.00)=-20.0\text{ dB}.$$

At 0.400 km,

$$\beta_{0.4}=10\log\left(\frac{0.121\text{ W/m}^2}{10^{-12}\text{ W/m}^2}\right)=110.8\text{ dB}.$$

At 4.00 km,

$$\beta_4=\beta_{0.4}+\Delta\beta=(110.8-20.0)\text{ dB}=90.8\text{ dB}.$$

Allowing for absorption of the wave over the distance traveled,

$$\beta_4'=\beta_4-(7.00\text{ dB/km})(3.60\text{ km})=\boxed{65.6\text{ dB}}.$$

This is equivalent to the sound intensity level of heavy traffic.

**P17.30** Let $r_1$ and $r_2$ be the distance from the speaker to the observer that hears 60.0 dB and 80.0 dB, respectively. Use the result of problem 28,

$\beta_2 - \beta_1 = 20\log\left(\dfrac{r_1}{r_2}\right)$, to obtain $80.0 - 60.0 = 20\log\left(\dfrac{r_1}{r_2}\right)$.

Thus, $\log\left(\dfrac{r_1}{r_2}\right) = 1$, so $r_1 = 10.0r_2$ . Also: $r_1 + r_2 = 110\text{ m}$, so

$10.0r_2 + r_2 = 110$ m giving $\boxed{r_2 = 10.0\text{ m}}$, and $\boxed{r_1 = 100\text{ m}}$.

**P17.31** We presume the speakers broadcast equally in all directions.

(a)
$$r_{AC} = \sqrt{3.00^2 + 4.00^2}\text{ m} = 5.00\text{ m}$$
$$I = \frac{\wp}{4\pi r^2} = \frac{1.00\times10^{-3}\text{ W}}{4\pi(5.00\text{ m})^2} = 3.18\times10^{-6}\ \text{W/m}^2$$
$$\beta = 10\text{ dB}\log\left(\frac{3.18\times10^{-6}\ \text{W/m}^2}{10^{-12}\ \text{W/m}^2}\right)$$
$$\beta = 10\text{ dB } 6.50 = \boxed{65.0\text{ dB}}$$

(b)
$$r_{BC} = 4.47\text{ m}$$
$$I = \frac{1.50\times10^{-3}\text{ W}}{4\pi(4.47\text{ m})^2} = 5.97\times10^{-6}\ \text{W/m}^2$$
$$\beta = 10\text{ dB}\log\left(\frac{5.97\times10^{-6}}{10^{-12}}\right)$$
$$\beta = \boxed{67.8\text{ dB}}$$

(c)
$$I = 3.18\ \mu\text{W/m}^2 + 5.97\ \mu\text{W/m}^2$$
$$\beta = 10\text{ dB}\log\left(\frac{9.15\times10^{-6}}{10^{-12}}\right) = \boxed{69.6\text{ dB}}$$

**P17.32** In $I = \dfrac{\wp}{4\pi r^2}$, intensity $I$ is proportional to $\dfrac{1}{r^2}$,

so between locations 1 and 2: $\dfrac{I_2}{I_1} = \dfrac{r_1^2}{r_2^2}$.

In $I = \dfrac{1}{2}\rho v(\omega s_{\max})^2$, intensity is proportional to $s_{\max}^2$, so $\dfrac{I_2}{I_1} = \dfrac{s_2^2}{s_1^2}$.

Then, $\left(\dfrac{s_2}{s_1}\right)^2 = \left(\dfrac{r_1}{r_2}\right)^2$ or $\left(\dfrac{1}{2}\right)^2 = \left(\dfrac{r_1}{r_2}\right)^2$, giving $r_2 = 2r_1 = 2(50.0\text{ m}) = 100\text{ m}$.

But, $r_2 = \sqrt{(50.0\text{ m})^2 + d^2}$ yields $d = \boxed{86.6\text{ m}}$.

**P17.33** $\beta = 10\log\left(\frac{I}{10^{-12}}\right) \qquad I = \left[10^{(\beta/10)}\right]\left(10^{-12}\right)\ \text{W/m}^2$

$I_{(120\ \text{dB})} = 1.00\ \text{W/m}^2$ ; $I_{(100\ \text{dB})} = 1.00 \times 10^{-2}\ \text{W/m}^2$ ; $I_{(10\ \text{dB})} = 1.00 \times 10^{-11}\ \text{W/m}^2$

(a) $\wp = 4\pi r^2 I$ so that $r_1^2 I_1 = r_2^2 I_2$

$$r_2 = r_1\left(\frac{I_1}{I_2}\right)^{1/2} = (3.00\ \text{m})\sqrt{\frac{1.00}{1.00 \times 10^{-2}}} = \boxed{30.0\ \text{m}}$$

(b) $$r_2 = r_1\left(\frac{I_1}{I_2}\right)^{1/2} = (3.00\ \text{m})\sqrt{\frac{1.00}{1.00 \times 10^{-11}}} = \boxed{9.49 \times 10^5\ \text{m}}$$

**P17.34** (a) $E = \wp t = 4\pi r^2 It = 4\pi(100\ \text{m})^2\left(7.00 \times 10^{-2}\ \text{W/m}^2\right)(0.200\ \text{s}) = \boxed{1.76\ \text{kJ}}$

(b) $\beta = 10\log\left(\frac{7.00 \times 10^{-2}}{1.00 \times 10^{-12}}\right) = \boxed{108\ \text{dB}}$

**P17.35** (a) The sound intensity inside the church is given by

$$\beta = 10\ln\left(\frac{I}{I_0}\right)$$

$$101\ \text{dB} = (10\ \text{dB})\ln\left(\frac{I}{10^{-12}\ \text{W/m}^2}\right)$$

$$I = 10^{10.1}\left(10^{-12}\ \text{W/m}^2\right) = 10^{-1.90}\ \text{W/m}^2 = 0.012\,6\ \text{W/m}^2$$

We suppose that sound comes perpendicularly out through the windows and doors. Then, the radiated power is

$$\wp = IA = \left(0.012\,6\ \text{W/m}^2\right)\left(22.0\ \text{m}^2\right) = 0.277\ \text{W}.$$

Are you surprised by how small this is? The energy radiated in 20.0 minutes is

$$E = \wp t = (0.277\ \text{J/s})(20.0\ \text{min})\left(\frac{60.0\ \text{s}}{1.00\ \text{min}}\right) = \boxed{332\ \text{J}}.$$

(b) If the ground reflects all sound energy headed downward, the sound power, $\wp = 0.277\ \text{W}$, covers the area of a hemisphere. One kilometer away, this area is
$A = 2\pi r^2 = 2\pi(1\,000\ \text{m})^2 = 2\pi \times 10^6\ \text{m}^2$.
The intensity at this distance is

$$I = \frac{\wp}{A} = \frac{0.277\ \text{W}}{2\pi \times 10^6\ \text{m}^2} = 4.41 \times 10^{-8}\ \text{W/m}^2$$

and the sound intensity level is

$$\beta = (10\ \text{dB})\ln\left(\frac{4.41 \times 10^{-8}\ \text{W/m}^2}{1.00 \times 10^{-12}\ \text{W/m}^2}\right) = \boxed{46.4\ \text{dB}}.$$

***P17.36** Assume you are 1 m away from your lawnmower and receiving 100 dB sound from it. The intensity of this sound is given by $100\text{ dB} = 10\text{ dB}\log\frac{I}{10^{-12}\text{ W/m}^2}$; $I = 10^{-2}\text{ W/m}^2$. If the lawnmower radiates as a point source, its sound power is given by $I = \frac{\wp}{4\pi r^2}$.

$$\wp = 4\pi(1\text{ m})^2 10^{-2}\text{ W/m}^2 = 0.126\text{ W}$$

Now let your neighbor have an identical lawnmower 20 m away. You receive from it sound with intensity $I = \frac{0.126\text{ W}}{4\pi(20\text{ m})^2} = 2.5\times10^{-5}\text{ W/m}^2$. The total sound intensity impinging on you is $10^{-2}\text{ W/m}^2 + 2.5\times10^{-5}\text{ W/m}^2 = 1.002\,5\times10^{-2}\text{ W/m}^2$. So its level is

$$10\text{ dB}\log\frac{1.002\,5\times10^{-2}}{10^{-12}} = 100.01\text{ dB}.$$

If the smallest noticeable difference is between 100 dB and 101 dB, this $\boxed{\text{cannot}}$ be heard as a change from 100 dB.

## Section 17.4 **The Doppler Effect**

**P17.37** $f' = f\frac{(v \pm v_O)}{(v \pm v_S)}$

(a) $f' = 320\frac{(343+40.0)}{(343+20.0)} = \boxed{338\text{ Hz}}$

(b) $f' = 510\frac{(343+20.0)}{(343+40.0)} = \boxed{483\text{ Hz}}$

**P17.38** (a) $\omega = 2\pi f = 2\pi\left(\frac{115/\text{min}}{60.0\text{ s/min}}\right) = 12.0\text{ rad/s}$

$v_{\max} = \omega A = (12.0\text{ rad/s})(1.80\times10^{-3}\text{ m}) = \boxed{0.021\,7\text{ m/s}}$

(b) The heart wall is a moving observer.

$$f' = f\left(\frac{v+v_O}{v}\right) = (2\,000\,000\text{ Hz})\left(\frac{1\,500+0.021\,7}{1\,500}\right) = \boxed{2\,000\,028.9\text{ Hz}}$$

(c) Now the heart wall is a moving source.

$$f'' = f'\left(\frac{v}{v-v_s}\right) = (2\,000\,029\text{ Hz})\left(\frac{1\,500}{1\,500-0.021\,7}\right) = \boxed{2\,000\,057.8\text{ Hz}}$$

**P17.39** Approaching ambulance: $f' = \dfrac{f}{(1 - v_S/v)}$

Departing ambulance: $f'' = \dfrac{f}{(1 - (-v_S/v))}$

Since $f' = 560$ Hz and $f'' = 480$ Hz $560\left(1 - \dfrac{v_S}{v}\right) = 480\left(1 + \dfrac{v_S}{v}\right)$

$$1\,040\frac{v_S}{v} = 80.0$$

$$v_S = \frac{80.0(343)}{1\,040}\text{ m/s} = \boxed{26.4\text{ m/s}}$$

**P17.40** (a) The maximum speed of the speaker is described by

$$\frac{1}{2}mv_{\max}^2 = \frac{1}{2}kA^2$$

$$v_{\max} = \sqrt{\frac{k}{m}}A = \sqrt{\frac{20.0\text{ N/m}}{5.00\text{ kg}}}(0.500\text{ m}) = 1.00\text{ m/s}$$

The frequencies heard by the stationary observer range from

$$f'_{\min} = f\left(\frac{v}{v + v_{\max}}\right)\text{ to }f'_{\max} = f\left(\frac{v}{v - v_{\max}}\right)$$

where $v$ is the speed of sound.

$$f'_{\min} = 440\text{ Hz}\left(\frac{343\text{ m/s}}{343\text{ m/s} + 1.00\text{ m/s}}\right) = \boxed{439\text{ Hz}}$$

$$f'_{\max} = 440\text{ Hz}\left(\frac{343\text{ m/s}}{343\text{ m/s} - 1.00\text{ m/s}}\right) = \boxed{441\text{ Hz}}$$

(b) $\beta = 10\text{ dB}\log\left(\dfrac{I}{I_0}\right) = 10\text{ dB}\log\left(\dfrac{\wp/4\pi r^2}{I_0}\right)$

The maximum intensity level (of 60.0 dB) occurs at $r = r_{\min} = 1.00$ m. The minimum intensity level occurs when the speaker is farthest from the listener (i.e., when $r = r_{\max} = r_{\min} + 2A = 2.00$ m).

Thus, $\beta_{\max} - \beta_{\min} = 10\text{ dB}\log\left(\dfrac{\wp}{4\pi I_0 r_{\min}^2}\right) - 10\text{ dB}\log\left(\dfrac{\wp}{4\pi I_0 r_{\max}^2}\right)$

or $\beta_{\max} - \beta_{\min} = 10\text{ dB}\log\left(\dfrac{\wp}{4\pi I_0 r_{\min}^2}\cdot\dfrac{4\pi I_0 r_{\max}^2}{\wp}\right) = 10\text{ dB}\log\left(\dfrac{r_{\max}^2}{r_{\min}^2}\right)$.

This gives: $60.0\text{ dB} - \beta_{\min} = 10\text{ dB}\log(4.00) = 6.02\text{ dB}$, and $\beta_{\min} = \boxed{54.0\text{ dB}}$.

**P17.41** $f' = f\left(\dfrac{v}{v - v_s}\right)$

$$485 = 512\left(\frac{340}{340 - (-9.80t_{\text{fall}})}\right)$$

$$485(340) + (485)\left(9.80t_f\right) = (512)(340)$$

$$t_f = \left(\frac{512 - 485}{485}\right)\frac{340}{9.80} = 1.93\text{ s}$$

$d_1 = \dfrac{1}{2}gt_f^2 = 18.3\text{ m}:$ $\qquad t_{\text{return}} = \dfrac{18.3}{340} = 0.053\,8\text{ s}$

The fork continues to fall while the sound returns.

$$t_{\text{total fall}} = t_f + t_{\text{return}} = 1.93\text{ s} + 0.053\,8\text{ s} = 1.985\text{ s}$$

$$d_{\text{total}} = \frac{1}{2}gt_{\text{total fall}}^2 = \boxed{19.3\text{ m}}$$

**P17.42** (a) $v = (331\text{ m/s}) + 0.6\dfrac{\text{m}}{\text{s}\cdot{}^\circ\text{C}}(-10^\circ\text{C}) = \boxed{325\text{ m/s}}$

(b) Approaching the bell, the athlete hears a frequency of $\quad f' = f\left(\dfrac{v + v_O}{v}\right)$

After passing the bell, she hears a lower frequency of $\quad f'' = f\left(\dfrac{v + (-v_O)}{v}\right)$

The ratio is $\quad \dfrac{f''}{f'} = \dfrac{v - v_O}{v + v_O} = \dfrac{5}{6}$

which gives $6v - 6v_o = 5v + 5v_o$ or $\quad v_O = \dfrac{v}{11} = \dfrac{325\text{ m/s}}{11} = \boxed{29.5\text{ m/s}}$

***P17.43** (a) Sound moves upwind with speed $(343 - 15)$ m/s. Crests pass a stationary upwind point at frequency 900 Hz.

Then $\quad \lambda = \dfrac{v}{f} = \dfrac{328\text{ m/s}}{900/\text{s}} = \boxed{0.364\text{ m}}$

(b) By similar logic, $\quad \lambda = \dfrac{v}{f} = \dfrac{(343 + 15)\text{ m/s}}{900/\text{s}} = \boxed{0.398\text{ m}}$

(c) The source is moving through the air at 15 m/s toward the observer. The observer is stationary relative to the air.

$$f' = f\left(\frac{v + v_o}{v - v_s}\right) = 900\text{ Hz}\left(\frac{343 + 0}{343 - 15}\right) = \boxed{941\text{ Hz}}$$

(d) The source is moving through the air at 15 m/s away from the downwind firefighter. Her speed relative to the air is 30 m/s toward the source.

$$f' = f\left(\frac{v + v_o}{v - v_s}\right) = 900\text{ Hz}\left(\frac{343 + 30}{343 - (-15)}\right) = 900\text{ Hz}\left(\frac{373}{358}\right) = \boxed{938\text{ Hz}}$$

***P17.44** The half-angle of the cone of the shock wave is $\theta$ where

$$\theta=\sin^{-1}\left(\frac{v_{\text{sound}}}{v_{\text{source}}}\right)=\sin^{-1}\left(\frac{1}{1.5}\right)=41.8°.$$

As shown in the sketch, the angle between the direction of propagation of the shock wave and the direction of the plane's velocity is

$$\phi=90°-\theta=90°-41.8°=\boxed{48.2°}.$$

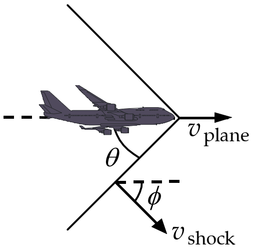


FIG. P17.44

**P17.45** The *half angle* of the shock wave cone is given by $\sin\theta=\frac{v_{\text{light}}}{v_S}$.

$$v_S=\frac{v_{\text{light}}}{\sin\theta}=\frac{2.25\times10^8\text{ m/s}}{\sin(53.0°)}=\boxed{2.82\times10^8\text{ m/s}}$$

**P17.46** $\theta=\sin^{-1}\frac{v}{v_S}=\sin^{-1}\frac{1}{1.38}=\boxed{46.4°}$

**P17.47** (b) $\sin\theta=\frac{v}{v_S}=\frac{1}{3.00}$; $\theta=19.5°$

$\tan\theta=\frac{h}{x}$; $x=\frac{h}{\tan\theta}$

$x=\frac{20\,000\text{ m}}{\tan19.5°}=5.66\times10^4\text{ m}=\boxed{56.6\text{ km}}$

(a) It takes the plane $t=\frac{x}{v_S}=\frac{5.66\times10^4\text{ m}}{3.00(335\text{ m/s})}=\boxed{56.3\text{ s}}$ to travel this distance.

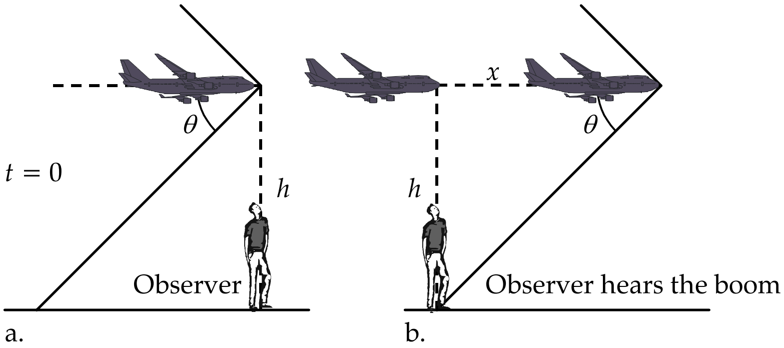


FIG. P17.47(a)

## Section 17.5 Digital Sound Recording

## Section 17.6 Motion Picture Sound

***P17.48** For a 40-dB sound,

$$40\text{ dB} = 10\text{ dB}\log\left[\frac{I}{10^{-12}\text{ W/m}^2}\right]$$

$$I = 10^{-8}\text{ W/m}^2 = \frac{\Delta P_{\max}^2}{2\rho v}$$

$$\Delta P_{\max} = \sqrt{2\rho v I} = \sqrt{2\left(1.20\text{ kg/m}^2\right)(343\text{ m/s})10^{-8}\text{ W/m}^2} = 2.87\times10^{-3}\text{ N/m}^2$$

(a) $$\text{code} = \frac{2.87\times10^{-3}\text{ N/m}^2}{28.7\text{ N/m}^2}65\,536 = \boxed{7}$$

(b) For sounds of 40 dB or softer, too few digital words are available to represent the wave form with good fidelity.

(c) In a sound wave $\Delta P$ is negative half of the time but this coding scheme has no words available for negative pressure variations.

***P17.49** If the source is to the left at angle $\theta$ from the direction you are facing, the sound must travel an extra distance $d\sin\theta$ to reach your right ear as shown, where $d$ is the distance between your ears. The delay time is $\Delta t$ in $v = \dfrac{d\sin\theta}{\Delta t}$. Then

$$\theta = \sin^{-1}\frac{v\Delta t}{d} = \sin^{-1}\frac{(343\text{ m/s})210\times10^{-6}\text{ s}}{0.19\text{ m}} = \boxed{22.3°\text{ left of center}}.$$

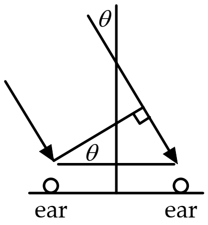


FIG. P17.49

***P17.50** $$103\text{ dB} = 10\text{ dB}\log\left[\frac{I}{10^{-12}\text{ W/m}^2}\right]$$

(a) $$I = 2.00\times10^{-2}\text{ W/m}^2 = \frac{\wp}{4\pi r^2} = \frac{\wp}{4\pi(1.6\text{ m})^2}$$

$$\wp = \boxed{0.642\text{ W}}$$

(b) $$\text{efficiency} = \frac{\text{sound output power}}{\text{total input power}} = \frac{0.642\text{ W}}{150\text{ W}} = \boxed{0.004\,28}$$

## Additional Problems

**P17.51** Model your loud, sharp sound impulse as a single narrow peak in a graph of air pressure versus time. It is a noise with no pitch, no frequency, wavelength, or period. It radiates away from you in all directions and some of it is incident on each one of the solid vertical risers of the bleachers. Suppose that, at the ambient temperature, sound moves at 340 m/s; and suppose that the horizontal width of each row of seats is 60 cm. Then there is a time delay of

$$\frac{0.6\text{ m}}{(340\text{ m/s})} = 0.002\text{ s}$$

*continued on next page*

between your sound impulse reaching each riser and the next. Whatever its material, each will reflect much of the sound that reaches it. The reflected wave sounds very different from the sharp pop you made. If there are twenty rows of seats, you hear from the bleachers a tone with twenty crests, each separated from the next in time by

$$\frac{2(0.6\text{ m})}{(340\text{ m/s})}=0.004\text{ s}.$$

This is the extra time for it to cross the width of one seat twice, once as an incident pulse and once again after its reflection. Thus, you hear a sound of definite pitch, with period about 0.004 s, frequency

$$\frac{1}{0.003\,5\text{ s}}\boxed{\sim 300\text{ Hz}}$$

wavelength

$$\lambda=\frac{v}{f}=\frac{(340\text{ m/s})}{(300/\text{s})}=1.2\text{ m}\sim\boxed{10^0\text{ m}}$$

and duration

$$20(0.004\text{ s})\sim\boxed{10^{-1}\text{ s}}.$$

**P17.52** (a) $\lambda=\frac{v}{f}=\frac{343\text{ m/s}}{1\,480\text{ s}^{-1}}=\boxed{0.232\text{ m}}$

(b) $\beta=81.0\text{ dB}=10\text{ dB}\log\left[\frac{I}{10^{-12}\text{ W/m}^2}\right]$

$$I=\left(10^{-12}\text{ W/m}^2\right)10^{8.10}=10^{-3.90}\text{ W/m}^2=1.26\times10^{-4}\text{ W/m}^2=\frac{1}{2}\rho v\omega^2 s_{\max}^2$$

$$s_{\max}=\sqrt{\frac{2I}{\rho v\omega^2}}=\sqrt{\frac{2\left(1.26\times10^{-4}\text{ W/m}^2\right)}{\left(1.20\text{ kg/m}^3\right)(343\text{ m/s})4\pi^2\left(1\,480\text{ s}^{-1}\right)^2}}=\boxed{8.41\times10^{-8}\text{ m}}$$

(c) $\lambda'=\frac{v}{f'}=\frac{343\text{ m/s}}{1\,397\text{ s}^{-1}}=0.246\text{ m}\quad\Delta\lambda=\lambda'-\lambda=\boxed{13.8\text{ mm}}$

**P17.53** Since $\cos^2\theta+\sin^2\theta=1$, $\sin\theta=\pm\sqrt{1-\cos^2\theta}$ (each sign applying half the time)

$$\Delta P=\Delta P_{\max}\sin(kx-\omega t)=\pm\rho v\omega s_{\max}\sqrt{1-\cos^2(kx-\omega t)}$$

Therefore $\Delta P=\pm\rho v\omega\sqrt{s_{\max}^2-s_{\max}^2\cos^2(kx-\omega t)}=\pm\rho v\omega\sqrt{s_{\max}^2-s^2}$

**P17.54** The trucks form a train analogous to a wave train of crests with speed $v=19.7\text{ m/s}$ and unshifted frequency $f=\frac{2}{3.00\text{ min}}=0.667\text{ min}^{-1}$.

(a) The cyclist as observer measures a lower Doppler-shifted frequency:

$$f'=f\left(\frac{v+v_o}{v}\right)=\left(0.667\text{ min}^{-1}\right)\left(\frac{19.7+(-4.47)}{19.7}\right)=\boxed{0.515/\text{min}}$$

(b) $f''=f\left(\frac{v+v_o'}{v}\right)=\left(0.667\text{ min}^{-1}\right)\left(\frac{19.7+(-1.56)}{19.7}\right)=\boxed{0.614/\text{min}}$

The cyclist's speed has decreased very significantly, but there is only a modest increase in the frequency of trucks passing him.

**P17.55** $v=\dfrac{2d}{t}$: $d=\dfrac{vt}{2}=\dfrac{1}{2}\left(6.50\times10^{3}\text{ m/s}\right)(1.85\text{ s})=\boxed{6.01\text{ km}}$

**P17.56** (a) The speed of a compression wave in a bar is

$$v=\sqrt{\frac{Y}{\rho}}=\sqrt{\frac{20.0\times10^{10}\text{ N/m}^2}{7\,860\text{ kg/m}^3}}=\boxed{5.04\times10^{3}\text{ m/s}}.$$

(b) The signal to stop passes between layers of atoms as a sound wave, reaching the back end of the bar in time

$$t=\frac{L}{v}=\frac{0.800\text{ m}}{5.04\times10^{3}\text{ m/s}}=\boxed{1.59\times10^{-4}\text{ s}}.$$

(c) As described by Newton's first law, the rearmost layer of steel has continued to move forward with its original speed $v_i$ for this time, compressing the bar by

$$\Delta L=v_it=(12.0\text{ m/s})\left(1.59\times10^{-4}\text{ s}\right)=1.90\times10^{-3}\text{ m}=\boxed{1.90\text{ mm}}.$$

(d) The strain in the rod is: $\dfrac{\Delta L}{L}=\dfrac{1.90\times10^{-3}\text{ m}}{0.800\text{ m}}=\boxed{2.38\times10^{-3}}$.

(e) The stress in the rod is:

$$\sigma=Y\left(\frac{\Delta L}{L}\right)=\left(20.0\times10^{10}\text{ N/m}^2\right)\left(2.38\times10^{-3}\right)=\boxed{476\text{ MPa}}.$$

Since $\sigma>400\text{ MPa}$, the rod will be permanently distorted.

(f) We go through the same steps as in parts (a) through (e), but use algebraic expressions rather than numbers:

The speed of sound in the rod is $v=\sqrt{\dfrac{Y}{\rho}}$.

The back end of the rod continues to move forward at speed $v_i$ for a time of $t=\dfrac{L}{v}=L\sqrt{\dfrac{\rho}{Y}}$, traveling distance $\Delta L=v_it$ after the front end hits the wall.

The strain in the rod is: $\dfrac{\Delta L}{L}=\dfrac{v_it}{L}=v_i\sqrt{\dfrac{\rho}{Y}}$.

The stress is then: $\sigma=Y\left(\dfrac{\Delta L}{L}\right)=Yv_i\sqrt{\dfrac{\rho}{Y}}=v_i\sqrt{\rho Y}$.

For this to be less than the yield stress, $\sigma_y$, it is necessary that

$$v_i\sqrt{\rho Y}<\sigma_y\text{ or }\boxed{v_i<\frac{\sigma_y}{\sqrt{\rho Y}}}.$$

With the given numbers, this speed is 10.1 m/s. The fact that the length of the rod divides out means that the steel will start to bend right away at the front end of the rod. There it will yield enough so that eventually the remainder of the rod will experience only stress within the elastic range. You can see this effect when sledgehammer blows give a mushroom top to a rod used as a tent stake.

**P17.57** (a) $f' = f\dfrac{v}{(v - v_{\text{diver}})}$

so $1 - \dfrac{v_{\text{diver}}}{v} = \dfrac{f}{f'}$

$\Rightarrow v_{\text{diver}} = v\left(1 - \dfrac{f}{f'}\right)$

with $v = 343\ \text{m/s}$, $f' = 1\,800\ \text{Hz}$ and $f = 2\,150\ \text{Hz}$

we find

$$v_{\text{diver}} = 343\left(1 - \frac{1\,800}{2\,150}\right) = \boxed{55.8\ \text{m/s}}.$$

(b) If the waves are reflected, and the skydiver is moving into them, we have

$$f'' = f'\frac{(v + v_{\text{diver}})}{v} \Rightarrow f'' = f\left[\frac{v}{(v - v_{\text{diver}})}\right]\frac{(v + v_{\text{diver}})}{v}$$

so $f'' = 1\,800\dfrac{(343 + 55.8)}{(343 - 55.8)} = \boxed{2\,500\ \text{Hz}}$.

**P17.58** (a) $f' = \dfrac{fv}{v - u}$ $\qquad f'' = \dfrac{fv}{v - (-u)}$ $\qquad f' - f'' = fv\left(\dfrac{1}{v - u} - \dfrac{1}{v + u}\right)$

$$\Delta f = \frac{fv(v + u - v + u)}{v^2 - u^2} = \frac{2uvf}{v^2\left(1 - \left(u^2/v^2\right)\right)} = \boxed{\frac{2(u/v)}{1 - \left(u^2/v^2\right)}f}$$

(b) $130\ \text{km/h} = 36.1\ \text{m/s}$ $\qquad \therefore \Delta f = \dfrac{2(36.1)(400)}{340\left[1 - (36.1)^2/340^2\right]} = \boxed{85.9\ \text{Hz}}$

**P17.59** When observer is moving in front of and in the same direction as the source, $f' = f\dfrac{v - v_O}{v - v_S}$ where $v_O$ and $v_S$ are measured relative to the medium in which the sound is propagated. In this case the ocean current is opposite the direction of travel of the ships and

$$v_O = 45.0\ \text{km/h} - (-10.0\ \text{km/h}) = 55.0\ \text{km/h} = 15.3\ \text{m/s}, \text{ and}$$
$$v_S = 64.0\ \text{km/h} - (-10.0\ \text{km/h}) = 74.0\ \text{km/h} = 20.55\ \text{m/s}$$

Therefore, $f' = (1\,200.0\ \text{Hz})\dfrac{1\,520\ \text{m/s} - 15.3\ \text{m/s}}{1\,520\ \text{m/s} - 20.55\ \text{m/s}} = \boxed{1\,204.2\ \text{Hz}}$.

**P17.60** Use the Doppler formula, and remember that the bat is a moving source.

If the velocity of the insect is $v_x$,

$$40.4 = 40.0\frac{(340+5.00)(340-v_x)}{(340-5.00)(340+v_x)}.$$

Solving,

$$v_x = 3.31 \text{ m/s}.$$

Therefore, $\boxed{\text{the bat is gaining on its prey at 1.69 m/s}}$.

**P17.61** $\sin\beta = \frac{v}{v_S} = \frac{1}{N_M}$

$h = v(12.8 \text{ s})$

$x = v_S(10.0 \text{ s})$

$\tan\beta = \frac{h}{x} = 1.28\frac{v}{v_S} = \frac{1.28}{N_M}$

$\cos\beta = \frac{\sin\beta}{\tan\beta} = \frac{1}{1.28}$

$\beta = 38.6°$

$N_M = \frac{1}{\sin\beta} = \boxed{1.60}$

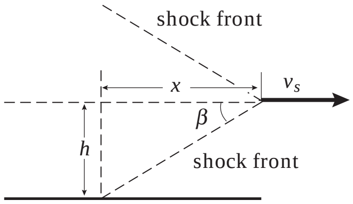


FIG. P17.61

**P17.62** (a)

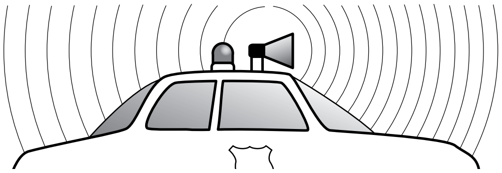


FIG. P17.62(a)

(b) $\lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{1\,000 \text{ s}^{-1}} = \boxed{0.343 \text{ m}}$

(c) $\lambda' = \frac{v}{f'} = \frac{v}{f}\left(\frac{v-v_S}{v}\right) = \frac{(343-40.0) \text{ m/s}}{1\,000 \text{ s}^{-1}} = \boxed{0.303 \text{ m}}$

(d) $\lambda'' = \frac{v}{f''} = \frac{v}{f}\left(\frac{v+v_S}{v}\right) = \frac{(343+40.0) \text{ m/s}}{1\,000 \text{ s}^{-1}} = \boxed{0.383 \text{ m}}$

(e) $f' = f\left(\frac{v-v_O}{v-v_S}\right) = (1\,000 \text{ Hz})\frac{(343-30.0) \text{ m/s}}{(343-40.0) \text{ m/s}} = \boxed{1.03 \text{ kHz}}$

**P17.63** $$\Delta t = L\left(\frac{1}{v_{\text{air}}} - \frac{1}{v_{\text{cu}}}\right) = L\frac{v_{\text{cu}} - v_{\text{air}}}{v_{\text{air}} v_{\text{cu}}}$$

$$L = \frac{v_{\text{air}} v_{\text{cu}}}{v_{\text{cu}} - v_{\text{air}}}\Delta t = \frac{(331\ \text{m/s})(3.56 \times 10^3\ \text{m/s})}{(3\,560 - 331)\ \text{m/s}}(6.40 \times 10^{-3}\ \text{s})$$

$$\boxed{L = 2.34\ \text{m}}$$

**P17.64** The shock wavefront connects all observers first hearing the plane, including our observer *O* and the plane *P*, so here it is vertical. The angle $\phi$ that the shock wavefront makes with the direction of the plane's line of travel is given by

$$\sin\phi = \frac{v}{v_S} = \frac{340\ \text{m/s}}{1\,963\ \text{m/s}} = 0.173$$

so $\phi = 9.97°$.

Using the right triangle *CPO*, the angle $\theta$ is seen to be

$$\theta = 90.0° - \phi = 90.0° - 9.97° = \boxed{80.0°}.$$

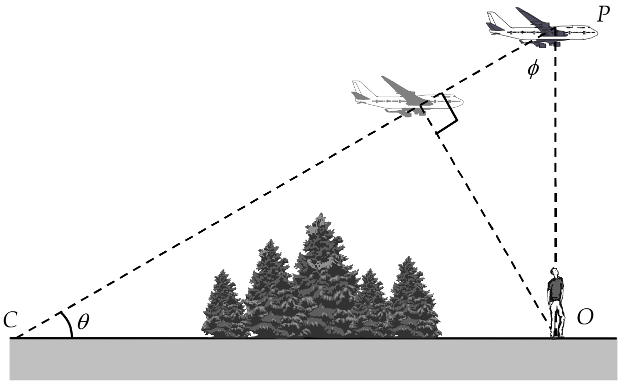


**FIG. P17.64**

**P17.65** (a) $$\theta = \sin^{-1}\left(\frac{v_{\text{sound}}}{v_{\text{obj}}}\right) = \sin^{-1}\left(\frac{331}{20.0 \times 10^3}\right) = \boxed{0.948°}$$

(b) $$\theta' = \sin^{-1}\left(\frac{1\,533}{20.0 \times 10^3}\right) = \boxed{4.40°}$$

**P17.66** $$\wp_2 = \frac{1}{20.0}\wp_1 \quad \beta_1 - \beta_2 = 10\log\frac{\wp_1}{\wp_2}$$

$$80.0 - \beta_2 = 10\log 20.0 = +13.0$$

$$\beta_2 = \boxed{67.0\ \text{dB}}$$

**P17.67** For the longitudinal wave $v_L = \left(\frac{Y}{\rho}\right)^{1/2}$.

For the transverse wave $v_T = \left(\frac{T}{\mu}\right)^{1/2}$.

If we require $\frac{v_L}{v_T} = 8.00$, we have $T = \frac{\mu Y}{64.0\rho}$ where $\mu = \frac{m}{L}$ and

$$\rho = \frac{\text{mass}}{\text{volume}} = \frac{m}{\pi r^2 L}.$$

This gives $T = \frac{\pi r^2 Y}{64.0} = \frac{\pi(2.00 \times 10^{-3}\ \text{m})^2(6.80 \times 10^{10}\ \text{N/m}^2)}{64.0} = \boxed{1.34 \times 10^4\ \text{N}}$.

**P17.68** The total output sound energy is $eE = \wp\Delta t$, where $\wp$ is the power radiated.

Thus, $\Delta t = \frac{eE}{\wp} = \frac{eE}{IA} = \frac{eE}{\left(4\pi r^2\right)I} = \frac{eE}{4\pi d^2 I}$.

But, $\beta = 10\log\left(\frac{I}{I_0}\right)$. Therefore, $I = I_0\left(10^{\beta/10}\right)$ and $\Delta t = \boxed{\frac{eE}{4\pi d^2 I_0 10^{\beta/10}}}$.

**P17.69** (a) If the source and the observer are moving away from each other, we have: $\theta_S - \theta_0 = 180°$, and since $\cos 180° = -1$, we get Equation 17.12 with negative values for both $v_O$ and $v_S$.

(b) If $v_O = 0$ m/s then $f' = \frac{v}{v - v_S\cos\theta_S}f$

Also, when the train is 40.0 m from the intersection, and the car is 30.0 m from the intersection,

$$\cos\theta_S = \frac{4}{5}$$

so $f' = \frac{343\text{ m/s}}{343\text{ m/s} - 0.800(25.0\text{ m/s})}(500\text{ Hz})$

or $f' = \boxed{531\text{ Hz}}$.

Note that as the train approaches, passes, and departs from the intersection, $\theta_S$ varies from 0° to 180° and the frequency heard by the observer varies from:

$$f'_{\max} = \frac{v}{v - v_S\cos 0°}f = \frac{343\text{ m/s}}{343\text{ m/s} - 25.0\text{ m/s}}(500\text{ Hz}) = 539\text{ Hz}$$

$$f'_{\min} = \frac{v}{v - v_S\cos 180°}f = \frac{343\text{ m/s}}{343\text{ m/s} + 25.0\text{ m/s}}(500\text{ Hz}) = 466\text{ Hz}$$

**P17.70** Let *T* represent the period of the source vibration, and *E* be the energy put into each wavefront. Then $\wp_{\text{av}} = \frac{E}{T}$. When the observer is at distance *r* in front of the source, he is receiving a spherical wavefront of radius *vt*, where *t* is the time since this energy was radiated, given by $vt - v_S t = r$. Then,

$$t = \frac{r}{v - v_S}.$$

The area of the sphere is $4\pi(vt)^2 = \frac{4\pi v^2 r^2}{\left(v - v_S\right)^2}$. The energy per unit area over the spherical wavefront is uniform with the value $\frac{E}{A} = \frac{\wp_{\text{av}}T\left(v - v_S\right)^2}{4\pi v^2 r^2}$.

The observer receives parcels of energy with the Doppler shifted frequency $f' = f\left(\frac{v}{v - v_S}\right) = \frac{v}{T\left(v - v_S\right)}$, so the observer receives a wave with intensity

$$I = \left(\frac{E}{A}\right)f' = \left(\frac{\wp_{\text{av}}T\left(v - v_S\right)^2}{4\pi v^2 r^2}\right)\left(\frac{v}{T\left(v - v_S\right)}\right) = \boxed{\frac{\wp_{\text{av}}}{4\pi r^2}\left(\frac{v - v_S}{v}\right)}.$$

**P17.71** (a) The time required for a sound pulse to travel distance $L$ at speed $v$ is given by $t = \frac{L}{v} = \frac{L}{\sqrt{Y/\rho}}$. Using this expression we find

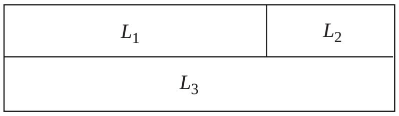


**FIG. P17.71**

$$t_1 = \frac{L_1}{\sqrt{Y_1/\rho_1}} = \frac{L_1}{\sqrt{\left(7.00\times10^{10}\ \text{N/m}^2\right)/\left(2\,700\ \text{kg/m}^3\right)}} = \left(1.96\times10^{-4}L_1\right)\text{ s}$$

$$t_2 = \frac{1.50\ \text{m} - L_1}{\sqrt{Y_2/\rho_2}} = \frac{1.50\ \text{m} - L_1}{\sqrt{\left(1.60\times10^{10}\ \text{N/m}^2\right)/\left(11.3\times10^3\ \text{kg/m}^3\right)}}$$

or $\quad t_2 = \left(1.26\times10^{-3} - 8.40\times10^{-4}L_1\right)\text{ s}$

$$t_3 = \frac{1.50\ \text{m}}{\sqrt{\left(11.0\times10^{10}\ \text{N/m}^3\right)/\left(8\,800\ \text{kg/m}^3\right)}}$$

$$t_3 = 4.24\times10^{-4}\ \text{s}$$

We require $t_1 + t_2 = t_3$, or

$$1.96\times10^{-4}L_1 + 1.26\times10^{-3} - 8.40\times10^{-4}L_1 = 4.24\times10^{-4}.$$

This gives $L_1 = 1.30\ \text{m}$ and $L_2 = 1.50 - 1.30 = 0.201\ \text{m}$.

The ratio of lengths is then $\frac{L_1}{L_2} = \boxed{6.45}$.

(b) The ratio of lengths $\frac{L_1}{L_2}$ is adjusted in part (a) so that $t_1 + t_2 = t_3$. Sound travels the two paths in equal time and the phase difference, $\boxed{\Delta\phi = 0}$.

**P17.72** To find the separation of adjacent molecules, use a model where each molecule occupies a sphere of radius $r$ given by

$$\rho_{\text{air}} = \frac{\text{average mass per molecule}}{\frac{4}{3}\pi r^3}$$

$$\text{or } 1.20\ \text{kg/m}^3 = \frac{4.82\times10^{-26}\ \text{kg}}{\frac{4}{3}\pi r^3},\ r = \left[\frac{3\left(4.82\times10^{-26}\ \text{kg}\right)}{4\pi\left(1.20\ \text{kg/m}^3\right)}\right]^{1/3} = 2.12\times10^{-9}\ \text{m}.$$

Intermolecular separation is $2r = 4.25\times10^{-9}\ \text{m}$, so the highest possible frequency sound wave is

$$f_{\max} = \frac{v}{\lambda_{\min}} = \frac{v}{2r} = \frac{343\ \text{m/s}}{4.25\times10^{-9}\ \text{m}} = 8.03\times10^{10}\ \text{Hz}\ \boxed{\sim10^{11}\ \text{Hz}}.$$

## ANSWERS TO EVEN PROBLEMS

| | |
|---|---|
| **P17.2** | $1.43\ \text{km/s}$ |
| **P17.4** | (a) 27.2 s; (b) longer than 25.7 s, because the air is cooler |
| **P17.6** | (a) $153\ \text{m/s}$; (b) 614 m |
| **P17.8** | (a) 4.16 m; (b) $0.455\ \mu\text{s}$; (c) 0.157 mm |
| **P17.10** | $1.55\times10^{-10}\ \text{m}$ |
| **P17.12** | (a) 1.27 Pa; (b) 170 Hz; (c) 2.00 m;<br>(d) $340\ \text{m/s}$ |
| **P17.14** | $s = 22.5\ \text{nm}\cos\left(62.8x - 2.16\times10^{4}t\right)$ |
| **P17.16** | (a) 4.63 mm; (b) $14.5\ \text{m/s}$;<br>(c) $4.73\times10^{9}\ \text{W/m}^2$ |
| **P17.18** | (a) $5.00\times10^{-17}\ \text{W}$; (b) $5.00\times10^{-5}\ \text{W}$ |
| **P17.20** | (a) $1.00\times10^{-5}\ \text{W/m}^2$; (b) 90.7 mPa |
| **P17.22** | (a) $I_2 = \left(\frac{f'}{f}\right)^2 I_1$; (b) $I_2 = I_1$ |
| **P17.24** | 21.2 W |
| **P17.26** | (a) 4.51 times larger in water than in air and 18.0 times larger in iron;<br>(b) 5.60 times larger in water than in iron and 331 times larger in air;<br>(c) 59.1 times larger in water than in air and 331 times larger in iron;<br>(d) 0.331 m; 1.49 m; 5.95 m; 10.9 nm; 184 pm; 32.9 pm; 29.2 mPa; 1.73 Pa; 9.67 Pa |
| **P17.28** | see the solution |
| **P17.30** | 10.0 m; 100 m |
| **P17.32** | 86.6 m |
| **P17.34** | (a) 1.76 kJ; (b) 108 dB |
| **P17.36** | no |
| **P17.38** | (a) $2.17\ \text{cm/s}$; (b) 2 000 028.9 Hz;<br>(c) 2 000 057.8 Hz |
| **P17.40** | (a) 441 Hz; 439 Hz; (b) 54.0 dB |
| **P17.42** | (a) $325\ \text{m/s}$; (b) $29.5\ \text{m/s}$ |
| **P17.44** | 48.2° |
| **P17.46** | 46.4° |
| **P17.48** | (a) 7; (b) and (c) see the solution |
| **P17.50** | (a) 0.642 W; (b) $0.004\,28 = 0.428\%$ |
| **P17.52** | (a) 0.232 m; (b) 84.1 nm; (c) 13.8 mm |
| **P17.54** | (a) 0.515/min; (b) 0.614/min |
| **P17.56** | (a) $5.04\ \text{km/s}$; (b) $159\ \mu\text{s}$; (c) 1.90 mm;<br>(d) 0.002 38; (e) 476 MPa;<br>(f) see the solution |
| **P17.58** | (a) see the solution; (b) 85.9 Hz |
| **P17.60** | The gap between bat and insect is closing at $1.69\ \text{m/s}$. |
| **P17.62** | (a) see the solution; (b) 0.343 m;<br>(c) 0.303 m; (d) 0.383 m; (e) 1.03 kHz |
| **P17.64** | 80.0° |
| **P17.66** | 67.0 dB |
| **P17.68** | $\Delta t = \frac{eE}{4\pi d^2 I_0 10^{\beta/10}}$ |
| **P17.70** | see the solution |
| **P17.72** | $\sim 10^{11}\ \text{Hz}$ |

# 18

# Superposition and Standing Waves

**CHAPTER OUTLINE**

18.1 Superposition and Interference
18.2 Standing Waves
18.3 Standing Waves in a String Fixed at Both Ends
18.4 Resonance
18.5 Standing Waves in Air Columns
18.6 Standing Waves in Rod and Plates
18.7 Beats: Interference in Time
18.8 Non-Sinusoidal Wave Patterns

## ANSWERS TO QUESTIONS

**Q18.1** No. Waves with other waveforms are also trains of disturbance that add together when waves from different sources move through the same medium at the same time.

**Q18.2** The energy has not disappeared, but is still carried by the wave pulses. Each particle of the string still has kinetic energy. This is similar to the motion of a simple pendulum. The pendulum does not stop at its equilibrium position during oscillation—likewise the particles of the string do not stop at the equilibrium position of the string when these two waves superimpose.

**Q18.3** No. A wave is not a solid object, but a chain of disturbance. As described by the principle of superposition, the waves move through each other.

**Q18.4** They can, wherever the two waves are nearly enough in phase that their displacements will add to create a total displacement greater than the amplitude of either of the two original waves.

When two one-dimensional sinusoidal waves of the same amplitude interfere, this condition is satisfied whenever the absolute value of the phase difference between the two waves is less than 120°.

**Q18.5** When the two tubes together are not an efficient transmitter of sound from source to receiver, they are an efficient reflector. The incoming sound is reflected back to the source. The waves reflected by the two tubes separately at the junction interfere constructively.

**Q18.6** No. The total energy of the pair of waves remains the same. Energy missing from zones of destructive interference appears in zones of constructive interference.

**Q18.7** Each of these standing wave patterns is made of two superimposed waves of identical frequencies traveling, and hence transferring energy, in opposite directions. Since the energy transfer of the waves are equal, then no net transfer of energy occurs.

**Q18.8** Damping, and non–linear effects in the vibration turn the energy of vibration into internal energy.

**Q18.9** The air in the shower stall can vibrate in standing wave patterns to intensify those frequencies in your voice which correspond to its free vibrations. The hard walls of the bathroom reflect sound very well to make your voice more intense at all frequencies, giving the room a longer reverberation time. The reverberant sound may help you to stay on key.

**Q18.10** The trombone slide and trumpet valves change the length of the air column inside the instrument, to change its resonant frequencies.

**Q18.11** The vibration of the air must have zero amplitude at the closed end. For air in a pipe closed at one end, the diagrams show how resonance vibrations have NA distances that are odd integer submultiples of the NA distance in the fundamental vibration. If the pipe is open, resonance vibrations have NA distances that are all integer submultiples of the NA distance in the fundamental.

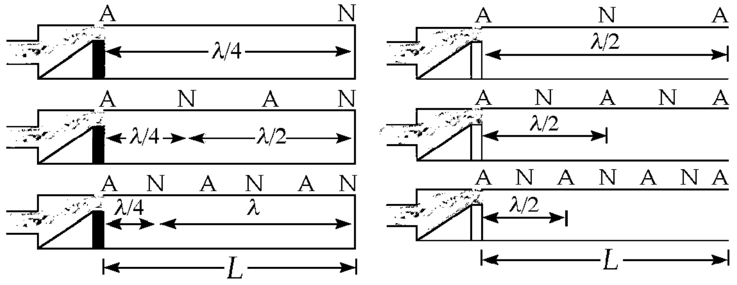


**FIG. Q18.11**

**Q18.12** What is needed is a tuning fork—or other pure-tone generator—of the desired frequency. Strike the tuning fork and pluck the corresponding string on the piano at the same time. If they are precisely in tune, you will hear a single pitch with no amplitude modulation. If the two pitches are a bit off, you will hear beats. As they vibrate, retune the piano string until the beat frequency goes to zero.

**Q18.13** Air blowing fast by a rim of the pipe creates a "shshshsh" sound called edgetone noise, a mixture of all frequencies, as the air turbulently switches between flowing on one side of the edge and the other. The air column inside the pipe finds one or more of its resonance frequencies in the noise. The air column starts vibrating with large amplitude in a standing wave vibration mode. It radiates sound into the surrounding air (and also locks the flapping airstream at the edge to its own frequency, making the noise disappear after just a few cycles).

**Q18.14** A typical standing–wave vibration possibility for a bell is similar to that for the glass shown in Figure 18.17. Here six node-to-node distances fit around the circumference of the rim. The circumference is equal to three times the wavelength of the transverse wave of in-and-out bending of the material. In other states the circumference is two, four, five, or higher integers times the wavelengths of the higher–frequency vibrations. (The circumference being equal to the wavelength would describe the bell moving from side to side without bending, which it can do without producing any sound.) A tuned bell is cast and shaped so that some of these vibrations will have their frequencies constitute higher harmonics of a musical note, the strike tone. This tuning is lost if a crack develops in the bell. The sides of the crack vibrate as antinodes. Energy of vibration may be rapidly converted into internal energy at the end of the crack, so the bell may not ring for so long a time.

**Q18.15** The bow string is pulled away from equilibrium and released, similar to the way that a guitar string is pulled and released when it is plucked. Thus, standing waves will be excited in the bow string. If the arrow leaves from the exact center of the string, then a series of odd harmonics will be excited. Even harmonies will not be excited because they have a node at the point where the string exhibits its maximum displacement.

**Q18.16** Walking makes the person's hand vibrate a little. If the frequency of this motion is equal to the natural frequency of coffee sloshing from side to side in the cup, then a large–amplitude vibration of the coffee will build up in resonance. To get off resonance and back to the normal case of a small-amplitude disturbance producing a small–amplitude result, the person can walk faster, walk slower, or get a larger or smaller cup. Alternatively, even at resonance he can reduce the amplitude by adding damping, as by stirring high–fiber quick–cooking oatmeal into the hot coffee.

**Q18.17** Beats. The propellers are rotating at slightly different frequencies.

**Q18.18** Instead of just radiating sound very softly into the surrounding air, the tuning fork makes the chalkboard vibrate. With its large area this stiff sounding board radiates sound into the air with higher power. So it drains away the fork's energy of vibration faster and the fork stops vibrating sooner. This process exemplifies conservation of energy, as the energy of vibration of the fork is transferred through the blackboard into energy of vibration of the air.

**Q18.19** The difference between static and kinetic friction makes your finger alternately slip and stick as it slides over the glass. Your finger produces a noisy vibration, a mixture of different frequencies, like new sneakers on a gymnasium floor. The glass finds one of its resonance frequencies in the noise. The thin stiff wall of the cup starts vibrating with large amplitude in a standing wave vibration mode. A typical possibility is shown in Figure 18.17. It radiates sound into the surrounding air, and also can lock your squeaking finger to its own frequency, making the noise disappear after just a few cycles. Get a lot of different thin–walled glasses of fine crystal and try them out. Each will generally produce a different note. You can tune them by adding wine.

**Q18.20** Helium is less dense than air. It carries sound at higher speed. Each cavity in your vocal apparatus has a standing-wave resonance frequency, and each of these frequencies is shifted to a higher value. Your vocal chords can vibrate at the same fundamental frequency, but your vocal tract amplifies by resonance a different set of higher frequencies. Then your voice has a different quacky quality.

*Warning*: Inhaling any pressurized gas can cause a gas embolism which can lead to stroke or death, regardless of your age or health status. If you plan to try this demonstration in class, inhale your helium from a balloon, not directly from a pressurized tank.

**Q18.21** Stick a bit of chewing gum to one tine of the second fork. If the beat frequency is then faster than 4 beats per second, the second has a lower frequency than the standard fork. If the beats have slowed down, the second fork has a higher frequency than the standard. Remove the gum, clean the fork, add or subtract 4 Hz according to what you found, and your answer will be the frequency of the second fork.

## SOLUTIONS TO PROBLEMS

### Section 18.1 Superposition and Interference

**P18.1** $y = y_1 + y_2 = 3.00\cos(4.00x - 1.60t) + 4.00\sin(5.0x - 2.00t)$ evaluated at the given $x$ values.

(a) $x = 1.00$, $t = 1.00$ $\quad y = 3.00\cos(2.40\text{ rad}) + 4.00\sin(+3.00\text{ rad}) = \boxed{-1.65\text{ cm}}$

(b) $x = 1.00$, $t = 0.500$ $\quad y = 3.00\cos(+3.20\text{ rad}) + 4.00\sin(+4.00\text{ rad}) = \boxed{-6.02\text{ cm}}$

(c) $x = 0.500$, $t = 0$ $\quad y = 3.00\cos(+2.00\text{ rad}) + 4.00\sin(+2.50\text{ rad}) = \boxed{1.15\text{ cm}}$

**P18.2**

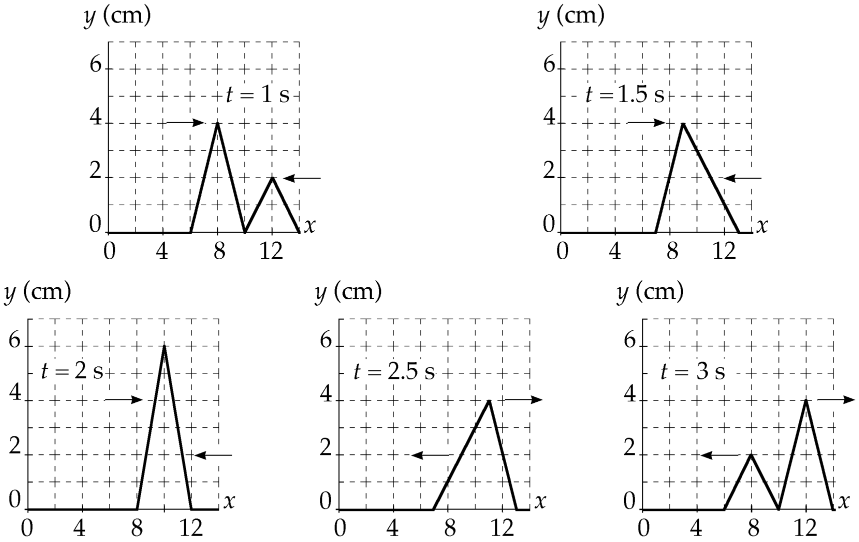


**FIG. P18.2**

**P18.3** (a) $y_1 = f(x - vt)$, so wave 1 travels in the $\boxed{+x \text{ direction}}$

$y_2 = f(x + vt)$, so wave 2 travels in the $\boxed{-x \text{ direction}}$

(b) To cancel, $y_1 + y_2 = 0$:

$$\frac{5}{(3x-4t)^2+2} = \frac{+5}{(3x+4t-6)^2+2}$$

$$(3x-4t)^2 = (3x+4t-6)^2$$

$$3x - 4t = \pm(3x+4t-6)$$

for the positive root, $8t = 6$ $\boxed{t = 0.750 \text{ s}}$

(at $t = 0.750$ s, the waves cancel everywhere)

(c) for the negative root, $6x = 6$ $\boxed{x = 1.00 \text{ m}}$

(at $x = 1.00$ m, the waves cancel always)

**P18.4** Suppose the waves are sinusoidal.

The sum is $(4.00 \text{ cm})\sin(kx - \omega t) + (4.00 \text{ cm})\sin(kx - \omega t + 90.0°)$

$$2(4.00 \text{ cm})\sin(kx - \omega t + 45.0°)\cos 45.0°$$

So the amplitude is $(8.00 \text{ cm})\cos 45.0° = \boxed{5.66 \text{ cm}}$.

**P18.5** The resultant wave function has the form

$$y = 2A_0 \cos\left(\frac{\phi}{2}\right)\sin\left(kx - \omega t + \frac{\phi}{2}\right)$$

(a) $A = 2A_0 \cos\left(\frac{\phi}{2}\right) = 2(5.00)\cos\left[\frac{-\pi/4}{2}\right] = \boxed{9.24 \text{ m}}$

(b) $f = \frac{\omega}{2\pi} = \frac{1\,200\pi}{2\pi} = \boxed{600 \text{ Hz}}$

**P18.6** $2A_0\cos\left(\frac{\phi}{2}\right)=A_0$ so $\frac{\phi}{2}=\cos^{-1}\left(\frac{1}{2}\right)=60.0°=\frac{\pi}{3}$

Thus, the phase difference is $\phi=120°=\frac{2\pi}{3}$

This phase difference results if the time delay is $\frac{T}{3}=\frac{1}{3f}=\frac{\lambda}{3v}$

$$\text{Time delay }=\frac{3.00\text{ m}}{3(2.00\text{ m/s})}=\boxed{0.500\text{ s}}$$

**P18.7** (a) If the end is fixed, there is inversion of the pulse upon reflection. Thus, when they meet, they cancel and the amplitude is $\boxed{\text{zero}}$.

(b) If the end is free, there is no inversion on reflection. When they meet, the amplitude is $2A=2(0.150\text{ m})=\boxed{0.300\text{ m}}$.

**P18.8** (a) $\Delta x=\sqrt{9.00+4.00}-3.00=\sqrt{13}-3.00=0.606\text{ m}$

The wavelength is $\lambda=\frac{v}{f}=\frac{343\text{ m/s}}{300\text{ Hz}}=1.14\text{ m}$

Thus, $\frac{\Delta x}{\lambda}=\frac{0.606}{1.14}=0.530\text{ of a wave},$

or $\Delta\phi=2\pi(0.530)=\boxed{3.33\text{ rad}}$

(b) For destructive interference, we want $\frac{\Delta x}{\lambda}=0.500=f\frac{\Delta x}{v}$

where $\Delta x$ is a constant in this set up. $f=\frac{v}{2\Delta x}=\frac{343}{2(0.606)}=\boxed{283\text{ Hz}}$

**P18.9** We suppose the man's ears are at the same level as the lower speaker. Sound from the upper speaker is delayed by traveling the extra distance $\sqrt{L^2+d^2}-L$.

He hears a minimum when this is $\frac{(2n-1)\lambda}{2}$ with $n=1,\ 2,\ 3,\ \ldots$

Then,
$$\sqrt{L^2+d^2}-L=\frac{(n-1/2)v}{f}$$
$$\sqrt{L^2+d^2}=\frac{(n-1/2)v}{f}+L$$
$$L^2+d^2=\frac{(n-1/2)^2v^2}{f^2}+L^2+\frac{2(n-1/2)vL}{f}$$
$$L=\frac{d^2-(n-1/2)^2v^2/f^2}{2(n-1/2)v/f}\qquad n=1,\ 2,\ 3,\ \ldots$$

This will give us the answer to (b). The path difference starts from nearly zero when the man is very far away and increases to $d$ when $L=0$. The number of minima he hears is the greatest integer solution to $d\geq\frac{(n-1/2)v}{f}$

$$n=\text{ greatest integer }\leq\frac{df}{v}+\frac{1}{2}.$$

*continued on next page*

(a) $$\frac{df}{v}+\frac{1}{2}=\frac{(4.00\text{ m})(200/\text{s})}{330\text{ m/s}}+\frac{1}{2}=2.92$$

He hears $\boxed{\text{two}}$ minima.

(b) With $n=1$,

$$L=\frac{d^2-(1/2)^2 v^2/f^2}{2(1/2)v/f}=\frac{(4.00\text{ m})^2-(330\text{ m/s})^2/4(200/\text{s})^2}{(330\text{ m/s})/200/\text{s}}$$

$$L=\boxed{9.28\text{ m}}$$

with $n=2$

$$L=\frac{d^2-(3/2)^2 v^2/f^2}{2(3/2)v/f}=\boxed{1.99\text{ m}}.$$

**P18.10** Suppose the man's ears are at the same level as the lower speaker. Sound from the upper speaker is delayed by traveling the extra distance $\Delta r=\sqrt{L^2+d^2}-L$.

He hears a minimum when $\Delta r=(2n-1)\left(\frac{\lambda}{2}\right)$ with $n=1, 2, 3, \ldots$

Then, $$\sqrt{L^2+d^2}-L=\left(n-\frac{1}{2}\right)\left(\frac{v}{f}\right)$$

$$\sqrt{L^2+d^2}=\left(n-\frac{1}{2}\right)\left(\frac{v}{f}\right)+L$$

$$L^2+d^2=\left(n-\frac{1}{2}\right)^2\left(\frac{v}{f}\right)^2+2\left(n-\frac{1}{2}\right)\left(\frac{v}{f}\right)L+L^2$$

$$d^2-\left(n-\frac{1}{2}\right)^2\left(\frac{v}{f}\right)^2=2\left(n-\frac{1}{2}\right)\left(\frac{v}{f}\right)L \qquad (1)$$

Equation 1 gives the distances from the lower speaker at which the man will hear a minimum. The path difference $\Delta r$ starts from nearly zero when the man is very far away and increases to $d$ when $L=0$.

(a) The number of minima he hears is the greatest integer value for which $L\geq 0$. This is the same as the greatest integer solution to $d\geq\left(n-\frac{1}{2}\right)\left(\frac{v}{f}\right)$, or

$$\boxed{\text{number of minima heard }=n_{\max}=\text{greatest integer }\leq d\left(\frac{f}{v}\right)+\frac{1}{2}}.$$

(b) From equation 1, the distances at which minima occur are given by

$$\boxed{L_n=\frac{d^2-(n-1/2)^2(v/f)^2}{2(n-1/2)(v/f)}\quad\text{where } n=1, 2, \ldots, n_{\max}}.$$

**P18.11** (a) $\phi_1 = (20.0\ \text{rad/cm})(5.00\ \text{cm}) - (32.0\ \text{rad/s})(2.00\ \text{s}) = 36.0\ \text{rad}$

$\phi_1 = (25.0\ \text{rad/cm})(5.00\ \text{cm}) - (40.0\ \text{rad/s})(2.00\ \text{s}) = 45.0\ \text{rad}$

$\Delta\phi = 9.00\ \text{radians} = 516° = \boxed{156°}$

(b) $\Delta\phi = \left|20.0x - 32.0t - [25.0x - 40.0t]\right| = \left|-5.00x + 8.00t\right|$

At $t = 2.00\ \text{s}$, the requirement is

$$\Delta\phi = \left|-5.00x + 8.00(2.00)\right| = (2n+1)\pi \text{ for any integer } n.$$

For $x < 3.20$, $-5.00x + 16.0$ is positive, so we have

$$-5.00x + 16.0 = (2n+1)\pi, \text{ or}$$

$$x = 3.20 - \frac{(2n+1)\pi}{5.00}$$

The smallest positive value of $x$ occurs for $n = 2$ and is

$$x = 3.20 - \frac{(4+1)\pi}{5.00} = 3.20 - \pi = \boxed{0.058\ 4\ \text{cm}}.$$

**P18.12** (a) First we calculate the wavelength: $\lambda = \frac{v}{f} = \frac{344\ \text{m/s}}{21.5\ \text{Hz}} = 16.0\ \text{m}$

Then we note that the path difference equals $9.00\ \text{m} - 1.00\ \text{m} = \boxed{\frac{1}{2}\lambda}$

Therefore, the receiver will record a minimum in sound intensity.

(b) We choose the origin at the midpoint between the speakers. If the receiver is located at point (*x*, *y*), then we must solve:

$$\sqrt{(x+5.00)^2 + y^2} - \sqrt{(x-5.00)^2 + y^2} = \frac{1}{2}\lambda$$

Then, $\sqrt{(x+5.00)^2 + y^2} = \sqrt{(x-5.00)^2 + y^2} + \frac{1}{2}\lambda$

Square both sides and simplify to get: $20.0x - \frac{\lambda^2}{4} = \lambda\sqrt{(x-5.00)^2 + y^2}$

Upon squaring again, this reduces to: $400x^2 - 10.0\lambda^2 x + \frac{\lambda^4}{16.0} = \lambda^2(x-5.00)^2 + \lambda^2 y^2$

Substituting $\lambda = 16.0\ \text{m}$, and reducing, $\boxed{9.00x^2 - 16.0y^2 = 144}$

or $\frac{x^2}{16.0} - \frac{y^2}{9.00} = 1$

(When plotted this yields a curve called a hyperbola.)

## Section 18.2 **Standing Waves**

**P18.13** $y = (1.50\ \text{m})\sin(0.400x)\cos(200t) = 2A_0 \sin kx \cos \omega t$

Therefore, $k = \dfrac{2\pi}{\lambda} = 0.400\ \text{rad/m}$ $\qquad \lambda = \dfrac{2\pi}{0.400\ \text{rad/m}} = \boxed{15.7\ \text{m}}$

and $\omega = 2\pi f$ so $\qquad f = \dfrac{\omega}{2\pi} = \dfrac{200\ \text{rad/s}}{2\pi\,\text{rad}} = \boxed{31.8\ \text{Hz}}$

The speed of waves in the medium is $v = \lambda f = \dfrac{\lambda}{2\pi} 2\pi f = \dfrac{\omega}{k} = \dfrac{200\ \text{rad/s}}{0.400\ \text{rad/m}} = \boxed{500\ \text{m/s}}$

**P18.14** $y = 0.030\,0\ \text{m}\cos\left(\dfrac{x}{2}\right)\cos(40t)$

(a) nodes occur where $y = 0$:

$$\frac{x}{2} = (2n+1)\frac{\pi}{2}$$

so $x = \boxed{(2n+1)\pi = \pi,\ 3\pi,\ 5\pi,\ \ldots}$.

(b) $y_{\text{max}} = 0.030\,0\ \text{m}\cos\left(\dfrac{0.400}{2}\right) = \boxed{0.029\,4\ \text{m}}$

**P18.15** The facing speakers produce a standing wave in the space between them, with the spacing between nodes being

$$d_{\text{NN}} = \frac{\lambda}{2} = \frac{v}{2f} = \frac{343\ \text{m/s}}{2\left(800\ \text{s}^{-1}\right)} = 0.214\ \text{m}$$

If the speakers vibrate in phase, the point halfway between them is an antinode of pressure at a distance from either speaker of

$$\frac{1.25\ \text{m}}{2} = 0.625\,.$$

Then there isa node at $\qquad 0.625 - \dfrac{0.214}{2} = \boxed{0.518\ \text{m}}$

a node at $\qquad 0.518\ \text{m} - 0.214\ \text{m} = \boxed{0.303\ \text{m}}$

a node at $\qquad 0.303\ \text{m} - 0.214\ \text{m} = \boxed{0.089\,1\ \text{m}}$

a node at $\qquad 0.518\ \text{m} + 0.214\ \text{m} = \boxed{0.732\ \text{m}}$

a node at $\qquad 0.732\ \text{m} + 0.214\ \text{m} = \boxed{0.947\ \text{m}}$

and a node at $\qquad 0.947\ \text{m} + 0.214\ \text{m} = \boxed{1.16\ \text{m}}$ from either speaker.

**P18.16** $y = 2A_0 \sin kx \cos \omega t$

$$\frac{\partial^2 y}{\partial x^2} = -2A_0 k^2 \sin kx \cos \omega t \qquad \frac{\partial^2 y}{\partial t^2} = -2A_0 \omega^2 \sin kx \cos \omega t$$

Substitution into the wave equation gives $-2A_0 k^2 \sin kx \cos \omega t = \left(\frac{1}{v^2}\right)\left(-2A_0 \omega^2 \sin kx \cos \omega t\right)$

This is satisfied, provided that $v = \frac{\omega}{k}$

**P18.17** $y_1 = 3.00 \sin\left[\pi(x + 0.600t)\right]$ cm; $y_2 = 3.00 \sin\left[\pi(x - 0.600t)\right]$ cm

$y = y_1 + y_2 = \left[3.00 \sin(\pi x)\cos(0.600\pi t) + 3.00 \sin(\pi x)\cos(0.600\pi t)\right]$ cm

$y = (6.00 \text{ cm})\sin(\pi x)\cos(0.600\pi t)$

(a) We can take $\cos(0.600\pi t) = 1$ to get the maximum $y$.

At $x = 0.250$ cm, $y_{\max} = (6.00 \text{ cm})\sin(0.250\pi) = \boxed{4.24 \text{ cm}}$

(b) At $x = 0.500$ cm, $y_{\max} = (6.00 \text{ cm})\sin(0.500\pi) = \boxed{6.00 \text{ cm}}$

(c) Now take $\cos(0.600\pi t) = -1$ to get $y_{\max}$:

At $x = 1.50$ cm, $y_{\max} = (6.00 \text{ cm})\sin(1.50\pi)(-1) = \boxed{6.00 \text{ cm}}$

(d) The antinodes occur when $x = \frac{n\lambda}{4}$ $(n = 1, 3, 5, \ldots)$

But $k = \frac{2\pi}{\lambda} = \pi$, so $\lambda = 2.00$ cm

and $x_1 = \frac{\lambda}{4} = \boxed{0.500 \text{ cm}}$ as in (b)

$x_2 = \frac{3\lambda}{4} = \boxed{1.50 \text{ cm}}$ as in (c)

$x_3 = \frac{5\lambda}{4} = \boxed{2.50 \text{ cm}}$

**P18.18** (a) The resultant wave is $y = 2A \sin\left(kx + \frac{\phi}{2}\right)\cos\left(\omega t - \frac{\phi}{2}\right)$

The nodes are located at $kx + \frac{\phi}{2} = n\pi$

so $x = \frac{n\pi}{k} - \frac{\phi}{2k}$

which means that each node is shifted $\frac{\phi}{2k}$ to the left.

(b) The separation of nodes is $\Delta x = \left[(n+1)\frac{\pi}{k} - \frac{\phi}{2k}\right] - \left[\frac{n\pi}{k} - \frac{\phi}{2k}\right]$ $\qquad \Delta x = \frac{\pi}{k} = \frac{\lambda}{2}$

The nodes are still separated by half a wavelength.

## Section 18.3 **Standing Waves in a String Fixed at Both Ends**

**P18.19** $L = 30.0 \text{ m}$; $\mu = 9.00 \times 10^{-3} \text{ kg/m}$; $T = 20.0 \text{ N}$; $f_1 = \dfrac{v}{2L}$

where $v = \left(\dfrac{T}{\mu}\right)^{1/2} = 47.1 \text{ m/s}$

so $f_1 = \dfrac{47.1}{60.0} = \boxed{0.786 \text{ Hz}}$ $\qquad f_2 = 2f_1 = \boxed{1.57 \text{ Hz}}$

$f_3 = 3f_1 = \boxed{2.36 \text{ Hz}}$ $\qquad f_4 = 4f_1 = \boxed{3.14 \text{ Hz}}$

***P18.20** The tension in the string is $T = (4 \text{ kg})(9.8 \text{ m/s}^2) = 39.2 \text{ N}$

Its linear density is $\mu = \dfrac{m}{L} = \dfrac{8 \times 10^{-3} \text{ kg}}{5 \text{ m}} = 1.6 \times 10^{-3} \text{ kg/m}$

and the wave speed on the string is $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{39.2 \text{ N}}{1.6 \times 10^{-3} \text{ kg/m}}} = 156.5 \text{ m/s}$

In its fundamental mode of vibration, we have $\lambda = 2L = 2(5 \text{ m}) = 10 \text{ m}$

Thus, $f = \dfrac{v}{\lambda} = \dfrac{156.5 \text{ m/s}}{10 \text{ m}} = \boxed{15.7 \text{ Hz}}$

**P18.21** (a) Let $n$ be the number of nodes in the standing wave resulting from the 25.0-kg mass. Then $n+1$ is the number of nodes for the standing wave resulting from the 16.0-kg mass. For standing waves, $\lambda = \dfrac{2L}{n}$, and the frequency is $f = \dfrac{v}{\lambda}$.

Thus, $f = \dfrac{n}{2L}\sqrt{\dfrac{T_n}{\mu}}$

and also $f = \dfrac{n+1}{2L}\sqrt{\dfrac{T_{n+1}}{\mu}}$

Thus, $\dfrac{n+1}{n} = \sqrt{\dfrac{T_n}{T_{n+1}}} = \sqrt{\dfrac{(25.0 \text{ kg})g}{(16.0 \text{ kg})g}} = \dfrac{5}{4}$

Therefore, $4n + 4 = 5n$, or $n = 4$

Then, $f = \dfrac{4}{2(2.00 \text{ m})}\sqrt{\dfrac{(25.0 \text{ kg})(9.80 \text{ m/s}^2)}{0.002\,00 \text{ kg/m}}} = \boxed{350 \text{ Hz}}$

(b) The largest mass will correspond to a standing wave of 1 loop

$(n = 1)$ so $350 \text{ Hz} = \dfrac{1}{2(2.00 \text{ m})}\sqrt{\dfrac{m(9.80 \text{ m/s}^2)}{0.002\,00 \text{ kg/m}}}$

yielding $m = \boxed{400 \text{ kg}}$

***P18.22** The first string has linear density

$$\mu_1 = \frac{1.56\times10^{-3}\text{ kg}}{0.658\text{ m}} = 2.37\times10^{-3}\ \text{kg/m}.$$

The second, $\mu_2 = \dfrac{6.75\times10^{-3}\text{ kg}}{0.950\text{ m}} = 7.11\times10^{-3}\ \text{kg/m}$.

The tension in both is $T = 6.93\text{ kg }9.8\ \text{m/s}^2 = 67.9\text{ N}$. The speed of waves in the first string is

$$v_1 = \sqrt{\frac{T}{\mu_1}} = \sqrt{\frac{67.9\text{ N}}{2.37\times10^{-3}\ \text{kg/m}}} = 169\ \text{m/s}$$

and in the second $v_2 = \sqrt{\dfrac{T}{\mu_2}} = 97.8\ \text{m/s}$. The two strings vibrate at the same frequency, according to $\dfrac{n_1 v_1}{2L_1} = \dfrac{n_2 v_2}{2L_2}$

$$\frac{n_1 169\ \text{m/s}}{2(0.658\text{ m})} = \frac{n_2 97.8\ \text{m/s}}{2(0.950\text{ m})}$$

$\dfrac{n_2}{n_1} = 2.50 = \dfrac{5}{2}$. Thus $n_1 = 2$ and $n_2 = 5$ are the number of antinodes on each string in the lowest resonance with a node at the junction.

(b) The first string has $2+1=3$ nodes and the second string 5 more nodes, for a total of 8, or $\boxed{6}$ other than the vibrator and pulley.

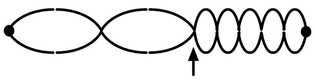


**FIG. P18.22(b)**

(a) The frequency is $\dfrac{2(169\ \text{m/s})}{2(0.658\text{ m})} = \boxed{257\text{ Hz}}$.

***P18.23** For the E-string on a guitar vibrating as a whole, $v = f\lambda = 330\text{ Hz}(2)64.0\text{ cm}$. When it is stopped at the first fret we have $\sqrt[12]{2}\,330\text{ Hz}(2)L_F = v = 330\text{ Hz}(2)64.0\text{ cm}$. So $L_F = \dfrac{64.0\text{ cm}}{\sqrt[12]{2}}$. Similarly for the second fret, $2^{2/12}330\text{ Hz}(2)L_{F\#} = v = 330\text{ Hz}(2)64.0\text{ cm}$. $L_{F\#} = \dfrac{64.0\text{ cm}}{2^{2/12}}$. The spacing between the first and second frets is

$$64.0\text{ cm}\left(\frac{1}{2^{1/12}} - \frac{1}{2^{2/12}}\right) = 64.0\text{ cm}\left(\frac{1}{1.059\,5} - \frac{1}{1.059\,5^2}\right) = 3.39\text{ cm}.$$

This is a more precise version of the answer to the example in the text.

Now the eighteenth fret is distant from the bridge by $L_{18} = \dfrac{64.0\text{ cm}}{2^{18/12}}$. And the nineteenth lets this much string vibrate: $L_{19} = \dfrac{64.0\text{ cm}}{2^{19/12}}$. The distance between them is

$$64.0\text{ cm}\left(\frac{1}{2^{18/12}} - \frac{1}{2^{19/12}}\right) = 64.0\text{ cm}\frac{1}{2^{1.5}}\left(1 - \frac{1}{2^{1/12}}\right) = \boxed{1.27\text{ cm}}.$$

***P18.24** For the whole string vibrating, $d_{NN} = 0.64 \text{ m} = \frac{\lambda}{2}$; $\lambda = 1.28 \text{ m}$. The speed of a pulse on the string is $v = f\lambda = 330\frac{1}{s}1.28 \text{ m} = 422 \text{ m/s}$.

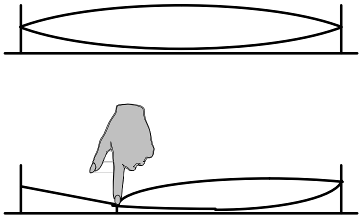


**FIG. P18.24(a)**

(a) When the string is stopped at the fret, $d_{NN} = \frac{2}{3}0.64 \text{ m} = \frac{\lambda}{2}$;

$\lambda = 0.853 \text{ m}$

$$f = \frac{v}{\lambda} = \frac{422 \text{ m/s}}{0.853 \text{ m}} = \boxed{495 \text{ Hz}}.$$

(b) The light touch at a point one third of the way along the string damps out vibration in the two lowest vibration states of the string as a whole. The whole string vibrates in its third resonance possibility: $3d_{NN} = 0.64 \text{ m} = 3\frac{\lambda}{2}$;

$\lambda = 0.427 \text{ m}$

$$f = \frac{v}{\lambda} = \frac{422 \text{ m/s}}{0.427 \text{ m}} = \boxed{990 \text{ Hz}}.$$

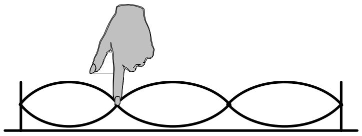


**FIG. P18.24(b)**

**P18.25** $f_1 = \frac{v}{2L}$, where $v = \left(\frac{T}{\mu}\right)^{1/2}$

(a) If $L$ is doubled, then $f_1 \propto L^{-1}$ will be reduced by a factor $\frac{1}{2}$.

(b) If $\mu$ is doubled, then $f_1 \propto \mu^{-1/2}$ will be reduced by a factor $\frac{1}{\sqrt{2}}$.

(c) If $T$ is doubled, then $f_1 \propto \sqrt{T}$ will increase by a factor of $\sqrt{2}$.

**P18.26** $L = 60.0 \text{ cm} = 0.600 \text{ m}$; $T = 50.0 \text{ N}$; $\mu = 0.100 \text{ g/cm} = 0.010\,0 \text{ kg/m}$

$$f_n = \frac{nv}{2L}$$

where

$$v = \left(\frac{T}{\mu}\right)^{1/2} = 70.7 \text{ m/s}$$

$$f_n = n\left(\frac{70.7}{1.20}\right) = 58.9n = 20\,000 \text{ Hz}$$

Largest $n = 339 \Rightarrow f = \boxed{19.976 \text{ kHz}}$.

**P18.27** $d_{\text{NN}} = 0.700 \text{ m}$

$\lambda = 1.40 \text{ m}$

$$f\lambda = v = 308 \text{ m/s} = \sqrt{\frac{T}{\left(1.20 \times 10^{-3}\right)/(0.700)}}$$

(a) $T = \boxed{163 \text{ N}}$

(b) $f_3 = \boxed{660 \text{ Hz}}$

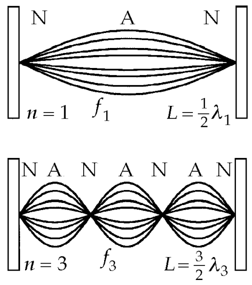

N A N
$n = 1$ $f_1$ $L = \frac{1}{2}\lambda_1$
N A N A N A N
$n = 3$ $f_3$ $L = \frac{3}{2}\lambda_3$

**FIG. P18.27**

**P18.28** $\lambda_G = 2(0.350\text{ m}) = \dfrac{v}{f_G}$; $\lambda_A = 2L_A = \dfrac{v}{f_A}$

$$L_G - L_A = L_G - \left(\frac{f_G}{f_A}\right)L_G = L_G\left(1 - \frac{f_G}{f_A}\right) = (0.350\text{ m})\left(1 - \frac{392}{440}\right) = 0.038\,2\text{ m}$$

Thus, $L_A = L_G - 0.038\,2\text{ m} = 0.350\text{ m} - 0.038\,2\text{ m} = 0.312\text{ m}$,

or the finger should be placed $\boxed{\text{31.2 cm from the bridge}}$.

$$L_A = \frac{v}{2f_A} = \frac{1}{2f_A}\sqrt{\frac{T}{\mu}};\; dL_A = \frac{dT}{4f_A\sqrt{T\mu}};\; \frac{dL_A}{L_A} = \frac{1}{2}\frac{dT}{T}$$

$$\frac{dT}{T} = 2\frac{dL_A}{L_A} = 2\frac{0.600\text{ cm}}{(35.0 - 3.82)\text{ cm}} = \boxed{3.84\%}$$

**P18.29** In the fundamental mode, the string above the rod has only two nodes, at A and B, with an anti-node halfway between A and B. Thus,

$$\frac{\lambda}{2} = \overline{AB} = \frac{L}{\cos\theta} \text{ or } \lambda = \frac{2L}{\cos\theta}.$$

Since the fundamental frequency is *f*, the wave speed in this segment of string is

$$v = \lambda f = \frac{2Lf}{\cos\theta}.$$

Also, $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{T}{m/\overline{AB}}} = \sqrt{\dfrac{TL}{m\cos\theta}}$

where *T* is the tension in this part of the string. Thus,

$$\frac{2Lf}{\cos\theta} = \sqrt{\frac{TL}{m\cos\theta}} \text{ or } \frac{4L^2f^2}{\cos^2\theta} = \frac{TL}{m\cos\theta}$$

and the mass of string above the rod is:

$$m = \frac{T\cos\theta}{4Lf^2} \qquad \text{[Equation 1]}$$

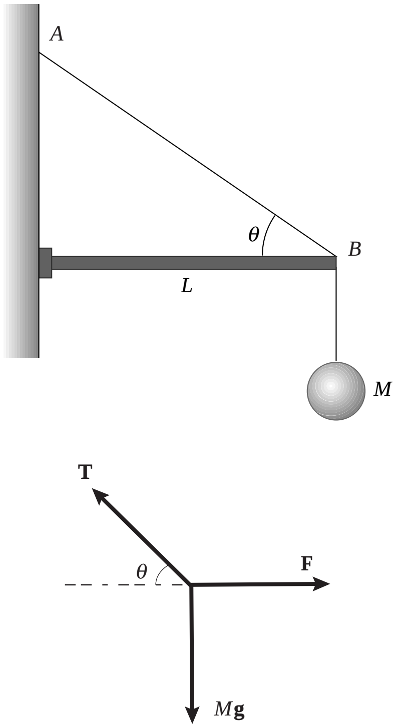


**FIG. P18.29**

Now, consider the tension in the string. The light rod would rotate about point P if the string exerted any vertical force on it. Therefore, recalling Newton's third law, the rod must exert only a horizontal force on the string. Consider a free-body diagram of the string segment in contact with the end of the rod.

$$\sum F_y = T\sin\theta - Mg = 0 \Rightarrow T = \frac{Mg}{\sin\theta}$$

Then, from Equation 1, the mass of string above the rod is

$$m = \left(\frac{Mg}{\sin\theta}\right)\frac{\cos\theta}{4Lf^2} = \boxed{\frac{Mg}{4Lf^2\tan\theta}}.$$

***P18.30** Let $m = \rho V$ represent the mass of the copper cylinder. The original tension in the wire is $T_1 = mg = \rho V g$. The water exerts a buoyant force $\rho_{\text{water}}\left(\frac{V}{2}\right)g$ on the cylinder, to reduce the tension to

$$T_2 = \rho V g - \rho_{\text{water}}\left(\frac{V}{2}\right)g = \left(\rho - \frac{\rho_{\text{water}}}{2}\right)Vg.$$

The speed of a wave on the string changes from $\sqrt{\frac{T_1}{\mu}}$ to $\sqrt{\frac{T_2}{\mu}}$. The frequency changes from

$$f_1 = \frac{v_1}{\lambda} = \sqrt{\frac{T_1}{\mu}}\frac{1}{\lambda} \text{ to } f_2 = \sqrt{\frac{T_2}{\mu}}\frac{1}{\lambda}$$

where we assume $\lambda = 2L$ is constant.

Then $$\frac{f_2}{f_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{\rho - \rho_{\text{water}}/2}{\rho}} = \sqrt{\frac{8.92 - 1.00/2}{8.92}}$$

$$f_2 = 300\text{ Hz}\sqrt{\frac{8.42}{8.92}} = \boxed{291\text{ Hz}}$$

***P18.31** Comparing $y = (0.002\text{ m})\sin\big((\pi\text{ rad/m})x\big)\cos\big((100\pi\text{ rad/s})t\big)$

with $y = 2A\sin kx\cos\omega t$

we find $k = \frac{2\pi}{\lambda} = \pi\,\text{m}^{-1}$, $\lambda = 2.00\text{ m}$, and $\omega = 2\pi f = 100\pi\,\text{s}^{-1}$: $f = 50.0\text{ Hz}$

(a) Then the distance between adjacent nodes is $d_{\text{NN}} = \frac{\lambda}{2} = 1.00\text{ m}$

and on the string are $\frac{L}{d_{\text{NN}}} = \frac{3.00\text{ m}}{1.00\text{ m}} = \boxed{3\text{ loops}}$

For the speed we have $v = f\lambda = \left(50\text{ s}^{-1}\right)2\text{ m} = 100\text{ m/s}$

(b) In the simplest standing wave vibration, $d_{\text{NN}} = 3.00\text{ m} = \frac{\lambda_b}{2}$, $\lambda_b = 6.00\text{ m}$

and $f_b = \frac{v_a}{\lambda_b} = \frac{100\text{ m/s}}{6.00\text{ m}} = \boxed{16.7\text{ Hz}}$

(c) In $v_0 = \sqrt{\frac{T_0}{\mu}}$, if the tension increases to $T_c = 9T_0$ and the string does not stretch, the speed increases to

$$v_c = \sqrt{\frac{9T_0}{\mu}} = 3\sqrt{\frac{T_0}{\mu}} = 3v_0 = 3(100\text{ m/s}) = 300\text{ m/s}$$

Then $\lambda_c = \frac{v_c}{f_a} = \frac{300\text{ m/s}}{50\text{ s}^{-1}} = 6.00\text{ m}$ $\quad d_{\text{NN}} = \frac{\lambda_c}{2} = 3.00\text{ m}$

and $\boxed{\text{one}}$ loop fits onto the string.

## Section 18.4 **Resonance**

**P18.32** The natural frequency is

$$f=\frac{1}{T}=\frac{1}{2\pi}\sqrt{\frac{g}{L}}=\frac{1}{2\pi}\sqrt{\frac{9.80\ \text{m/s}^2}{2.00\ \text{m}}}=0.352\ \text{Hz}.$$

The big brother must push at this same frequency of $\boxed{0.352\ \text{Hz}}$ to produce resonance.

**P18.33** (a) The wave speed is $v=\dfrac{9.15\ \text{m}}{2.50\ \text{s}}=\boxed{3.66\ \text{m/s}}$

(b) From the figure, there are antinodes at both ends of the pond, so the distance between adjacent antinodes

is $d_{\text{AA}}=\dfrac{\lambda}{2}=9.15\ \text{m}$,

and the wavelength is $\lambda=18.3\ \text{m}$

The frequency is then $f=\dfrac{v}{\lambda}=\dfrac{3.66\ \text{m/s}}{18.3\ \text{m}}=\boxed{0.200\ \text{Hz}}$

We have assumed the wave speed is the same for all wavelengths.

**P18.34** The wave speed is $v=\sqrt{gd}=\sqrt{\left(9.80\ \text{m/s}^2\right)(36.1\ \text{m})}=18.8\ \text{m/s}$

The bay has one end open and one closed. Its simplest resonance is with a node of horizontal velocity, which is also an antinode of vertical displacement, at the head of the bay and an antinode of velocity, which is a node of displacement, at the mouth. The vibration of the water in the bay is like that in one half of the pond shown in Figure P18.33.

Then, $d_{\text{NA}}=210\times10^3\ \text{m}=\dfrac{\lambda}{4}$

and $\lambda=840\times10^3\ \text{m}$

Therefore, the period is $T=\dfrac{1}{f}=\dfrac{\lambda}{v}=\dfrac{840\times10^3\ \text{m}}{18.8\ \text{m/s}}=4.47\times10^4\ \text{s}=\boxed{12\ \text{h}\ 24\ \text{min}}$

$\boxed{\text{This agrees precisely with the period of the lunar excitation}}$, so we identify the extra-high tides as amplified by resonance.

**P18.35** The distance between adjacent nodes is one-quarter of the circumference.

$$d_{\text{NN}}=d_{\text{AA}}=\frac{\lambda}{2}=\frac{20.0\ \text{cm}}{4}=5.00\ \text{cm}$$

so $\lambda=10.0\ \text{cm}$ and $f=\dfrac{v}{\lambda}=\dfrac{900\ \text{m/s}}{0.100\ \text{m}}=9\,000\ \text{Hz}=\boxed{9.00\ \text{kHz}}$.

The singer must match this frequency quite precisely for some interval of time to feed enough energy into the glass to crack it.

## Section 18.5 **Standing Waves in Air Columns**

**P18.36** $d_{AA} = 0.320\text{ m};\ \lambda = 0.640\text{ m}$

(a) $f = \dfrac{v}{\lambda} = \boxed{531\text{ Hz}}$

(b) $\lambda = 0.085\,0\text{ m};\ d_{AA} = \boxed{42.5\text{ mm}}$

**P18.37** (a) For the fundamental mode in a closed pipe, $\lambda = 4L$, as in the diagram.

But $v = f\lambda$, therefore $L = \dfrac{v}{4f}$

So, $L = \dfrac{343\text{ m/s}}{4\left(240\text{ s}^{-1}\right)} = \boxed{0.357\text{ m}}$

(b) For an open pipe, $\lambda = 2L$, as in the diagram.

So, $L = \dfrac{v}{2f} = \dfrac{343\text{ m/s}}{2\left(240\text{ s}^{-1}\right)} = \boxed{0.715\text{ m}}$

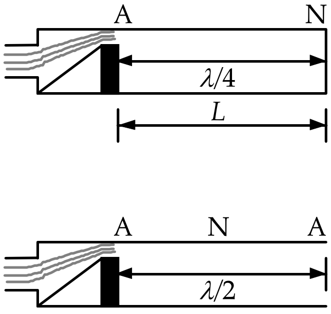


**FIG. P18.37**

**P18.38** The wavelength is $\lambda = \dfrac{v}{f} = \dfrac{343\text{ m/s}}{261.6/\text{s}} = 1.31\text{ m}$

so the length of the open pipe vibrating in its simplest (A-N-A) mode is

$$d_{\text{A to A}} = \frac{1}{2}\lambda = \boxed{0.656\text{ m}}$$

A closed pipe has (N-A) for its simplest resonance,

(N-A-N-A) for the second,

and (N-A-N-A-N-A) for the third.

Here, the pipe length is $5d_{\text{N to A}} = \dfrac{5\lambda}{4} = \dfrac{5}{4}(1.31\text{ m}) = \boxed{1.64\text{ m}}$

***P18.39** Assuming an air temperature of $T = 37°\text{C} = 310\text{ K}$, the speed of sound inside the pipe is

$$v = (331\text{ m/s})\sqrt{\frac{310\text{ K}}{273\text{ K}}} = 353\text{ m/s}.$$

In the fundamental resonant mode, the wavelength of sound waves in a pipe closed at one end is $\lambda = 4L$. Thus, for the whooping crane

$$\lambda = 4(5.0\text{ ft}) = 2.0\times10^{1}\text{ ft and } f = \frac{v}{\lambda} = \frac{(353\text{ m/s})}{2.0\times10^{1}\text{ ft}}\left(\frac{3.281\text{ ft}}{1\text{ m}}\right) = \boxed{57.9\text{ Hz}}.$$

**P18.40** The air in the auditory canal, about 3 cm long, can vibrate with a node at the closed end and antinode at the open end,

with $d_{\text{N to A}} = 3\text{ cm} = \dfrac{\lambda}{4}$

so $\lambda = 0.12\text{ m}$

and $f = \dfrac{v}{\lambda} = \dfrac{343\text{ m/s}}{0.12\text{ m}} \approx \boxed{3\text{ kHz}}$

> A small-amplitude external excitation at this frequency can, over time, feed energy into a larger-amplitude resonance vibration of the air in the canal, making it audible.

**P18.41** For a closed box, the resonant frequencies will have nodes at both sides, so the permitted wavelengths will be $L = \frac{1}{2}n\lambda$, $(n = 1, 2, 3, \ldots)$.

i.e., $L = \dfrac{n\lambda}{2} = \dfrac{nv}{2f}$ and $f = \dfrac{nv}{2L}$.

Therefore, with $L = 0.860\text{ m}$ and $L' = 2.10\text{ m}$, the resonant frequencies are

$f_n = \boxed{n(206\text{ Hz})}$ for $L = 0.860\text{ m}$ for each $n$ from 1 to 9

and $f_n' = \boxed{n(84.5\text{ Hz})}$ for $L' = 2.10\text{ m}$ for each $n$ from 2 to 23.

**P18.42** The wavelength of sound is $\lambda = \dfrac{v}{f}$

The distance between water levels at resonance is $d = \dfrac{v}{2f}$ $\quad \therefore Rt = \pi r^2 d = \dfrac{\pi r^2 v}{2f}$

and $t = \boxed{\dfrac{\pi r^2 v}{2Rf}}$.

**P18.43** For both open and closed pipes, resonant frequencies are equally spaced as numbers. The set of resonant frequencies then must be 650 Hz, 550 Hz, 450 Hz, 350 Hz, 250 Hz, 150 Hz, 50 Hz. These are odd-integer multipliers of the fundamental frequency of $\boxed{50.0\text{ Hz}}$. Then the pipe length is

$d_{NA} = \dfrac{\lambda}{4} = \dfrac{v}{4f} = \dfrac{340\text{ m/s}}{4(50/\text{s})} = \boxed{1.70\text{ m}}$.

**P18.44** $\dfrac{\lambda}{2} = d_{\text{AA}} = \dfrac{L}{n}$ or $\quad L = \dfrac{n\lambda}{2} \quad$ for $n = 1, 2, 3, \ldots$

Since $\lambda = \dfrac{v}{f}$ $\quad L = n\left(\dfrac{v}{2f}\right) \quad$ for $n = 1, 2, 3, \ldots$

With $v = 343\text{ m/s}$ and $\quad f = 680\text{ Hz}$,

$$L = n\left(\frac{343\text{ m/s}}{2(680\text{ Hz})}\right) = n(0.252\text{ m}) \quad \text{for } n = 1, 2, 3, \ldots$$

Possible lengths for resonance are: $L = \boxed{0.252\text{ m}, 0.504\text{ m}, 0.757\text{ m}, \ldots, n(0.252)\text{ m}}$.

**P18.45** For resonance in a narrow tube open at one end,

$$f = n\frac{v}{4L}\left(n = 1,\ 3,\ 5,\ \ldots\right).$$

(a) Assuming $n = 1$ and $n = 3$,

$$384 = \frac{v}{4(0.228)} \text{ and } 384 = \frac{3v}{4(0.683)}.$$

In either case, $v = \boxed{350\ \text{m/s}}$.

(b) For the next resonance $n = 5$, and $L = \frac{5v}{4f} = \frac{5(350\ \text{m/s})}{4\left(384\ \text{s}^{-1}\right)} = \boxed{1.14\ \text{m}}$.

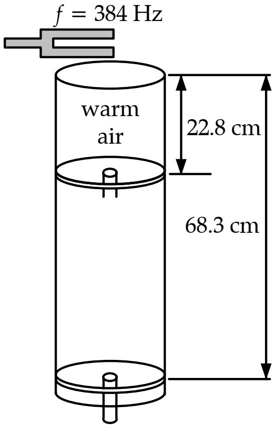


**FIG. P18.45**

**P18.46** The length corresponding to the fundamental satisfies $f = \frac{v}{4L}$: $L_1 = \frac{v}{4f} = \frac{34}{4(512)} = 0.167\ \text{m}$.

Since $L > 20.0$ cm, the *next* two modes will be observed, corresponding to $f = \frac{3v}{4L_2}$ and $f = \frac{5v}{4L_3}$.

or $L_2 = \frac{3v}{4f} = \boxed{0.502\ \text{m}}$ and $L_3 = \frac{5v}{4f} = \boxed{0.837\ \text{m}}$.

**P18.47** We suppose these are the lowest resonances of the enclosed air columns.

For one, $\lambda = \frac{v}{f} = \frac{343\ \text{m/s}}{256\ \text{s}^{-1}} = 1.34\ \text{m}$ length $= d_{AA} = \frac{\lambda}{2} = 0.670\ \text{m}$

For the other, $\lambda = \frac{v}{f} = \frac{343\ \text{m/s}}{440\ \text{s}^{-1}} = 0.780\ \text{m}$ length $= 0.390\ \text{m}$

So,

(b) original length $= \boxed{1.06\ \text{m}}$

$\lambda = 2d_{AA} = 2.12\ \text{m}$

(a) $f = \frac{343\ \text{m/s}}{2.12\ \text{m}} = \boxed{162\ \text{Hz}}$

**P18.48** (a) For the fundamental mode of an open tube,

$$L = \frac{\lambda}{2} = \frac{v}{2f} = \frac{343\ \text{m/s}}{2\left(880\ \text{s}^{-1}\right)} = \boxed{0.195\ \text{m}}.$$

(b) $v = 331\ \text{m/s}\sqrt{1 + \frac{(-5.00)}{273}} = 328\ \text{m/s}$

We ignore the thermal expansion of the metal.

$$f = \frac{v}{\lambda} = \frac{v}{2L} = \frac{328\ \text{m/s}}{2(0.195\ \text{m})} = \boxed{841\ \text{Hz}}$$

The flute is flat by a semitone.

## Section 18.6 **Standing Waves in Rod and Plates**

**P18.49** (a) $f=\dfrac{v}{2L}=\dfrac{5\,100}{(2)(1.60)}=\boxed{1.59\text{ kHz}}$

(b) Since it is held in the center, there must be a node in the center as well as antinodes at the ends. The even harmonics have an antinode at the center so only $\boxed{\text{the odd harmonics}}$ are present.

(c) $f=\dfrac{v'}{2L}=\dfrac{3\,560}{(2)(1.60)}=\boxed{1.11\text{ kHz}}$

**P18.50** When the rod is clamped at one-quarter of its length, the vibration pattern reads ANANA and the rod length is $L=2d_{\text{AA}}=\lambda$.

Therefore, $L=\dfrac{v}{f}=\dfrac{5\,100\text{ m/s}}{4\,400\text{ Hz}}=\boxed{1.16\text{ m}}$

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## Section 18.7 **Beats: Interference in Time**

**P18.51** $f\propto v\propto\sqrt{T}$ $\qquad f_{\text{new}}=110\sqrt{\dfrac{540}{600}}=104.4\text{ Hz}$

$\Delta f=\boxed{5.64\text{ beats/s}}$

**P18.52** (a) The string could be tuned to either $\boxed{521\text{ Hz or }525\text{ Hz}}$ from this evidence.

(b) Tightening the string raises the wave speed and frequency. If the frequency were originally 521 Hz, the beats would slow down.

Instead, the frequency must have started at 525 Hz to become $\boxed{526\text{ Hz}}$.

(c) From $f=\dfrac{v}{\lambda}=\dfrac{\sqrt{T/\mu}}{2L}=\dfrac{1}{2L}\sqrt{\dfrac{T}{\mu}}$

$$\frac{f_2}{f_1}=\sqrt{\frac{T_2}{T_1}}\text{ and }T_2=\left(\frac{f_2}{f_1}\right)^2T_1=\left(\frac{523\text{ Hz}}{526\text{ Hz}}\right)^2T_1=0.989T_1.$$

The fractional change that should be made in the tension is then

$$\text{fractional change}=\frac{T_1-T_2}{T_1}=1-0.989=0.011\,4=1.14\%\text{ lower.}$$

The tension should be $\boxed{\text{reduced by }1.14\%}$.

**P18.53** For an echo $f' = f\dfrac{(v+v_s)}{(v-v_s)}$ the beat frequency is $f_b = |f' - f|$.

Solving for $f_b$.

gives $f_b = f\dfrac{(2v_s)}{(v-v_s)}$ when approaching wall.

(a) $$f_b = (256)\frac{2(1.33)}{(343-1.33)} = \boxed{1.99\text{ Hz}}\text{ beat frequency}$$

(b) When he is moving away from the wall, $v_s$ changes sign. Solving for $v_s$ gives

$$v_s = \frac{f_b v}{2f - f_b} = \frac{(5)(343)}{(2)(256)-5} = \boxed{3.38\text{ m/s}}.$$

***P18.54** Using the $\boxed{4\text{ and }2\frac{2}{3}\text{- foot pipes}}$ produces actual frequencies of 131 Hz and 196 Hz and a combination tone at $(196-131)\text{Hz} = 65.4\text{ Hz}$, so this pair supplies the so-called missing fundamental. The 4 and 2-foot pipes produce a combination tone $(262-131)\text{Hz} = 131\text{ Hz}$, so this does not work.

The $\boxed{2\frac{2}{3}\text{ and 2-foot pipes}}$ produce a combination tone at $(262-196)\text{Hz} = 65.4\text{ Hz}$, so this works.

Also, $\boxed{4,\ 2\frac{2}{3},\text{ and 2-foot pipes}}$ all playing together produce the 65.4-Hz combination tone.

## Section 18.8 **Non-Sinusoidal Wave Patterns**

**P18.55** We list the frequencies of the harmonics of each note in Hz:

| | Harmonic | | | | |
|---|---|---|---|---|---|
| Note | 1 | 2 | 3 | 4 | 5 |
| A | 440.00 | 880.00 | 1 320.0 | 1 760.0 | 2 200.0 |
| C# | 554.37 | 1 108.7 | 1 663.1 | 2 217.5 | 2 771.9 |
| E | 659.26 | 1 318.5 | 1 977.8 | 2 637.0 | 3 296.3 |

The second harmonic of E is close the the third harmonic of A, and the fourth harmonic of C# is close to the fifth harmonic of A.

**P18.56** We evaluate

$$s = 100\sin\theta + 157\sin 2\theta + 62.9\sin 3\theta + 105\sin 4\theta$$
$$+51.9\sin 5\theta + 29.5\sin 6\theta + 25.3\sin 7\theta$$

where $s$ represents particle displacement in nanometers and $\theta$ represents the phase of the wave in radians. As $\theta$ advances by $2\pi$, time advances by (1/523) s. Here is the result:

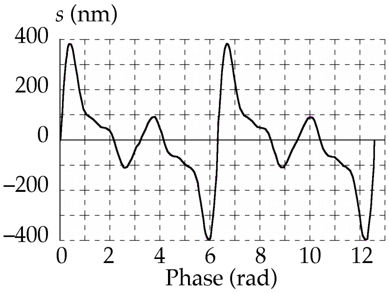

**FIG. P18.56**

## Additional Problems

**P18.57** $f = 87.0\text{ Hz}$

speed of sound in air: $v_a = 340\text{ m/s}$

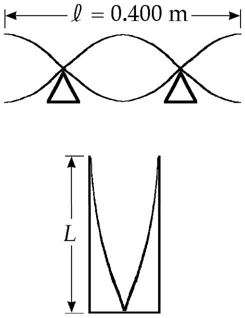

**FIG. P18.57**

(a) $\lambda_b = \ell$ $\quad v = f\lambda_b = \left(87.0\text{ s}^{-1}\right)(0.400\text{ m})$

$$v = \boxed{34.8\text{ m/s}}$$

(b) $\left.\begin{aligned}\lambda_a &= 4L\\ v_a &= \lambda_a f\end{aligned}\right\}$ $\quad L = \dfrac{v_a}{4f} = \dfrac{340\text{ m/s}}{4\left(87.0\text{ s}^{-1}\right)} = \boxed{0.977\text{ m}}$

***P18.58** (a) Use the Doppler formula

$$f' = f\frac{(v \pm v_0)}{(v \mp v_s)}.$$

With $f_1' =$ frequency of the speaker in front of student and
$f_2' =$ frequency of the speaker behind the student.

$$f_1' = (456\text{ Hz})\frac{(343\text{ m/s} + 1.50\text{ m/s})}{(343\text{ m/s} - 0)} = 458\text{ Hz}$$

$$f_2' = (456\text{ Hz})\frac{(343\text{ m/s} - 1.50\text{ m/s})}{(343\text{ m/s} + 0)} = 454\text{ Hz}$$

Therefore, $f_b = f_1' - f_2' = \boxed{3.99\text{ Hz}}$.

(b) The waves broadcast by both speakers have $\lambda = \dfrac{v}{f} = \dfrac{343\text{ m/s}}{456/\text{s}} = 0.752\text{ m}$. The standing wave between them has $d_{AA} = \dfrac{\lambda}{2} = 0.376\text{ m}$. The student walks from one maximum to the next in time $\Delta t = \dfrac{0.376\text{ m}}{1.50\text{ m/s}} = 0.251\text{ s}$, so the frequency at which she hears maxima is $f = \dfrac{1}{T} = \boxed{3.99\text{ Hz}}$.

**P18.59** Moving away from station, frequency is depressed:

$f' = 180 - 2.00 = 178\text{ Hz}:$ $\quad 178 = 180\dfrac{343}{343 - (-v)}$

Solving for $v$ gives $\quad v = \dfrac{(2.00)(343)}{178}$

Therefore, $\quad v = \boxed{3.85\text{ m/s away from station}}$

Moving toward the station, the frequency is enhanced:

$f' = 180 + 2.00 = 182\text{ Hz}:$ $\quad 182 = 180\dfrac{343}{343 - v}$

Solving for $v$ gives $\quad 4 = \dfrac{(2.00)(343)}{182}$

Therefore, $\quad v = \boxed{3.77\text{ m/s toward the station}}$

**P18.60** $v=\sqrt{\dfrac{(48.0)(2.00)}{4.80\times10^{-3}}}=141\ \text{m/s}$

$d_{\text{NN}}=1.00\ \text{m};\ \lambda=2.00\ \text{m};\ f=\dfrac{v}{\lambda}=70.7\ \text{Hz}$

$\lambda_a=\dfrac{v_a}{f}=\dfrac{343\ \text{m/s}}{70.7\ \text{Hz}}=\boxed{4.85\ \text{m}}$

**P18.61** Call $L$ the depth of the well and $v$ the speed of sound.

Then for some integer $n$ $\quad L=(2n-1)\dfrac{\lambda_1}{4}=(2n-1)\dfrac{v}{4f_1}=\dfrac{(2n-1)(343\ \text{m/s})}{4\left(51.5\ \text{s}^{-1}\right)}$

and for the next resonance $\quad L=\left[2(n+1)-1\right]\dfrac{\lambda_2}{4}=(2n+1)\dfrac{v}{4f_2}=\dfrac{(2n+1)(343\ \text{m/s})}{4\left(60.0\ \text{s}^{-1}\right)}$

Thus, $\quad \dfrac{(2n-1)(343\ \text{m/s})}{4\left(51.5\ \text{s}^{-1}\right)}=\dfrac{(2n+1)(343\ \text{m/s})}{4\left(60.0\ \text{s}^{-1}\right)}$

and we require an *integer* solution to $\quad \dfrac{2n+1}{60.0}=\dfrac{2n-1}{51.5}$

The equation gives $n=\dfrac{111.5}{17}=6.56$, so the best fitting integer is $n=7$.

Then $\quad L=\dfrac{\left[2(7)-1\right](343\ \text{m/s})}{4\left(51.5\ \text{s}^{-1}\right)}=21.6\ \text{m}$

and $\quad L=\dfrac{\left[2(7)+1\right](343\ \text{m/s})}{4\left(60.0\ \text{s}^{-1}\right)}=21.4\ \text{m}$

suggest the best value for the depth of the well is $\boxed{21.5\ \text{m}}$.

**P18.62** The second standing wave mode of the air in the pipe reads ANAN, with $d_{\text{NA}}=\dfrac{\lambda}{4}=\dfrac{1.75\ \text{m}}{3}$

so $\quad \lambda=2.33\ \text{m}$

and $\quad f=\dfrac{v}{\lambda}=\dfrac{343\ \text{m/s}}{2.33\ \text{m}}=147\ \text{Hz}$

For the string, $\lambda$ and $v$ are different but $f$ is the same.

$$\frac{\lambda}{2}=d_{\text{NN}}=\frac{0.400\ \text{m}}{2}$$

so $\quad \lambda=0.400\ \text{m}$

$v=\lambda f=(0.400\ \text{m})(147\ \text{Hz})=58.8\ \text{m/s}=\sqrt{\dfrac{T}{\mu}}$

$T=\mu v^2=\left(9.00\times10^{-3}\ \text{kg/m}\right)(58.8\ \text{m/s})^2=\boxed{31.1\ \text{N}}$

**P18.63** (a) Since the first node is at the weld, the wavelength in the thin wire is 2*L* or 80.0 cm. The frequency and tension are the same in both sections, so

$$f=\frac{1}{2L}\sqrt{\frac{T}{\mu}}=\frac{1}{2(0.400)}\sqrt{\frac{4.60}{2.00\times10^{-3}}}=\boxed{59.9\text{ Hz}}.$$

(b) As the thick wire is twice the diameter, the linear density is 4 times that of the thin wire.

$\mu'=8.00\text{ g/m}$

so $L'=\frac{1}{2f}\sqrt{\frac{T}{\mu'}}$ $\qquad L'=\left[\frac{1}{(2)(59.9)}\right]\sqrt{\frac{4.60}{8.00\times10^{-3}}}=\boxed{20.0\text{ cm}}$ half the length of the thin wire.

**P18.64** (a) For the block:

$$\sum F_x=T-Mg\sin30.0°=0$$

so $T=Mg\sin30.0°=\boxed{\frac{1}{2}Mg}$.

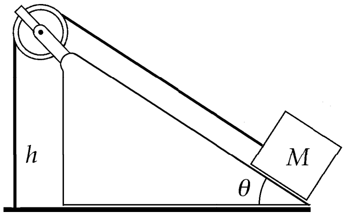

**FIG. P18.64**

(b) The length of the section of string parallel to the incline is $\frac{h}{\sin30.0°}=2h$. The total length of the string is then $\boxed{3h}$.

(c) The mass per unit length of the string is $\qquad \mu=\boxed{\frac{m}{3h}}$

(d) The speed of waves in the string is $\qquad v=\sqrt{\frac{T}{\mu}}=\sqrt{\left(\frac{Mg}{2}\right)\left(\frac{3h}{m}\right)}=\boxed{\sqrt{\frac{3Mgh}{2m}}}$

(e) In the fundamental mode, the segment of length *h* vibrates as one loop. The distance between adjacent nodes is then $d_{\text{NN}}=\frac{\lambda}{2}=h$, so the wavelength is $\lambda=2h$.

The frequency is $\qquad f=\frac{v}{\lambda}=\frac{1}{2h}\sqrt{\frac{3Mgh}{2m}}=\boxed{\sqrt{\frac{3Mg}{8mh}}}$

(g) When the vertical segment of string vibrates with 2 loops (i.e., 3 nodes), then $h=2\left(\frac{\lambda}{2}\right)$ and the wavelength is $\qquad \lambda=\boxed{h}$.

(f) The period of the standing wave of 3 nodes (or two loops) is

$$T=\frac{1}{f}=\frac{\lambda}{v}=h\sqrt{\frac{2m}{3Mgh}}=\boxed{\sqrt{\frac{2mh}{3Mg}}}$$

(h) $f_b=1.02f-f=\left(2.00\times10^{-2}\right)f=\boxed{\left(2.00\times10^{-2}\right)\sqrt{\frac{3Mg}{8mh}}}$

**P18.65** (a) $$f=\frac{n}{2L}\sqrt{\frac{T}{\mu}}$$

so $\frac{f'}{f}=\frac{L}{L'}=\frac{L}{2L}=\frac{1}{2}$

The $\boxed{\text{frequency should be halved}}$ to get the same number of antinodes for twice the length.

(b) $\frac{n'}{n}=\sqrt{\frac{T}{T'}}$ so $\frac{T'}{T}=\left(\frac{n}{n'}\right)^2=\left[\frac{n}{n+1}\right]^2$

The tension must be $T'=\boxed{\left[\frac{n}{n+1}\right]^2 T}$

(c) $\frac{f'}{f}=\frac{n'L}{nL'}\sqrt{\frac{T'}{T}}$ so $\frac{T'}{T}=\left(\frac{nf'L'}{n'fL}\right)^2$

$\frac{T'}{T}=\left(\frac{3}{2\cdot 2}\right)^2$ $\boxed{\frac{T'}{T}=\frac{9}{16}}$ to get twice as many antinodes.

**P18.66** For the wire, $\mu=\frac{0.010\,0\text{ kg}}{2.00\text{ m}}=5.00\times10^{-3}\text{ kg/m}$: $v=\sqrt{\frac{T}{\mu}}=\sqrt{\frac{\left(200\text{ kg}\cdot\text{m/s}^2\right)}{5.00\times10^{-3}\text{ kg/m}}}$

$v=200\text{ m/s}$

If it vibrates in its simplest state, $d_{\text{NN}}=2.00\text{ m}=\frac{\lambda}{2}$: $f=\frac{v}{\lambda}=\frac{(200\text{ m/s})}{4.00\text{ m}}=50.0\text{ Hz}$

(a) The tuning fork can have frequencies $\boxed{45.0\text{ Hz or }55.0\text{ Hz}}$.

(b) If $f=45.0\text{ Hz}$, $v=f\lambda=(45.0/\text{s})4.00\text{ m}=180\text{ m/s}$.

Then, $T=v^2\mu=(180\text{ m/s})^2\left(5.00\times10^{-3}\text{ kg/m}\right)=\boxed{162\text{ N}}$

$\boxed{\text{or}}$ if $f=55.0\text{ Hz}$, $T=v^2\mu=f^2\lambda^2\mu=(55.0/\text{s})^2(4.00\text{ m})^2\left(5.00\times10^{-3}\text{ kg/m}\right)=\boxed{242\text{ N}}$.

**P18.67** We look for a solution of the form

$$\begin{aligned}5.00\sin(2.00x-10.0t)+10.0\cos(2.00x-10.0t)&=A\sin(2.00x-10.0t+\phi)\\&=A\sin(2.00x-10.0t)\cos\phi+A\cos(2.00x-10.0t)\sin\phi\end{aligned}$$

This will be true if both $5.00=A\cos\phi$ and $10.0=A\sin\phi$,

requiring $(5.00)^2+(10.0)^2=A^2$

$A=11.2$ and $\phi=63.4°$

The resultant wave $\boxed{11.2\sin(2.00x-10.0t+63.4°)}$ is sinusoidal.

**P18.68** (a) With $k = \frac{2\pi}{\lambda}$ and $\omega = 2\pi f = \frac{2\pi v}{\lambda}$: $y(x,\,t) = 2A\sin kx\cos\omega t = \boxed{2A\sin\left(\frac{2\pi x}{\lambda}\right)\cos\left(\frac{2\pi vt}{\lambda}\right)}$

(b) For the fundamental vibration, $\lambda_1 = 2L$

so $y_1(x,\,t) = \boxed{2A\sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{\pi vt}{L}\right)}$

(c) For the second harmonic $\lambda_2 = L$ and $y_2(x,\,t) = \boxed{2A\sin\left(\frac{2\pi x}{L}\right)\cos\left(\frac{2\pi vt}{L}\right)}$

(d) In general, $\lambda_n = \frac{2L}{n}$ and $y_n(x,\,t) = \boxed{2A\sin\left(\frac{n\pi x}{L}\right)\cos\left(\frac{n\pi vt}{L}\right)}$

**P18.69** (a) Let $\theta$ represent the angle each slanted rope makes with the vertical.

In the diagram, observe that:

$$\sin\theta = \frac{1.00\text{ m}}{1.50\text{ m}} = \frac{2}{3}$$

or $\theta = 41.8°$.

Considering the mass,

$\sum F_y = 0$: $2T\cos\theta = mg$

or $T = \frac{(12.0\text{ kg})(9.80\text{ m/s}^2)}{2\cos 41.8°} = \boxed{78.9\text{ N}}$

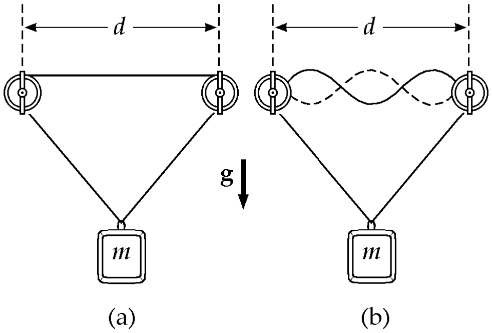

**FIG. P18.69**

(b) The speed of transverse waves in the string is $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{78.9\text{ N}}{0.001\,00\text{ kg/m}}} = 281\text{ m/s}$

For the standing wave pattern shown (3 loops), $d = \frac{3}{2}\lambda$

or $\lambda = \frac{2(2.00\text{ m})}{3} = 1.33\text{ m}$

Thus, the required frequency is $f = \frac{v}{\lambda} = \frac{281\text{ m/s}}{1.33\text{ m}} = \boxed{211\text{ Hz}}$

***P18.70** $d_{AA} = \frac{\lambda}{2} = 7.05\times10^{-3}\text{ m}$ is the distance between antinodes.

Then $\lambda = 14.1\times10^{-3}\text{ m}$

and $f = \frac{v}{\lambda} = \frac{3.70\times10^3\text{ m/s}}{14.1\times10^{-3}\text{ m}} = \boxed{2.62\times10^5\text{ Hz}}$.

The crystal can be tuned to vibrate at $2^{18}$ Hz, so that binary counters can derive from it a signal at precisely 1 Hz.

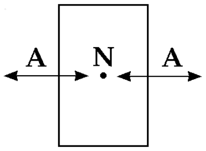

**FIG. P18.70**

## ANSWERS TO EVEN PROBLEMS

| | |
|---|---|
| **P18.2** | see the solution |
| **P18.4** | 5.66 cm |
| **P18.6** | 0.500 s |
| **P18.8** | (a) 3.33 rad; (b) 283 Hz |
| **P18.10** | (a) The number is the greatest integer $\le d\left(\frac{f}{v}\right)+\frac{1}{2}$;<br>(b) $L_n=\frac{d^2-(n-1/2)^2(v/f)^2}{2(n-1/2)(v/f)}$ where $n=1,\ 2,\ \ldots,\ n_{\max}$ |
| **P18.12** | (a) $\Delta x=\frac{\lambda}{2}$;<br>(b) along the hyperbola $9x^2-16y^2=144$ |
| **P18.14** | (a) $(2n+1)\pi$ m for $n=0,\ 1,\ 2,\ 3,\ \ldots$;<br>(b) 0.029 4 m |
| **P18.16** | see the solution |
| **P18.18** | see the solution |
| **P18.20** | 15.7 Hz |
| **P18.22** | (a) 257 Hz; (b) 6 |
| **P18.24** | (a) 495 Hz; (b) 990 Hz |
| **P18.26** | 19.976 kHz |
| **P18.28** | 3.84% |
| **P18.30** | 291 Hz |
| **P18.32** | 0.352 Hz |
| **P18.34** | see the solution |
| **P18.36** | (a) 531 Hz; (b) 42.5 mm |
| **P18.38** | 0.656 m; 1.64 m |
| **P18.40** | 3 kHz; see the solution |
| **P18.42** | $\Delta t=\frac{\pi r^2 v}{2Rf}$ |
| **P18.44** | $L=0.252$ m, 0.504 m, 0.757 m, …,<br>$n(0.252)$ m for $n=1,\ 2,\ 3,\ \ldots$ |
| **P18.46** | 0.502 m; 0.837 m |
| **P18.48** | (a) 0.195 m; (b) 841 m |
| **P18.50** | 1.16 m |
| **P18.52** | (a) 521 Hz or 525 Hz; (b) 526 Hz;<br>(c) reduce by 1.14% |
| **P18.54** | 4-foot and $2\frac{2}{3}$-foot; $2\frac{2}{3}$ and 2-foot; and all three together |
| **P18.56** | see the solution |
| **P18.58** | (a) and (b) 3.99 beats/s |
| **P18.60** | 4.85 m |
| **P18.62** | 31.1 N |
| **P18.64** | (a) $\frac{1}{2}Mg$; (b) $3h$; (c) $\frac{m}{3h}$; (d) $\sqrt{\frac{3Mgh}{2m}}$;<br>(e) $\sqrt{\frac{3Mg}{8mh}}$; (f) $\sqrt{\frac{2mh}{3Mg}}$; (g) $h$;<br>(h) $\left(2.00\times10^{-2}\right)\sqrt{\frac{3Mg}{8mh}}$ |
| **P18.66** | (a) 45.0 Hz or 55.0 Hz; (b) 162 N or 242 N |
| **P18.68** | see the solution |
| **P18.70** | 262 kHz |

# 19

# Temperature

**CHAPTER OUTLINE**

19.1 Temperature and the Zeroth Law of Thermodynamics
19.2 Thermometers and the Celsius Temperature Scale
19.3 The Constant-Volume Gas Thermometer and the Absolute Temperature Scale
19.4 Thermal Expansion of Solids and Liquids
19.5 Macroscopic Description of an Ideal Gas

## ANSWERS TO QUESTIONS

**Q19.1** Two objects in thermal equilibrium need not be in contact. Consider the two objects that are in thermal equilibrium in Figure 19.1(c). The act of separating them by a small distance does not affect how the molecules are moving inside either object, so they will still be in thermal equilibrium.

**Q19.2** The copper's temperature drops and the water temperature rises until both temperatures are the same. Then the metal and the water are in thermal equilibrium.

**Q19.3** The astronaut is referring to the temperature of the lunar surface, specifically a 400°F difference. A thermometer would register the temperature of the thermometer liquid. Since there is no atmosphere in the moon, the thermometer will not read a realistic temperature unless it is placed into the lunar soil.

**Q19.4** Rubber contracts when it is warmed.

**Q19.5** Thermal expansion of the glass bulb occurs first, since the wall of the bulb is in direct contact with the hot water. Then the mercury heats up, and it expands.

**Q19.6** If the amalgam had a larger coefficient of expansion than your tooth, it would expand more than the cavity in your tooth when you take a sip of your ever-beloved coffee, resulting in a broken or cracked tooth! As you ice down your now excruciatingly painful broken tooth, the amalgam would contract more than the cavity in your tooth and fall out, leaving the nerve roots exposed. Isn't it nice that your dentist knows thermodynamics?

**Q19.7** The measurements made with the heated steel tape will be too short—but only by a factor of $5\times10^{-5}$ of the measured length.

**Q19.8** (a) One mole of $H_2$ has a mass of 2.016 0 g.

(b) One mole of He has a mass of 4.002 6 g.

(c) One mole of CO has a mass of 28.010 g.

**Q19.9** The ideal gas law, $PV = nRT$ predicts zero volume at absolute zero. This is incorrect because the ideal gas law cannot work all the way down to or below the temperature at which gas turns to liquid, or in the case of $CO_2$, a solid.

**Q19.10** Call the process isobaric cooling or isobaric contraction. The rubber wall is easy to stretch. The air inside is nearly at atmospheric pressure originally and stays at atmospheric pressure as the wall moves in, just maintaining equality of pressure outside and inside. The air is nearly an ideal gas to start with, but $PV = nRT$ soon fails. Volume will drop by a larger factor than temperature as the water vapor liquefies and then freezes, as the carbon dioxide turns to snow, as the argon turns to slush, and as the oxygen liquefies. From the outside, you see contraction to a small fraction of the original volume.

**Q19.11** Cylinder A must be at lower pressure. If the gas is thin, it will be at one-third the absolute pressure of B.

**Q19.12** At high temperature and pressure, the steam inside exerts large forces on the pot and cover. Strong latches hold them together, but they would explode apart if you tried to open the hot cooker.

**Q19.13** (a) The water level in the cave rises by a smaller distance than the water outside, as the trapped air is compressed. Air can escape from the cave if the rock is not completely airtight, and also by dissolving in the water.

(b) The ideal cave stays completely full of water at low tide. The water in the cave is supported by atmospheric pressure on the free water surface outside.

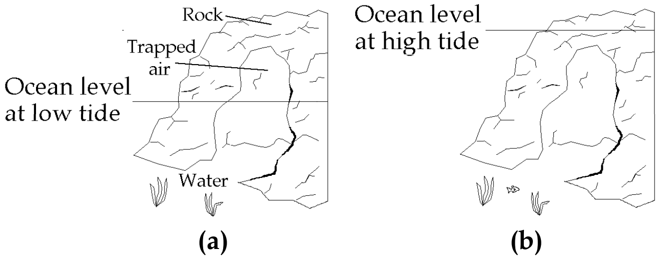

**FIG. Q19.13**

**Q19.14** Absolute zero is a natural choice for the zero of a temperature scale. If an alien race had bodies that were mostly liquid water—or if they just liked its taste or its cleaning properties—it is conceivable that they might place one hundred degrees between its freezing and boiling points. It is very unlikely, on the other hand, that these would be our familiar "normal" ice and steam points, because atmospheric pressure would surely be different where the aliens come from.

**Q19.15** As the temperature increases, the brass expands. This would effectively increase the distance, *d*, from the pivot point to the center of mass of the pendulum, and also increase the moment of inertia of the pendulum. Since the moment of inertia is proportional to $d^2$, and the period of a physical pendulum is $T = 2\pi\sqrt{\frac{I}{mgd}}$, the period would increase, and the clock would run slow.

**Q19.16** As the water rises in temperature, it expands. The excess volume would spill out of the cooling system. Modern cooling systems have an overflow reservoir to take up excess volume when the coolant heats up and expands.

**Q19.17** The coefficient of expansion of metal is larger than that of glass. When hot water is run over the jar, both the glass and the lid expand, but at different rates. Since *all* dimensions expand, there will be a certain temperature at which the inner diameter of the lid has expanded more than the top of the jar, and the lid will be easier to remove.

**Q19.18** The sphere expands when heated, so that it no longer fits through the ring. With the sphere still hot, you can separate the sphere and ring by heating the ring. This more surprising result occurs because the thermal expansion of the ring is not like the inflation of a blood-pressure cuff. Rather, it is like a photographic enlargement; every linear dimension, including the hole diameter, increases by the same factor. The reason for this is that the atoms everywhere, including those around the inner circumference, push away from each other. The only way that the atoms can accommodate the greater distances is for the circumference—and corresponding diameter—to grow. This property was once used to fit metal rims to wooden wagon and horse-buggy wheels. If the ring is heated and the sphere left at room temperature, the sphere would pass through the ring with more space to spare.

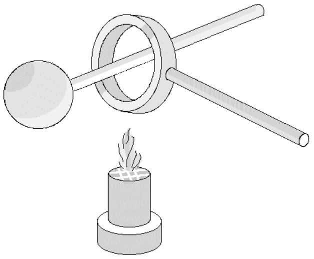

**FIG. Q19.18**

## SOLUTIONS TO PROBLEMS

### Section 19.1 Temperature and the Zeroth Law of Thermodynamics

No problems in this section

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### Section 19.2 Thermometers and the Celsius Temperature Scale

### Section 19.3 The Constant-Volume Gas Thermometer and the Absolute Temperature Scale

**P19.1** Since we have a linear graph, the pressure is related to the temperature as $P = A + BT$, where $A$ and $B$ are constants. To find $A$ and $B$, we use the data

$$0.900\text{ atm} = A + (-80.0^\circ\text{C})B \qquad (1)$$

$$1.635\text{ atm} = A + (78.0^\circ\text{C})B \qquad (2)$$

Solving (1) and (2) simultaneously,

we find $A = 1.272\text{ atm}$

and $B = 4.652 \times 10^{-3}\ \text{atm/}^\circ\text{C}$

Therefore, $P = 1.272\text{ atm} + \left(4.652 \times 10^{-3}\ \text{atm/}^\circ\text{C}\right)T$

(a) At absolute zero $P = 0 = 1.272\text{ atm} + \left(4.652 \times 10^{-3}\ \text{atm/}^\circ\text{C}\right)T$

which gives $\boxed{T = -274^\circ\text{C}}$.

(b) At the freezing point of water $P = 1.272\text{ atm} + 0 = \boxed{1.27\text{ atm}}$.

(c) And at the boiling point $P = 1.272\text{ atm} + \left(4.652 \times 10^{-3}\ \text{atm/}^\circ\text{C}\right)(100^\circ\text{C}) = \boxed{1.74\text{ atm}}$.

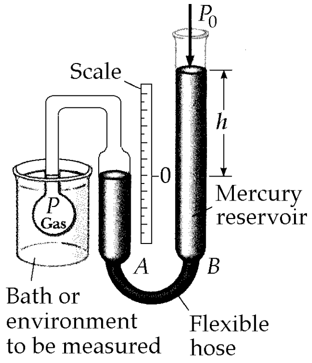

**FIG. P19.2**

**P19.2** $P_1V = nRT_1$

and $P_2V = nRT_2$

imply that $\frac{P_2}{P_1} = \frac{T_2}{T_1}$

(a) $$P_2 = \frac{P_1T_2}{T_1} = \frac{(0.980\text{ atm})(273\text{ K} + 45.0\text{ K})}{(273 + 20.0)\text{ K}} = \boxed{1.06\text{ atm}}$$

(b) $$T_3 = \frac{T_1P_3}{P_1} = \frac{(293\text{ K})(0.500\text{ atm})}{0.980\text{ atm}} = 149\text{ K} = \boxed{-124°\text{C}}$$

**P19.3** (a) $$T_F = \frac{9}{5}T_C + 32.0°\text{F} = \frac{9}{5}(-195.81) + 32.0 = \boxed{-320°\text{F}}$$

(b) $$T = T_C + 273.15 = -195.81 + 273.15 = \boxed{77.3\text{ K}}$$

**P19.4** (a) To convert from Fahrenheit to Celsius, we use $T_C = \frac{5}{9}(T_F - 32.0) = \frac{5}{9}(98.6 - 32.0) = \boxed{37.0°\text{C}}$

and the Kelvin temperature is found as $T = T_C + 273 = \boxed{310\text{ K}}$

(b) In a fashion identical to that used in (a), we find $T_C = \boxed{-20.6°\text{C}}$

and $T = \boxed{253\text{ K}}$

**P19.5** (a) $$\Delta T = 450°\text{C} = 450°\text{C}\left(\frac{212°\text{F} - 32.0°\text{F}}{100°\text{C} - 0.00°\text{C}}\right) = \boxed{810°\text{F}}$$

(b) $$\Delta T = 450°\text{C} = \boxed{450\text{ K}}$$

**P19.6** Require $0.00°\text{C} = a(-15.0°\text{S}) + b$

$100°\text{C} = a(60.0°\text{S}) + b$

Subtracting, $100°\text{C} = a(75.0°\text{S})$

$$a = 1.33\ \text{C°/S°}.$$

Then $0.00°\text{C} = 1.33(-15.0°\text{S})\text{C°} + b$

$$b = 20.0°\text{C}.$$

So the conversion is $\boxed{T_C = (1.33\ \text{C°/S°})T_S + 20.0°\text{C}}$.

**P19.7** (a) $T = 1\,064 + 273 = \boxed{1\,337\text{ K}}$ melting point

$T = 2\,660 + 273 = \boxed{2\,933\text{ K}}$ boiling point

(b) $\Delta T = \boxed{1\,596°\text{C}} = \boxed{1\,596\text{ K}}$. The differences are the same.

## Section 19.4 **Thermal Expansion of Solids and Liquids**

**P19.8** $\alpha = 1.10 \times 10^{-5}\,{}^\circ\text{C}^{-1}$ for steel

$$\Delta L = 518\text{ m}\left(1.10\times10^{-5}\,{}^\circ\text{C}^{-1}\right)\left[35.0^\circ\text{C}-(-20.0^\circ\text{C})\right] = \boxed{0.313\text{ m}}$$

**P19.9** The wire is 35.0 m long when $T_C = -20.0^\circ\text{C}$.

$$\Delta L = L_i\bar{\alpha}(T - T_i)$$

$\bar{\alpha} = \alpha(20.0^\circ\text{C}) = 1.70\times10^{-5}\ (\text{C}^\circ)^{-1}$ for Cu.

$$\Delta L = (35.0\text{ m})\left(1.70\times10^{-5}\ (\text{C}^\circ)^{-1}\right)\left(35.0^\circ\text{C}-(-20.0^\circ\text{C})\right) = \boxed{+3.27\text{ cm}}$$

**P19.10** $\Delta L = L_i\alpha\,\Delta T = (25.0\text{ m})\left(12.0\times10^{-6}/\text{C}^\circ\right)(40.0^\circ\text{C}) = \boxed{1.20\text{ cm}}$

**P19.11** For the dimensions to increase, $\Delta L = \alpha L_i\Delta T$

$$1.00\times10^{-2}\text{ cm} = 1.30\times10^{-4}\,{}^\circ\text{C}^{-1}(2.20\text{ cm})(T-20.0^\circ\text{C})$$

$$T = \boxed{55.0^\circ\text{C}}$$

***P19.12** $\Delta L = \alpha L_i\Delta T = \left(22\times10^{-6}/\text{C}^\circ\right)(2.40\text{ cm})(30^\circ\text{C}) = \boxed{1.58\times10^{-3}\text{ cm}}$

**P19.13** (a) $\Delta L = \alpha L_i\Delta T = 9.00\times10^{-6}\,{}^\circ\text{C}^{-1}(30.0\text{ cm})(65.0^\circ\text{C}) = \boxed{0.176\text{ mm}}$

(b) $\Delta L = \alpha L_i\Delta T = 9.00\times10^{-6}\,{}^\circ\text{C}^{-1}(1.50\text{ cm})(65.0^\circ\text{C}) = \boxed{8.78\times10^{-4}\text{ cm}}$

(c) $\Delta V = 3\alpha V_i\Delta T = 3\left(9.00\times10^{-6}\,{}^\circ\text{C}^{-1}\right)\left(\dfrac{30.0(\pi)(1.50)^2}{4}\text{ cm}^3\right)(65.0^\circ\text{C}) = \boxed{0.093\,0\text{ cm}^3}$

***P19.14** The horizontal section expands according to $\Delta L = \alpha L_i\Delta T$.

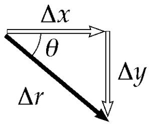

**FIG. P19.14**

$$\Delta x = \left(17\times10^{-6}\,{}^\circ\text{C}^{-1}\right)(28.0\text{ cm})(46.5^\circ\text{C}-18.0^\circ\text{C}) = 1.36\times10^{-2}\text{ cm}$$

The vertical section expands similarly by

$$\Delta y = \left(17\times10^{-6}\,{}^\circ\text{C}^{-1}\right)(134\text{ cm})(28.5^\circ\text{C}) = 6.49\times10^{-2}\text{ cm}.$$

The vector displacement of the pipe elbow has magnitude

$$\Delta r = \sqrt{\Delta x^2+\Delta y^2} = \sqrt{(0.136\text{ mm})^2+(0.649\text{ mm})^2} = 0.663\text{ mm}$$

and is directed to the right below the horizontal at angle

$$\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right) = \tan^{-1}\left(\frac{0.649\text{ mm}}{0.136\text{ mm}}\right) = 78.2^\circ$$

$$\boxed{\Delta r = 0.663\text{ mm to the right at }78.2^\circ\text{ below the horizontal}}$$

**P19.15** (a)

$$L_{\text{Al}}(1+\alpha_{\text{Al}}\Delta T)=L_{\text{Brass}}(1+\alpha_{\text{Brass}}\Delta T)$$

$$\Delta T=\frac{L_{\text{Al}}-L_{\text{Brass}}}{L_{\text{Brass}}\alpha_{\text{Brass}}-L_{\text{Al}}\alpha_{\text{Al}}}$$

$$\Delta T=\frac{(10.01-10.00)}{(10.00)\left(19.0\times10^{-6}\right)-(10.01)\left(24.0\times10^{-6}\right)}$$

$$\Delta T=-199°\text{C so } T=\boxed{-179°\text{C. This is attainable.}}$$

(b)

$$\Delta T=\frac{(10.02-10.00)}{(10.00)\left(19.0\times10^{-6}\right)-(10.02)\left(24.0\times10^{-6}\right)}$$

$$\Delta T=-396°\text{C so } T=\boxed{-376°\text{C which is below 0 K so it cannot be reached.}}$$

**P19.16** (a)

$$\Delta A=2\alpha A_i\Delta T:\quad \Delta A=2\left(17.0\times10^{-6}\,°\text{C}^{-1}\right)(0.080\,0\text{ m})^2(50.0°\text{C})$$

$$\Delta A=1.09\times10^{-5}\text{ m}^2=\boxed{0.109\text{ cm}^2}$$

(b) The length of each side of the hole has increased. Thus, this represents an $\boxed{\text{increase}}$ in the area of the hole.

**P19.17**

$$\Delta V=(\beta-3\alpha)V_i\Delta T=\left(5.81\times10^{-4}-3\left(11.0\times10^{-6}\right)\right)(50.0\text{ gal})(20.0)=\boxed{0.548\text{ gal}}$$

**P19.18** (a)

$$L=L_i(1+\alpha\Delta T):\quad 5.050\text{ cm}=5.000\text{ cm}\left[1+24.0\times10^{-6}\,°\text{C}^{-1}(T-20.0°\text{C})\right]$$

$$T=\boxed{437°\text{C}}$$

(b) We must get $L_{\text{Al}}=L_{\text{Brass}}$ for some $\Delta T$, or

$$L_{i,\text{ Al}}(1+\alpha_{\text{Al}}\Delta T)=L_{i,\text{ Brass}}(1+\alpha_{\text{Brass}}\Delta T)$$

$$5.000\text{ cm}\left[1+\left(24.0\times10^{-6}\,°\text{C}^{-1}\right)\Delta T\right]=5.050\text{ cm}\left[1+\left(19.0\times10^{-6}\,°\text{C}^{-1}\right)\Delta T\right]$$

Solving for $\Delta T$, $\Delta T=2\,080°\text{C}$,

so $\boxed{T=3\,000°\text{C}}$

This will not work because $\boxed{\text{aluminum melts at 660°C}}$.

**P19.19** (a)

$$V_f=V_i(1+\beta\Delta T)=100\left[1+1.50\times10^{-4}(-15.0)\right]=\boxed{99.8\text{ mL}}$$

(b)

$$\Delta V_{\text{acetone}}=(\beta V_i\Delta T)_{\text{acetone}}$$

$$\Delta V_{\text{flask}}=(\beta V_i\Delta T)_{\text{Pyrex}}=(3\alpha V_i\Delta T)_{\text{Pyrex}}$$

for same $V_i$, $\Delta T$,

$$\frac{\Delta V_{\text{acetone}}}{\Delta V_{\text{flask}}}=\frac{\beta_{\text{acetone}}}{\beta_{\text{flask}}}=\frac{1.50\times10^{-4}}{3\left(3.20\times10^{-6}\right)}=\frac{1}{6.40\times10^{-2}}$$

The volume change of flask is

$\boxed{\text{about 6\% of the change in the acetone's volume}}$.

**P19.20** (a),(b) The material would expand by $\Delta L = \alpha L_i \Delta T$,

$$\frac{\Delta L}{L_i} = \alpha \Delta T, \text{ but instead feels stress}$$

$$\frac{F}{A} = \frac{Y\Delta L}{L_i} = Y\alpha\Delta T = \left(7.00\times 10^{9}\ \text{N/m}^2\right)12.0\times 10^{-6}(\text{C}^\circ)^{-1}(30.0^\circ\text{C})$$

$$= \boxed{2.52\times 10^{6}\ \text{N/m}^2}\text{. This will } \boxed{\text{not break}} \text{ concrete.}$$

**P19.21** (a)

$$\Delta V = V_t\beta_t\Delta T - V_{\text{Al}}\beta_{\text{Al}}\Delta T = \left(\beta_t - 3\alpha_{\text{Al}}\right)V_i\Delta T$$

$$= \left(9.00\times 10^{-4} - 0.720\times 10^{-4}\right)^\circ\text{C}^{-1}\left(2\,000\ \text{cm}^3\right)(60.0^\circ\text{C})$$

$$\Delta V = \boxed{99.4\ \text{cm}^3} \text{ overflows.}$$

(b) The whole new volume of turpentine is

$$2\,000\ \text{cm}^3 + 9.00\times 10^{-4}\,{}^\circ\text{C}^{-1}\left(2\,000\ \text{cm}^3\right)(60.0^\circ\text{C}) = 2\,108\ \text{cm}^3$$

so the fraction lost is $\dfrac{99.4\ \text{cm}^3}{2\,108\ \text{cm}^3} = 4.71\times 10^{-2}$

and this fraction of the cylinder's depth will be empty upon cooling:

$$4.71\times 10^{-2}(20.0\ \text{cm}) = \boxed{0.943\ \text{cm}}.$$

***P19.22** The volume of the sphere is

$$V_{\text{Pb}} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(2\ \text{cm})^3 = 33.5\ \text{cm}^3.$$

The amount of mercury overflowing is

$$\text{overflow} = \Delta V_{\text{Hg}} + \Delta V_{\text{Pb}} - \Delta V_{\text{glass}} = \left(\beta_{\text{Hg}}V_{\text{Hg}} + \beta_{\text{Pb}}V_{\text{Pb}} - \beta_{\text{glass}}V_{\text{glass}}\right)\Delta T$$

where $V_{\text{glass}} = V_{\text{Hg}} + V_{\text{Pb}}$ is the initial volume. Then

$$\text{overflow} = \left[\left(\beta_{\text{Hg}} - \beta_{\text{glass}}\right)V_{\text{Hg}} + \left(\beta_{\text{Pb}} - \beta_{\text{glass}}\right)V_{\text{Pb}}\right]\Delta T = \left[\left(\beta_{\text{Hg}} - 3\alpha_{\text{glass}}\right)V_{\text{Hg}} + \left(3\alpha_{\text{Pb}} - 3\alpha_{\text{glass}}\right)V_{\text{Pb}}\right]\Delta T$$

$$= \left[(182-27)10^{-6}\frac{1}{\text{C}^\circ}118\ \text{cm}^3 + (87-27)10^{-6}\frac{1}{\text{C}^\circ}33.5\ \text{cm}^3\right]40^\circ\text{C} = \boxed{0.812\ \text{cm}^3}$$

**P19.23** In $\dfrac{F}{A} = \dfrac{Y\Delta L}{L_i}$ require $\Delta L = \alpha L_i \Delta T$

$$\frac{F}{A} = Y\alpha\Delta T$$

$$\Delta T = \frac{F}{AY\alpha} = \frac{500\ \text{N}}{\left(2.00\times 10^{-4}\ \text{m}^2\right)\left(20.0\times 10^{10}\ \text{N/m}^2\right)\left(11.0\times 10^{-6}/\text{C}^\circ\right)}$$

$$\Delta T = \boxed{1.14^\circ\text{C}}$$

***P19.24** Model the wire as contracting according to $\Delta L = \alpha L_i \Delta T$ and then stretching according to stress $= \dfrac{F}{A} = Y\dfrac{\Delta L}{L_i} = \dfrac{Y}{L_i}\alpha L_i \Delta T = Y\alpha\Delta T$.

(a) $$F = YA\alpha\Delta T = \left(20\times10^{10}\ \text{N/m}^2\right)4\times10^{-6}\ \text{m}^2\ 11\times10^{-6}\ \frac{1}{\text{C}^\circ}45^\circ\text{C} = \boxed{396\ \text{N}}$$

(b) $$\Delta T = \frac{\text{stress}}{Y\alpha} = \frac{3\times10^{8}\ \text{N/m}^2}{\left(20\times10^{10}\ \text{N/m}^2\right)11\times10^{-6}/\text{C}^\circ} = 136^\circ\text{C}$$

To increase the stress the temperature must decrease to $35^\circ\text{C} - 136^\circ\text{C} = \boxed{-101^\circ\text{C}}$.

(c) The original length divides out, so the answers would not change.

***P19.25** The area of the chip decreases according to $\Delta A = \gamma A_1 \Delta T = A_f - A_i$

$$A_f = A_i(1+\gamma\Delta T) = A_i(1+2\alpha\Delta T)$$

The star images are scattered uniformly, so the number $N$ of stars that fit is proportional to the area. Then $N_f = N_i(1+2\alpha\Delta T) = 5\,342\left[1+2\left(4.68\times10^{-6}\ {}^\circ\text{C}^{-1}\right)(-100^\circ\text{C}-20^\circ\text{C})\right] = \boxed{5\,336\ \text{star images}}$.

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## Section 19.5 **Macroscopic Description of an Ideal Gas**

**P19.26** (a) $$n = \frac{PV}{RT} = \frac{(9.00\ \text{atm})\left(1.013\times10^{5}\ \text{Pa/atm}\right)\left(8.00\times10^{-3}\ \text{m}^3\right)}{(8.314\ \text{N}\cdot\text{mol K})(293\ \text{K})} = \boxed{2.99\ \text{mol}}$$

(b) $$N = nN_\text{A} = (2.99\ \text{mol})\left(6.02\times10^{23}\ \text{molecules/mol}\right) = \boxed{1.80\times10^{24}\ \text{molecules}}$$

**P19.27** (a) Initially, $P_iV_i = n_iRT_i$ $\qquad (1.00\ \text{atm})V_i = n_iR(10.0+273.15)\ \text{K}$

Finally, $P_fV_f = n_fRT_f$ $\qquad P_f(0.280V_i) = n_iR(40.0+273.15)\ \text{K}$

Dividing these equations, $\qquad \dfrac{0.280P_f}{1.00\ \text{atm}} = \dfrac{313.15\ \text{K}}{283.15\ \text{K}}$

giving $\qquad P_f = 3.95\ \text{atm}$

or $\qquad P_f = \boxed{4.00\times10^{5}\ \text{Pa(abs.)}}$.

(b) After being driven $\qquad P_d(1.02)(0.280V_i) = n_iR(85.0+273.15)\ \text{K}$

$$P_d = 1.121P_f = \boxed{4.49\times10^{5}\ \text{Pa}}$$

**P19.28** $PV = NP'V' = \dfrac{4}{3}\pi r^3 NP'$: $\qquad N = \dfrac{3PV}{4\pi r^3 P'} = \dfrac{3(150)(0.100)}{4\pi(0.150)^3(1.20)} = \boxed{884\ \text{balloons}}$

If we have no special means for squeezing the last 100 L of helium out of the tank, the tank will be full of helium at 1.20 atm when the last balloon is inflated. The number of balloons is then reduced to to $884 - \dfrac{\left(0.100\ \text{m}^3\right)3}{4\pi(0.15\ \text{m})^3} = 877$.

**P19.29** The equation of state of an ideal gas is $PV = nRT$ so we need to solve for the number of moles to find $N$.

$$n = \frac{PV}{RT} = \frac{\left(1.01\times10^{5}\ \text{N/m}^2\right)\left[(10.0\ \text{m})(20.0\ \text{m})(30.0\ \text{m})\right]}{(8.314\ \text{J/mol}\cdot\text{K})(293\ \text{K})} = 2.49\times10^{5}\ \text{mol}$$

$$N = nN_A = 2.49\times10^{5}\ \text{mol}\left(6.022\times10^{23}\ \text{molecules/mol}\right) = \boxed{1.50\times10^{29}\ \text{molecules}}$$

***P19.30** (a)

$$P_iV_i = n_iRT_i = \frac{m_i}{M}RT_i$$

$$m_i = \frac{MP_iV_i}{RT_i} = \frac{4.00\times10^{-3}\ \text{kg}\ 1.013\times10^{5}\ \text{N}\ 4\pi\left(6.37\times10^{6}\ \text{m}\right)^3\ \text{mole}\cdot\text{K}}{\text{mole}\qquad\qquad \text{m}^2 3\qquad\qquad 8.314\ \text{Nm}\ \ 50\ \text{K}}$$

$$= \boxed{1.06\times10^{21}\ \text{kg}}$$

(b)

$$\frac{P_fV_f}{P_iV_i} = \frac{n_fRT_f}{n_iRT_i}$$

$$2\cdot1 = \left(\frac{1.06\times10^{21}\ \text{kg} + 8.00\times10^{20}\ \text{kg}}{1.06\times10^{21}\ \text{kg}}\right)\frac{T_f}{50\ \text{K}}$$

$$T_f = 100\ \text{K}\left(\frac{1}{1.76}\right) = \boxed{56.9\ \text{K}}$$

**P19.31**

$$P = \frac{nRT}{V} = \left(\frac{9.00\ \text{g}}{18.0\ \text{g/mol}}\right)\left(\frac{8.314\ \text{J}}{\text{mol K}}\right)\left(\frac{773\ \text{K}}{2.00\times10^{-3}\ \text{m}^3}\right) = \boxed{1.61\ \text{MPa}} = 15.9\ \text{atm}$$

**P19.32** (a)

$$T_2 = T_1\frac{P_2}{P_1} = (300\ \text{K})(3) = \boxed{900\ \text{K}}$$

(b)

$$T_2 = T_1\frac{P_2V_2}{P_1V_1} = 300(2)(2) = \boxed{1\,200\ \text{K}}$$

**P19.33** $\sum F_y = 0$:

$$\rho_{\text{out}}gV - \rho_{\text{in}}gV - (200\ \text{kg})g = 0$$

$$\left(\rho_{\text{out}} - \rho_{\text{in}}\right)\left(400\ \text{m}^3\right) = 200\ \text{kg}$$

The density of the air outside is 1.25 $\text{kg/m}^3$.

From $PV = nRT$, $\frac{n}{V} = \frac{P}{RT}$

The density is inversely proportional to the temperature, and the density of the hot air is

$$\rho_{\text{in}} = \left(1.25\ \text{kg/m}^3\right)\left(\frac{283\ \text{K}}{T_{\text{in}}}\right)$$

Then

$$\left(1.25\ \text{kg/m}^3\right)\left(1 - \frac{283\ \text{K}}{T_{\text{in}}}\right)\left(400\ \text{m}^3\right) = 200\ \text{kg}$$

$$1 - \frac{283\ \text{K}}{T_{\text{in}}} = 0.400$$

$$0.600 = \frac{283\ \text{K}}{T_{\text{in}}}\qquad T_{\text{in}} = \boxed{472\ \text{K}}$$

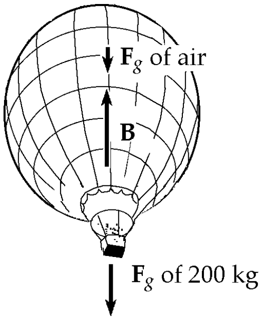

**FIG. P19.33**

***P19.34** Consider the air in the tank during one discharge process. We suppose that the process is slow enough that the temperature remains constant. Then as the pressure drops from 2.40 atm to 1.20 atm, the volume of the air doubles. During the first discharge, the air volume changes from 1 L to 2 L. Just 1 L of water is expelled and 3 L remains. In the second discharge, the air volume changes from 2 L to 4 L and 2 L of water is sprayed out. In the third discharge, only the last 1 L of water comes out. Were it not for male pattern dumbness, each person could more efficiently use his device by starting with the tank half full of water.

**P19.35** (a) $PV = nRT$

$$n = \frac{PV}{RT} = \frac{(1.013\times10^5\ \text{Pa})(1.00\ \text{m}^3)}{(8.314\ \text{J/mol}\cdot\text{K})(293\ \text{K})} = \boxed{41.6\ \text{mol}}$$

(b) $m = nM = (41.6\ \text{mol})(28.9\ \text{g/mol}) = \boxed{1.20\ \text{kg}}$, in agreement with the tabulated density of $1.20\ \text{kg/m}^3$ at 20.0°C.

***P19.36** The void volume is $0.765V_{\text{total}} = 0.765\pi r^2\ell = 0.765\pi\left(1.27\times10^{-2}\ \text{m}\right)^2 0.2\ \text{m} = 7.75\times10^{-5}\ \text{m}^3$. Now for the gas remaining $PV = nRT$

$$n = \frac{PV}{RT} = \frac{12.5\left(1.013\times10^5\ \text{N/m}^2\right)7.75\times10^{-5}\ \text{m}^3}{(8.314\ \text{Nm/mole K})(273+25)\ \text{K}} = \boxed{3.96\times10^{-2}\ \text{mol}}$$

**P19.37** (a) $PV = nRT \qquad n = \frac{PV}{RT}$

$$m = nM = \frac{PVM}{RT} = \frac{1.013\times10^5\ \text{Pa}(0.100\ \text{m})^3\left(28.9\times10^{-3}\ \text{kg/mol}\right)}{(8.314\ \text{J/mol}\cdot\text{K})(300\ \text{K})}$$

$$m = \boxed{1.17\times10^{-3}\ \text{kg}}$$

(b) $F_g = mg = 1.17\times10^{-3}\ \text{kg}\left(9.80\ \text{m/s}^2\right) = \boxed{11.5\ \text{mN}}$

(c) $F = PA = \left(1.013\times10^5\ \text{N/m}^2\right)(0.100\ \text{m})^2 = \boxed{1.01\ \text{kN}}$

(d) The $\boxed{\text{molecules must be moving very fast}}$ to hit the walls hard.

**P19.38** At depth, $P = P_0 + \rho gh$ and $PV_i = nRT_i$

At the surface, $P_0V_f = nRT_f$: $\qquad \frac{P_0V_f}{(P_0+\rho gh)V_i} = \frac{T_f}{T_i}$

Therefore $\quad V_f = V_i\left(\frac{T_f}{T_i}\right)\left(\frac{P_0+\rho gh}{P_0}\right)$

$$V_f = 1.00\ \text{cm}^3\left(\frac{293\ \text{K}}{278\ \text{K}}\right)\left(\frac{1.013\times10^5\ \text{Pa} + \left(1\,025\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)(25.0\ \text{m})}{1.013\times10^5\ \text{Pa}}\right)$$

$$V_f = \boxed{3.67\ \text{cm}^3}$$

**P19.39** $PV = nRT$: $$\frac{m_f}{m_i} = \frac{n_f}{n_i} = \frac{P_f V_f}{RT_f}\frac{RT_i}{P_i V_i} = \frac{P_f}{P_i}$$

so $$m_f = m_i\left(\frac{P_f}{P_i}\right)$$

$$|\Delta m| = m_i - m_f = m_i\left(\frac{P_i - P_f}{P_i}\right) = 12.0\text{ kg}\left(\frac{41.0\text{ atm} - 26.0\text{ atm}}{41.0\text{ atm}}\right) = \boxed{4.39\text{ kg}}$$

**P19.40** My bedroom is 4 m long, 4 m wide, and 2.4 m high, enclosing air at 100 kPa and $20°\text{C} = 293\text{ K}$. Think of the air as 80.0% $N_2$ and 20.0% $O_2$.

Avogadro's number of molecules has mass

$$(0.800)(28.0\text{ g/mol}) + (0.200)(32.0\text{ g/mol}) = 0.028\,8\text{ kg/mol}$$

Then $$PV = nRT = \left(\frac{m}{M}\right)RT$$

gives $$m = \frac{PVM}{RT} = \frac{(1.00\times10^5\text{ N/m}^2)(38.4\text{ m}^3)(0.028\,8\text{ kg/mol})}{(8.314\text{ J/mol}\cdot\text{K})(293\text{ K})} = 45.4\text{ kg}\boxed{\sim10^2\text{ kg}}$$

***P19.41** The $CO_2$ is far from liquefaction, so after it comes out of solution it behaves as an ideal gas. Its molar mass is $M = 12.0\text{ g/mol} + 2(16.0\text{ g/mol}) = 44.0\text{ g/mol}$. The quantity of gas in the cylinder is

$$n = \frac{m_{\text{sample}}}{M} = \frac{6.50\text{ g}}{44.0\text{ g/mol}} = 0.148\text{ mol}$$

Then $$PV = nRT$$

gives $$V = \frac{nRT}{P} = \frac{0.148\text{ mol}(8.314\text{ J/mol}\cdot\text{K})(273\text{ K} + 20\text{ K})}{1.013\times10^5\text{ N/m}^2}\left(\frac{1\text{ N}\cdot\text{m}}{1\text{ J}}\right)\left(\frac{10^3\text{ L}}{1\text{ m}^3}\right) = \boxed{3.55\text{ L}}$$

**P19.42** $$N = \frac{PVN_A}{RT} = \frac{(10^{-9}\text{ Pa})(1.00\text{ m}^3)(6.02\times10^{23}\text{ molecules/mol})}{(8.314\text{ J/K}\cdot\text{mol})(300\text{ K})} = \boxed{2.41\times10^{11}\text{ molecules}}$$

**P19.43** $$P_0V = n_1RT_1 = \left(\frac{m_1}{M}\right)RT_1$$

$$P_0V = n_2RT_2 = \left(\frac{m_2}{M}\right)RT_2$$

$$\boxed{m_1 - m_2 = \frac{P_0VM}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)}$$

**P19.44** (a) Initially the air in the bell satisfies $P_0V_{\text{bell}} = nRT_i$

or $P_0\left[(2.50\text{ m})A\right] = nRT_i$ (1)

When the bell is lowered, the air in the bell satisfies

$$P_{\text{bell}}(2.50\text{ m} - x)A = nRT_f \qquad (2)$$

where $x$ is the height the water rises in the bell. Also, the pressure in the bell, once it is lowered, is equal to the sea water pressure at the depth of the water level in the bell.

$$P_{\text{bell}} = P_0 + \rho g(82.3\text{ m} - x) \approx P_0 + \rho g(82.3\text{ m}) \qquad (3)$$

The approximation is good, as $x < 2.50$ m. Substituting (3) into (2) and substituting $nR$ from (1) into (2),

$$\left[P_0 + \rho g(82.3\text{ m})\right](2.50\text{ m} - x)A = P_0V_{\text{bell}}\frac{T_f}{T_i}.$$

Using $P_0 = 1\text{ atm} = 1.013 \times 10^5\text{ Pa}$ and $\rho = 1.025 \times 10^3\ \text{kg/m}^3$

$$x = (2.50\text{ m})\left[1 - \frac{T_f}{T_0}\left(1 + \frac{\rho g(82.3\text{ m})}{P_0}\right)^{-1}\right]$$

$$= (2.50\text{ m})\left[1 - \frac{277.15\text{ K}}{293.15\text{ K}}\left(1 + \frac{\left(1.025 \times 10^3\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)(82.3\text{ m})}{1.013 \times 10^5\ \text{N/m}^2}\right)^{-1}\right]$$

$$x = \boxed{2.24\text{ m}}$$

(b) If the water in the bell is to be expelled, the air pressure in the bell must be raised to the water pressure at the bottom of the bell. That is,

$$P_{\text{bell}} = P_0 + \rho g(82.3\text{ m})$$

$$= 1.013 \times 10^5\text{ Pa} + \left(1.025 \times 10^3\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)(82.3\text{ m})$$

$$P_{\text{bell}} = 9.28 \times 10^5\text{ Pa} = \boxed{9.16\text{ atm}}$$

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## Additional Problems

**P19.45** The excess expansion of the brass is $\Delta L_{\text{rod}} - \Delta L_{\text{tape}} = \left(\alpha_{\text{brass}} - \alpha_{\text{steel}}\right)L_i\Delta T$

$$\Delta(\Delta L) = (19.0 - 11.0) \times 10^{-6}\ (°\text{C})^{-1}(0.950\text{ m})(35.0°\text{C})$$

$$\Delta(\Delta L) = 2.66 \times 10^{-4}\text{ m}$$

(a) The rod contracts more than tape to

a length reading $0.950\,0\text{ m} - 0.000\,266\text{ m} = \boxed{0.949\,7\text{ m}}$

(b) $0.950\,0\text{ m} + 0.000\,266\text{ m} = \boxed{0.950\,3\text{ m}}$

**P19.46** At 0°C, 10.0 gallons of gasoline has mass,

from $$\rho=\frac{m}{V}$$

$$m=\rho V=\left(730\ \text{kg/m}^3\right)(10.0\ \text{gal})\left(\frac{0.003\,80\ \text{m}^3}{1.00\ \text{gal}}\right)=27.7\ \text{kg}$$

The gasoline will expand in volume by

$$\Delta V=\beta V_i\Delta T=9.60\times10^{-4}\ {}^\circ\text{C}^{-1}(10.0\ \text{gal})(20.0^\circ\text{C}-0.0^\circ\text{C})=0.192\ \text{gal}$$

At 20.0°C, $10.192\ \text{gal}=27.7\ \text{kg}$

$$10.0\ \text{gal}=27.7\ \text{kg}\left(\frac{10.0\ \text{gal}}{10.192\ \text{gal}}\right)=27.2\ \text{kg}$$

The extra mass contained in 10.0 gallons at 0.0°C is

$$27.7\ \text{kg}-27.2\ \text{kg}=\boxed{0.523\ \text{kg}}.$$

**P19.47** Neglecting the expansion of the glass,

$$\Delta h=\frac{V}{A}\beta\Delta T$$

$$\Delta h=\frac{\frac{4}{3}\pi(0.250\ \text{cm}/2)^3}{\pi\left(2.00\times10^{-3}\ \text{cm}\right)^2}\left(1.82\times10^{-4}\ {}^\circ\text{C}^{-1}\right)(30.0^\circ\text{C})=\boxed{3.55\ \text{cm}}$$

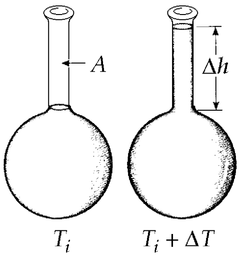


**FIG. P19.47**

**P19.48** (a) The volume of the liquid increases as $\Delta V_\ell=V_i\beta\Delta T$. The volume of the flask increases as $\Delta V_g=3\alpha V_i\Delta T$. Therefore, the overflow in the capillary is $V_c=V_i\Delta T(\beta-3\alpha)$; and in the capillary $V_c=A\Delta h$.

Therefore, $\boxed{\Delta h=\frac{V_i}{A}(\beta-3\alpha)\Delta T}$.

(b) For a mercury thermometer $\beta(\text{Hg})=1.82\times10^{-4}\ {}^\circ\text{C}^{-1}$

and for glass, $3\alpha=3\times3.20\times10^{-6}\ {}^\circ\text{C}^{-1}$

Thus $\beta-3\alpha\approx\beta$

or $\boxed{\alpha<<\beta}$.

**P19.49** The frequency played by the cold-walled flute is $f_i = \frac{v}{\lambda_i} = \frac{v}{2L_i}$.

When the instrument warms up

$$f_f = \frac{v}{\lambda_f} = \frac{v}{2L_f} = \frac{v}{2L_i(1+\alpha\Delta T)} = \frac{f_i}{1+\alpha\Delta T}.$$

The final frequency is lower. The change in frequency is

$$\Delta f = f_i - f_f = f_i\left(1 - \frac{1}{1+\alpha\Delta T}\right)$$

$$\Delta f = \frac{v}{2L_i}\left(\frac{\alpha\Delta T}{1+\alpha\Delta T}\right) \approx \frac{v}{2L_i}(\alpha\Delta T)$$

$$\Delta f \approx \frac{(343\ \text{m/s})\left(24.0\times10^{-6}/\text{C}^\circ\right)(15.0^\circ\text{C})}{2(0.655\ \text{m})} = \boxed{0.094\,3\ \text{Hz}}$$

This change in frequency is imperceptibly small.

**P19.50** (a)

$$\frac{P_0V}{T} = \frac{P'V'}{T'}$$

$$V' = V + Ah$$

$$P' = P_0 + \frac{kh}{A}$$

$$\left(P_0 + \frac{kh}{A}\right)(V+Ah) = P_0V\left(\frac{T'}{T}\right)$$

$$\left(1.013\times10^5\ \text{N/m}^2 + 2.00\times10^5\ \text{N/m}^3\ h\right)\left(5.00\times10^{-3}\ \text{m}^3 + \left(0.010\,0\ \text{m}^2\right)h\right) = \left(1.013\times10^5\ \text{N/m}^2\right)\left(5.00\times10^{-3}\ \text{m}^3\right)\left(\frac{523\ \text{K}}{293\ \text{K}}\right)$$

$$2\,000h^2 + 2\,013h - 397 = 0$$

$$h = \frac{-2\,013 \pm 2\,689}{4\,000} = \boxed{0.169\ \text{m}}$$

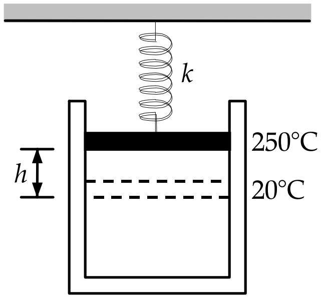


FIG. P19.50

(b)

$$P' = P + \frac{kh}{A} = 1.013\times10^5\ \text{Pa} + \frac{\left(2.00\times10^3\ \text{N/m}\right)(0.169)}{0.010\,0\ \text{m}^2}$$

$$P' = \boxed{1.35\times10^5\ \text{Pa}}$$

**P19.51** (a) $\rho = \frac{m}{V}$ and $d\rho = -\frac{m}{V^2}dV$

For very small changes in $V$ and $\rho$, this can be expressed as

$$\Delta\rho = -\frac{m}{V}\frac{\Delta V}{V} = -\rho\beta\Delta T.$$

The negative sign means that any increase in temperature causes the density to decrease and vice versa.

(b) For water we have $\beta = \left|\frac{\Delta\rho}{\rho\Delta T}\right| = \left|\frac{1.000\,0\ \text{g/cm}^3 - 0.999\,7\ \text{g/cm}^3}{\left(1.000\,0\ \text{g/cm}^3\right)(10.0°\text{C} - 4.0°\text{C})}\right| = \boxed{5\times 10^{-5}\ °\text{C}^{-1}}$.

***P19.52** The astronauts exhale this much $CO_2$:

$$n = \frac{m_{\text{sample}}}{M} = \frac{1.09\ \text{kg}}{\text{astronaut}\cdot\text{day}}\left(\frac{1\,000\ \text{g}}{1\ \text{kg}}\right)(3\ \text{astronauts})(7\ \text{days})\left(\frac{1\ \text{mol}}{44.0\ \text{g}}\right) = 520\ \text{mol}.$$

Then 520 mol of methane is generated. It is far from liquefaction and behaves as an ideal gas.

$$P = \frac{nRT}{V} = \frac{520\ \text{mol}(8.314\ \text{J/mol}\cdot\text{K})(273\ \text{K} - 45\ \text{K})}{150\times 10^{-3}\ \text{m}^3} = \boxed{6.57\times 10^6\ \text{Pa}}$$

**P19.53** (a) We assume that air at atmospheric pressure is above the piston.

In equilibrium $P_{\text{gas}} = \frac{mg}{A} + P_0$

Therefore, $\frac{nRT}{hA} = \frac{mg}{A} + P_0$

or $\boxed{h = \frac{nRT}{mg + P_0A}}$

where we have used $V = hA$ as the volume of the gas.

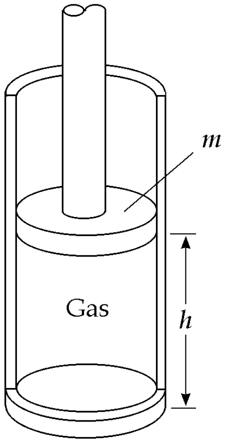


**FIG. P19.53**

(b) From the data given,

$$h = \frac{0.200\ \text{mol}(8.314\ \text{J/K}\cdot\text{mol})(400\ \text{K})}{20.0\ \text{kg}\left(9.80\ \text{m/s}^2\right) + \left(1.013\times 10^5\ \text{N/m}^2\right)\left(0.008\,00\ \text{m}^2\right)}$$

$$= \boxed{0.661\ \text{m}}$$

**P19.54** The angle of bending $\theta$, between tangents to the two ends of the strip, is equal to the angle the strip subtends at its center of curvature. (The angles are equal because their sides are perpendicular, right side to the right side and left side to left side.)

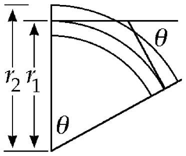


**FIG. P19.54**

(a) The definition of radian measure gives $L_i + \Delta L_1 = \theta r_1$

and $L_i + \Delta L_2 = \theta r_2$

By subtraction, $\Delta L_2 - \Delta L_1 = \theta(r_2 - r_1)$

$$\alpha_2 L_i \Delta T - \alpha_1 L_i \Delta T = \theta\,\Delta r$$

$$\boxed{\theta = \frac{(\alpha_2 - \alpha_1) L_i \Delta T}{\Delta r}}$$

(b) In the expression from part (a), $\theta$ is directly proportional to $\Delta T$ and also to $(\alpha_2 - \alpha_1)$. Therefore $\theta$ is zero when either of these quantities becomes zero.

(c) The material that expands more when heated contracts more when cooled, so the bimetallic strip bends the other way. It is fun to demonstrate this with liquid nitrogen.

(d)
$$\theta = \frac{2(\alpha_2 - \alpha_1) L_i \Delta T}{2\Delta r} = \frac{2\left(\left(19 \times 10^{-6} - 0.9 \times 10^{-6}\right)^\circ\text{C}^{-1}\right)(200\text{ mm})(1^\circ\text{C})}{0.500\text{ mm}}$$
$$= 1.45 \times 10^{-2} = 1.45 \times 10^{-2}\text{ rad}\left(\frac{180^\circ}{\pi\,\text{rad}}\right) = \boxed{0.830^\circ}$$

**P19.55** From the diagram we see that the change in area is

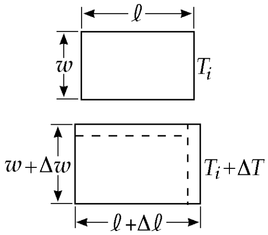


**FIG. P19.55**

$$\Delta A = \ell \Delta w + w \Delta\ell + \Delta w \Delta\ell\,.$$

Since $\Delta\ell$ and $\Delta w$ are each small quantities, the product $\Delta w \Delta\ell$ will be very small. Therefore, we assume $\Delta w \Delta\ell \approx 0$.

Since $\Delta w = w\alpha\Delta T$ and $\Delta\ell = \ell\alpha\Delta T$,

we then have $\Delta A = \ell w \alpha \Delta T + \ell w \alpha \Delta T$

and since $A = \ell w$, $\boxed{\Delta A = 2\alpha A \Delta T}$.

The approximation assumes $\Delta w \Delta\ell \approx 0$, or $\alpha\Delta T \approx 0$. Another way of stating this is $\boxed{\alpha\Delta T << 1}$.

**P19.56** (a) $T_i = 2\pi\sqrt{\frac{L_i}{g}}$ so $L_i = \frac{T_i^2 g}{4\pi^2} = \frac{(1.000\text{ s})^2\left(9.80\text{ m/s}^2\right)}{4\pi^2} = 0.248\,2\text{ m}$

$$\Delta L = \alpha L_i \Delta T = 19.0\times10^{-6}\,{}^\circ\text{C}^{-1}(0.284\,2\text{ m})(10.0^\circ\text{C}) = 4.72\times10^{-5}\text{ m}$$

$$T_f = 2\pi\sqrt{\frac{L_i+\Delta L}{g}} = 2\pi\sqrt{\frac{0.248\,3\text{ m}}{9.80\text{ m/s}^2}} = 1.000\,095\,0\text{ s}$$

$$\Delta T = \boxed{9.50\times10^{-5}\text{ s}}$$

(b) In one week, the time lost is time lost $= 1\text{ week}\left(9.50\times10^{-5}\text{ s lost per second}\right)$

$$\text{time lost} = (7.00\text{ d/week})\left(\frac{86\,400\text{ s}}{1.00\text{ d}}\right)\left(9.50\times10^{-5}\ \frac{\text{s lost}}{\text{s}}\right)$$

$$\text{time lost} = \boxed{57.5\text{ s lost}}$$

**P19.57** $I = \int r^2 dm$ and since $r(T) = r(T_i)(1+\alpha\Delta T)$

for $\alpha\Delta T \ll 1$ we find $\frac{I(T)}{I(T_i)} = (1+\alpha\Delta T)^2$

thus $\frac{I(T)-I(T_i)}{I(T_i)} \approx 2\alpha\Delta T$

(a) With $\alpha = 17.0\times10^{-6}\,{}^\circ\text{C}^{-1}$ and $\Delta T = 100^\circ\text{C}$

we find for Cu: $\frac{\Delta I}{I} = 2\left(17.0\times10^{-6}\,{}^\circ\text{C}^{-1}\right)(100^\circ\text{C}) = \boxed{0.340\%}$

(b) With $\alpha = 24.0\times10^{-6}\,{}^\circ\text{C}^{-1}$

and $\Delta T = 100^\circ\text{C}$

we find for Al: $\frac{\Delta I}{I} = 2\left(24.0\times10^{-6}\,{}^\circ\text{C}^{-1}\right)(100^\circ\text{C}) = \boxed{0.480\%}$

**P19.58** (a) $B = \rho g V'$ $P' = P_0 + \rho g d$ $P'V' = P_0 V_i$

$$B = \frac{\rho g P_0 V_i}{P'} = \boxed{\frac{\rho g P_0 V_i}{(P_0+\rho g d)}}$$

(b) Since *d* is in the denominator, *B* must $\boxed{\text{decrease}}$ as the depth increases. (The volume of the balloon becomes smaller with increasing pressure.)

(c) $$\frac{1}{2} = \frac{B(d)}{B(0)} = \frac{\rho g P_0 V_i/(P_0+\rho g d)}{\rho g P_0 V_i/P_0} = \frac{P_0}{P_0+\rho g d}$$

$$P_0 + \rho g d = 2P_0$$

$$d = \frac{P_0}{\rho g} = \frac{1.013\times10^5\text{ N/m}^2}{\left(1.00\times10^3\text{ kg/m}^3\right)\left(9.80\text{ m/s}^2\right)} = \boxed{10.3\text{ m}}$$

***P19.59** The effective coefficient is defined by $\Delta L_{\text{total}} = \alpha_{\text{effective}} L_{\text{total}} \Delta T$ where $\Delta L_{\text{total}} = \Delta L_{\text{Cu}} + \Delta L_{\text{Pb}}$ and $L_{\text{total}} = L_{\text{Cu}} + L_{\text{Pb}} = xL_{\text{total}} + (1-x)L_{\text{total}}$. Then by substitution

$$\alpha_{\text{Cu}} L_{\text{Cu}} \Delta T + \alpha_{\text{Pb}} L_{\text{Pb}} \Delta T = \alpha_{\text{eff}} (L_{\text{Cu}} + L_{\text{Pb}}) \Delta T$$
$$\alpha_{\text{Cu}} x + \alpha_{\text{Pb}} (1-x) = \alpha_{\text{eff}}$$
$$(\alpha_{\text{Cu}} - \alpha_{\text{Pb}}) x = \alpha_{\text{eff}} - \alpha_{\text{Pb}}$$
$$x = \frac{20 \times 10^{-6}\ 1/\text{C}° - 29 \times 10^{-6}\ 1/\text{C}°}{17 \times 10^{-6}\ 1/\text{C}° - 29 \times 10^{-6}\ 1/\text{C}°} = \frac{9}{12} = \boxed{0.750}$$

***P19.60** (a) No torque acts on the disk so its angular momentum is constant. Its moment of inertia decreases as it contracts so its angular speed must $\boxed{\text{increase}}$.

(b)
$$I_i \omega_i = I_f \omega_f = \frac{1}{2} M R_i^2 \omega_i = \frac{1}{2} M R_f^2 \omega_f = \frac{1}{2} M \left[ R_i + R_i \alpha \Delta T \right]^2 \omega_f = \frac{1}{2} M R_i^2 \left[ 1 - \alpha |\Delta T| \right]^2 \omega_f$$
$$\omega_f = \omega_i \left[ 1 - \alpha |\Delta T| \right]^{-2} = \frac{25.0\ \text{rad/s}}{\left(1 - \left(17 \times 10^{-6}\ 1/\text{C}°\right) 830°\text{C}\right)^2} = \frac{25.0\ \text{rad/s}}{0.972} = \boxed{25.7\ \text{rad/s}}$$

**P19.61** After expansion, the length of one of the spans is

$$L_f = L_i (1 + \alpha \Delta T) = 125\ \text{m} \left[ 1 + 12 \times 10^{-6}\ °\text{C}^{-1} (20.0°\text{C}) \right] = 125.03\ \text{m}.$$

$L_f$, $y$, and the original 125 m length of this span form a right triangle with $y$ as the altitude. Using the Pythagorean theorem gives:

$$(125.03\ \text{m})^2 = y^2 + (125\ \text{m})^2$$

yielding $y = \boxed{2.74\ \text{m}}$.

**P19.62** After expansion, the length of one of the spans is $L_f = L(1 + \alpha \Delta T)$. $L_f$, $y$, and the original length $L$ of this span form a right triangle with $y$ as the altitude. Using the Pythagorean theorem gives

$$L_f^2 = L^2 + y^2, \qquad \text{or} \qquad y = \sqrt{L_f^2 - L^2} = L\sqrt{(1 + \alpha \Delta T)^2 - 1} = L\sqrt{2\alpha \Delta T + (\alpha \Delta T)^2}$$

Since $\alpha \Delta T << 1$, $y \approx \boxed{L\sqrt{2\alpha \Delta T}}$.

**P19.63** (a) Let $m$ represent the sample mass. The number of moles is $n = \frac{m}{M}$ and the density is $\rho = \frac{m}{V}$.

So $PV = nRT$ becomes $PV = \frac{m}{M} RT$ or $PM = \frac{m}{V} RT$.

Then, $\rho = \frac{m}{V} = \boxed{\frac{PM}{RT}}$.

(b)
$$\rho = \frac{PM}{RT} = \frac{\left(1.013 \times 10^5\ \text{N/m}^2\right)(0.032\,0\ \text{kg/mol})}{(8.314\ \text{J/mol} \cdot \text{K})(293\ \text{K})} = \boxed{1.33\ \text{kg/m}^3}$$

**P19.64** (a) From $PV = nRT$, the volume is: $V = \left(\frac{nR}{P}\right)T$

Therefore, when pressure is held constant, $\frac{dV}{dT} = \frac{nR}{P} = \frac{V}{T}$

Thus, $\beta \equiv \left(\frac{1}{V}\right)\frac{dV}{dT} = \left(\frac{1}{V}\right)\frac{V}{T'}$, or $\beta = \boxed{\frac{1}{T}}$

(b) At $T = 0°\text{C} = 273\text{ K}$, this predicts $\beta = \frac{1}{273\text{ K}} = \boxed{3.66 \times 10^{-3}\text{ K}^{-1}}$

Experimental values are: $\beta_{\text{He}} = 3.665 \times 10^{-3}\text{ K}^{-1}$ and $\beta_{\text{air}} = 3.67 \times 10^{-3}\text{ K}^{-1}$

They agree within 0.06% and 0.2%, respectively.

**P19.65** For each gas alone, $P_1 = \frac{N_1 kT}{V}$ and $P_2 = \frac{N_2 kT}{V}$ and $P_3 = \frac{N_3 kT}{V}$, etc.

For all gases

$$P_1V_1 + P_2V_2 + P_3V_3 \ldots = (N_1 + N_2 + N_3 \ldots)kT \text{ and}$$
$$(N_1 + N_2 + N_3 \ldots)kT = PV$$

Also, $V_1 = V_2 = V_3 = \ldots = V$, therefore $\boxed{P = P_1 + P_2 + P_3 \ldots}$.

**P19.66** (a) Using the Periodic Table, we find the molecular masses of the air components to be

$$M(\text{N}_2) = 28.01\text{ u},\ M(\text{O}_2) = 32.00\text{ u},\ M(\text{Ar}) = 39.95\text{ u}$$

and $M(\text{CO}_2) = 44.01\text{ u}$.

Thus, the number of moles of each gas in the sample is

$$n(\text{N}_2) = \frac{75.52\text{ g}}{28.01\text{ g/mol}} = 2.696\text{ mol}$$
$$n(\text{O}_2) = \frac{23.15\text{ g}}{32.00\text{ g/mol}} = 0.723\,4\text{ mol}$$
$$n(\text{Ar}) = \frac{1.28\text{ g}}{39.95\text{ g/mol}} = 0.032\,0\text{ mol}$$
$$n(\text{CO}_2) = \frac{0.05\text{ g}}{44.01\text{ g/mol}} = 0.001\,1\text{ mol}$$

The total number of moles is $n_0 = \sum n_i = 3.453\text{ mol}$. Then, the partial pressure of $\text{N}_2$ is

$$P(\text{N}_2) = \frac{2.696\text{ mol}}{3.453\text{ mol}}\left(1.013 \times 10^5\text{ Pa}\right) = \boxed{79.1\text{ kPa}}.$$

Similarly,

$$P(\text{O}_2) = \boxed{21.2\text{ kPa}} \qquad P(\text{Ar}) = \boxed{940\text{ Pa}} \qquad P(\text{CO}_2) = \boxed{33.3\text{ Pa}}$$

*continued on next page*

(b) Solving the ideal gas law equation for $V$ and using $T = 273.15 + 15.00 = 288.15\text{ K}$, we find

$$V = \frac{n_0 RT}{P} = \frac{(3.453\text{ mol})(8.314\text{ J/mol}\cdot\text{K})(288.15\text{ K})}{1.013\times10^5\text{ Pa}} = \boxed{8.166\times10^{-2}\text{ m}^3}.$$

Then, $\rho = \frac{m}{V} = \frac{100\times10^{-3}\text{ kg}}{8.166\times10^{-2}\text{ m}^3} = \boxed{1.22\text{ kg/m}^3}$.

(c) The 100 g sample must have an appropriate molar mass to yield $n_0$ moles of gas: that is

$$M(\text{air}) = \frac{100\text{ g}}{3.453\text{ mol}} = \boxed{29.0\text{ g/mol}}.$$

***P19.67** Consider a spherical steel shell of inner radius $r$ and much smaller thickness $t$, containing helium at pressure $P$. When it contains so much helium that it is on the point of bursting into two hemispheres, we have $P\pi r^2 = \left(5\times10^8\text{ N/m}^2\right)2\pi rt$. The mass of the steel is $\rho_s V = \rho_s 4\pi r^2 t = \rho_s 4\pi r^2 \frac{Pr}{10^9\text{ Pa}}$. For the helium in the tank, $PV = nRT$ becomes

$$P\frac{4}{3}\pi r^3 = nRT = \frac{m_{\text{He}}}{M_{\text{He}}}RT = 1\text{ atm}V_{\text{balloon}}.$$

The buoyant force on the balloon is the weight of the air it displaces, which is described by $1\text{ atm}V_{\text{balloon}} = \frac{m_{\text{air}}}{M_{\text{air}}}RT = P\frac{4}{3}\pi r^3$. The net upward force on the balloon with the steel tank hanging from it is

$$+m_{\text{air}}g - m_{\text{He}}g - m_s g = \frac{M_{\text{air}}P4\pi r^3 g}{3RT} - \frac{M_{\text{He}}P4\pi r^3 g}{3RT} - \frac{\rho_s P4\pi r^3 g}{10^9\text{ Pa}}$$

The balloon will or will not lift the tank depending on whether this quantity is positive or negative, which depends on the sign of $\frac{(M_{\text{air}} - M_{\text{He}})}{3RT} - \frac{\rho_s}{10^9\text{ Pa}}$. At 20°C this quantity is

$$\begin{aligned}&= \frac{(28.9 - 4.00)\times10^{-3}\text{ kg/mol}}{3(8.314\text{ J/mol}\cdot\text{K})293\text{ K}} - \frac{7\,860\text{ kg/m}^3}{10^9\text{ N/m}^2}\\ &= 3.41\times10^{-6}\text{ s}^2/\text{m}^2 - 7.86\times10^{-6}\text{ s}^2/\text{m}^2\end{aligned}$$

where we have used the density of iron. The net force on the balloon is downward so the helium balloon is $\boxed{\text{not able to lift}}$ its tank.

**P19.68** With piston alone: $T = \text{constant, so } PV = P_0V_0$

or $P(Ah_i) = P_0(Ah_0)$

With $A$ = constant, $P = P_0\left(\dfrac{h_0}{h_i}\right)$

But, $P = P_0 + \dfrac{m_pg}{A}$

where $m_p$ is the mass of the piston.

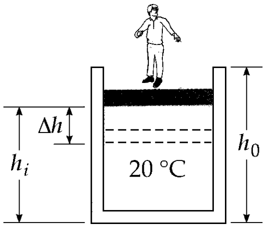


FIG. P19.68

Thus, $P_0 + \dfrac{m_pg}{A} = P_0\left(\dfrac{h_0}{h_i}\right)$

which reduces to $h_i = \dfrac{h_0}{1+\frac{m_pg}{P_0A}} = \dfrac{50.0\text{ cm}}{1+\frac{20.0\text{ kg}(9.80\text{ m/s}^2)}{1.013\times10^5\text{ Pa}\left[\pi(0.400\text{ m})^2\right]}} = 49.81\text{ cm}$

With the man of mass $M$ on the piston, a very similar calculation (replacing $m_p$ by $m_p+M$) gives:

$$h' = \frac{h_0}{1+\frac{(m_p+M)g}{P_0A}} = \frac{50.0\text{ cm}}{1+\frac{95.0\text{ kg}(9.80\text{ m/s}^2)}{1.013\times10^5\text{ Pa}\left[\pi(0.400\text{ m})^2\right]}} = 49.10\text{ cm}$$

Thus, when the man steps on the piston, it moves downward by

$$\Delta h = h_i - h' = 49.81\text{ cm} - 49.10\text{ cm} = 0.706\text{ cm} = \boxed{7.06\text{ mm}}.$$

(b) $P$ = const, so $\dfrac{V}{T} = \dfrac{V'}{T_i}$ or $\dfrac{Ah_i}{T} = \dfrac{Ah'}{T_i}$

giving $T = T_i\left(\dfrac{h_i}{h'}\right) = 293\text{ K}\left(\dfrac{49.81}{49.10}\right) = \boxed{297\text{ K}}$ (or 24°C)

**P19.69** (a) $\dfrac{dL}{L} = \alpha dT: \quad \displaystyle\int_{T_i}^{T_i}\alpha dT = \int_{L_i}^{L_i}\frac{dL}{L} \Rightarrow \ln\left(\frac{L_f}{L_i}\right) = \alpha\Delta T \Rightarrow \boxed{L_f = L_ie^{\alpha\Delta T}}$

(b) $L_f = (1.00\text{ m})e^{\left[2.00\times10^{-5}\,°\text{C}^{-1}(100°\text{C})\right]} = 1.002\,002\text{ m}$

$L_f' = 1.00\text{ m}\left[1+2.00\times10^{-5}\,°\text{C}^{-1}(100°\text{C})\right] = 1.002\,000\text{ m}: \quad \dfrac{L_f - L_f'}{L_f} = 2.00\times10^{-6} = \boxed{2.00\times10^{-4}\%}$

$L_f = (1.00\text{ m})e^{\left[2.00\times10^{-2}\,°\text{C}^{-1}(100°\text{C})\right]} = 7.389\text{ m}$

$L_f' = 1.00\text{ m}\left[1+0.020\,0°\text{C}^{-1}(100°\text{C})\right] = 3.000\text{ m}: \quad \dfrac{L_f - L_f'}{L_f} = \boxed{59.4\%}$

**P19.70** At 20.0°C, the unstretched lengths of the steel and copper wires are

$$L_s(20.0°\text{C}) = (2.000\ \text{m})\left[1 + 11.0\times10^{-6}(\text{C}°)^{-1}(-20.0°\text{C})\right] = 1.999\,56\ \text{m}$$
$$L_c(20.0°\text{C}) = (2.000\ \text{m})\left[1 + 17.0\times10^{-6}(\text{C}°)^{-1}(-20.0°\text{C})\right] = 1.999\,32\ \text{m}$$

Under a tension $F$, the length of the steel and copper wires are

$$L_s' = L_s\left[1 + \frac{F}{YA}\right]_s \qquad L_c' = L_c\left[1 + \frac{F}{YA}\right]_c \qquad \text{where } L_s' + L_c' = 4.000\ \text{m}.$$

Since the tension, $F$, must be the same in each wire, solve for $F$:

$$F = \frac{(L_s' + L_c') - (L_s + L_c)}{\frac{L_s}{Y_sA_s} + \frac{L_c}{Y_cA_c}}.$$

When the wires are stretched, their areas become

$$A_s = \pi\left(1.000\times10^{-3}\ \text{m}\right)^2\left[1 + \left(11.0\times10^{-6}\right)(-20.0)\right]^2 = 3.140\times10^{-6}\ \text{m}^2$$
$$A_c = \pi\left(1.000\times10^{-3}\ \text{m}\right)^2\left[1 + \left(17.0\times10^{-6}\right)(-20.0)\right]^2 = 3.139\times10^{-6}\ \text{m}^2$$

Recall $Y_s = 20.0\times10^{10}\ \text{Pa}$ and $Y_c = 11.0\times10^{10}\ \text{Pa}$. Substituting into the equation for $F$, we obtain

$$F = \frac{4.000\ \text{m} - (1.999\,56\ \text{m} + 1.999\,32\ \text{m})}{[1.999\,56\ \text{m}]\big/\left[\left(20.0\times10^{10}\ \text{Pa}\right)\left(3.140\times10^{-6}\right)\text{m}^2\right] + [1.999\,32\ \text{m}]\big/\left[\left(11.0\times10^{10}\ \text{Pa}\right)\left(3.139\times10^{-6}\right)\text{m}^2\right]}$$
$$F = \boxed{125\ \text{N}}$$

To find the $x$-coordinate of the junction,

$$L_s' = (1.999\,56\ \text{m})\left[1 + \frac{125\ \text{N}}{\left(20.0\times10^{10}\ \text{N/m}^2\right)\left(3.140\times10^{-6}\ \text{m}^2\right)}\right] = 1.999\,958\ \text{m}$$

Thus the $x$-coordinate is $-2.000 + 1.999\,958 = \boxed{-4.20\times10^{-5}\ \text{m}}$.

**P19.71** (a) $\mu = \pi r^2 \rho = \pi\left(5.00\times10^{-4}\text{ m}\right)^2\left(7.86\times10^3\text{ kg/m}^3\right) = \boxed{6.17\times10^{-3}\text{ kg/m}}$

(b) $f_1 = \dfrac{v}{2L}$ and $v = \sqrt{\dfrac{T}{\mu}}$ so $f_1 = \dfrac{1}{2L}\sqrt{\dfrac{T}{\mu}}$

Therefore, $T = \mu(2Lf_1)^2 = \left(6.17\times10^{-3}\right)(2\times0.800\times200)^2 = \boxed{632\text{ N}}$

(c) First find the unstressed length of the string at 0°C:

$$L = L_{\text{natural}}\left(1+\frac{T}{AY}\right)\text{ so } L_{\text{natural}} = \frac{L}{1+T/AY}$$

$$A = \pi\left(5.00\times10^{-4}\text{ m}\right)^2 = 7.854\times10^{-7}\text{ m}^2 \text{ and } Y = 20.0\times10^{10}\text{ Pa}$$

Therefore, $\dfrac{T}{AY} = \dfrac{632}{\left(7.854\times10^{-7}\right)\left(20.0\times10^{10}\right)} = 4.02\times10^{-3}$, and

$$L_{\text{natural}} = \frac{(0.800\text{ m})}{\left(1+4.02\times10^{-3}\right)} = 0.796\,8\text{ m}.$$

The unstressed length at 30.0°C is $L_{30°\text{C}} = L_{\text{natural}}\left[1+\alpha(30.0°\text{C}-0.0°\text{C})\right]$,

or $L_{30°\text{C}} = (0.796\,8\text{ m})\left[1+\left(11.0\times10^{-6}\right)(30.0)\right] = 0.797\,06\text{ m}$.

Since $L = L_{30°\text{C}}\left[1+\dfrac{T'}{AY}\right]$, where $T'$ is the tension in the string at 30.0°C,

$$T' = AY\left[\frac{L}{L_{30°\text{C}}}-1\right] = \left(7.854\times10^{-7}\right)\left(20.0\times10^{10}\right)\left[\frac{0.800}{0.797\,06}-1\right] = 580\text{ N}.$$

To find the frequency at 30.0°C, realize that

$$\frac{f_1'}{f_1} = \sqrt{\frac{T'}{T}}\text{ so } f_1' = (200\text{ Hz})\sqrt{\frac{580\text{ N}}{632\text{ N}}} = \boxed{192\text{ Hz}}.$$

***P19.72** Some gas will pass through the porous plug from the reaction chamber 1 to the reservoir 2 as the reaction chamber is heated, but the net quantity of gas stays constant according to

$$n_{i1}+n_{i2} = n_{f1}+n_{f2}.$$

Assuming the gas is ideal, we apply $n = \dfrac{PV}{RT}$ to each term:

$$\frac{P_iV_0}{(300\text{ K})R}+\frac{P_i(4V_0)}{(300\text{ K})R} = \frac{P_fV_0}{(673\text{ K})R}+\frac{P_f(4V_0)}{(300\text{ K})R}$$

$$1\text{ atm}\left(\frac{5}{300\text{ K}}\right) = P_f\left(\frac{1}{673\text{ K}}+\frac{4}{300\text{ K}}\right) \qquad \boxed{P_f = 1.12\text{ atm}}$$

**P19.73** Let $2\theta$ represent the angle the curved rail subtends. We have

$$L_i + \Delta L = 2\theta R = L_i(1+\alpha\Delta T)$$

and
$$\sin\theta = \frac{\frac{L_i}{2}}{R} = \frac{L_i}{2R}$$

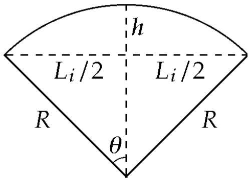


**FIG. P19.73**

Thus,
$$\theta = \frac{L_i}{2R}(1+\alpha\Delta T) = (1+\alpha\Delta T)\sin\theta$$

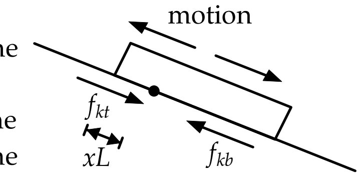
and we must solve the transcendental equation $\theta = (1+\alpha\Delta T)\sin\theta = (1.000\,005\,5)\sin\theta$

Homing in on the non-zero solution gives, to four digits, $\theta = 0.018\,16 \text{ rad} = 1.040\,5°$

Now,
$$h = R - R\cos\theta = \frac{L_i(1-\cos\theta)}{2\sin\theta}$$

This yields $\boxed{h = 4.54 \text{ m}}$, a remarkably large value compared to $\Delta L = 5.50 \text{ cm}$.

***P19.74** (a) Let $xL$ represent the distance of the stationary line below the top edge of the plate. The normal force on the lower part of the plate is $mg(1-x)\cos\theta$ and the force of kinetic friction on it is $\mu_k mg(1-x)\cos\theta$ up the roof. Again, $\mu_k mgx\cos\theta$ acts down the roof on the upper part of the plate. The near-equilibrium of the plate requires $\sum F_x = 0$



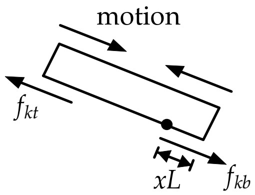
temperature rising

**FIG. P19.74(a)**

$$-\mu_k mgx\cos\theta + \mu_k mg(1-x)\cos\theta - mg\sin\theta = 0$$
$$-2\mu_k mgx\cos\theta = mg\sin\theta - \mu_k mg\cos\theta$$
$$2\mu_k x = \mu_k - \tan\theta$$
$$x = \frac{1}{2} - \frac{\tan\theta}{2\mu_k}$$

and the stationary line is indeed below the top edge by $xL = \frac{L}{2}\left(1 - \frac{\tan\theta}{\mu_k}\right)$.

(b) With the temperature falling, the plate contracts faster than the roof. The upper part slides down and feels an upward frictional force $\mu_k mg(1-x)\cos\theta$. The lower part slides up and feels downward frictional force $\mu_k mgx\cos\theta$. The equation $\sum F_x = 0$ is then the same as in part (a) and the stationary line is above the bottom edge by $xL = \frac{L}{2}\left(1 - \frac{\tan\theta}{\mu_k}\right)$.



temperature falling

**FIG. P19.74(b)**

(c) Start thinking about the plate at dawn, as the temperature starts to rise. As in part (a), a line at distance $xL$ below the top edge of the plate stays stationary relative to the roof as long as the temperature rises. The point $P$ on the plate at distance $xL$ above the bottom edge is destined to become the fixed point when the temperature starts falling. As the temperature rises, this point moves down the roof because of the expansion of the central part of the plate. Its displacement for the day is

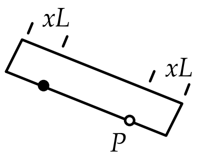


**FIG. P19.74(c)**

*continued on next page*

$$\begin{aligned}\Delta L &= (\alpha_2-\alpha_1)(L-xL-xL)\Delta T\\ &= (\alpha_2-\alpha_1)\left[L-2\frac{L}{2}\left(1-\frac{\tan\theta}{\mu_k}\right)\right](T_h-T_c)\\ &= (\alpha_2-\alpha_1)\left(\frac{L\tan\theta}{\mu_k}\right)(T_h-T_c).\end{aligned}$$

At dawn the next day the point $P$ is farther down the roof by the distance $\Delta L$. It represents the displacement of every other point on the plate.

(d) $$(\alpha_2-\alpha_1)\left(\frac{L\tan\theta}{\mu_k}\right)(T_h-T_c)=\left(24\times10^{-6}\,\frac{1}{\text{C}^\circ}-15\times10^{-6}\,\frac{1}{\text{C}^\circ}\right)\frac{1.20\text{ m}\tan 18.5^\circ}{0.42}32^\circ\text{C}=\boxed{0.275\text{ mm}}$$

(e) If $\alpha_2<\alpha_1$, the diagram in part (a) applies to temperature falling and the diagram in part (b) applies to temperature rising. The weight of the plate still pulls it step by step down the roof. The same expression describes how far it moves each day.

## ANSWERS TO EVEN PROBLEMS

| | |
|---|---|
| **P19.2** | (a) 1.06 atm; (b) $-124^\circ$C |
| **P19.4** | (a) $37.0^\circ\text{C}=310$ K; (b) $-20.6^\circ\text{C}=253$ K |
| **P19.6** | $T_C=(1.33\ \text{C}^\circ/\text{S}^\circ)T_S+20.0^\circ\text{C}$ |
| **P19.8** | 0.313 m |
| **P19.10** | 1.20 cm |
| **P19.12** | $15.8\,\mu$m |
| **P19.14** | 0.663 mm to the right at 78.2° below the horizontal |
| **P19.16** | (a) 0.109 $\text{cm}^2$; (b) increase |
| **P19.18** | (a) 437°C; (b) 3 000°C; no |
| **P19.20** | (a) $2.52\times10^6\ \text{N/m}^2$; (b) no |
| **P19.22** | 0.812 $\text{cm}^3$ |
| **P19.24** | (a) 396 N; (b) $-101^\circ$C; (c) no change |
| **P19.26** | (a) 2.99 mol; (b) $1.80\times10^{24}$ molecules |
| **P19.28** | 884 balloons |
| **P19.30** | (a) $1.06\times10^{21}$ kg; (b) 56.9 K |
| **P19.32** | (a) 900 K; (b) 1 200 K |
| **P19.34** | see the solution |
| **P19.36** | $3.96\times10^{-2}$ mol |
| **P19.38** | 3.67 $\text{cm}^3$ |
| **P19.40** | between $10^1$ kg and $10^2$ kg |
| **P19.42** | $2.41\times10^{11}$ molecules |
| **P19.44** | (a) 2.24 m; (b) $9.28\times10^5$ Pa |
| **P19.46** | 0.523 kg |
| **P19.48** | (a) see the solution; (b) $\alpha \ll \beta$ |
| **P19.50** | (a) 0.169 m; (b) $1.35\times10^5$ Pa |
| **P19.52** | 6.57 MPa |
| **P19.54** | (a) $\theta=\dfrac{(\alpha_2-\alpha_1)L_i\Delta T}{\Delta r}$; (b) see the solution; (c) it bends the other way; (d) 0.830° |
| **P19.56** | (a) increase by 95.0 $\mu$s; (b) loses 57.5 s |
| **P19.58** | (a) $B=\rho g P_0 V_i(P_0+\rho g d)^{-1}$ up; (b) decrease; (c) 10.3 m |

**P19.60** (a) yes; see the solution; (b) 25.7 rad/s

**P19.62** $y \approx L(2\alpha\Delta T)^{1/2}$

**P19.64** (a) see the solution;
(b) $3.66 \times 10^{-3}\ \text{K}^{-1}$, within 0.06% and 0.2% of the experimental values

**P19.66** (a) 79.1 kPa for $N_2$; 21.2 kPa for $O_2$;
940 Pa for Ar; 33.3 Pa for $CO_2$;
(b) 81.7 L; 1.22 $\text{kg/m}^3$; (c) 29.0 g/mol

**P19.68** (a) 7.06 mm; (b) 297 K

**P19.70** 125 N; $-42.0\ \mu\text{m}$

**P19.72** 1.12 atm

**P19.74** (a), (b), (c) see the solution; (d) 0.275 mm;
(e) see the solution

# 20

# Heat and the First Law of Thermodynamics

### CHAPTER OUTLINE

20.1 Heat and Internal Energy
20.2 Specific Heat and Calorimetry
20.3 Latent Heat
20.4 Work and Heat in Thermodynamic Processes
20.5 The First Law of Thermodynamics
20.6 Some Applications of the First Law of Thermodynamics
20.7 Energy Transfer Mechanisms

## ANSWERS TO QUESTIONS

**Q20.1** Temperature is a measure of molecular motion. Heat is energy in the process of being transferred between objects by random molecular collisions. Internal energy is an object's energy of random molecular motion and molecular interaction.

**Q20.2** The $\Delta T$ is twice as great in the ethyl alcohol.

**Q20.3** The final equilibrium temperature will show no significant increase over the initial temperature of the water.

**Q20.4** Some water may boil away. You would have to very precisely measure how much, and very quickly measure the temperature of the steam; it is not necessarily $100^\circ\text{C}$.

**Q20.5** The fingers are wetted to create a layer of steam between the fingers and the molten lead. The steam acts as an insulator and can prevent or delay serious burns. The molten lead demonstration is dangerous, and we do not recommend it.

**Q20.6** Heat is energy being transferred, not energy contained in an object. Further, a large-mass object, or an object made of a material with high specific heat, can contain more internal energy than a higher-temperature object.

**Q20.7** There are three properties to consider here: thermal conductivity, specific heat, and mass. With dry aluminum, the thermal conductivity of aluminum is much greater than that of (dry) skin. This means that the internal energy in the aluminum can more readily be transferred to the atmosphere than to your fingers. In essence, your skin acts as a thermal insulator to some degree (pun intended). If the aluminum is wet, it can wet the outer layer of your skin to make it into a good conductor of heat; then more internal energy from the aluminum can get into you. Further, the water itself, with additional mass and with a relatively large specific heat compared to aluminum, can be a significant source of extra energy to burn you. In practical terms, when you let go of a hot, dry piece of aluminum foil, the heat transfer immediately ends. When you let go of a hot *and* wet piece of aluminum foil, the hot water sticks to your skin, continuing the heat transfer, and resulting in more energy transfer to you!

**Q20.8** Write $1\,000\text{ kg}(4\,186\text{ J/kg}\cdot{}^\circ\text{C})(1^\circ\text{C}) = V\left(1.3\text{ kg/m}^3\right)(1\,000\text{ J/kg}\cdot{}^\circ\text{C})(1^\circ\text{C})$ to find $V = 3.2\times10^3\text{ m}^3$.

**Q20.9** The large amount of energy stored in concrete during the day as the sun falls on it is released at night, resulting in an higher average evening temperature than the countryside. The cool air in the surrounding countryside exerts a buoyant force on the warmer air in the city, pushing it upward and moving into the city in the process. Thus, evening breezes tend to blow from country to city.

**Q20.10** If the system is isolated, no energy enters or leaves the system by heat, work, or other transfer processes. Within the system energy can change from one form to another, but since energy is conserved these transformations cannot affect the total amount of energy. The total energy is constant.

**Q20.11** (a) and (b) both increase by minuscule amounts.

**Q20.12** The steam locomotive engine is a perfect example of turning internal energy into mechanical energy. Liquid water is heated past the point of vaporization. Through a controlled mechanical process, the expanding water vapor is allowed to push a piston. The translational kinetic energy of the piston is usually turned into rotational kinetic energy of the drive wheel.

**Q20.13** Yes. If you know the different specific heats of zinc and copper, you can determine the fraction of each by heating a known mass of pennies to a specific initial temperature, say $100°\text{C}$, and dumping them into a known quantity of water, at say $20°\text{C}$. The final temperature $T$ will reveal the metal content:

$$m_{\text{pennies}}\left[xc_{\text{Cu}}+(1-x)c_{\text{Zn}}\right](100°\text{C}-T)=m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}}(T-20°\text{C}).$$

Since all quantities are known, except $x$, the fraction of the penny that is copper will be found by putting in the experimental numbers $m_{\text{pennies}}$, $m_{\text{H}_2\text{O}}$, $T(\text{final})$, $c_{\text{Zn}}$, and $c_{\text{Cu}}$.

**Q20.14** The materials used to make the support structure of the roof have a higher thermal conductivity than the insulated spaces in between. The heat from the barn conducts through the rafters and melts the snow.

**Q20.15** The tile is a better thermal conductor than carpet. Thus, energy is conducted away from your feet more rapidly by the tile than by the carpeted floor.

**Q20.16** The question refers to baking in a conventional oven, not to microwaving. The metal has much higher thermal conductivity than the potato. The metal quickly conducts energy from the hot oven into the center of potato.

**Q20.17** Copper has a higher thermal conductivity than the wood. Heat from the flame is conducted through the copper away from the paper, so that the paper need not reach its kindling temperature. The wood does not conduct the heat away from the paper as readily as the copper, so the energy in the paper can increase enough to make it ignite.

**Q20.18** In winter the interior of the house is warmer than the air outside. On a summer day we want the interior to stay cooler than the exterior. Heavy draperies over the windows can slow down energy transfer by conduction, by convection, and by radiation, to make it easier to maintain the desired difference in temperature.

**Q20.19** You must allow time for the flow of energy into the center of the piece of meat. To avoid burning the outside, the meat should be relatively far from the flame. If the outer layer does char, the carbon will slow subsequent energy flow to the interior.

**Q20.20** At night, the Styrofoam beads would decrease the overall thermal conductivity of the windows, and thus decrease the amount of heat conducted from inside to outside. The air pockets in the Styrofoam are an efficient insulator. During the winter day, the influx of sunlight coming through the window warms the living space.

An interesting aside—the majority of the energy that goes into warming a home from sunlight through a window is *not* the infrared light given off by the sun. Glass is a relatively good insulator of infrared. If not, the window on your cooking oven might as well be just an open hole! Glass is opaque to a large portion of the ultraviolet range. The glass molecules absorb ultraviolet light from the sun and re-emit the energy in the infrared region. It is this re-emitted infrared radiation that contributes to warming your home, along with visible light.

**Q20.21** In winter the produce is protected from freezing. The heat capacity of the earth is so high that soil freezes only to a depth of a few decimeters in temperate regions. Throughout the year the temperature will stay nearly constant all day and night. Factors to be considered are the insulating properties of soil, the absence of a path for energy to be radiated away from or to the vegetables, and the hindrance to the formation of convection currents in the small, enclosed space.

**Q20.22** The high mass and specific heat of the barrel of water and its high heat of fusion mean that a large amount of energy would have to leak out of the cellar before the water and the produce froze solid. Evaporation of the water keeps the relative humidity high to protect foodstuffs from drying out.

**Q20.23** The sunlight hitting the peaks warms the air immediately around them. This air, which is slightly warmer and less dense than the surrounding air, rises, as it is buoyed up by cooler air from the valley below. The air from the valley flows up toward the sunny peaks, creating the morning breeze.

**Q20.24** Sunlight hits the earth and warms the air immediately above it. This warm, less-dense air rises, creating an up-draft. Many raptors, like eagles, hawks and falcons use updrafts to aid in hunting. These birds can often be seen flying without flapping their wings—just sitting in an updraft with wings extended.

**Q20.25** The bit of water immediately over the flame warms up and expands. It is buoyed up and rises through the rest of the water. Colder, more dense water flows in to take its place. Convection currents are set up. This effectively warms the bulk of the water all at once, much more rapidly than it would be by heat being conducted through the water from the flame.

**Q20.26** The porcelain of the teacup is a thermal insulator. That is, it is a thermal conductor of relatively low conductivity. When you wrap your hands around a cup of hot tea, you make $A$ large and $L$ small in the equation $\mathcal{P} = kA\frac{T_h - T_c}{L}$ for the rate of energy transfer by heat from tea into you. When you hold the cup by the handle, you make the rate of energy transfer much smaller by reducing $A$ and increasing $L$. The air around the cup handle will also reduce the temperature where you are touching it. A paper cup can be fitted into a tubular jacket of corrugated cardboard, with the channels running vertically, for remarkably effective insulation, according to the same principles.

**Q20.27** As described in the answer to question 20.25, convection currents in the water serve to bring more of the heat into the water from the paper cup than the specific heats and thermal conductivities of paper and water would suggest. Since the boiling point of water is far lower than the kindling temperature of the cup, the extra energy goes into boiling the water.

**Q20.28** Keep them dry. The air pockets in the pad conduct energy by heat, but only slowly. Wet pads would absorb some energy in warming up themselves, but the pot would still be hot and the water would quickly conduct and convect a lot of energy right into you.

**Q20.29** The person should add the cream immediately when the coffee is poured. Then the smaller temperature difference between coffee and environment will reduce the rate of energy loss during the several minutes.

**Q20.30** The cup without the spoon will be warmer. Heat is conducted from the coffee up through the metal. The energy then radiates and convects into the atmosphere.

**Q20.31** Convection. The bridge deck loses energy rapidly to the air both above it and below it.

**Q20.32** The marshmallow has very small mass compared to the saliva in the teacher's mouth and the surrounding tissues. Mostly air and sugar, the marshmallow also has a low specific heat compared to living matter. Then the marshmallow can zoom up through a large temperature change while causing only a small temperature drop of the teacher's mouth. The marshmallow is a foam with closed cells and it carries very little liquid nitrogen into the mouth. The liquid nitrogen still on the marshmallow comes in contact with the much hotter saliva and immediately boils into cold gaseous nitrogen. This nitrogen gas has very low thermal conductivity. It creates an insulating thermal barrier between the marshmallow and the teacher's mouth (the Leydenfrost effect). A similar effect can be seen when water droplets are put on a hot skillet. Each one dances around as it slowly shrinks, because it is levitated on a thin film of steam. The most extreme demonstration of this effect is pouring liquid nitrogen into one's mouth and blowing out a plume of nitrogen gas. We strongly recommended that you read of Jearl Walker's adventures with this demonstration rather than trying it.

**Q20.33** (a) Warm a pot of coffee on a hot stove.

(b) Place an ice cube at $0°\text{C}$ in warm water—the ice will absorb energy while melting, but not increase in temperature.

(c) Let a high-pressure gas at room temperature slowly expand by pushing on a piston. Work comes out of the gas in a constant-temperature expansion as the same quantity of heat flows in from the surroundings.

(d) Warm your hands by rubbing them together. Heat your tepid coffee in a microwave oven. Energy input by work, by electromagnetic radiation, or by other means, can all alike produce a temperature increase.

(e) Davy's experiment is an example of this process.

(f) This is not necessarily true. Consider some supercooled liquid water, unstable but with temperature below $0°\text{C}$. Drop in a snowflake or a grain of dust to trigger its freezing into ice, and the loss of internal energy measured by its latent heat of fusion can actually push its temperature up.

**Q20.34** Heat is conducted from the warm oil to the pipe that carries it. That heat is then conducted to the cooling fins and up through the solid material of the fins. The energy then radiates off in all directions and is efficiently carried away by convection into the air. The ground below is left frozen.

## SOLUTIONS TO PROBLEMS

### Section 20.1 **Heat and Internal Energy**

**P20.1** Taking $m = 1.00\text{ kg}$, we have

$$\Delta U_g = mgh = (1.00\text{ kg})\left(9.80\text{ m/s}^2\right)(50.0\text{ m}) = 490\text{ J}.$$

But $\Delta U_g = Q = mc\Delta T = (1.00\text{ kg})(4\,186\text{ J/kg}\cdot{}^\circ\text{C})\Delta T = 490\text{ J}$ so $\Delta T = 0.117^\circ\text{C}$

$$T_f = T_i + \Delta T = \boxed{(10.0 + 0.117)^\circ\text{C}}$$

**P20.2** The container is thermally insulated, so no energy flows by heat:

$$Q = 0$$

and $\Delta E_{\text{int}} = Q + W_{\text{input}} = 0 + W_{\text{input}} = 2mgh$

The work on the falling weights is equal to the work done on the water in the container by the rotating blades. This work results in an increase in internal energy of the water:

$$2mgh = \Delta E_{\text{int}} = m_{\text{water}} c\Delta T$$

$$\Delta T = \frac{2mgh}{m_{\text{water}}c} = \frac{2\times 1.50\text{ kg}\left(9.80\text{ m/s}^2\right)(3.00\text{ m})}{0.200\text{ kg}(4\,186\text{ J/kg}\cdot{}^\circ\text{C})} = \frac{88.2\text{ J}}{837\text{ J/}^\circ\text{C}}$$

$$= \boxed{0.105^\circ\text{C}}$$

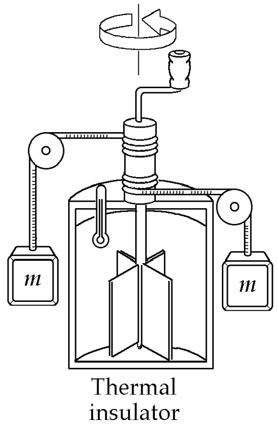


**FIG. P20.2**

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### Section 20.2 **Specific Heat and Calorimetry**

**P20.3**

$$\Delta Q = mc_{\text{silver}}\Delta T$$

$$1.23\text{ kJ} = (0.525\text{ kg})c_{\text{silver}}(10.0^\circ\text{C})$$

$$c_{\text{silver}} = \boxed{0.234\text{ kJ/kg}\cdot{}^\circ\text{C}}$$

**P20.4** From $Q = mc\Delta T$

we find $\Delta T = \dfrac{Q}{mc} = \dfrac{1\,200\text{ J}}{0.050\,0\text{ kg}(387\text{ J/kg}\cdot{}^\circ\text{C})} = 62.0^\circ\text{C}$

Thus, the final temperature is $\boxed{87.0^\circ\text{C}}$.

***P20.5** We imagine the stone energy reservoir has a large area in contact with air and is always at nearly the same temperature as the air. Its overnight loss of energy is described by

$$\mathcal{P} = \frac{Q}{\Delta t} = \frac{mc\Delta T}{\Delta t}$$

$$m = \frac{\mathcal{P}\Delta t}{c\Delta T} = \frac{(-6\,000\text{ J/s})(14\text{ h})(3\,600\text{ s/h})}{(850\text{ J/kg}\cdot{}^\circ\text{C})(18^\circ\text{C} - 38^\circ\text{C})} = \frac{3.02\times 10^8\text{ J}\cdot\text{kg}\cdot{}^\circ\text{C}}{850\text{ J}(20^\circ\text{C})} = \boxed{1.78\times 10^4\text{ kg}}$$

***P20.6** The laser energy output:

$$\mathscr{P}\Delta t = \left(1.60\times10^{13}\ \text{J/s}\right)2.50\times10^{-9}\ \text{s} = 4.00\times10^{4}\ \text{J}.$$

The teakettle input:

$$Q = mc\Delta T = 0.800\ \text{kg}\left(4\ 186\ \text{J/kg}\cdot{}^\circ\text{C}\right)80^\circ\text{C} = 2.68\times10^{5}\ \text{J}.$$

This is larger by 6.70 times.

**P20.7** $Q_{\text{cold}} = -Q_{\text{hot}}$

$(mc\Delta T)_{\text{water}} = -(mc\Delta T)_{\text{iron}}$

$20.0\ \text{kg}\left(4\,186\ \text{J/kg}\cdot{}^\circ\text{C}\right)\left(T_f - 25.0^\circ\text{C}\right) = -(1.50\ \text{kg})(448\ \text{J/kg}\cdot{}^\circ\text{C})\left(T_f - 600^\circ\text{C}\right)$

$T_f = \boxed{29.6^\circ\text{C}}$

**P20.8** Let us find the energy transferred in one minute.

$$Q = \left[m_{\text{cup}}c_{\text{cup}} + m_{\text{water}}c_{\text{water}}\right]\Delta T$$
$$Q = \left[(0.200\ \text{kg})(900\ \text{J/kg}\cdot{}^\circ\text{C}) + (0.800\ \text{kg})(4\,186\ \text{J/kg}\cdot{}^\circ\text{C})\right](-1.50^\circ\text{C}) = -5\ 290\ \text{J}$$

If this much energy is removed from the system each minute, the rate of removal is

$$\mathscr{P} = \frac{|Q|}{\Delta t} = \frac{5\ 290\ \text{J}}{60.0\ \text{s}} = 88.2\ \text{J/s} = \boxed{88.2\ \text{W}}.$$

**P20.9** (a) $Q_{\text{cold}} = -Q_{\text{hot}}$

$(m_w c_w + m_c c_c)\left(T_f - T_c\right) = -m_{\text{Cu}}c_{\text{Cu}}\left(T_f - T_{\text{Cu}}\right) - m_{unk}c_{unk}\left(T_f - T_{unk}\right)$

where *w* is for water, *c* the calorimeter, Cu the copper sample, and *unk* the unknown.

$$\left[250\ \text{g}(1.00\ \text{cal/g}\cdot{}^\circ\text{C}) + 100\ \text{g}(0.215\ \text{cal/g}\cdot{}^\circ\text{C})\right](20.0-10.0)^\circ\text{C}$$
$$= -(50.0\ \text{g})(0.092\,4\ \text{cal/g}\cdot{}^\circ\text{C})(20.0-80.0)^\circ\text{C} - (70.0\ \text{g})c_{unk}(20.0-100)^\circ\text{C}$$
$$2.44\times10^{3}\ \text{cal} = \left(5.60\times10^{3}\ \text{g}\cdot{}^\circ\text{C}\right)c_{unk}$$

or $c_{unk} = \boxed{0.435\ \text{cal/g}\cdot{}^\circ\text{C}}$.

(b) The material of the sample is $\boxed{\text{beryllium}}$.

**P20.10** (a) $(f)(mgh) = mc\Delta T$

$$\frac{(0.600)\left(3.00\times10^{-3}\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)(50.0\ \text{m})}{4.186\ \text{J/cal}} = (3.00\ \text{g})(0.092\,4\ \text{cal/g}\cdot{}^\circ\text{C})(\Delta T)$$

$$\Delta T = 0.760^\circ\text{C};\ \boxed{T = 25.8^\circ\text{C}}$$

(b) $\boxed{\text{No}}$. Both the change in potential energy and the heat absorbed are proportional to the mass; hence, the mass cancels in the energy relation.

***P20.11** We do not know whether the aluminum will rise or drop in temperature. The energy the water can absorb in rising to 26°C is $mc\Delta T = 0.25\ \text{kg}\ 4\,186\frac{\text{J}}{\text{kg}\ {}^\circ\text{C}}6^\circ\text{C} = 6\,279\ \text{J}$. The energy the copper can put out in dropping to 26°C is $mc\Delta T = 0.1\ \text{kg}\ 387\frac{\text{J}}{\text{kg}\ {}^\circ\text{C}}74^\circ\text{C} = 2\,864\ \text{J}$. Since 6 279 J > 2 864 J, the final temperature is less than 26°C. We can write $Q_h = -Q_c$ as

$$Q_{\text{water}} + Q_{\text{Al}} + Q_{\text{Cu}} = 0$$

$$0.25\ \text{kg}\ 4\,186\frac{\text{J}}{\text{kg}\ {}^\circ\text{C}}\left(T_f - 20^\circ\text{C}\right) + 0.4\ \text{kg}\ 900\frac{\text{J}}{\text{kg}\ {}^\circ\text{C}}\left(T_f - 26^\circ\text{C}\right)$$

$$+0.1\ \text{kg}\ 387\frac{\text{J}}{\text{kg}\ {}^\circ\text{C}}\left(T_f - 100^\circ\text{C}\right) = 0$$

$$1\,046.5T_f - 20\,930^\circ\text{C} + 360T_f - 9\,360^\circ\text{C} + 38.7T_f - 3\,870^\circ\text{C} = 0$$

$$1\,445.2T_f = 34\,160^\circ\text{C}$$

$$T_f = \boxed{23.6^\circ\text{C}}$$

**P20.12** $Q_{\text{cold}} = -Q_{\text{hot}}$

$$m_{\text{Al}}c_{\text{Al}}\left(T_f - T_c\right) + m_c c_w\left(T_f - T_c\right) = -m_h c_w\left(T_f - T_h\right)$$

$$\left(m_{\text{Al}}c_{\text{Al}} + m_c c_w\right)T_f - \left(m_{\text{Al}}c_{\text{Al}} + m_c c_w\right)T_c = -m_h c_w T_f + m_h c_w T_h$$

$$\left(m_{\text{Al}}c_{\text{Al}} + m_c c_w + m_h c_w\right)T_f = \left(m_{\text{Al}}c_{\text{Al}} + m_c c_w\right)T_c + m_h c_w T_h$$

$$T_f = \boxed{\frac{\left(m_{\text{Al}}c_{\text{Al}} + m_c c_w\right)T_c + m_h c_w T_h}{m_{\text{Al}}c_{\text{Al}} + m_c c_w + m_h c_w}}$$

**P20.13** The rate of collection of energy is $\mathcal{P} = 550\ \text{W/m}^2\left(6.00\ \text{m}^2\right) = 3\,300\ \text{W}$. The amount of energy required to raise the temperature of 1 000 kg of water by 40.0°C is:

$$Q = mc\Delta T = 1\,000\ \text{kg}(4\,186\ \text{J/kg}\cdot{}^\circ\text{C})(40.0^\circ\text{C}) = 1.67\times10^8\ \text{J}$$

Thus, $\mathcal{P}\Delta t = 1.67\times10^8\ \text{J}$

or $\Delta t = \dfrac{1.67\times10^8\ \text{J}}{3\,300\ \text{W}} = \boxed{50.7\ \text{ks}} = 14.1\ \text{h}$.

***P20.14** Vessel one contains oxygen according to $PV = nRT$:

$$n_c = \frac{PV}{RT} = \frac{1.75\left(1.013\times10^5\ \text{Pa}\right)16.8\times10^{-3}\ \text{m}^3}{8.314\ \text{Nm/mol}\cdot\text{K}\ 300\ \text{K}} = 1.194\ \text{mol}.$$

Vessel two contains this much oxygen:

$$n_h = \frac{2.25\left(1.013\times10^5\right)22.4\times10^{-3}}{8.314(450)}\ \text{mol} = 1.365\ \text{mol}.$$

(a) The gas comes to an equilibrium temperature according to

$$(mc\Delta T)_{\text{cold}} = -(mc\Delta T)_{\text{hot}}$$
$$n_c Mc\left(T_f - 300\ \text{K}\right) + n_h Mc\left(T_f - 450\ \text{K}\right) = 0$$

The molar mass $M$ and specific heat divide out:

$$1.194T_f - 358.2\ \text{K} + 1.365T_f - 614.1\ \text{K} = 0$$
$$T_f = \frac{972.3\ \text{K}}{2.559} = \boxed{380\ \text{K}}$$

(b) The pressure of the whole sample in its final state is

$$P = \frac{nRT}{V} = \frac{2.559\ \text{mol}\ 8.314\ \text{J}\ 380\ \text{K}}{\text{mol K}(22.4+16.8)\times10^{-3}\ \text{m}^3} = \boxed{2.06\times10^5\ \text{Pa}} = 2.04\ \text{atm}.$$

## Section 20.3 **Latent Heat**

**P20.15** The heat needed is the sum of the following terms:

$$Q_{\text{needed}} = (\text{heat to reach melting point}) + (\text{heat to melt})$$
$$+(\text{heat to reach melting point}) + (\text{heat to vaporize}) + (\text{heat to reach } 110^\circ\text{C})$$

Thus, we have

$$Q_{\text{needed}} = 0.040\,0\ \text{kg}\Big[(2\,090\ \text{J/kg}\cdot{}^\circ\text{C})(10.0^\circ\text{C}) + \left(3.33\times10^5\ \text{J/kg}\right)$$
$$+(4\,186\ \text{J/kg}\cdot{}^\circ\text{C})(100^\circ\text{C}) + \left(2.26\times10^6\ \text{J/kg}\right) + (2\,010\ \text{J/kg}\cdot{}^\circ\text{C})(10.0^\circ\text{C})\Big]$$
$$Q_{\text{needed}} = \boxed{1.22\times10^5\ \text{J}}$$

**P20.16** $Q_{\text{cold}} = -Q_{\text{hot}}$

$$(m_w c_w + m_c c_c)\left(T_f - T_i\right) = -m_s\left[-L_v + c_w\left(T_f - 100\right)\right]$$
$$\left[0.250\ \text{kg}(4\,186\ \text{J/kg}\cdot{}^\circ\text{C}) + 0.050\,0\ \text{kg}(387\ \text{J/kg}\cdot{}^\circ\text{C})\right](50.0^\circ\text{C} - 20.0^\circ\text{C})$$
$$= -m_s\left[-2.26\times10^6\ \text{J/kg} + (4\,186\ \text{J/kg}\cdot{}^\circ\text{C})(50.0^\circ\text{C} - 100^\circ\text{C})\right]$$
$$m_s = \frac{3.20\times10^4\ \text{J}}{2.47\times10^6\ \text{J/kg}} = 0.012\,9\ \text{kg} = \boxed{12.9\ \text{g steam}}$$

**P20.17** The bullet will not melt all the ice, so its final temperature is 0°C.

Then $\left(\frac{1}{2}mv^2+mc|\Delta T|\right)_{\text{bullet}}=m_wL_f$

where $m_w$ is the melt water mass

$$m_w=\frac{0.500\left(3.00\times10^{-3}\text{ kg}\right)(240\text{ m/s})^2+3.00\times10^{-3}\text{ kg}(128\text{ J/kg}\cdot{}^\circ\text{C})(30.0^\circ\text{C})}{3.33\times10^5\text{ J/kg}}$$

$$m_w=\frac{86.4\text{ J}+11.5\text{ J}}{333\,000\text{ J/kg}}=\boxed{0.294\text{ g}}$$

**P20.18** (a) $Q_1=$ heat to melt all the ice $=\left(50.0\times10^{-3}\text{ kg}\right)\left(3.33\times10^5\text{ J/kg}\right)=1.67\times10^4\text{ J}$

$Q_2=(\text{heat to raise temp of ice to }100^\circ\text{C})$

$=\left(50.0\times10^{-3}\text{ kg}\right)(4\,186\text{ J/kg}\cdot{}^\circ\text{C})(100^\circ\text{C})=2.09\times10^4\text{ J}$

Thus, the total heat to melt ice and raise temp to 100°C $=3.76\times10^4$ J

$Q_3=\dfrac{\text{heat available}}{\text{as steam condenses}}=\left(10.0\times10^{-3}\text{ kg}\right)\left(2.26\times10^6\text{ J/kg}\right)=2.26\times10^4\text{ J}$

Thus, we see that $Q_3>Q_1$, but $Q_3<Q_1+Q_2$.

Therefore, all the ice melts but $T_f<100^\circ\text{C}$. Let us now find $T_f$

$$Q_{\text{cold}}=-Q_{\text{hot}}$$
$$\left(50.0\times10^{-3}\text{ kg}\right)\left(3.33\times10^5\text{ J/kg}\right)+\left(50.0\times10^{-3}\text{ kg}\right)(4\,186\text{ J/kg}\cdot{}^\circ\text{C})\left(T_f-0^\circ\text{C}\right)$$
$$=-\left(10.0\times10^{-3}\text{ kg}\right)\left(-2.26\times10^6\text{ J/kg}\right)-\left(10.0\times10^{-3}\text{ kg}\right)(4\,186\text{ J/kg}\cdot{}^\circ\text{C})\left(T_f-100^\circ\text{C}\right)$$

From which, $\boxed{T_f=40.4^\circ\text{C}}$.

(b) $Q_1=$ heat to melt all ice $=1.67\times10^4$ J [See part (a)]

$Q_2=\dfrac{\text{heat given up}}{\text{as steam condenses}}=\left(10^{-3}\text{ kg}\right)\left(2.26\times10^6\text{ J/kg}\right)=2.26\times10^3\text{ J}$

$Q_3=\dfrac{\text{heat given up as condensed}}{\text{steam cools to }0^\circ\text{C}}=\left(10^{-3}\text{ kg}\right)(4\,186\text{ J/kg}\cdot{}^\circ\text{C})(100^\circ\text{C})=419\text{ J}$

Note that $Q_2+Q_3<Q_1$. Therefore, the final temperature will be 0°C with some ice remaining. Let us find the mass of ice which must melt to condense the steam and cool the condensate to 0°C.

$$mL_f=Q_2+Q_3=2.68\times10^3\text{ J}$$

Thus, $m=\dfrac{2.68\times10^3\text{ J}}{3.33\times10^5\text{ J/kg}}=8.04\times10^{-3}\text{ kg}=8.04\text{ g}$.

Therefore, there is $\boxed{\text{42.0 g of ice left over}}$.

**P20.19** $Q = m_{\text{Cu}} c_{\text{Cu}} \Delta T = m_{\text{N}_2} \left(L_{\text{vap}}\right)_{\text{N}_2}$

$1.00 \text{ kg}(0.092\,0 \text{ cal/g}\cdot°\text{C})(293 - 77.3)°\text{C} = m(48.0 \text{ cal/g})$

$m = \boxed{0.414 \text{ kg}}$

***P20.20** The original gravitational energy of the hailstone-Earth system changes entirely into additional internal energy in the hailstone, to produce its phase change. No temperature change occurs, either in the hailstone, in the air, or in sidewalk. Then

$$mgy = mL$$

$$y = \frac{L}{g} = \frac{3.33 \times 10^5 \text{ J/kg}}{9.8 \text{ m/s}^2}\left(\frac{1 \text{ kg}\cdot\text{m}^2/\text{s}^2}{1 \text{ J}}\right) = \boxed{3.40 \times 10^4 \text{ m}}$$

**P20.21** (a) Since the heat required to melt 250 g of ice at 0°C *exceeds* the heat required to cool 600 g of water from 18°C to 0°C, the final temperature of the system (water + ice) must be $\boxed{0°\text{C}}$.

(b) Let $m$ represent the mass of ice that melts before the system reaches equilibrium at 0°C.

$$Q_{\text{cold}} = -Q_{\text{hot}}$$

$$mL_f = -m_w c_w (0°\text{C} - T_i)$$

$$m\left(3.33 \times 10^5 \text{ J/kg}\right) = -(0.600 \text{ kg})(4\,186 \text{ J/kg}\cdot°\text{C})(0°\text{C} - 18.0°\text{C})$$

$$m = 136 \text{ g, so the ice remaining} = 250 \text{ g} - 136 \text{ g} = \boxed{114 \text{ g}}$$

**P20.22** The original kinetic energy all becomes thermal energy:

$$\frac{1}{2}mv^2 + \frac{1}{2}mv^2 = 2\left(\frac{1}{2}\right)\left(5.00 \times 10^{-3} \text{ kg}\right)(500 \text{ m/s})^2 = 1.25 \text{ kJ}.$$

Raising the temperature to the melting point requires

$$Q = mc\Delta T = 10.0 \times 10^{-3} \text{ kg}(128 \text{ J/kg}\cdot°\text{C})(327°\text{C} - 20.0°\text{C}) = 393 \text{ J}.$$

Since $1\,250 \text{ J} > 393 \text{ J}$, the lead starts to melt. Melting it all requires

$$Q = mL = \left(10.0 \times 10^{-3} \text{ kg}\right)\left(2.45 \times 10^4 \text{ J/kg}\right) = 245 \text{ J}.$$

Since $1\,250 \text{ J} > 393 + 245 \text{ J}$, it all melts. If we assume $\boxed{\text{liquid lead}}$ has the same specific heat as solid lead, the final temperature is given by

$$1.25 \times 10^3 \text{ J} = 393 \text{ J} + 245 \text{ J} + 10.0 \times 10^{-3} \text{ kg}(128 \text{ J/kg}\cdot°\text{C})\left(T_f - 327°\text{C}\right)$$

$$\boxed{T_f = 805°\text{C}}$$

## Section 20.4 Work and Heat in Thermodynamic Processes

**P20.23** $W_{if} = -\int_i^f P\,dV$

The work done on the gas is the negative of the area under the curve $P = \alpha V^2$ between $V_i$ and $V_f$.

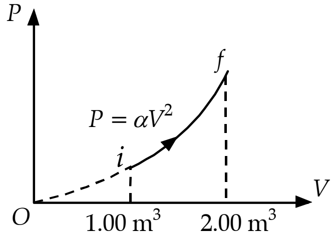

FIG. P20.23

$$W_{if} = -\int_i^f \alpha V^2 dV = -\frac{1}{3}\alpha\left(V_f^3 - V_i^3\right)$$

$$V_f = 2V_i = 2\left(1.00\text{ m}^3\right) = 2.00\text{ m}^3$$

$$W_{if} = -\frac{1}{3}\left[\left(5.00\text{ atm/m}^6\right)\left(1.013\times10^5\text{ Pa/atm}\right)\right]\left[\left(2.00\text{ m}^3\right)^3 + \left(1.00\text{ m}^3\right)^3\right] = \boxed{-1.18\text{ MJ}}$$

**P20.24** (a)

$$W = -\int P\,dV$$

$$W = -\left(6.00\times10^6\text{ Pa}\right)(2.00-1.00)\text{ m}^3 + -\left(4.00\times10^6\text{ Pa}\right)(3.00-2.00)\text{ m}^3 + -\left(2.00\times10^6\text{ Pa}\right)(4.00-3.00)\text{ m}^3$$

$$W_{i\to f} = \boxed{-12.0\text{ MJ}}$$

(b)

$$W_{f\to i} = \boxed{+12.0\text{ MJ}}$$

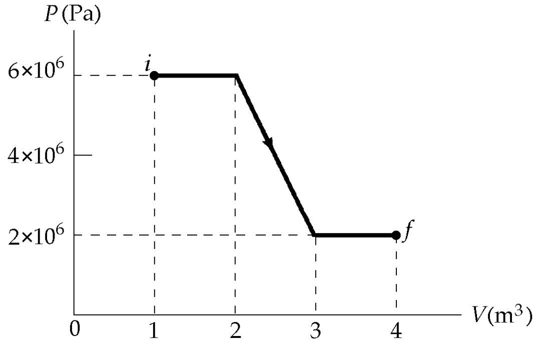

FIG. P20.24

**P20.25** $W = -P\Delta V = -P\left(\frac{nR}{P}\right)\left(T_f - T_i\right) = -nR\Delta T = -(0.200)(8.314)(280) = \boxed{-466\text{ J}}$

**P20.26** $W = -\int_i^f P\,dV = -P\int_i^f dV = -P\Delta V = -nR\Delta T = \boxed{-nR\left(T_2 - T_1\right)}$

**P20.27** During the heating process $P = \left(\frac{P_i}{V_i}\right)V$.

(a)

$$W = -\int_i^f P\,dV = -\int_{V_i}^{3V_i}\left(\frac{P_i}{V_i}\right)V\,dV$$

$$W = -\left(\frac{P_i}{V_i}\right)\frac{V^2}{2}\Bigg|_{V_i}^{3V_i} = -\frac{P_i}{2V_i}\left(9V_i^2 - V_i^2\right) = \boxed{-4P_iV_i}$$

(b)

$$PV = nRT$$

$$\left[\left(\frac{P_i}{V_i}\right)V\right]V = nRT$$

$$\boxed{T = \left(\frac{P_i}{nRV_i}\right)V^2}$$

Temperature must be proportional to the square of volume, rising to nine times its original value.

## Section 20.5 **The First Law of Thermodynamics**

**P20.28** (a) $W = -P\Delta V = -(0.800\text{ atm})(-7.00\text{ L})\left(1.013\times10^{5}\text{ Pa/atm}\right)\left(10^{-3}\text{ m}^3/\text{L}\right) = \boxed{+567\text{ J}}$

(b) $\Delta E_{\text{int}} = Q + W = -400\text{ J} + 567\text{ J} = \boxed{167\text{ J}}$

**P20.29** $\Delta E_{\text{int}} = Q + W$

$Q = \Delta E_{\text{int}} - W = -500\text{ J} - 220\text{ J} = \boxed{-720\text{ J}}$

The negative sign indicates that positive energy is transferred *from* the system by heat.

**P20.30** (a) $Q = -W =$ Area of triangle

$$Q = \frac{1}{2}\left(4.00\text{ m}^3\right)(6.00\text{ kPa}) = \boxed{12.0\text{ kJ}}$$

(b) $Q = -W = \boxed{-12.0\text{ kJ}}$

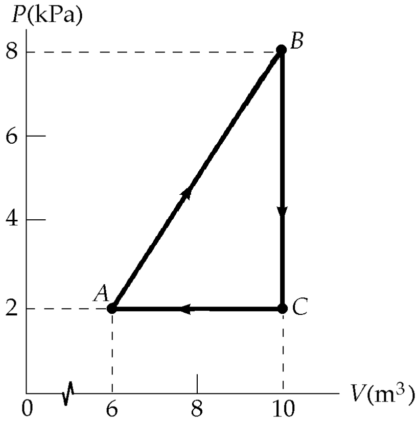

**FIG. P20.30**

**P20.31**

| | $Q$ | $W$ | $\Delta E_{\text{int}}$ | |
|---|---|---|---|---|
| $BC$ | – | 0 | – | $(Q = \Delta E_{\text{int}}\text{ since }W_{BC} = 0)$ |
| $CA$ | – | + | – | $(\Delta E_{\text{int}} < 0\text{ and }W > 0,\text{ so }Q < 0)$ |
| $AB$ | + | – | + | $(W < 0,\ \Delta E_{\text{int}} > 0\text{ since }\Delta E_{\text{int}} < 0\text{ for }B\to C\to A;\text{ so }Q > 0)$ |

**P20.32** $W_{\text{BC}} = -P_{\text{B}}\left(V_{\text{C}} - V_{\text{B}}\right) = -3.00\text{ atm}(0.400 - 0.090\,0)\text{ m}^3$

$= -94.2\text{ kJ}$

$\Delta E_{\text{int}} = Q + W$

$E_{\text{int, C}} - E_{\text{int, B}} = (100 - 94.2)\text{ kJ}$

$E_{\text{int, C}} - E_{\text{int, B}} = 5.79\text{ kJ}$

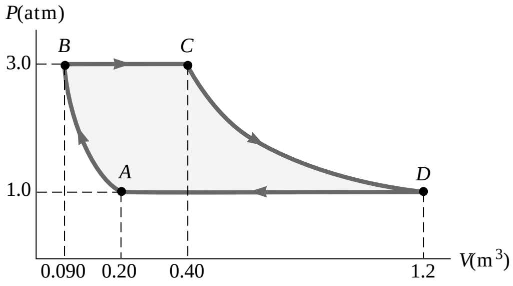

**FIG. P20.32**

Since $T$ is constant,

$E_{\text{int, D}} - E_{\text{int, C}} = 0$

$W_{\text{DA}} = -P_{\text{D}}\left(V_{\text{A}} - V_{\text{D}}\right) = -1.00\text{ atm}(0.200 - 1.20)\text{ m}^3$

$= +101\text{ kJ}$

$E_{\text{int, A}} - E_{\text{int, D}} = -150\text{ kJ} + (+101\text{ kJ}) = -48.7\text{ kJ}$

Now, $E_{\text{int, B}} - E_{\text{int, A}} = -\left[\left(E_{\text{int, C}} - E_{\text{int, B}}\right) + \left(E_{\text{int, D}} - E_{\text{int, C}}\right) + \left(E_{\text{int, A}} - E_{\text{int, D}}\right)\right]$

$E_{\text{int, B}} - E_{\text{int, A}} = -[5.79\text{ kJ} + 0 - 48.7\text{ kJ}] = \boxed{42.9\text{ kJ}}$

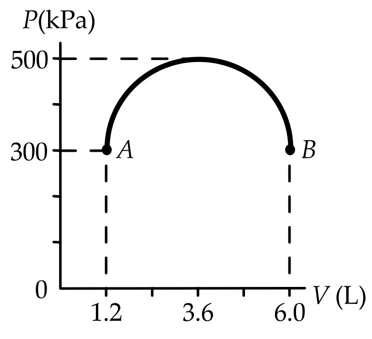

**FIG. P20.33**

***P20.33** The area of a true semicircle is $\frac{1}{2}\pi r^2$. The arrow in Figure P20.33 looks like a semicircle when the scale makes 1.2 L fill the same space as 100 kPa. Its area is

$$\frac{1}{2}\pi(2.4\text{ L})(200\text{ kPa})=\frac{1}{2}\pi\left(2.4\times10^{-3}\text{ m}^3\right)\left(2\times10^{5}\text{ N/m}^2\right).$$

The work on the gas is

$$\begin{aligned}W&=-\int_A^B PdV=-\text{area under the arch shown in the graph}\\&=-\left(\frac{1}{2}\pi 2.4(200)\text{ J}+3\times10^{5}\text{ N/m}^2\ 4.8\times10^{-3}\text{ m}^3\right)\\&=-(754\text{ J}+1\,440\text{ J})=-2\,190\text{ J}\end{aligned}$$

$$\Delta E_{\text{int}}=Q+W=5\,790\text{ J}-2\,190\text{ J}=\boxed{3.60\text{ kJ}}$$

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## Section 20.6 **Some Applications of the First Law of Thermodynamics**

**P20.34** (a) $W=-nRT\ln\left(\frac{V_f}{V_i}\right)=-P_fV_f\ln\left(\frac{V_f}{V_i}\right)$

so $V_i=V_f\exp\left(+\frac{W}{P_fV_f}\right)=(0.025\,0)\exp\left[\frac{-3\,000}{0.025\,0\left(1.013\times10^{5}\right)}\right]=\boxed{0.007\,65\text{ m}^3}$

(b) $T_f=\frac{P_fV_f}{nR}=\frac{1.013\times10^{5}\text{ Pa}\left(0.025\,0\text{ m}^3\right)}{1.00\text{ mol}(8.314\text{ J/K}\cdot\text{mol})}=\boxed{305\text{ K}}$

**P20.35** (a) $\Delta E_{\text{int}}=Q-P\Delta V=12.5\text{ kJ}-2.50\text{ kPa}(3.00-1.00)\text{ m}^3=\boxed{7.50\text{ kJ}}$

(b) $\frac{V_1}{T_1}=\frac{V_2}{T_2}$

$T_2=\frac{V_2}{V_1}T_1=\frac{3.00}{1.00}(300\text{ K})=\boxed{900\text{ K}}$

**P20.36** (a) $W=-P\Delta V=-P[3\alpha V\Delta T]$

$$=-\left(1.013\times10^{5}\text{ N/m}^2\right)\left[3\left(24.0\times10^{-6}\,{}^\circ\text{C}^{-1}\right)\left(\frac{1.00\text{ kg}}{2.70\times10^{3}\text{ kg/m}^3}\right)(18.0^\circ\text{C})\right]$$

$W=\boxed{-48.6\text{ mJ}}$

(b) $Q=cm\Delta T=(900\text{ J/kg}\cdot{}^\circ\text{C})(1.00\text{ kg})(18.0^\circ\text{C})=\boxed{16.2\text{ kJ}}$

(c) $\Delta E_{\text{int}}=Q+W=16.2\text{ kJ}-48.6\text{ mJ}=\boxed{16.2\text{ kJ}}$

**P20.37**
$$W=-P\Delta V=-P\left(V_s-V_w\right)=-\frac{P(nRT)}{P}+P\left[\frac{18.0\text{ g}}{\left(1.00\text{ g/cm}^3\right)\left(10^6\text{ cm}^3/\text{m}^3\right)}\right]$$

$$W=-(1.00\text{ mol})(8.314\text{ J/K}\cdot\text{mol})(373\text{ K})+\left(1.013\times10^5\text{ N/m}^2\right)\left(\frac{18.0\text{ g}}{10^6\text{ g/m}^3}\right)=\boxed{-3.10\text{ kJ}}$$

$$Q=mL_v=0.018\,0\text{ kg}\left(2.26\times10^6\text{ J/kg}\right)=40.7\text{ kJ}$$

$$\Delta E_{\text{int}}=Q+W=\boxed{37.6\text{ kJ}}$$

**P20.38** (a) The work done during each step of the cycle equals the negative of the area under that segment of the *PV* curve.

$$W=W_{DA}+W_{AB}+W_{BC}+W_{CD}$$

$$W=-P_i\left(V_i-3V_i\right)+0-3P_i\left(3V_i-V_i\right)+0=\boxed{-4P_iV_i}$$

(b) The initial and final values of $T$ for the system are equal. Therefore, $\Delta E_{\text{int}}=0$ and $Q=-W=\boxed{4P_iV_i}$.

(c) $W=-4P_iV_i=-4nRT_i=-4(1.00)(8.314)(273)=\boxed{-9.08\text{ kJ}}$

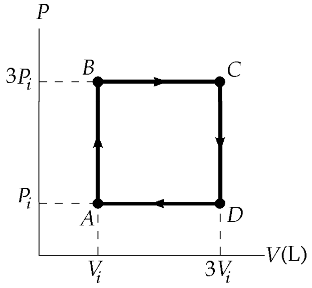

**FIG. P20.38**

**P20.39** (a) $P_iV_i=P_fV_f=nRT=2.00\text{ mol}(8.314\text{ J/K}\cdot\text{mol})(300\text{ K})=4.99\times10^3\text{ J}$

$$V_i=\frac{nRT}{P_i}=\frac{4.99\times10^3\text{ J}}{0.400\text{ atm}}$$

$$V_f=\frac{nRT}{P_f}=\frac{4.99\times10^3\text{ J}}{1.20\text{ atm}}=\frac{1}{3}V_i=\boxed{0.041\,0\text{ m}^3}$$

(b) $W=-\int PdV=-nRT\ln\left(\frac{V_f}{V_i}\right)=-\left(4.99\times10^3\right)\ln\left(\frac{1}{3}\right)=\boxed{+5.48\text{ kJ}}$

(c) $\Delta E_{\text{int}}=0=Q+W$

$Q=\boxed{-5.48\text{ kJ}}$

**P20.40** $\Delta E_{\text{int},\,ABC}=\Delta E_{\text{int},\,AC}$ (conservation of energy)

(a) $\Delta E_{\text{int},\,ABC}=Q_{ABC}+W_{ABC}$ (First Law)

$Q_{ABC}=800\text{ J}+500\text{ J}=\boxed{1\,300\text{ J}}$

(b) $W_{CD}=-P_C\Delta V_{CD}$, $\Delta V_{AB}=-\Delta V_{CD}$, and $P_A=5P_C$

Then, $W_{CD}=\frac{1}{5}P_A\Delta V_{AB}=-\frac{1}{5}W_{AB}=\boxed{100\text{ J}}$

(+ means that work is done on the system)

(c) $W_{CDA}=W_{CD}$ so that $Q_{CA}=\Delta E_{\text{int},\,CA}-W_{CDA}=-800\text{ J}-100\text{ J}=\boxed{-900\text{ J}}$

(– means that energy must be removed from the system by heat)

(d) $\Delta E_{\text{int},\,CD}=\Delta E_{\text{int},\,CDA}-\Delta E_{\text{int},\,DA}=-800\text{ J}-500\text{ J}=-1\,300\text{ J}$

and $Q_{CD}=\Delta E_{\text{int},\,CD}-W_{CD}=-1\,300\text{ J}-100\text{ J}=\boxed{-1\,400\text{ J}}$

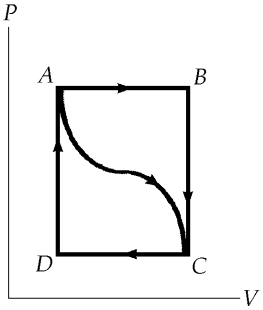

**FIG. P20.40**

## Section 20.7 **Energy Transfer Mechanisms**

**P20.41** $\mathcal{P}=kA\dfrac{\Delta T}{L}$

$$k=\frac{\mathcal{P}L}{A\Delta T}=\frac{10.0\text{ W}(0.040\,0\text{ m})}{1.20\text{ m}^2(15.0^\circ\text{C})}=\boxed{2.22\times 10^{-2}\ \text{W/m}\cdot{}^\circ\text{C}}$$

**P20.42** $$\mathcal{P}=\frac{kA\Delta T}{L}=\frac{(0.800\ \text{W/m}\cdot{}^\circ\text{C})\left(3.00\text{ m}^2\right)(25.0^\circ\text{C})}{6.00\times 10^{-3}\ \text{m}}=1.00\times 10^4\ \text{W}=\boxed{10.0\text{ kW}}$$

**P20.43** In the steady state condition, $\mathcal{P}_{\text{Au}}=\mathcal{P}_{\text{Ag}}$

so that $k_{\text{Au}}A_{\text{Au}}\left(\dfrac{\Delta T}{\Delta x}\right)_{\text{Au}}=k_{\text{Ag}}A_{\text{Ag}}\left(\dfrac{\Delta T}{\Delta x}\right)_{\text{Ag}}$

In this case $A_{\text{Au}}=A_{\text{Ag}}$

$\Delta x_{\text{Au}}=\Delta x_{\text{Ag}}$

$\Delta T_{\text{Au}}=(80.0-T)$

and $\Delta T_{\text{Ag}}=(T-30.0)$

where $T$ is the temperature of the junction.

Therefore, $k_{\text{Au}}(80.0-T)=k_{\text{Ag}}(T-30.0)$

And $\boxed{T=51.2^\circ\text{C}}$

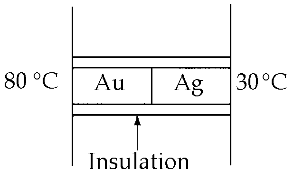

**FIG. P20.43**

**P20.44** $$\mathcal{P}=\frac{A\Delta T}{\sum_i \frac{L_i}{k_i}}=\frac{\left(6.00\text{ m}^2\right)(50.0^\circ\text{C})}{\left[2\left(4.00\times 10^{-3}\ \text{m}\right)\right]/\left[0.800\ \text{W/m}\cdot{}^\circ\text{C}\right]+\left[5.00\times 10^{-3}\ \text{m}\right]/\left[0.023\,4\ \text{W/m}\cdot{}^\circ\text{C}\right]}=\boxed{1.34\text{ kW}}$$

***P20.45** We suppose that the area of the transistor is so small that energy flow by heat from the transistor directly to the air is negligible compared to energy conduction through the mica.

$$\mathcal{P}=kA\frac{(T_h-T_c)}{L}$$

$$T_h=T_c+\frac{\mathcal{P}L}{kA}=35.0^\circ\text{C}+\frac{1.50\text{ W}\left(0.085\,2\times 10^{-3}\ \text{m}\right)}{(0.075\,3\ \text{W/m}\cdot{}^\circ\text{C})(8.25\times 6.25)10^{-6}\ \text{m}^2}=\boxed{67.9^\circ\text{C}}$$

**P20.46** From Table 20.4,

(a) $R=\boxed{0.890\text{ ft}^2\cdot{}^\circ\text{F}\cdot\text{h/Btu}}$

(b) The insulating glass in the table must have sheets of glass less than $\frac{1}{8}$ inch thick. So we estimate the $R$-value of a 0.250-inch air space as $\frac{0.250}{3.50}$ times that of the thicker air space. Then for the double glazing

$$R_b=\left[0.890+\left(\frac{0.250}{3.50}\right)1.01+0.890\right]\frac{\text{ft}^2\cdot{}^\circ\text{F}\cdot\text{h}}{\text{Btu}}=\boxed{1.85\frac{\text{ft}^2\cdot{}^\circ\text{F}\cdot\text{h}}{\text{Btu}}}.$$

(c) Since $A$ and $(T_2-T_1)$ are constants, heat flow is reduced by a factor of $\dfrac{1.85}{0.890}=\boxed{2.08}$.

**P20.47** $\mathcal{P}=\sigma AeT^4=\left(5.669\,6\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)\left[4\pi\left(6.96\times10^8\ \text{m}\right)^2\right](0.965)(5\,800\ \text{K})^4$

$\mathcal{P}=\boxed{3.77\times10^{26}\ \text{W}}$

**P20.48** Suppose the pizza is 70 cm in diameter and $\ell=2.0$ cm thick, sizzling at 100°C. It cannot lose heat by conduction or convection. It radiates according to $\mathcal{P}=\sigma AeT^4$. Here, $A$ is its surface area,

$$A=2\pi r^2+2\pi r\ell=2\pi(0.35\ \text{m})^2+2\pi(0.35\ \text{m})(0.02\ \text{m})=0.81\ \text{m}^2.$$

Suppose it is dark in the infrared, with emissivity about 0.8. Then

$$\mathcal{P}=\left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)\left(0.81\ \text{m}^2\right)(0.80)(373\ \text{K})^4=710\ \text{W}\boxed{\sim10^3\ \text{W}}.$$

If the density of the pizza is half that of water, its mass is

$$m=\rho V=\rho\pi r^2\ell=\left(500\ \text{kg/m}^3\right)\pi(0.35\ \text{m})^2(0.02\ \text{m})=4\ \text{kg}.$$

Suppose its specific heat is $c=0.6\ \text{cal/g}\cdot°\text{C}$. The drop in temperature of the pizza is described by:

$$Q=mc\left(T_f-T_i\right)$$
$$\mathcal{P}=\frac{dQ}{dt}=mc\frac{dT_f}{dt}-0$$
$$\frac{dT_f}{dt}=\frac{\mathcal{P}}{mc}=\frac{710\ \text{J/s}}{(4\ \text{kg})(0.6\cdot4\,186\ \text{J/kg}\cdot°\text{C})}=0.07\ °\text{C/s}\boxed{\sim10^{-1}\ \text{K/s}}$$

**P20.49** $\mathcal{P}=\sigma AeT^4$

$2.00\ \text{W}=\left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)\left(0.250\times10^{-6}\ \text{m}^2\right)(0.950)T^4$

$T=\left(1.49\times10^{14}\ \text{K}^4\right)^{1/4}=\boxed{3.49\times10^3\ \text{K}}$

**P20.50** We suppose the earth below is an insulator. The square meter must radiate in the infrared as much energy as it absorbs, $\mathcal{P}=\sigma AeT^4$. Assuming that $e=1.00$ for blackbody blacktop:

$1\,000\ \text{W}=\left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)\left(1.00\ \text{m}^2\right)(1.00)T^4$

$T=\left(1.76\times10^{10}\ \text{K}^4\right)^{1/4}=\boxed{364\ \text{K}}$ (You can cook an egg on it.)

**P20.51** The sphere of radius $R$ absorbs sunlight over the area of its day hemisphere, projected as a flat circle perpendicular to the light: $\pi R^2$. It radiates in all directions, over area $4\pi R^2$. Then, in steady state,

$$\mathcal{P}_{\text{in}}=\mathcal{P}_{\text{out}}$$
$$e\left(1\,340\ \text{W/m}^2\right)\pi R^2=e\sigma\left(4\pi R^2\right)T^4$$

The emissivity $e$, the radius $R$, and $\pi$ all cancel.

Therefore, $T=\left[\dfrac{1\,340\ \text{W/m}^2}{4\left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)}\right]^{1/4}=\boxed{277\ \text{K}}=4°\text{C}$.

## Additional Problems

**P20.52** 77.3 K = –195.8°C is the boiling point of nitrogen. It gains no heat to warm as a liquid, but gains heat to vaporize:

$$Q = mL_v = (0.100\text{ kg})\left(2.01\times10^5\text{ J/kg}\right) = 2.01\times10^4\text{ J}.$$

The water first loses heat by cooling. Before it starts to freeze, it can lose

$$Q = mc\Delta T = (0.200\text{ kg})(4\,186\text{ J/kg}\cdot°\text{C})(5.00°\text{C}) = 4.19\times10^3\text{ J}.$$

The remaining $\left(2.01\times10^4 - 4.19\times10^3\right)\text{ J} = 1.59\times10^4\text{ J}$ that is removed from the water can freeze a mass $x$ of water:

$$Q = mL_f$$
$$1.59\times10^4\text{ J} = x\left(3.33\times10^5\text{ J/kg}\right)$$
$$x = 0.047\,7\text{ kg} = \boxed{47.7\text{ g}}\text{ of water can be frozen}$$

**P20.53** The increase in internal energy required to melt 1.00 kg of snow is

$$\Delta E_{\text{int}} = (1.00\text{ kg})\left(3.33\times10^5\text{ J/kg}\right) = 3.33\times10^5\text{ J}$$

The force of friction is $f = \mu n = \mu mg = 0.200(75.0\text{ kg})\left(9.80\text{ m/s}^2\right) = 147\text{ N}$

According to the problem statement, the loss of mechanical energy of the skier is assumed to be equal to the increase in internal energy of the snow. This increase in internal energy is

$$\Delta E_{\text{int}} = f\Delta r = (147\text{ N})\Delta r = 3.33\times10^5\text{ J}$$

and $\Delta r = \boxed{2.27\times10^3\text{ m}}$.

**P20.54** (a) The energy thus far gained by the copper equals the energy loss by the silver. Your down parka is an excellent insulator.

$$Q_{\text{cold}} = -Q_{\text{hot}}$$

or

$$m_{\text{Cu}}c_{\text{Cu}}\left(T_f - T_i\right)_{\text{Cu}} = -m_{\text{Ag}}c_{\text{Ag}}\left(T_f - T_i\right)_{\text{Ag}}$$

$$(9.00\text{ g})(387\text{ J/kg}\cdot°\text{C})(16.0°\text{C}) = -(14.0\text{ g})(234\text{ J/kg}\cdot°\text{C})\left(T_f - 30.0°\text{C}\right)_{\text{Ag}}$$

$$\left(T_f - 30.0°\text{C}\right)_{\text{Ag}} = -17.0°\text{C}$$

so $T_{f,\text{Ag}} = \boxed{13.0°\text{C}}$.

(b) Differentiating the energy gain-and-loss equation gives: $m_{\text{Ag}}c_{\text{Ag}}\left(\dfrac{dT}{dt}\right)_{\text{Ag}} = -m_{\text{Cu}}c_{\text{Cu}}\left(\dfrac{dT}{dt}\right)_{\text{Cu}}$

$$\left(\frac{dT}{dt}\right)_{\text{Ag}} = -\frac{m_{\text{Cu}}c_{\text{Cu}}}{m_{\text{Ag}}c_{\text{Ag}}}\left(\frac{dT}{dt}\right)_{\text{Cu}} = -\frac{9.00\text{ g}(387\text{ J/kg}\cdot°\text{C})}{14.0\text{ g}(234\text{ J/kg}\cdot°\text{C})}(+0.500\,°\text{C/s})$$

$$\left(\frac{dT}{dt}\right)_{\text{Ag}} = \boxed{-0.532\,°\text{C/s}}\ (\text{negative sign} \Rightarrow \text{decreasing temperature})$$

**P20.55** (a) Before conduction has time to become important, the energy lost by the rod equals the energy gained by the helium. Therefore,

$$(mL_v)_{\text{He}} = (mc|\Delta T|)_{\text{Al}}$$

or $$(\rho V L_v)_{\text{He}} = (\rho V c|\Delta T|)_{\text{Al}}$$

so $$V_{\text{He}} = \frac{(\rho V c|\Delta T|)_{\text{Al}}}{(\rho L_v)_{\text{He}}}$$

$$V_{\text{He}} = \frac{(2.70\ \text{g/cm}^3)(62.5\ \text{cm}^3)(0.210\ \text{cal/g}\cdot{}^\circ\text{C})(295.8^\circ\text{C})}{(0.125\ \text{g/cm}^3)(2.09\times10^4\ \text{J/kg})(1.00\ \text{cal}/4.186\ \text{J})(1.00\ \text{kg}/1\,000\ \text{g})}$$

$$V_{\text{He}} = 1.68\times10^4\ \text{cm}^3 = \boxed{16.8\ \text{liters}}$$

(b) The rate at which energy is supplied to the rod in order to maintain constant temperatures is given by

$$\mathcal{P} = kA\left(\frac{dT}{dx}\right) = (31.0\ \text{J/s}\cdot\text{cm}\cdot\text{K})(2.50\ \text{cm}^2)\left(\frac{295.8\ \text{K}}{25.0\ \text{cm}}\right) = 917\ \text{W}$$

This power supplied to the helium will produce a "boil-off" rate of

$$\frac{\mathcal{P}}{\rho L_v} = \frac{(917\ \text{W})(10^3\ \text{g/kg})}{(0.125\ \text{g/cm}^3)(2.09\times10^4\ \text{J/kg})} = 351\ \text{cm}^3/\text{s} = \boxed{0.351\ \text{L/s}}$$

***P20.56** At the equilibrium temperature $T_{eq}$ the diameters of the sphere and ring are equal:

$$d_s + d_s\alpha_{\text{Al}}(T_{\text{eq}} - T_i) = d_r + d_r\alpha_{\text{Cu}}(T_{\text{eq}} - 15^\circ\text{C})$$

$$5.01\ \text{cm} + 5.01\ \text{cm}(2.40\times10^{-5}\ 1/^\circ\text{C})(T_{\text{eq}} - T_i) = 5.00\ \text{cm} + 5.00\ \text{cm}(1.70\times10^{-5}\ 1/^\circ\text{C})(T_{\text{eq}} - 15^\circ\text{C})$$

$$0.01^\circ\text{C} + 1.202\,4\times10^{-4}T_{\text{eq}} - 1.202\,4\times10^{-4}T_i = 8.5\times10^{-5}T_{\text{eq}} - 1.275\times10^{-3}\,^\circ\text{C}$$

$$1.127\,5\times10^{-2}\,^\circ\text{C} + 3.524\times10^{-5}T_{\text{eq}} = 1.202\,4\times10^{-4}T_i$$

$$319.95^\circ\text{C} + T_{\text{eq}} = 3.412\,0T_i$$

At the equilibrium temperature, the energy lost is equal to the energy gained:

$$m_s c_{\text{Al}}(T_{\text{eq}} - T_i) = -m_r c_{\text{Cu}}(T_{\text{eq}} - 15^\circ\text{C})$$

$$10.9\ \text{g}\ 0.215\ \text{cal/g}\cdot{}^\circ\text{C}(T_{\text{eq}} - T_i) = -25\ \text{g}\ 0.092\,4\ \text{cal/g}\cdot{}^\circ\text{C}(T_{\text{eq}} - 15^\circ\text{C})$$

$$2.343\,5T_{\text{eq}} - 2.343\,5T_i = 34.65^\circ\text{C} - 2.31T_{\text{eq}}$$

$$4.653\,5T_{\text{eq}} = 34.65^\circ\text{C} + 2.343\,5T_i$$

Solving by substitution,

$$4.653\,5(3.412\,0T_i - 319.95^\circ\text{C}) = 34.65^\circ\text{C} + 2.343\,5T_i$$

$$15.877\,7T_i - 1\,488.89^\circ\text{C} = 34.65^\circ\text{C} + 2.343\,5T_i$$

(b) $$T_i = \frac{1\,523.54^\circ\text{C}}{13.534} = \boxed{113^\circ\text{C}}$$

(a) $$T_{\text{eq}} = -319.95 + 3.412\,0(112.57) = \boxed{64.1^\circ\text{C}}$$

**P20.57** $Q = mc\Delta T = (\rho V)c\Delta T$ so that when a constant temperature difference $\Delta T$ is maintained,

the rate of adding energy to the liquid is $\mathscr{P} = \frac{dQ}{dt} = \rho\left(\frac{dV}{dt}\right)c\Delta T = \rho Rc\Delta T$

and the specific heat of the liquid is $c = \boxed{\frac{\mathscr{P}}{\rho R\Delta T}}$.

**P20.58** (a) Work done by the gas is the negative of the area under the *PV* curve

$$W = -P_i\left(\frac{V_i}{2} - V_i\right) = \boxed{+\frac{P_iV_i}{2}}.$$

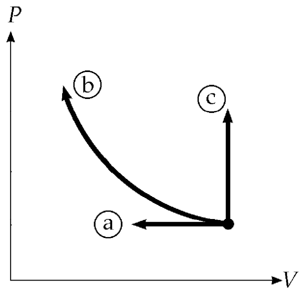

**FIG. P20.58**

(b) In this case the area under the curve is $W = -\int P\,dV$. Since the process is isothermal,

$$PV = P_iV_i = 4P_i\left(\frac{V_i}{4}\right) = nRT_i$$

and $$W = -\int_{V_i}^{V_i/4}\left(\frac{dV}{V}\right)(P_iV_i) = -P_iV_i\ln\left(\frac{V_i/4}{V_i}\right) = P_iV_i\ln 4$$

$$= \boxed{+1.39P_iV_i}$$

(c) The area under the curve is 0 and $\boxed{W = 0}$.

**P20.59** Call the initial pressure $P_1$. In the constant volume process $1 \to 2$ the work is zero.

$$P_1V_1 = nRT_1$$
$$P_2V_2 = nRT_2$$

so $$\frac{P_2V_2}{P_1V_1} = \frac{T_2}{T_1};\ T_2 = 300\text{ K}\left(\frac{1}{4}\right)(1) = 75.0\text{ K}$$

Now in $2 \to 3$

$$W = -\int_2^3 P\,dV = -P_2(V_3 - V_2) = -P_3V_3 + P_2V_2$$
$$W = -nRT_3 + nRT_2 = -(1.00\text{ mol})(8.314\text{ J/mol}\cdot\text{K})(300\text{ K} - 75.0\text{ K})$$
$$W = \boxed{-1.87\text{ kJ}}$$

***P20.60** The initial moment of inertia of the disk is

$$\frac{1}{2}MR^2=\frac{1}{2}\rho VR^2=\frac{1}{2}\rho\pi R^2tR^2=\frac{1}{2}\left(8\,920\ \text{kg/m}^3\right)\pi(28\ \text{m})^4 1.2\ \text{m}=1.033\times10^{10}\ \text{kg}\cdot\text{m}^2$$

The rotation speeds up as the disk cools off, according to

$$I_i\omega_i=I_f\omega_f$$

$$\frac{1}{2}MR_i^2\omega_i=\frac{1}{2}MR_f^2\omega_f=\frac{1}{2}MR_i^2\left(1-\alpha|\Delta T|\right)^2\omega_f$$

$$\omega_f=\omega_i\frac{1}{\left(1-\alpha|\Delta T|\right)^2}=25\ \text{rad/s}\frac{1}{\left[1-\left(17\times10^{-6}\ 1/^\circ\text{C}\right)830^\circ\text{C}\right]^2}=25.720\,7\ \text{rad/s}$$

(a) The kinetic energy increases by

$$\frac{1}{2}I_f\omega_f^2-\frac{1}{2}I_i\omega_i^2=\frac{1}{2}I_i\omega_i\omega_f-\frac{1}{2}I_i\omega_i^2=\frac{1}{2}I_i\omega_i\left(\omega_f-\omega_i\right)$$

$$=\frac{1}{2}1.033\times10^{10}\ \text{kg}\cdot\text{m}^2(25\ \text{rad/s})0.720\,7\ \text{rad/s}=\boxed{9.31\times10^{10}\ \text{J}}$$

(b) $\Delta E_{\text{int}}=mc\Delta T=2.64\times10^7\ \text{kg}\left(387\ \text{J/kg}\cdot^\circ\text{C}\right)(20^\circ\text{C}-850^\circ\text{C})=\boxed{-8.47\times10^{12}\ \text{J}}$

(c) As $8.47\times10^{12}$ J leaves the fund of internal energy, $9.31\times10^{10}$ J changes into extra kinetic energy, and the rest, $\boxed{8.38\times10^{12}\ \text{J}}$ is radiated.

***P20.61** The loss of mechanical energy is

$$\frac{1}{2}mv_i^2+\frac{GM_Em}{R_E}=\frac{1}{2}670\ \text{kg}\left(1.4\times10^4\ \text{m/s}\right)^2+\frac{6.67\times10^{-11}\ \text{Nm}^2\ 5.98\times10^{24}\ \text{kg}\ 670\ \text{kg}}{\text{kg}^2\ 6.37\times10^6\ \text{m}}$$

$$=6.57\times10^{10}\ \text{J}+4.20\times10^{10}\ \text{J}=1.08\times10^{11}\ \text{J}$$

One half becomes extra internal energy in the aluminum: $\Delta E_{\text{int}}=5.38\times10^{10}\ \text{J}$. To raise its temperature to the melting point requires energy

$$mc\Delta T=670\ \text{kg}\ 900\frac{\text{J}}{\text{kg}^\circ\text{C}}\left(660-(-15^\circ\text{C})\right)=4.07\times10^8\ \text{J}.$$

To melt it, $mL=670\ \text{kg}\ 3.97\times10^5\ \text{J/kg}=2.66\times10^8\ \text{J}$. To raise it to the boiling point, $mc\Delta T=670(1\,170)(2\,450-600)\text{J}=1.40\times10^9\ \text{J}$. To boil it, $mL=670\ \text{kg}\ 1.14\times10^7\ \text{J/kg}=7.64\times10^9\ \text{J}$. Then

$$5.38\times10^{10}\ \text{J}=9.71\times10^9\ \text{J}+670(1\,170)\left(T_f-2\,450^\circ\text{C}\right)\text{J}/^\circ\text{C}$$

$$T_f=\boxed{5.87\times10^4\ ^\circ\text{C}}$$

**P20.62** (a) $Fv = (50.0\text{ N})(40.0\text{ m/s}) = \boxed{2\,000\text{ W}}$

(b) Energy received by each object is $(1\,000\text{ W})(10\text{ s}) = 10^4\text{ J} = 2\,389\text{ cal}$. The specific heat of iron is $0.107\text{ cal/g}\cdot{}^\circ\text{C}$, so the heat capacity of each object is $5.00\times10^3\times0.107 = 535.0\text{ cal/}^\circ\text{C}$.

$$\Delta T = \frac{2\,389\text{ cal}}{535.0\text{ cal/}^\circ\text{C}} = \boxed{4.47^\circ\text{C}}$$

**P20.63** The power incident on the solar collector is

$$\mathcal{P}_i = IA = \left(600\text{ W/m}^2\right)\left[\pi(0.300\text{ m})^2\right] = 170\text{ W}.$$

For a 40.0% reflector, the collected power is $\mathcal{P}_c = 67.9\text{ W}$. The total energy required to increase the temperature of the water to the boiling point and to evaporate it is $Q = cm\Delta T + mL_V$:

$$Q = 0.500\text{ kg}\left[(4\,186\text{ J/kg}\cdot{}^\circ\text{C})(80.0^\circ\text{C}) + 2.26\times10^6\text{ J/kg}\right] = 1.30\times10^6\text{ J}.$$

The time interval required is $\Delta t = \frac{Q}{\mathcal{P}_c} = \frac{1.30\times10^6\text{ J}}{67.9\text{ W}} = \boxed{5.31\text{ h}}$.

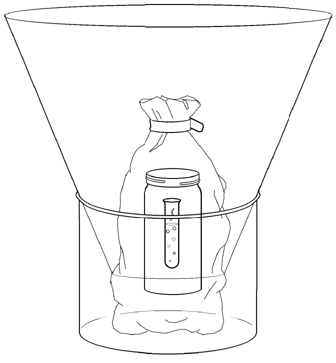

FIG. P20.63

**P20.64** From $Q = mL_V$ the rate of boiling is described by

$$\mathcal{P} = \frac{Q}{\Delta t} = \frac{L_V m}{\Delta t} \qquad \therefore \frac{m}{\Delta t} = \frac{\mathcal{P}}{L_V}$$

Model the water vapor as an ideal gas

$$P_0V_0 = nRT = \left(\frac{m}{M}\right)RT$$

$$\frac{P_0V}{\Delta t} = \frac{m}{\Delta t}\left(\frac{RT}{M}\right)$$

$$P_0Av = \frac{\mathcal{P}}{L_V}\left(\frac{RT}{M}\right)$$

$$v = \frac{\mathcal{P}RT}{ML_VP_0A} = \frac{1\,000\text{ W}(8.314\text{ J/mol}\cdot\text{K})(373\text{ K})}{(0.018\,0\text{ kg/mol})\left(2.26\times10^6\text{ J/kg}\right)\left(1.013\times10^5\text{ N/m}^2\right)\left(2.00\times10^{-4}\text{ m}^2\right)}$$

$$v = \boxed{3.76\text{ m/s}}$$

**P20.65** Energy goes in at a constant rate $\mathcal{P}$. For the period from

$$50.0\text{ min to }60.0\text{ min, } Q = mc\Delta T$$

$$\mathcal{P}(10.0\text{ min}) = (10\text{ kg} + m_i)(4\,186\text{ J/kg}\cdot°\text{C})(2.00°\text{C} - 0°\text{C})$$

$$\mathcal{P}(10.0\text{ min}) = 83.7\text{ kJ} + (8.37\text{ kJ/kg})m_i \qquad (1)$$

For the period from 0 to 50.0 min, $Q = m_i L_f$

$$\mathcal{P}(50.0\text{ min}) = m_i\left(3.33\times 10^5\text{ J/kg}\right)$$

Substitute $\mathcal{P} = \dfrac{m_i\left(3.33\times 10^5\text{ J/kg}\right)}{50.0\text{ min}}$ into Equation (1) to find

$$\frac{m_i\left(3.33\times 10^5\text{ J/kg}\right)}{5.00} = 83.7\text{ kJ} + (8.37\text{ kJ/kg})m_i$$

$$m_i = \frac{83.7\text{ kJ}}{(66.6 - 8.37)\text{ kJ/kg}} = \boxed{1.44\text{ kg}}$$

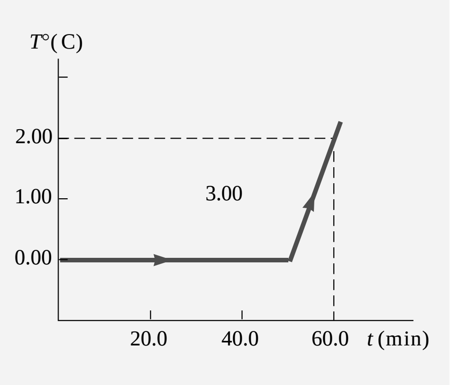

**FIG. P20.65**

**P20.66** (a) The block starts with $K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(1.60\text{ kg})(2.50\text{ m/s})^2 = 5.00\text{ J}$

All this becomes extra internal energy in ice, melting some according to $\text{“}Q\text{”} = m_{\text{ice}}L_f$. Thus, the mass of ice that melts is

$$m_{\text{ice}} = \frac{\text{“}Q\text{”}}{L_f} = \frac{K_i}{L_f} = \frac{5.00\text{ J}}{3.33\times 10^5\text{ J/kg}} = 1.50\times 10^{-5}\text{ kg} = \boxed{15.0\text{ mg}}.$$

For the block: $Q = 0$ (no energy flows by heat since there is no temperature difference)

$W = -5.00\text{ J}$

$\Delta E_{\text{int}} = 0$ (no temperature change)

and $\Delta K = -5.00\text{ J}$

For the ice, $Q = 0$

$W = +5.00\text{ J}$

$\Delta E_{\text{int}} = +5.00\text{ J}$

and $\Delta K = 0$

(b) Again, $K_i = 5.00\text{ J}$ and $m_{\text{ice}} = \boxed{15.0\text{ mg}}$

For the block of ice: $Q = 0$; $\Delta E_{\text{int}} = +5.00\text{ J}$; $\Delta K = -5.00\text{ J}$

so $W = 0$.

For the copper, nothing happens: $Q = \Delta E_{\text{int}} = \Delta K = W = 0$.

*continued on next page*

(c) Again, $K_i = 5.00\text{ J}$. Both blocks must rise equally in temperature.

$$\text{“}Q\text{”} = mc\Delta T: \qquad \Delta T = \frac{\text{“}Q\text{”}}{mc} = \frac{5.00\text{ J}}{2(1.60\text{ kg})(387\ \text{J/kg}\cdot\text{°C})} = \boxed{4.04\times10^{-3}\,\text{°C}}$$

At any instant, the two blocks are at the same temperature, so for both $Q = 0$.

For the moving block: $\Delta K = -5.00\text{ J}$

and $\Delta E_{\text{int}} = +2.50\text{ J}$

so $W = -2.50\text{ J}$

For the stationary block: $\Delta K = 0$

and $\Delta E_{\text{int}} = +2.50\text{ J}$

so $W = +2.50\text{ J}$

For each object in each situation, the general continuity equation for energy, in the form $\Delta K + \Delta E_{\text{int}} = W + Q$, correctly describes the relationship between energy transfers and changes in the object's energy content.

**P20.67** $A = A_{\text{end walls}} + A_{\text{ends of attic}} + A_{\text{side walls}} + A_{\text{roof}}$

$$A = 2(8.00\text{ m}\times5.00\text{ m}) + 2\left[2\times\frac{1}{2}\times4.00\text{ m}\times(4.00\text{ m})\tan37.0°\right]$$

$$+2(10.0\text{ m}\times5.00\text{ m}) + 2(10.0\text{ m})\left(\frac{4.00\text{ m}}{\cos37.0°}\right)$$

$$A = 304\text{ m}^2$$

$$\mathcal{P} = \frac{kA\Delta T}{L} = \frac{(4.80\times10^{-4}\ \text{kW/m}\cdot\text{°C})(304\text{ m}^2)(25.0\text{°C})}{0.210\text{ m}} = 17.4\text{ kW} = 4.15\ \text{kcal/s}$$

Thus, the energy lost per day by heat is $(4.15\ \text{kcal/s})(86\,400\text{ s}) = 3.59\times10^5\ \text{kcal/day}$.

The gas needed to replace this loss is $\dfrac{3.59\times10^5\ \text{kcal/day}}{9\,300\ \text{kcal/m}^3} = \boxed{38.6\ \text{m}^3\text{/day}}$.

**P20.68** $$\frac{L\rho A dx}{dt} = kA\left(\frac{\Delta T}{x}\right)$$

$$L\rho\int_{4.00}^{8.00} x\,dx = k\Delta T\int_0^{\Delta t} dt$$

$$L\rho\left.\frac{x^2}{2}\right|_{4.00}^{8.00} = k\Delta T\Delta t$$

$$(3.33\times10^5\ \text{J/kg})(917\ \text{kg/m}^3)\left(\frac{(0.080\,0\text{ m})^2 - (0.040\,0\text{ m})^2}{2}\right) = (2.00\ \text{W/m}\cdot\text{°C})(10.0\text{°C})\Delta t$$

$$\Delta t = 3.66\times10^4\ \text{s} = \boxed{10.2\text{ h}}$$

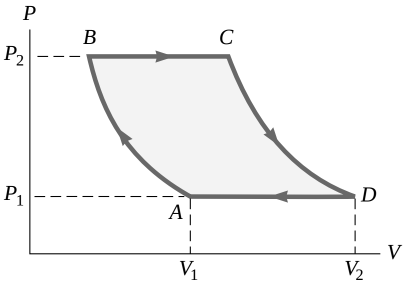

**FIG. P20.69**

**P20.69** $W = W_{AB} + W_{BC} + W_{CD} + W_{DA}$

$$W = -\int_A^B P\,dV - \int_B^C P\,dV - \int_C^D P\,dV - \int_D^A P\,dV$$

$$W = -nRT_1 \int_A^B \frac{dV}{V} - P_2 \int_B^C dV - nRT_2 \int_C^D \frac{dV}{V} - P_1 \int_D^A dV$$

$$W = -nRT_1 \ln\left(\frac{V_B}{V_1}\right) - P_2(V_C - V_B) - nRT_2 \ln\left(\frac{V_2}{V_C}\right) - P_1(V_A - V_D)$$

Now $P_1V_A = P_2V_B$ and $P_2V_C = P_1V_D$, so only the logarithmic terms do not cancel out.

Also, $\frac{V_B}{V_1} = \frac{P_1}{P_2}$ and $\frac{V_2}{V_C} = \frac{P_2}{P_1}$

$$\sum W = -nRT_1 \ln\left(\frac{P_1}{P_2}\right) - nRT_2 \ln\left(\frac{P_2}{P_1}\right) = +nRT_1 \ln\left(\frac{P_2}{P_1}\right) - nRT_2 \ln\left(\frac{P_2}{P_1}\right) = -nR(T_2 - T_1)\ln\left(\frac{P_2}{P_1}\right)$$

Moreover $P_1V_2 = nRT_2$ and $P_1V_1 = nRT_1$

$$\sum W = \boxed{-P_1(V_2 - V_1)\ln\left(\frac{P_2}{P_1}\right)}$$

**P20.70** For a cylindrical shell of radius *r*, height *L*, and thickness *dr*, the equation for thermal conduction,

$$\frac{dQ}{dt} = -kA\frac{dT}{dx} \qquad \text{becomes} \qquad \frac{dQ}{dt} = -k(2\pi rL)\frac{dT}{dr}$$

Under equilibrium conditions, $\frac{dQ}{dt}$ is constant; therefore,

$$dT = -\frac{dQ}{dt}\left(\frac{1}{2\pi kL}\right)\left(\frac{dr}{r}\right) \qquad \text{and} \qquad T_b - T_a = -\frac{dQ}{dt}\left(\frac{1}{2\pi kL}\right)\ln\left(\frac{b}{a}\right)$$

But $T_a > T_b$, so $\boxed{\frac{dQ}{dt} = \frac{2\pi kL(T_a - T_b)}{\ln(b/a)}}$

**P20.71** From problem 70, the rate of energy flow through the wall is

$$\frac{dQ}{dt} = \frac{2\pi kL(T_a - T_b)}{\ln(b/a)}$$

$$\frac{dQ}{dt} = \frac{2\pi\left(4.00\times10^{-5}\ \text{cal/s}\cdot\text{cm}\cdot{}^\circ\text{C}\right)(3\,500\ \text{cm})(60.0^\circ\text{C})}{\ln(256\ \text{cm}/250\ \text{cm})}$$

$$\frac{dQ}{dt} = 2.23\times10^3\ \text{cal/s} = \boxed{9.32\ \text{kW}}$$

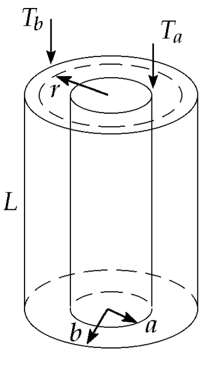

**FIG. P20.71**

This is the rate of energy loss from the plane by heat, and consequently is the rate at which energy must be supplied in order to maintain a constant temperature.

**P20.72** $Q_{\text{cold}} = -Q_{\text{hot}}$

or $Q_{\text{Al}} = -\left(Q_{\text{water}} + Q_{\text{calo}}\right)$

$$m_{\text{Al}}c_{\text{Al}}\left(T_f - T_i\right)_{\text{Al}} = -\left(m_w c_w + m_c c_c\right)\left(T_f - T_i\right)_w$$

$$(0.200\text{ kg})c_{\text{Al}}(+39.3°\text{C}) = -\left[0.400\text{ kg}(4\,186\text{ J/kg}\cdot°\text{C}) + 0.040\,0\text{ kg}(630\text{ J/kg}\cdot°\text{C})\right](-3.70°\text{C})$$

$$c_{\text{Al}} = \frac{6.29\times10^3\text{ J}}{7.86\text{ kg}\cdot°\text{C}} = \boxed{800\text{ J/kg}\cdot°\text{C}}$$

***P20.73** (a) $\mathscr{P} = \sigma AeT^4 = \left(5.67\times10^{-8}\text{ W/m}^2\text{K}^4\right)5.1\times10^{14}\text{ m}^2\ (0.965)(5\,800\text{ K})^4 = \boxed{3.16\times10^{22}\text{ W}}$

(b) $T_{\text{avg}} = 0.1(4\,800\text{ K}) + 0.9(5\,890\text{ K}) = \boxed{5.78\times10^3\text{ K}}$

This is cooler than 5 800 K by $\dfrac{5\,800 - 5\,781}{5\,800} = 0.327\%$.

(c) $\mathscr{P} = \left(5.67\times10^{-8}\text{ W/m}^2\text{K}^4\right)0.1\left(5.1\times10^{14}\text{ m}^2\right)0.965(4\,800\text{ K})^4$

$+5.67\times10^{-8}\text{ W }0.9\left(5.1\times10^{14}\right)0.965(5\,890)^4 = \boxed{3.17\times10^{22}\text{ W}}$

This is larger than $3.158\times10^{22}$ W by $\dfrac{1.29\times10^{20}\text{ W}}{3.16\times10^{22}\text{ W}} = 0.408\%$.

## ANSWERS TO EVEN PROBLEMS

**P20.2** 0.105°C

**P20.4** 87.0°C

**P20.6** The energy input to the water is 6.70 times larger than the laser output of 40.0 kJ.

**P20.8** 88.2 W

**P20.10** (a) 25.8°C; (b) no

**P20.12** $T_f = \dfrac{\left(m_{\text{Al}}c_{\text{Al}} + m_c c_w\right)T_c + m_h c_w T_h}{m_{\text{Al}}c_{\text{Al}} + m_c c_w + m_h c_w}$

**P20.14** (a) 380 K; (b) 206 kPa

**P20.16** 12.9 g

**P20.18** (a) all the ice melts; 40.4°C;
(b) 8.04 g melts; 0°C

**P20.20** 34.0 km

**P20.22** liquid lead at 805°C

**P20.24** (a) –12.0 MJ; (b) +12.0 MJ

**P20.26** $-nR\left(T_2 - T_1\right)$

**P20.28** (a) 567 J; (b) 167 J

**P20.30** (a) 12.0 kJ; (b) –12.0 kJ

**P20.32** 42.9 kJ

**P20.34** (a) 7.65 L; (b) 305 K

**P20.36** (a) –48.6 mJ; (b) 16.2 kJ; (c) 16.2 kJ

**P20.38** (a) $-4P_iV_i$; (b) $+4P_iV_i$; (c) –9.08 kJ

**P20.40** (a) 1 300 J; (b) 100 J; (c) –900 J; (d) –1 400 J

**P20.42** 10.0 kW

| | |
|---|---|
| **P20.44** | 1.34 kW |
| **P20.46** | (a) $0.890\ \text{ft}^2\cdot{}^\circ\text{F}\cdot\text{h/Btu}$; (b) $1.85\,\frac{\text{ft}^2\cdot{}^\circ\text{F}\cdot\text{h}}{\text{Btu}}$;<br>(c) 2.08 |
| **P20.48** | (a) $\sim 10^3\ \text{W}$; (b) $\sim -10^{-1}\ \text{K/s}$ |
| **P20.50** | 364 K |
| **P20.52** | 47.7 g |
| **P20.54** | (a) $13.0^\circ\text{C}$; (b) $-0.532\ {}^\circ\text{C/s}$ |
| **P20.56** | (a) $64.1^\circ\text{C}$; (b) $113^\circ\text{C}$ |
| **P20.58** | see the solution (a) $\frac{1}{2}P_iV_i$; (b) $1.39P_iV_i$; (c) 0 |
| **P20.60** | (a) $9.31\times10^{10}\ \text{J}$; (b) $-8.47\times10^{12}\ \text{J}$;<br>(c) $8.38\times10^{12}\ \text{J}$ |
| **P20.62** | (a) 2 000 W; (b) $4.47^\circ\text{C}$ |
| **P20.64** | 3.76 m/s |
| **P20.66** | (a) 15.0 mg; block: $Q=0$; $W=-5.00\ \text{J}$;<br>$\Delta E_{\text{int}}=0$; $\Delta K=-5.00\ \text{J}$;<br>ice: $Q=0$; $W=5.00\ \text{J}$; $\Delta E_{\text{int}}=5.00\ \text{J}$; $\Delta K=0$<br>(b) 15.0 mg; block: $Q=0$; $W=0$;<br>$\Delta E_{\text{int}}=5.00\ \text{J}$; $\Delta K=-5.00\ \text{J}$;<br>metal: $Q=0$; $W=0$; $\Delta E_{\text{int}}=0$; $\Delta K=0$<br>(c) $0.004\,04^\circ\text{C}$; moving block: $Q=0$;<br>$W=-2.50\ \text{J}$; $\Delta E_{\text{int}}=2.50\ \text{J}$; $\Delta K=-5.00\ \text{J}$;<br>stationary block: $Q=0$; $W=2.50\ \text{J}$;<br>$\Delta E_{\text{int}}=2.50\ \text{J}$; $\Delta K=0$ |
| **P20.68** | 10.2 h |
| **P20.70** | see the solution |
| **P20.72** | 800 J/kg·°C |

# 21

# The Kinetic Theory of Gases

**CHAPTER OUTLINE**

21.1 Molecular Model of an Ideal Gas
21.2 Molar Specific Heat of an Ideal Gas
21.3 Adiabatic Processes for an Ideal Gas
21.4 The Equipartition of Energy
21.5 The Boltzmann Distribution Law
21.6 Distribution of Molecular Speeds
21.7 Mean Free Path

## ANSWERS TO QUESTIONS

**Q21.1** The molecules of all different kinds collide with the walls of the container, so molecules of all different kinds exert partial pressures that contribute to the total pressure. The molecules can be so small that they collide with one another relatively rarely and each kind exerts partial pressure as if the other kinds of molecules were absent. If the molecules collide with one another often, the collisions exactly conserve momentum and so do not affect the net force on the walls.

**Q21.2** The helium must have the higher rms speed. According to Equation 21.4, the gas with the smaller mass per atom must have the higher average speed-squared and thus the higher rms speed.

**Q21.3** Yes. As soon as the gases are mixed, they come to thermal equilibrium. Equation 21.4 predicts that the lighter helium atoms will on average have a greater speed than the heavier nitrogen molecules. Collisions between the different kinds of molecules gives each kind the same average kinetic energy of translation.

**Q21.4** If the average velocity were non-zero, then the bulk sample of gas would be moving in the direction of the average velocity. In a closed tank, this motion would result in a pressure difference within the tank that could not be sustained.

**Q21.5** The alcohol evaporates, absorbing energy from the skin to lower the skin temperature.

**Q21.6** Partially evacuating the container is equivalent to letting the remaining gas expand. This means that the gas does work, making its internal energy and hence its temperature decrease. The liquid in the container will eventually reach thermal equilibrium with the low pressure gas. This effect of an expanding gas decreasing in temperature is a key process in your refrigerator or air conditioner.

**Q21.7** Since the volume is fixed, the density of the cooled gas cannot change, so the mean free path does not change. The collision frequency decreases since each molecule of the gas has a lower average speed.

**Q21.8** The mean free path decreases as the density of the gas increases.

**Q21.9** The volume of the balloon will decrease. The pressure inside the balloon is nearly equal to the constant exterior atmospheric pressure. Then from $PV = nRT$, volume must decrease in proportion to the absolute temperature. Call the process isobaric contraction.

**Q21.10** The dry air is more dense. Since the air and the water vapor are at the same temperature, they have the same kinetic energy per molecule. For a controlled experiment, the humid and dry air are at the same pressure, so the number of molecules per unit volume must be the same for both. The water molecule has a smaller molecular mass (18.0 u) than any of the gases that make up the air, so the humid air must have the smaller mass per unit volume.

**Q21.11** Suppose the balloon rises into air uniform in temperature. The air cannot be uniform in pressure because the lower layers support the weight of all the air above them. The rubber in a typical balloon is easy to stretch and stretches or contracts until interior and exterior pressures are nearly equal. So as the balloon rises it expands. This is an isothermal expansion, with $P$ decreasing as $V$ increases by the same factor in $PV = nRT$. If the rubber wall is very strong it will eventually contain the helium at higher pressure than the air outside but at the same density, so that the balloon will stop rising. More likely, the rubber will stretch and break, releasing the helium to keep rising and "boil out" of the Earth's atmosphere.

**Q21.12** A diatomic gas has more degrees of freedom—those of vibration and rotation—than a monatomic gas. The energy content per mole is proportional to the number of degrees of freedom.

**Q21.13** (a) Average molecular kinetic energy increases by a factor of 3.

(b) The rms speed increases by a factor of $\sqrt{3}$.

(c) Average momentum change increases by $\sqrt{3}$.

(d) Rate of collisions increases by a factor of $\sqrt{3}$ since the mean free path remains unchanged.

(e) Pressure increases by a factor of 3.

**Q21.14** They can, as this possibility is not contradicted by any of our descriptions of the motion of gases. If the vessel contains more than a few molecules, it is highly improbable that all will have the same speed. Collisions will make their speeds scatter according to the Boltzmann distribution law.

**Q21.15** Collisions between molecules are mediated by electrical interactions among their electrons. On an atomic level, collisions of billiard balls work the same way. Collisions between gas molecules are perfectly elastic. Collisions between macroscopic spheres can be very nearly elastic. So the hard-sphere model is very good. On the other hand, an atom is not 'solid,' but has small-mass electrons moving through empty space as they orbit the nucleus.

**Q21.16** As a parcel of air is pushed upward, it moves into a region of lower pressure, so it expands and does work on its surroundings. Its fund of internal energy drops, and so does its temperature. As mentioned in the question, the low thermal conductivity of air means that very little heat will be conducted into the now-cool parcel from the denser but warmer air below it.

**Q21.17** A more massive diatomic or polyatomic molecule will generally have a lower frequency of vibration. At room temperature, vibration has a higher probability of being excited than in a less massive molecule. The absorption of energy into vibration shows up in higher specific heats.

## SOLUTIONS TO PROBLEMS

### Section 21.1 Molecular Model of an Ideal Gas

**P21.1**
$$\overline{F} = Nm\frac{\Delta v}{\Delta t} = 500\left(5.00\times10^{-3}\text{ kg}\right)\frac{\left[8.00\sin 45.0°-(-8.00\sin 45.0°)\right]\text{ m/s}}{30.0\text{ s}} = \boxed{0.943\text{ N}}$$

$$P = \frac{\overline{F}}{A} = 1.57\text{ N/m}^2 = \boxed{1.57\text{ Pa}}$$

**P21.2** $$\overline{F}=\frac{\left(5.00\times10^{23}\right)\left[2\left(4.68\times10^{-26}\ \text{kg}\right)(300\ \text{m/s})\right]}{1.00\ \text{s}}=14.0\ \text{N}$$

and $P=\dfrac{\overline{F}}{A}=\dfrac{14.0\ \text{N}}{8.00\times10^{-4}\ \text{m}^2}=\boxed{17.6\ \text{kPa}}$.

**P21.3** We first find the pressure exerted by the gas on the wall of the container.

$$P=\frac{NkT}{V}=\frac{3N_{\text{A}}k_{\text{B}}T}{V}=\frac{3RT}{V}=\frac{3(8.314\ \text{N}\cdot\text{m/mol}\cdot\text{K})(293\ \text{K})}{8.00\times10^{-3}\ \text{m}^3}=9.13\times10^5\ \text{Pa}$$

Thus, the force on one of the walls of the cubical container is

$$F=PA=\left(9.13\times10^5\ \text{Pa}\right)\left(4.00\times10^{-2}\ \text{m}^2\right)=\boxed{3.65\times10^4\ \text{N}}.$$

**P21.4** Use Equation 21.2, $P=\dfrac{2N}{3V}\left(\dfrac{mv^2}{2}\right)$, so that

$$K_{\text{av}}=\frac{mv^2}{2}=\frac{3PV}{2N}\quad\text{where } N=nN_{\text{A}}=2N_{\text{A}}$$

$$K_{\text{av}}=\frac{3PV}{2(2N_{\text{A}})}=\frac{3(8.00\ \text{atm})\left(1.013\times10^5\ \text{Pa/atm}\right)\left(5.00\times10^{-3}\ \text{m}^3\right)}{2(2\ \text{mol})\left(6.02\times10^{23}\ \text{molecules/mol}\right)}$$

$$K_{\text{av}}=\boxed{5.05\times10^{-21}\ \text{J/molecule}}$$

**P21.5** $P=\dfrac{2}{3}\dfrac{N}{V}\left(\overline{KE}\right)$ Equation 21.2

$$N=\frac{3}{2}\frac{PV}{\left(\overline{KE}\right)}=\frac{3}{2}\frac{\left(1.20\times10^5\right)\left(4.00\times10^{-3}\right)}{\left(3.60\times10^{-22}\right)}=2.00\times10^{24}\ \text{molecules}$$

$$n=\frac{N}{N_{\text{A}}}=\frac{2.00\times10^{24}\ \text{molecules}}{6.02\times10^{23}\ \text{molecules/mol}}=\boxed{3.32\ \text{mol}}$$

**P21.6** One mole of helium contains Avogadro's number of molecules and has a mass of 4.00 g. Let us call $m$ the mass of one atom, and we have

$$N_{\text{A}}m=4.00\ \text{g/mol}$$

or $$m=\frac{4.00\ \text{g/mol}}{6.02\times10^{23}\ \text{molecules/mol}}=6.64\times10^{-24}\ \text{g/molecule}$$

$$m=\boxed{6.64\times10^{-27}\ \text{kg}}$$

**P21.7** (a) $PV=Nk_BT$: $N=\dfrac{PV}{k_BT}=\dfrac{1.013\times10^5\ \text{Pa}\left[\frac{4}{3}\pi(0.150\ \text{m})^3\right]}{\left(1.38\times10^{-23}\ \text{J/K}\right)(293\ \text{K})}=\boxed{3.54\times10^{23}\ \text{atoms}}$

(b) $\overline{K}=\dfrac{3}{2}k_BT=\dfrac{3}{2}\left(1.38\times10^{-23}\right)(293)\ \text{J}=\boxed{6.07\times10^{-21}\ \text{J}}$

(c) For helium, the atomic mass is $m=\dfrac{4.00\ \text{g/mol}}{6.02\times10^{23}\ \text{molecules/mol}}=6.64\times10^{-24}\ \text{g/molecule}$

$$m=6.64\times10^{-27}\ \text{kg/molecule}$$

$\dfrac{1}{2}m\overline{v^2}=\dfrac{3}{2}k_BT$: $\therefore v_{\text{rms}}=\sqrt{\dfrac{3k_BT}{m}}=\boxed{1.35\ \text{km/s}}$

**P21.8** $v = \sqrt{\dfrac{3k_BT}{m}}$

$$\frac{v_{\text{O}}}{v_{\text{He}}} = \sqrt{\frac{M_{\text{He}}}{M_{\text{O}}}} = \sqrt{\frac{4.00}{32.0}} = \sqrt{\frac{1}{8.00}}$$

$$v_{\text{O}} = \frac{1\,350 \text{ m/s}}{\sqrt{8.00}} = \boxed{477 \text{ m/s}}$$

**P21.9** (a) $\overline{K} = \dfrac{3}{2}k_BT = \dfrac{3}{2}\left(1.38\times10^{-23} \text{ J/K}\right)(423 \text{ K}) = \boxed{8.76\times10^{-21} \text{ J}}$

(b) $\overline{K} = \dfrac{1}{2}mv_{rms}^2 = 8.76\times10^{-21} \text{ J}$

so $$v_{rms} = \sqrt{\frac{1.75\times10^{-20} \text{ J}}{m}} \qquad (1)$$

For helium, $m = \dfrac{4.00 \text{ g/mol}}{6.02\times10^{23} \text{ molecules/mol}} = 6.64\times10^{-24} \text{ g/molecule}$

$m = 6.64\times10^{-27} \text{ kg/molecule}$

Similarly for argon, $m = \dfrac{39.9 \text{ g/mol}}{6.02\times10^{23} \text{ molecules/mol}} = 6.63\times10^{-23} \text{ g/molecule}$

$m = 6.63\times10^{-26} \text{ kg/molecule}$

Substituting in (1) above,
we find for helium, $\boxed{v_{rms} = 1.62 \text{ km/s}}$

and for argon, $\boxed{v_{rms} = 514 \text{ m/s}}$

**P21.10** (a) $PV = nRT = \dfrac{Nmv^2}{3}$

The total translational kinetic energy is $\dfrac{Nmv^2}{2} = E_{\text{trans}}$:

$E_{\text{trans}} = \dfrac{3}{2}PV = \dfrac{3}{2}\left(3.00\times1.013\times10^5\right)\left(5.00\times10^{-3}\right) = \boxed{2.28 \text{ kJ}}$

(b) $\dfrac{mv^2}{2} = \dfrac{3k_BT}{2} = \dfrac{3RT}{2N_{\text{A}}} = \dfrac{3(8.314)(300)}{2\left(6.02\times10^{23}\right)} = \boxed{6.21\times10^{-21} \text{ J}}$

**P21.11** (a) $1 \text{ Pa} = (1 \text{ Pa})\left(\dfrac{1 \text{ N/m}^2}{1 \text{ Pa}}\right)\left(\dfrac{1 \text{ J}}{1 \text{ N}\cdot\text{m}}\right) = \boxed{1 \text{ J/m}^3}$

(b) For a monatomic ideal gas, $E_{\text{int}} = \dfrac{3}{2}nRT$

For any ideal gas, the energy of molecular translation is the same,

$$E_{\text{trans}} = \frac{3}{2}nRT = \frac{3}{2}PV.$$

Thus, the energy per volume is $\dfrac{E_{\text{trans}}}{V} = \boxed{\dfrac{3}{2}P}$.

## Section 21.2 **Molar Specific Heat of an Ideal Gas**

**P21.12** $E_{\text{int}} = \frac{3}{2}nRT$

$$\Delta E_{\text{int}} = \frac{3}{2}nR\Delta T = \frac{3}{2}(3.00\text{ mol})(8.314\text{ J/mol}\cdot\text{K})(2.00\text{ K}) = \boxed{74.8\text{ J}}$$

**P21.13** We us the tabulated values for $C_P$ and $C_V$

(a) $Q = nC_P\Delta T = 1.00\text{ mol}(28.8\text{ J/mol}\cdot\text{K})(420-300)\text{ K} = \boxed{3.46\text{ kJ}}$

(b) $\Delta E_{\text{int}} = nC_V\Delta T = 1.00\text{ mol}(20.4\text{ J/mol}\cdot\text{K})(120\text{ K}) = \boxed{2.45\text{ kJ}}$

(c) $W = -Q + \Delta E_{\text{int}} = -3.46\text{ kJ} + 2.45\text{ kJ} = \boxed{-1.01\text{ kJ}}$

**P21.14** The piston moves to keep pressure constant. Since $V = \frac{nRT}{P}$, then

$$\Delta V = \frac{nR\Delta T}{P} \text{ for a constant pressure process.}$$

$$Q = nC_P\Delta T = n(C_V + R)\Delta T \text{ so } \Delta T = \frac{Q}{n(C_V + R)} = \frac{Q}{n(5R/2 + R)} = \frac{2Q}{7nR}$$

and $\Delta V = \frac{nR}{P}\left(\frac{2Q}{7nR}\right) = \frac{2Q}{7P} = \frac{2}{7}\frac{QV}{nRT}$

$$\Delta V = \frac{2}{7}\frac{\left(4.40\times 10^3\text{ J}\right)(5.00\text{ L})}{(1.00\text{ mol})(8.314\text{ J/mol}\cdot\text{K})(300\text{ K})} = 2.52\text{ L}$$

Thus, $V_f = V_i + \Delta V = 5.00\text{ L} + 2.52\text{ L} = \boxed{7.52\text{ L}}$

**P21.15** $n = 1.00\text{ mol}$, $T_i = 300\text{ K}$

(b) Since $V = \text{constant}$, $W = \boxed{0}$

(a) $\Delta E_{\text{int}} = Q + W = 209\text{ J} + 0 = \boxed{209\text{ J}}$

(c) $\Delta E_{\text{int}} = nC_V\Delta T = n\left(\frac{3}{2}R\right)\Delta T$

so $$\Delta T = \frac{2\Delta E_{\text{int}}}{3nR} = \frac{2(209\text{ J})}{3(1.00\text{ mol})(8.314\text{ J/mol}\cdot\text{K})} = 16.8\text{ K}$$

$$T = T_i + \Delta T = 300\text{ K} + 16.8\text{ K} = \boxed{317\text{ K}}$$

**P21.16** (a) Consider heating it at constant pressure. Oxygen and nitrogen are diatomic, so $C_P = \frac{7R}{2}$

$$Q = nC_P\Delta T = \frac{7}{2}nR\Delta T = \frac{7}{2}\left(\frac{PV}{T}\right)\Delta T$$

$$Q = \frac{7}{2}\frac{\left(1.013\times10^5\ \text{N/m}^2\right)\left(100\ \text{m}^3\right)}{300\ \text{K}}(1.00\ \text{K}) = \boxed{118\ \text{kJ}}$$

(b) $U_g = mgy$

$$m = \frac{U_g}{gy} = \frac{1.18\times10^5\ \text{J}}{\left(9.80\ \text{m/s}^2\right)2.00\ \text{m}} = \boxed{6.03\times10^3\ \text{kg}}$$

***P21.17** (a) We assume that the bulb does not expand. Then this is a constant-volume heating process. The quantity of the gas is $n = \frac{P_iV}{RT_i}$. The energy input is $Q = \mathscr{P}\Delta t = nC_V\Delta T$ so

$$\Delta T = \frac{\mathscr{P}\Delta t}{nC_V} = \frac{\mathscr{P}\Delta tRT_i}{P_iVC_V}.$$

The final temperature is $T_f = T_i + \Delta T = T_i\left(1 + \frac{\mathscr{P}\Delta tR}{P_iVC_V}\right)$.

The final pressure is $P_f = P_i\frac{T_f}{T_i} = P_i\left(1 + \frac{\mathscr{P}\Delta tR}{P_iVC_V}\right)$.

(b) $$P_f = 1\ \text{atm}\left(1 + \frac{3.60\ \text{J}\ 4\ \text{s}\ 8.314\ \text{J}\cdot\text{m}^2\ 3\ \text{mol}\cdot\text{K}}{\text{s}\cdot\text{mol}\cdot\text{K}\ 1.013\times10^5\ \text{N}\ 4\pi(0.05\ \text{m})^3\ 12.5\ \text{J}}\right) = \boxed{1.18\ \text{atm}}$$

**P21.18** (a) $$C_V = \frac{5}{2}R = \frac{5}{2}(8.314\ \text{J/mol}\cdot\text{K})\left(\frac{1.00\ \text{mol}}{0.028\,9\ \text{kg}}\right) = 719\ \text{J/kg}\cdot\text{K} = \boxed{0.719\ \text{kJ/kg}\cdot\text{K}}$$

(b) $$m = Mn = M\left(\frac{PV}{RT}\right)$$

$$m = (0.028\,9\ \text{kg/mol})\left(\frac{200\times10^3\ \text{Pa}\left(0.350\ \text{m}^3\right)}{(8.314\ \text{J/mol}\cdot\text{K})(300\ \text{K})}\right) = \boxed{0.811\ \text{kg}}$$

(c) We consider a constant volume process where no work is done.

$$Q = mC_V\Delta T = 0.811\ \text{kg}(0.719\ \text{kJ/kg}\cdot\text{K})(700\ \text{K} - 300\ \text{K}) = \boxed{233\ \text{kJ}}$$

(d) We now consider a constant pressure process where the internal energy of the gas is increased and work is done.

$$Q = mC_P\Delta T = m(C_V + R)\Delta T = m\left(\frac{7R}{2}\right)\Delta T = m\left(\frac{7C_V}{5}\right)\Delta T$$

$$Q = 0.811\ \text{kg}\left[\frac{7}{5}(0.719\ \text{kJ/kg}\cdot\text{K})\right](400\ \text{K}) = \boxed{327\ \text{kJ}}$$

**P21.19** Consider 800 $\text{cm}^3$ of (flavored) water at 90.0 °C mixing with 200 $\text{cm}^3$ of diatomic ideal gas at 20.0°C:

$$Q_{\text{cold}} = -Q_{\text{hot}}$$

or

$$m_{\text{air}} c_{P,\,\text{air}}\left(T_f - T_{i,\,\text{air}}\right) = -m_w c_w (\Delta T)_w$$

$$(\Delta T)_w = \frac{-m_{\text{air}} c_{P,\,\text{air}}\left(T_f - T_{i,\,\text{air}}\right)}{m_w c_w} = \frac{-(\rho V)_{\text{air}} c_{P,\,\text{air}}(90.0°\text{C} - 20.0°\text{C})}{(\rho_w V_w) c_w}$$

where we have anticipated that the final temperature of the mixture will be close to 90.0°C.

The molar specific heat of air is $C_{P,\,\text{air}} = \frac{7}{2}R$

So the specific heat per gram is $c_{P,\,\text{air}} = \frac{7}{2}\left(\frac{R}{M}\right) = \frac{7}{2}(8.314\ \text{J/mol}\cdot\text{K})\left(\frac{1.00\ \text{mol}}{28.9\ \text{g}}\right) = 1.01\ \text{J/g}\cdot°\text{C}$

$$(\Delta T)_w = -\frac{\left[\left(1.20\times10^{-3}\ \text{g/cm}^3\right)\left(200\ \text{cm}^3\right)\right](1.01\ \text{J/g}\cdot°\text{C})(70.0°\text{C})}{\left[\left(1.00\ \text{g/cm}^3\right)\left(800\ \text{cm}^3\right)\right](4.186\ \text{J/kg}\cdot°\text{C})}$$

or $(\Delta T)_w \approx -5.05\times10^{-3}\,°\text{C}$

The change of temperature for the water is $\boxed{\text{between } 10^{-3}\,°\text{C and } 10^{-2}\,°\text{C}}$.

**P21.20** $Q = (nC_P\Delta T)_{\text{isobaric}} + (nC_V\Delta T)_{\text{isovolumetric}}$

In the isobaric process, $V$ doubles so $T$ must double, to $2T_i$.

In the isovolumetric process, $P$ triples so $T$ changes from $2T_i$ to $6T_i$.

$$Q = n\left(\frac{7}{2}R\right)(2T_i - T_i) + n\left(\frac{5}{2}R\right)(6T_i - 2T_i) = 13.5nRT_i = \boxed{13.5PV}$$

**P21.21** In the isovolumetric process $A \to B$, $W = 0$ and $Q = nC_V\Delta T = 500\ \text{J}$

$$500\ \text{J} = n\left(\frac{3R}{2}\right)(T_B - T_A) \text{ or } T_B = T_A + \frac{2(500\ \text{J})}{3nR}$$

$$T_B = 300\ \text{K} + \frac{2(500\ \text{J})}{3(1.00\ \text{mol})(8.314\ \text{J/mol}\cdot\text{K})} = 340\ \text{K}$$

In the isobaric process $B \to C$,

$$Q = nC_P\Delta T = \frac{5nR}{2}(T_C - T_B) = -500\ \text{J}.$$

Thus,

(a) $$T_C = T_B - \frac{2(500\ \text{J})}{5nR} = 340\ \text{K} - \frac{1\,000\ \text{J}}{5(1.00\ \text{mol})(8.314\ \text{J/mol}\cdot\text{K})} = \boxed{316\ \text{K}}$$

(b) The work done on the gas during the isobaric process is

$$W_{BC} = -P_B\Delta V = -nR(T_C - T_B) = -(1.00\ \text{mol})(8.314\ \text{J/mol}\cdot\text{K})(316\ \text{K} - 340\ \text{J})$$

or $W_{BC} = +200\ \text{J}$

The work done on the gas in the isovolumetric process is zero, so in total

$$W_{\text{on gas}} = \boxed{+200\ \text{J}}.$$

***P21.22** (a) At any point in the heating process, $P_i = kV_i$ and $P = kV = \frac{P_i}{V_i}V = \frac{nRT_i}{V_i^2}V$. At the end,

$$P_f = \frac{nRT_i}{V_i^2}2V_i = 2P_i \text{ and } T_f = \frac{P_f V_f}{nR} = \frac{2P_i 2V_i}{nR} = \boxed{4T_i}.$$

(b) The work input is $W = -\int_i^f P\,dV = -\int_{V_i}^{2V_i} \frac{nRT_i}{V_i^2}V\,dV = -\frac{nRT_i}{V_i^2}\frac{V^2}{2}\Big|_{V_i}^{2V_i} = -\frac{nRT_i}{2V_i^2}\left(4V_i^2 - V_i^2\right) = -\frac{3}{2}nRT_i$.

The change in internal energy, is $\Delta E_{\text{int}} = nC_V\Delta T = n\frac{5}{2}R(4T_i - T_i) = +\frac{15}{2}nRT_i$. The heat input is $Q = \Delta E_{\text{int}} - W = \frac{18}{2}nRT_i = \boxed{9(1\text{ mol})RT_i}$.

**P21.23** (a) The heat required to produce a temperature change is

$$Q = n_1C_1\Delta T + n_2C_2\Delta T$$

The number of molecules is $N_1 + N_2$, so the number of "moles of the mixture" is $n_1 + n_2$ and $Q = (n_1 + n_2)C\Delta T$,

so $$C = \boxed{\frac{n_1C_1 + n_2C_2}{n_1 + n_2}}.$$

(b) $$Q = \sum_{i=1}^{m} n_iC_i\Delta T = \left(\sum_{i=1}^{m} n_i\right)C\Delta T$$

$$C = \boxed{\frac{\sum_{i=1}^{m} n_iC_i}{\sum_{i=1}^{m} n_i}}$$

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## Section 21.3 **Adiabatic Processes for an Ideal Gas**

**P21.24** (a) $P_iV_i^\gamma = P_fV_f^\gamma$ so $\frac{V_f}{V_i} = \left(\frac{P_i}{P_f}\right)^{1/\gamma} = \left(\frac{1.00}{20.0}\right)^{5/7} = \boxed{0.118}$

(b) $\frac{T_f}{T_i} = \frac{P_fV_f}{P_iV_i} = \left(\frac{P_f}{P_i}\right)\left(\frac{V_f}{V_i}\right) = (20.0)(0.118)$ $\frac{T_f}{T_i} = \boxed{2.35}$

(c) Since the process is adiabatic, $\boxed{Q = 0}$

Since $\gamma = 1.40 = \frac{C_P}{C_V} = \frac{R + C_V}{C_V}$, $C_V = \frac{5}{2}R$ and $\Delta T = 2.35T_i - T_i = 1.35T_i$

$$\Delta E_{\text{int}} = nC_V\Delta T = (0.016\,0\text{ mol})\left(\frac{5}{2}\right)(8.314\text{ J/mol}\cdot\text{K})\left[1.35(300\text{ K})\right] = \boxed{135\text{ J}}$$

and $W = -Q + \Delta E_{\text{int}} = 0 + 135\text{ J} = \boxed{+135\text{ J}}$.

**P21.25** (a) $P_iV_i^{\gamma} = P_fV_f^{\gamma}$

$$P_f = P_i\left(\frac{V_i}{V_f}\right)^{\gamma} = 5.00\text{ atm}\left(\frac{12.0}{30.0}\right)^{1.40} = \boxed{1.39\text{ atm}}$$

(b) $$T_i = \frac{P_iV_i}{nR} = \frac{5.00\left(1.013\times10^5\text{ Pa}\right)\left(12.0\times10^{-3}\text{ m}^3\right)}{2.00\text{ mol}(8.314\text{ J/mol}\cdot\text{K})} = \boxed{365\text{ K}}$$

$$T_f = \frac{P_fV_f}{nR} = \frac{1.39\left(1.013\times10^5\text{ Pa}\right)\left(30.0\times10^{-3}\text{ m}^3\right)}{2.00\text{ mol}(8.314\text{ J/mol}\cdot\text{K})} = \boxed{253\text{ K}}$$

(c) The process is adiabatic: $\boxed{Q=0}$

$$\gamma = 1.40 = \frac{C_P}{C_V} = \frac{R+C_V}{C_V},\ C_V = \frac{5}{2}R$$

$$\Delta E_{\text{int}} = nC_V\Delta T = 2.00\text{ mol}\left(\frac{5}{2}(8.314\text{ J/mol}\cdot\text{K})\right)(253\text{ K}-365\text{ K}) = \boxed{-4.66\text{ kJ}}$$

$$W = \Delta E_{\text{int}} - Q = -4.66\text{ kJ} - 0 = \boxed{-4.66\text{ kJ}}$$

**P21.26** $$V_i = \pi\left(\frac{2.50\times10^{-2}\text{ m}}{2}\right)^2 0.500\text{ m} = 2.45\times10^{-4}\text{ m}^3$$

The quantity of air we find from $P_iV_i = nRT_i$

$$n = \frac{P_iV_i}{RT_i} = \frac{\left(1.013\times10^5\text{ Pa}\right)\left(2.45\times10^{-4}\text{ m}^3\right)}{(8.314\text{ J/mol}\cdot\text{K})(300\text{ K})}$$

$$n = 9.97\times10^{-3}\text{ mol}$$

Adiabatic compression: $P_f = 101.3\text{ kPa} + 800\text{ kPa} = 901.3\text{ kPa}$

(a) $P_iV_i^{\gamma} = P_fV_f^{\gamma}$

$$V_f = V_i\left(\frac{P_i}{P_f}\right)^{1/\gamma} = 2.45\times10^{-4}\text{ m}^3\left(\frac{101.3}{901.3}\right)^{5/7}$$

$$V_f = \boxed{5.15\times10^{-5}\text{ m}^3}$$

(b) $P_fV_f = nRT_f$

$$T_f = T_i\frac{P_fV_f}{P_iV_i} = T_i\frac{P_f}{P_i}\left(\frac{P_i}{P_f}\right)^{1/\gamma} = T_i\left(\frac{P_i}{P_f}\right)^{(1/\gamma-1)}$$

$$T_f = 300\text{ K}\left(\frac{101.3}{901.3}\right)^{(5/7-1)} = \boxed{560\text{ K}}$$

(c) The work put into the gas in compressing it is $\Delta E_{\text{int}} = nC_V\Delta T$

$$W = \left(9.97\times10^{-3}\text{ mol}\right)\frac{5}{2}(8.314\text{ J/mol}\cdot\text{K})(560-300)\text{ K}$$

$$W = 53.9\text{ J}$$

*continued on next page*

Now imagine this energy being shared with the inner wall as the gas is held at constant volume. The pump wall has outer diameter $25.0\text{ mm}+2.00\text{ mm}+2.00\text{ mm}=29.0\text{ mm}$, and volume

$$\left[\pi\left(14.5\times10^{-3}\text{ m}\right)^2-\pi\left(12.5\times10^{-3}\text{ m}\right)^2\right]4.00\times10^{-2}\text{ m}=6.79\times10^{-6}\text{ m}^3$$

and mass $\rho V=\left(7.86\times10^3\text{ kg/m}^3\right)\left(6.79\times10^{-6}\text{ m}^3\right)=53.3\text{ g}$

The overall warming process is described by

$$\begin{aligned}53.9\text{ J}&=nC_V\Delta T+mc\Delta T\\53.9\text{ J}&=\left(9.97\times10^{-3}\text{ mol}\right)\frac{5}{2}(8.314\text{ J/mol}\cdot\text{K})\left(T_{ff}-300\text{ K}\right)\\&\quad+\left(53.3\times10^{-3}\text{ kg}\right)(448\text{ J/kg}\cdot\text{K})\left(T_{ff}-300\text{ K}\right)\\53.9\text{ J}&=(0.207\text{ J/K}+23.9\text{ J/K})\left(T_{ff}-300\text{ K}\right)\\T_{ff}-300\text{ K}&=\boxed{2.24\text{ K}}\end{aligned}$$

**P21.27** $\quad\dfrac{T_f}{T_i}=\left(\dfrac{V_i}{V_f}\right)^{\gamma-1}=\left(\dfrac{1}{2}\right)^{0.400}$

If $T_i=300\text{ K}$, then $T_f=\boxed{227\text{ K}}$.

***P21.28** (a) In $P_iV_i^\gamma=P_fV_f^\gamma$ we have $P_f=P_i\left(\dfrac{V_i}{V_f}\right)^\gamma$

$$P_f=P_i\left(\frac{0.720\text{ m}^3}{0.240\text{ m}^3}\right)^{1.40}=4.66P_i$$

Then $\dfrac{P_iV_i}{T_i}=\dfrac{P_fV_f}{T_f}$ $\qquad T_f=T_i\dfrac{P_fV_f}{P_iV_i}=T_i(4.66)\dfrac{1}{3}=1.55$

The factor of increase in temperature is the same as the factor of increase in internal energy, according to $E_{\text{int}}=nC_VT$. Then $\dfrac{E_{\text{int},f}}{E_{\text{int},i}}=\boxed{1.55}$.

(b) In $\dfrac{T_f}{T_i}=\dfrac{P_fV_f}{P_iV_i}=\left(\dfrac{V_i}{V_f}\right)^\gamma\dfrac{V_f}{V_i}=\left(\dfrac{V_i}{V_f}\right)^{\gamma-1}$ we have

$$2=\left(\frac{0.720\text{ m}^3}{V_f}\right)^{0.40}$$

$$\frac{0.720\text{ m}^3}{V_f}=2^{1/0.4}=2^{2.5}=5.66$$

$$V_f=\frac{0.720\text{ m}^3}{5.66}=\boxed{0.127\text{ m}^3}$$

**P21.29** (a) See the diagram at the right.

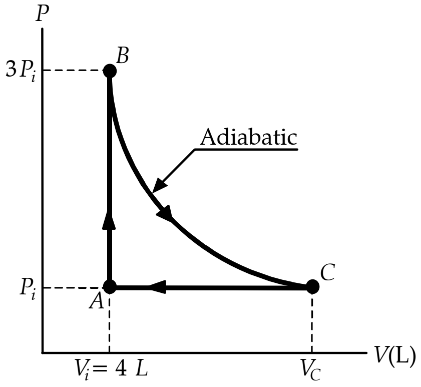

**FIG. P21.29**

(b) $P_B V_B^\gamma = P_C V_C^\gamma$

$3P_i V_i^\gamma = P_i V_C^\gamma$

$V_C = \left(3^{1/\gamma}\right)V_i = \left(3^{5/7}\right)V_i = 2.19V_i$

$V_C = 2.19(4.00 \text{ L}) = \boxed{8.77 \text{ L}}$

(c) $P_B V_B = nRT_B = 3P_i V_i = 3nRT_i$

$T_B = 3T_i = 3(300 \text{ K}) = \boxed{900 \text{ K}}$

(d) After one whole cycle, $T_A = T_i = \boxed{300 \text{ K}}$.

(e) In *AB*, $Q_{AB} = nC_V \Delta V = n\left(\frac{5}{2}R\right)(3T_i - T_i) = (5.00)nRT_i$

$Q_{BC} = 0$ as this process is adiabatic

$P_C V_C = nRT_C = P_i(2.19V_i) = (2.19)nRT_i$

so $T_C = 2.19T_i$

$Q_{CA} = nC_P \Delta T = n\left(\frac{7}{2}R\right)(T_i - 2.19T_i) = (-4.17)nRT_i$

For the whole cycle,

$Q_{ABCA} = Q_{AB} + Q_{BC} + Q_{CA} = (5.00 - 4.17)nRT_i = (0.829)nRT_i$

$(\Delta E_{\text{int}})_{ABCA} = 0 = Q_{ABCA} + W_{ABCA}$

$W_{ABCA} = -Q_{ABCA} = -(0.829)nRT_i = -(0.829)P_i V_i$

$W_{ABCA} = -(0.829)\left(1.013 \times 10^5 \text{ Pa}\right)\left(4.00 \times 10^{-3} \text{ m}^3\right) = \boxed{-336 \text{ J}}$

**P21.30** (a) See the diagram at the right.

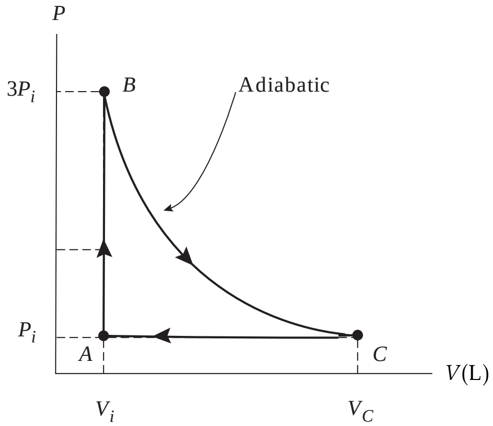

**FIG. P21.30**

(b) $P_B V_B^\gamma = P_C V_C^\gamma$

$3P_i V_i^\gamma = P_i V_C^\gamma$

$V_C = 3^{1/\gamma} V_i = 3^{5/7} V_i = \boxed{2.19V_i}$

(c) $P_B V_B = nRT_B = 3P_i V_i = 3nRT_i$

$T_B = \boxed{3T_i}$

(d) After one whole cycle, $T_A = \boxed{T_i}$

*continued on next page*

(e) In $AB$, $Q_{AB} = nC_V\Delta T = n\left(\frac{5}{2}R\right)(3T_i - T_i) = (5.00)nRT_i$

$$Q_{BC} = 0 \text{ as this process is abiabatic}$$

$$P_C V_C = nRT_C = P_i(2.19V_i) = 2.19nRT_i \text{ so } T_C = 2.19T_i$$

$$Q_{CA} = nC_P\Delta T = n\left(\frac{7}{2}R\right)(T_i - 2.19T_i) = -4.17nRT_i$$

For the whole cycle,

$$Q_{ABCA} = Q_{AB} + Q_{BC} + Q_{CA} = (5.00 - 4.17)nRT_i = 0.830nRT_i$$

$$(\Delta E_{\text{int}})_{ABCA} = 0 = Q_{ABCA} + W_{ABCA}$$

$$W_{ABCA} = -Q_{ABCA} = -0.830nRT_i = \boxed{-0.830P_iV_i}$$

**P21.31** (a) The work done *on* the gas is

$$W_{ab} = -\int_{V_a}^{V_b} PdV.$$

For the isothermal process,

$$W_{ab'} = -nRT_a \int_{V_a}^{V_{b'}} \left(\frac{1}{V}\right) dV$$

$$W_{ab'} = -nRT_a \ln\left(\frac{V_{b'}}{V_a}\right) = nRT \ln\left(\frac{V_a}{V_{b'}}\right).$$

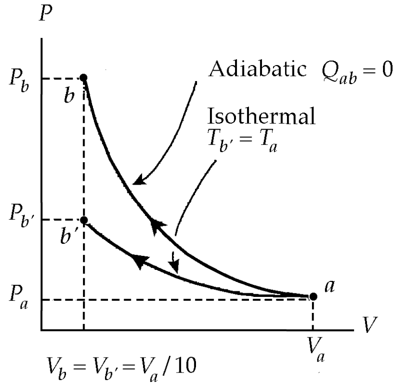

**FIG. P21.31**

Thus, $W_{ab'} = 5.00 \text{ mol}(8.314 \text{ J/mol}\cdot\text{K})(293 \text{ K})\ln(10.0)$

$$W_{ab'} = \boxed{28.0 \text{ kJ}}.$$

(b) For the adiabatic process, we must first find the final temperature, $T_b$. Since air consists primarily of diatomic molecules, we shall use

$$\gamma_{\text{air}} = 1.40 \text{ and } C_{V,\text{ air}} = \frac{5R}{2} = \frac{5(8.314)}{2} = 20.8 \text{ J/mol}\cdot\text{K}.$$

Then, for the adiabatic preocess

$$T_b = T_a\left(\frac{V_a}{V_b}\right)^{\gamma-1} = 293 \text{ K}(10.0)^{0.400} = 736 \text{ K}.$$

Thus, the work done on the gas during the adiabatic process is

$$W_{ab}(-Q + \Delta E_{\text{int}})_{ab} = (-0 + nC_V\Delta T)_{ab} = nC_V(T_b - T_a)$$

or $W_{ab} = 5.00 \text{ mol}(20.8 \text{ J/mol}\cdot\text{K})(736 - 293) \text{ K} = \boxed{46.0 \text{ kJ}}$.

*continued on next page*

(c) For the isothermal process, we have

$$P_{b'}V_{b'} = P_a V_a .$$

Thus, $P_{b'} = P_a\left(\dfrac{V_a}{V_{b'}}\right) = 1.00 \text{ atm}(10.0) = \boxed{10.0 \text{ atm}}$.

For the adiabatic process, we have $P_b V_b^{\gamma} = P_a V_a^{\gamma}$.

Thus, $P_b = P_a\left(\dfrac{V_a}{V_b}\right)^{\gamma} = 1.00 \text{ atm}(10.0)^{1.40} = \boxed{25.1 \text{ atm}}$.

**P21.32** We suppose the air plus burnt gasoline behaves like a diatomic ideal gas. We find its final absolute pressure:

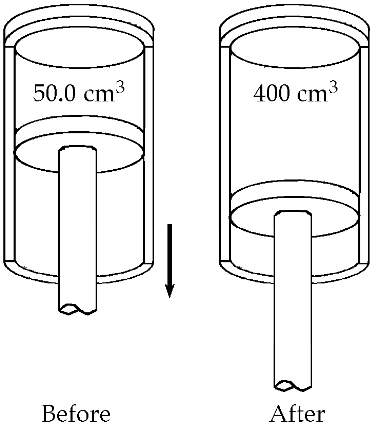

**FIG. P21.32**

$$21.0 \text{ atm}\left(50.0 \text{ cm}^3\right)^{7/5} = P_f\left(400 \text{ cm}^3\right)^{7/5}$$

$$P_f = 21.0 \text{ atm}\left(\frac{1}{8}\right)^{7/5} = 1.14 \text{ atm}$$

Now $Q = 0$

and $W = \Delta E_{\text{int}} = nC_V\left(T_f - T_i\right)$

$$\therefore W = \frac{5}{2}nRT_f - \frac{5}{2}nRT_i = \frac{5}{2}\left(P_f V_f - P_i V_i\right)$$

$$W = \frac{5}{2}\left[1.14 \text{ atm}\left(400 \text{ cm}^3\right) - 21.0 \text{ atm}\left(50.0 \text{ cm}^3\right)\right]\left(\frac{1.013\times10^5 \text{ N/m}^2}{1 \text{ atm}}\right)\left(10^{-6} \text{ m}^3/\text{cm}^3\right)$$

$$W = -150 \text{ J}$$

The output work is $-W = +150$ J

The time for this stroke is $\dfrac{1}{4}\left(\dfrac{1 \text{ min}}{2\,500}\right)\left(\dfrac{60 \text{ s}}{1 \text{ min}}\right) = 6.00\times10^{-3}$ s

$$\mathcal{P} = \frac{-W}{\Delta t} = \frac{150 \text{ J}}{6.00\times10^{-3} \text{ s}} = \boxed{25.0 \text{ kW}}$$

## Section 21.4 **The Equipartition of Energy**

**P21.33** The heat capacity at constant volume is $nC_V$. An ideal gas of diatomic molecules has three degrees of freedom for translation in the $x$, $y$, and $z$ directions. If we take the $y$ axis along the axis of a molecule, then outside forces cannot excite rotation about this axis, since they have no lever arms. Collisions will set the molecule spinning only about the $x$ and $z$ axes.

(a) If the molecules do not vibrate, they have five degrees of freedom. Random collisions put equal amounts of energy $\frac{1}{2}k_BT$ into all five kinds of motion. The average energy of one molecule is $\frac{5}{2}k_BT$. The internal energy of the two-mole sample is

$$N\left(\frac{5}{2}k_BT\right)=nN_A\left(\frac{5}{2}k_BT\right)=n\left(\frac{5}{2}R\right)T=nC_VT.$$

The molar heat capacity is $C_V=\frac{5}{2}R$ and the sample's heat capacity is

$$nC_V=n\left(\frac{5}{2}R\right)=2\text{ mol}\left(\frac{5}{2}(8.314\text{ J/mol}\cdot\text{K})\right)$$

$$\boxed{nC_V=41.6\text{ J/K}}$$

For the heat capacity at constant pressure we have

$$nC_P=n(C_V+R)=n\left(\frac{5}{2}R+R\right)=\frac{7}{2}nR=2\text{ mol}\left(\frac{7}{2}(8.314\text{ J/mol}\cdot\text{K})\right)$$

$$\boxed{nC_P=58.2\text{ J/K}}$$

(b) In vibration with the center of mass fixed, both atoms are always moving in opposite directions with equal speeds. Vibration adds two more degrees of freedom for two more terms in the molecular energy, for kinetic and for elastic potential energy. We have

$$nC_V=n\left(\frac{7}{2}R\right)=\boxed{58.2\text{ J/K}}$$

and $$nC_P=n\left(\frac{9}{2}R\right)=\boxed{74.8\text{ J/K}}$$

**P21.34** (1) $$E_{\text{int}}=Nf\left(\frac{k_BT}{2}\right)=f\left(\frac{nRT}{2}\right)$$

(2) $$C_V=\frac{1}{n}\left(\frac{dE_{\text{int}}}{dT}\right)=\frac{1}{2}fR$$

(3) $$C_P=C_V+R=\frac{1}{2}(f+2)R$$

(4) $$\gamma=\frac{C_P}{C_V}=\frac{f+2}{f}$$

**P21.35** Rotational Kinetic Energy $= \frac{1}{2} I\omega^2$

$I = 2mr^2$, $m = 35.0 \times 1.67 \times 10^{-27}$ kg, $r = 10^{-10}$ m

$I = 1.17 \times 10^{-45}\ \text{kg} \cdot \text{m}^2$ $\qquad \omega = 2.00 \times 10^{12}\ \text{s}^{-1}$

$\therefore K_{\text{rot}} = \frac{1}{2} I\omega^2 = \boxed{2.33 \times 10^{-21}\ \text{J}}$

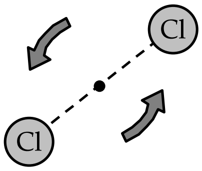

**FIG. P21.35**

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### Section 21.5 **The Boltzmann Distribution Law**

### Section 21.6 **Distribution of Molecular Speeds**

**P21.36** (a) The ratio of the number at higher energy to the number at lower energy is $e^{-\Delta E/k_B T}$ where $\Delta E$ is the energy difference. Here,

$$\Delta E = (10.2\ \text{eV})\left(\frac{1.60 \times 10^{-19}\ \text{J}}{1\ \text{eV}}\right) = 1.63 \times 10^{-18}\ \text{J}$$

and at 0°C,

$$k_B T = \left(1.38 \times 10^{-23}\ \text{J/K}\right)(273\ \text{K}) = 3.77 \times 10^{-21}\ \text{J}.$$

Since this is much less than the excitation energy, nearly all the atoms will be in the ground state and the number excited is

$$\left(2.70 \times 10^{25}\right)\exp\left(\frac{-1.63 \times 10^{-18}\ \text{J}}{3.77 \times 10^{-21}\ \text{J}}\right) = \left(2.70 \times 10^{25}\right)e^{-433}.$$

This number is much less than one, so almost all of the time $\boxed{\text{no atom is excited}}$.

(b) At 10 000°C,

$$k_B T = \left(1.38 \times 10^{-23}\ \text{J/K}\right)10\,273\ \text{K} = 1.42 \times 10^{-19}\ \text{J}.$$

The number excited is

$$\left(2.70 \times 10^{25}\right)\exp\left(\frac{-1.63 \times 10^{-18}\ \text{J}}{1.42 \times 10^{-19}\ \text{J}}\right) = \left(2.70 \times 10^{25}\right)e^{-11.5} = \boxed{2.70 \times 10^{20}}.$$

**P21.37** (a) $$v_{\text{av}} = \frac{\sum n_i v_i}{N} = \frac{1}{15}\left[1(2)+2(3)+3(5)+4(7)+3(9)+2(12)\right] = \boxed{6.80\ \text{m/s}}$$

(b) $$\left(v^2\right)_{\text{av}} = \frac{\sum n_i v_i^2}{N} = 54.9\ \text{m}^2/\text{s}^2$$

so $v_{\text{rms}} = \sqrt{\left(v^2\right)_{\text{av}}} = \sqrt{54.9} = \boxed{7.41\ \text{m/s}}$

(c) $$v_{\text{mp}} = \boxed{7.00\ \text{m/s}}$$

**P21.38** (a) $$\frac{V_{\text{rms, }35}}{V_{\text{rms, }37}} = \frac{\sqrt{\frac{3RT}{M_{35}}}}{\sqrt{\frac{3RT}{M_{37}}}} = \left(\frac{37.0\ \text{g/mol}}{35.0\ \text{g/mol}}\right)^{1/2} = \boxed{1.03}$$

(b) The lighter atom, $\boxed{{}^{35}\text{Cl}}$, moves faster.

**P21.39** In the Maxwell Boltzmann speed distribution function take $\frac{dN_v}{dv} = 0$ to find

$$4\pi N\left(\frac{m}{2\pi k_B T}\right)^{3/2} \exp\left(-\frac{mv^2}{2k_B T}\right)\left(2v - \frac{2mv^3}{2k_B T}\right) = 0$$

and solve for $v$ to find the most probable speed.

Reject as solutions $v = 0$ and $v = \infty$

Retain only $2 - \frac{mv^2}{k_B T} = 0$

Then $v_{\text{mp}} = \sqrt{\frac{2k_B T}{m}}$

**P21.40** The most probable speed is $v_{\text{mp}} = \sqrt{\frac{2k_B T}{m}} = \sqrt{\frac{2\left(1.38\times10^{-23}\ \text{J/K}\right)(4.20\ \text{K})}{6.64\times10^{-27}\ \text{kg}}} = \boxed{132\ \text{m/s}}$.

**P21.41** (a) From $v_{\text{av}} = \sqrt{\frac{8k_B T}{\pi m}}$

we find the temperature as $T = \frac{\pi\left(6.64\times10^{-27}\ \text{kg}\right)\left(1.12\times10^{4}\ \text{m/s}\right)^2}{8\left(1.38\times10^{-23}\ \text{J/mol}\cdot\text{K}\right)} = \boxed{2.37\times10^{4}\ \text{K}}$

(b) $$T = \frac{\pi\left(6.64\times10^{-27}\ \text{kg}\right)\left(2.37\times10^{3}\ \text{m/s}\right)^2}{8\left(1.38\times10^{-23}\ \text{J/mol}\cdot\text{K}\right)} = \boxed{1.06\times10^{3}\ \text{K}}$$

**P21.42** At 0°C, $\frac{1}{2}mv_{\text{rms0}}^2 = \frac{3}{2}k_B T_0$

At the higher temperature, $\frac{1}{2}m\left(2v_{\text{rms0}}\right)^2 = \frac{3}{2}k_B T$

$$T = 4T_0 = 4(273\ \text{K}) = 1\,092\ \text{K} = \boxed{819°\text{C}}.$$

***P21.43** (a) From the Boltzmann distribution law, the number density of molecules with gravitational energy $mgy$ is $n_0 e^{-mgy/k_BT}$. These are the molecules with height $y$, so this is the number per volume at height $y$ as a function of $y$.

(b)

$$\frac{n(y)}{n_0} = e^{-mgy/k_BT} = e^{-Mgy/N_Ak_BT} = e^{-Mgy/RT}$$

$$= e^{-\left(28.9\times10^{-3}\text{ kg/mol}\right)\left(9.8\text{ m/s}^2\right)\left(11\times10^{3}\text{ m}\right)/\left(8.314\text{ J/mol}\cdot\text{K}\right)\left(293\text{ K}\right)}$$

$$= e^{-1.279} = \boxed{0.278}$$

***P21.44** (a) We calculate

$$\int_0^\infty e^{-mgy/k_BT}\,dy = \int_{y=0}^{\infty} e^{-mgy/k_BT}\left(-\frac{mg\,dy}{k_BT}\right)\left(-\frac{k_BT}{mg}\right)$$

$$= -\frac{k_BT}{mg}e^{-mgy/k_BT}\Big|_0^\infty = -\frac{k_BT}{mg}(0-1) = \frac{k_BT}{mg}$$

Using Table B.6 in the appendix

$$\int_0^\infty y e^{-mgy/k_BT}\,dy = \frac{1!}{\left(mg/k_BT\right)^2} = \left(\frac{k_BT}{mg}\right)^2.$$

Then $\bar{y} = \dfrac{\int_0^\infty y e^{-mgy/k_BT}\,dy}{\int_0^\infty e^{-mgy/k_BT}\,dy} = \dfrac{\left(k_BT/mg\right)^2}{k_BT/mg} = \dfrac{k_BT}{mg}.$

(b)

$$\bar{y} = \frac{k_BT}{\left(M/N_A\right)g} = \frac{RT}{Mg} = \frac{8.314\text{ J }283\text{ K s}^2}{\text{mol}\cdot\text{K }28.9\times10^{-3}\text{ kg }9.8\text{ m}} = \boxed{8.31\times10^{3}\text{ m}}$$

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## Section 21.7 **Mean Free Path**

**P21.45** (a) $PV = \left(\dfrac{N}{N_A}\right)RT$ and $N = \dfrac{PVN_A}{RT}$ so that

$$N = \frac{\left(1.00\times10^{-10}\right)(133)(1.00)\left(6.02\times10^{23}\right)}{(8.314)(300)} = \boxed{3.21\times10^{12}\text{ molecules}}$$

(b)

$$\ell = \frac{1}{n_V\pi d^2 2^{1/2}} = \frac{V}{N\pi d^2 2^{1/2}} = \frac{1.00\text{ m}^3}{\left(3.21\times10^{12}\text{ molecules}\right)\pi\left(3.00\times10^{-10}\text{ m}\right)^2(2)^{1/2}}$$

$$\ell = \boxed{779\text{ km}}$$

(c)

$$f = \frac{v}{\ell} = \boxed{6.42\times10^{-4}\text{ s}^{-1}}$$

**P21.46** The average molecular speed is

$$v=\sqrt{\frac{8k_BT}{\pi m}}=\sqrt{\frac{8k_BN_AT}{\pi N_Am}}$$

$$v=\sqrt{\frac{8RT}{\pi M}}$$

$$v=\sqrt{\frac{8(8.314\ \text{J/mol}\cdot\text{K})3.00\ \text{K}}{\pi\left(2.016\times10^{-3}\ \text{kg/mol}\right)}}$$

$$v=178\ \text{m/s}$$

(a) The mean free path is

$$\ell=\frac{1}{\sqrt{2}\pi d^2n_V}=\frac{1}{\sqrt{2}\pi\left(0.200\times10^{-9}\ \text{m}\right)^2\ 1/\text{m}^3}$$

$$\ell=\boxed{5.63\times10^{18}\ \text{m}}$$

The mean free time is

$$\frac{\ell}{v}=\frac{5.63\times10^{18}\ \text{m}}{178\ \text{m/s}}=3.17\times10^{16}\ \text{s}=\boxed{1.00\times10^{9}\ \text{yr}}.$$

(b) Now $n_V$ is $10^6$ times larger, to make $\ell$ smaller by $10^6$ times:

$$\ell=\boxed{5.63\times10^{12}\ \text{m}}.$$

Thus, $\frac{\ell}{v}=3.17\times10^{10}\ \text{s}=\boxed{1.00\times10^{3}\ \text{yr}}$.

**P21.47** From Equation 21.30, $\ell=\frac{1}{\sqrt{2}\pi d^2n_V}$

For an ideal gas, $n_V=\frac{N}{V}=\frac{P}{k_BT}$

Therefore, $\ell=\frac{k_BT}{\sqrt{2}\pi d^2P}$, as required.

**P21.48** $\ell=\left[\sqrt{2}\pi d^2n_V\right]^{-1}$ $\qquad$ $n_V=\frac{P}{k_BT}$

$d=3.60\times10^{-10}\ \text{m}$ $\qquad$ $n_V=\frac{1.013\times10^5}{\left(1.38\times10^{-23}\right)(293)}=2.51\times10^{25}/\text{m}^3$

$\therefore\ \ell=6.93\times10^{-8}\ \text{m}$, or about $\boxed{\text{193 molecular diameters}}$.

**P21.49** Using $P = n_V k_B T$, Equation 21.30 becomes $\ell = \dfrac{k_B T}{\sqrt{2}\pi P d^2}$ (1)

(a) $$\ell = \frac{\left(1.38 \times 10^{-23}\ \text{J/K}\right)(293\ \text{K})}{\sqrt{2}\pi\left(1.013 \times 10^{5}\ \text{Pa}\right)\left(3.10 \times 10^{-10}\ \text{m}\right)^2} = \boxed{9.36 \times 10^{-8}\ \text{m}}$$

(b) Equation (1) shows that $P_1\ell_1 = P_2\ell_2$. Taking $P_1\ell_1$ from (a) and with $\ell_2 = 1.00$ m, we find

$$P_2 = \frac{(1.00\ \text{atm})\left(9.36 \times 10^{-8}\ \text{m}\right)}{1.00\ \text{m}} = \boxed{9.36 \times 10^{-8}\ \text{atm}}.$$

(c) For $\ell_3 = 3.10 \times 10^{-10}$ m, we have

$$P_3 = \frac{(1.00\ \text{atm})\left(9.36 \times 10^{-8}\ \text{m}\right)}{3.10 \times 10^{-10}\ \text{m}} = \boxed{302\ \text{atm}}.$$

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## Additional Problems

**P21.50** (a) $$n = \frac{PV}{RT} = \frac{(1.013 \times 10^5\ \text{Pa})(4.20\ \text{m} \times 3.00\ \text{m} \times 2.50\ \text{m})}{(8.314\ \text{J/mol}\cdot\text{K})(293\ \text{K})} = 1.31 \times 10^3\ \text{mol}$$

$$N = nN_A = \left(1.31 \times 10^3\ \text{mol}\right)\left(6.02 \times 10^{23}\ \text{molecules/mol}\right)$$

$$N = \boxed{7.89 \times 10^{26}\ \text{molecules}}$$

(b) $$m = nM = \left(1.31 \times 10^3\ \text{mol}\right)(0.028\,9\ \text{kg/mol}) = \boxed{37.9\ \text{kg}}$$

(c) $$\frac{1}{2}m_0 v^2 = \frac{3}{2}k_B T = \frac{3}{2}\left(1.38 \times 10^{-23}\ \text{J/k}\right)(293\ \text{K}) = \boxed{6.07 \times 10^{-21}\ \text{J/molecule}}$$

(d) For one molecule,

$$m_0 = \frac{M}{N_A} = \frac{0.028\,9\ \text{kg/mol}}{6.02 \times 10^{23}\ \text{molecules/mol}} = 4.80 \times 10^{-26}\ \text{kg/molecule}$$

$$v_{\text{rms}} = \sqrt{\frac{2\left(6.07 \times 10^{-21}\ \text{J/molecule}\right)}{4.80 \times 10^{-26}\ \text{kg/molecule}}} = \boxed{503\ \text{m/s}}$$

(e),(f) $$E_{\text{int}} = nC_V T = n\left(\frac{5}{2}R\right)T = \frac{5}{2}PV$$

$$E_{\text{int}} = \frac{5}{2}\left(1.013 \times 10^5\ \text{Pa}\right)\left(31.5\ \text{m}^3\right) = \boxed{7.98\ \text{MJ}}$$

**P21.51** (a) $P_f = \boxed{100 \text{ kPa}}$ $T_f = \boxed{400 \text{ K}}$

$$V_f = \frac{nRT_f}{P_f} = \frac{2.00 \text{ mol}(8.314 \text{ J/mol}\cdot\text{K})(400 \text{ K})}{100\times10^3 \text{ Pa}} = 0.066\,5 \text{ m}^3 = \boxed{66.5 \text{ L}}$$

$$\Delta E_{\text{int}} = (3.50)nR\Delta T = 3.50(2.00 \text{ mol})(8.314 \text{ J/mol}\cdot\text{K})(100 \text{ K}) = \boxed{5.82 \text{ kJ}}$$

$$W = -P\Delta V = -nR\Delta T = -(2.00 \text{ mol})(8.314 \text{ J/mol}\cdot\text{K})(100 \text{ K}) = \boxed{-1.66 \text{ kJ}}$$

$$Q = \Delta E_{\text{int}} - W = 5.82 \text{ kJ} + 1.66 \text{ kJ} = \boxed{7.48 \text{ kJ}}$$

(b) $T_f = \boxed{400 \text{ K}}$ $V_f = V_i = \dfrac{nRT_i}{P_i} = \dfrac{2.00 \text{ mol}(8.314 \text{ J/mol}\cdot\text{K})(300 \text{ K})}{100\times10^3 \text{ Pa}} = 0.049\,9 \text{ m}^3 = \boxed{49.9 \text{ L}}$

$P_f = P_i\left(\dfrac{T_f}{T_i}\right) = 100 \text{ kPa}\left(\dfrac{400 \text{ K}}{300 \text{ K}}\right) = \boxed{133 \text{ kPa}}$ $W = -\int P\,dV = \boxed{0}$ since $V = \text{constant}$

$\Delta E_{\text{int}} = \boxed{5.82 \text{ kJ}}$ as in part (a) $Q = \Delta E_{\text{int}} - W = 5.82 \text{ kJ} - 0 = \boxed{5.82 \text{ kJ}}$

(c) $P_f = \boxed{120 \text{ kPa}}$ $T_f = \boxed{300 \text{ K}}$

$V_f = V_i\left(\dfrac{P_i}{P_f}\right) = 49.9 \text{ L}\left(\dfrac{100 \text{ kPa}}{120 \text{ kPa}}\right) = \boxed{41.6 \text{ L}}$ $\Delta E_{\text{int}} = (3.50)nR\Delta T = \boxed{0}$ since $T = \text{constant}$

$$W = -\int P\,dV = -nRT_i\int_{V_i}^{V_f}\frac{dV}{V} = -nRT_i\ln\left(\frac{V_f}{V_i}\right) = -nRT_i\ln\left(\frac{P_i}{P_f}\right)$$

$$W = -(2.00 \text{ mol})(8.314 \text{ J/mol}\cdot\text{K})(300 \text{ K})\ln\left(\frac{100 \text{ kPa}}{120 \text{ kPa}}\right) = \boxed{+909 \text{ J}}$$

$$Q = \Delta E_{\text{int}} - W = 0 - 910 \text{ J} = \boxed{-909 \text{ J}}$$

(d) $P_f = \boxed{120 \text{ kPa}}$ $\gamma = \dfrac{C_P}{C_V} = \dfrac{C_V + R}{C_V} = \dfrac{3.50R + R}{3.50R} = \dfrac{4.50}{3.50} = \dfrac{9}{7}$

$P_fV_f^{\gamma} = P_iV_i^{\gamma}$: so $V_f = V_i\left(\dfrac{P_i}{P_f}\right)^{1/\gamma} = 49.9 \text{ L}\left(\dfrac{100 \text{ kPa}}{120 \text{ kPa}}\right)^{7/9} = \boxed{43.3 \text{ L}}$

$$T_f = T_i\left(\frac{P_fV_f}{P_iV_i}\right) = 300 \text{ K}\left(\frac{120 \text{ kPa}}{100 \text{ kPa}}\right)\left(\frac{43.3 \text{ L}}{49.9 \text{ L}}\right) = \boxed{312 \text{ K}}$$

$$\Delta E_{\text{int}} = (3.50)nR\Delta T = 3.50(2.00 \text{ mol})(8.314 \text{ J/mol}\cdot\text{K})(12.4 \text{ K}) = \boxed{722 \text{ J}}$$

$Q = \boxed{0}$ (adiabatic process)

$$W = -Q + \Delta E_{\text{int}} = 0 + 722 \text{ J} = \boxed{+722 \text{ J}}$$

**P21.52** (a) The average speed $v_{\text{av}}$ is just the weighted average of all the speeds.

$$v_{\text{av}} = \frac{\left[2(v)+3(2v)+5(3v)+4(4v)+3(5v)+2(6v)+1(7v)\right]}{(2+3+5+4+3+2+1)} = \boxed{3.65v}$$

(b) First find the average of the square of the speeds,

$$v_{\text{av}}^2 = \frac{\left[2(v)^2+3(2v)^2+5(3v)^2+4(4v)^2+3(5v)^2+2(6v)^2+1(7v)^2\right]}{2+3+5+4+3+2+1} = 15.95v^2.$$

The root-mean square speed is then $v_{\text{rms}} = \sqrt{v_{\text{av}}^2} = \boxed{3.99v}$.

(c) The most probable speed is the one that most of the particles have;
i.e., five particles have speed $\boxed{3.00v}$.

(d) $PV = \frac{1}{3}Nmv_{\text{av}}^2$

Therefore, $P = \frac{20}{3}\frac{\left[m(15.95)v^2\right]}{V} = \boxed{106\left(\frac{mv^2}{V}\right)}$.

(e) The average kinetic energy for each particle is
$\overline{K} = \frac{1}{2}mv_{\text{av}}^2 = \frac{1}{2}m\left(15.95v^2\right) = \boxed{7.98mv^2}$.

**P21.53** (a) $PV^{\gamma} = k$. So, $W = -\int_i^f PdV = -k\int_i^f \frac{dV}{V^{\gamma}} = \frac{P_fV_f - P_iV_i}{\gamma - 1}$

(b) $dE_{\text{int}} = dQ + dW$ and $dQ = 0$ for an adiabatic process.

Therefore, $W = +\Delta E_{\text{int}} = nC_V\left(T_f - T_i\right)$.

To show consistency between these 2 equations, consider that $\gamma = \frac{C_P}{C_V}$ and $C_P - C_V = R$.

Therefore, $\frac{1}{\gamma - 1} = \frac{C_V}{R}$.

Using this, the result found in part (a) becomes

$$W = \left(P_fV_f - P_iV_i\right)\frac{C_V}{R}.$$

Also, for an ideal gas $\frac{PV}{R} = nT$ so that $W = nC_V\left(T_f - T_i\right)$.

***P21.54** (a)

$$W = nC_V\left(T_f - T_i\right)$$

$$-2\,500\text{ J} = 1\text{ mol}\frac{3}{2}8.314\text{ J/mol}\cdot\text{K}\left(T_f - 500\text{ K}\right)$$

$$T_f = \boxed{300\text{ K}}$$

(b)

$$P_i V_i^\gamma = P_f V_f^\gamma$$

$$P_i\left(\frac{nRT_i}{P_i}\right)^\gamma = P_f\left(\frac{nRT_f}{P_f}\right)^\gamma \qquad T_i^\gamma P_i^{1-\gamma} = T_f^\gamma P_f^{1-\gamma}$$

$$\frac{T_i^{\gamma/(\gamma-1)}}{P_i} = \frac{T_f^{\gamma/(\gamma-1)}}{P_f} \qquad P_f = P_i\left(\frac{T_f}{T_i}\right)^{\gamma/(\gamma-1)}$$

$$P_f = P_i\left(\frac{T_f}{T_i}\right)^{(5/3)(3/2)} = 3.60\text{ atm}\left(\frac{300}{500}\right)^{5/2} = \boxed{1.00\text{ atm}}$$

***P21.55** Let the subscripts '1' and '2' refer to the hot and cold compartments, respectively. The pressure is higher in the hot compartment, therefore the hot compartment expands and the cold compartment contracts. The work done by the adiabatically expanding gas is equal and opposite to the work done by the adiabatically compressed gas.

$$\frac{nR}{\gamma-1}\left(T_{1i} - T_{1f}\right) = -\frac{nR}{\gamma-1}\left(T_{2i} - T_{2f}\right)$$

$$\therefore T_{1f} + T_{2f} = T_{1i} + T_{2i} = 800\text{ K} \qquad (1)$$

Consider the adiabatic changes of the gases.

$P_{1i}V_{1i}^\gamma = P_{1f}V_{1f}^\gamma$ and $P_{2i}V_{2i}^\gamma = P_{2f}V_{2f}^\gamma$

$$\therefore \frac{P_{1i}V_{1i}^\gamma}{P_{2i}V_{2i}^\gamma} = \frac{P_{1f}V_{1f}^\gamma}{P_{2f}V_{2f}^\gamma}$$

$\therefore \dfrac{P_{1i}}{P_{2i}} = \left(\dfrac{V_{1f}}{V_{2f}}\right)^\gamma$, since $V_{1i} = V_{2i}$ and $P_{1f} = P_{2f}$

$\therefore \dfrac{nRT_{1i}/V_{1i}}{nRT_{2i}/V_{2i}} = \left(\dfrac{nRT_{1f}/P_{1f}}{nRT_{2f}/P_{2f}}\right)^\gamma$, using the ideal gas law

$\therefore \dfrac{T_{1i}}{T_{2i}} = \left(\dfrac{T_{1f}}{T_{2f}}\right)^\gamma$, since $V_{1i} = V_{2i}$ and $P_{1f} = P_{2f}$

$\therefore \dfrac{T_{1f}}{T_{2f}} = \left(\dfrac{T_{1i}}{T_{2i}}\right)^{1/\gamma} = \left(\dfrac{550\text{ K}}{250\text{ K}}\right)^{1/1.4} = 1.756 \quad (2)$

Solving equations (1) and (2) simultaneously gives

$$\boxed{T_{1f} = 510\text{ K},\ T_{2f} = 290\text{ K}}.$$

***P21.56** The work done by the gas on the bullet becomes its kinetic energy:

$$\frac{1}{2}mv^2=\frac{1}{2}1.1\times10^{-3}\text{ kg}(120\text{ m/s})^2=7.92\text{ J}.$$

The work on the gas is

$$\frac{1}{\gamma-1}\left(P_fV_f-P_iV_i\right)=-7.92\text{ J}.$$

Also $P_fV_f^\gamma=P_iV_i^\gamma \qquad P_f=P_i\left(\dfrac{V_i}{V_f}\right)^\gamma$.

So $-7.92\text{ J}=\dfrac{1}{0.40}P_i\left[V_f\left(\dfrac{V_i}{V_f}\right)^\gamma-V_i\right]$.

And $V_f=12\text{ cm}^3+50\text{ cm }0.03\text{ cm}^2=13.5\text{ cm}^3$.

Then $P_i=\dfrac{-7.92\text{ J}(0.40)10^6\text{ cm}^3/\text{m}^3}{\left[13.5\text{ cm}^3\left(\frac{12}{13.5}\right)^{1.40}-12\text{ cm}^3\right]}=\boxed{5.74\times10^6\text{ Pa}}=56.6\text{ atm}$.

**P21.57** The pressure of the gas in the lungs of the diver must be the same as the absolute pressure of the water at this depth of 50.0 meters. This is:

$$P=P_0+\rho gh=1.00\text{ atm}+\left(1.03\times10^3\text{ kg/m}^3\right)\left(9.80\text{ m/s}^2\right)(50.0\text{ m})$$

or $\quad P=1.00\text{ atm}+5.05\times10^5\text{ Pa}\left(\dfrac{1.00\text{ atm}}{1.013\times10^5\text{ Pa}}\right)=5.98\text{ atm}$

If the partial pressure due to the oxygen in the gas mixture is to be 1.00 atmosphere (or the fraction $\frac{1}{5.98}$ of the total pressure) oxygen molecules should make up only $\frac{1}{5.98}$ of the total number of molecules. This will be true if 1.00 mole of oxygen is used for every 4.98 mole of helium. The ratio by weight is then

$$\frac{(4.98\text{ mol He})(4.003\text{ g/mol He})g}{(1.00\text{ mol O}_2)(2\times15.999\text{ g/mol O}_2)g}=\boxed{0.623}.$$

**P21.58** (a) Maxwell's speed distribution function is

$$N_v=4\pi N\left(\frac{m}{2\pi k_BT}\right)^{3/2}v^2e^{-mv^2/2k_BT}$$

With $\quad N=1.00\times10^4$,

$m=\dfrac{M}{N_A}=\dfrac{0.032\text{ kg}}{6.02\times10^{23}}=5.32\times10^{-26}\text{ kg}$

$T=500\text{ K}$

and $\quad k_B=1.38\times10^{-23}\text{ J/molecule}\cdot\text{K}$

this becomes $\quad N_v=\left(1.71\times10^{-4}\right)v^2e^{-\left(3.85\times10^{-6}\right)v^2}$

To the right is a plot of this function for the range $0\le v\le1\,500\text{ m/s}$.

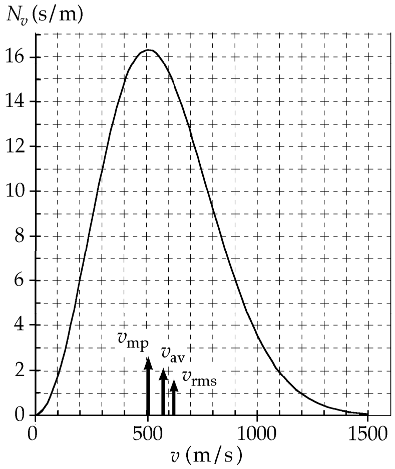


**FIG. P21.58(a)**

*continued on next page*

(b) The most probable speed occurs where $N_v$ is a maximum.

From the graph, $\boxed{v_{\text{mp}} \approx 510 \text{ m/s}}$

(c) $$v_{\text{av}} = \sqrt{\frac{8k_BT}{\pi m}} = \sqrt{\frac{8\left(1.38\times10^{-23}\right)(500)}{\pi\left(5.32\times10^{-26}\right)}} = \boxed{575 \text{ m/s}}$$

Also,

$$v_{\text{rms}} = \sqrt{\frac{3k_BT}{m}} = \sqrt{\frac{3\left(1.38\times10^{-23}\right)(500)}{5.32\times10^{-26}}} = \boxed{624 \text{ m/s}}$$

(d) The fraction of particles in the range $300 \text{ m/s} \le v \le 600 \text{ m/s}$

is $$\frac{\int_{300}^{600} N_v dv}{N}$$

where $N = 10^4$

and the integral of $N_v$ is read from the graph as the area under the curve.

This is approximately 4 400 and the fraction is 0.44 or $\boxed{44\%}$.

**P21.59** (a) Since $\boxed{\text{pressure increases as volume decreases}}$ (and vice versa),

$$\frac{dV}{dP} < 0 \text{ and } -\frac{1}{V}\left[\frac{dV}{dP}\right] > 0.$$

(b) For an ideal gas, $V = \frac{nRT}{P}$ and $\kappa_1 = -\frac{1}{V}\frac{d}{dP}\left(\frac{nRT}{P}\right)$.

If the compression is isothermal, $T$ is constant and

$$\kappa_1 = -\frac{nRT}{V}\left(-\frac{1}{P^2}\right) = \frac{1}{P}.$$

(c) For an adiabatic compression, $PV^\gamma = C$ (where $C$ is a constant) and

$$\kappa_2 = -\frac{1}{V}\frac{d}{dP}\left(\frac{C}{P}\right)^{1/\gamma} = \frac{1}{V}\left(\frac{1}{\gamma}\right)\frac{C^{1/\gamma}}{P^{(1/\gamma)+1}} = \frac{P^{1/\gamma}}{\gamma P^{1/\gamma+1}} = \frac{1}{\gamma P}.$$

(d) $$\kappa_1 = \frac{1}{P} = \frac{1}{(2.00 \text{ atm})} = \boxed{0.500 \text{ atm}^{-1}}$$

$\gamma = \frac{C_P}{C_V}$ and for a monatomic ideal gas, $\gamma = \frac{5}{3}$, so that

$$\kappa_2 = \frac{1}{\gamma P} = \frac{1}{\frac{5}{3}(2.00 \text{ atm})} = \boxed{0.300 \text{ atm}^{-1}}$$

**P21.60** (a) The speed of sound is $v=\sqrt{\frac{B}{\rho}}$ where $B=-V\frac{dP}{dV}$.

According to Problem 59, in an adiabatic process, this is $B=\frac{1}{\kappa_2}=\gamma P$.

Also, $\rho=\frac{m_s}{V}=\frac{nM}{V}=\frac{(nRT)M}{V(RT)}=\frac{PM}{RT}$ where $m_s$ is the sample mass. Then, the speed of sound in the ideal gas is $v=\sqrt{\frac{B}{\rho}}=\sqrt{\gamma P\left(\frac{RT}{PM}\right)}=\boxed{\sqrt{\frac{\gamma RT}{M}}}$.

(b) $$v=\sqrt{\frac{1.40(8.314\ \text{J/mol}\cdot\text{K})(293\ \text{K})}{0.028\,9\ \text{kg/mol}}}=\boxed{344\ \text{m/s}}$$

This nearly agrees with the 343 m/s listed in Table 17.1.

(c) We use $k_B=\frac{R}{N_A}$ and $M=mN_A$: $v=\sqrt{\frac{\gamma RT}{M}}=\sqrt{\frac{\gamma\, k_B N_A T}{mN_A}}=\sqrt{\frac{\gamma\, k_B T}{m}}$.

The most probable molecular speed is $\sqrt{\frac{2k_BT}{m}}$,

the average speed is $\sqrt{\frac{8k_BT}{\pi m}}$, and the rms speed is $\sqrt{\frac{3k_BT}{m}}$.

$\boxed{\text{All are somewhat larger}}$ than the speed of sound.

**P21.61** $$n=\frac{m}{M}=\frac{1.20\ \text{kg}}{0.028\,9\ \text{kg/mol}}=41.5\ \text{mol}$$

(a) $$V_i=\frac{nRT_i}{P_i}=\frac{(41.5\ \text{mol})(8.314\ \text{J/mol}\cdot\text{K})(298\ \text{K})}{200\times10^3\ \text{Pa}}=\boxed{0.514\ \text{m}^3}$$

(b) $$\frac{P_f}{P_i}=\frac{\sqrt{V_f}}{\sqrt{V_i}}\ \text{so}\ V_f=V_i\left(\frac{P_f}{P_i}\right)^2=\left(0.514\ \text{m}^3\right)\left(\frac{400}{200}\right)^2=\boxed{2.06\ \text{m}^3}$$

(c) $$T_f=\frac{P_fV_f}{nR}=\frac{\left(400\times10^3\ \text{Pa}\right)\left(2.06\ \text{m}^3\right)}{(41.5\ \text{mol})(8.314\ \text{J/mol}\cdot\text{K})}=\boxed{2.38\times10^3\ \text{K}}$$

(d) $$W=-\int_{V_i}^{V_f}PdV=-C\int_{V_i}^{V_f}V^{1/2}dV=-\left(\frac{P_i}{V_i^{1/2}}\right)\frac{2V^{3/2}}{3}\Bigg|_{V_i}^{V_f}=-\frac{2}{3}\left(\frac{P_i}{V_i^{1/2}}\right)\left(V_f^{3/2}-V_i^{3/2}\right)$$

$$W=-\frac{2}{3}\left(\frac{200\times10^3\ \text{Pa}}{\sqrt{0.514\ \text{m}}}\right)\left[\left(2.06\ \text{m}^3\right)^{3/2}-(0.514\ \text{m})^{3/2}\right]=\boxed{-4.80\times10^5\ \text{J}}$$

(e) $$\Delta E_{\text{int}}=nC_V\Delta T=(41.5\ \text{mol})\left[\frac{5}{2}(8.314\ \text{J/mol}\cdot\text{K})\right]\left(2.38\times10^3-298\right)\text{K}$$

$$\Delta E_{\text{int}}=1.80\times10^6\ \text{J}$$

$$Q=\Delta E_{\text{int}}-W=1.80\times10^6\ \text{J}+4.80\times10^5\ \text{J}=2.28\times10^6\ \text{J}=\boxed{2.28\ \text{MJ}}$$

**P21.62** The ball loses energy $\frac{1}{2}mv_i^2-\frac{1}{2}mv_f^2=\frac{1}{2}(0.142\text{ kg})\left[(47.2)^2-(42.5)^2\right]\text{ m}^2/\text{s}^2=29.9\text{ J}$

The air volume is $V=\pi(0.037\,0\text{ m})^2(19.4\text{ m})=0.083\,4\text{ m}^3$

and its quantity is $n=\frac{PV}{RT}=\frac{1.013\times10^5\text{ Pa}\left(0.083\,4\text{ m}^3\right)}{(8.314\text{ J/mol}\cdot\text{K})(293\text{ K})}=3.47\text{ mol}$

The air absorbs energy according to

$$Q=nC_P\Delta T$$

So $\Delta T=\frac{Q}{nC_P}=\frac{29.9\text{ J}}{3.47\text{ mol}\left(\frac{7}{2}\right)(8.314\text{ J/mol}\cdot\text{K})}=\boxed{0.296^\circ\text{C}}$

**P21.63** $N_v(v)=4\pi N\left(\frac{m}{2\pi k_BT}\right)^{3/2}v^2\exp\left(\frac{-mv^2}{2k_BT}\right)$

Note that $v_{\text{mp}}=\left(\frac{2k_BT}{m}\right)^{1/2}$

Thus, $N_v(v)=4\pi N\left(\frac{m}{2\pi k_BT}\right)^{3/2}v^2e^{\left(-v^2/v_{\text{mp}}^2\right)}$

And $\frac{N_v(v)}{N_v\left(v_{\text{mp}}\right)}=\left(\frac{v}{v_{\text{mp}}}\right)^2e^{\left(1-v^2/v_{\text{mp}}^2\right)}$

For $v=\frac{v_{\text{mp}}}{50}$

$$\frac{N_v(v)}{N_v\left(v_{\text{mp}}\right)}=\left(\frac{1}{50}\right)^2e^{\left[1-(1/50)^2\right]}=1.09\times10^{-3}$$

The other values are computed similarly, with the following results:

| $\frac{v}{v_{\text{mp}}}$ | $\frac{N_v(v)}{N_v\left(v_{\text{mp}}\right)}$ |
|---|---|
| $\frac{1}{50}$ | $1.09\times10^{-3}$ |
| $\frac{1}{10}$ | $2.69\times10^{-2}$ |
| $\frac{1}{2}$ | 0.529 |
| 1 | 1.00 |
| 2 | 0.199 |
| 10 | $1.01\times10^{-41}$ |
| 50 | $1.25\times10^{-1\,082}$ |

To find the last value, note:

$$(50)^2e^{1-2\,500}=2\,500e^{-2\,499}$$

$$10^{\log 2\,500}e^{(\ln10)(-2\,499/\ln10)}=10^{\log 2\,500}10^{-2\,499/\ln10}=10^{\log 2\,500-2\,499/\ln10}=10^{-1\,081.904}$$

**P21.64** (a) The effect of high angular speed is like the effect of a very high gravitational field on an atmosphere. The result is:

$\boxed{\text{The larger-mass molecules settle to the outside}}$ while the region at smaller $r$ has a higher concentration of low-mass molecules.

(b) Consider a single kind of molecules, all of mass $m$. To cause the centripetal acceleration of the molecules between $r$ and $r+dr$, the pressure must increase outward according to $\sum F_r = ma_r$. Thus,

$$PA-(P+dP)A=-(nmA\,dr)\left(r\omega^2\right)$$

where $n$ is the number of molecules per unit volume and $A$ is the area of any cylindrical surface. This reduces to $dP = nm\omega^2 r dr$.

But also $P = nk_BT$, so $dP = k_BTdn$. Therefore, the equation becomes

$$\frac{dn}{n}=\frac{m\omega^2}{k_BT}rdr \text{ giving } \int_{n_0}^{n}\frac{dn}{n}=\frac{m\omega^2}{k_BT}\int_0^r rdr \text{ or}$$

$$\ln(n)\Big|_{n_0}^{n}=\frac{m\omega^2}{k_BT}\left(\frac{r^2}{2}\right)\Bigg|_0^r$$

$$\ln\left(\frac{n}{n_0}\right)=\frac{m\omega^2}{2k_BT}r^2 \text{ and solving for } n\text{: } \boxed{n=n_0e^{mr^2\omega^2/2k_BT}}.$$

**P21.65** First find $v_{\text{av}}^2$ as $v_{\text{av}}^2=\frac{1}{N}\int_0^\infty v^2N_v dv$. Let $a=\frac{m}{2k_BT}$.

Then, $v_{\text{av}}^2=\frac{\left[4N\pi^{-1/2}a^{3/2}\right]}{N}\int_0^\infty v^4e^{-av^2dv}=\left[4a^{3/2}\pi^{-1/2}\right]\frac{3}{8a^2}\sqrt{\frac{\pi}{a}}=\frac{3k_BT}{m}$

The root-mean square speed is then $v_{\text{rms}}=\sqrt{v_{\text{av}}^2}=\boxed{\sqrt{\frac{3k_BT}{m}}}$.

To find the average speed, we have

$$v_{\text{av}}=\frac{1}{N}\int_0^\infty vN_v dv=\frac{\left(4Na^{3/2}\pi^{-1/2}\right)}{N}\int_0^\infty v^3e^{-av^2}dv=\frac{4a^{3/2}\pi^{-1/2}}{2a^2}=\boxed{\sqrt{\frac{8k_BT}{\pi m}}}.$$

***P21.66** We want to evaluate $\frac{dP}{dV}$ for the function implied by $PV=nRT=\text{constant}$, and also for the different function implied by $PV^\gamma=\text{constant}$. We can use implicit differentiation:

From $PV=\text{constant}$ $\quad P\frac{dV}{dV}+V\frac{dP}{dV}=0 \quad \left(\frac{dP}{dV}\right)_{\text{isotherm}}=-\frac{P}{V}$

From $PV^\gamma=\text{constant}$ $\quad P\gamma V^{\gamma-1}+V^\gamma\frac{dP}{dV}=0 \quad \left(\frac{dP}{dV}\right)_{\text{adiabat}}=-\frac{\gamma P}{V}$

Therefore, $\quad \left(\frac{dP}{dV}\right)_{\text{adiabat}}=\gamma\left(\frac{dP}{dV}\right)_{\text{isotherm}}$

The theorem is proved.

**P21.67** (a) $$n=\frac{PV}{RT}=\frac{\left(1.013\times10^{5}\text{ Pa}\right)\left(5.00\times10^{-3}\text{ m}^{3}\right)}{(8.314\text{ J/mol}\cdot\text{K})(300\text{ K})}=\boxed{0.203\text{ mol}}$$

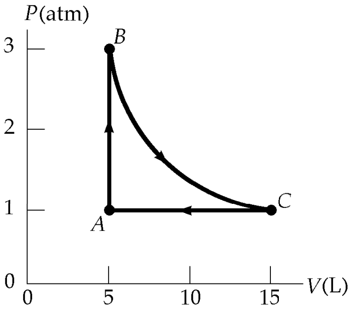


**FIG. P21.67**

(b) $$T_B=T_A\left(\frac{P_B}{P_A}\right)=300\text{ K}\left(\frac{3.00}{1.00}\right)=\boxed{900\text{ K}}$$

$$T_C=T_B=\boxed{900\text{ K}}$$

$$V_C=V_A\left(\frac{T_C}{T_A}\right)=5.00\text{ L}\left(\frac{900}{300}\right)=\boxed{15.0\text{ L}}$$

(c) $$E_{\text{int},A}=\frac{3}{2}nRT_A=\frac{3}{2}(0.203\text{ mol})(8.314\text{ J/mol}\cdot\text{K})(300\text{ K})=\boxed{760\text{ J}}$$

$$E_{\text{int},B}=E_{\text{int},C}=\frac{3}{2}nRT_B=\frac{3}{2}(0.203\text{ mol})(8.314\text{ J/mol}\cdot\text{K})(900\text{ K})=\boxed{2.28\text{ kJ}}$$

(d)

| | $P$ (atm) | $V$(L) | $T$(K) | $E_{\text{int}}$ (kJ) |
|---|---|---|---|---|
| $A$ | 1.00 | 5.00 | 300 | 0.760 |
| $B$ | 3.00 | 5.00 | 900 | 2.28 |
| $C$ | 1.00 | 15.00 | 900 | 2.28 |

(e) For the process *AB*, lock the piston in place and put the cylinder into an oven at 900 K. For *BC*, keep the sample in the oven while gradually letting the gas expand to lift a load on the piston as far as it can. For *CA*, carry the cylinder back into the room at 300 K and let the gas cool without touching the piston.

(f) For *AB*: $W=\boxed{0}$ $\Delta E_{\text{int}}=E_{\text{int},B}-E_{\text{int},A}=(2.28-0.760)\text{ kJ}=\boxed{1.52\text{ kJ}}$

$$Q=\Delta E_{\text{int}}-W=\boxed{1.52\text{ kJ}}$$

For *BC*: $\Delta E_{\text{int}}=\boxed{0}$, $W=-nRT_B\ln\left(\frac{V_C}{V_B}\right)$

$$W=-(0.203\text{ mol})(8.314\text{ J/mol}\cdot\text{K})(900\text{ K})\ln(3.00)=\boxed{-1.67\text{ kJ}}$$

$$Q=\Delta E_{\text{int}}-W=\boxed{1.67\text{ kJ}}$$

For *CA*: $\Delta E_{\text{int}}=E_{\text{int},A}-E_{\text{int},C}=(0.760-2.28)\text{ kJ}=\boxed{-1.52\text{ kJ}}$

$$W=-P\Delta V=-nR\Delta T=-(0.203\text{ mol})(8.314\text{ J/mol}\cdot\text{K})(-600\text{ K})=\boxed{1.01\text{ kJ}}$$

$$Q=\Delta E_{\text{int}}-W=-1.52\text{ kJ}-1.01\text{ kJ}=\boxed{-2.53\text{ kJ}}$$

(g) We add the amounts of energy for each process to find them for the whole cycle.

$$Q_{ABCA}=+1.52\text{ kJ}+1.67\text{ kJ}-2.53\text{ kJ}=\boxed{0.656\text{ kJ}}$$

$$W_{ABCA}=0-1.67\text{ kJ}+1.01\text{ kJ}=\boxed{-0.656\text{ kJ}}$$

$$\left(\Delta E_{\text{int}}\right)_{ABCA}=+1.52\text{ kJ}+0-1.52\text{ kJ}=\boxed{0}$$

**P21.68** (a) $$(10\,000\text{ g})\left(\frac{1.00\text{ mol}}{18.0\text{ g}}\right)\left(\frac{6.02\times10^{23}\text{ molecules}}{1.00\text{ mol}}\right)=\boxed{3.34\times10^{26}\text{ molecules}}$$

(b) After one day, $10^{-1}$ of the original molecules would remain. After two days, the fraction would be $10^{-2}$, and so on. After 26 days, only 3 of the original molecules would likely remain, and after $\boxed{27\text{ days}}$, likely none.

(c) The soup is this fraction of the hydrosphere: $\left(\dfrac{10.0\text{ kg}}{1.32\times10^{21}\text{ kg}}\right)$.

Therefore, today's soup likely contains this fraction of the original molecules. The number of original molecules likely in the pot again today is:

$$\left(\frac{10.0\text{ kg}}{1.32\times10^{21}\text{ kg}}\right)\left(3.34\times10^{26}\text{ molecules}\right)=\boxed{2.53\times10^{6}\text{ molecules}}.$$

**P21.69** (a) For escape, $\frac{1}{2}mv^2=\frac{GmM}{R_E}$. Since the free-fall acceleration at the surface is $g=\frac{GM}{R_E^2}$, this can also be written as: $\frac{1}{2}mv^2=\frac{GmM}{R_E}=\boxed{mgR_E}$.

(b) For $O_2$, the mass of one molecule is

$$m=\frac{0.032\,0\text{ kg/mol}}{6.02\times10^{23}\text{ molecules/mol}}=5.32\times10^{-26}\text{ kg/molecule}.$$

Then, if $mgR_E=10\left(\frac{3k_BT}{2}\right)$, the temperature is

$$T=\frac{mgR_E}{15k_B}=\frac{\left(5.32\times10^{-26}\text{ kg}\right)\left(9.80\text{ m/s}^2\right)\left(6.37\times10^{6}\text{ m}\right)}{15\left(1.38\times10^{-23}\text{ J/mol}\cdot\text{K}\right)}=\boxed{1.60\times10^{4}\text{ K}}.$$

**P21.70** (a) For sodium atoms (with a molar mass $M=32.0\text{ g/mol}$)

$$\frac{1}{2}mv^2=\frac{3}{2}k_BT$$

$$\frac{1}{2}\left(\frac{M}{N_A}\right)v^2=\frac{3}{2}k_BT$$

$$v_{\text{rms}}=\sqrt{\frac{3RT}{M}}=\sqrt{\frac{3(8.314\text{ J/mol}\cdot\text{K})\left(2.40\times10^{-4}\text{ K}\right)}{23.0\times10^{-3}\text{ kg}}}=\boxed{0.510\text{ m/s}}$$

(b) $$t=\frac{d}{v_{\text{rms}}}=\frac{0.010\text{ m}}{0.510\text{ m/s}}=\boxed{20\text{ ms}}$$

## ANSWERS TO EVEN PROBLEMS

| | |
|---|---|
| **P21.2** | 17.6 kPa |
| **P21.4** | $5.05 \times 10^{-21}$ J/molecule |
| **P21.6** | $6.64 \times 10^{-27}$ kg |
| **P21.8** | 477 m/s |
| **P21.10** | (a) 2.28 kJ; (b) $6.21 \times 10^{-21}$ J |
| **P21.12** | 74.8 J |
| **P21.14** | 7.52 L |
| **P21.16** | (a) 118 kJ; (b) $6.03 \times 10^{3}$ kg |
| **P21.18** | (a) 719 J/kg·K; (b) 0.811 kg; (c) 233 kJ;<br>(d) 327 kJ |
| **P21.20** | $13.5PV$ |
| **P21.22** | (a) $4T_i$; (b) $9(1\text{ mol})RT_i$ |
| **P21.24** | (a) 0.118; (b) 2.35; (c) 0; 135 J; 135 J |
| **P21.26** | (a) $5.15 \times 10^{-5}$ m$^3$; (b) 560 K; (c) 2.24 K |
| **P21.28** | (a) 1.55; (b) 0.127 m$^3$ |
| **P21.30** | (a) see the solution; (b) $2.19V_i$; (c) $3T_i$;<br>(d) $T_i$; (e) $-0.830P_iV_i$ |
| **P21.32** | 25.0 kW |
| **P21.34** | see the solution |
| **P21.36** | (a) No atom, almost all the time;<br>(b) $2.70 \times 10^{20}$ |
| **P21.38** | (a) 1.03; (b) $^{35}$Cl |
| **P21.40** | 132 m/s |
| **P21.42** | 819°C |
| **P21.44** | (a) see the solution; (b) 8.31 km |
| **P21.46** | (a) $5.63 \times 10^{18}$ m; $1.00 \times 10^{9}$ yr;<br>(b) $5.63 \times 10^{12}$ m; $1.00 \times 10^{3}$ yr |
| **P21.48** | 193 molecular diameters |
| **P21.50** | (a) $7.89 \times 10^{26}$ molecules; (b) 37.9 kg;<br>(c) $6.07 \times 10^{-21}$ J/molecule; (d) 503 m/s;<br>(e) 7.98 MJ; (f) 7.98 MJ |
| **P21.52** | (a) $3.65v$; (b) $3.99v$; (c) $3.00v$;<br>(d) $106\left(\frac{mv^2}{V}\right)$; (e) $7.98mv^2$ |
| **P21.54** | (a) 300 K; (b) 1.00 atm |
| **P21.56** | $5.74 \times 10^{6}$ Pa |
| **P21.58** | (a) see the solution; (b) $5.1 \times 10^{2}$ m/s;<br>(c) $v_{\text{av}} = 575$ m/s; $v_{\text{rms}} = 624$ m/s; (d) 44% |
| **P21.60** | (a) see the solution; (b) 344 m/s nearly agreeing with the tabulated value;<br>(c) see the solution; somewhat smaller than each |
| **P21.62** | 0.296°C |
| **P21.64** | see the solution |
| **P21.66** | see the solution |
| **P21.68** | (a) $3.34 \times 10^{26}$ molecules; (b) during the 27th day; (c) $2.53 \times 10^{6}$ molecules |
| **P21.70** | (a) 0.510 m/s; (b) 20 ms |

# 22

# Heat Engines, Entropy, and the Second Law of Thermodynamics

## CHAPTER OUTLINE

22.1 Heat Engines and the Second Law of Thermodynamics
22.2 Heat Pumps and Refrigerators
22.3 Reversible and Irreversible Processes
22.4 The Carnot Engine
22.5 Gasoline and Diesel Engines
22.6 Entropy
22.7 Entropy Changes in Irreversible Processes
22.8 Entropy on a Microscopic Scale

## ANSWERS TO QUESTIONS

**Q22.1** First, the efficiency of the automobile engine cannot exceed the Carnot efficiency: it is limited by the temperature of burning fuel and the temperature of the environment into which the exhaust is dumped. Second, the engine block cannot be allowed to go over a certain temperature. Third, any practical engine has friction, incomplete burning of fuel, and limits set by timing and energy transfer by heat.

**Q22.2** It is easier to control the temperature of a hot reservoir. If it cools down, then heat can be added through some external means, like an exothermic reaction. If it gets too hot, then heat can be allowed to “escape” into the atmosphere. To maintain the temperature of a cold reservoir, one must remove heat if the reservoir gets too hot. Doing this requires either an “even colder” reservoir, which you also must maintain, or an endothermic process.

**Q22.3** A higher steam temperature means that more energy can be extracted from the steam. For a constant temperature heat sink at $T_c$, and steam at $T_h$, the efficiency of the power plant goes as $\frac{T_h - T_c}{T_h} = 1 - \frac{T_c}{T_h}$ and is maximized for a high $T_h$.

**Q22.4** No. Any heat engine takes in energy by heat and must also put out energy by heat. The energy that is dumped as exhaust into the low-temperature sink will always be thermal pollution in the outside environment. So-called ‘steady growth’ in human energy use cannot continue.

**Q22.5** No. The first law of thermodynamics is a statement about energy conservation, while the second is a statement about stable thermal equilibrium. They are by no means mutually exclusive. For the particular case of a cycling heat engine, the first law implies $|Q_h| = W_{eng} + |Q_c|$, and the second law implies $|Q_c| > 0$.

**Q22.6** Take an automobile as an example. According to the first law or the idea of energy conservation, it must take in all the energy it puts out. Its energy source is chemical energy in gasoline. During the combustion process, some of that energy goes into moving the pistons and eventually into the mechanical motion of the car. Clearly much of the energy goes into heat, which, through the cooling system, is dissipated into the atmosphere. Moreover, there are numerous places where friction, both mechanical and fluid, turns mechanical energy into heat. In even the most efficient internal combustion engine cars, less than 30% of the energy from the fuel actually goes into moving the car. The rest ends up as useless heat in the atmosphere.

**Q22.7** Suppose the ambient temperature is 20°C. A gas can be heated to the temperature of the bottom of the pond, and allowed to cool as it blows through a turbine. The Carnot efficiency of such an engine is about $e_c = \frac{\Delta T}{T_h} = \frac{80}{373} = 22\%$.

**Q22.8** No, because the work done to run the heat pump represents energy transferred into the house by heat.

**Q22.9** A slice of hot pizza cools off. Road friction brings a skidding car to a stop. A cup falls to the floor and shatters. Your cat dies. Any process is irreversible if it looks funny or frightening when shown in a videotape running backwards. The free flight of a projectile is nearly reversible.

**Q22.10** Below the frost line, the winter temperature is much higher than the air or surface temperature. The earth is a huge reservoir of internal energy, but digging a lot of deep trenches is much more expensive than setting a heat-exchanger out on a concrete pad. A heat pump can have a much higher coefficient of performance when it is transferring energy by heat between reservoirs at close to the same temperature.

**Q22.11** (a) When the two sides of the semiconductor are at different temperatures, an electric potential (voltage) is generated across the material, which can drive electric current through an external circuit. The two cups at 50°C contain the same amount of internal energy as the pair of hot and cold cups. But no energy flows by heat through the converter bridging between them and no voltage is generated across the semiconductors.

(b) A heat engine must put out exhaust energy by heat. The cold cup provides a sink to absorb output or wasted energy by heat, which has nowhere to go between two cups of equally warm water.

**Q22.12** Energy flows by heat from a hot bowl of chili into the cooler surrounding air. Heat lost by the hot stuff is equal to heat gained by the cold stuff, but the entropy decrease of the hot stuff is less than the entropy increase of the cold stuff. As you inflate a soft car tire at a service station, air from a tank at high pressure expands to fill a larger volume. That air increases in entropy and the surrounding atmosphere undergoes no significant entropy change. The brakes of your car get warm as you come to a stop. The shoes and drums increase in entropy and nothing loses energy by heat, so nothing decreases in entropy.

**Q22.13** (a) For an expanding ideal gas at constant temperature, $\Delta S = \frac{\Delta Q}{T} = nR\ln\left(\frac{V_2}{V_1}\right)$.

(b) For a reversible adiabatic expansion $\Delta Q = 0$, and $\Delta S = 0$. An ideal gas undergoing an irreversible adiabatic expansion can have any positive value for $\Delta S$ up to the value given in part (a).

**Q22.14** The rest of the Universe must have an entropy change of +8.0 J/K, or more.

**Q22.15** Even at essentially constant temperature, energy must flow by heat out of the solidifying sugar into the surroundings, to raise the entropy of the environment. The water molecules become less ordered as they leave the liquid in the container to mix into the whole atmosphere and hydrosphere. Thus the entropy of the surroundings increases, and the second law describes the situation correctly.

**Q22.16** To increase its entropy, raise its temperature. To decrease its entropy, lower its temperature. "Remove energy from it by heat" is not such a good answer, for if you hammer on it or rub it with a blunt file and at the same time remove energy from it by heat into a constant temperature bath, its entropy can stay constant.

**Q22.17** An analogy used by Carnot is instructive: A waterfall continuously converts mechanical energy into internal energy. It continuously creates entropy as the organized motion of the falling water turns into disorganized molecular motion. We humans put turbines into the waterfall, diverting some of the energy stream to our use. Water flows spontaneously from high to low elevation and energy spontaneously flows by heat from high to low temperature. Into the great flow of solar radiation from Sun to Earth, living things put themselves. They live on energy flow, more than just on energy. A basking snake diverts energy from a high-temperature source (the Sun) through itself temporarily, before the energy inevitably is radiated from the body of the snake to a low-temperature sink (outer space). A tree builds organized cellulose molecules and we build libraries and babies who look like their grandmothers, all out of a thin diverted stream in the universal flow of energy crashing down to disorder. We do not violate the second law, for we build local reductions in the entropy of one thing within the inexorable increase in the total entropy of the Universe. Your roommate's exercise puts energy into the room by heat.

**Q22.18** (a) Entropy increases as the yeast dies and as energy is transferred from the hot oven into the originally cooler dough and then from the hot bread into the surrounding air.

(b) Entropy increases some more as you metabolize the starches, converting chemical energy into internal energy.

**Q22.19** Either statement can be considered an instructive analogy. We choose to take the first view. All processes require energy, either as energy content or as energy input. The kinetic energy which it possessed at its formation continues to make the Earth go around. Energy released by nuclear reactions in the core of the Sun drives weather on the Earth and essentially all processes in the biosphere. The energy intensity of sunlight controls how lush a forest or jungle can be and how warm a planet is. Continuous energy input is not required for the motion of the planet. Continuous energy input is required for life because energy tends to be continuously degraded, as heat flows into lower-temperature sinks. The continuously increasing entropy of the Universe is the index to energy-transfers completed.

**Q22.20** The statement is not true. Although the probability is not exactly zero that this will happen, the probability of the concentration of air in one corner of the room is very nearly zero. If some billions of molecules are heading toward that corner just now, other billions are heading away from the corner in their random motion. Spontaneous compression of the air would violate the second law of thermodynamics. It would be a spontaneous departure from thermal and mechanical equilibrium.

**Q22.21** Shaking opens up spaces between jellybeans. The smaller ones more often can fall down into spaces below them. The accumulation of larger candies on top and smaller ones on the bottom implies a small increase in order, a small decrease in one contribution to the total entropy, but the second law is not violated. The total entropy increases as the system warms up, its increase in internal energy coming from the work put into shaking the box and also from a bit of gravitational energy loss as the beans settle compactly together.

## SOLUTIONS TO PROBLEMS

### Section 22.1 **Heat Engines and the Second Law of Thermodynamics**

**P22.1** (a) $e=\dfrac{W_{\text{eng}}}{|Q_h|}=\dfrac{25.0\text{ J}}{360\text{ J}}=\boxed{0.069\,4}$ or $\boxed{6.94\%}$

(b) $|Q_c|=|Q_h|-W_{\text{eng}}=360\text{ J}-25.0\text{ J}=\boxed{335\text{ J}}$

**P22.2** $W_{\text{eng}}=|Q_h|-|Q_c|=200\text{ J}$ (1)

$$e=\frac{W_{\text{eng}}}{|Q_h|}=1-\frac{|Q_c|}{|Q_h|}=0.300 \quad (2)$$

From (2), $|Q_c|=0.700|Q_h|$ (3)

Solving (3) and (1) simultaneously,

we have

(a) $\boxed{|Q_h|=667\text{ J}}$ and

(b) $\boxed{|Q_c|=467\text{ J}}$.

**P22.3** (a) We have $e=\dfrac{W_{\text{eng}}}{|Q_h|}=\dfrac{|Q_h|-|Q_c|}{|Q_h|}=1-\dfrac{|Q_c|}{|Q_h|}=0.250$

with $|Q_c|=8\,000\text{ J}$, we have $|Q_h|=\boxed{10.7\text{ kJ}}$

(b) $W_{\text{eng}}=|Q_h|-|Q_c|=2\,667\text{ J}$

and from $\mathcal{P}=\dfrac{W_{\text{eng}}}{\Delta t}$, we have $\Delta t=\dfrac{W_{\text{eng}}}{\mathcal{P}}=\dfrac{2\,667\text{ J}}{5\,000\text{ J/s}}=\boxed{0.533\text{ s}}$.

***P22.4** We have $Q_{hx}=4Q_{hy}$, $W_{\text{eng}\,x}=2W_{\text{eng}\,y}$ and $Q_{cx}=7Q_{cy}$. As well as $Q_{hx}=W_{\text{eng}\,x}+Q_{cx}$ and $Q_{hy}=W_{\text{eng}\,y}+Q_{cy}$. Substituting, $4Q_{hy}=2W_{\text{eng}\,y}+7Q_{cy}$

$$4Q_{hy}=2W_{\text{eng}\,y}+7Q_{hy}-7W_{\text{eng}\,y}$$
$$5W_{\text{eng}\,y}=3Q_{hy}$$

(b) $e_y=\dfrac{W_{\text{eng}\,y}}{Q_{hy}}=\dfrac{3}{5}=\boxed{60.0\%}$

(a) $e_x=\dfrac{W_{\text{eng}\,x}}{Q_{hx}}=\dfrac{2W_{\text{eng}\,y}}{4Q_{hy}}=\dfrac{2}{4}(0.600)=0.300=\boxed{30.0\%}$

***P22.5** (a) The input energy each hour is

$$\left(7.89\times10^{3}\text{ J/revolution}\right)\left(2\,500\text{ rev/min}\right)\frac{60\text{ min}}{1\text{ h}}=1.18\times10^{9}\text{ J/h}$$

implying fuel input $\left(1.18\times10^{9}\text{ J/h}\right)\left(\frac{1\text{ L}}{4.03\times10^{7}\text{ J}}\right)=\boxed{29.4\text{ L/h}}$

(b) $Q_h=W_{\text{eng}}+\left|Q_c\right|$. For a continuous-transfer process we may divide by time to have

$$\frac{Q_h}{\Delta t}=\frac{W_{\text{eng}}}{\Delta t}+\frac{\left|Q_c\right|}{\Delta t}$$

$$\text{Useful power output}=\frac{W_{\text{eng}}}{\Delta t}=\frac{Q_h}{\Delta t}-\frac{\left|Q_c\right|}{\Delta t}$$

$$=\left(\frac{7.89\times10^{3}\text{ J}}{\text{revolution}}-\frac{4.58\times10^{3}\text{ J}}{\text{revolution}}\right)\frac{2\,500\text{ rev}}{1\text{ min}}\frac{1\text{ min}}{60\text{ s}}=1.38\times10^{5}\text{ W}$$

$$\mathcal{P}_{\text{eng}}=1.38\times10^{5}\text{ W}\left(\frac{1\text{ hp}}{746\text{ W}}\right)=\boxed{185\text{ hp}}$$

(c) $\mathcal{P}_{\text{eng}}=\tau\omega\Rightarrow\tau=\frac{\mathcal{P}_{\text{eng}}}{\omega}=\frac{1.38\times10^{5}\text{ J/s}}{\left(2\,500\text{ rev/60 s}\right)}\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right)=\boxed{527\text{ N}\cdot\text{m}}$

(d) $\frac{\left|Q_c\right|}{\Delta t}=\frac{4.58\times10^{3}\text{ J}}{\text{revolution}}\left(\frac{2\,500\text{ rev}}{60\text{ s}}\right)=\boxed{1.91\times10^{5}\text{ W}}$

**P22.6** The heat to melt 15.0 g of Hg is $\left|Q_c\right|=mL_f=\left(15\times10^{-3}\text{ kg}\right)\left(1.18\times10^{4}\text{ J/kg}\right)=177\text{ J}$

The energy absorbed to freeze 1.00 g of aluminum is

$$\left|Q_h\right|=mL_f=\left(10^{-3}\text{ kg}\right)\left(3.97\times10^{5}\text{ J/kg}\right)=397\text{ J}$$

and the work output is $W_{\text{eng}}=\left|Q_h\right|-\left|Q_c\right|=220\text{ J}$

$$e=\frac{W_{\text{eng}}}{\left|Q_h\right|}=\frac{220\text{ J}}{397\text{ J}}=0.554,\text{ or }\boxed{55.4\%}$$

The theoretical (Carnot) efficiency is $\frac{T_h-T_c}{T_h}=\frac{933\text{ K}-243.1\text{ K}}{933\text{ K}}=0.749=74.9\%$

## Section 22.2 **Heat Pumps and Refrigerators**

**P22.7** $\text{COP(refrigerator)}=\frac{Q_c}{W}$

(a) If $Q_c=120\text{ J}$ and $\text{COP}=5.00$, then $\boxed{W=24.0\text{ J}}$

(b) Heat expelled = Heat removed + Work done.

$$Q_h=Q_c+W=120\text{ J}+24\text{ J}=\boxed{144\text{ J}}$$

**P22.8** $\text{COP} = 3.00 = \dfrac{Q_c}{W}$. Therefore, $W = \dfrac{Q_c}{3.00}$.

The heat removed each minute is

$$\frac{Q_C}{t} = (0.030\,0\text{ kg})(4\,186\text{ J/kg}\,{}^\circ\text{C})(22.0^\circ\text{C}) + (0.030\,0\text{ kg})\left(3.33\times10^5\text{ J/kg}\right)$$
$$+(0.030\,0\text{ kg})(2\,090\text{ J/kg}\,{}^\circ\text{C})(20.0^\circ\text{C}) = 1.40\times10^4\text{ J/min}$$

or, $\dfrac{Q_c}{t} = 233\text{ J/s}$.

Thus, the work done per sec $= \mathscr{P} = \dfrac{233\text{ J/s}}{3.00} = \boxed{77.8\text{ W}}$.

**P22.9** (a) $\left(10.0\,\dfrac{\text{Btu}}{\text{h}\cdot\text{W}}\right)\left(\dfrac{1055\text{ J}}{1\text{ Btu}}\right)\left(\dfrac{1\text{ h}}{3\,600\text{ s}}\right)\left(\dfrac{1\text{ W}}{1\text{ J/s}}\right) = \boxed{2.93}$

(b) Coefficient of performance for a refrigerator: $\boxed{(\text{COP})_{\text{refrigerator}}}$

(c) With EER 5, $5\,\dfrac{\text{Btu}}{\text{h}\cdot\text{W}} = \dfrac{10\,000\text{ Btu/h}}{\mathscr{P}}$: $\mathscr{P} = \dfrac{10\,000\text{ Btu/h}}{\frac{5\text{ Btu}}{\text{h}\cdot\text{W}}} = 2\,000\text{ W} = 2.00\text{ kW}$

Energy purchased is $\mathscr{P}\Delta t = (2.00\text{ kW})(1\,500\text{ h}) = 3.00\times10^3\text{ kWh}$

$\text{Cost} = \left(3.00\times10^3\text{ kWh}\right)(0.100\text{ \$/kWh}) = \$300$

With EER 10, $10\,\dfrac{\text{Btu}}{\text{h}\cdot\text{W}} = \dfrac{10\,000\text{ Btu/h}}{\mathscr{P}}$: $\mathscr{P} = \dfrac{10\,000\text{ Btu/h}}{\frac{10\text{ Btu}}{\text{h}\cdot\text{W}}} = 1\,000\text{ W} = 1.00\text{ kW}$

Energy purchased is $\mathscr{P}\Delta t = (1.00\text{ kW})(1\,500\text{ h}) = 1.50\times10^3\text{ kWh}$

$\text{Cost} = \left(1.50\times10^3\text{ kWh}\right)(0.100\text{ \$/kWh}) = \$150$

Thus, the cost for air conditioning is $\boxed{\text{half as much with EER 10}}$

## Section 22.3 **Reversible and Irreversible Processes**

No problems in this section

## Section 22.4 **The Carnot Engine**

**P22.10** When $e = e_c$, $1 - \dfrac{T_c}{T_h} = \dfrac{W_{\text{eng}}}{|Q_h|}$ and $\dfrac{\frac{W_{\text{eng}}}{\Delta t}}{\frac{|Q_h|}{\Delta t}} = 1 - \dfrac{T_c}{T_h}$

(a) $|Q_h| = \dfrac{\left(\frac{W_{\text{eng}}}{\Delta t}\right)\Delta t}{1 - \frac{T_c}{T_h}} = \dfrac{\left(1.50\times10^5\text{ W}\right)(3\,600\text{ s})}{1 - \frac{293}{773}}$

$|Q_h| = 8.69\times10^8\text{ J} = \boxed{869\text{ MJ}}$

(b) $|Q_c| = |Q_h| - \left(\dfrac{W_{\text{eng}}}{\Delta t}\right)\Delta t = 8.69\times10^8 - \left(1.50\times10^5\right)(3\,600) = 3.30\times10^8\text{ J} = \boxed{330\text{ MJ}}$

**P22.11** $T_c = 703\text{ K} \qquad T_h = 2\,143\text{ K}$

(a) $e_c = \dfrac{\Delta T}{T_h} = \dfrac{1\,440}{2\,143} = \boxed{67.2\%}$

(b) $|Q_h| = 1.40 \times 10^5\text{ J},\ W_{\text{eng}} = 0.420|Q_h|$

$$\mathcal{P} = \frac{W_{\text{eng}}}{\Delta t} = \frac{5.88 \times 10^4\text{ J}}{1\text{ s}} = \boxed{58.8\text{ kW}}$$

**P22.12** The Carnot efficiency of the engine is $e_c = \dfrac{\Delta T}{T_h} = \dfrac{120\text{ K}}{473\text{ K}} = 0.253$

At 20.0% of this maximum efficiency, $e = 0.200(0.253) = 0.050\,6$

From the definition of efficiency $W_{\text{eng}} = |Q_h|e$

and $|Q_h| = \dfrac{W_{\text{eng}}}{e} = \dfrac{10.0\text{ kJ}}{0.050\,6} = \boxed{197\text{ kJ}}$

**P22.13** Isothermal expansion at $T_h = 523\text{ K}$
Isothermal compression at $T_c = 323\text{ K}$
Gas absorbs 1 200 J during expansion.

(a) $|Q_c| = |Q_h|\left(\dfrac{T_c}{T_h}\right) = 1\,200\text{ J}\left(\dfrac{323}{523}\right) = \boxed{741\text{ J}}$

(b) $W_{\text{eng}} = |Q_h| - |Q_c| = (1\,200 - 741)\text{ J} = \boxed{459\text{ J}}$

**P22.14** We use $e_c = 1 - \dfrac{T_c}{T_h}$

as $0.300 = 1 - \dfrac{573\text{ K}}{T_h}$

From which, $T_h = 819\text{ K} = \boxed{546°\text{C}}$

***P22.15** The efficiency is $e_c = 1 - \dfrac{T_c}{T_h} = 1 - \dfrac{|Q_c|}{|Q_h|}$

Then $\dfrac{T_c}{T_h} = \dfrac{\frac{|Q_c|}{\Delta t}}{\frac{|Q_h|}{\Delta t}}$

$$\frac{|Q_h|}{\Delta t} = \frac{|Q_c|}{\Delta t}\frac{T_h}{T_c} = 15.4\text{ W}\frac{(273+100)\text{ K}}{(273+20)\text{ K}} = 19.6\text{ W}$$

(a) $|Q_h| = W_{\text{eng}} + |Q_c|$

The useful power output is $\dfrac{W_{\text{eng}}}{\Delta t} = \dfrac{|Q_h|}{\Delta t} - \dfrac{|Q_c|}{\Delta t} = 19.6\text{ W} - 15.4\text{ W} = \boxed{4.20\text{ W}}$

(b) $|Q_h| = \left(\dfrac{|Q_h|}{\Delta t}\right)\Delta t = mL_V \qquad m = \dfrac{|Q_h|}{\Delta t}\dfrac{\Delta t}{L_V} = (19.6\text{ J/s})\left(\dfrac{3\,600\text{ s}}{2.26 \times 10^6\text{ J/kg}}\right) = \boxed{3.12 \times 10^{-2}\text{ kg}}$

**P22.16** The Carnot summer efficiency is $e_{c,s}=1-\frac{T_c}{T_h}=1-\frac{(273+20)\text{ K}}{(273+350)\text{ K}}=0.530$

And in winter, $e_{c,w}=1-\frac{283}{623}=0.546$

Then the actual winter efficiency is $0.320\left(\frac{0.546}{0.530}\right)=\boxed{0.330}$ or $\boxed{33.0\%}$

**P22.17** (a) In an adiabatic process, $P_fV_f^\gamma=P_iV_i^\gamma$. Also, $\left(\frac{P_fV_f}{T_f}\right)^\gamma=\left(\frac{P_iV_i}{T_i}\right)^\gamma$.

Dividing the second equation by the first yields $T_f=T_i\left(\frac{P_f}{P_i}\right)^{(\gamma-1)/\gamma}$.

Since $\gamma=\frac{5}{3}$ for Argon, $\frac{\gamma-1}{\gamma}=\frac{2}{5}=0.400$ and we have

$$T_f=(1\,073\text{ K})\left(\frac{300\times10^3\text{ Pa}}{1.50\times10^6\text{ Pa}}\right)^{0.400}=\boxed{564\text{ K}}.$$

(b) $\Delta E_{\text{int}}=nC_V\Delta T=Q-W_{\text{eng}}=0-W_{\text{eng}}$, so $W_{\text{eng}}=-nC_V\Delta T$,

and the power output is

$$\mathscr{P}=\frac{W_{\text{eng}}}{t}=\frac{-nC_V\Delta T}{t}\text{ or}$$

$$=\frac{(-80.0\text{ kg})\left(\frac{1.00\text{ mol}}{0.039\,9\text{ kg}}\right)\left(\frac{3}{2}\right)(8.314\text{ J/mol}\cdot\text{K})(564-1\,073)\text{K}}{60.0\text{ s}}$$

$$\mathscr{P}=2.12\times10^5\text{ W}=\boxed{212\text{ kW}}$$

(c) $e_C=1-\frac{T_c}{T_h}=1-\frac{564\text{ K}}{1\,073\text{ K}}=0.475$ or $\boxed{47.5\%}$

**P22.18** (a) $e_{\max}=1-\frac{T_c}{T_h}=1-\frac{278}{293}=5.12\times10^{-2}=\boxed{5.12\%}$

(b) $\mathscr{P}=\frac{W_{\text{eng}}}{\Delta t}=75.0\times10^6\text{ J/s}$

Therefore, $W_{\text{eng}}=(75.0\times10^6\text{ J/s})(3\,600\text{ s/h})=2.70\times10^{11}\text{ J/h}$

From $e=\frac{W_{\text{eng}}}{|Q_h|}$ we find $|Q_h|=\frac{W_{\text{eng}}}{e}=\frac{2.70\times10^{11}\text{ J/h}}{5.12\times10^{-2}}=5.27\times10^{12}\text{ J/h}=\boxed{5.27\text{ TJ/h}}$

(c) As fossil-fuel prices rise, this way to use solar energy will become a good buy.

***P22.19** (a) $$e = \frac{W_{\text{eng}1} + W_{\text{eng}2}}{Q_{h1}} = \frac{e_1 Q_{1h} + e_2 Q_{2h}}{Q_{h1}}$$

Now $Q_{2h} = Q_{1c} = Q_{1h} - W_{\text{eng}1} = Q_{h1} - e_1 Q_{1h}$.

So $e = \frac{e_1 Q_{1h} + e_2 (Q_{1h} - e_1 Q_{1h})}{Q_{1h}} = \boxed{e_1 + e_2 - e_1 e_2}$.

(b) $$e = e_1 + e_2 - e_1 e_2 = 1 - \frac{T_i}{T_h} + 1 - \frac{T_c}{T_i} - \left(1 - \frac{T_i}{T_h}\right)\left(1 - \frac{T_c}{T_i}\right) = 2 - \frac{T_i}{T_h} - \frac{T_c}{T_i} - 1 + \frac{T_i}{T_h} + \frac{T_c}{T_i} - \frac{T_c}{T_h} = \boxed{1 - \frac{T_c}{T_h}}$$

The combination of reversible engines is itself a reversible engine so it has the Carnot efficiency.

(c) With $W_{\text{eng}2} = W_{\text{eng}1}$, $e = \frac{W_{\text{eng}1} + W_{\text{eng}2}}{Q_{1h}} = \frac{2W_{\text{eng}1}}{Q_{1h}} = 2e_1$

$$1 - \frac{T_c}{T_h} = 2\left(1 - \frac{T_i}{T_h}\right)$$

$$0 - \frac{T_c}{T_h} = 1 - \frac{2T_i}{T_h}$$

$$2T_i = T_h + T_c$$

$$\boxed{T_i = \frac{1}{2}(T_h + T_c)}$$

(d) $$e_1 = e_2 = 1 - \frac{T_i}{T_h} = 1 - \frac{T_c}{T_i}$$

$$T_i^2 = T_c T_h$$

$$\boxed{T_i = (T_h T_c)^{1/2}}$$

**P22.20** The work output is $W_{\text{eng}} = \frac{1}{2} m_{\text{train}} (5.00 \text{ m/s})^2$.

We are told $e = \frac{W_{\text{eng}}}{Q_h}$

$$0.200 = \frac{1}{2} m_t \frac{(5.00 \text{ m/s})^2}{Q_h}$$

and $e_C = 1 - \frac{300 \text{ K}}{T_h} = \frac{1}{2} m_t \frac{(6.50 \text{ m/s})^2}{Q_h}$.

Substitute $Q_h = \frac{1}{2} m_t \frac{(5.00 \text{ m/s})^2}{0.200}$.

Then, $1 - \frac{300 \text{ K}}{T_h} = 0.200 \left( \frac{\frac{1}{2} m_t (6.50 \text{ m/s})^2}{\frac{1}{2} m_t (5.00 \text{ m/s})^2} \right)$

$$1 - \frac{300 \text{ K}}{T_h} = 0.338$$

$$T_h = \frac{300 \text{ K}}{0.662} = \boxed{453 \text{ K}}$$

**P22.21** For the Carnot engine, $e_c = 1 - \frac{T_c}{T_h} = 1 - \frac{300\text{ K}}{750\text{ K}} = 0.600$.

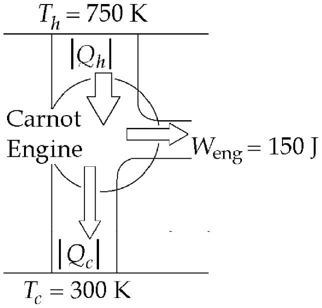


**FIG. P22.21**

Also, $e_c = \frac{W_{\text{eng}}}{|Q_h|}$.

so $|Q_h| = \frac{W_{\text{eng}}}{e_c} = \frac{150\text{ J}}{0.600} = 250\text{ J}$.

and $|Q_c| = |Q_h| - W_{\text{eng}} = 250\text{ J} - 150\text{ J} = 100\text{ J}$.

(a) $|Q_h| = \frac{W_{\text{eng}}}{e_S} = \frac{150\text{ J}}{0.700} = \boxed{214\text{ J}}$

$|Q_c| = |Q_h| - W_{\text{eng}} = 214\text{ J} - 150\text{ J} = \boxed{64.3\text{ J}}$

(b) $|Q_{h,\text{net}}| = 214\text{ J} - 250\text{ J} = \boxed{-35.7\text{ J}}$

$|Q_{c,\text{net}}| = 64.3\text{ J} - 100\text{ J} = \boxed{-35.7\text{ J}}$

The net flow of energy by heat from the cold to the hot reservoir without work input, is impossible.

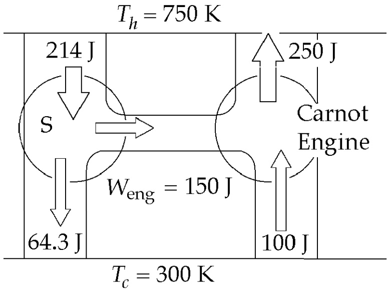


**FIG. P22.21(b)**

(c) For engine *S*: $|Q_c| = |Q_h| - W_{\text{eng}} = \frac{W_{\text{eng}}}{e_S} - W_{\text{eng}}$.

so $W_{\text{eng}} = \frac{|Q_c|}{\frac{1}{e_S} - 1} = \frac{100\text{ J}}{\frac{1}{0.700} - 1} = \boxed{233\text{ J}}$.

and $|Q_h| = |Q_c| + W_{\text{eng}} = 233\text{ J} + 100\text{ J} = \boxed{333\text{ J}}$.

(d) $|Q_{h,\text{net}}| = 333\text{ J} - 250\text{ J} = \boxed{83.3\text{ J}}$

$W_{\text{net}} = 233\text{ J} - 150\text{ J} = \boxed{83.3\text{ J}}$

$|Q_{c,\text{net}}| = \boxed{0}$

The output of 83.3 J of energy from the heat engine by work in a cyclic process without any exhaust by heat is impossible.

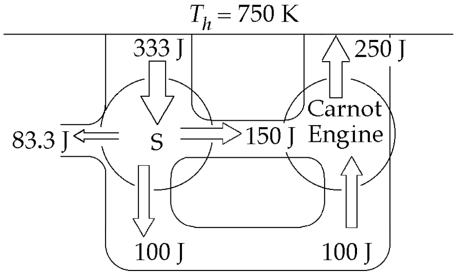


**FIG. P22.21(d)**

(e) Both engines operate in cycles, so $\Delta S_S = \Delta S_{\text{Carnot}} = 0$.

For the reservoirs, $\Delta S_h = -\frac{|Q_h|}{T_h}$ and $\Delta S_c = +\frac{|Q_c|}{T_c}$.

Thus, $\Delta S_{\text{total}} = \Delta S_S + \Delta S_{\text{Carnot}} + \Delta S_h + \Delta S_c = 0 + 0 - \frac{83.3\text{ J}}{750\text{ K}} + \frac{0}{300\text{ K}} = \boxed{-0.111\text{ J/K}}$.

A decrease in total entropy is impossible.

**P22.22** (a) First, consider the adiabatic process $D \to A$:

$P_D V_D^\gamma = P_A V_A^\gamma$ so $$P_D = P_A\left(\frac{V_A}{V_D}\right)^\gamma = 1\,400\text{ kPa}\left(\frac{10.0\text{ L}}{15.0\text{ L}}\right)^{5/3} = \boxed{712\text{ kPa}}.$$

Also $$\left(\frac{nRT_D}{V_D}\right)V_D^\gamma = \left(\frac{nRT_A}{V_A}\right)V_A^\gamma$$

or $$T_D = T_A\left(\frac{V_A}{V_D}\right)^{\gamma-1} = 720\text{ K}\left(\frac{10.0}{15.0}\right)^{2/3} = \boxed{549\text{ K}}.$$

Now, consider the isothermal process $C \to D$: $T_C = T_D = \boxed{549\text{ K}}$.

$$P_C = P_D\left(\frac{V_D}{V_C}\right) = \left[P_A\left(\frac{V_A}{V_D}\right)^\gamma\right]\left(\frac{V_D}{V_C}\right) = \frac{P_A V_A^\gamma}{V_C V_D^{\gamma-1}}$$

$$P_C = \frac{1\,400\text{ kPa}(10.0\text{ L})^{5/3}}{24.0\text{ L}(15.0\text{ L})^{2/3}} = \boxed{445\text{ kPa}}$$

Next, consider the adiabatic process $B \to C$: $P_B V_B^\gamma = P_C V_C^\gamma$.

But, $P_C = \dfrac{P_A V_A^\gamma}{V_C V_D^{\gamma-1}}$ from above. Also considering the isothermal process, $P_B = P_A\left(\dfrac{V_A}{V_B}\right)$.

Hence, $P_A\left(\dfrac{V_A}{V_B}\right)V_B^\gamma = \left(\dfrac{P_A V_A^\gamma}{V_C V_D^{\gamma-1}}\right)V_C^\gamma$ which reduces to $V_B = \dfrac{V_A V_C}{V_D} = \dfrac{10.0\text{ L}(24.0\text{ L})}{15.0\text{ L}} = \boxed{16.0\text{ L}}$.

Finally, $P_B = P_A\left(\dfrac{V_A}{V_B}\right) = 1\,400\text{ kPa}\left(\dfrac{10.0\text{ L}}{16.0\text{ L}}\right) = \boxed{875\text{ kPa}}$.

| State | $P$(kPa) | $V$(L) | $T$(K) |
|---|---|---|---|
| $A$ | 1 400 | 10.0 | 720 |
| $B$ | 875 | 16.0 | 720 |
| $C$ | 445 | 24.0 | 549 |
| $D$ | 712 | 15.0 | 549 |

(b) For the isothermal process $A \to B$: $\Delta E_{\text{int}} = nC_V\Delta T = \boxed{0}$

so $Q = -W = nRT\ln\left(\dfrac{V_B}{V_A}\right) = 2.34\text{ mol}(8.314\text{ J/mol}\cdot\text{K})(720\text{ K})\ln\left(\dfrac{16.0}{10.0}\right) = \boxed{+6.58\text{ kJ}}$.

For the adiabatic process $B \to C$: $Q = \boxed{0}$

$\Delta E_{\text{int}} = nC_V(T_C - T_B) = 2.34\text{ mol}\left[\dfrac{3}{2}(8.314\text{ J/mol}\cdot\text{K})\right](549 - 720)\text{ K} = \boxed{-4.98\text{ kJ}}$

and $W = -Q + \Delta E_{\text{int}} = 0 + (-4.98\text{ kJ}) = \boxed{-4.98\text{ kJ}}$.

*continued on next page*

For the isothermal process $C \to D$: $\Delta E_{\text{int}} = nC_V \Delta T = \boxed{0}$

and $Q = -W = nRT \ln\left(\frac{V_D}{V_C}\right) = 2.34 \text{ mol}(8.314 \text{ J/mol}\cdot\text{K})(549 \text{ K})\ln\left(\frac{15.0}{24.0}\right) = \boxed{-5.02 \text{ kJ}}$.

Finally, for the adiabatic process $D \to A$: $Q = \boxed{0}$

$$\Delta E_{\text{int}} = nC_V\left(T_A - T_D\right) = 2.34 \text{ mol}\left[\frac{3}{2}(8.314 \text{ J/mol}\cdot\text{K})\right](720 - 549) \text{ K} = \boxed{+4.98 \text{ kJ}}$$

and $W = -Q + \Delta E_{\text{int}} = 0 + 4.98 \text{ kJ} = \boxed{+4.98 \text{ kJ}}$.

| Process | $Q$(kJ) | $W$(kJ) | $\Delta E_{\text{int}}$ (kJ) |
|---|---|---|---|
| $A \to B$ | +6.58 | –6.58 | 0 |
| $B \to C$ | 0 | –4.98 | –4.98 |
| $C \to D$ | –5.02 | +5.02 | 0 |
| $D \to A$ | 0 | +4.98 | +4.98 |
| $ABCDA$ | +1.56 | –1.56 | 0 |

The work done *by* the engine is the negative of the work input. The output work $W_{\text{eng}}$ is given by the work column in the table with all signs reversed.

(c) $e = \frac{W_{\text{eng}}}{|Q_h|} = \frac{-W_{ABCD}}{Q_{A\to B}} = \frac{1.56 \text{ kJ}}{6.58 \text{ kJ}} = 0.237$ or $\boxed{23.7\%}$

$e_c = 1 - \frac{T_c}{T_h} = 1 - \frac{549}{720} = 0.237$ or $\boxed{23.7\%}$

**P22.23** $(\text{COP})_{\text{refrig}} = \frac{T_c}{\Delta T} = \frac{270}{30.0} = \boxed{9.00}$

**P22.24** $(\text{COP})_{\text{heat pump}} = \frac{|Q_c| + W}{W} = \frac{T_h}{\Delta T} = \frac{295}{25} = \boxed{11.8}$

**P22.25** (a) For a complete cycle, $\Delta E_{\text{int}} = 0$ and $W = |Q_h| - |Q_c| = |Q_c|\left[\frac{|(Q_h)|}{|Q_c|} - 1\right]$.

We have already shown that for a Carnot cycle (and only for a Carnot cycle) $\frac{|Q_h|}{|Q_c|} = \frac{T_h}{T_c}$.

Therefore, $W = \boxed{|Q_c|\left[\frac{T_h - T_c}{T_c}\right]}$.

(b) We have the definition of the coefficient of performance for a refrigerator, $\text{COP} = \frac{|Q_c|}{W}$.

Using the result from part (a), this becomes $\text{COP} = \boxed{\frac{T_c}{T_h - T_c}}$.

**P22.26** $\text{COP} = 0.100\,\text{COP}_{\text{Carnot cycle}}$

or $$\frac{|Q_h|}{W} = 0.100\left(\frac{|Q_h|}{W}\right)_{\text{Carnot cycle}} = 0.100\left(\frac{1}{\text{Carnot efficiency}}\right)$$

$$\frac{|Q_h|}{W} = 0.100\left(\frac{T_h}{T_h - T_c}\right) = 0.100\left(\frac{293\text{ K}}{293\text{ K} - 268\text{ K}}\right) = 1.17$$

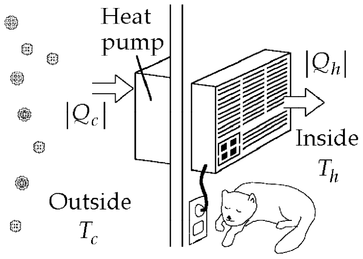


**FIG. P22.26**

Thus, $\boxed{\text{1.17 joules of energy enter the room by heat for each joule of work done.}}$

**P22.27** $$(\text{COP})_{\text{Carnot refrig}} = \frac{T_c}{\Delta T} = \frac{4.00}{289} = 0.013\,8 = \frac{|Q_c|}{W}$$

$\therefore W = \boxed{72.2\text{ J}}$ per 1 J energy removed by heat.

**P22.28** A Carnot refrigerator runs on minimum power.

For it: $\frac{Q_h}{T_h} = \frac{Q_c}{T_c}$ so $\frac{Q_h/t}{T_h} = \frac{Q_c/t}{T_c}$.

Solving part (b) first:

(b) $$\frac{Q_h}{t} = \frac{Q_c}{t}\left(\frac{T_h}{T_c}\right) = (8.00\ \text{MJ/h})\left(\frac{298\text{ K}}{273\text{ K}}\right) = \left(8.73\times10^{6}\ \text{J/h}\right)\left(\frac{1\text{ h}}{3\,600\text{ s}}\right) = \boxed{2.43\text{ kW}}$$

(a) $$\frac{W}{t} = \frac{Q_h}{t} - \frac{Q_c}{t} = 2.43\text{ kW} - \frac{8.00\times10^{6}\ \text{J/h}}{3\,600\ \text{s/h}} = \boxed{204\text{ W}}$$

**P22.29** $e = \frac{W}{Q_h} = 0.350 \qquad W = 0.350Q_h$

$Q_h = W + Q_c \qquad Q_c = 0.650Q_h$

$$\text{COP(refrigerator)} = \frac{Q_c}{W} = \frac{0.650Q_h}{0.350Q_h} = \boxed{1.86}$$

***P22.30** To have the same efficiencies as engines, $1 - \frac{T_{\text{cp}}}{T_{\text{hp}}} = 1 - \frac{T_{\text{cr}}}{T_{\text{hr}}}$ the pump and refrigerator must operate between reservoirs with the same ratio $\frac{T_{\text{cp}}}{T_{\text{hp}}} = \frac{T_{\text{cr}}}{T_{\text{hr}}}$, which we define as $r$. Now $\text{COP}_{\text{p}} = 1.50\,\text{COP}_{\text{r}}$ becomes $\frac{T_{\text{hp}}}{T_{\text{hp}} - T_{\text{cp}}} = \frac{3}{2}\frac{T_{\text{cr}}}{T_{\text{hr}} - T_{\text{cr}}}$ or $\frac{T_{\text{hp}}}{T_{\text{hp}} - rT_{\text{hp}}} = \frac{3}{2}\frac{rT_{\text{hr}}}{T_{\text{hr}} - rT_{\text{hr}}}$, $\frac{2}{1-r} = \frac{3r}{1-r}$, $r = \frac{2}{3}$.

(a) $$\text{COP}_{\text{r}} = \frac{r}{1-r} = \frac{\frac{2}{3}}{1-\frac{2}{3}} = \boxed{2.00}$$

(b) $$\text{COP}_{\text{p}} = \frac{1}{1-r} = \frac{1}{1-\frac{2}{3}} = \boxed{3.00}$$

(c) $$e = 1 - r = 1 - \frac{2}{3} = \boxed{33.3\%}$$

## Section 22.5 **Gasoline and Diesel Engines**

**P22.31** (a) $P_iV_i^\gamma = P_fV_f^\gamma$

$$P_f = P_i\left(\frac{V_i}{V_f}\right)^\gamma = \left(3.00\times10^6\ \text{Pa}\right)\left(\frac{50.0\ \text{cm}^3}{300\ \text{cm}^3}\right)^{1.40} = \boxed{244\ \text{kPa}}$$

(b) $W = \int_{V_i}^{V_i} PdV \qquad P = P_i\left(\frac{V_i}{V}\right)^\gamma$

Integrating,

$$W = \left(\frac{1}{\gamma-1}\right)P_iV_i\left[1-\left(\frac{V_i}{V_f}\right)^{\gamma-1}\right] = (2.50)\left(3.00\times10^6\ \text{Pa}\right)\left(5.00\times10^{-5}\ \text{m}^3\right)\left[1-\left(\frac{50.0\ \text{cm}^3}{300\ \text{cm}^3}\right)^{0.400}\right]$$

$$= \boxed{192\ \text{J}}$$

**P22.32** Compression ratio $= 6.00$, $\gamma = 1.40$

(a) Efficiency of an Otto-engine $e = 1-\left(\frac{V_2}{V_1}\right)^{\gamma-1}$

$$e = 1-\left(\frac{1}{6.00}\right)^{0.400} = \boxed{51.2\%}.$$

(b) If actual efficiency $e' = 15.0\%$ losses in system are $e - e' = \boxed{36.2\%}$.

**P22.33** $e_{\text{Otto}} = 1-\frac{1}{(V_1/V_2)^{\gamma-1}} = 1-\frac{1}{(6.20)^{(7/5-1)}} = 1-\frac{1}{(6.20)^{0.400}}$

$e_{\text{Otto}} = 0.518$

We have assumed the fuel-air mixture to behave like a diatomic gas.

Now $e = \frac{W_{\text{eng}}}{Q_h} = \frac{W_{\text{eng}}/t}{Q_h/t}$

$$\frac{Q_h}{t} = \frac{W_{\text{eng}}/t}{e} = 102\ \text{hp}\frac{746\ \text{W}/1\ \text{hp}}{0.518}$$

$$\frac{Q_h}{t} = \boxed{146\ \text{kW}}$$

$$Q_h = W_{\text{eng}} + |Q_c|$$

$$\frac{|Q_c|}{t} = \frac{Q_h}{t} - \frac{W_{\text{eng}}}{t}$$

$$\frac{|Q_c|}{t} = 146\times10^3\ \text{W} - 102\ \text{hp}\left(\frac{746\ \text{W}}{1\ \text{hp}}\right) = \boxed{70.8\ \text{kW}}$$

**P22.34** (a), (b) The quantity of gas is

$$n=\frac{P_AV_A}{RT_A}=\frac{\left(100\times10^3\ \text{Pa}\right)\left(500\times10^{-6}\ \text{m}^3\right)}{(8.314\ \text{J/mol}\cdot\text{K})(293\ \text{K})}=0.020\,5\ \text{mol}$$

$$E_{\text{int},\,A}=\frac{5}{2}nRT_A=\frac{5}{2}P_AV_A=\frac{5}{2}\left(100\times10^3\ \text{Pa}\right)\left(500\times10^{-6}\ \text{m}^3\right)=\boxed{125\ \text{J}}$$

In process $AB$, $P_B=P_A\left(\frac{V_A}{V_B}\right)^{\gamma}=\left(100\times10^3\ \text{Pa}\right)(8.00)^{1.40}=\boxed{1.84\times10^6\ \text{Pa}}$

$$T_B=\frac{P_BV_B}{nR}=\frac{\left(1.84\times10^6\ \text{Pa}\right)\left(500\times10^{-6}\ \text{m}^3/8.00\right)}{(0.020\,5\ \text{mol})(8.314\ \text{J/mol}\cdot\text{K})}=\boxed{673\ \text{K}}$$

$$E_{\text{int},\,B}=\frac{5}{2}nRT_B=\frac{5}{2}(0.020\,5\ \text{mol})(8.314\ \text{J/mol}\cdot\text{K})(673\ \text{K})=\boxed{287\ \text{J}}$$

so $\Delta E_{\text{int},\,AB}=287\ \text{J}-125\ \text{J}=\boxed{162\ \text{J}}=Q-W_{\text{out}}=0-W_{\text{out}}$ $W_{AB}=\boxed{-162\ \text{J}}$

Process $BC$ takes us to:

$$P_C=\frac{nRT_C}{V_C}=\frac{(0.020\,5\ \text{mol})(8.314\ \text{J/mol}\cdot\text{K})(1\,023\ \text{K})}{62.5\times10^{-6}\ \text{m}^3}=\boxed{2.79\times10^6\ \text{Pa}}$$

$$E_{\text{int},\,\text{C}}=\frac{5}{2}nRT_C=\frac{5}{2}(0.020\,5\ \text{mol})(8.314\ \text{J/mol}\cdot\text{K})(1\,023\ \text{K})=\boxed{436\ \text{J}}$$

$$E_{\text{int},\,BC}=436\ \text{J}-287\ \text{J}=\boxed{149\ \text{J}}=Q-W_{\text{out}}=Q-0$$

$$Q_{BC}=\boxed{149\ \text{J}}$$

In process $CD$:

$$P_D=P_C\left(\frac{V_C}{V_D}\right)^{\gamma}=\left(2.79\times10^6\ \text{Pa}\right)\left(\frac{1}{8.00}\right)^{1.40}=\boxed{1.52\times10^5\ \text{Pa}}$$

$$T_D=\frac{P_DV_D}{nR}=\frac{\left(1.52\times10^5\ \text{Pa}\right)\left(500\times10^{-6}\ \text{m}^3\right)}{(0.020\,5\ \text{mol})(8.314\ \text{J/mol}\cdot\text{K})}=\boxed{445\ \text{K}}$$

$$E_{\text{int},\,D}=\frac{5}{2}nRT_D=\frac{5}{2}(0.020\,5\ \text{mol})(8.314\ \text{J/mol}\cdot\text{K})(445\ \text{K})=\boxed{190\ \text{J}}$$

$$\Delta E_{\text{int},\,CD}=190\ \text{J}-436\ \text{J}=\boxed{-246\ \text{J}}=Q-W_{\text{out}}=0-W_{\text{out}}$$

$$W_{CD}=\boxed{246\ \text{J}}$$

and $\Delta E_{\text{int},\,DA}=E_{\text{int},\,A}-E_{\text{int},\,D}=125\ \text{J}-190\ \text{J}=\boxed{-65.0\ \text{J}}=Q-W_{\text{out}}=Q-0$

$$Q_{DA}=\boxed{-65.0\ \text{J}}$$

*continued on next page*

For the entire cycle, $\Delta E_{\text{int, net}} = 162\text{ J} + 149 - 246 - 65.0 = \boxed{0}$. The net work is

$$W_{\text{eng}} = -162\text{ J} + 0 + 246\text{ J} + 0 = \boxed{84.3\text{ J}}$$

$$Q_{\text{net}} = 0 + 149\text{ J} + 0 - 65.0\text{ J} = \boxed{84.3\text{ J}}$$

The tables look like:

| State | $T$(K) | $P$(kPa) | $V(\text{cm}^3)$ | $E_{\text{int}}$ (J) |
|---|---|---|---|---|
| *A* | 293 | 100 | 500 | 125 |
| *B* | 673 | 1 840 | 62.5 | 287 |
| *C* | 1 023 | 2 790 | 62.5 | 436 |
| *D* | 445 | 152 | 500 | 190 |
| *A* | 293 | 100 | 500 | 125 |

| Process | $Q$(J) | output $W$(J) | $\Delta E_{\text{int}}$ (J) |
|---|---|---|---|
| *AB* | 0 | –162 | 162 |
| *BC* | 149 | 0 | 149 |
| *CD* | 0 | 246 | –246 |
| *DA* | –65.0 | 0 | –65.0 |
| *ABCDA* | 84.3 | 84.3 | 0 |

(c) The input energy is $Q_h = \boxed{149\text{ J}}$, the waste is $|Q_c| = \boxed{65.0\text{ J}}$, and $W_{\text{eng}} = \boxed{84.3\text{ J}}$.

(d) The efficiency is: $e = \dfrac{W_{\text{eng}}}{Q_h} = \dfrac{84.3\text{ J}}{149\text{ J}} = \boxed{0.565}$.

(e) Let $f$ represent the angular speed of the crankshaft. Then $\dfrac{f}{2}$ is the frequency at which we obtain work in the amount of 84.3 J/cycle:

$$1\,000\text{ J/s} = \left(\frac{f}{2}\right)(84.3\text{ J/cycle})$$

$$f = \frac{2\,000\text{ J/s}}{84.3\text{ J/cycle}} = 23.7\text{ rev/s} = \boxed{1.42\times10^3\text{ rev/min}}$$

## Section 22.6 **Entropy**

**P22.35** For a freezing process,

$$\Delta S = \frac{\Delta Q}{T} = \frac{-(0.500\text{ kg})\left(3.33\times10^5\text{ J/kg}\right)}{273\text{ K}} = \boxed{-610\text{ J/K}}.$$

**P22.36** At a constant temperature of 4.20 K,

$$\Delta S = \frac{\Delta Q}{T} = \frac{L_v}{4.20\text{ K}} = \frac{20.5\text{ kJ/kg}}{4.20\text{ K}}$$
$$\Delta S = \boxed{4.88\text{ kJ/kg}\cdot\text{K}}$$

**P22.37** $\Delta S = \int_i^f \frac{dQ}{T} = \int_{T_i}^{T_f} \frac{mcdT}{T} = mc\ln\left(\frac{T_f}{T_i}\right)$

$$\Delta S = 250\text{ g}(1.00\text{ cal/g}\cdot{}^\circ\text{C})\ln\left(\frac{353}{293}\right) = 46.6\text{ cal/K} = \boxed{195\text{ J/K}}$$

***P22.38** (a) The process is $\boxed{\text{isobaric}}$ because it takes place under constant atmospheric pressure. As described by Newton's third law, the stewing syrup must exert the same force on the air as the air exerts on it. The heating process is not adiabatic (energy goes in by heat), isothermal (*T* goes up), isovolumetic (it likely expands a bit), cyclic (it is different at the end), or isentropic (entropy increases). It could be made as nearly reversible as you wish, by not using a kitchen stove but a heater kept always just incrementally higher in temperature than the syrup. The process would then also be eternal, and impractical for food production.

(b) The final temperature is

$$220^\circ\text{F} = 212^\circ\text{F} + 8^\circ\text{F} = 100^\circ\text{C} + 8^\circ\text{F}\left(\frac{100-0^\circ\text{C}}{212-32^\circ\text{F}}\right) = 104^\circ\text{C}.$$

For the mixture,

$$Q = m_1c_1\Delta T + m_2c_2\Delta T = (900\text{ g }1\text{ cal/g}\cdot{}^\circ\text{C} + 930\text{ g }0.299\text{ cal/g}\cdot{}^\circ\text{C})(104.4^\circ\text{C} - 23^\circ\text{C})$$
$$= 9.59\times10^4\text{ cal} = \boxed{4.02\times10^5\text{ J}}$$

(c) Consider the reversible heating process described in part (a):

$$\Delta S = \int_i^f \frac{dQ}{T} = \int_i^f \frac{(m_1c_1 + m_2c_2)dT}{T} = (m_1c_1 + m_2c_2)\ln\frac{T_f}{T_i}$$
$$= [900(1) + 930(0.299)](\text{cal}/{}^\circ\text{C})\left(\frac{4.186\text{ J}}{1\text{ cal}}\right)\left(\frac{1^\circ\text{C}}{1\text{ K}}\right)\ln\left(\frac{273+104}{273+23}\right)$$
$$= (4\,930\text{ J/K})0.243 = \boxed{1.20\times10^3\text{ J/K}}$$

***P22.39** We take data from the description of Figure 20.2 in section 20.3, and we assume a constant specific heat for each phase. As the ice is warmed from –12°C to 0°C, its entropy increases by

$$\Delta S = \int_i^f \frac{dQ}{T} = \int_{261\text{ K}}^{273\text{ K}} \frac{mc_{\text{ice}}dT}{T} = mc_{\text{ice}} \int_{261\text{ K}}^{273\text{ K}} T^{-1}dT = mc_{\text{ice}} \ln T\Big|_{261\text{ K}}^{273\text{ K}}$$

$$\Delta S = 0.027\,0\text{ kg}\left(2\,090\text{ J/kg}\cdot{}^\circ\text{C}\right)(\ln 273\text{ K} - \ln 261\text{ K}) = 0.027\,0\text{ kg}\left(2\,090\text{ J/kg}\cdot{}^\circ\text{C}\right)\left(\ln\left(\frac{273}{261}\right)\right)$$

$$\Delta S = 2.54\text{ J/K}$$

As the ice melts its entropy change is

$$\Delta S = \frac{Q}{T} = \frac{mL_f}{T} = \frac{0.027\,0\text{ kg}\left(3.33\times 10^5\text{ J/kg}\right)}{273\text{ K}} = 32.9\text{ J/K}$$

As liquid water warms from 273 K to 373 K,

$$\Delta S = \int_i^f \frac{mc_{\text{liquid}}dT}{T} = mc_{\text{liquid}} \ln\left(\frac{T_f}{T_i}\right) = 0.027\,0\text{ kg}\left(4\,186\text{ J/kg}\cdot{}^\circ\text{C}\right)\ln\left(\frac{373}{273}\right) = 35.3\text{ J/K}$$

As the water boils and the steam warms,

$$\Delta S = \frac{mL_v}{T} + mc_{\text{steam}} \ln\left(\frac{T_f}{T_i}\right)$$

$$\Delta S = \frac{0.027\,0\text{ kg}\left(2.26\times 10^6\text{ J/kg}\right)}{373\text{ K}} + 0.027\,0\text{ kg}\left(2\,010\text{ J/kg}\cdot{}^\circ\text{C}\right)\ln\left(\frac{388}{373}\right) = 164\text{ J/K} + 2.14\text{ J/K}$$

The total entropy change is

$$(2.54 + 32.9 + 35.3 + 164 + 2.14)\text{ J/K} = \boxed{236\text{ J/K}}.$$

We could equally well have taken the values for specific heats and latent heats from Tables 20.1 and 20.2. For steam at constant pressure, the molar specific heat in Table 21.2 implies a specific heat of $(35.4\text{ J/mol}\cdot\text{K})\left(\frac{1\text{ mol}}{0.018\text{ kg}}\right) = 1\,970\text{ J/kg}\cdot\text{K}$, nearly agreeing with $2\,010\text{ J/kg}\cdot\text{K}$.

## Section 22.7 **Entropy Changes in Irreversible Processes**

**P22.40** $$\Delta S = \frac{Q_2}{T_2} - \frac{Q_1}{T_1} = \left(\frac{1\,000}{290} - \frac{1\,000}{5\,700}\right)\text{ J/K} = \boxed{3.27\text{ J/K}}$$

**P22.41** The car ends up in the same thermodynamic state as it started, so it undergoes zero changes in entropy. The original kinetic energy of the car is transferred by heat to the surrounding air, adding to the internal energy of the air. Its change in entropy is

$$\Delta S = \frac{\frac{1}{2}mv^2}{T} = \frac{750(20.0)^2}{293}\text{ J/K} = \boxed{1.02\text{ kJ/K}}.$$

**P22.42** $c_{\text{iron}} = 448 \text{ J/kg}\cdot{}^\circ\text{C}$; $c_{\text{water}} = 4\,186 \text{ J/kg}\cdot{}^\circ\text{C}$

$Q_{\text{cold}} = -Q_{\text{hot}}$: $4.00 \text{ kg}(4\,186 \text{ J/kg}\cdot{}^\circ\text{C})(T_f - 10.0^\circ\text{C}) = -(1.00 \text{ kg})(448 \text{ J/kg}\cdot{}^\circ\text{C})(T_f - 900^\circ\text{C})$

which yields $T_f = 33.2^\circ\text{C} = 306.2 \text{ K}$

$$\Delta S = \int_{283 \text{ K}}^{306.2 \text{ K}} \frac{c_{\text{water}} m_{\text{water}} dT}{T} + \int_{1\,173 \text{ K}}^{306.2 \text{ K}} \frac{c_{\text{iron}} m_{\text{iron}} dT}{T}$$

$$\Delta S = c_{\text{water}} m_{\text{water}} \ln\left(\frac{306.2}{283}\right) + c_{\text{iron}} m_{\text{iron}} \ln\left(\frac{306.2}{1\,173}\right)$$

$$\Delta S = (4\,186 \text{ J/kg}\cdot\text{K})(4.00 \text{ kg})(0.078\,8) + (448 \text{ J/kg}\cdot\text{K})(1.00 \text{ kg})(-1.34)$$

$$\Delta S = \boxed{718 \text{ J/K}}$$

**P22.43** Sitting here writing, I convert chemical energy, in ordered molecules in food, into internal energy that leaves my body by heat into the room-temperature surroundings. My rate of energy output is equal to my metabolic rate,

$$2\,500 \text{ kcal/d} = \frac{2\,500 \times 10^3 \text{ cal}}{86\,400 \text{ s}}\left(\frac{4.186 \text{ J}}{1 \text{ cal}}\right) = 120 \text{ W}.$$

My body is in steady state, changing little in entropy, as the environment increases in entropy at the rate

$$\frac{\Delta S}{\Delta t} = \frac{Q/T}{\Delta t} = \frac{Q/\Delta t}{T} = \frac{120 \text{ W}}{293 \text{ K}} = 0.4 \text{ W/K} \sim \boxed{1 \text{ W/K}}.$$

When using powerful appliances or an automobile, my personal contribution to entropy production is much greater than the above estimate, based only on metabolism.

**P22.44** (a) $V = \dfrac{nRT_i}{P_i} = \dfrac{(40.0 \text{ g})(8.314 \text{ J/mol}\cdot\text{K})(473 \text{ K})}{(39.9 \text{ g/mol})(100 \times 10^3 \text{ Pa})} = 39.4 \times 10^{-3} \text{ m}^3 = \boxed{39.4 \text{ L}}$

(b) $\Delta E_{\text{int}} = nC_V \Delta T = \left(\dfrac{40.0 \text{ gm}}{39.9 \text{ g/mol}}\right)\left[\dfrac{3}{2}(8.314 \text{ J/mol}\cdot\text{K})\right](-200^\circ\text{C}) = \boxed{-2.50 \text{ kJ}}$

(c) $W = 0$ so $Q = \Delta E_{\text{int}} = \boxed{-2.50 \text{ kJ}}$

(d) $\Delta S_{\text{argon}} = \displaystyle\int_i^f \frac{dQ}{T} = nC_V \ln\left(\frac{T_f}{T_i}\right)$

$$= \left(\frac{40.0 \text{ g}}{39.9 \text{ g/mol}}\right)\left[\frac{3}{2}(8.314 \text{ J/mol}\cdot\text{K})\right]\ln\left(\frac{273}{473}\right) = \boxed{-6.87 \text{ J/K}}$$

(e) $\Delta S_{\text{bath}} = \dfrac{2.50 \text{ kJ}}{273 \text{ K}} = \boxed{+9.16 \text{ J/K}}$

The total change in entropy is

$$\Delta S_{\text{total}} = \Delta S_{\text{argon}} + \Delta S_{\text{bath}} = -6.87 \text{ J/K} + 9.16 \text{ J/K} = +2.29 \text{ J/K}$$

$\Delta S_{\text{total}} > 0$ for this irreversible process.

**P22.45** $\Delta S = nR\ln\left(\frac{V_f}{V_i}\right) = R\ln 2 = \boxed{5.76\ \text{J/K}}$

There is $\boxed{\text{no change in temperature}}$.

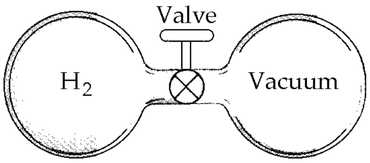


FIG. P22.45

**P22.46** $\Delta S = nR\ln\left(\frac{V_f}{V_i}\right) = (0.044\,0)(2)R\ln 2$

$\Delta S = 0.088\,0(8.314)\ln 2 = \boxed{0.507\ \text{J/K}}$

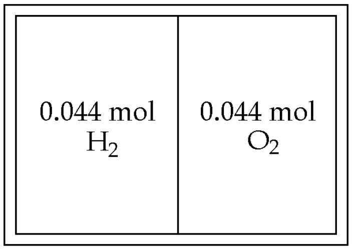


FIG. P22.46

**P22.47** For any infinitesimal step in a process on an ideal gas,

$dE_{\text{int}} = dQ + dW$: $\quad dQ = dE_{\text{int}} - dW = nC_V dT + PdV = nC_V dT + \frac{nRTdV}{V}$

and $\quad \frac{dQ}{T} = nC_V\frac{dT}{T} + nR\frac{dV}{V}$

If the whole process is reversible, $\quad \Delta S = \int_i^f \frac{dQ_r}{T} = \int_i^f\left(nC_V\frac{dT}{T} + nR\frac{dV}{V}\right) = nC_V\ln\left(\frac{T_f}{T_i}\right) + nR\ln\left(\frac{V_f}{V_i}\right)$

Also, from the ideal gas law, $\quad \frac{T_f}{T_i} = \frac{P_fV_f}{P_iV_i}$

$$\Delta S = (1.00\ \text{mol})\left[\frac{3}{2}(8.314\ \text{J/mol}\cdot\text{K})\right]\ln\left(\frac{(2.00)(0.040\,0)}{(1.00)(0.025\,0)}\right) + (1.00\ \text{mol})(8.314\ \text{J/mol}\cdot\text{K})\ln\left(\frac{0.040\,0}{0.025\,0}\right)$$

$$= \boxed{18.4\ \text{J/K}}$$

**P22.48** $\Delta S = nC_V\ln\left(\frac{T_f}{T_i}\right) + nR\ln\left(\frac{V_f}{V_i}\right)$

$$= (1.00\ \text{mol})\left[\frac{5}{2}(8.314\ \text{J/mol}\cdot\text{K})\right]\ln\left(\frac{2P\cdot 2V}{PV}\right) + (1.00\ \text{mol})(8.314\ \text{J/mol}\cdot\text{K})\ln\left(\frac{2V}{V}\right)$$

$$\Delta S = \boxed{34.6\ \text{J/K}}$$

## Section 22.8 Entropy on a Microscopic Scale

**P22.49** (a) A 12 can only be obtained $\boxed{\text{one}}$ way $6+6$

(b) A 7 can be obtained $\boxed{\text{six}}$ ways: $6+1$, $5+2$, $4+3$, $3+4$, $2+5$, $1+6$

**P22.50** (a) The table is shown below. On the basis of the table, the most probable result of a toss is $\boxed{\text{2 heads and 2 tails}}$.

(b) The most ordered state is the least likely state. Thus, on the basis of the table this is $\boxed{\text{either all heads or all tails}}$.

(c) The most disordered is the most likely state. Thus, this is $\boxed{\text{2 heads and 2 tails}}$.

| Result | Possible Combinations | Total |
|---|---|---|
| All heads | HHHH | 1 |
| 3H, 1T | THHH, HTHH, HHTH, HHHT | 4 |
| 2H, 2T | TTHH, THTH, THHT, HTTH, HTHT, HHTT | 6 |
| 1H, 3T | HTTT, THTT, TTHT, TTTH | 4 |
| All tails | TTTT | 1 |

**P22.51** (a)

| Result | Possible Combinations | Total |
|---|---|---|
| All red | RRR | 1 |
| 2R, 1G | RRG, RGR, GRR | 3 |
| 1R, 2G | RGG, GRG, GGR | 3 |
| All green | GGG | 1 |

(b)

| Result | Possible Combinations | Total |
|---|---|---|
| All red | RRRRR | 1 |
| 4R, 1G | RRRRG, RRRGR, RRGRR, RGRRR, GRRRR | 5 |
| 3R, 2G | RRRGG, RRGRG, RGRRG, GRRRG, RRGGR, RGRGR, GRRGR, RGGRR, GRGRR, GGRRR | 10 |
| 2R, 3G | GGGRR, GGRGR, GRGGR, RGGGR, GGRRG, GRGRG, RGGRG, GRRGG, RGRGG, RRGGG | 10 |
| 1R, 4G | RGGGG, GRGGG, GGRGG, GGGRG, GGGGR | 5 |
| All green | GGGGG | 1 |

## Additional Problems

**P22.52** The conversion of gravitational potential energy into kinetic energy as the water falls is reversible. But the subsequent conversion into internal energy is not. We imagine arriving at the same final state by adding energy by heat, in amount $mgy$, to the water from a stove at a temperature infinitesimally above 20.0°C. Then,

$$\Delta S=\int\frac{dQ}{T}=\frac{Q}{T}=\frac{mgy}{T}=\frac{5\,000\ \text{m}^3\left(1\,000\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)(50.0\ \text{m})}{293\ \text{K}}=\boxed{8.36\times10^6\ \text{J/K}}.$$

**P22.53** (a) $\mathscr{P}_{\text{electric}} = \dfrac{H_{ET}}{\Delta t}$ so if all the electric energy is converted into internal energy, the steady-state condition of the house is described by $H_{ET} = |Q|$.

Therefore, $\mathscr{P}_{\text{electric}} = \dfrac{Q}{\Delta t} = \boxed{5\,000 \text{ W}}$

(b) For a heat pump, $(\text{COP})_{\text{Carnot}} = \dfrac{T_h}{\Delta T} = \dfrac{295 \text{ K}}{27 \text{ K}} = 10.92$

$$\text{Actual COP} = 0.6(10.92) = 6.55 = \frac{|Q_h|}{W} = \frac{|Q_h|/\Delta t}{W/\Delta t}$$

Therefore, to bring 5 000 W of energy into the house only requires input power

$$\mathscr{P}_{\text{heat pump}} = \frac{W}{\Delta t} = \frac{|Q_h|/\Delta t}{\text{COP}} = \frac{5\,000 \text{ W}}{6.56} = \boxed{763 \text{ W}}$$

**P22.54** $|Q_c| = mc\Delta T + mL + mc\Delta T =$

$$|Q_c| = 0.500 \text{ kg}(4\,186 \text{ J/kg}\cdot{}^\circ\text{C})(10^\circ\text{C}) + 0.500 \text{ kg}\left(3.33\times10^5 \text{ J/kg}\right) + 0.500 \text{ kg}(2\,090 \text{ J/kg}\cdot{}^\circ\text{C})(20^\circ\text{C})$$

$$|Q_c| = 2.08\times10^5 \text{ J}$$

$$\frac{|Q_c|}{W} = \text{COP}_c(\text{refrigerator}) = \frac{T_c}{T_h - T_c}$$

$$W = \frac{|Q_c|(T_h - T_c)}{T_c} = \frac{\left(2.08\times10^5 \text{ J}\right)\left[20.0^\circ\text{C} - (-20.0^\circ\text{C})\right]}{(273 - 20.0) \text{ K}} = \boxed{32.9 \text{ kJ}}$$

**P22.55** $\Delta S_{\text{hot}} = \dfrac{-1\,000 \text{ J}}{600 \text{ K}}$

$\Delta S_{\text{cold}} = \dfrac{+750 \text{ J}}{350 \text{ K}}$

(a) $\Delta S_U = \Delta S_{\text{hot}} + \Delta S_{\text{cold}} = \boxed{0.476 \text{ J/K}}$

(b) $e_c = 1 - \dfrac{T_1}{T_2} = 0.417$

$W_{\text{eng}} = e_c|Q_h| = 0.417(1\,000 \text{ J}) = \boxed{417 \text{ J}}$

(c) $W_{\text{net}} = 417 \text{ J} - 250 \text{ J} = 167 \text{ J}$

$T_1\Delta S_U = 350 \text{ K}(0.476 \text{ J/K}) = \boxed{167 \text{ J}}$

***P22.56** (a) The energy put into the engine by the hot reservoir is $dQ_h = mcdT_h$. The energy put into the cold reservoir by the engine is $|dQ_c| = -mcdT_c = (1-e)dQ_h = \left[1-\left(1-\frac{T_c}{T_h}\right)\right]mcdT_h$. Then

$$-\frac{dT_c}{T_c} = \frac{dT_h}{T_h}$$

$$\int_{T_c}^{T_f} -\frac{dT}{T} = \int_{T_h}^{T_f} \frac{dT}{T}$$

$$-\ln T\Big|_{T_c}^{T_f} = \ln T\Big|_{T_h}^{T_f}$$

$$\ln\frac{T_c}{T_f} = \ln\frac{T_f}{T_h}$$

$$T_f^2 = T_c T_h$$

$$T_f = \left(T_h T_c\right)^{1/2}$$

(b) The hot reservoir loses energy $|Q_h| = mc\left(T_h - T_f\right)$. The cold reservoir gains $|Q_c| = mc\left(T_f - T_c\right)$. Then $|Q_h| = W_{\text{eng}} + |Q_c|$.

$$\begin{aligned} W_{\text{eng}} &= mc\left(T_h - T_f\right) - mc\left(T_f - T_c\right) \\ &= mc\left(T_h - \sqrt{T_h T_c} - \sqrt{T_h T_c} + T_c\right) \\ &= mc\left(T_h - 2\sqrt{T_h}\sqrt{T_c} + T_c\right) = mc\left(\sqrt{T_h} - \sqrt{T_c}\right)^2 \end{aligned}$$

**P22.57** (a) For an isothermal process, $Q = nRT\ln\left(\frac{V_2}{V_1}\right)$

Therefore, $Q_1 = nR(3T_i)\ln 2$

and $Q_3 = nR(T_i)\ln\left(\frac{1}{2}\right)$

For the constant volume processes, $Q_2 = \Delta E_{\text{int},2} = \frac{3}{2}nR(T_i - 3T_i)$

and $Q_4 = \Delta E_{\text{int},4} = \frac{3}{2}nR(3T_i - T_i)$

The net energy by heat transferred is then

$$Q = Q_1 + Q_2 + Q_3 + Q_4$$

or $Q = \boxed{2nRT_i\ln 2}$.

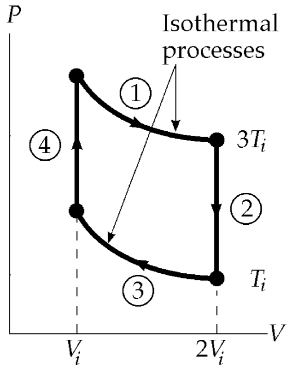


FIG. P22.57

(b) A positive value for heat represents energy transferred into the system.

Therefore, $|Q_h| = Q_1 + Q_4 = 3nRT_i(1 + \ln 2)$

Since the change in temperature for the complete cycle is zero,

$$\Delta E_{\text{int}} = 0 \text{ and } W_{\text{eng}} = Q$$

Therefore, the efficiency is $e_c = \frac{W_{\text{eng}}}{|Q_h|} = \frac{Q}{|Q_h|} = \frac{2\ln 2}{3(1+\ln 2)} = \boxed{0.273}$

**P22.58** (a) $$\frac{W_{\text{eng}}}{t}=1.50\times10^{8}\ \text{W}_{(\text{electrical})},\ Q=mL=\left[\frac{\frac{W_{\text{eng}}}{t}}{0.150}\right]\Delta t,$$

and $L=33.0\ \text{kJ/g}=33.0\times10^{6}\ \text{J/kg}$

$$m=\left[\frac{W_{\text{eng}}/t}{0.150}\right]\frac{\Delta t}{L}$$

$$m=\frac{\left(1.50\times10^{8}\ \text{W}\right)(86\,400\ \text{s/day})}{0.150\left(33.0\times10^{6}\ \text{J/kg}\right)\left(10^{3}\ \text{kg/metric ton}\right)}=\boxed{2\,620\ \text{metric tons/day}}$$

(b) Cost = (\$8.00/metric ton)(2 618 metric tons/day)(365 days/yr)

Cost = $\boxed{\$7.65\ \text{million/year}}$

(c) First find the rate at which heat energy is discharged into the water. If the plant is 15.0% efficient in producing electrical energy then the rate of heat production is

$$\frac{|Q_c|}{t}=\left(\frac{W_{\text{eng}}}{t}\right)\left(\frac{1}{e}-1\right)=\left(1.50\times10^{8}\ \text{W}\right)\left(\frac{1}{0.150}-1\right)=8.50\times10^{8}\ \text{W}.$$

Then, $\frac{|Q_c|}{t}=\frac{mc\Delta T}{t}$ and

$$\frac{m}{t}=\frac{\frac{|Q_c|}{t}}{c\Delta T}=\frac{8.50\times10^{8}\ \text{J/s}}{(4\,186\ \text{J/kg}\cdot{}^{\circ}\text{C})(5.00^{\circ}\text{C})}=\boxed{4.06\times10^{4}\ \text{kg/s}}.$$

**P22.59** $$e_c=1-\frac{T_c}{T_h}=\frac{W_{\text{eng}}}{|Q_h|}=\frac{\frac{W_{\text{eng}}}{\Delta t}}{\frac{|Q_h|}{\Delta t}}:\quad \frac{|Q_h|}{\Delta t}=\frac{\mathcal{P}}{(1-T_c/T_h)}=\frac{\mathcal{P}T_h}{T_h-T_c}$$

$$|Q_h|=W_{\text{eng}}+|Q_c|:\quad \frac{|Q_c|}{\Delta t}=\frac{|Q_h|}{\Delta t}-\frac{W_{\text{eng}}}{\Delta t}$$

$$\frac{|Q_c|}{\Delta t}=\frac{\mathcal{P}T_h}{T_h-T_c}-\mathcal{P}=\frac{\mathcal{P}T_c}{T_h-T_c}$$

$$|Q_c|=mc\Delta T:\quad \frac{|Q_c|}{\Delta t}=\left(\frac{\Delta m}{\Delta t}\right)c\Delta T=\frac{\mathcal{P}T_c}{T_h-T_c}$$

$$\frac{\Delta m}{\Delta t}=\frac{\mathcal{P}T_c}{(T_h-T_c)c\Delta T}$$

$$\frac{\Delta m}{\Delta t}=\frac{\left(1.00\times10^{9}\ \text{W}\right)(300\ \text{K})}{200\ \text{K}(4\,186\ \text{J/kg}\cdot{}^{\circ}\text{C})(6.00^{\circ}\text{C})}=\boxed{5.97\times10^{4}\ \text{kg/s}}$$

**P22.60** $$e_c = 1 - \frac{T_c}{T_h} = \frac{W_{\text{eng}}}{|Q_h|} = \frac{\frac{W_{\text{eng}}}{\Delta t}}{\frac{|Q_h|}{\Delta t}} \qquad \frac{|Q_h|}{\Delta t} = \frac{\mathcal{P}}{\left(1 - \frac{T_c}{T_h}\right)} = \frac{\mathcal{P} T_h}{T_h - T_c}$$

$$\frac{|Q_c|}{\Delta t} = \left(\frac{|Q_h|}{\Delta t}\right) - \mathcal{P} = \frac{\mathcal{P} T_c}{T_h - T_c}$$

$|Q_c| = mc\Delta T$, where $c$ is the specific heat of water.

Therefore, $$\frac{|Q_c|}{\Delta t} = \left(\frac{\Delta m}{\Delta t}\right) c\Delta T = \frac{\mathcal{P} T_c}{T_h - T_c}$$

and $$\frac{\Delta m}{\Delta t} = \boxed{\frac{\mathcal{P} T_c}{(T_h - T_c)c\Delta T}}$$

**P22.61** (a) $$35.0°\text{F} = \frac{5}{9}(35.0 - 32.0)°\text{C} = (1.67 + 273.15)\text{ K} = 274.82\text{ K}$$

$$98.6°\text{F} = \frac{5}{9}(98.6 - 32.0)°\text{C} = (37.0 + 273.15)\text{ K} = 310.15\text{ K}$$

$$\Delta S_{\text{ice water}} = \int \frac{dQ}{T} = (453.6\text{ g})(1.00\text{ cal/g}\cdot\text{K}) \times \int_{274.82}^{310.15} \frac{dT}{T} = 453.6 \ln\left(\frac{310.15}{274.82}\right) = 54.86\text{ cal/K}$$

$$\Delta S_{\text{body}} = -\frac{|Q|}{T_{\text{body}}} = -(453.6)(1.00)\frac{(310.15 - 274.82)}{310.15} = -51.67\text{ cal/K}$$

$$\Delta S_{\text{system}} = 54.86 - 51.67 = \boxed{3.19\text{ cal/K}}$$

(b) $$(453.6)(1)(T_F - 274.82) = \left(70.0 \times 10^3\right)(1)(310.15 - T_F)$$

Thus,

$$(70.0 + 0.453\,6) \times 10^3 T_F = \left[(70.0)(310.15) + (0.453\,6)(274.82)\right] \times 10^3$$

and $T_F = 309.92\text{ K} = 36.77°\text{C} = \boxed{98.19°\text{F}}$

$$\Delta S'_{\text{ice water}} = 453.6 \ln\left(\frac{309.92}{274.82}\right) = 54.52\text{ cal/K}$$

$$\Delta S'_{\text{body}} = -\left(70.0 \times 10^3\right)\ln\left(\frac{310.15}{309.92}\right) = -51.93\text{ cal/K}$$

$\Delta S'_{\text{sys}} = 54.52 - 51.93 = \boxed{2.59\text{ cal/K}}$ which is less than the estimate in part (a).

**P22.62** (a) For the isothermal process *AB*, the work on the gas is

$$W_{AB} = -P_A V_A \ln\left(\frac{V_B}{V_A}\right)$$

$$W_{AB} = -5\left(1.013\times10^5\ \text{Pa}\right)\left(10.0\times10^{-3}\ \text{m}^3\right)\ln\left(\frac{50.0}{10.0}\right)$$

$$W_{AB} = -8.15\times10^3\ \text{J}$$

where we have used $1.00\ \text{atm} = 1.013\times10^5\ \text{Pa}$

and $1.00\ \text{L} = 1.00\times10^{-3}\ \text{m}^3$

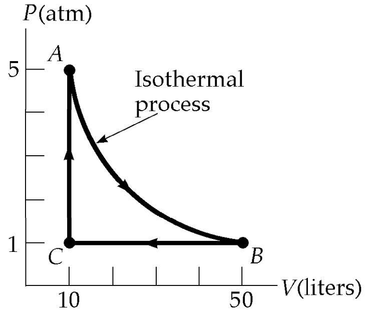


**FIG. P22.62**

$$W_{BC} = -P_B\Delta V = -\left(1.013\times10^5\ \text{Pa}\right)\left[(10.0-50.0)\times10^{-3}\right]\text{m}^3 = +4.05\times10^3\ \text{J}$$

$$W_{CA} = 0 \text{ and } W_{\text{eng}} = -W_{AB} - W_{BC} = 4.11\times10^3\ \text{J} = \boxed{4.11\ \text{kJ}}$$

(b) Since *AB* is an isothermal process, $\Delta E_{\text{int},\,AB} = 0$

and $Q_{AB} = -W_{AB} = 8.15\times10^3\ \text{J}$

For an ideal monatomic gas, $C_V = \frac{3R}{2}$ and $C_P = \frac{5R}{2}$

$$T_B = T_A = \frac{P_B V_B}{nR} = \frac{\left(1.013\times10^5\right)\left(50.0\times10^{-3}\right)}{R} = \frac{5.05\times10^3}{R}$$

Also, $$T_C = \frac{P_C V_C}{nR} = \frac{\left(1.013\times10^5\right)\left(10.0\times10^{-3}\right)}{R} = \frac{1.01\times10^3}{R}$$

$$Q_{CA} = nC_V\Delta T = 1.00\left(\frac{3}{2}R\right)\left(\frac{5.05\times10^3 - 1.01\times10^3}{R}\right) = 6.08\ \text{kJ}$$

so the total energy absorbed by heat is $Q_{AB} + Q_{CA} = 8.15\ \text{kJ} + 6.08\ \text{kJ} = \boxed{14.2\ \text{kJ}}$.

(c) $$Q_{BC} = nC_P\Delta T = \frac{5}{2}(nR\Delta T) = \frac{5}{2}P_B\Delta V_{BC}$$

$$Q_{BC} = \frac{5}{2}\left(1.013\times10^5\right)\left[(10.0-50.0)\times10^{-3}\right] = -1.01\times10^4\ \text{J} = \boxed{-10.1\ \text{kJ}}$$

(d) $$e = \frac{W_{\text{eng}}}{|Q_h|} = \frac{W_{\text{eng}}}{Q_{AB}+Q_{CA}} = \frac{4.11\times10^3\ \text{J}}{1.42\times10^4\ \text{J}} = 0.289 \text{ or } \boxed{28.9\%}$$

***P22.63** Like a refrigerator, an air conditioner has as its purpose the removal of energy by heat from the cold reservoir.

Its ideal COP is $$\mathrm{COP}_{\text{Carnot}}=\frac{T_c}{T_h-T_c}=\frac{280\text{ K}}{20\text{ K}}=14.0$$

(a) Its actual COP is $$0.400(14.0)=5.60=\frac{|Q_c|}{|Q_h|-|Q_c|}=\frac{|Q_c/\Delta t|}{|Q_h/\Delta t|-|Q_c/\Delta t|}$$

$$5.60\left|\frac{Q_h}{\Delta t}\right|-5.60\left|\frac{Q_c}{\Delta t}\right|=\left|\frac{Q_c}{\Delta t}\right|$$

$$5.60(10.0\text{ kW})=6.60\left|\frac{Q_c}{\Delta t}\right| \text{ and } \left|\frac{Q_c}{\Delta t}\right|=\boxed{8.48\text{ kW}}$$

(b) $|Q_h|=W_{\text{eng}}+|Q_c|$: $$\frac{W_{\text{eng}}}{\Delta t}=\left|\frac{Q_h}{\Delta t}\right|-\left|\frac{Q_c}{\Delta t}\right|=10.0\text{ kW}-8.48\text{ kW}=\boxed{1.52\text{ kW}}$$

(c) The air conditioner operates in a cycle, so the entropy of the working fluid does not change. The hot reservoir increases in entropy by

$$\frac{|Q_h|}{T_h}=\frac{(10.0\times10^3\text{ J/s})(3\,600\text{ s})}{300\text{ K}}=1.20\times10^5\text{ J/K}$$

The cold room decreases in entropy by

$$\Delta S=-\frac{|Q_c|}{T_c}=-\frac{(8.48\times10^3\text{ J/s})(3\,600\text{ s})}{280\text{ K}}=-1.09\times10^5\text{ J/K}$$

The net entropy change is positive, as it must be:

$$+1.20\times10^5\text{ J/K}-1.09\times10^5\text{ J/K}=\boxed{1.09\times10^4\text{ J/K}}$$

(d) The new ideal COP is $$\mathrm{COP}_{\text{Carnot}}=\frac{T_c}{T_h-T_c}=\frac{280\text{ K}}{25\text{ K}}=11.2$$

We suppose the actual COP is $0.400(11.2)=4.48$

As a fraction of the original 5.60, this is $\frac{4.48}{5.60}=0.800$, so the fractional change is to $\boxed{\text{drop by }20.0\%}$.

**P22.64** (a) $$W=\int_{V_i}^{V_f}PdV=nRT\int_{V_i}^{2V_i}\frac{dV}{V}=(1.00)RT\ln\left(\frac{2V_i}{V_i}\right)=\boxed{RT\ln2}$$

(b) $\boxed{\text{The second law refers to cycles.}}$

**P22.65** At point $A$, $P_iV_i = nRT_i$ and $n = 1.00$ mol

At point $B$, $3P_iV_i = nRT_B$ so $T_B = 3T_i$

At point $C$, $(3P_i)(2V_i) = nRT_C$ and $T_C = 6T_i$

At point $D$, $P_i(2V_i) = nRT_D$ so $T_D = 2T_i$

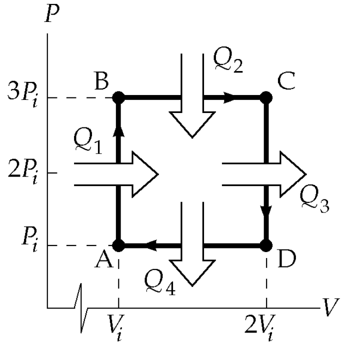


FIG. P22.65

The heat for each step in the cycle is found using $C_V = \frac{3R}{2}$ and $C_P = \frac{5R}{2}$:

$$Q_{AB} = nC_V(3T_i - T_i) = 3nRT_i$$
$$Q_{BC} = nC_P(6T_i - 3T_i) = 7.50nRT_i$$
$$Q_{CD} = nC_V(2T_i - 6T_i) = -6nRT_i$$
$$Q_{DA} = nC_P(T_i - 2T_i) = -2.50nRT_i$$

(a) Therefore, $Q_{\text{entering}} = |Q_h| = Q_{AB} + Q_{BC} = \boxed{10.5nRT_i}$

(b) $Q_{\text{leaving}} = |Q_c| = |Q_{CD} + Q_{DA}| = \boxed{8.50nRT_i}$

(c) Actual efficiency, $e = \frac{|Q_h| - |Q_c|}{|Q_h|} = \boxed{0.190}$

(d) Carnot efficiency, $e_c = 1 - \frac{T_c}{T_h} = 1 - \frac{T_i}{6T_i} = \boxed{0.833}$

***P22.66** $\Delta S = \int_i^f \frac{dQ}{T} = \int_i^f \frac{nC_P dT}{T} = nC_P \int_i^f T^{-1} dT = nC_P \ln T\Big|_{T_i}^{T_f} = nC_P(\ln T_f - \ln T_i) = nC_P \ln\left(\frac{T_f}{T_i}\right)$

$\Delta S = nC_P \ln\left(\frac{PV_f}{nR}\frac{nR}{PV_i}\right) = \boxed{nC_P \ln 3}$

***P22.67** (a) The ideal gas at constant temperature keeps constant internal energy. As it puts out energy by work in expanding it must take in an equal amount of energy by heat. Thus its entropy increases. Let $P_i$, $V_i$, $T_i$ represent the state of the gas before the isothermal expansion. Let $P_C$, $V_C$, $T_i$ represent the state after this process, so that $P_iV_i = P_CV_C$. Let $P_i$, $3V_i$, $T_f$ represent the state after the adiabatic compression.

Then $P_CV_C^{\gamma} = P_i(3V_i)^{\gamma}$

Substituting $P_C = \frac{P_iV_i}{V_C}$

gives $P_iV_iV_C^{\gamma-1} = P_i(3^{\gamma}V_i^{\gamma})$

Then $V_C^{\gamma-1} = 3^{\gamma}V_i^{\gamma-1}$ and $\frac{V_C}{V_i} = 3^{\gamma/(\gamma-1)}$

*continued on next page*

The work output in the isothermal expansion is

$$W=\int_i^C PdV=nRT_i\int_i^C V^{-1}dV=nRT_i\ln\left(\frac{V_C}{V_i}\right)=nRT_i\ln\left(3^{\gamma/(\gamma-1)}\right)=nRT_i\left(\frac{\gamma}{\gamma-1}\right)\ln 3$$

This is also the input heat, so the entropy change is

$$\Delta S=\frac{Q}{T}=nR\left(\frac{\gamma}{\gamma-1}\right)\ln 3$$

Since $C_P=\gamma C_V=C_V+R$

we have $(\gamma-1)C_V=R,\ C_V=\dfrac{R}{\gamma-1}$

and $C_P=\dfrac{\gamma R}{\gamma-1}$

Then the result is $\boxed{\Delta S=nC_P\ln 3}$

(b) The pair of processes considered here carry the gas from the initial state in Problem 66 to the final state there. Entropy is a function of state. Entropy change does not depend on path. Therefore the entropy change in Problem 66 equals $\Delta S_{\text{isothermal}}+\Delta S_{\text{adiabatic}}$ in this problem. Since $\Delta S_{\text{adiabatic}}=0$, the answers to Problems 66 and 67 (a) must be the same.

**P22.68** Simply evaluate the maximum (Carnot) efficiency.

$$e_C=\frac{\Delta T}{T_h}=\frac{4.00\text{ K}}{277\text{ K}}=\boxed{0.014\ 4}$$

The proposal does not merit serious consideration.

**P22.69** The heat transfer over the paths *CD* and *BA* is zero since they are adiabatic.

Over path *BC*: $Q_{BC}=nC_P(T_C-T_B)>0$

Over path *DA*: $Q_{DA}=nC_V(T_A-T_D)<0$

Therefore, $|Q_c|=|Q_{DA}|$ and $Q_h=Q_{BC}$

The efficiency is then

$$e=1-\frac{|Q_c|}{Q_h}=1-\frac{(T_D-T_A)C_V}{(T_C-T_B)C_P}$$

$$e=1-\frac{1}{\gamma}\left[\frac{T_D-T_A}{T_C-T_B}\right]$$

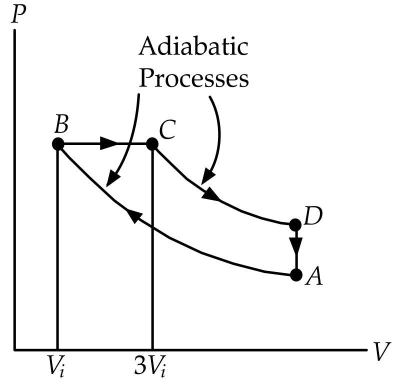


FIG. P22.69

**P22.70** (a) Use the equation of state for an ideal gas

$$V=\frac{nRT}{P}$$

$$V_A=\frac{1.00(8.314)(600)}{25.0\left(1.013\times10^5\right)}=\boxed{1.97\times10^{-3}\ \text{m}^3}$$

$$V_C=\frac{1.00(8.314)(400)}{1.013\times10^5}=\boxed{32.8\times10^{-3}\ \text{m}^3}$$

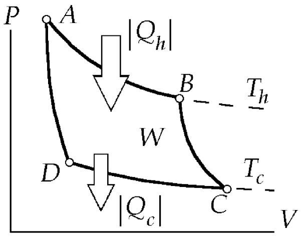


FIG. P22.70

Since *AB* is isothermal, $P_AV_A=P_BV_B$

and since *BC* is adiabatic, $P_BV_B^\gamma=P_CV_C^\gamma$

Combining these expressions, $$V_B=\left[\left(\frac{P_C}{P_A}\right)\frac{V_C^\gamma}{V_A}\right]^{1/(\gamma-1)}=\left[\left(\frac{1.00}{25.0}\right)\frac{\left(32.8\times10^{-3}\ \text{m}^3\right)^{1.40}}{1.97\times10^{-3}\ \text{m}^3}\right]^{(1/0.400)}$$

$$V_B=\boxed{11.9\times10^{-3}\ \text{m}^3}$$

Similarly, $$V_D=\left[\left(\frac{P_A}{P_C}\right)\frac{V_A^\gamma}{V_C}\right]^{1/(\gamma-1)}=\left[\left(\frac{25.0}{1.00}\right)\frac{\left(1.97\times10^{-3}\ \text{m}^3\right)^{1.40}}{32.8\times10^{-3}\ \text{m}^3}\right]^{(1/0.400)}$$

or $$V_D=\boxed{5.44\times10^{-3}\ \text{m}^3}$$

Since *AB* is isothermal, $P_AV_A=P_BV_B$

and $$P_B=P_A\left(\frac{V_A}{V_B}\right)=25.0\ \text{atm}\left(\frac{1.97\times10^{-3}\ \text{m}^3}{11.9\times10^{-3}\ \text{m}^3}\right)=\boxed{4.14\ \text{atm}}$$

Also, *CD* is an isothermal and $$P_D=P_C\left(\frac{V_C}{V_D}\right)=1.00\ \text{atm}\left(\frac{32.8\times10^{-3}\ \text{m}^3}{5.44\times10^{-3}\ \text{m}^3}\right)=\boxed{6.03\ \text{atm}}$$

Solving part (c) before part (b):

(c) For this Carnot cycle, $$e_c=1-\frac{T_c}{T_h}=1-\frac{400\ \text{K}}{600\ \text{K}}=\boxed{0.333}$$

(b) Energy is added by heat to the gas during the process *AB*. For the isothermal process, $\Delta E_{\text{int}}=0$.

and the first law gives $$Q_{AB}=-W_{AB}=nRT_h\ln\left(\frac{V_B}{V_A}\right)$$

or $$|Q_h|=Q_{AB}=1.00\ \text{mol}(8.314\ \text{J/mol}\cdot\text{K})(600\ \text{K})\ln\left(\frac{11.9}{1.97}\right)=8.97\ \text{kJ}$$

Then, from $$e=\frac{W_{\text{eng}}}{|Q_h|}$$

the net work done per cycle is $W_{\text{eng}}=e_c|Q_h|=0.333(8.97\ \text{kJ})=\boxed{2.99\ \text{kJ}}$.

**P22.71** (a) $\boxed{20.0°\text{C}}$

(b) $$\Delta S = mc\ln\frac{T_f}{T_1} + mc\ln\frac{T_f}{T_2} = 1.00\text{ kg}(4.19\text{ kJ/kg}\cdot\text{K})\left[\ln\frac{T_f}{T_1}+\ln\frac{T_f}{T_2}\right] = (4.19\text{ kJ/K})\ln\left(\frac{293}{283}\cdot\frac{293}{303}\right)$$

(c) $\Delta S = \boxed{+4.88\text{ J/K}}$

(d) $\boxed{\text{Yes}}$. Entropy has increased.

## ANSWERS TO EVEN PROBLEMS

**P22.2** (a) 667 J; (b) 467 J

**P22.4** (a) 30.0%; (b) 60.0%

**P22.6** 55.4%

**P22.8** 77.8 W

**P22.10** (a) 869 MJ; (b) 330 MJ

**P22.12** 197 kJ

**P22.14** 546°C

**P22.16** 33.0%

**P22.18** (a) 5.12%; (b) 5.27 TJ/h;
(c) see the solution

**P22.20** 453 K

**P22.22** (a), (b) see the solution;
(c) 23.7%; see the solution

**P22.24** 11.8

**P22.26** 1.17 J

**P22.28** (a) 204 W; (b) 2.43 kW

**P22.30** (a) 2.00; (b) 3.00; (c) 33.3%

**P22.32** (a) 51.2%; (b) 36.2%

**P22.34** (a), (b) see the solution;
(c) $Q_h = 149$ J; $|Q_c| = 65.0$ J; $W_{\text{eng}} = 84.3$ J;
(d) 56.5%; (e) $1.42\times10^3$ rev/min

**P22.36** 4.88 kJ/kg·K

**P22.38** (a) isobaric; (b) 402 kJ; (c) 1.20 kJ/K

**P22.40** 3.27 J/K

**P22.42** 718 J/K

**P22.44** (a) 39.4 L; (b) −2.50 kJ; (c) −2.50 kJ;
(d) −6.87 J/K; (e) +9.16 J/K

**P22.46** 0.507 J/K

**P22.48** 34.6 J/K

**P22.50** (a) 2 heads and 2 tails;
(b) All heads or all tails;
(c) 2 heads and 2 tails

**P22.52** 8.36 MJ/K

**P22.54** 32.9 kJ

**P22.56** see the solution

**P22.58** (a) $2.62\times10^3$ tons/d; (b) $7.65 million/yr;
(c) $4.06\times10^4$ kg/s

**P22.60** $\frac{\mathcal{P} T_c}{(T_h - T_c) c \Delta T}$

**P22.62** (a) 4.11 kJ; (b) 14.2 kJ; (c) 10.1 kJ; (d) 28.9%

**P22.64** see the solution

**P22.66** $nC_P \ln 3$

**P22.68** no; see the solution

**P22.70** (a)

| | $P$, atm | $V$, L |
|---|---|---|
| A | 25.0 | 1.97 |
| B | 4.14 | 11.9 |
| C | 1.00 | 32.8 |
| D | 6.03 | 5.44 |

(b) 2.99 kJ; (c) 33.3%

# 23

# Electric Fields

**CHAPTER OUTLINE**

23.1 Properties of Electric Charges
23.2 Charging Objects by Induction
23.3 Coulomb's Law
23.4 The Electric Field
23.5 Electric Field of a Continuous Charge Distribution
23.6 Electric Field Lines
23.7 Motion of Charged Particles in a Uniform Electric Field

## ANSWERS TO QUESTIONS

**Q23.1** A neutral atom is one that has no net charge. This means that it has the same number of electrons orbiting the nucleus as it has protons in the nucleus. A negatively charged atom has one or more excess electrons.

**Q23.2** When the comb is nearby, molecules in the paper are polarized, similar to the molecules in the wall in Figure 23.5a, and the paper is attracted. During contact, charge from the comb is transferred to the paper by conduction. Then the paper has the same charge as the comb, and is repelled.

**Q23.3** The clothes dryer rubs dissimilar materials together as it tumbles the clothes. Electrons are transferred from one kind of molecule to another. The charges on pieces of cloth, or on nearby objects charged by induction, can produce strong electric fields that promote the ionization process in the surrounding air that is necessary for a spark to occur. Then you hear or see the sparks.

**Q23.4** To avoid making a spark. Rubber-soled shoes acquire a charge by friction with the floor and could discharge with a spark, possibly causing an explosion of any flammable material in the oxygen-enriched atmosphere.

**Q23.5** Electrons are less massive and more mobile than protons. Also, they are more easily detached from atoms than protons.

**Q23.6** The electric field due to the charged rod induces charges on near and far sides of the sphere. The attractive Coulomb force of the rod on the dissimilar charge on the close side of the sphere is larger than the repulsive Coulomb force of the rod on the like charge on the far side of the sphere. The result is a net attraction of the sphere to the rod. When the sphere touches the rod, charge is conducted between the rod and the sphere, leaving both the rod and the sphere like-charged. This results in a repulsive Coulomb force.

**Q23.7** All of the constituents of air are nonpolar except for water. The polar water molecules in the air quite readily "steal" charge from a charged object, as any physics teacher trying to perform electrostatics demonstrations in the summer well knows. As a result—it is difficult to accumulate large amounts of excess charge on an object in a humid climate. During a North American winter, the cold, dry air allows accumulation of significant excess charge, giving the potential (pun intended) for a shocking (pun also intended) introduction to static electricity sparks.

**Q23.8** Similarities: A force of gravity is proportional to the product of the intrinsic properties (masses) of two particles, and inversely proportional to the square of the separation distance. An electrical force exhibits the same proportionalities, with charge as the intrinsic property.

Differences: The electrical force can either attract or repel, while the gravitational force as described by Newton's law can only attract. The electrical force between elementary particles is vastly stronger than the gravitational force.

**Q23.9** No. The balloon induces polarization of the molecules in the wall, so that a layer of positive charge exists near the balloon. This is just like the situation in Figure 23.5a, except that the signs of the charges are reversed. The attraction between these charges and the negative charges on the balloon is stronger than the repulsion between the negative charges on the balloon and the negative charges in the polarized molecules (because they are farther from the balloon), so that there is a net attractive force toward the wall. Ionization processes in the air surrounding the balloon provide ions to which excess electrons in the balloon can transfer, reducing the charge on the balloon and eventually causing the attractive force to be insufficient to support the weight of the balloon.

**Q23.10** The electric field due to the charged rod induces a charge in the aluminum foil. If the rod is brought towards the aluminum from above, the top of the aluminum will have a negative charge induced on it, while the parts draping over the pencil can have a positive charge induced on them. These positive induced charges on the two parts give rise to a repulsive Coulomb force. If the pencil is a good insulator, the net charge on the aluminum can be zero.

**Q23.11** So the electric field created by the test charge does not distort the electric field you are trying to measure, by moving the charges that create it.

**Q23.12** With a very high budget, you could send first a proton and then an electron into an evacuated region in which the field exists. If the field is gravitational, both particles will experience a force in the same direction, while they will experience forces in opposite directions if the field is electric.

On a more practical scale, stick identical pith balls on each end of a toothpick. Charge one pith ball + and the other –, creating a large-scale dipole. Carefully suspend this dipole about its center of mass so that it can rotate freely. When suspended in the field in question, the dipole will rotate to align itself with an electric field, while it will not for a gravitational field. If the test device does not rotate, be sure to insert it into the field in more than one orientation in case it was aligned with the electric field when you inserted it on the first trial.

**Q23.13** The student standing on the insulating platform is held at the same electrical potential as the generator sphere. Charge will only flow when there is a difference in potential. The student who unwisely touches the charged sphere is near zero electrical potential when compared to the charged sphere. When the student comes in contact with the sphere, charge will flow from the sphere to him or her until they are at the same electrical potential.

**Q23.14** An electric field once established by a positive or negative charge extends in all directions from the charge. Thus, it can exist in empty space if that is what surrounds the charge. There is no material at point $A$ in Figure 23.23(a), so there is no charge, nor is there a force. There would be a force if a charge were present at point $A$, however. A field does exist at point $A$.

**Q23.15** If a charge distribution is small compared to the distance of a field point from it, the charge distribution can be modeled as a single particle with charge equal to the net charge of the distribution. Further, if a charge distribution is spherically symmetric, it will create a field at exterior points just as if all of its charge were a point charge at its center.

**Q23.16** The direction of the electric field is the direction in which a positive test charge would feel a force when placed in the field. A charge will not experience two electrical forces at the same time, but the vector sum of the two. If electric field lines crossed, then a test charge placed at the point at which they cross would feel a force in two directions. Furthermore, the path that the test charge would follow if released at the point where the field lines cross would be indeterminate.

**Q23.17** Both figures are drawn correctly. $\mathbf{E}_1$ and $\mathbf{E}_2$ are the electric fields separately created by the point charges $q_1$ and $q_2$ in Figure 23.14 or $q$ and $-q$ in Figure 23.15, respectively. The net electric field is the vector sum of $\mathbf{E}_1$ and $\mathbf{E}_2$, shown as **E**. Figure 23.21 shows only one electric field line at each point away from the charge. At the point location of an object modeled as a point charge, the direction of the field is undefined, and so is its magnitude.

**Q23.18** The electric forces on the particles have the same magnitude, but are in opposite directions. The electron will have a much larger acceleration (by a factor of about 2 000) than the proton, due to its much smaller mass.

**Q23.19** The electric field around a point charge approaches infinity as $r$ approaches zero.

**Q23.20** Vertically downward.

**Q23.21** Four times as many electric field lines start at the surface of the larger charge as end at the smaller charge. The extra lines extend away from the pair of charges. They may never end, or they may terminate on more distant negative charges. Figure 23.24 shows the situation for charges $+2q$ and $-q$.

**Q23.22** At a point exactly midway between the two changes.

**Q23.23** Linear charge density, $\lambda$, is charge per unit length. It is used when trying to determine the electric field created by a charged rod.

Surface charge density, $\sigma$, is charge per unit area. It is used when determining the electric field above a charged sheet or disk.

Volume charge density, $\rho$, is charge per unit volume. It is used when determining the electric field due to a uniformly charged sphere made of insulating material.

**Q23.24** Yes, the path would still be parabolic. The electrical force on the electron is in the downward direction. This is similar to throwing a ball from the roof of a building horizontally or at some angle with the vertical. In both cases, the acceleration due to gravity is downward, giving a parabolic trajectory.

**Q23.25** No. Life would be no different if electrons were + charged and protons were – charged. Opposite charges would still attract, and like charges would repel. The naming of + and – charge is merely a convention.

**Q23.26** If the antenna were not grounded, electric charges in the atmosphere during a storm could place the antenna at a high positive or negative potential. The antenna would then place the television set inside the house at the high voltage, to make it a shock hazard. The wire to the ground keeps the antenna, the television set, and even the air around the antenna at close to zero potential.

**Q23.27** People are all attracted to the Earth. If the force were electrostatic, people would all carry charge with the same sign and would repel each other. This repulsion is not observed. When we changed the charge on a person, as in the chapter-opener photograph, the person's weight would change greatly in magnitude or direction. We could levitate an airplane simply by draining away its electric charge. The failure of such experiments gives evidence that the attraction to the Earth is not due to electrical forces.

**Q23.28** In special orientations the force between two dipoles can be zero or a force of repulsion. In general each dipole will exert a torque on the other, tending to align its axis with the field created by the first dipole. After this alignment, each dipole exerts a force of attraction on the other.

## SOLUTIONS TO PROBLEMS

### Section 23.1 **Properties of Electric Charges**

***P23.1** (a) The mass of an average neutral hydrogen atom is 1.007 9u. Losing one electron reduces its mass by a negligible amount, to

$$1.007\,9\left(1.660\times10^{-27}\text{ kg}\right)-9.11\times10^{-31}\text{ kg}=\boxed{1.67\times10^{-27}\text{ kg}}.$$

Its charge, due to loss of one electron, is

$$0-1\left(-1.60\times10^{-19}\text{ C}\right)=\boxed{+1.60\times10^{-19}\text{ C}}.$$

(b) By similar logic, charge $=\boxed{+1.60\times10^{-19}\text{ C}}$

mass $=22.99\left(1.66\times10^{-27}\text{ kg}\right)-9.11\times10^{-31}\text{ kg}=\boxed{3.82\times10^{-26}\text{ kg}}$

(c) charge of $\text{Cl}^-=\boxed{-1.60\times10^{-19}\text{ C}}$

mass $=35.453\left(1.66\times10^{-27}\text{ kg}\right)+9.11\times10^{-31}\text{ kg}=\boxed{5.89\times10^{-26}\text{ kg}}$

(d) charge of $\text{Ca}^{++}=-2\left(-1.60\times10^{-19}\text{ C}\right)=\boxed{+3.20\times10^{-19}\text{ C}}$

mass $=40.078\left(1.66\times10^{-27}\text{ kg}\right)-2\left(9.11\times10^{-31}\text{ kg}\right)=\boxed{6.65\times10^{-26}\text{ kg}}$

(e) charge of $\text{N}^{3-}=3\left(-1.60\times10^{-19}\text{ C}\right)=\boxed{-4.80\times10^{-19}\text{ C}}$

mass $=14.007\left(1.66\times10^{-27}\text{ kg}\right)+3\left(9.11\times10^{-31}\text{ kg}\right)=\boxed{2.33\times10^{-26}\text{ kg}}$

(f) charge of $\text{N}^{4+}=4\left(1.60\times10^{-19}\text{ C}\right)=\boxed{+6.40\times10^{-19}\text{ C}}$

mass $=14.007\left(1.66\times10^{-27}\text{ kg}\right)-4\left(9.11\times10^{-31}\text{ kg}\right)=\boxed{2.32\times10^{-26}\text{ kg}}$

(g) We think of a nitrogen nucleus as a seven-times ionized nitrogen atom.

charge $=7\left(1.60\times10^{-19}\text{ C}\right)=\boxed{1.12\times10^{-18}\text{ C}}$

mass $=14.007\left(1.66\times10^{-27}\text{ kg}\right)-7\left(9.11\times10^{-31}\text{ kg}\right)=\boxed{2.32\times10^{-26}\text{ kg}}$

(h) charge $=\boxed{-1.60\times10^{-19}\text{ C}}$

mass $=\left[2(1.007\,9)+15.999\right]1.66\times10^{-27}\text{ kg}+9.11\times10^{-31}\text{ kg}=\boxed{2.99\times10^{-26}\text{ kg}}$

**P23.2** (a) $$N=\left(\frac{10.0\text{ grams}}{107.87\text{ grams/mol}}\right)\left(6.02\times10^{23}\ \frac{\text{atoms}}{\text{mol}}\right)\left(47\ \frac{\text{electrons}}{\text{atom}}\right)=\boxed{2.62\times10^{24}}$$

(b) $$\#\text{ electrons added}=\frac{Q}{e}=\frac{1.00\times10^{-3}\text{ C}}{1.60\times10^{-19}\text{ C/electron}}=6.25\times10^{15}$$

or $\boxed{2.38\text{ electrons for every }10^{9}\text{ already present}}$.

Section 23.2 **Charging Objects by Induction**

Section 23.3 **Coulomb's Law**

**P23.3** If each person has a mass of $\approx 70\text{ kg}$ and is (almost) composed of water, then each person contains

$$N\cong\left(\frac{70\,000\text{ grams}}{18\text{ grams/mol}}\right)\left(6.02\times10^{23}\ \frac{\text{molecules}}{\text{mol}}\right)\left(10\ \frac{\text{protons}}{\text{molecule}}\right)\cong 2.3\times10^{28}\text{ protons}.$$

With an excess of 1% electrons over protons, each person has a charge

$$q=0.01\left(1.6\times10^{-19}\text{ C}\right)\left(2.3\times10^{28}\right)=3.7\times10^{7}\text{ C}.$$

So $$F=k_e\frac{q_1q_2}{r^2}=\left(9\times10^{9}\right)\frac{\left(3.7\times10^{7}\right)^2}{0.6^2}\text{ N}=4\times10^{25}\text{ N}\boxed{\sim10^{26}\text{ N}}.$$

This force is almost enough to lift a weight equal to that of the Earth:

$$Mg=6\times10^{24}\text{ kg}\left(9.8\text{ m/s}^2\right)=6\times10^{25}\text{ N}\sim10^{26}\text{ N}.$$

***P23.4** The force on one proton is $\mathbf{F}=\frac{k_eq_1q_2}{r^2}$ away from the other proton. Its magnitude is

$$\left(8.99\times10^{9}\text{ N}\cdot\text{m/C}^2\right)\left(\frac{1.6\times10^{-19}\text{ C}}{2\times10^{-15}\text{ m}}\right)^2=\boxed{57.5\text{ N}}.$$

**P23.5** (a) $$F_e=\frac{k_eq_1q_2}{r^2}=\frac{\left(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\text{ C}\right)^2}{\left(3.80\times10^{-10}\text{ m}\right)^2}=\boxed{1.59\times10^{-9}\text{ N}}\text{ (repulsion)}$$

(b) $$F_g=\frac{Gm_1m_2}{r^2}=\frac{\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.67\times10^{-27}\text{ kg}\right)^2}{\left(3.80\times10^{-10}\text{ m}\right)^2}=\boxed{1.29\times10^{-45}\text{ N}}$$

The electric force is $\boxed{\text{larger by }1.24\times10^{36}\text{ times}}$.

(c) If $k_e\frac{q_1q_2}{r^2}=G\frac{m_1m_2}{r^2}$ with $q_1=q_2=q$ and $m_1=m_2=m$, then

$$\frac{q}{m}=\sqrt{\frac{G}{k_e}}=\sqrt{\frac{6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2}{8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2}}=\boxed{8.61\times10^{-11}\text{ C/kg}}.$$

**P23.6** We find the equal-magnitude charges on both spheres:

$$F = k_e \frac{q_1 q_2}{r^2} = k_e \frac{q^2}{r^2} \qquad \text{so} \qquad q = r\sqrt{\frac{F}{k_e}} = (1.00 \text{ m})\sqrt{\frac{1.00\times 10^{4} \text{ N}}{8.99\times 10^{9} \text{ N}\cdot\text{m}^2/\text{C}^2}} = 1.05\times 10^{-3} \text{ C}.$$

The number of electron transferred is then

$$N_{\text{xfer}} = \frac{1.05\times 10^{-3} \text{ C}}{1.60\times 10^{-19} \text{ C}/e^-} = 6.59\times 10^{15} \text{ electrons}.$$

The whole number of electrons in each sphere is

$$N_{\text{tot}} = \left(\frac{10.0 \text{ g}}{107.87 \text{ g/mol}}\right)\left(6.02\times 10^{23} \text{ atoms/mol}\right)\left(47 \ e^-/\text{atom}\right) = 2.62\times 10^{24} \ e^-.$$

The fraction transferred is then

$$f = \frac{N_{\text{xfer}}}{N_{\text{tot}}} = \left(\frac{6.59\times 10^{15}}{2.62\times 10^{24}}\right) = \boxed{2.51\times 10^{-9}} = 2.51 \text{ charges in every billion.}$$

**P23.7**

$$F_1 = k_e \frac{q_1 q_2}{r^2} = \frac{\left(8.99\times 10^{9} \text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(7.00\times 10^{-6} \text{ C}\right)\left(2.00\times 10^{-6} \text{ C}\right)}{(0.500 \text{ m})^2} = 0.503 \text{ N}$$

$$F_2 = k_e \frac{q_1 q_2}{r^2} = \frac{\left(8.99\times 10^{9} \text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(7.00\times 10^{-6} \text{ C}\right)\left(4.00\times 10^{-6} \text{ C}\right)}{(0.500 \text{ m})^2} = 1.01 \text{ N}$$

$$F_x = 0.503\cos 60.0° + 1.01\cos 60.0° = 0.755 \text{ N}$$
$$F_y = 0.503\sin 60.0° - 1.01\sin 60.0° = -0.436 \text{ N}$$
$$\mathbf{F} = (0.755 \text{ N})\hat{\mathbf{i}} - (0.436 \text{ N})\hat{\mathbf{j}} = \boxed{0.872 \text{ N at an angle of } 330°}$$

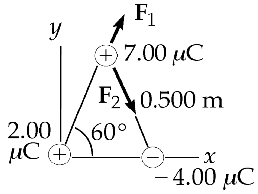


**FIG. P23.7**

**P23.8**

$$F = k_e \frac{q_1 q_2}{r^2} = \frac{\left(8.99\times 10^{9} \text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times 10^{-19} \text{ C}\right)^2\left(6.02\times 10^{23}\right)^2}{\left[2\left(6.37\times 10^{6} \text{ m}\right)\right]^2} = \boxed{514 \text{ kN}}$$

**P23.9** (a) The force is one of $\boxed{\text{attraction}}$. The distance *r* in Coulomb's law is the distance between centers. The magnitude of the force is

$$F = \frac{k_e q_1 q_2}{r^2} = \left(8.99\times 10^{9} \text{ N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(12.0\times 10^{-9} \text{ C}\right)\left(18.0\times 10^{-9} \text{ C}\right)}{(0.300 \text{ m})^2} = \boxed{2.16\times 10^{-5} \text{ N}}.$$

(b) The net charge of $-6.00\times 10^{-9}$ C will be equally split between the two spheres, or $-3.00\times 10^{-9}$ C on each. The force is one of $\boxed{\text{repulsion}}$, and its magnitude is

$$F = \frac{k_e q_1 q_2}{r^2} = \left(8.99\times 10^{9} \text{ N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(3.00\times 10^{-9} \text{ C}\right)\left(3.00\times 10^{-9} \text{ C}\right)}{(0.300 \text{ m})^2} = \boxed{8.99\times 10^{-7} \text{ N}}.$$

**P23.10** Let the third bead have charge $Q$ and be located distance $x$ from the left end of the rod. This bead will experience a net force given by

$$\mathbf{F}=\frac{k_e(3q)Q}{x^2}\hat{\mathbf{i}}+\frac{k_e(q)Q}{(d-x)^2}\left(-\hat{\mathbf{i}}\right).$$

The net force will be zero if $\frac{3}{x^2}=\frac{1}{(d-x)^2}$, or $d-x=\frac{x}{\sqrt{3}}$.

This gives an equilibrium position of the third bead of $x=\boxed{0.634d}$.

The equilibrium is $\boxed{\text{stable if the third bead has positive charge}}$.

**P23.11** (a) $$F=\frac{k_e e^2}{r^2}=\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(1.60\times10^{-19}\ \text{C}\right)^2}{\left(0.529\times10^{-10}\ \text{m}\right)^2}=\boxed{8.22\times10^{-8}\ \text{N}}$$

(b) We have $F=\frac{mv^2}{r}$ from which

$$v=\sqrt{\frac{Fr}{m}}=\sqrt{\frac{8.22\times10^{-8}\ \text{N}\left(0.529\times10^{-10}\ \text{m}\right)}{9.11\times10^{-31}\ \text{kg}}}=\boxed{2.19\times10^6\ \text{m/s}}.$$

**P23.12** The top charge exerts a force on the negative charge $\frac{k_e qQ}{\left(\frac{d}{2}\right)^2+x^2}$ which is directed upward and to the left, at an angle of $\tan^{-1}\left(\frac{d}{2x}\right)$ to the $x$-axis. The two positive charges together exert force

$$\left(\frac{2k_e qQ}{\left(\frac{d^2}{4}+x^2\right)}\right)\left(\frac{(-x)\hat{\mathbf{i}}}{\left(\frac{d^2}{4}+x^2\right)^{1/2}}\right)=m\mathbf{a}\ \text{ or for }\ x<<\frac{d}{2},\ \mathbf{a}\approx\frac{-2k_e qQ}{md^3/8}\mathbf{x}.$$

(a) The acceleration is equal to a negative constant times the excursion from equilibrium, as in $\mathbf{a}=-\omega^2\mathbf{x}$, so we have Simple Harmonic Motion with $\omega^2=\frac{16k_e qQ}{md^3}$.

$T=\frac{2\pi}{\omega}=\boxed{\frac{\pi}{2}\sqrt{\frac{md^3}{k_e qQ}}}$, where $m$ is the mass of the object with charge $-Q$.

(b) $$v_{\max}=\omega A=\boxed{4a\sqrt{\frac{k_e qQ}{md^3}}}$$

## Section 23.4 **The Electric Field**

**P23.13** For equilibrium, $\mathbf{F}_e = -\mathbf{F}_g$

or $q\mathbf{E} = -mg\left(-\hat{\mathbf{j}}\right)$.

Thus, $\mathbf{E} = \dfrac{mg}{q}\hat{\mathbf{j}}$.

(a) $$\mathbf{E} = \frac{mg}{q}\hat{\mathbf{j}} = \frac{\left(9.11\times10^{-31}\text{ kg}\right)\left(9.80\text{ m/s}^2\right)}{\left(-1.60\times10^{-19}\text{ C}\right)}\hat{\mathbf{j}} = \boxed{-\left(5.58\times10^{-11}\text{ N/C}\right)\hat{\mathbf{j}}}$$

(b) $$\mathbf{E} = \frac{mg}{q}\hat{\mathbf{j}} = \frac{\left(1.67\times10^{-27}\text{ kg}\right)\left(9.80\text{ m/s}^2\right)}{\left(1.60\times10^{-19}\text{ C}\right)}\hat{\mathbf{j}} = \boxed{\left(1.02\times10^{-7}\text{ N/C}\right)\hat{\mathbf{j}}}$$

**P23.14** $\sum F_y = 0:\ QE\hat{\mathbf{j}} + mg\left(-\hat{\mathbf{j}}\right) = 0$

$$\therefore m = \frac{QE}{g} = \frac{\left(24.0\times10^{-6}\text{ C}\right)\left(610\text{ N/C}\right)}{9.80\text{ m/s}^2} = \boxed{1.49\text{ grams}}$$

**P23.15** The point is designated in the sketch. The magnitudes of the electric fields, $E_1$, (due to the $-2.50\times10^{-6}$ C charge) and $E_2$ (due to the $6.00\times10^{-6}$ C charge) are

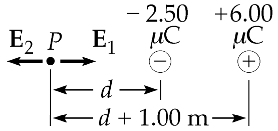


**FIG. P23.15**

$$E_1 = \frac{k_e q}{r^2} = \frac{\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(2.50\times10^{-6}\text{ C}\right)}{d^2} \qquad (1)$$

$$E_2 = \frac{k_e q}{r^2} = \frac{\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(6.00\times10^{-6}\text{ C}\right)}{(d+1.00\text{ m})^2} \qquad (2)$$

Equate the right sides of (1) and (2)

to get $(d+1.00\text{ m})^2 = 2.40d^2$

or $d + 1.00\text{ m} = \pm1.55d$

which yields $d = 1.82$ m

or $d = -0.392$ m.

The negative value for *d* is unsatisfactory because that locates a point between the charges where both fields are in the same direction.

Thus, $d = \boxed{1.82\text{ m to the left of the }-2.50\ \mu\text{C charge}}$.

**P23.16** If we treat the concentrations as point charges,

$$\mathbf{E}_+ = k_e\frac{q}{r^2} = \left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\frac{(40.0\ \mathrm{C})}{(1\,000\ \mathrm{m})^2}\left(-\hat{\mathbf{j}}\right) = 3.60\times10^5\ \mathrm{N/C}\left(-\hat{\mathbf{j}}\right)(\text{downward})$$

$$\mathbf{E}_- = k_e\frac{q}{r^2} = \left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\frac{(40.0\ \mathrm{C})}{(1\,000\ \mathrm{m})^2}\left(-\hat{\mathbf{j}}\right) = 3.60\times10^5\ \mathrm{N/C}\left(-\hat{\mathbf{j}}\right)(\text{downward})$$

$$\mathbf{E} = \mathbf{E}_+ + \mathbf{E}_- = \boxed{7.20\times10^5\ \mathrm{N/C\ downward}}$$

***P23.17** The first charge creates at the origin field $\frac{k_eQ}{a^2}$ to the right. Suppose the total field at the origin is to the right. Then $q$ must be negative:

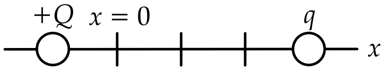


**FIG. P23.17**

$$\frac{k_eQ}{a^2}\hat{\mathbf{i}} + \frac{k_eq}{(3a)^2}\left(-\hat{\mathbf{i}}\right) = \frac{2k_eQ}{a^2}\hat{\mathbf{i}} \qquad \boxed{q = -9Q}.$$

In the alternative, the total field at the origin is to the left:

$$\frac{k_eQ}{a^2}\hat{\mathbf{i}} + \frac{k_eq}{9a^2}\left(-\hat{\mathbf{i}}\right) = \frac{2k_eQ}{a^2}\left(-\hat{\mathbf{i}}\right) \qquad \boxed{q = +27Q}.$$

**P23.18** (a)

$$E_1 = \frac{k_eq}{r^2} = \frac{\left(8.99\times10^9\right)\left(7.00\times10^{-6}\right)}{(0.500)^2} = 2.52\times10^5\ \mathrm{N/C}$$

$$E_2 = \frac{k_eq}{r^2} = \frac{\left(8.99\times10^9\right)\left(4.00\times10^{-6}\right)}{(0.500)^2} = 1.44\times10^5\ \mathrm{N/C}$$

$$E_x = E_2 - E_1\cos60° = 1.44\times10^5 - 2.52\times10^5\cos60.0° = 18.0\times10^3\ \mathrm{N/C}$$

$$E_y = -E_1\sin60.0° = -2.52\times10^5\sin60.0° = -218\times10^3\ \mathrm{N/C}$$

$$\mathbf{E} = \left[18.0\hat{\mathbf{i}} - 218\hat{\mathbf{j}}\right]\times10^3\ \mathrm{N/C} = \boxed{\left[18.0\hat{\mathbf{i}} - 218\hat{\mathbf{j}}\right]\ \mathrm{kN/C}}$$

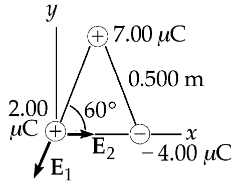


**FIG. P23.18**

(b)

$$\mathbf{F} = q\mathbf{E} = \left(2.00\times10^{-6}\ \mathrm{C}\right)\left(18.0\hat{\mathbf{i}} - 218\hat{\mathbf{j}}\right)\times10^3\ \mathrm{N/C} = \left(36.0\hat{\mathbf{i}} - 436\hat{\mathbf{j}}\right)\times10^{-3}\ \mathrm{N} = \boxed{\left(36.0\hat{\mathbf{i}} - 436\hat{\mathbf{j}}\right)\ \mathrm{mN}}$$

**P23.19** (a)

$$\mathbf{E}_1 = \frac{k_e|q_1|}{r_1^2}\left(-\hat{\mathbf{j}}\right) = \frac{\left(8.99\times10^9\right)\left(3.00\times10^{-9}\right)}{(0.100)^2}\left(-\hat{\mathbf{j}}\right) = -\left(2.70\times10^3\ \mathrm{N/C}\right)\hat{\mathbf{j}}$$

$$\mathbf{E}_2 = \frac{k_e|q_2|}{r_2^2}\left(-\hat{\mathbf{i}}\right) = \frac{\left(8.99\times10^9\right)\left(6.00\times10^{-9}\right)}{(0.300)^2}\left(-\hat{\mathbf{i}}\right) = -\left(5.99\times10^2\ \mathrm{N/C}\right)\hat{\mathbf{i}}$$

$$\mathbf{E} = \mathbf{E}_2 + \mathbf{E}_1 = \boxed{-\left(5.99\times10^2\ \mathrm{N/C}\right)\hat{\mathbf{i}} - \left(2.70\times10^3\ \mathrm{N/C}\right)\hat{\mathbf{j}}}$$

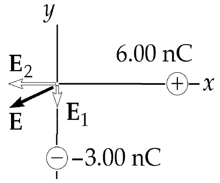


**FIG. P23.19**

(b)

$$\mathbf{F} = q\mathbf{E} = \left(5.00\times10^{-9}\ \mathrm{C}\right)\left(-599\hat{\mathbf{i}} - 2\,700\hat{\mathbf{j}}\right)\ \mathrm{N/C}$$

$$\mathbf{F} = \left(-3.00\times10^{-6}\hat{\mathbf{i}} - 13.5\times10^{-6}\hat{\mathbf{j}}\right)\ \mathrm{N} = \boxed{\left(-3.00\hat{\mathbf{i}} - 13.5\hat{\mathbf{j}}\right)\ \mu\mathrm{N}}$$

**P23.20** (a) $$E = \frac{k_e q}{r^2} = \frac{\left(8.99\times10^{9}\right)\left(2.00\times10^{-6}\right)}{(1.12)^2} = 14\,400\ \text{N/C}$$

$$E_x = 0 \qquad \text{and} \qquad E_y = 2(14\,400)\sin 26.6° = 1.29\times10^{4}\ \text{N/C}$$

so $\boxed{\mathbf{E} = 1.29\times10^{4}\,\hat{\mathbf{j}}\ \text{N/C}}$.

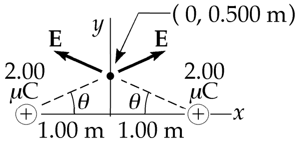


**FIG. P23.20**

(b) $$\mathbf{F} = q\mathbf{E} = \left(-3.00\times10^{-6}\right)\left(1.29\times10^{4}\,\hat{\mathbf{j}}\right) = \boxed{-3.86\times10^{-2}\,\hat{\mathbf{j}}\ \text{N}}$$

**P23.21** (a) $$\mathbf{E} = \frac{k_e q_1}{r_1^2}\hat{\mathbf{r}}_1 + \frac{k_e q_2}{r_2^2}\hat{\mathbf{r}}_2 + \frac{k_e q_3}{r_3^2}\hat{\mathbf{r}}_3 = \frac{k_e(2q)}{a^2}\hat{\mathbf{i}} + \frac{k_e(3q)}{2a^2}\left(\hat{\mathbf{i}}\cos 45.0° + \hat{\mathbf{j}}\sin 45.0°\right) + \frac{k_e(4q)}{a^2}\hat{\mathbf{j}}$$

$$\mathbf{E} = 3.06\frac{k_e q}{a^2}\hat{\mathbf{i}} + 5.06\frac{k_e q}{a^2}\hat{\mathbf{j}} = \boxed{5.91\frac{k_e q}{a^2}\ \text{at } 58.8°}$$

(b) $$\mathbf{F} = q\mathbf{E} = \boxed{5.91\frac{k_e q^2}{a^2}\ \text{at } 58.8°}$$

**P23.22** The electric field at any point $x$ is

$$E = \frac{k_e q}{(x-a)^2} - \frac{k_e q}{\left(x-(-a)\right)^2} = \frac{k_e q(4ax)}{\left(x^2-a^2\right)^2}.$$

When $x$ is much, much greater than $a$, we find $E \cong \boxed{\dfrac{4a(k_e q)}{x^3}}$.

**P23.23** (a) One of the charges creates at $P$ a field $\mathbf{E} = \dfrac{k_e Q/n}{R^2+x^2}$ at an angle $\theta$ to the $x$-axis as shown.

When all the charges produce field, for $n > 1$, the components perpendicular to the $x$-axis add to zero.

The total field is $\dfrac{nk_e(Q/n)\hat{\mathbf{i}}}{R^2+x^2}\cos\theta = \boxed{\dfrac{k_e Qx\hat{\mathbf{i}}}{\left(R^2+x^2\right)^{3/2}}}$.

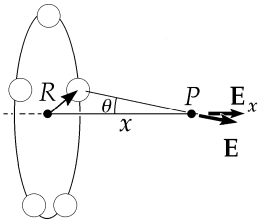


**FIG. P23.23**

(b) A circle of charge corresponds to letting $n$ grow beyond all bounds, but the result does not depend on $n$. Smearing the charge around the circle does not change its amount or its distance from the field point, so it $\boxed{\text{does not change the field}}$.

**P23.24** $$\mathbf{E} = \sum\frac{k_e q}{r^2}\hat{\mathbf{r}} = \frac{k_e q}{a^2}\left(-\hat{\mathbf{i}}\right) + \frac{k_e q}{(2a)^2}\left(-\hat{\mathbf{i}}\right) + \frac{k_e q}{(3a)^2}\left(-\hat{\mathbf{i}}\right) + \ldots = \frac{-k_e q\hat{\mathbf{i}}}{a^2}\left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\right) = \boxed{-\frac{\pi^2 k_e q}{6a^2}\hat{\mathbf{i}}}$$

## Section 23.5 **Electric Field of a Continuous Charge Distribution**

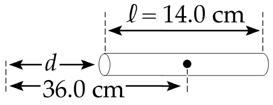


**FIG. P23.25**

**P23.25** $$E = \frac{k_e \lambda \ell}{d(\ell + d)} = \frac{k_e (Q/\ell)\ell}{d(\ell + d)} = \frac{k_e Q}{d(\ell + d)} = \frac{\left(8.99 \times 10^9\right)\left(22.0 \times 10^{-6}\right)}{(0.290)(0.140 + 0.290)}$$

$$\mathbf{E} = \boxed{1.59 \times 10^6 \text{ N/C, directed toward the rod.}}$$

**P23.26** $E = \int \frac{k_e dq}{x^2}$, where $dq = \lambda_0 dx$

$$E = k_e \lambda_0 \int_{x_0}^{\infty} \frac{dx}{x^2} = k_e \lambda_0 \left(-\frac{1}{x}\right)\Bigg|_{x_0}^{\infty} = \boxed{\frac{k_e \lambda_0}{x_0}}$$ $\boxed{\text{The direction is } -\hat{\mathbf{i}} \text{ or left for } \lambda_0 > 0}$

**P23.27** $$E = \frac{k_e x Q}{\left(x^2 + a^2\right)^{3/2}} = \frac{\left(8.99 \times 10^9\right)\left(75.0 \times 10^{-6}\right)x}{\left(x^2 + 0.100^2\right)^{3/2}} = \frac{6.74 \times 10^5 x}{\left(x^2 + 0.010\,0\right)^{3/2}}$$

(a) At $x = 0.010\,0$ m, $\mathbf{E} = 6.64 \times 10^6 \hat{\mathbf{i}}$ N/C $= \boxed{6.64\hat{\mathbf{i}} \text{ MN/C}}$

(b) At $x = 0.050\,0$ m, $\mathbf{E} = 2.41 \times 10^7 \hat{\mathbf{i}}$ N/C $= \boxed{24.1\hat{\mathbf{i}} \text{ MN/C}}$

(c) At $x = 0.300$ m, $\mathbf{E} = 6.40 \times 10^6 \hat{\mathbf{i}}$ N/C $= \boxed{6.40\hat{\mathbf{i}} \text{ MN/C}}$

(d) At $x = 1.00$ m, $\mathbf{E} = 6.64 \times 10^5 \hat{\mathbf{i}}$ N/C $= \boxed{0.664\hat{\mathbf{i}} \text{ MN/C}}$

**P23.28** $$\mathbf{E} = \int d\mathbf{E} = \int_{x_0}^{\infty} \left[\frac{k_e \lambda_0 x_0 dx\left(-\hat{\mathbf{i}}\right)}{x^3}\right] = -k_e \lambda_0 x_0 \hat{\mathbf{i}} \int_{x_0}^{\infty} x^{-3} dx = -k_e \lambda_0 x_0 \hat{\mathbf{i}} \left(-\frac{1}{2x^2}\Bigg|_{x_0}^{\infty}\right) = \boxed{\frac{k_e \lambda_0}{2x_0}\left(-\hat{\mathbf{i}}\right)}$$

**P23.29** $$E = \frac{k_e Q x}{\left(x^2 + a^2\right)^{3/2}}$$

For a maximum, $$\frac{dE}{dx} = Qk_e \left[\frac{1}{\left(x^2 + a^2\right)^{3/2}} - \frac{3x^2}{\left(x^2 + a^2\right)^{5/2}}\right] = 0$$

$x^2 + a^2 - 3x^2 = 0$ or $x = \frac{a}{\sqrt{2}}$.

Substituting into the expression for $E$ gives

$$E = \frac{k_e Q a}{\sqrt{2}\left(\frac{3}{2}a^2\right)^{3/2}} = \frac{k_e Q}{3\frac{\sqrt{3}}{2}a^2} = \boxed{\frac{2k_e Q}{3\sqrt{3}a^2}} = \boxed{\frac{Q}{6\sqrt{3}\pi \in_0 a^2}}.$$

**P23.30** $E = 2\pi k_e \sigma \left(1 - \dfrac{x}{\sqrt{x^2 + R^2}}\right)$

$$E = 2\pi\left(8.99 \times 10^9\right)\left(7.90 \times 10^{-3}\right)\left(1 - \frac{x}{\sqrt{x^2 + (0.350)^2}}\right) = 4.46 \times 10^8 \left(1 - \frac{x}{\sqrt{x^2 + 0.123}}\right)$$

(a) At $x = 0.050\,0$ m, $E = 3.83 \times 10^8$ N/C $= \boxed{383\ \text{MN/C}}$

(b) At $x = 0.100$ m, $E = 3.24 \times 10^8$ N/C $= \boxed{324\ \text{MN/C}}$

(c) At $x = 0.500$ m, $E = 8.07 \times 10^7$ N/C $= \boxed{80.7\ \text{MN/C}}$

(d) At $x = 2.00$ m, $E = 6.68 \times 10^8$ N/C $= \boxed{6.68\ \text{MN/C}}$

**P23.31** (a) From Example 23.9: $E = 2\pi k_e \sigma \left(1 - \dfrac{x}{\sqrt{x^2 + R^2}}\right)$

$$\sigma = \frac{Q}{\pi R^2} = 1.84 \times 10^{-3}\ \text{C/m}^2$$

$$E = \left(1.04 \times 10^8\ \text{N/C}\right)(0.900) = 9.36 \times 10^7\ \text{N/C} = \boxed{93.6\ \text{MN/C}}$$

appx: $E = 2\pi k_e \sigma = \boxed{104\ \text{MN/C (about 11\% high)}}$

(b) $$E = \left(1.04 \times 10^8\ \text{N/C}\right)\left(1 - \frac{30.0\ \text{cm}}{\sqrt{30.0^2 + 3.00^2}\ \text{cm}}\right) = \left(1.04 \times 10^8\ \text{N/C}\right)(0.004\,96) = \boxed{0.516\ \text{MN/C}}$$

appx: $E = k_e \dfrac{Q}{r^2} = \left(8.99 \times 10^9\right)\dfrac{5.20 \times 10^{-6}}{(0.30)^2} = \boxed{0.519\ \text{MN/C (about 0.6\% high)}}$

**P23.32** The electric field at a distance $x$ is

$$E_x = 2\pi k_e \sigma \left[1 - \frac{x}{\sqrt{x^2 + R^2}}\right]$$

This is equivalent to

$$E_x = 2\pi k_e \sigma \left[1 - \frac{1}{\sqrt{1 + R^2/x^2}}\right]$$

For large $x$, $\dfrac{R^2}{x^2} << 1$ and

$$\sqrt{1 + \frac{R^2}{x^2}} \approx 1 + \frac{R^2}{2x^2}$$

so

$$E_x = 2\pi k_e \sigma \left(1 - \frac{1}{\left[1 + R^2/\left(2x^2\right)\right]}\right) = 2\pi k_e \sigma \frac{\left(1 + R^2/\left(2x^2\right) - 1\right)}{\left[1 + R^2/\left(2x^2\right)\right]}$$

Substitute $\sigma = \dfrac{Q}{\pi R^2}$,

$$E_x = \frac{k_e Q\left(1/x^2\right)}{\left[1 + R^2/\left(2x^2\right)\right]} = k_e Q\left(x^2 + \frac{R^2}{2}\right)$$

But for $x >> R$, $\dfrac{1}{x^2 + R^2/2} \approx \dfrac{1}{x^2}$, so

$$\boxed{E_x \approx \frac{k_e Q}{x^2}\ \text{for a disk at large distances}}$$

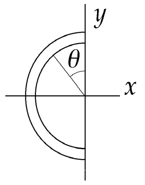


**FIG. P23.33**

**P23.33** Due to symmetry $E_y = \int dE_y = 0$, and $E_x = \int dE\sin\theta = k_e \int \frac{dq\sin\theta}{r^2}$

where $dq = \lambda ds = \lambda r d\theta$,

so that, $E_x = \frac{k_e\lambda}{r}\int_0^{\pi}\sin\theta d\theta = \frac{k_e\lambda}{r}(-\cos\theta)\Big|_0^{\pi} = \frac{2k_e\lambda}{r}$

where $\lambda = \frac{q}{L}$ and $r = \frac{L}{\pi}$.

Thus, $E_x = \frac{2k_e q\pi}{L^2} = \frac{2\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(7.50\times10^{-6}\ \text{C}\right)\pi}{(0.140\ \text{m})^2}$.

Solving, $E_x = 2.16\times10^7\ \text{N/C}$.

Since the rod has a negative charge, $\mathbf{E} = \left(-2.16\times10^7\,\hat{\mathbf{i}}\right)\ \text{N/C} = \boxed{-21.6\hat{\mathbf{i}}\ \text{MN/C}}$.

**P23.34** (a) We define $x = 0$ at the point where we are to find the field. One ring, with thickness *dx*, has charge $\frac{Qdx}{h}$ and produces, at the chosen point, a field

$$d\mathbf{E} = \frac{k_e x}{\left(x^2+R^2\right)^{3/2}}\frac{Qdx}{h}\hat{\mathbf{i}}.$$

The total field is

$$\mathbf{E} = \int_{\text{all charge}} d\mathbf{E} = \int_d^{d+h}\frac{k_e Qxdx}{h\left(x^2+R^2\right)^{3/2}}\hat{\mathbf{i}} = \frac{k_e Q\hat{\mathbf{i}}}{2h}\int_{x=d}^{d+h}\left(x^2+R^2\right)^{-3/2}2xdx$$

$$\mathbf{E} = \frac{k_e Q\hat{\mathbf{i}}}{2h}\frac{\left(x^2+R^2\right)^{-1/2}}{(-1/2)}\Bigg|_{x=d}^{d+h} = \boxed{\frac{k_e Q\hat{\mathbf{i}}}{h}\left[\frac{1}{\left(d^2+R^2\right)^{1/2}} - \frac{1}{\left((d+h)^2+R^2\right)^{1/2}}\right]}$$

(b) Think of the cylinder as a stack of disks, each with thickness *dx*, charge $\frac{Qdx}{h}$, and charge-per-area $\sigma = \frac{Qdx}{\pi R^2 h}$. One disk produces a field

$$d\mathbf{E} = \frac{2\pi k_e Qdx}{\pi R^2 h}\left(1 - \frac{x}{\left(x^2+R^2\right)^{1/2}}\right)\hat{\mathbf{i}}.$$

So, $$\mathbf{E} = \int_{\text{all charge}} d\mathbf{E} = \int_{x=d}^{d+h}\frac{2k_e Qdx}{R^2 h}\left(1 - \frac{x}{\left(x^2+R^2\right)^{1/2}}\right)\hat{\mathbf{i}}$$

$$\mathbf{E} = \frac{2k_e Q\hat{\mathbf{i}}}{R^2 h}\left[\int_d^{d+h}dx - \frac{1}{2}\int_{x=d}^{d+h}\left(x^2+R^2\right)^{-1/2}2xdx\right] = \frac{2k_e Q\hat{\mathbf{i}}}{R^2 h}\left[x\Big|_d^{d+h} - \frac{1}{2}\frac{\left(x^2+R^2\right)^{1/2}}{1/2}\Bigg|_d^{d+h}\right]$$

$$\mathbf{E} = \frac{2k_e Q\hat{\mathbf{i}}}{R^2 h}\left[d+h-d-\left((d+h)^2+R^2\right)^{1/2} + \left(d^2+R^2\right)^{1/2}\right]$$

$$\mathbf{E} = \boxed{\frac{2k_e Q\hat{\mathbf{i}}}{R^2 h}\left[h + \left(d^2+R^2\right)^{1/2} - \left((d+h)^2+R^2\right)^{1/2}\right]}$$

**P23.35** (a) The electric field at point $P$ due to each element of length $dx$, is $dE = \frac{k_e dq}{x^2 + y^2}$ and is directed along the line joining the element to point $P$. By symmetry,

$$E_x = \int dE_x = 0 \qquad \text{and since} \qquad dq = \lambda dx ,$$

$$E = E_y = \int dE_y = \int dE\cos\theta \qquad \text{where} \qquad \cos\theta = \frac{y}{\sqrt{x^2 + y^2}} .$$

Therefore, $E = 2k_e\lambda y \int_0^{\ell/2} \frac{dx}{\left(x^2 + y^2\right)^{3/2}} = \boxed{\frac{2k_e\lambda\sin\theta_0}{y}}$.

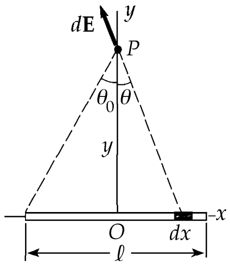


FIG. P23.35

(b) For a bar of infinite length, $\theta_0 = 90°$ and $E_y = \boxed{\frac{2k_e\lambda}{y}}$.

**P23.36** (a) The whole surface area of the cylinder is $A = 2\pi r^2 + 2\pi rL = 2\pi r(r + L)$.

$$Q = \sigma A = \left(15.0 \times 10^{-9}\ \text{C/m}^2\right)2\pi(0.025\,0\ \text{m})\left[0.025\,0\ \text{m} + 0.060\,0\ \text{m}\right] = \boxed{2.00 \times 10^{-10}\ \text{C}}$$

(b) For the curved lateral surface only, $A = 2\pi rL$.

$$Q = \sigma A = \left(15.0 \times 10^{-9}\ \text{C/m}^2\right)\left[2\pi(0.025\,0\ \text{m})(0.060\,0\ \text{m})\right] = \boxed{1.41 \times 10^{-10}\ \text{C}}$$

(c) $$Q = \rho V = \rho\pi r^2 L = \left(500 \times 10^{-9}\ \text{C/m}^3\right)\left[\pi(0.025\,0\ \text{m})^2(0.060\,0\ \text{m})\right] = \boxed{5.89 \times 10^{-11}\ \text{C}}$$

**P23.37** (a) Every object has the same volume, $V = 8(0.030\,0\ \text{m})^3 = 2.16 \times 10^{-4}\ \text{m}^3$.

For each, $Q = \rho V = \left(400 \times 10^{-9}\ \text{C/m}^3\right)\left(2.16 \times 10^{-4}\ \text{m}^3\right) = \boxed{8.64 \times 10^{-11}\ \text{C}}$

(b) We must count the $9.00\ \text{cm}^2$ squares painted with charge:

(i) $6 \times 4 = 24$ squares

$$Q = \sigma A = \left(15.0 \times 10^{-9}\ \text{C/m}^2\right)24.0\left(9.00 \times 10^{-4}\ \text{m}^2\right) = \boxed{3.24 \times 10^{-10}\ \text{C}}$$

(ii) 34 squares exposed

$$Q = \sigma A = \left(15.0 \times 10^{-9}\ \text{C/m}^2\right)34.0\left(9.00 \times 10^{-4}\ \text{m}^2\right) = \boxed{4.59 \times 10^{-10}\ \text{C}}$$

(iii) 34 squares

$$Q = \sigma A = \left(15.0 \times 10^{-9}\ \text{C/m}^2\right)34.0\left(9.00 \times 10^{-4}\ \text{m}^2\right) = \boxed{4.59 \times 10^{-10}\ \text{C}}$$

(iv) 32 squares

$$Q = \sigma A = \left(15.0 \times 10^{-9}\ \text{C/m}^2\right)32.0\left(9.00 \times 10^{-4}\ \text{m}^2\right) = \boxed{4.32 \times 10^{-10}\ \text{C}}$$

(c) (i) total edge length: $\ell = 24 \times (0.030\,0\ \text{m})$

$$Q = \lambda\ell = \left(80.0 \times 10^{-12}\ \text{C/m}\right)24 \times (0.030\,0\ \text{m}) = \boxed{5.76 \times 10^{-11}\ \text{C}}$$

(ii) $$Q = \lambda\ell = \left(80.0 \times 10^{-12}\ \text{C/m}\right)44 \times (0.030\,0\ \text{m}) = \boxed{1.06 \times 10^{-10}\ \text{C}}$$

*continued on next page*

(iii) $Q=\lambda\ell=\left(80.0\times10^{-12}\ \text{C/m}\right)64\times\left(0.030\,0\ \text{m}\right)=\boxed{1.54\times10^{-10}\ \text{C}}$

(iv) $Q=\lambda\ell=\left(80.0\times10^{-12}\ \text{C/m}\right)40\times\left(0.030\,0\ \text{m}\right)=\boxed{0.960\times10^{-10}\ \text{C}}$

## Section 23.6 **Electric Field Lines**

**P23.38**

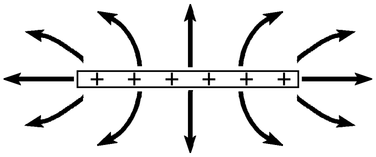


**FIG. P23.38**

**P23.39**

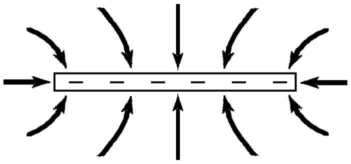


**FIG. P23.39**

**P23.40** (a) $\dfrac{q_1}{q_2}=\dfrac{-6}{18}=\boxed{-\dfrac{1}{3}}$

(b) $\boxed{q_1 \text{ is negative, } q_2 \text{ is positive}}$

**P23.41** (a) The electric field has the general appearance shown. It is zero $\boxed{\text{at the center}}$, where (by symmetry) one can see that the three charges individually produce fields that cancel out.

In addition to the center of the triangle, the electric field lines in the second figure to the right indicate three other points near the middle of each leg of the triangle where $E=0$, but they are more difficult to find mathematically.

(b) You may need to review vector addition in Chapter Three. The electric field at point $P$ can be found by adding the electric field vectors due to each of the two lower point charges: $\mathbf{E}=\mathbf{E}_1+\mathbf{E}_2$.

The electric field from a point charge is $\mathbf{E}=k_e\dfrac{q}{r^2}\hat{\mathbf{r}}$.

As shown in the solution figure at right,

$\mathbf{E}_1=k_e\dfrac{q}{a^2}$ to the right and upward at 60°

$\mathbf{E}_2=k_e\dfrac{q}{a^2}$ to the left and upward at 60°

$$\mathbf{E}=\mathbf{E}_1+\mathbf{E}_2=k_e\frac{q}{a^2}\left[\left(\cos60°\,\hat{\mathbf{i}}+\sin60°\,\hat{\mathbf{j}}\right)+\left(-\cos60°\,\hat{\mathbf{i}}+\sin60°\,\hat{\mathbf{j}}\right)\right]=k_e\frac{q}{a^2}\left[2\left(\sin60°\,\hat{\mathbf{j}}\right)\right]$$

$$=\boxed{1.73k_e\frac{q}{a^2}\hat{\mathbf{j}}}$$

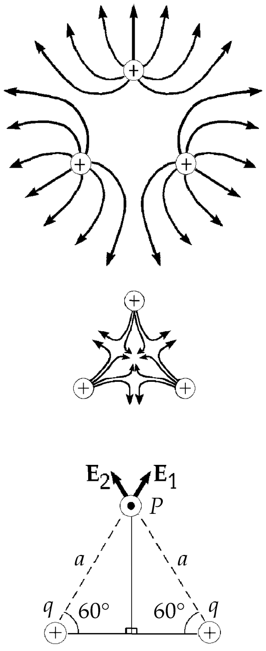


**FIG. P23.41**

## Section 23.7 **Motion of Charged Particles in a Uniform Electric Field**

**P23.42** $F = qE = ma \qquad a = \dfrac{qE}{m}$

$v_f = v_i + at \qquad v_f = \dfrac{qEt}{m}$

electron: $v_e = \dfrac{\left(1.602 \times 10^{-19}\right)(520)\left(48.0 \times 10^{-9}\right)}{9.11 \times 10^{-31}} = \boxed{4.39 \times 10^{6}\ \text{m/s}}$

in a direction opposite to the field

proton: $v_p = \dfrac{\left(1.602 \times 10^{-19}\right)(520)\left(48.0 \times 10^{-9}\right)}{1.67 \times 10^{-27}} = \boxed{2.39 \times 10^{3}\ \text{m/s}}$

in the same direction as the field

**P23.43** (a) $a = \dfrac{qE}{m} = \dfrac{1.602 \times 10^{-19}(640)}{1.67 \times 10^{-27}} = \boxed{6.14 \times 10^{10}\ \text{m/s}^2}$

(b) $v_f = v_i + at \qquad 1.20 \times 10^{6} = \left(6.14 \times 10^{10}\right)t \qquad t = \boxed{1.95 \times 10^{-5}\ \text{s}}$

(c) $x_f - x_i = \dfrac{1}{2}\left(v_i + v_f\right)t \qquad x_f = \dfrac{1}{2}\left(1.20 \times 10^{6}\right)\left(1.95 \times 10^{-5}\right) = \boxed{11.7\ \text{m}}$

(d) $K = \dfrac{1}{2}mv^2 = \dfrac{1}{2}\left(1.67 \times 10^{-27}\ \text{kg}\right)\left(1.20 \times 10^{6}\ \text{m/s}\right)^2 = \boxed{1.20 \times 10^{-15}\ \text{J}}$

**P23.44** (a) $|a| = \dfrac{qE}{m} = \dfrac{\left(1.602 \times 10^{-19}\right)\left(6.00 \times 10^{5}\right)}{\left(1.67 \times 10^{-27}\right)} = 5.76 \times 10^{13}\ \text{m/s so } \mathbf{a} = \boxed{-5.76 \times 10^{13}\,\hat{\mathbf{i}}\ \text{m/s}^2}$

(b) $v_f = v_i + 2a\left(x_f - x_i\right)$

$0 = v_i^2 + 2\left(-5.76 \times 10^{13}\right)(0.070\,0) \qquad \boxed{\mathbf{v}_i = 2.84 \times 10^{6}\,\hat{\mathbf{i}}\ \text{m/s}}$

(c) $v_f = v_i + at$

$0 = 2.84 \times 10^{6} + \left(-5.76 \times 10^{13}\right)t \qquad t = \boxed{4.93 \times 10^{-8}\ \text{s}}$

**P23.45** The required electric field will be $\boxed{\text{in the direction of motion}}$.

Work done $= \Delta K$

so, $-Fd = -\dfrac{1}{2}mv_i^2$ (since the final velocity $= 0$)

which becomes $eEd = K$

and $E = \boxed{\dfrac{K}{ed}}$.

**P23.46** The acceleration is given by

$$v_f^2 = v_i^2 + 2a\left(x_f - x_i\right) \quad \text{or} \qquad v_f^2 = 0 + 2a(-h).$$

Solving $$a = -\frac{v_f^2}{2h}.$$

Now $$\sum \mathbf{F} = m\mathbf{a}: \quad -mg\hat{\mathbf{j}} + q\mathbf{E} = -\frac{mv_f^2\hat{\mathbf{j}}}{2h}.$$

Therefore $$q\mathbf{E} = \left(-\frac{mv_f^2}{2h} + mg\right)\hat{\mathbf{j}}.$$

(a) Gravity alone would give the bead downward impact velocity

$$\sqrt{2\left(9.80\ \text{m/s}^2\right)(5.00\ \text{m})} = 9.90\ \text{m/s}.$$

To change this to 21.0 m/s down, a $\boxed{\text{downward}}$ electric field must exert a downward electric force.

(b) $$q = \frac{m}{E}\left(\frac{v_f^2}{2h} - g\right) = \frac{1.00\times10^{-3}\ \text{kg}}{1.00\times10^{4}\ \text{N/C}}\left(\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}\right)\left[\frac{(21.0\ \text{m/s})^2}{2(5.00\ \text{m})} - 9.80\ \text{m/s}^2\right] = \boxed{3.43\ \mu\text{C}}$$

**P23.47** (a) $$t = \frac{x}{v_x} = \frac{0.050\,0}{4.50\times10^{5}} = 1.11\times10^{-7}\ \text{s} = \boxed{111\ \text{ns}}$$

(b) $$a_y = \frac{qE}{m} = \frac{\left(1.602\times10^{-19}\right)\left(9.60\times10^{3}\right)}{\left(1.67\times10^{-27}\right)} = 9.21\times10^{11}\ \text{m/s}^2$$

$$y_f - y_i = v_{yi}t + \frac{1}{2}a_y t^2: \qquad y_f = \frac{1}{2}\left(9.21\times10^{11}\right)\left(1.11\times10^{-7}\right)^2 = 5.68\times10^{-3}\ \text{m} = \boxed{5.68\ \text{mm}}$$

(c) $$v_x = \boxed{4.50\times10^{5}\ \text{m/s}} \qquad v_{yf} = v_{yi} + a_y t = \left(9.21\times10^{11}\right)\left(1.11\times10^{-7}\right) = \boxed{1.02\times10^{5}\ \text{m/s}}$$

***P23.48** The particle feels a constant force: $\mathbf{F} = q\mathbf{E} = \left(1\times10^{-6}\ \text{C}\right)(2\,000\ \text{N/C})\left(-\hat{\mathbf{j}}\right) = 2\times10^{-3}\ \text{N}\left(-\hat{\mathbf{j}}\right)$

and moves with acceleration: $$\mathbf{a} = \frac{\sum\mathbf{F}}{m} = \frac{\left(2\times10^{-3}\ \text{kg}\cdot\text{m/s}^2\right)\left(-\hat{\mathbf{j}}\right)}{2\times10^{-16}\ \text{kg}} = \left(1\times10^{13}\ \text{m/s}^2\right)\left(-\hat{\mathbf{j}}\right).$$

Its $x$-component of velocity is constant at $\left(1.00\times10^{5}\ \text{m/s}\right)\cos 37° = 7.99\times10^{4}\ \text{m/s}$. Thus it moves in a parabola opening downward. The maximum height it attains above the bottom plate is described by

$$v_{yf}^2 = v_{yi}^2 + 2a_y\left(y_f - y_i\right): \qquad 0 = \left(6.02\times10^{4}\ \text{m/s}\right)^2 - \left(2\times10^{13}\ \text{m/s}^2\right)\left(y_f - 0\right)$$

$$y_f = 1.81\times10^{-4}\ \text{m}.$$

*continued on next page*

Since this is less than 10 mm, the particle does not strike the top plate, but moves in a symmetric parabola and strikes the bottom plate after a time given by

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 \qquad 0 = 0 + \left(6.02\times10^4 \text{ m/s}\right)t + \frac{1}{2}\left(-1\times10^{13} \text{ m/s}^2\right)t^2$$

since $t > 0$, $\quad t = 1.20\times10^{-8}$ s.

The particle's range is $\quad x_f = x_i + v_x t = 0 + \left(7.99\times10^4 \text{ m/s}\right)\left(1.20\times10^{-8} \text{ s}\right) = 9.61\times10^{-4}$ m.

In sum,

$\boxed{\text{The particle strikes the negative plate after moving in a parabola with a height of 0.181 mm and a width of 0.961 mm.}}$

**P23.49** $\quad v_i = 9.55\times10^3$ m/s

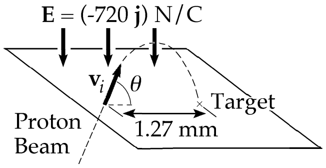


**FIG. P23.49**

(a) $$a_y = \frac{eE}{m} = \frac{\left(1.60\times10^{-19}\right)(720)}{\left(1.67\times10^{-27}\right)} = 6.90\times10^{10} \text{ m/s}^2$$

$$R = \frac{v_i^2 \sin 2\theta}{a_y} = 1.27\times10^{-3} \text{ m so that}$$

$$\frac{\left(9.55\times10^3\right)^2 \sin 2\theta}{6.90\times10^{10}} = 1.27\times10^{-3}$$

$\sin 2\theta = 0.961 \qquad \theta = \boxed{36.9°} \qquad 90.0° - \theta = \boxed{53.1°}$

(b) $$t = \frac{R}{v_{ix}} = \frac{R}{v_i \cos\theta}$$ If $\theta = 36.9°$, $t = \boxed{167 \text{ ns}}$. $\qquad$ If $\theta = 53.1°$, $t = \boxed{221 \text{ ns}}$.

## Additional Problems

***P23.50** The two given charges exert equal-size forces of attraction on each other. If a third charge, positive or negative, were placed between them they could not be in equilibrium. If the third charge were at a point $x > 15$ cm, it would exert a stronger force on the 45 $\mu$C than on the $-12$ $\mu$C, and could not produce equilibrium for both. Thus the third charge must be at $x = -d < 0$. Its equilibrium requires

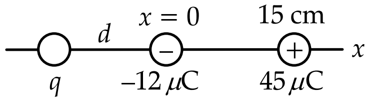


**FIG. P23.50**

$$\frac{k_e q(12\ \mu\text{C})}{d^2} = \frac{k_e q(45\ \mu\text{C})}{(15 \text{ cm} + d)^2} \qquad \left(\frac{15 \text{ cm} + d}{d}\right)^2 = \frac{45}{12} = 3.75$$

$15 \text{ cm} + d = 1.94d \qquad d = 16.0$ cm.

The third charge is at $\boxed{x = -16.0 \text{ cm}}$. The equilibrium of the $-12$ $\mu$C requires

$$\frac{k_e q(12\ \mu\text{C})}{(16.0 \text{ cm})^2} = \frac{k_e(45\ \mu\text{C})12\ \mu\text{C}}{(15 \text{ cm})^2} \qquad \boxed{q = 51.3\ \mu\text{C}}.$$

All six individual forces are now equal in magnitude, so we have equilibrium as required, and this is the only solution.

**P23.51** The proton moves with acceleration $\left|a_p\right|=\frac{qE}{m}=\frac{\left(1.60\times10^{-19}\text{ C}\right)(640\text{ N/C})}{1.673\times10^{-27}\text{ kg}}=6.13\times10^{10}\text{ m/s}^2$

while the $e^-$ has acceleration $\left|a_e\right|=\frac{\left(1.60\times10^{-19}\text{ C}\right)(640\text{ N/C})}{9.110\times10^{-31}\text{ kg}}=1.12\times10^{14}\text{ m/s}^2=1\,836a_p$.

(a) We want to find the distance traveled by the proton (i.e., $d=\frac{1}{2}a_pt^2$), knowing:

$$4.00\text{ cm}=\frac{1}{2}a_pt^2+\frac{1}{2}a_et^2=1\,837\left(\frac{1}{2}a_pt^2\right).$$

Thus, $d=\frac{1}{2}a_pt^2=\frac{4.00\text{ cm}}{1\,837}=\boxed{21.8\ \mu\text{m}}$.

(b) The distance from the positive plate to where the meeting occurs equals the distance the sodium ion travels (i.e., $d_{\text{Na}}=\frac{1}{2}a_{\text{Na}}t^2$). This is found from:

$4.00\text{ cm}=\frac{1}{2}a_{\text{Na}}t^2+\frac{1}{2}a_{\text{Cl}}t^2$: $\quad 4.00\text{ cm}=\frac{1}{2}\left(\frac{eE}{22.99\text{ u}}\right)t^2+\frac{1}{2}\left(\frac{eE}{35.45\text{ u}}\right)t^2$.

This may be written as $\quad 4.00\text{ cm}=\frac{1}{2}a_{\text{Na}}t^2+\frac{1}{2}(0.649a_{\text{Na}})t^2=1.65\left(\frac{1}{2}a_{\text{Na}}t^2\right)$

so $\quad d_{\text{Na}}=\frac{1}{2}a_{\text{Na}}t^2=\frac{4.00\text{ cm}}{1.65}=\boxed{2.43\text{ cm}}$.

**P23.52** (a) The field, $E_1$, due to the $4.00\times10^{-9}$ C charge is in the $-x$ direction.

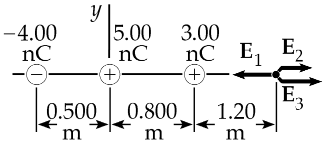


**FIG. P23.52(a)**

$$\mathbf{E}_1=\frac{k_eq}{r^2}\hat{\mathbf{r}}=\frac{\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(-4.00\times10^{-9}\text{ C}\right)}{(2.50\text{ m})^2}\hat{\mathbf{i}}$$

$$=-5.75\hat{\mathbf{i}}\text{ N/C}$$

Likewise, $E_2$ and $E_3$, due to the $5.00\times10^{-9}$ C charge and the $3.00\times10^{-9}$ C charge are

$$\mathbf{E}_2=\frac{k_eq}{r^2}\hat{\mathbf{r}}=\frac{\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(5.00\times10^{-9}\text{ C}\right)}{(2.00\text{ m})^2}\hat{\mathbf{i}}=11.2\text{ N/C }\hat{\mathbf{i}}$$

$$\mathbf{E}_3=\frac{\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(3.00\times10^{-9}\text{ C}\right)}{(1.20\text{ m})^2}\hat{\mathbf{i}}=18.7\text{ N/C }\hat{\mathbf{i}}$$

$\mathbf{E}_R=\mathbf{E}_1+\mathbf{E}_2+\mathbf{E}_3=\boxed{24.2\text{ N/C}}$ in $+x$ direction.

(b)

$$\mathbf{E}_1=\frac{k_eq}{r^2}\hat{\mathbf{r}}=(-8.46\text{ N/C})\left(0.243\hat{\mathbf{i}}+0.970\hat{\mathbf{j}}\right)$$

$$\mathbf{E}_2=\frac{k_eq}{r^2}\hat{\mathbf{r}}=(11.2\text{ N/C})\left(+\hat{\mathbf{j}}\right)$$

$$\mathbf{E}_3=\frac{k_eq}{r^2}\hat{\mathbf{r}}=(5.81\text{ N/C})\left(-0.371\hat{\mathbf{i}}+0.928\hat{\mathbf{j}}\right)$$

$E_x=E_{1x}+E_{3x}=-4.21\hat{\mathbf{i}}\text{ N/C}\qquad E_y=E_{1y}+E_{2y}+E_{3y}=8.43\hat{\mathbf{j}}\text{ N/C}$

$E_R=\boxed{9.42\text{ N/C}}\qquad \theta=\boxed{63.4°\text{ above }-x\text{ axis}}$

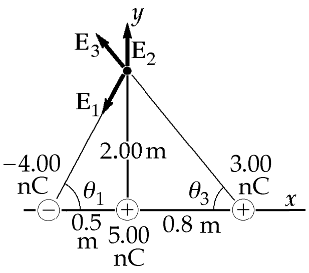


**FIG. P23.52(b)**

***P23.53** (a) Each ion moves in a quarter circle. The electric force causes the centripetal acceleration.

$$\sum F = ma \qquad qE = \frac{mv^2}{R} \qquad \boxed{E = \frac{mv^2}{qR}}$$

(b) For the $x$-motion, $v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$

$$0 = v^2 + 2a_x R \qquad a_x = -\frac{v^2}{2R} = \frac{F_x}{m} = \frac{qE_x}{m}$$

$E_x = -\frac{mv^2}{2qR}$. Similarly for the $y$-motion,

$$v^2 = 0 + 2a_y R \qquad a_y = +\frac{v^2}{2R} = \frac{qE_y}{m} \qquad E_y = \frac{mv^2}{2qR}$$

The magnitude of the field is

$$\sqrt{E_x^2 + E_y^2} = \boxed{\frac{mv^2}{\sqrt{2}qR} \text{ at } 135° \text{ counterclockwise from the } x\text{-axis}}\,.$$

**P23.54** From the free-body diagram shown,

$\sum F_y = 0:$ $\qquad T\cos 15.0° = 1.96 \times 10^{-2}$ N.

So $\qquad T = 2.03 \times 10^{-2}$ N.

From $\sum F_x = 0$, we have $\qquad qE = T\sin 15.0°$

or $$q = \frac{T\sin 15.0°}{E} = \frac{\left(2.03 \times 10^{-2}\text{ N}\right)\sin 15.0°}{1.00 \times 10^3 \text{ N/C}} = 5.25 \times 10^{-6}\text{ C} = \boxed{5.25\ \mu\text{C}}\,.$$

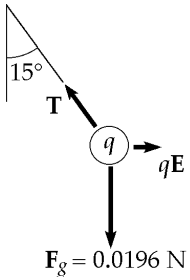


FIG. P23.54

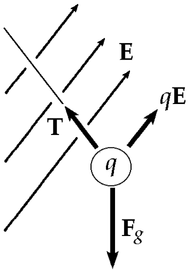
**P23.55** (a) Let us sum force components to find

$$\sum F_x = qE_x - T\sin\theta = 0, \text{ and } \sum F_y = qE_y + T\cos\theta - mg = 0\,.$$

Combining these two equations, we get

$$q = \frac{mg}{\left(E_x\cot\theta + E_y\right)} = \frac{\left(1.00 \times 10^{-3}\right)(9.80)}{(3.00\cot 37.0° + 5.00) \times 10^5} = 1.09 \times 10^{-8}\text{ C}$$

$$= \boxed{10.9\text{ nC}}$$



**Free Body Diagram**

FIG. P23.55

(b) From the two equations for $\sum F_x$ and $\sum F_y$ we also find

$$T = \frac{qEx}{\sin 37.0°} = 5.44 \times 10^{-3}\text{ N} = \boxed{5.44\text{ mN}}\,.$$

**P23.56** This is the general version of the preceding problem. The known quantities are $A$, $B$, $m$, $g$, and $\theta$. The unknowns are $q$ and $T$.

The approach to this problem should be the same as for the last problem, but without numbers to substitute for the variables. Likewise, we can use the free body diagram given in the solution to problem 55.

Again, Newton's second law: $\sum F_x = -T\sin\theta + qA = 0$ (1)

and $\sum F_y = +T\cos\theta + qB - mg = 0$ (2)

(a) Substituting $T = \frac{qA}{\sin\theta}$, into Eq. (2), $\frac{qA\cos\theta}{\sin\theta} + qB = mg$.

Isolating $q$ on the left, $\boxed{q = \frac{mg}{(A\cot\theta + B)}}$.

(b) Substituting this value into Eq. (1), $\boxed{T = \frac{mgA}{(A\cos\theta + B\sin\theta)}}$.

If we had solved this general problem first, we would only need to substitute the appropriate values in the equations for $q$ and $T$ to find the numerical results needed for problem 55. If you find this problem more difficult than problem 55, the little list at the first step is useful. It shows what symbols to think of as known data, and what to consider unknown. The list is a guide for deciding what to solve for in the analysis step, and for recognizing when we have an answer.

**P23.57** $F = \frac{k_e q_1 q_2}{r^2}$: $\tan\theta = \frac{15.0}{60.0}$

$\theta = 14.0°$

$$F_1 = \frac{\left(8.99\times10^9\right)\left(10.0\times10^{-6}\right)^2}{(0.150)^2} = 40.0\text{ N}$$

$$F_3 = \frac{\left(8.99\times10^9\right)\left(10.0\times10^{-6}\right)^2}{(0.600)^2} = 2.50\text{ N}$$

$$F_2 = \frac{\left(8.99\times10^9\right)\left(10.0\times10^{-6}\right)^2}{(0.619)^2} = 2.35\text{ N}$$

$$F_x = -F_3 - F_2\cos14.0° = -2.50 - 2.35\cos14.0° = -4.78\text{ N}$$

$$F_y = -F_1 - F_2\sin14.0° = -40.0 - 2.35\sin14.0° = -40.6\text{ N}$$

$$F_{\text{net}} = \sqrt{F_x^2 + F_y^2} = \sqrt{(4.78)^2 + (40.6)^2} = \boxed{40.9\text{ N}}$$

$$\tan\phi = \frac{F_y}{F_x} = \frac{-40.6}{-4.78}$$

$$\phi = \boxed{263°}$$

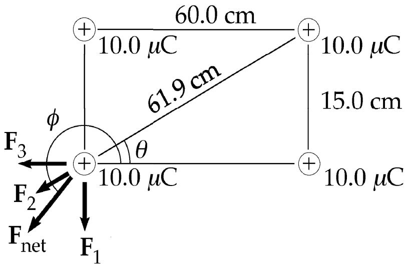


FIG. P23.57

**P23.58** From Figure A: $d\cos 30.0° = 15.0 \text{ cm}$,

or $d = \dfrac{15.0 \text{ cm}}{\cos 30.0°}$

From Figure B: $\theta = \sin^{-1}\left(\dfrac{d}{50.0 \text{ cm}}\right)$

$$\theta = \sin^{-1}\left(\frac{15.0 \text{ cm}}{50.0 \text{ cm}(\cos 30.0°)}\right) = 20.3°$$

$$\frac{F_q}{mg} = \tan\theta$$

or $F_q = mg\tan 20.3°$ (1)

From Figure C: $F_q = 2F\cos 30.0°$

$$F_q = 2\left[\frac{k_e q^2}{(0.300 \text{ m})^2}\right]\cos 30.0° \qquad (2)$$

Combining equations (1) and (2),

$$2\left[\frac{k_e q^2}{(0.300 \text{ m})^2}\right]\cos 30.0° = mg\tan 20.3°$$

$$q^2 = \frac{mg(0.300 \text{ m})^2 \tan 20.3°}{2k_e\cos 30.0°}$$

$$q^2 = \frac{\left(2.00\times10^{-3} \text{ kg}\right)\left(9.80 \text{ m/s}^2\right)(0.300 \text{ m})^2 \tan 20.3°}{2\left(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)\cos 30.0°}$$

$$q = \sqrt{4.20\times10^{-14} \text{ C}^2} = 2.05\times10^{-7} \text{ C} = \boxed{0.205 \ \mu\text{C}}$$

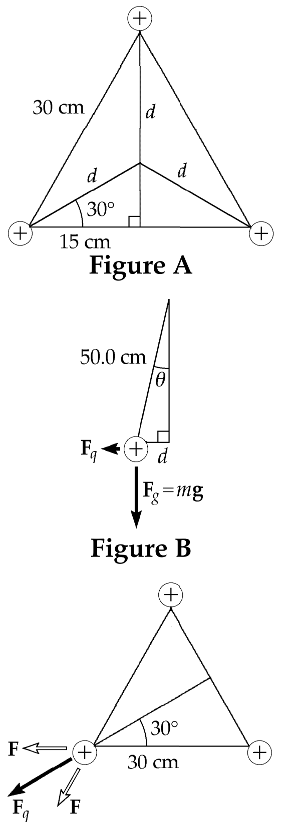


Figure A

Figure B

Figure C

FIG. P23.58

**P23.59** Charge $\dfrac{Q}{2}$ resides on each block, which repel as point charges: $F = \dfrac{k_e(Q/2)(Q/2)}{L^2} = k(L-L_i)$.

Solving for $Q$, $Q = \boxed{2L\sqrt{\dfrac{k(L-L_i)}{k_e}}}$.

***P23.60** If we place one more charge $q$ at the 29th vertex, the total force on the central charge will add up to zero: $\mathbf{F}_{28 \text{ charges}} + \dfrac{k_e qQ}{a^2}$ away from vertex $29 = 0$ $\mathbf{F}_{28 \text{ charges}} = \boxed{\dfrac{\text{k}_\text{e} qQ}{a^2} \text{ toward vertex 29}}$.

**P23.61** According to the result of Example 23.7, the left-hand rod creates this field at a distance $d$ from its right-hand end:

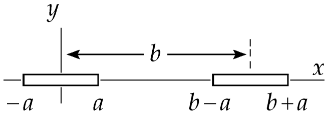


FIG. P23.61

$$E = \frac{k_e Q}{d(2a+d)}$$

$$dF = \frac{k_e QQ}{2a}\frac{dx}{d(d+2a)}$$

$$F = \frac{k_e Q^2}{2a}\int_{x=b-2a}^{b}\frac{dx}{x(x+2a)} = \frac{k_e Q^2}{2a}\left(-\frac{1}{2a}\ln\frac{2a+x}{x}\right)_{b-2a}^{b}$$

$$F = \frac{+k_e Q^2}{4a^2}\left(-\ln\frac{2a+b}{b} + \ln\frac{b}{b-2a}\right) = \frac{k_e Q^2}{4a^2}\ln\frac{b^2}{(b-2a)(b+2a)} = \boxed{\left(\frac{k_e Q^2}{4a^2}\right)\ln\left(\frac{b^2}{b^2-4a^2}\right)}$$

**P23.62** At equilibrium, the distance between the charges is $r = 2(0.100\text{ m})\sin 10.0° = 3.47\times10^{-2}\text{ m}$

Now consider the forces on the sphere with charge $+q$, and use $\sum F_y = 0$:

$$\sum F_y = 0: \qquad T\cos 10.0° = mg\text{, or } T = \frac{mg}{\cos 10.0°} \qquad (1)$$

$$\sum F_x = 0: \qquad F_{\text{net}} = F_2 - F_1 = T\sin 10.0° \qquad (2)$$

$F_{\text{net}}$ is the net electrical force on the charged sphere. Eliminate $T$ from (2) by use of (1).

$$F_{\text{net}} = \frac{mg\sin 10.0°}{\cos 10.0°} = mg\tan 10.0° = \left(2.00\times10^{-3}\text{ kg}\right)\left(9.80\text{ m/s}^2\right)\tan 10.0° = 3.46\times10^{-3}\text{ N}$$

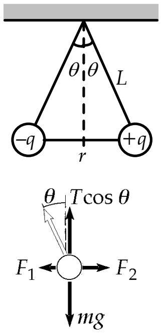


**FIG. P23.62**

$F_{\text{net}}$ is the resultant of two forces, $F_1$ and $F_2$. $F_1$ is the attractive force on $+q$ exerted by $-q$, and $F_2$ is the force exerted on $+q$ by the external electric field.

$F_{\text{net}} = F_2 - F_1$ or $F_2 = F_{\text{net}} + F_1$

$$F_1 = \left(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(5.00\times10^{-8}\text{ C}\right)\left(5.00\times10^{-8}\text{ C}\right)}{\left(3.47\times10^{-3}\text{ m}\right)^2} = 1.87\times10^{-2}\text{ N}$$

Thus, $F_2 = F_{\text{net}} + F_1$ yields $F_2 = 3.46\times10^{-3}\text{ N} + 1.87\times10^{-2}\text{ N} = 2.21\times10^{-2}\text{ N}$

and $F_2 = qE$, or $E = \dfrac{F_2}{q} = \dfrac{2.21\times10^{-2}\text{ N}}{5.00\times10^{-8}\text{ C}} = 4.43\times10^{5}\text{ N/C} = \boxed{443\text{ kN/C}}$.

**P23.63** $Q = \int \lambda d\ell = \int_{-90.0°}^{90.0°} \lambda_0 \cos\theta R d\theta = \lambda_0 R\sin\theta\Big|_{-90.0°}^{90.0°} = \lambda_0 R[1-(-1)] = 2\lambda_0 R$

$Q = 12.0\ \mu\text{C} = (2\lambda_0)(0.600)\text{ m} = 12.0\ \mu\text{C}$ so $\lambda_0 = 10.0\ \mu\text{C/m}$

$$dF_y = \frac{1}{4\pi\epsilon_0}\left(\frac{(3.00\ \mu\text{C})(\lambda d\ell)}{R^2}\right)\cos\theta = \frac{1}{4\pi\epsilon_0}\left(\frac{(3.00\ \mu\text{C})\left(\lambda_0\cos^2\theta R d\theta\right)}{R^2}\right)$$

$$F_y = \int_{-90.0°}^{90.0°}\left(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(3.00\times10^{-6}\text{ C}\right)\left(10.0\times10^{-6}\text{ C/m}\right)}{(0.600\text{ m})}\cos^2\theta d\theta$$

$$F_y = \frac{8.99(30.0)}{0.600}\left(10^{-3}\text{ N}\right)\int_{-\pi/2}^{\pi/2}\left(\frac{1}{2}+\frac{1}{2}\cos 2\theta\right)d\theta$$

$$F_y = (0.450\text{ N})\left(\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\Big|_{-\pi/2}^{\pi/2}\right) = \boxed{0.707\text{ N}}\text{ Downward.}$$

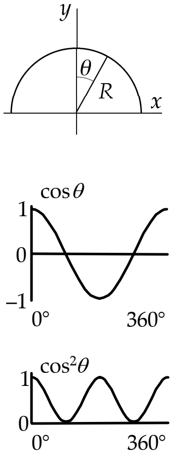


**FIG. P23.63**

Since the leftward and rightward forces due to the two halves of the semicircle cancel out, $F_x = 0$.

**P23.64** At an equilibrium position, the net force on the charge $Q$ is zero. The equilibrium position can be located by determining the angle $\theta$ corresponding to equilibrium.

In terms of lengths $s$, $\frac{1}{2}a\sqrt{3}$, and $r$, shown in Figure P23.64, the charge at the origin exerts an attractive force

$$\frac{k_e Qq}{\left(s+\frac{1}{2}a\sqrt{3}\right)^2}$$

*continued on next page*

The other two charges exert equal repulsive forces of magnitude $\frac{k_eQq}{r^2}$. The horizontal components of the two repulsive forces add, balancing the attractive force,

$$F_{\text{net}} = k_eQq\left[\frac{2\cos\theta}{r^2} - \frac{1}{\left(s+\frac{1}{2}a\sqrt{3}\right)^2}\right] = 0$$

From Figure P23.64 $\quad r = \frac{\frac{1}{2}a}{\sin\theta} \qquad s = \frac{1}{2}a\cot\theta$

The equilibrium condition, in terms of $\theta$, is $\quad F_{\text{net}} = \left(\frac{4}{a^2}\right)k_eQq\left(2\cos\theta\sin^2\theta - \frac{1}{\left(\sqrt{3}+\cot\theta\right)^2}\right) = 0$.

Thus the equilibrium value of $\theta$ satisfies $\quad 2\cos\theta\sin^2\theta\left(\sqrt{3}+\cot\theta\right)^2 = 1$.

One method for solving for $\theta$ is to tabulate the left side. To three significant figures a value of $\theta$ corresponding to equilibrium is 81.7°.

The distance from the vertical side of the triangle to the equilibrium position is

$$s = \frac{1}{2}a\cot 81.7° = \boxed{0.072\,9a}.$$

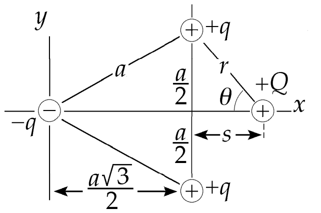


**FIG. P23.64**

| $\theta$ | $2\cos\theta\sin^2\theta\left(\sqrt{3}+\cot\theta\right)^2$ |
|---|---|
| 60° | 4 |
| 70° | 2.654 |
| 80° | 1.226 |
| 90° | 0 |
| 81° | 1.091 |
| 81.5° | 1.024 |
| 81.7° | 0.997 |

A second zero-field point is on the negative side of the $x$-axis, where $\theta = -9.16°$ and $s = -3.10a$.

**P23.65** (a) From the 2$Q$ charge we have $F_e - T_2\sin\theta_2 = 0$ and $mg - T_2\cos\theta_2 = 0$.

Combining these we find $\quad \frac{F_e}{mg} = \frac{T_2\sin\theta_2}{T_2\cos\theta_2} = \tan\theta_2$.

From the $Q$ charge we have $F_e = T_1\sin\theta_1 = 0$ and $mg - T_1\cos\theta_1 = 0$.

Combining these we find $\quad \frac{F_e}{mg} = \frac{T_1\sin\theta_1}{T_1\cos\theta_1} = \tan\theta_1$ or $\boxed{\theta_2 = \theta_1}$.

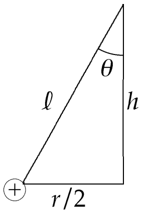


**FIG. P23.65**

(b) $\quad F_e = \frac{k_e 2QQ}{r^2} = \frac{2k_eQ^2}{r^2}$

If we assume $\theta$ is small then $\quad \tan\theta \approx \frac{r/2}{\ell}$.

Substitute expressions for $F_e$ and $\tan\theta$ into either equation found in part (a) and solve for $r$.

$\frac{F_e}{mg} = \tan\theta$ then $\frac{2k_eQ^2}{r^2}\left(\frac{1}{mg}\right) \approx \frac{r}{2\ell}$ and solving for $r$ we find $r \approx \left(\frac{4k_eQ^2\ell}{mg}\right)^{1/3}$.

**P23.66** (a) The distance from each corner to the center of the square is

$$\sqrt{\left(\frac{L}{2}\right)^2+\left(\frac{L}{2}\right)^2}=\frac{L}{\sqrt{2}}.$$

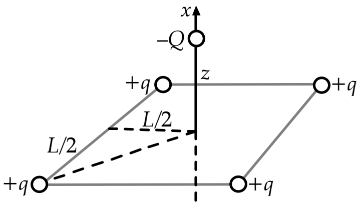


**FIG. P23.66**

The distance from each positive charge to $-Q$ is then $\sqrt{z^2+\frac{L^2}{2}}$. Each positive charge exerts a force directed along the line joining $q$ and $-Q$, of magnitude $\frac{k_eQq}{z^2+L^2/2}$.

The line of force makes an angle with the $z$-axis whose cosine is $\frac{z}{\sqrt{z^2+L^2/2}}$

The four charges together exert forces whose $x$ and $y$ components add to zero, while the $z$-components add to $\mathbf{F}=\boxed{-\frac{4k_eQqz}{\left(z^2+L^2/2\right)^{3/2}}\hat{\mathbf{k}}}$

(b) For $z \gg L$, the magnitude of this force is $F_z=-\frac{4k_eQqz}{\left(L^2/2\right)^{3/2}}=-\left(\frac{4(2)^{3/2}k_eQq}{L^3}\right)z=ma_z$

Therefore, the object's vertical acceleration is of the form $\boxed{a_z=-\omega^2z}$

with $\omega^2=\frac{4(2)^{3/2}k_eQq}{mL^3}=\frac{k_eQq\sqrt{128}}{mL^3}$.

Since the acceleration of the object is always oppositely directed to its excursion from equilibrium and in magnitude proportional to it, the object will execute simple harmonic motion with a period given by

$$T=\frac{2\pi}{\omega}=\frac{2\pi}{(128)^{1/4}}\sqrt{\frac{mL^3}{k_eQq}}=\boxed{\frac{\pi}{(8)^{1/4}}\sqrt{\frac{mL^3}{k_eQq}}}.$$

**P23.67** (a) The total non-contact force on the cork ball is: $F=qE+mg=m\left(g+\frac{qE}{m}\right)$,

which is constant and directed downward. Therefore, it behaves like a simple pendulum in the presence of a modified uniform gravitational field with a period given by:

$$T=2\pi\sqrt{\frac{L}{g+qE/m}}=2\pi\sqrt{\frac{0.500\text{ m}}{9.80\text{ m/s}^2+\left[\left(2.00\times10^{-6}\text{ C}\right)\left(1.00\times10^{5}\text{ N/C}\right)/1.00\times10^{-3}\text{ kg}\right]}}$$

$$=\boxed{0.307\text{ s}}$$

(b) $\boxed{\text{Yes}}$. Without gravity in part (a), we get $T=2\pi\sqrt{\frac{L}{qE/m}}$

$$T=2\pi\sqrt{\frac{0.500\text{ m}}{\left(2.00\times10^{-6}\text{ C}\right)\left(1.00\times10^{5}\text{ N/C}\right)/1.00\times10^{-3}\text{ kg}}}=0.314\text{ s (a 2.28\% difference).}$$

**P23.68** The bowl exerts a normal force on each bead, directed along the radius line or at 60.0° above the horizontal. Consider the free-body diagram of the bead on the left:

$$\sum F_y = n\sin 60.0° - mg = 0\,,$$

or $$n = \frac{mg}{\sin 60.0°}\,.$$

Also, $$\sum F_x = -F_e + n\cos 60.0° = 0\,,$$

or $$\frac{k_e q^2}{R^2} = n\cos 60.0° = \frac{mg}{\tan 60.0°} = \frac{mg}{\sqrt{3}}\,.$$

Thus, $$q = \boxed{R\left(\frac{mg}{k_e\sqrt{3}}\right)^{1/2}}\,.$$

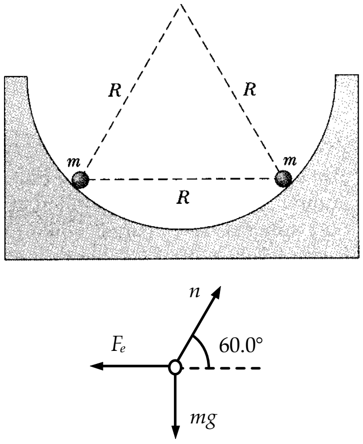


**FIG. P23.68**

**P23.69** (a) There are 7 terms which contribute:

3 are $s$ away (along sides)

3 are $\sqrt{2}s$ away (face diagonals) and $\sin\theta = \frac{1}{\sqrt{2}} = \cos\theta$

1 is $\sqrt{3}s$ away (body diagonal) and $\sin\phi = \frac{1}{\sqrt{3}}$.

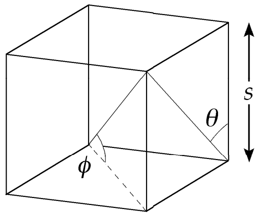


**FIG. P23.69**

The component in each direction is the same by symmetry.

$$\mathbf{F} = \frac{k_e q^2}{s^2}\left[1 + \frac{2}{2\sqrt{2}} + \frac{1}{3\sqrt{3}}\right]\left(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}\right) = \boxed{\frac{k_e q^2}{s^2}(1.90)\left(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}\right)}$$

(b) $$F = \sqrt{F_x^2 + F_y^2 + F_z^2} = \boxed{3.29\frac{k_e q^2}{s^2}\text{ away from the origin}}$$

**P23.70** (a) Zero contribution from the same face due to symmetry, opposite face contributes

$$4\left(\frac{k_e q}{r^2}\sin\phi\right) \quad \text{where} \quad r = \sqrt{\left(\frac{s}{2}\right)^2 + \left(\frac{s}{2}\right)^2 + s^2} = \sqrt{1.5}s = 1.22s$$

$$\sin\phi = \frac{s}{r} \qquad E = 4\frac{k_e q s}{r^3} = \frac{4}{(1.22)^3}\frac{k_e q}{s^2} = \boxed{2.18\frac{k_e q}{s^2}}$$

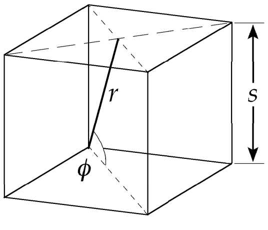


**FIG. P23.70**

(b) $\boxed{\text{The direction is the } \hat{\mathbf{k}} \text{ direction.}}$

**P23.71** The field on the axis of the ring is calculated in Example 23.8, $E = E_x = \dfrac{k_e x Q}{\left(x^2+a^2\right)^{3/2}}$

The force experienced by a charge $-q$ placed along the axis of the ring is $F = -k_e Qq\left[\dfrac{x}{\left(x^2+a^2\right)^{3/2}}\right]$

and when $x \ll a$, this becomes $F = -\left(\dfrac{k_e Qq}{a^3}\right)x$

This expression for the force is in the form of Hooke's law, with an effective spring constant of $k = \dfrac{k_e Qq}{a^3}$

Since $\omega = 2\pi f = \sqrt{\dfrac{k}{m}}$, we have $f = \boxed{\dfrac{1}{2\pi}\sqrt{\dfrac{k_e Qq}{ma^3}}}$.

**P23.72**

$$d\mathbf{E} = \frac{k_e\,dq}{x^2+(0.150\text{ m})^2}\left(\frac{-x\hat{\mathbf{i}}+0.150\text{ m}\hat{\mathbf{j}}}{\sqrt{x^2+(0.150\text{ m})^2}}\right) = \frac{k_e\lambda\left(-x\hat{\mathbf{i}}+0.150\text{ m}\hat{\mathbf{j}}\right)dx}{\left[x^2+(0.150\text{ m})^2\right]^{3/2}}$$

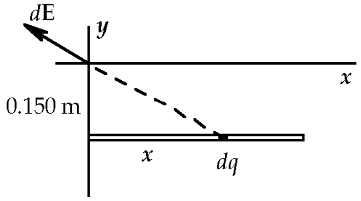


FIG. P23.72

$$\mathbf{E} = \int_{\text{all charge}} d\mathbf{E} = k_e\lambda \int_{x=0}^{0.400\text{ m}} \frac{\left(-x\hat{\mathbf{i}}+0.150\text{ m}\hat{\mathbf{j}}\right)dx}{\left[x^2+(0.150\text{ m})^2\right]^{3/2}}$$

$$\mathbf{E} = k_e\lambda\left[\left.\frac{+\hat{\mathbf{i}}}{\sqrt{x^2+(0.150\text{ m})^2}}\right|_0^{0.400\text{ m}} + \left.\frac{(0.150\text{ m})\hat{\mathbf{j}}x}{(0.150\text{ m})^2\sqrt{x^2+(0.150\text{ m})^2}}\right|_0^{0.400\text{ m}}\right]$$

$$\mathbf{E} = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(35.0\times10^{-9}\ \text{C/m}\right)\left[\hat{\mathbf{i}}(2.34-6.67)\ \text{m}^{-1}+\hat{\mathbf{j}}(6.24-0)\ \text{m}^{-1}\right]$$

$$\mathbf{E} = \left(-1.36\hat{\mathbf{i}}+1.96\hat{\mathbf{j}}\right)\times10^3\ \text{N/C} = \boxed{\left(-1.36\hat{\mathbf{i}}+1.96\hat{\mathbf{j}}\right)\ \text{kN/C}}$$

**P23.73** The electrostatic forces exerted on the two charges result in a net torque $\tau = -2Fa\sin\theta = -2Eqa\sin\theta$.

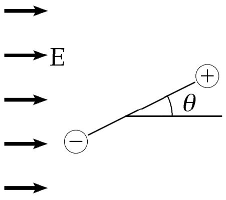


FIG. P23.73

For small $\theta$, $\sin\theta \approx \theta$ and using $p = 2qa$, we have $\tau = -Ep\theta$.

The torque produces an angular acceleration given by $\tau = I\alpha = I\dfrac{d^2\theta}{dt^2}$.

Combining these two expressions for torque, we have $\dfrac{d^2\theta}{dt^2}+\left(\dfrac{Ep}{I}\right)\theta = 0$.

This equation can be written in the form $\dfrac{d^2\theta}{dt^2} = -\omega^2\theta$ where $\omega^2 = \dfrac{Ep}{I}$.

This is the same form as Equation 15.5 and the frequency of oscillation is found by comparison with Equation 15.11, or $f = \dfrac{1}{2\pi}\sqrt{\dfrac{pE}{I}} = \boxed{\dfrac{1}{2\pi}\sqrt{\dfrac{2qaE}{I}}}$.

## ANSWERS TO EVEN PROBLEMS

**P23.2** (a) $2.62\times10^{24}$; (b) 2.38 electrons for every $10^{9}$ present

**P23.4** 57.5 N

**P23.6** $2.51\times10^{-9}$

**P23.8** 514 kN

**P23.10** $x=0.634d$. The equilibrium is stable if the third bead has positive charge.

**P23.12** (a) period $=\dfrac{\pi}{2}\sqrt{\dfrac{md^3}{k_eqQ}}$ where $m$ is the mass of the object with charge $-Q$; (b) $4a\sqrt{\dfrac{k_eqQ}{md^3}}$

**P23.14** 1.49 g

**P23.16** 720 kN/C down

**P23.18** (a) $\left[18.0\hat{\mathbf{i}}-218\hat{\mathbf{j}}\right]$ kN/C;
(b) $\left(36.0\hat{\mathbf{i}}-436\hat{\mathbf{j}}\right)$ mN

**P23.20** (a) $12.9\hat{\mathbf{j}}$ kN/C; (b) $-38.6\hat{\mathbf{j}}$ mN

**P23.22** see the solution

**P23.24** $-\dfrac{\pi^2k_eq}{6a^2}\hat{\mathbf{i}}$

**P23.26** $\dfrac{k_e\lambda_0}{x_0}\left(-\hat{\mathbf{i}}\right)$

**P23.28** $\dfrac{k_e\lambda_0}{2x_0}\left(-\hat{\mathbf{i}}\right)$

**P23.30** (a) 383 MN/C away; (b) 324 MN/C away; (c) 80.7 MN/C away; (d) 6.68 MN/C away

**P23.32** see the solution

**P23.34** (a) $\dfrac{k_eQ\hat{\mathbf{i}}}{h}\left[\left(d^2+R^2\right)^{-1/2}-\left((d+h)^2+R^2\right)^{-1/2}\right]$;
(b) $\dfrac{2k_eQ\hat{\mathbf{i}}}{R^2h}\left[h+\left(d^2+R^2\right)^{1/2}-\left((d+h)^2+R^2\right)^{1/2}\right]$

**P23.36** (a) 200 pC; (b) 141 pC; (c) 58.9 pC

**P23.38** see the solution

**P23.40** (a) $-\dfrac{1}{3}$; (b) $q_1$ is negative and $q_2$ is positive

**P23.42** electron: 4.39 Mm/s; proton: 2.39 km/s

**P23.44** (a) $-57.6\hat{\mathbf{i}}$ Tm/s$^2$; (b) $2.84\hat{\mathbf{i}}$ Mm/s; (c) 49.3 ns

**P23.46** (a) down; (b) 3.43 $\mu$C

**P23.48** The particle strikes the negative plate after moving in a parabola 0.181 mm high and 0.961 mm.

**P23.50** Possible only with $+51.3\ \mu$C at $x=-16.0$ cm

**P23.52** (a) 24.2 N/C at 0°; (b) 9.42 N/C at 117°

**P23.54** 5.25 $\mu$C

**P23.56** (a) $\dfrac{mg}{A\cot\theta+B}$; (b) $\dfrac{mgA}{A\cos\theta+B\sin\theta}$

**P23.58** 0.205 $\mu$C

**P23.60** $\dfrac{k_eqQ}{a^2}$ toward the 29th vertex

**P23.62** $443\,\hat{\mathbf{i}}$ kN/C

**P23.64** $0.072\,9a$

**P23.66** see the solution; the period is $\dfrac{\pi}{8^{1/4}}\sqrt{\dfrac{mL^3}{k_eQq}}$

**P23.68** $R\left(\dfrac{mg}{k_e\sqrt{3}}\right)^{1/2}$

**P23.70** (a) see the solution; (b) $\hat{\mathbf{k}}$

**P23.72** $\left(-1.36\hat{\mathbf{i}}+1.96\hat{\mathbf{j}}\right)$ kN/C

# 24

# Gauss's Law

## CHAPTER OUTLINE

24.1 Electric Flux
24.2 Gauss's Law
24.3 Application of Gauss's Law to Various Charge Distributions
24.4 Conductors in Electrostatic Equilibrium
24.5 Formal Derivation of Gauss's Law

## ANSWERS TO QUESTIONS

**Q24.1** The luminous flux on a given area is less when the sun is low in the sky, because the angle between the rays of the sun and the local area vector, $d\mathbf{A}$, is greater than zero. The cosine of this angle is reduced. The decreased flux results, on the average, in colder weather.

**Q24.2** If the region is just a point, line, or plane, no. Consider two protons in otherwise empty space. The electric field is zero at the midpoint of the line joining the protons. If the field-free region is three-dimensional, then it can contain no charges, but it might be surrounded by electric charge. Consider the interior of a metal sphere carrying static charge.

**Q24.3** The surface must enclose a positive total charge.

**Q24.4** The net flux through any gaussian surface is zero. We can argue it two ways. Any surface contains zero charge so Gauss's law says the total flux is zero. The field is uniform, so the field lines entering one side of the closed surface come out the other side and the net flux is zero.

**Q24.5** Gauss's law cannot tell the different values of the electric field at different points on the surface. When $E$ is an unknown number, then we can say $\int E\cos\theta dA = E\int\cos\theta dA$. When $E(x,\ y,\ z)$ is an unknown function, then there is no such simplification.

**Q24.6** The electric flux through a sphere around a point charge is independent of the size of the sphere. A sphere of larger radius has a larger area, but a smaller field at its surface, so that the product of field strength and area is independent of radius. If the surface is not spherical, some parts are closer to the charge than others. In this case as well, smaller projected areas go with stronger fields, so that the net flux is unaffected.

**Q24.7** Faraday's visualization of electric field lines lends insight to this question. Consider a section of a vertical sheet carrying charge +1 coulomb. It has $\frac{1}{\epsilon_0}$ field lines pointing out from it horizontally to the right and left, all uniformly spaced. The lines have the same uniform spacing close to the sheet and far away, showing that the field has the same value at all distances.

**Q24.8** Consider any point, zone, or object where electric field lines begin. Surround it with a close-fitting gaussian surface. The lines will go outward through the surface to constitute positive net flux. Then Gauss's law asserts that positive net charge must be inside the surface: it is where the lines begin. Similarly, any place where electric field lines end must be just inside a gaussian surface passing net negative flux, and must be a negative charge.

**Q24.9** Inject some charge at arbitrary places within a conducting object. Every bit of the charge repels every other bit, so each bit runs away as far as it can, stopping only when it reaches the outer surface of the conductor.

**Q24.10** If the person is uncharged, the electric field inside the sphere is zero. The interior wall of the shell carries no charge. The person is not harmed by touching this wall. If the person carries a (small) charge $q$, the electric field inside the sphere is no longer zero. Charge $-q$ is induced on the inner wall of the sphere. The person will get a (small) shock when touching the sphere, as all the charge on his body jumps to the metal.

**Q24.11** The electric fields outside are identical. The electric fields inside are very different. We have $\mathbf{E} = 0$ everywhere inside the conducting sphere while $E$ decreases gradually as you go below the surface of the sphere with uniform volume charge density.

**Q24.12** There is zero force. The huge charged sheet creates a uniform field. The field can polarize the neutral sheet, creating in effect a film of opposite charge on the near face and a film with an equal amount of like charge on the far face of the neutral sheet. Since the field is uniform, the films of charge feel equal-magnitude forces of attraction and repulsion to the charged sheet. The forces add to zero.

**Q24.13** Gauss's law predicts, as described in section 24.4, that excess charge on a conductor will reside on the surface of the conductor. If a car is left charged by a lightning strike, then that charge will remain on the outside of the car, not harming the occupants. It turns out that during the lightning strike, the current also remains on the outside of the conductor. Note that it is not necessarily safe to be in a fiberglass car or a convertible during a thunderstorm.

## SOLUTIONS TO PROBLEMS

### Section 24.1 **Electric Flux**

**P24.1** (a) $\Phi_E = EA\cos\theta = \left(3.50\times10^3\right)(0.350\times0.700)\cos 0° = \boxed{858\ \text{N}\cdot\text{m}^2/\text{C}}$

(b) $\theta = 90.0°$ $\quad\boxed{\Phi_E = 0}$

(c) $\Phi_E = \left(3.50\times10^3\right)(0.350\times0.700)\cos 40.0° = \boxed{657\ \text{N}\cdot\text{m}^2/\text{C}}$

**P24.2** $\Phi_E = EA\cos\theta = \left(2.00\times10^4\ \text{N/C}\right)\left(18.0\ \text{m}^2\right)\cos 10.0° = \boxed{355\ \text{kN}\cdot\text{m}^2/\text{C}}$

**P24.3** $\Phi_E = EA\cos\theta$ $\qquad A = \pi r^2 = \pi(0.200)^2 = 0.126\ \text{m}^2$

$5.20\times10^5 = E(0.126)\cos 0°$ $\qquad E = 4.14\times10^6\ \text{N/C} = \boxed{4.14\ \text{MN/C}}$

**P24.4** (a) $A' = (10.0\text{ cm})(30.0\text{ cm})$

$A' = 300\text{ cm}^2 = 0.030\,0\text{ m}^2$

$\Phi_{E,\,A'} = EA'\cos\theta$

$\Phi_{E,\,A'} = \left(7.80\times10^4\right)(0.030\,0)\cos 180°$

$\Phi_{E,\,A'} = \boxed{-2.34\text{ kN}\cdot\text{m}^2/\text{C}}$

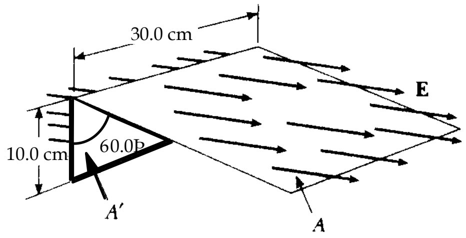


**FIG. P24.4**

(b) $\Phi_{E,\,A} = EA\cos\theta = \left(7.80\times10^4\right)(A)\cos 60.0°$

$A = (30.0\text{ cm})(w) = (30.0\text{ cm})\left(\dfrac{10.0\text{ cm}}{\cos 60.0°}\right) = 600\text{ cm}^2 = 0.060\,0\text{ m}^2$

$\Phi_{E,\,A} = \left(7.80\times10^4\right)(0.060\,0)\cos 60.0° = \boxed{+2.34\text{ kN}\cdot\text{m}^2/\text{C}}$

(c) The bottom and the two triangular sides all lie *parallel* to **E**, so $\Phi_E = 0$ for each of these. Thus,

$\Phi_{E,\,\text{total}} = -2.34\text{ kN}\cdot\text{m}^2/\text{C} + 2.34\text{ kN}\cdot\text{m}^2/\text{C} + 0 + 0 + 0 = \boxed{0}$.

**P24.5** (a) $\Phi_E = \mathbf{E}\cdot\mathbf{A} = \left(a\hat{\mathbf{i}} + b\hat{\mathbf{j}}\right)\cdot A\hat{\mathbf{i}} = \boxed{aA}$

(b) $\Phi_E = \left(a\hat{\mathbf{i}} + b\hat{\mathbf{j}}\right)\cdot A\hat{\mathbf{j}} = \boxed{bA}$

(c) $\Phi_E = \left(a\hat{\mathbf{i}} + b\hat{\mathbf{j}}\right)\cdot A\hat{\mathbf{k}} = \boxed{0}$

**P24.6** Only the charge inside radius *R* contributes to the total flux.

$\Phi_E = \boxed{\dfrac{q}{\epsilon_0}}$

**P24.7** $\Phi_E = EA\cos\theta$ through the base

$\Phi_E = (52.0)(36.0)\cos 180° = -1.87\text{ kN}\cdot\text{m}^2/\text{C}$.

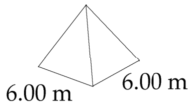


**FIG. P24.7**

Note the same number of electric field lines go through the base as go through the pyramid's surface (not counting the base).

For the slanting surfaces, $\boxed{\Phi_E = +1.87\text{ kN}\cdot\text{m}^2/\text{C}}$.

**P24.8** The flux entering the closed surface equals the flux exiting the surface. The flux entering the left side of the cone is $\Phi_E = \int \mathbf{E}\cdot d\mathbf{A} = \boxed{ERh}$. This is the same as the flux that exits the right side of the cone. Note that for a uniform field only the cross sectional area matters, not shape.

## Section 24.2 **Gauss's Law**

**P24.9** (a) $\Phi_E = \dfrac{q_{\text{in}}}{\epsilon_0} = \dfrac{(+5.00\ \mu\text{C} - 9.00\ \mu\text{C} + 27.0\ \mu\text{C} - 84.0\ \mu\text{C})}{8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = -6.89 \times 10^6\ \text{N}\cdot\text{m}^2/\text{C}^2$

$\Phi_E = \boxed{-6.89\ \text{MN}\cdot\text{m}^2/\text{C}}$

(b) Since the net electric flux is negative, more lines enter than leave the surface.

**P24.10** (a) $E = \dfrac{k_e Q}{r^2}$: $\quad 8.90 \times 10^2 = \dfrac{(8.99 \times 10^9)Q}{(0.750)^2}$

But $Q$ is negative since **E** points inward. $\quad Q = -5.56 \times 10^{-8}\ \text{C} = \boxed{-55.6\ \text{nC}}$

(b) The $\boxed{\text{negative}}$ charge has a $\boxed{\text{spherically symmetric}}$ charge distribution.

**P24.11** $\Phi_E = \dfrac{q_{\text{in}}}{\epsilon_0}$

Through $S_1$ $\quad \Phi_E = \dfrac{-2Q + Q}{\epsilon_0} = \boxed{-\dfrac{Q}{\epsilon_0}}$

Through $S_2$ $\quad \Phi_E = \dfrac{+Q - Q}{\epsilon_0} = \boxed{0}$

Through $S_3$ $\quad \Phi_E = \dfrac{-2Q + Q - Q}{\epsilon_0} = \boxed{-\dfrac{2Q}{\epsilon_0}}$

Through $S_4$ $\quad \Phi_E = \boxed{0}$

**P24.12** (a) One-half of the total flux created by the charge $q$ goes through the plane. Thus,

$\Phi_{E,\text{ plane}} = \dfrac{1}{2}\Phi_{E,\text{ total}} = \dfrac{1}{2}\left(\dfrac{q}{\epsilon_0}\right) = \boxed{\dfrac{q}{2\,\epsilon_0}}$.

(b) The square looks like an infinite plane to a charge *very close* to the surface. Hence,

$\Phi_{E,\text{ square}} \approx \Phi_{E,\text{ plane}} = \boxed{\dfrac{q}{2\,\epsilon_0}}$.

(c) $\boxed{\text{The plane and the square look the same to the charge.}}$

**P24.13** The flux through the curved surface is equal to the flux through the flat circle, $\boxed{E_0 \pi r^2}$.

**P24.14** (a) $\Phi_{E,\text{ shell}} = \dfrac{q_{\text{in}}}{\epsilon_0} = \dfrac{12.0 \times 10^{-6}}{8.85 \times 10^{-12}} = 1.36 \times 10^6\ \text{N}\cdot\text{m}^2/\text{C} = \boxed{1.36\ \text{MN}\cdot\text{m}^2/\text{C}}$

(b) $\Phi_{E,\text{ half shell}} = \dfrac{1}{2}\left(1.36 \times 10^6\ \text{N}\cdot\text{m}^2/\text{C}\right) = 6.78 \times 10^5\ \text{N}\cdot\text{m}^2/\text{C} = \boxed{678\ \text{kN}\cdot\text{m}^2/\text{C}}$

(c) $\boxed{\text{No,}}$ the same number of field lines will pass through each surface, no matter how the radius changes.

**P24.15** (a) With $\delta$ very small, all points on the hemisphere are nearly at a distance $R$ from the charge, so the field everywhere on the curved surface is $\frac{k_e Q}{R^2}$ radially outward (normal to the surface). Therefore, the flux is this field strength times the area of half a sphere:

$$\Phi_{\text{curved}} = \int \mathbf{E} \cdot d\mathbf{A} = E_{\text{local}} A_{\text{hemisphere}}$$

$$\Phi_{\text{curved}} = \left(k_e \frac{Q}{R^2}\right)\left(\frac{1}{2} 4\pi R^2\right) = \frac{1}{4\pi \in_0} Q(2\pi) = \boxed{\frac{+Q}{2 \in_0}}$$

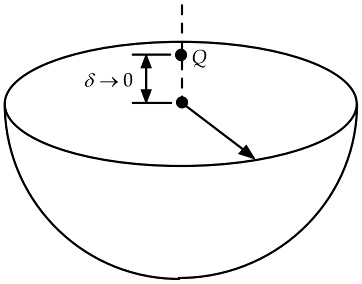


FIG. P24.15

(b) The closed surface encloses zero charge so Gauss's law gives

$$\Phi_{\text{curved}} + \Phi_{\text{flat}} = 0 \qquad \text{or} \qquad \Phi_{\text{flat}} = -\Phi_{\text{curved}} = \boxed{\frac{-Q}{2 \in_0}}.$$

***P24.16** Consider as a gaussian surface a box with horizontal area $A$, lying between 500 and 600 m elevation.

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\in_0}: \qquad (+120 \text{ N/C})A + (-100 \text{ N/C})A = \frac{\rho A(100 \text{ m})}{\in_0}$$

$$\rho = \frac{(20 \text{ N/C})\left(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2\right)}{100 \text{ m}} = \boxed{1.77 \times 10^{-12} \text{ C/m}^3}$$

The charge is $\boxed{\text{positive}}$, to produce the net outward flux of electric field.

**P24.17** The total charge is $Q - 6|q|$. The total outward flux from the cube is $\frac{Q - 6|q|}{\in_0}$, of which one-sixth goes through each face:

$$(\Phi_E)_{\text{one face}} = \frac{Q - 6|q|}{6 \in_0}$$

$$(\Phi_E)_{\text{one face}} = \frac{Q - 6|q|}{6 \in_0} = \frac{(5.00 - 6.00) \times 10^{-6} \text{ C} \cdot \text{N} \cdot \text{m}^2}{6 \times 8.85 \times 10^{-12} \text{ C}^2} = \boxed{-18.8 \text{ kN} \cdot \text{m}^2/\text{C}}.$$

**P24.18** The total charge is $Q - 6|q|$. The total outward flux from the cube is $\frac{Q - 6|q|}{\in_0}$, of which one-sixth goes through each face:

$$(\Phi_E)_{\text{one face}} = \boxed{\frac{Q - 6|q|}{6 \in_0}}.$$

**P24.19** If $R \le d$, the sphere encloses no charge and $\Phi_E = \frac{q_{\text{in}}}{\in_0} = \boxed{0}$.

If $R > d$, the length of line falling within the sphere is $2\sqrt{R^2 - d^2}$

so $\Phi_E = \boxed{\frac{2\lambda\sqrt{R^2 - d^2}}{\in_0}}$.

**P24.20** $$\Phi_{E,\,\text{hole}} = \mathbf{E}\cdot\mathbf{A}_{\text{hole}} = \left(\frac{k_eQ}{R^2}\right)\left(\pi r^2\right) = \left(\frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(10.0\times10^{-6}\ \text{C}\right)}{(0.100\ \text{m})^2}\right)\pi\left(1.00\times10^{-3}\ \text{m}\right)^2$$

$$\Phi_{E,\,\text{hole}} = \boxed{28.2\ \text{N}\cdot\text{m}^2/\text{C}}$$

**P24.21** $$\Phi_E = \frac{q_{\text{in}}}{\epsilon_0} = \frac{170\times10^{-6}\ \text{C}}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = 1.92\times10^{7}\ \text{N}\cdot\text{m}^2/\text{C}$$

(a) $$\left(\Phi_E\right)_{\text{one face}} = \frac{1}{6}\Phi_E = \frac{1.92\times10^{7}\ \text{N}\cdot\text{m}^2/\text{C}}{6} \qquad \left(\Phi_E\right)_{\text{one face}} = \boxed{3.20\ \text{MN}\cdot\text{m}^2/\text{C}}$$

(b) $$\Phi_E = \boxed{19.2\ \text{MN}\cdot\text{m}^2/\text{C}}$$

(c) The answer to (a) would change because the flux through each face of the cube would not be equal with an asymmetric charge distribution. The sides of the cube nearer the charge would have more flux and the ones further away would have less. The answer to (b) would remain the same, since the overall flux would remain the same.

**P24.22** No charge is inside the cube. The net flux through the cube is zero. Positive flux comes out through the three faces meeting at $g$. These three faces together fill solid angle equal to one-eighth of a sphere as seen from $q$, and together pass flux $\frac{1}{8}\left(\frac{q}{\epsilon_0}\right)$. Each face containing $a$ intercepts equal flux going into the cube:

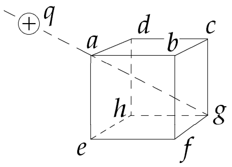


**FIG. P24.22**

$$0 = \Phi_{E,\,\text{net}} = 3\Phi_{E,\,\text{abcd}} + \frac{q}{8\,\epsilon_0}$$

$$\Phi_{E,\,\text{abcd}} = \boxed{\frac{-q}{24\,\epsilon_0}}$$

## Section 24.3 **Application of Gauss's Law to Various Charge Distributions**

**P24.23** The charge distributed through the nucleus creates a field at the surface equal to that of a point charge at its center: $E = \frac{k_eq}{r^2}$

$$E = \frac{\left(8.99\times10^{9}\ \text{Nm}^2/\text{C}^2\right)\left(82\times1.60\times10^{-19}\ \text{C}\right)}{\left[(208)^{1/3}1.20\times10^{-15}\ \text{m}\right]^2}$$

$E = \boxed{2.33\times10^{21}\ \text{N/C}}$ away from the nucleus

**P24.24** (a) $E = \frac{k_e Qr}{a^3} = \boxed{0}$

(b) $E = \frac{k_e Qr}{a^3} = \frac{(8.99 \times 10^9)(26.0 \times 10^{-6})(0.100)}{(0.400)^3} = \boxed{365 \text{ kN/C}}$

(c) $E = \frac{k_e Q}{r^2} = \frac{(8.99 \times 10^9)(26.0 \times 10^{-6})}{(0.400)^2} = \boxed{1.46 \text{ MN/C}}$

(d) $E = \frac{k_e Q}{r^2} = \frac{(8.99 \times 10^9)(26.0 \times 10^{-6})}{(0.600)^2} = \boxed{649 \text{ kN/C}}$

The direction for each electric field is $\boxed{\text{radially outward}}$.

***P24.25** $mg = qE = q\left(\frac{\sigma}{2 \in_0}\right) = q\left(\frac{Q/A}{2 \in_0}\right)$ $\frac{Q}{A} = \frac{2 \in_0 mg}{q} = \frac{2(8.85 \times 10^{-12})(0.01)(9.8)}{-0.7 \times 10^{-6}} = \boxed{-2.48 \ \mu\text{C/m}^2}$

**P24.26** (a) $E = \frac{2k_e \lambda}{r}$ $3.60 \times 10^4 = \frac{2(8.99 \times 10^9)(Q/2.40)}{0.190}$

$Q = +9.13 \times 10^{-7} \text{ C} = \boxed{+913 \text{ nC}}$

(b) $\mathbf{E} = \boxed{0}$

***P24.27** The volume of the spherical shell is

$\frac{4}{3}\pi\left[(0.25 \text{ m})^3 - (0.20 \text{ m})^3\right] = 3.19 \times 10^{-2} \text{ m}^3$.

Its charge is

$\rho V = (-1.33 \times 10^{-6} \text{ C/m}^3)(3.19 \times 10^{-2} \text{ m}^3) = -4.25 \times 10^{-8} \text{ C}$.

The net charge inside a sphere containing the proton's path as its equator is

$-60 \times 10^{-9} \text{ C} - 4.25 \times 10^{-8} \text{ C} = -1.02 \times 10^{-7} \text{ C}$.

The electric field is radially inward with magnitude

$\frac{k_e|q|}{r^2} = \frac{|q|}{\in_0 4\pi r^2} = \frac{8.99 \times 10^9 \text{ Nm}^2(1.02 \times 10^{-7} \text{ C})}{\text{C}^2(0.25 \text{ m})^2} = 1.47 \times 10^4 \text{ N/C}$.

For the proton

$\sum F = ma$ $eE = \frac{mv^2}{r}$

$v = \left(\frac{eEr}{m}\right)^{1/2} = \left(\frac{1.60 \times 10^{-19} \text{ C}(1.47 \times 10^4 \text{ N/C})0.25 \text{ m}}{1.67 \times 10^{-27} \text{ kg}}\right)^{1/2} = \boxed{5.94 \times 10^5 \text{ m/s}}$.

**P24.28** $\sigma = \left(8.60\times10^{-6}\ \text{C/cm}^2\right)\left(\dfrac{100\ \text{cm}}{\text{m}}\right)^2 = 8.60\times10^{-2}\ \text{C/m}^2$

$$E = \frac{\sigma}{2\in_0} = \frac{8.60\times10^{-2}}{2\left(8.85\times10^{-12}\right)} = \boxed{4.86\times10^{9}\ \text{N/C away from the wall}}$$

The field is essentially uniform as long as the distance from the center of the wall to the field point is much less than the dimensions of the wall.

**P24.29** If $\rho$ is positive, the field must be radially outward. Choose as the gaussian surface a cylinder of length $L$ and radius $r$, contained inside the charged rod. Its volume is $\pi r^2 L$ and it encloses charge $\rho\pi r^2 L$. Because the charge distribution is long, no electric flux passes through the circular end caps; $\mathbf{E}\cdot d\mathbf{A} = EdA\cos 90.0° = 0$. The curved surface has $\mathbf{E}\cdot d\mathbf{A} = EdA\cos 0°$, and $E$ must be the same strength everywhere over the curved surface.

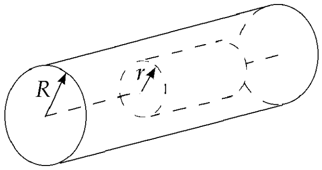


**FIG. P24.29**

Gauss's law, $\oint \mathbf{E}\cdot d\mathbf{A} = \dfrac{q}{\in_0}$, becomes $E\displaystyle\int_{\substack{\text{Curved}\\\text{Surface}}} dA = \frac{\rho\pi r^2 L}{\in_0}$.

Now the lateral surface area of the cylinder is $2\pi rL$:

$$E(2\pi r)L = \frac{\rho\pi r^2 L}{\in_0}. \qquad \text{Thus,} \qquad \mathbf{E} = \boxed{\frac{\rho r}{2\in_0}\ \text{radially away from the cylinder axis}}.$$

***P24.30** Let $\rho$ represent the charge density. For the field inside the sphere at $r_1 = 5$ cm we have

$$E_1 4\pi r_1^2 = \frac{q_{\text{inside}}}{\in_0} = \frac{4\pi r_1^3 \rho}{3\in_0} \qquad E_1 = \frac{r_1\rho}{3\in_0}$$

$$\rho = \frac{3\in_0 E_1}{r_1} = \frac{3\left(8.85\times10^{-12}\ \text{C}^2\right)\left(-86\times10^3\ \text{N}\right)}{0.05\ \text{m}\quad \text{Nm}^2\quad \text{C}} = -4.57\times10^{-5}\ \text{C/m}^3.$$

Now for the field outside at $r_3 = 15$ cm

$$E_3 4\pi r_3^2 = \frac{4\pi r_2^3 \rho}{3\in_0}$$

$$E_3 = \frac{k_e}{r_3^2}\frac{4}{3}\frac{\pi(0.10\ \text{m})^3\left(-4.57\times10^{-5}\ \text{C}\right)}{\text{m}^3} = \frac{8.99\times10^9\ \text{Nm}^2\left(-1.91\times10^{-7}\ \text{C}\right)}{(0.15\ \text{m})^2\text{C}^2} = -7.64\times10^4\ \text{N/C}$$

$\mathbf{E}_3 = \boxed{76.4\ \text{kN/C}}$ radially inward

**P24.31** (a) $\mathbf{E} = \boxed{0}$

(b) $E = \dfrac{k_e Q}{r^2} = \dfrac{\left(8.99\times10^9\right)\left(32.0\times10^{-6}\right)}{(0.200)^2} = 7.19\ \text{MN/C}$ $\qquad \mathbf{E} = \boxed{7.19\ \text{MN/C radially outward}}$

**P24.32** The distance between centers is $2\times5.90\times10^{-15}$ m. Each produces a field as if it were a point charge at its center, and each feels a force as if all its charge were a point at its center.

$$F = \frac{k_e q_1 q_2}{r^2} = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{(46)^2\left(1.60\times10^{-19}\ \text{C}\right)^2}{\left(2\times5.90\times10^{-15}\ \text{m}\right)^2} = 3.50\times10^3\ \text{N} = \boxed{3.50\ \text{kN}}$$

**P24.33** Consider two balloons of diameter 0.2 m, each with mass 1 g, hanging apart with a 0.05 m separation on the ends of strings making angles of 10° with the vertical.

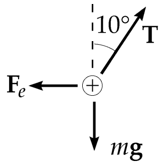


FIG. P24.33

(a)
$$\sum F_y = T\cos 10° - mg = 0 \Rightarrow T = \frac{mg}{\cos 10°}$$

$\sum F_x = T\sin 10° - F_e = 0 \Rightarrow F_e = T\sin 10°$, so

$$F_e = \left(\frac{mg}{\cos 10°}\right)\sin 10° = mg\tan 10° = (0.001\text{ kg})\left(9.8\text{ m/s}^2\right)\tan 10°$$

$$F_e \approx 2\times 10^{-3}\text{ N}\ \boxed{\sim 10^{-3}\text{ N or 1 mN}}$$

(b)
$$F_e = \frac{k_e q^2}{r^2}$$

$$2\times 10^{-3}\text{ N} \approx \frac{\left(8.99\times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)q^2}{(0.25\text{ m})^2}$$

$$q \approx 1.2\times 10^{-7}\text{ C}\ \boxed{\sim 10^{-7}\text{ C or 100 nC}}$$

(c)
$$E = \frac{k_e q}{r^2} \approx \frac{\left(8.99\times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.2\times 10^{-7}\text{ C}\right)}{(0.25\text{ m})^2} \approx 1.7\times 10^4\text{ N/C}\ \boxed{\sim 10\text{ kN/C}}$$

(d)
$$\Phi_E = \frac{q}{\epsilon_0} \approx \frac{1.2\times 10^{-7}\text{ C}}{8.85\times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2} = 1.4\times 10^4\text{ N}\cdot\text{m}^2/\text{C}\ \boxed{\sim 10\text{ kN}\cdot\text{m}^2/\text{C}}$$

***P24.34** The charge density is determined by $Q = \frac{4}{3}\pi a^3 \rho$ $\qquad \rho = \frac{3Q}{4\pi a^3}$

(a) The flux is that created by the enclosed charge within radius $r$:

$$\Phi_E = \frac{q_{\text{in}}}{\epsilon_0} = \frac{4\pi r^3 \rho}{3\epsilon_0} = \frac{4\pi r^3 3Q}{3\epsilon_0 4\pi a^3} = \boxed{\frac{Qr^3}{\epsilon_0 a^3}}$$

(b) $\Phi_E = \boxed{\frac{Q}{\epsilon_0}}$. Note that the answers to parts (a) and (b) agree at $r = a$.

(c)

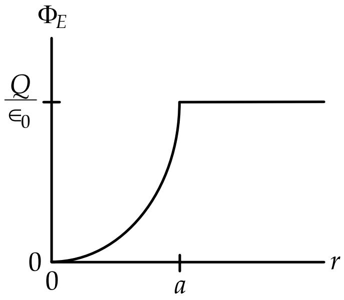


FIG. P24.34(c)

**P24.35** (a) $$E=\frac{2k_e\lambda}{r}=\frac{2\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left[\left(2.00\times10^{-6}\ \mathrm{C}\right)/7.00\ \mathrm{m}\right]}{0.100\ \mathrm{m}}$$

$$E=\boxed{51.4\ \mathrm{kN/C},\ \text{radially outward}}$$

(b) $$\Phi_E=EA\cos\theta=E(2\pi r\ell)\cos0°$$

$$\Phi_E=\left(5.14\times10^4\ \mathrm{N/C}\right)2\pi(0.100\ \mathrm{m})(0.020\,0\ \mathrm{m})(1.00)=\boxed{646\ \mathrm{N\cdot m^2/C}}$$

**P24.36** (a) $$\rho=\frac{Q}{\frac{4}{3}\pi a^3}=\frac{5.70\times10^{-6}}{\frac{4}{3}\pi(0.040\,0)^3}=2.13\times10^{-2}\ \mathrm{C/m^3}$$

$$q_{\text{in}}=\rho\left(\frac{4}{3}\pi r^3\right)=\left(2.13\times10^{-2}\right)\left(\frac{4}{3}\pi\right)(0.020\,0)^3=7.13\times10^{-7}\ \mathrm{C}=\boxed{713\ \mathrm{nC}}$$

(b) $$q_{\text{in}}=\rho\left(\frac{4}{3}\pi r^3\right)=\left(2.13\times10^{-2}\right)\left(\frac{4}{3}\pi\right)(0.040\,0)^3=\boxed{5.70\ \mu\mathrm{C}}$$

**P24.37** $$E=\frac{\sigma}{2\epsilon_0}=\frac{9.00\times10^{-6}\ \mathrm{C/m^2}}{2\left(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)}=\boxed{508\ \mathrm{kN/C},\ \text{upward}}$$

**P24.38** Note that the electric field in each case is directed radially inward, toward the filament.

(a) $$E=\frac{2k_e\lambda}{r}=\frac{2\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(90.0\times10^{-6}\ \mathrm{C/m}\right)}{0.100\ \mathrm{m}}=\boxed{16.2\ \mathrm{MN/C}}$$

(b) $$E=\frac{2k_e\lambda}{r}=\frac{2\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(90.0\times10^{-6}\ \mathrm{C/m}\right)}{0.200\ \mathrm{m}}=\boxed{8.09\ \mathrm{MN/C}}$$

(c) $$E=\frac{2k_e\lambda}{r}=\frac{2\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(90.0\times10^{-6}\ \mathrm{C/m}\right)}{1.00\ \mathrm{m}}=\boxed{1.62\ \mathrm{MN/C}}$$

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## Section 24.4 **Conductors in Electrostatic Equilibrium**

**P24.39** $$\oint EdA=E(2\pi rl)=\frac{q_{\text{in}}}{\epsilon_0}\qquad E=\frac{q_{\text{in}}/l}{2\pi\epsilon_0 r}=\frac{\lambda}{2\pi\epsilon_0 r}$$

(a) $r=3.00$ cm $\qquad \mathbf{E}=\boxed{0}$

(b) $r=10.0$ cm $$\mathbf{E}=\frac{30.0\times10^{-9}}{2\pi\left(8.85\times10^{-12}\right)(0.100)}=\boxed{5\,400\ \mathrm{N/C},\ \text{outward}}$$

(c) $r=100$ cm $$\mathbf{E}=\frac{30.0\times10^{-9}}{2\pi\left(8.85\times10^{-12}\right)(1.00)}=\boxed{540\ \mathrm{N/C},\ \text{outward}}$$

**P24.40** From Gauss's Law, $EA = \dfrac{Q}{\epsilon_0}$

$$\sigma = \frac{Q}{A} = \epsilon_0 E = \left(8.85\times10^{-12}\right)(-130) = -1.15\times10^{-9}\ \text{C/m}^2 = \boxed{-1.15\ \text{nC/m}^2}$$

**P24.41** The fields are equal. The Equation 24.9 $E = \dfrac{\sigma_{\text{conductor}}}{\epsilon_0}$ for the field outside the aluminum looks different from Equation 24.8 $E = \dfrac{\sigma_{\text{insulator}}}{2\epsilon_0}$ for the field around glass. But its charge will spread out to cover both sides of the aluminum plate, so the density is $\sigma_{\text{conductor}} = \dfrac{Q}{2A}$. The glass carries charge only on area $A$, with $\sigma_{\text{insulator}} = \dfrac{Q}{A}$. The two fields are $\dfrac{Q}{2A\epsilon_0}$ the same in magnitude, and both are perpendicular to the plates, vertically upward if $Q$ is positive.

***P24.42** (a) All of the charge sits on the surface of the copper sphere at radius 15 cm. The field inside is $\boxed{\text{zero}}$.

(b) The charged sphere creates field at exterior points as if it were a point charge at the center:

$$\mathbf{E} = \frac{k_e q}{r^2}\ \text{away} = \frac{\left(8.99\times10^9\ \text{Nm}^2\right)\left(40\times10^{-9}\ \text{C}\right)}{\text{C}^2(0.17\ \text{m})^2}\ \text{outward} = \boxed{1.24\times10^4\ \text{N/C outward}}$$

(c) $$\mathbf{E} = \frac{\left(8.99\times10^9\ \text{Nm}^2\right)\left(40\times10^{-9}\ \text{C}\right)}{\text{C}^2(0.75\ \text{m})^2}\ \text{outward} = \boxed{639\ \text{N/C outward}}$$

(d) All three answers would be the same.

**P24.43** (a) $E = \dfrac{\sigma}{\epsilon_0}$ $\qquad \sigma = \left(8.00\times10^4\right)\left(8.85\times10^{-12}\right) = 7.08\times10^{-7}\ \text{C/m}^2$

$\sigma = \boxed{708\ \text{nC/m}^2}$, positive on one face and negative on the other.

(b) $\sigma = \dfrac{Q}{A}$ $\qquad Q = \sigma A = \left(7.08\times10^{-7}\right)(0.500)^2\ \text{C}$

$Q = 1.77\times10^{-7}\ \text{C} = \boxed{177\ \text{nC}}$, positive on one face and negative on the other.

**P24.44** (a) $\mathbf{E} = \boxed{0}$

(b) $$E = \frac{k_e Q}{r^2} = \frac{\left(8.99\times10^9\right)\left(8.00\times10^{-6}\right)}{(0.030\,0)^2} = 7.99\times10^7\ \text{N/C} \qquad \mathbf{E} = \boxed{79.9\ \text{MN/C radially outward}}$$

(c) $\mathbf{E} = \boxed{0}$

(d) $$E = \frac{k_e Q}{r^2} = \frac{\left(8.99\times10^9\right)\left(4.00\times10^{-6}\right)}{(0.070\,0)^2} = 7.34\times10^6\ \text{N/C} \qquad \mathbf{E} = \boxed{7.34\ \text{MN/C radially outward}}$$

**P24.45** The charge divides equally between the identical spheres, with charge $\frac{Q}{2}$ on each. Then they repel like point charges at their centers:

$$F=\frac{k_e(Q/2)(Q/2)}{(L+R+R)^2}=\frac{k_eQ^2}{4(L+2R)^2}=\frac{8.99\times10^9\ \text{N}\cdot\text{m}^2\left(60.0\times10^{-6}\ \text{C}\right)^2}{4\ \text{C}^2(2.01\ \text{m})^2}=\boxed{2.00\ \text{N}}.$$

**P24.46** The electric field on the surface of a conductor varies inversely with the radius of curvature of the surface. Thus, the field is most intense where the radius of curvature is smallest and vice-versa. The local charge density and the electric field intensity are related by

$$E=\frac{\sigma}{\epsilon_0}\qquad\text{or}\qquad\sigma=\epsilon_0 E.$$

(a) Where the radius of curvature is the greatest,

$$\sigma=\epsilon_0 E_{\text{min}}=\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(2.80\times10^4\ \text{N/C}\right)=\boxed{248\ \text{nC/m}^2}.$$

(b) Where the radius of curvature is the smallest,

$$\sigma=\epsilon_0 E_{\text{max}}=\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(5.60\times10^4\ \text{N/C}\right)=\boxed{496\ \text{nC/m}^2}.$$

**P24.47** (a) Inside surface: consider a cylindrical surface within the metal. Since $E$ inside the conducting shell is zero, the total charge inside the gaussian surface must be zero, so the inside charge/length $=-\lambda$.

$0=\lambda\ell+q_{\text{in}}$ so $\frac{q_{\text{in}}}{\ell}=\boxed{-\lambda}$

Outside surface: The total charge on the metal cylinder is $2\lambda\ell=q_{\text{in}}+q_{\text{out}}$

$q_{\text{out}}=2\lambda\ell+\lambda\ell$ so the outside charge/length is $\boxed{3\lambda}$.

(b) $$E=\frac{2k_e(3\lambda)}{r}=\frac{6k_e\lambda}{r}=\boxed{\frac{3\lambda}{2\pi\epsilon_0 r}\ \text{radially outward}}$$

**P24.48** (a) $$E=\frac{k_eQ}{r^2}=\frac{\left(8.99\times10^9\right)\left(6.40\times10^{-6}\right)}{(0.150)^2}=\boxed{2.56\ \text{MN/C, radially inward}}$$

(b) $\boxed{E=0}$

**P24.49** (a) The charge density on each of the surfaces (upper and lower) of the plate is:

$$\sigma=\frac{1}{2}\left(\frac{q}{A}\right)=\frac{1}{2}\frac{\left(4.00\times10^{-8}\ \text{C}\right)}{(0.500\ \text{m})^2}=8.00\times10^{-8}\ \text{C/m}^2=\boxed{80.0\ \text{nC/m}^2}.$$

(b) $$\mathbf{E}=\left(\frac{\sigma}{\epsilon_0}\right)\hat{\mathbf{k}}=\left(\frac{8.00\times10^{-8}\ \text{C/m}^2}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2}\right)\hat{\mathbf{k}}=\boxed{(9.04\ \text{kN/C})\hat{\mathbf{k}}}$$

(c) $\mathbf{E}=\boxed{(-9.04\ \text{kN/C})\hat{\mathbf{k}}}$

**P24.50** (a) The charge $+q$ at the center induces charge $-q$ on the inner surface of the conductor, where its surface density is:

$$\sigma_a = \boxed{\frac{-q}{4\pi a^2}}.$$

(b) The outer surface carries charge $Q+q$ with density

$$\sigma_b = \boxed{\frac{Q+q}{4\pi b^2}}.$$

**P24.51** Use Gauss's Law to evaluate the electric field in each region, recalling that the electric field is zero everywhere within conducting materials. The results are:

$\boxed{E = 0 \text{ inside the sphere and within the material of the shell}}$

$\boxed{E = k_e \frac{Q}{r^2} \text{ between the sphere and shell, directed radially inward}}$

$\boxed{E = k_e \frac{2Q}{r^2} \text{ outside the shell, directed radially outward}}$.

Charge $\boxed{-Q \text{ is on the outer surface of the sphere}}$.

Charge $\boxed{+Q \text{ is on the inner surface of the shell}}$,

and $\boxed{+2Q \text{ is on the outer surface of the shell.}}$

**P24.52** An approximate sketch is given at the right. Note that the electric field lines should be perpendicular to the conductor both inside and outside.

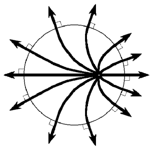

**FIG. P24.52**

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Section 24.5 **Formal Derivation of Gauss's Law**

**P24.53** (a) Uniform **E**, pointing radially outward, so $\Phi_E = EA$. The arc length is $ds = R\,d\theta$, and the circumference is $2\pi r = 2\pi R \sin\theta$

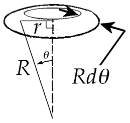

**FIG. P24.53**

$$A = \int 2\pi r\,ds = \int_0^\theta (2\pi R\sin\theta) R\,d\theta = 2\pi R^2 \int_0^\theta \sin\theta\,d\theta = 2\pi R^2 (-\cos\theta)\Big|_0^\theta = 2\pi R^2 (1-\cos\theta)$$

$$\Phi_E = \frac{1}{4\pi \in_0} \frac{Q}{R^2} \cdot 2\pi R^2 (1-\cos\theta) = \boxed{\frac{Q}{2\in_0}(1-\cos\theta)} \quad \text{[independent of R!]}$$

(b) For $\theta = 90.0°$ (hemisphere): $\Phi_E = \frac{Q}{2\in_0}(1-\cos 90°) = \boxed{\frac{Q}{2\in_0}}$.

(c) For $\theta = 180°$ (entire sphere): $\Phi_E = \frac{Q}{2\in_0}(1-\cos 180°) = \boxed{\frac{Q}{\in_0}}$ [Gauss's Law].

## Additional Problems

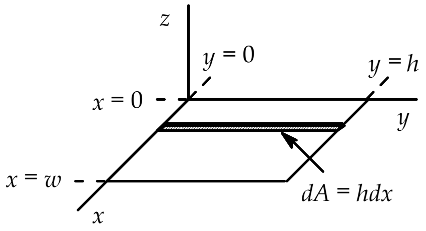

FIG. P24.54

**P24.54** In general, $\mathbf{E} = ay\hat{\mathbf{i}} + bz\hat{\mathbf{j}} + cx\hat{\mathbf{k}}$

In the $xy$ plane, $z = 0$ and $\mathbf{E} = ay\hat{\mathbf{i}} + cx\hat{\mathbf{k}}$

$$\Phi_E = \int \mathbf{E} \cdot d\mathbf{A} = \int \left(ay\hat{\mathbf{i}} + cx\hat{\mathbf{k}}\right) \cdot \hat{\mathbf{k}} dA$$

$$\Phi_E = ch \int_{x=0}^{w} x dx = ch \left.\frac{x^2}{2}\right|_{x=0}^{w} = \boxed{\frac{chw^2}{2}}$$

**P24.55** (a) $q_{\text{in}} = +3Q - Q = \boxed{+2Q}$

(b) The charge distribution is spherically symmetric and $q_{\text{in}} > 0$. Thus, the field is directed $\boxed{\text{radially outward}}$.

(c) $E = \frac{k_e q_{\text{in}}}{r^2} = \boxed{\frac{2k_e Q}{r^2}}$ for $r \geq c$.

(d) Since all points within this region are located inside conducting material, $\boxed{E = 0}$ for $b < r < c$.

(e) $\Phi_E = \int \mathbf{E} \cdot d\mathbf{A} = 0 \Rightarrow q_{\text{in}} = \epsilon_0 \Phi_E = \boxed{0}$

(f) $q_{\text{in}} = \boxed{+3Q}$

(g) $E = \frac{k_e q_{\text{in}}}{r^2} = \boxed{\frac{3k_e Q}{r^2}}$ (radially outward) for $a \leq r < b$.

(h) $q_{\text{in}} = \rho V = \left(\frac{+3Q}{\frac{4}{3}\pi a^3}\right)\left(\frac{4}{3}\pi r^3\right) = \boxed{+3Q\frac{r^3}{a^3}}$

(i) $E = \frac{k_e q_{\text{in}}}{r^2} = \frac{k_e}{r^2}\left(+3Q\frac{r^3}{a^3}\right) = \boxed{3k_e Q\frac{r}{a^3}}$ (radially outward) for $0 \leq r \leq a$.

(j) From part (d), $E = 0$ for $b < r < c$. Thus, for a spherical gaussian surface with $b < r < c$, $q_{\text{in}} = +3Q + q_{\text{inner}} = 0$ where $q_{\text{inner}}$ is the charge on the inner surface of the conducting shell. This yields $q_{\text{inner}} = \boxed{-3Q}$.

(k) Since the total charge on the conducting shell is $q_{\text{net}} = q_{\text{outer}} + q_{\text{inner}} = -Q$, we have $q_{\text{outer}} = -Q - q_{\text{inner}} = -Q - (-3Q) = \boxed{+2Q}$.

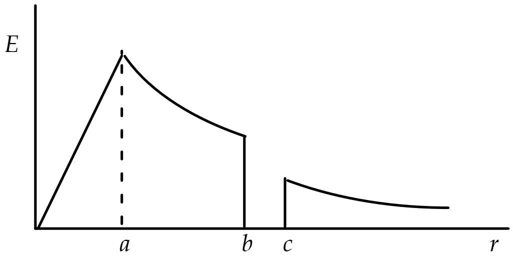

FIG. P24.55(l)

(l) This is shown in the figure to the right.

**P24.56** The sphere with large charge creates a strong field to polarize the other sphere. That means it pushes the excess charge over to the far side, leaving charge of the opposite sign on the near side. This patch of opposite charge is smaller in amount but located in a stronger external field, so it can feel a force of attraction that is larger than the repelling force felt by the larger charge in the weaker field on the other side.

**P24.57** (a) $\oint \mathbf{E}\cdot d\mathbf{A} = E\left(4\pi r^2\right) = \dfrac{q_{\text{in}}}{\epsilon_0}$

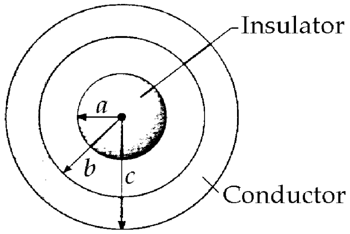

**FIG. P24.57**

For $r < a$, $\quad q_{\text{in}} = \rho\left(\dfrac{4}{3}\pi r^3\right)$

so $\quad E = \boxed{\dfrac{\rho r}{3\,\epsilon_0}}$.

For $a < r < b$ and $c < r$, $\quad q_{\text{in}} = Q$.

So $\quad E = \boxed{\dfrac{Q}{4\pi r^2\,\epsilon_0}}$.

For $b \le r \le c$, $\quad E = 0$, since $\boxed{E = 0}$ inside a conductor.

(b) Let $q_1 =$ induced charge on the inner surface of the hollow sphere. Since $E = 0$ inside the conductor, the total charge enclosed by a spherical surface of radius $b \le r \le c$ must be zero.

Therefore, $\quad q_1 + Q = 0 \quad$ and $\quad \sigma_1 = \dfrac{q_1}{4\pi b^2} = \boxed{\dfrac{-Q}{4\pi b^2}}$.

Let $q_2 =$ induced charge on the outside surface of the hollow sphere. Since the hollow sphere is uncharged, we require

$$q_1 + q_2 = 0 \quad \text{and} \quad \sigma_2 = \frac{q_1}{4\pi c^2} = \boxed{\frac{Q}{4\pi c^2}}.$$

**P24.58** $\oint \mathbf{E}\cdot d\mathbf{A} = E\left(4\pi r^2\right) = \dfrac{q_{\text{in}}}{\epsilon_0}$

(a) $\left(-3.60\times10^3\ \text{N/C}\right)4\pi(0.100\ \text{m})^2 = \dfrac{Q}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} \qquad (a < r < b)$

$Q = -4.00\times10^{-9}\ \text{C} = \boxed{-4.00\ \text{nC}}$

(b) We take $Q'$ to be the net charge on the hollow sphere. Outside $c$,

$\left(+2.00\times10^2\ \text{N/C}\right)4\pi(0.500\ \text{m})^2 = \dfrac{Q+Q'}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} \qquad (r > c)$

$Q + Q' = +5.56\times10^{-9}\ \text{C}$, so $Q' = +9.56\times10^{-9}\ \text{C} = \boxed{+9.56\ \text{nC}}$

(c) For $b < r < c$: $E = 0$ and $q_{\text{in}} = Q + Q_1 = 0$ where $Q_1$ is the total charge on the inner surface of the hollow sphere. Thus, $Q_1 = -Q = \boxed{+4.00\ \text{nC}}$.

Then, if $Q_2$ is the total charge on the outer surface of the hollow sphere,

$Q_2 = Q' - Q_1 = 9.56\ \text{nC} - 4.0\ \text{nC} = \boxed{+5.56\ \text{nC}}$.

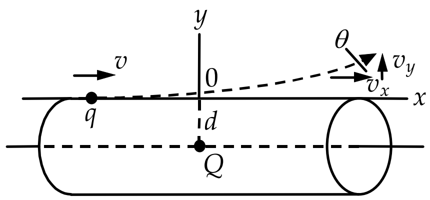

**FIG. P24.59**

***P24.59** The vertical velocity component of the moving charge increases according to

$$m\frac{dv_y}{dt}=F_y \qquad m\frac{dv_y}{dx}\frac{dx}{dt}=qE_y.$$

Now $\frac{dx}{dt}=v_x$ has the nearly constant value $v$. So

$$dv_y=\frac{q}{mv}E_y dx \qquad v_y=\int_0^{v_y}dv_y=\frac{q}{mv}\int_{-\infty}^{\infty}E_y dx.$$

The radially outward compnent of the electric field varies along the $x$ axis, but is described by

$$\int_{-\infty}^{\infty}E_y dA=\int_{-\infty}^{\infty}E_y(2\pi d)dx=\frac{Q}{\epsilon_0}.$$

So $\int_{-\infty}^{\infty}E_y dx=\frac{Q}{2\pi d\,\epsilon_0}$ and $v_y=\frac{qQ}{mv2\pi d\,\epsilon_0}$. The angle of deflection is described by

$$\tan\theta=\frac{v_y}{v}=\frac{qQ}{2\pi\,\epsilon_0\,dmv^2} \qquad \boxed{\theta=\tan^{-1}\frac{qQ}{2\pi\,\epsilon_0\,dmv^2}}.$$

**P24.60** First, consider the field at distance $r<R$ from the center of a uniform sphere of positive charge $(Q=+e)$ with radius $R$.

$$\left(4\pi r^2\right)E=\frac{q_{\text{in}}}{\epsilon_0}=\frac{\rho V}{\epsilon_0}=\left(\frac{+e}{\frac{4}{3}\pi R^3}\right)\frac{\frac{4}{3}\pi r^3}{\epsilon_0} \quad \text{so } E=\left(\frac{e}{4\pi\,\epsilon_0\,R^3}\right)r \text{ directed outward}$$

(a) The force exerted on a point charge $q=-e$ located at distance $r$ from the center is then

$$F=qE=-e\left(\frac{e}{4\pi\,\epsilon_0\,R^3}\right)r=-\left(\frac{e^2}{4\pi\,\epsilon_0\,R^3}\right)r=\boxed{-Kr}.$$

(b) $$K=\frac{e^2}{4\pi\,\epsilon_0\,R^3}=\boxed{\frac{k_e e^2}{R^3}}$$

(c) $$F_r=m_e a_r=-\left(\frac{k_e e^2}{R^3}\right)r\text{, so } a_r=-\left(\frac{k_e e^2}{m_e R^3}\right)r=-\omega^2 r$$

Thus, the motion is simple harmonic with frequency $f=\frac{\omega}{2\pi}=\boxed{\frac{1}{2\pi}\sqrt{\frac{k_e e^2}{m_e R^3}}}$.

(d) $$f=2.47\times10^{15}\text{ Hz}=\frac{1}{2\pi}\sqrt{\frac{\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\text{ C}\right)^2}{\left(9.11\times10^{-31}\text{ kg}\right)R^3}}$$

which yields $R^3=1.05\times10^{-30}\text{ m}^3$, or $R=1.02\times10^{-10}\text{ m}=\boxed{102\text{ pm}}$.

**P24.61** The field direction is radially outward perpendicular to the axis. The field strength depends on $r$ but not on the other cylindrical coordinates $\theta$ or $z$. Choose a Gaussian cylinder of radius $r$ and length $L$. If $r < a$,

$$\Phi_E = \frac{q_{\text{in}}}{\epsilon_0} \qquad \text{and} \qquad E(2\pi rL) = \frac{\lambda L}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi r\,\epsilon_0} \qquad \text{or} \qquad \boxed{\mathbf{E} = \frac{\lambda}{2\pi r\,\epsilon_0}\hat{\mathbf{r}} \qquad (r < a)}\,.$$

If $a < r < b$, $$E(2\pi rL) = \frac{\lambda L + \rho\pi\left(r^2 - a^2\right)L}{\epsilon_0}$$

$$\boxed{\mathbf{E} = \frac{\lambda + \rho\pi\left(r^2 - a^2\right)}{2\pi r\,\epsilon_0}\hat{\mathbf{r}} \qquad (a < r < b)}\,.$$

If $r > b$, $$E(2\pi rL) = \frac{\lambda L + \rho\pi\left(b^2 - a^2\right)L}{\epsilon_0}$$

$$\boxed{\mathbf{E} = \frac{\lambda + \rho\pi\left(b^2 - a^2\right)}{2\pi r\,\epsilon_0}\hat{\mathbf{r}} \qquad (r > b)}\,.$$

**P24.62** Consider the field due to a single sheet and let $E_+$ and $E_-$ represent the fields due to the positive and negative sheets. The field at any distance from each sheet has a magnitude given by Equation 24.8:

$$|E_+| = |E_-| = \frac{\sigma}{2\,\epsilon_0}.$$

(a) To the left of the positive sheet, $E_+$ is directed toward the left and $E_-$ toward the right and the net field over this region is $\mathbf{E} = \boxed{0}$.

(b) In the region between the sheets, $E_+$ and $E_-$ are both directed toward the right and the net field is

$$\mathbf{E} = \boxed{\frac{\sigma}{\epsilon_0} \text{ to the right}}\,.$$

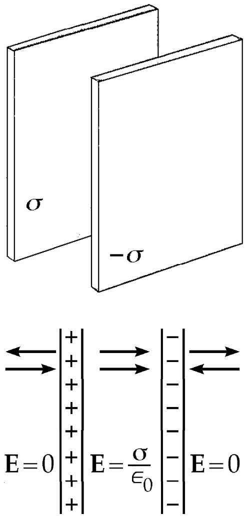

FIG. P24.62

(c) To the right of the negative sheet, $E_+$ and $E_-$ are again oppositely directed and $\mathbf{E} = \boxed{0}$.

**P24.63** The magnitude of the field due to the each sheet given by Equation 24.8 is

$$E=\frac{\sigma}{2\epsilon_0} \text{ directed perpendicular to the sheet.}$$

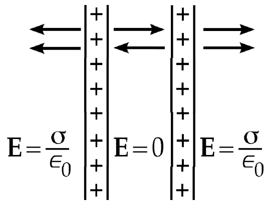

**FIG. P24.63**

(a) In the region to the left of the pair of sheets, both fields are directed toward the left and the net field is

$$\mathbf{E}=\boxed{\frac{\sigma}{\epsilon_0}\text{ to the left}}.$$

(b) In the region between the sheets, the fields due to the individual sheets are oppositely directed and the net field is

$$\mathbf{E}=\boxed{0}.$$

(c) In the region to the right of the pair of sheets, both are fields are directed toward the right and the net field is

$$\mathbf{E}=\boxed{\frac{\sigma}{\epsilon_0}\text{ to the right}}.$$

**P24.64** The resultant field within the cavity is the superposition of two fields, one $\mathbf{E}_+$ due to a uniform sphere of positive charge of radius $2a$, and the other $\mathbf{E}_-$ due to a sphere of negative charge of radius $a$ centered within the cavity.

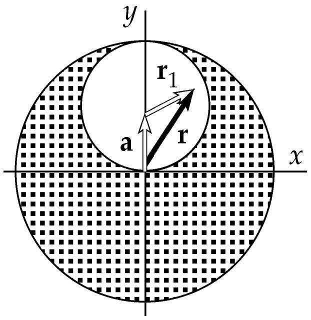

**FIG. P24.64**

$$\frac{4}{3}\left(\frac{\pi r^3\rho}{\epsilon_0}\right)=4\pi r^2E_+ \qquad \text{so} \qquad \mathbf{E}_+=\frac{\rho r}{3\epsilon_0}\hat{\mathbf{r}}=\frac{\rho\mathbf{r}}{3\epsilon_0}$$

$$- -\frac{4}{3}\left(\frac{\pi r_1^3\rho}{\epsilon_0}\right)=4\pi r_1^2E_- \qquad \text{so} \qquad \mathbf{E}_-=\frac{\rho r_1}{3\epsilon_0}(-\hat{\mathbf{r}}_1)=\frac{-\rho}{3\epsilon_0}\mathbf{r}_1.$$

Since $\mathbf{r}=\mathbf{a}+\mathbf{r}_1$, $\quad \mathbf{E}_-=\frac{-\rho(\mathbf{r}-\mathbf{a})}{3\epsilon_0}$

$$\mathbf{E}=\mathbf{E}_++\mathbf{E}_-=\frac{\rho\mathbf{r}}{3\epsilon_0}-\frac{\rho\mathbf{r}}{3\epsilon_0}+\frac{\rho\mathbf{a}}{3\epsilon_0}=\frac{\rho\mathbf{a}}{3\epsilon_0}=0\hat{\mathbf{i}}+\frac{\rho a}{3\epsilon_0}\hat{\mathbf{j}}.$$

Thus, $\boxed{E_x=0}$

and $\boxed{E_y=\frac{\rho a}{3\epsilon_0}}$ at all points within the cavity.

***P24.65** Consider the charge distribution to be an unbroken charged spherical shell with uniform charge density $\sigma$ and a circular disk with charge per area $-\sigma$. The total field is that due to the whole sphere, $\frac{Q}{4\pi\epsilon_0R^2}=\frac{4\pi R^2\sigma}{4\pi\varepsilon_0R^2}=\frac{\sigma}{\epsilon_0}$ outward plus the field of the disk $-\frac{\sigma}{2\epsilon_0}=\frac{\sigma}{2\epsilon_0}$ radially inward. The total field is $\frac{\sigma}{\epsilon_0}-\frac{\sigma}{2\epsilon_0}=\boxed{\frac{\sigma}{2\epsilon_0}\text{ outward}}$.

**P24.66** The electric field throughout the region is directed along $x$; therefore, $\mathbf{E}$ will be perpendicular to $dA$ over the four faces of the surface which are perpendicular to the $yz$ plane, and $E$ will be parallel to $dA$ over the two faces which are parallel to the $yz$ plane. Therefore,

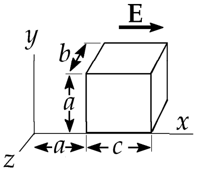

**FIG. P24.66**

$$\Phi_E = -\left(E_x\big|_{x=a}\right)A + \left(E_x\big|_{x=a+c}\right)A = -\left(3+2a^2\right)ab + \left(3+2(a+c)^2\right)ab = 2abc(2a+c).$$

Substituting the given values for $a$, $b$, and $c$, we find $\Phi_E = \boxed{0.269\ \text{N}\cdot\text{m}^2/\text{C}}$.

$Q = \epsilon_0 \Phi_E = 2.38\times 10^{-12}\ \text{C} = \boxed{2.38\ \text{pC}}$

**P24.67** $\oint \mathbf{E}\cdot d\mathbf{A} = E\left(4\pi r^2\right) = \dfrac{q_{\text{in}}}{\epsilon_0}$

(a) For $r > R$, $\quad q_{\text{in}} = \displaystyle\int_0^R Ar^2\left(4\pi r^2\right)dr = 4\pi\frac{AR^5}{5}$

and $\quad E = \boxed{\dfrac{AR^5}{5\epsilon_0 r^2}}$.

(b) For $r < R$, $\quad q_{\text{in}} = \displaystyle\int_0^r Ar^2\left(4\pi r^2\right)dr = \frac{4\pi Ar^5}{5}$

and $\quad E = \boxed{\dfrac{Ar^3}{5\epsilon_0}}$.

**P24.68** The total flux through a surface enclosing the charge $Q$ is $\dfrac{Q}{\epsilon_0}$. The flux through the disk is

$$\Phi_{\text{disk}} = \int \mathbf{E}\cdot d\mathbf{A}$$

where the integration covers the area of the disk. We must evaluate this integral and set it equal to $\dfrac{\frac{1}{4}Q}{\epsilon_0}$ to find how $b$ and $R$ are related. In the figure, take $d\mathbf{A}$ to be the area of an annular ring of radius $s$ and width $ds$. The flux through $d\mathbf{A}$ is

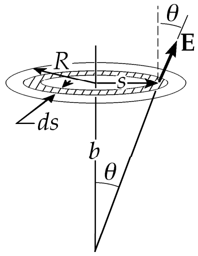

**FIG. P24.68**

$$\mathbf{E}\cdot d\mathbf{A} = EdA\cos\theta = E(2\pi s\,ds)\cos\theta.$$

The magnitude of the electric field has the same value at all points within the annular ring,

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{Q}{s^2+b^2} \quad \text{and} \quad \cos\theta = \frac{b}{r} = \frac{b}{\left(s^2+b^2\right)^{1/2}}.$$

Integrate from $s = 0$ to $s = R$ to get the flux through the entire disk.

$$\Phi_{E,\text{ disk}} = \frac{Qb}{2\epsilon_0}\int_0^R \frac{s\,ds}{\left(s^2+b^2\right)^{3/2}} = \frac{Qb}{2\epsilon_0}\left[-\left(s^2+b^2\right)^{1/2}\right]_0^R = \frac{Q}{2\epsilon_0}\left[1-\frac{b}{\left(R^2+b^2\right)^{1/2}}\right]$$

The flux through the disk equals $\dfrac{Q}{4\epsilon_0}$ provided that $\dfrac{b}{\left(R^2+b^2\right)^{1/2}} = \dfrac{1}{2}$.

This is satisfied if $\boxed{R = \sqrt{3}b}$.

**P24.69** $\oint \mathbf{E}\cdot d\mathbf{A} = \frac{q_{\text{in}}}{\epsilon_0} = \frac{1}{\epsilon_0}\int_0^r \frac{a}{r}4\pi r^2 dr$

$$E4\pi r^2 = \frac{4\pi a}{\epsilon_0}\int_0^r r dr = \frac{4\pi a}{\epsilon_0}\frac{r^2}{2}$$

$$\boxed{E = \frac{a}{2\epsilon_0}} = \text{constant magnitude}$$

(The direction is radially outward from center for positive *a*; radially inward for negative *a*.)

**P24.70** In this case the charge density is *not uniform*, and Gauss's law is written as $\oint \mathbf{E}\cdot d\mathbf{A} = \frac{1}{\epsilon_0}\int \rho dV$. We use a gaussian surface which is a cylinder of radius *r*, length $\ell$, and is coaxial with the charge distribution.

(a) When $r < R$, this becomes $E(2\pi r\ell) = \frac{\rho_0}{\epsilon_0}\int_0^r \left(a - \frac{r}{b}\right)dV$. The element of volume is a cylindrical shell of radius *r*, length $\ell$, and thickness *dr* so that $dV = 2\pi r\ell dr$.

$E(2\pi r\ell) = \left(\frac{2\pi r^2\ell\rho_0}{\epsilon_0}\right)\left(\frac{a}{2} - \frac{r}{3b}\right)$ so inside the cylinder, $E = \boxed{\frac{\rho_0 r}{2\epsilon_0}\left(a - \frac{2r}{3b}\right)}$.

(b) When $r > R$, Gauss's law becomes

$E(2\pi r\ell) = \frac{\rho_0}{\epsilon_0}\int_0^R \left(a - \frac{r}{b}\right)(2\pi r\ell dr)$ or outside the cylinder, $E = \boxed{\frac{\rho_0 R^2}{2\epsilon_0 r}\left(a - \frac{2R}{3b}\right)}$.

**P24.71** (a) Consider a cylindrical shaped gaussian surface perpendicular to the *yz* plane with one end in the *yz* plane and the other end containing the point *x*:

Use Gauss's law: $\oint \mathbf{E}\cdot d\mathbf{A} = \frac{q_{\text{in}}}{\epsilon_0}$

By symmetry, the electric field is zero in the *yz* plane and is perpendicular to $d\mathbf{A}$ over the wall of the gaussian cylinder. Therefore, the only contribution to the integral is over the end cap containing the point *x*:

$\oint \mathbf{E}\cdot d\mathbf{A} = \frac{q_{\text{in}}}{\epsilon_0}$ or $EA = \frac{\rho(Ax)}{\epsilon_0}$

so that at distance *x* from the mid-line of the slab, $\boxed{E = \frac{\rho x}{\epsilon_0}}$.

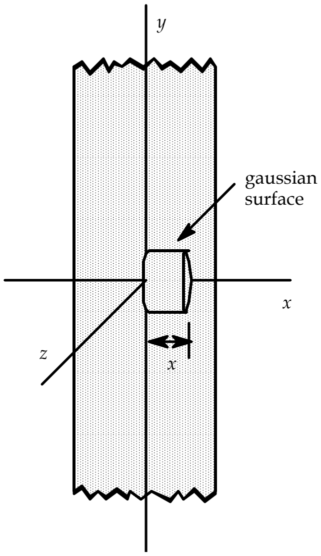

FIG. P24.71

(b) $a = \frac{F}{m_e} = \frac{(-e)E}{m_e} = -\left(\frac{\rho e}{m_e\epsilon_0}\right)x$

The acceleration of the electron is of the form $a = -\omega^2 x$ with $\omega = \sqrt{\frac{\rho e}{m_e\epsilon_0}}$.

Thus, the motion is simple harmonic with frequency $f = \frac{\omega}{2\pi} = \boxed{\frac{1}{2\pi}\sqrt{\frac{\rho e}{m_e\epsilon_0}}}$.

**P24.72** Consider the gaussian surface described in the solution to problem 71.

(a) For $x > \frac{d}{2}$, $dq = \rho\, dV = \rho A\, dx = CAx^2\, dx$

$$\int \mathbf{E}\cdot d\mathbf{A} = \frac{1}{\epsilon_0}\int dq$$

$$EA = \frac{CA}{\epsilon_0}\int_0^{d/2} x^2\, dx = \frac{1}{3}\left(\frac{CA}{\epsilon_0}\right)\left(\frac{d^3}{8}\right)$$

$$E = \frac{Cd^3}{24\epsilon_0} \quad \text{or} \quad \boxed{\mathbf{E} = \frac{Cd^3}{24\epsilon_0}\hat{\mathbf{i}} \text{ for } x > \frac{d}{2}; \qquad \mathbf{E} = -\frac{Cd^3}{24\epsilon_0}\hat{\mathbf{i}} \text{ for } x < -\frac{d}{2}}$$

(b) For $-\frac{d}{2} < x < \frac{d}{2}$ $\int \mathbf{E}\cdot d\mathbf{A} = \frac{1}{\epsilon_0}\int dq = \frac{CA}{\epsilon_0}\int_0^x x^2\, dx = \frac{CAx^3}{3\epsilon_0}$

$$\boxed{\mathbf{E} = \frac{Cx^3}{3\epsilon_0}\hat{\mathbf{i}} \text{ for } x > 0; \qquad \mathbf{E} = -\frac{Cx^3}{3\epsilon_0}\hat{\mathbf{i}} \text{ for } x < 0}$$

**P24.73** (a) A point mass $m$ creates a gravitational acceleration $\mathbf{g} = -\frac{Gm}{r^2}\hat{\mathbf{r}}$ at a distance $r$.

The flux of this field through a sphere is $\oint \mathbf{g}\cdot d\mathbf{A} = -\frac{Gm}{r^2}\left(4\pi r^2\right) = -4\pi Gm$.

Since the $r$ has divided out, we can visualize the field as unbroken field lines. The same flux would go through any other closed surface around the mass. If there are several or no masses inside a closed surface, each creates field to make its own contribution to the net flux according to

$$\boxed{\oint \mathbf{g}\cdot d\mathbf{A} = -4\pi Gm_{\text{in}}}.$$

(b) Take a spherical gaussian surface of radius $r$. The field is inward so $\oint \mathbf{g}\cdot d\mathbf{A} = g4\pi r^2 \cos 180° = -g4\pi r^2$

and $-4\pi Gm_{\text{in}} = -4\pi G\frac{4}{3}\pi r^3\rho.$

Then, $-g4\pi r^2 = -4\pi G\frac{4}{3}\pi r^3\rho$ and $g = \frac{4}{3}\pi r\rho G$.

Or, since $\rho = \frac{M_E}{\frac{4}{3}\pi R_E^3}$, $g = \frac{M_E Gr}{R_E^3}$ or $\boxed{\mathbf{g} = \frac{M_E Gr}{R_E^3} \text{ inward}}$.

## ANSWERS TO EVEN PROBLEMS

**P24.2** $355\ \text{kN}\cdot\text{m}^2/\text{C}$

**P24.4** (a) $-2.34\ \text{kN}\cdot\text{m}^2/\text{C}$; (b) $+2.34\ \text{kN}\cdot\text{m}^2/\text{C}$; (c) 0

**P24.6** $\frac{q}{\epsilon_0}$

**P24.8** $ERh$

**P24.10** (a) $-55.6\ \text{nC}$; (b) The negative charge has a spherically symmetric distribution.

**P24.12** (a) $\frac{q}{2\epsilon_0}$; (b) $\frac{q}{2\epsilon_0}$; (c) Plane and square both subtend a solid angle of a hemisphere at the charge.

**P24.14** (a) $1.36\ \text{MN}\cdot\text{m}^2/\text{C}$; (b) $678\ \text{kN}\cdot\text{m}^2/\text{C}$; (c) No; see the solution.

**P24.16** 1.77 $\text{pC/m}^3$ positive

**P24.18** $\dfrac{Q-6|q|}{6\epsilon_0}$

**P24.20** 28.2 $\text{N}\cdot\text{m}^2/\text{C}$

**P24.22** $\dfrac{-q}{24\epsilon_0}$

**P24.24** (a) 0; (b) 365 kN/C; (c) 1.46 MN/C; (d) 649 kN/C

**P24.26** (a) 913 nC; (b) 0

**P24.28** 4.86 GN/C away from the wall. It is constant close to the wall

**P24.30** 76.4 kN/C radially inward

**P24.32** 3.50 kN

**P24.34** (a) $\dfrac{Qr^3}{\epsilon_0 a^3}$; (b) $\dfrac{Q}{\epsilon_0}$; (c) see the solution

**P24.36** 713 nC; (b) 5.70 $\mu$C

**P24.38** (a) 16.2 MN/C toward the filament; (b) 8.09 MN/C toward the filament; (c) 1.62 MN/C toward the filament

**P24.40** −1.15 $\text{nC/m}^2$

**P24.42** (a) 0; (b) 12.4 kN/C radially outward; (c) 639 N/C radially outward; (d) Nothing would change.

**P24.44** (a) 0; (b) 79.9 MN/C radially outward; (c) 0; (d) 7.34 MN/C radially outward

**P24.46** (a) 248 $\text{nC/m}^2$; (b) 496 $\text{nC/m}^2$

**P24.48** (a) 2.56 MN/C radially inward; (b) 0

**P24.50** (a) $\dfrac{-q}{4\pi a^2}$; (b) $\dfrac{Q+q}{4\pi b^2}$

**P24.52** see the solution

**P24.54** $\dfrac{chw^2}{2}$

**P24.56** see the solution

**P24.58** (a) −4.00 nC; (b) +9.56 nC; (c) +4.00 nC and +5.56 nC

**P24.60** (a, b) see the solution; (c) $\dfrac{1}{2\pi}\sqrt{\dfrac{k_e e^2}{m_e R^3}}$; (d) 102 pm

**P24.62** (a) 0; (b) $\dfrac{\sigma}{\epsilon_0}$ to the right; (c) 0

**P24.64** see the solution

**P24.66** 0.269 $\text{N}\cdot\text{m}^2/\text{C}$; 2.38 pC

**P24.68** see the solution

**P24.70** (a) $\dfrac{\rho_0 r}{2\epsilon_0}\left(a-\dfrac{2r}{3b}\right)$; (b) $\dfrac{\rho_0 R^2}{2\epsilon_0 r}\left(a-\dfrac{2R}{3b}\right)$

**P24.72** (a) $\mathbf{E}=\dfrac{Cd^3}{24\epsilon_0}\hat{\mathbf{i}}$ for $x>\dfrac{d}{2}$;

$\mathbf{E}=-\dfrac{Cd^3}{24\epsilon_0}\hat{\mathbf{i}}$ for $x<-\dfrac{d}{2}$;

(b) $\mathbf{E}=\dfrac{Cx^3}{3\epsilon_0}\hat{\mathbf{i}}$ for $x>0$; $\mathbf{E}=-\dfrac{Cx^3}{3\epsilon_0}\hat{\mathbf{i}}$ for $x<0$

# 25

# Electric Potential

**CHAPTER OUTLINE**

25.1 Potential Difference and Electric Potential
25.2 Potential Difference in a Uniform Electric Field
25.3 Electric Potential and Potential Energy Due to Point Charges
25.4 Obtaining the Value of the Electric Field from the Electric Potential
25.5 Electric Potential Due to Continuous Charge Distributions
25.6 Electric Potential Due to a Charged Conductor
25.7 The Milliken Oil Drop Experiment
25.8 Application of Electrostatistics

## ANSWERS TO QUESTIONS

**Q25.1** When one object *B* with electric charge is immersed in the electric field of another charge or charges *A*, the system possesses electric potential energy. The energy can be measured by seeing how much work the field does on the charge *B* as it moves to a reference location. We choose not to visualize *A*'s effect on *B* as an action-at-a-distance, but as the result of a two-step process: Charge *A* creates electric potential throughout the surrounding space. Then the potential acts on *B* to inject the system with energy.

**Q25.2** The potential energy increases. When an outside agent makes it move in the direction of the field, the charge moves to a region of lower electric potential. Then the product of its negative charge with a lower number of volts gives a higher number of joules. Keep in mind that a negative charge feels an electric force in the *opposite* direction to the field, while the potential is the work done on the charge to move it in a field per unit charge.

**Q25.3** To move like charges together from an infinite separation, at which the potential energy of the system of two charges is zero, requires *work* to be done on the system by an outside agent. Hence energy is stored, and potential energy is positive. As charges with opposite signs move together from an infinite separation, energy is released, and the potential energy of the set of charges becomes negative.

**Q25.4** The charge can be moved along any path parallel to the $y$-$z$ plane, namely perpendicular to the field.

**Q25.5** The electric field always points in the direction of the greatest change in electric potential. This is implied by the relationships $E_x = -\frac{\partial V}{\partial x}$, $E_y = -\frac{\partial V}{\partial y}$ and $E_z = -\frac{\partial V}{\partial z}$.

**Q25.6** (a) The equipotential surfaces are nesting coaxial cylinders around an infinite line of charge.

(b) The equipotential surfaces are nesting concentric spheres around a uniformly charged sphere.

**Q25.7** If there were a potential difference between two points on the conductor, the free electrons in the conductor would move until the potential difference disappears.

**Q25.8** No. The uniformly charged sphere, whether hollow or solid metal, is an equipotential volume. Since there is no electric field, this means that there is no *change* in electrical potential. The potential at every point inside is the same as the value of the potential at the surface.

**Q25.9** Infinitely far away from a line of charge, the line will not look like a point. In fact, without any distinguishing features, it is not possible to tell the distance from an infinitely long line of charge. Another way of stating the answer: The potential would diverge to infinity at any finite distance, if it were zero infinitely far away.

**Q25.10** The smaller sphere will. In the solution to the example referred to, equation 1 states that each will have the same ratio of charge to radius, $\frac{q}{r}$. In this case, the charge density is a surface charge density, $\frac{q}{4\pi r^2}$, so the smaller-radius sphere will have the greater charge density.

**Q25.11** The main factor is the radius of the dome. One often overlooked aspect is also the humidity of the air—drier air has a larger dielectric breakdown strength, resulting in a higher attainable electric potential. If other grounded objects are nearby, the maximum potential might be reduced.

**Q25.12** The intense—often oscillating—electric fields around high voltage lines is large enough to ionize the air surrounding the cables. When the molecules recapture their electrons, they release that energy in the form of light.

**Q25.13** A sharp point in a charged conductor would imply a large electric field in that region. An electric discharge could most easily take place at that sharp point.

**Q25.14** Use a conductive box to shield the equipment. Any stray electric field will cause charges on the outer surface of the conductor to rearrange and cancel the stray field inside the volume it encloses.

**Q25.15** No charge stays on the inner sphere in equilibrium. If there were any, it would create an electric field in the wire to push more charge to the outer sphere. All of the charge is on the outer sphere. Therefore, zero charge is on the inner sphere and 10.0 $\mu$C is on the outer sphere.

**Q25.16** The grounding wire can be touched equally well to any point on the sphere. Electrons will drain away into the ground and the sphere will be left positively charged. The ground, wire, and sphere are all conducting. They together form an equipotential volume at zero volts during the contact. However close the grounding wire is to the negative charge, electrons have no difficulty in moving within the metal through the grounding wire to ground. The ground can act as an infinite source or sink of electrons. In this case, it is an electron sink.

## SOLUTIONS TO PROBLEMS

### Section 25.1 Potential Difference and Electric Potential

**P25.1** $\Delta V = -14.0\text{ V}$ and $Q = -N_A e = -\left(6.02\times10^{23}\right)\left(1.60\times10^{-19}\right) = -9.63\times10^{4}\text{ C}$

$\Delta V = \frac{W}{Q}$, so $W = Q\Delta V = \left(-9.63\times10^{4}\text{ C}\right)\left(-14.0\text{ J/C}\right) = \boxed{1.35\text{ MJ}}$

**P25.2** $\Delta K = q|\Delta V|$ $\qquad 7.37 \times 10^{-17} = q(115)$

$\boxed{q = 6.41 \times 10^{-19}\ \text{C}}$

**P25.3** (a) Energy of the proton-field system is conserved as the proton moves from high to low potential, which can be defined for this problem as moving from 120 V down to 0 V.

$$K_i + U_i + \Delta E_{\text{mech}} = K_f + U_f \qquad 0 + qV + 0 = \frac{1}{2}mv_p^2 + 0$$

$$\left(1.60 \times 10^{-19}\ \text{C}\right)(120\ \text{V})\left(\frac{1\ \text{J}}{1\ \text{V}\cdot\text{C}}\right) = \frac{1}{2}\left(1.67 \times 10^{-27}\ \text{kg}\right)v_p^2$$

$$v_p = \boxed{1.52 \times 10^5\ \text{m/s}}$$

(b) The electron will gain speed in moving the other way,

from $V_i = 0$ to $V_f = 120$ V : $\qquad K_i + U_i + \Delta E_{\text{mech}} = K_f + U_f$

$$0 + 0 + 0 = \frac{1}{2}mv_e^2 + qV$$

$$0 = \frac{1}{2}\left(9.11 \times 10^{-31}\ \text{kg}\right)v_e^2 + \left(-1.60 \times 10^{-19}\ \text{C}\right)(120\ \text{J/C})$$

$$v_e = \boxed{6.49 \times 10^6\ \text{m/s}}$$

**P25.4** $W = \Delta K = -q\Delta V$

$$0 - \frac{1}{2}\left(9.11 \times 10^{-31}\ \text{kg}\right)\left(4.20 \times 10^5\ \text{m/s}\right)^2 = -\left(-1.60 \times 10^{-19}\ \text{C}\right)\Delta V$$

From which, $\Delta V = \boxed{-0.502\ \text{V}}$.

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## Section 25.2 **Potential Difference in a Uniform Electric Field**

**P25.5** (a) We follow the path from (0, 0) to (20.0 cm, 0) to (20.0 cm, 50.0 cm).

$\Delta U = -$(work done)

$\Delta U = -$(work from origin to (20.0 cm, 0)) – (work from (20.0 cm, 0) to (20.0 cm, 50.0 cm))

Note that the last term is equal to 0 because the force is perpendicular to the displacement.

$$\Delta U = -(qE_x)\Delta x = -\left(12.0 \times 10^{-6}\ \text{C}\right)(250\ \text{V/m})(0.200\ \text{m}) = \boxed{-6.00 \times 10^{-4}\ \text{J}}$$

(b) $$\Delta V = \frac{\Delta U}{q} = -\frac{6.00 \times 10^{-4}\ \text{J}}{12.0 \times 10^{-6}\ \text{C}} = -50.0\ \text{J/C} = \boxed{-50.0\ \text{V}}$$

**P25.6** $$E = \frac{|\Delta V|}{d} = \frac{25.0 \times 10^3\ \text{J/C}}{1.50 \times 10^{-2}\ \text{m}} = 1.67 \times 10^6\ \text{N/C} = \boxed{1.67\ \text{MN/C}}$$

**P25.7** $\Delta U=-\frac{1}{2}m\left(v_f^2-v_i^2\right)=-\frac{1}{2}\left(9.11\times10^{-31}\text{ kg}\right)\left[\left(1.40\times10^5\text{ m/s}\right)^2-\left(3.70\times10^6\text{ m/s}\right)^2\right]=6.23\times10^{-18}\text{ J}$

$\Delta U=q\Delta V$: $\quad +6.23\times10^{-18}=\left(-1.60\times10^{-19}\right)\Delta V$

$\Delta V=\boxed{-38.9\text{ V. The origin is at highest potential.}}$

**P25.8** (a) $|\Delta V|=Ed=\left(5.90\times10^3\text{ V/m}\right)(0.010\,0\text{ m})=\boxed{59.0\text{ V}}$

(b) $\frac{1}{2}mv_f^2=|q\Delta V|$: $\quad \frac{1}{2}\left(9.11\times10^{-31}\right)v_f^2=\left(1.60\times10^{-19}\right)(59.0)$

$v_f=\boxed{4.55\times10^6\text{ m/s}}$

**P25.9** $V_B-V_A=-\int_A^B\mathbf{E}\cdot d\mathbf{s}=-\int_A^C\mathbf{E}\cdot d\mathbf{s}-\int_C^B\mathbf{E}\cdot d\mathbf{s}$

$V_B-V_A=(-E\cos180°)\int_{-0.300}^{0.500}dy-(E\cos90.0°)\int_{-0.200}^{0.400}dx$

$V_B-V_A=(325)(0.800)=\boxed{+260\text{ V}}$

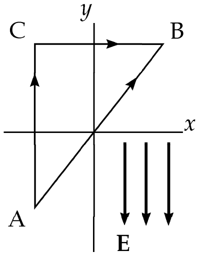

**FIG. P25.9**

***P25.10** Assume the opposite. Then at some point $A$ on some equipotential surface the electric field has a nonzero component $E_p$ in the plane of the surface. Let a test charge start from point $A$ and move some distance on the surface in the direction of the field component. Then $\Delta V=-\int_A^B\mathbf{E}\cdot d\mathbf{s}$ is nonzero. The electric potential charges across the surface and it is not an equipotential surface. The contradiction shows that our assumption is false, that $E_p=0$, and that the field is perpendicular to the equipotential surface.

**P25.11** (a) Arbitrarily choose $V=0$ at 0. Then at other points

$V=-Ex$ and $U_e=QV=-QEx$.

Between the endpoints of the motion,

$(K+U_s+U_e)_i=(K+U_s+U_e)_f$

$0+0+0=0+\frac{1}{2}kx_{\max}^2-QEx_{\max}$ so $x_{\max}=\boxed{\frac{2QE}{k}}$.

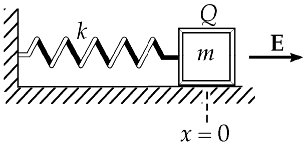

**FIG. P25.11**

(b) At equilibrium,

$\sum F_x=-F_s+F_e=0$ or $kx=QE$.

So the equilibrium position is at $x=\boxed{\frac{QE}{k}}$.

*continued on next page*

(c) The block's equation of motion is $\sum F_x = -kx + QE = m\frac{d^2x}{dt^2}$.

Let $x' = x - \frac{QE}{k}$, or $x = x' + \frac{QE}{k}$,

so the equation of motion becomes:

$$-k\left(x' + \frac{QE}{k}\right) + QE = m\frac{d^2(x + QE/k)}{dt^2}, \text{ or } \frac{d^2x'}{dt^2} = -\left(\frac{k}{m}\right)x'.$$

This is the equation for simple harmonic motion $a_{x'} = -\omega^2 x'$

with $\omega = \sqrt{\frac{k}{m}}$.

The period of the motion is then $T = \frac{2\pi}{\omega} = \boxed{2\pi\sqrt{\frac{m}{k}}}$.

(d) $(K + U_s + U_e)_i + \Delta E_{\text{mech}} = (K + U_s + U_e)_f$

$$0 + 0 + 0 - \mu_k mgx_{\max} = 0 + \frac{1}{2}kx_{\max}^2 - QEx_{\max}$$

$$x_{\max} = \boxed{\frac{2(QE - \mu_k mg)}{k}}$$

**P25.12** For the entire motion, $y_f - y_i = v_{yi}t + \frac{1}{2}a_y t^2$

$0 - 0 = v_i t + \frac{1}{2}a_y t^2$ so $a_y = -\frac{2v_i}{t}$

$\sum F_y = ma_y$: $-mg - qE = -\frac{2mv_i}{t}$

$E = \frac{m}{q}\left(\frac{2v_i}{t} - g\right)$ and $\mathbf{E} = -\frac{m}{q}\left(\frac{2v_i}{t} - g\right)\hat{\mathbf{j}}$.

For the upward flight: $v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$

$0 = v_i^2 + 2\left(-\frac{2v_i}{t}\right)(y_{\max} - 0)$ and $y_{\max} = \frac{1}{4}v_i t$

$$\Delta V = -\int_0^{y_{\max}} \mathbf{E}\cdot d\mathbf{y} = +\frac{m}{q}\left(\frac{2v_i}{t} - g\right)y\bigg|_0^{y_{\max}} = \frac{m}{q}\left(\frac{2v_i}{t} - g\right)\left(\frac{1}{4}v_i t\right)$$

$$\Delta V = \frac{2.00\text{ kg}}{5.00\times10^{-6}\text{ C}}\left(\frac{2(20.1\text{ m/s})}{4.10\text{ s}} - 9.80\text{ m/s}^2\right)\left[\frac{1}{4}(20.1\text{ m/s})(4.10\text{ s})\right] = \boxed{40.2\text{ kV}}$$

**P25.13** Arbitrarily take $V = 0$ at the initial point. Then at distance *d* downfield, where *L* is the rod length, $V = -Ed$ and $U_e = -\lambda LEd$.

(a) $(K + U)_i = (K + U)_f$

$$0 + 0 = \frac{1}{2}\mu Lv^2 - \lambda LEd$$

$$v = \sqrt{\frac{2\lambda Ed}{\mu}} = \sqrt{\frac{2(40.0\times10^{-6}\text{ C/m})(100\text{ N/C})(2.00\text{ m})}{(0.100\text{ kg/m})}} = \boxed{0.400\text{ m/s}}$$

(b) $\boxed{\text{The same.}}$

**P25.14** Arbitrarily take $V=0$ at point $P$. Then (from Equation 25.8) the potential at the original position of the charge is $-\mathbf{E}\cdot\mathbf{s}=-EL\cos\theta$. At the final point $a$, $V=-EL$. Suppose the table is frictionless: $(K+U)_i=(K+U)_f$

$$0-qEL\cos\theta=\frac{1}{2}mv^2-qEL$$

$$v=\sqrt{\frac{2qEL(1-\cos\theta)}{m}}=\sqrt{\frac{2\left(2.00\times10^{-6}\ \text{C}\right)(300\ \text{N/C})(1.50\ \text{m})(1-\cos 60.0°)}{0.010\,0\ \text{kg}}}=\boxed{0.300\ \text{m/s}}$$

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## Section 25.3 Electric Potential and Potential Energy Due to Point Charges

**P25.15** (a) The potential at 1.00 cm is $V_1=k_e\frac{q}{r}=\frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)}{1.00\times10^{-2}\ \text{m}}=\boxed{1.44\times10^{-7}\ \text{V}}$.

(b) The potential at 2.00 cm is $V_2=k_e\frac{q}{r}=\frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)}{2.00\times10^{-2}\ \text{m}}=0.719\times10^{-7}\ \text{V}$.

Thus, the difference in potential between the two points is $\Delta V=V_2-V_1=\boxed{-7.19\times10^{-8}\ \text{V}}$.

(c) The approach is the same as above except the charge is $-1.60\times10^{-19}\ \text{C}$. This changes the sign of each answer, with its magnitude remaining the same.

That is, the potential at 1.00 cm is $\boxed{-1.44\times10^{-7}\ \text{V}}$.

The potential at 2.00 cm is $-0.719\times10^{-7}\ \text{V}$, so $\Delta V=V_2-V_1=\boxed{7.19\times10^{-8}\ \text{V}}$.

**P25.16** (a) Since the charges are equal and placed symmetrically, $\boxed{F=0}$.

(b) Since $F=qE=0$, $\boxed{E=0}$.

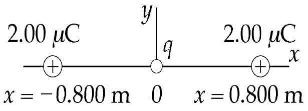

**FIG. P25.16**

(c) $V=2k_e\frac{q}{r}=2\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{2.00\times10^{-6}\ \text{C}}{0.800\ \text{m}}\right)$

$V=4.50\times10^4\ \text{V}=\boxed{45.0\ \text{kV}}$

**P25.17** (a) $E=\frac{|Q|}{4\pi\in_0 r^2}$

$V=\frac{Q}{4\pi\in_0 r}$

$r=\frac{|V|}{|E|}=\frac{3\,000\ \text{V}}{500\ \text{V/m}}=\boxed{6.00\ \text{m}}$

(b) $V=-3\,000\ \text{V}=\frac{Q}{4\pi\in_0(6.00\ \text{m})}$

$Q=\frac{-3\,000\ \text{V}}{\left(8.99\times10^9\ \text{V}\cdot\text{m/C}\right)}(6.00\ \text{m})=\boxed{-2.00\ \mu\text{C}}$

**P25.18** (a) $E_x = \frac{k_e q_1}{x^2} + \frac{k_e q_2}{(x-2.00)^2} = 0$ becomes $E_x = k_e\left(\frac{+q}{x^2} + \frac{-2q}{(x-2.00)^2}\right) = 0$.

Dividing by $k_e$, $2qx^2 = q(x-2.00)^2$ $x^2 + 4.00x - 4.00 = 0$.

Therefore $E = 0$ when $x = \frac{-4.00 \pm \sqrt{16.0+16.0}}{2} = \boxed{-4.83 \text{ m}}$.

(Note that the positive root does not correspond to a physically valid situation.)

(b) $V = \frac{k_e q_1}{x} + \frac{k_e q_2}{2.00 - x} = 0$ or $V = k_e\left(\frac{+q}{x} - \frac{2q}{2.00-x}\right) = 0$.

Again solving for *x*, $2qx = q(2.00 - x)$.

For $0 \le x \le 2.00$ $V = 0$ when $x = \boxed{0.667 \text{ m}}$

and $\frac{q}{|x|} = \frac{-2q}{|2-x|}$. For $x < 0$ $x = \boxed{-2.00 \text{ m}}$.

**P25.19**

$$V = \sum_i k\frac{q_i}{r_i}$$

$$V = \left(8.99\times10^9\right)\left(7.00\times10^{-6}\right)\left[\frac{-1}{0.010\,0} - \frac{1}{0.010\,0} + \frac{1}{0.038\,7}\right]$$

$$V = \boxed{-1.10\times10^7 \text{ V} = -11.0 \text{ MV}}$$

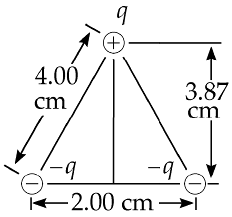

**FIG. P25.19**

**P25.20** (a) $U = \frac{qQ}{4\pi \in_0 r} = \frac{\left(5.00\times10^{-9} \text{ C}\right)\left(-3.00\times10^{-9} \text{ C}\right)\left(8.99\times10^9 \text{ V}\cdot\text{m/C}\right)}{(0.350 \text{ m})} = \boxed{-3.86\times10^{-7} \text{ J}}$

The minus sign means it takes $3.86\times10^{-7}$ J to pull the two charges apart from 35 cm to a much larger separation.

(b)

$$V = \frac{Q_1}{4\pi \in_0 r_1} + \frac{Q_2}{4\pi \in_0 r_2}$$

$$= \frac{\left(5.00\times10^{-9} \text{ C}\right)\left(8.99\times10^9 \text{ V}\cdot\text{m/C}\right)}{0.175 \text{ m}} + \frac{\left(-3.00\times10^{-9} \text{ C}\right)\left(8.99\times10^9 \text{ V}\cdot\text{m/C}\right)}{0.175 \text{ m}}$$

$$V = \boxed{103 \text{ V}}$$

**P25.21** $$U_e = q_4V_1 + q_4V_2 + q_4V_3 = q_4\left(\frac{1}{4\pi \in_0}\right)\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3}\right)$$

$$U_e = \left(10.0\times10^{-6}\text{ C}\right)^2\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{1}{0.600\text{ m}} + \frac{1}{0.150\text{ m}} + \frac{1}{\sqrt{(0.600\text{ m})^2 + (0.150\text{ m})^2}}\right)$$

$$U_e = \boxed{8.95\text{ J}}$$

**P25.22** (a) $$V = \frac{k_eq_1}{r_1} + \frac{k_eq_2}{r_2} = 2\left(\frac{k_eq}{r}\right)$$

$$V = 2\left(\frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(2.00\times10^{-6}\text{ C}\right)}{\sqrt{(1.00\text{ m})^2 + (0.500\text{ m})^2}}\right)$$

$$V = 3.22\times10^4\text{ V} = \boxed{32.2\text{ kV}}$$

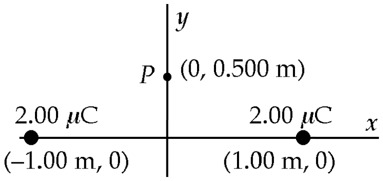

**FIG. P25.22**

(b) $$U = qV = \left(-3.00\times10^{-6}\text{ C}\right)\left(3.22\times10^4\ \text{J/C}\right) = \boxed{-9.65\times10^{-2}\text{ J}}$$

**P25.23** $$U = U_1 + U_2 + U_3 + U_4$$

$$U = 0 + U_{12} + \left(U_{13} + U_{23}\right) + \left(U_{14} + U_{24} + U_{34}\right)$$

$$U = 0 + \frac{k_eQ^2}{s} + \frac{k_eQ^2}{s}\left(\frac{1}{\sqrt{2}} + 1\right) + \frac{k_eQ^2}{s}\left(1 + \frac{1}{\sqrt{2}} + 1\right)$$

$$U = \frac{k_eQ^2}{s}\left(4 + \frac{2}{\sqrt{2}}\right) = \boxed{5.41\frac{k_eQ^2}{s}}$$

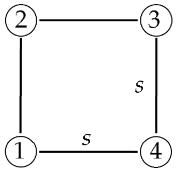

**FIG. P25.23**

An alternate way to get the term $\left(4 + \frac{2}{\sqrt{2}}\right)$ is to recognize that there are 4 side pairs and 2 face diagonal pairs.

**P25.24** Each charge creates equal potential at the center. The total potential is:

$$V = 5\left[\frac{k_e(-q)}{R}\right] = \boxed{-\frac{5k_eq}{R}}.$$

**P25.25** (a) Each charge separately creates positive potential everywhere. The total potential produced by the three charges together is then the sum of three positive terms. There is $\boxed{\text{no point}}$ located at a finite distance from the charges, at which this total potential is zero.

(b) $$V = \frac{k_eq}{a} + \frac{k_eq}{a} = \boxed{\frac{2k_eq}{a}}$$

**P25.26** Consider the two spheres as a system.

(a) Conservation of momentum: $0 = m_1 v_1 \hat{\mathbf{i}} + m_2 v_2 \left(-\hat{\mathbf{i}}\right)$ or $v_2 = \frac{m_1 v_1}{m_2}$

By conservation of energy, $0 = \frac{k_e(-q_1)q_2}{d} = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + \frac{k_e(-q_1)q_2}{r_1 + r_2}$

and $\frac{k_e q_1 q_2}{r_1 + r_2} - \frac{k_e q_1 q_2}{d} = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}\frac{m_1^2 v_1^2}{m_2}$

$$v_1 = \sqrt{\frac{2m_2 k_e q_1 q_2}{m_1(m_1 + m_2)}\left(\frac{1}{r_1 + r_2} - \frac{1}{d}\right)}$$

$$v_1 = \sqrt{\frac{2(0.700\text{ kg})\left(8.99 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(2 \times 10^{-6}\text{ C}\right)\left(3 \times 10^{-6}\text{ C}\right)}{(0.100\text{ kg})(0.800\text{ kg})}\left(\frac{1}{8 \times 10^{-3}\text{ m}} - \frac{1}{1.00\text{ m}}\right)}$$

$$= \boxed{10.8\text{ m/s}}$$

$$v_2 = \frac{m_1 v_1}{m_2} = \frac{0.100\text{ kg}(10.8\text{ m/s})}{0.700\text{ kg}} = \boxed{1.55\text{ m/s}}$$

(b) If the spheres are metal, electrons will move around on them with negligible energy loss to place the centers of excess charge on the insides of the spheres. Then just before they touch, the effective distance between charges will be less than $r_1 + r_2$ and the spheres will really be moving $\boxed{\text{faster than calculated in (a)}}$.

**P25.27** Consider the two spheres as a system.

(a) Conservation of momentum: $0 = m_1 v_1 \hat{\mathbf{i}} + m_2 v_2 \left(-\hat{\mathbf{i}}\right)$

or $v_2 = \frac{m_1 v_1}{m_2}$.

By conservation of energy, $0 = \frac{k_e(-q_1)q_2}{d} = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + \frac{k_e(-q_1)q_2}{r_1 + r_2}$

and $\frac{k_e q_1 q_2}{r_1 + r_2} - \frac{k_e q_1 q_2}{d} = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}\frac{m_1^2 v_1^2}{m_2}$.

$$v_1 = \boxed{\sqrt{\frac{2m_2 k_e q_1 q_2}{m_1(m_1 + m_2)}\left(\frac{1}{r_1 + r_2} - \frac{1}{d}\right)}}$$

$$v_2 = \left(\frac{m_1}{m_2}\right)v_1 = \boxed{\sqrt{\frac{2m_1 k_e q_1 q_2}{m_2(m_1 + m_2)}\left(\frac{1}{r_1 + r_2} - \frac{1}{d}\right)}}$$

(b) If the spheres are metal, electrons will move around on them with negligible energy loss to place the centers of excess charge on the insides of the spheres. Then just before they touch, the effective distance between charges will be less than $r_1 + r_2$ and the spheres will really be moving $\boxed{\text{faster than calculated in (a)}}$.

***P25.28** (a) In an empty universe, the 20-nC charge can be placed at its location with no energy investment. At a distance of 4 cm, it creates a potential

$$V_1 = \frac{k_e q_1}{r} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(20\times10^{-9}\ \text{C}\right)}{0.04\ \text{m}} = 4.50\ \text{kV}.$$

To place the 10-nC charge there we must put in energy

$$U_{12} = q_2 V_1 = \left(10\times10^{-9}\ \text{C}\right)\left(4.5\times10^3\ \text{V}\right) = 4.50\times10^{-5}\ \text{J}.$$

Next, to bring up the –20-nC charge requires energy

$$U_{23} + U_{13} = q_3 V_2 + q_3 V_1 = q_3\left(V_2 + V_1\right)$$

$$= -20\times10^{-9}\ \text{C}\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{10\times10^{-9}\ \text{C}}{0.04\ \text{m}} + \frac{20\times10^{-9}\ \text{C}}{0.08\ \text{m}}\right)$$

$$= -4.50\times10^{-5}\ \text{J} - 4.50\times10^{-5}\ \text{J}$$

The total energy of the three charges is

$$U_{12} + U_{23} + U_{13} = \boxed{-4.50\times10^{-5}\ \text{J}}.$$

(b) The three fixed charges create this potential at the location where the fourth is released:

$$V = V_1 + V_2 + V_3 = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{20\times10^{-9}}{\sqrt{0.04^2 + 0.03^2}} + \frac{10\times10^{-9}}{0.03} - \frac{20\times10^{-9}}{0.05}\right)\ \text{C/m}$$

$$V = 3.00\times10^3\ \text{V}$$

Energy of the system of four charged objects is conserved as the fourth charge flies away:

$$\left(\frac{1}{2}mv^2 + qV\right)_i = \left(\frac{1}{2}mv^2 + qV\right)_f$$

$$0 + \left(40\times10^{-9}\ \text{C}\right)\left(3.00\times10^3\ \text{V}\right) = \frac{1}{2}\left(2.00\times10^{-13}\ \text{kg}\right)v^2 + 0$$

$$v = \sqrt{\frac{2\left(1.20\times10^{-4}\ \text{J}\right)}{2\times10^{-13}\ \text{kg}}} = \boxed{3.46\times10^4\ \text{m/s}}$$

***P25.29** The original electrical potential energy is

$$U_e = qV = q\frac{k_e q}{d}.$$

In the final configuration we have mechanical equilibrium. The spring and electrostatic forces on each charge are $-k(2d) + q\dfrac{k_e q}{(3d)^2} = 0$. Then $k = \dfrac{k_e q^2}{18d^3}$. In the final configuration the total potential energy is $\dfrac{1}{2}kx^2 + qV = \dfrac{1}{2}\dfrac{k_e q^2}{18d^3}(2d)^2 + q\dfrac{k_e q}{3d} = \dfrac{4}{9}\dfrac{k_e q^2}{d}$. The missing energy must have become internal energy, as the system is isolated: $\dfrac{k_e q^2}{d} = \dfrac{4k_e q^2}{9d} + \Delta E_{\text{int}}$

$$\boxed{\Delta E_{\text{int}} = \frac{5}{9}\frac{k_e q^2}{d}}.$$

**P25.30** (a) $$V(x)=\frac{k_eQ_1}{r_1}+\frac{k_eQ_2}{r_2}=\frac{k_e(+Q)}{\sqrt{x^2+a^2}}+\frac{k_e(+Q)}{\sqrt{x^2+(-a)^2}}$$

$$V(x)=\frac{2k_eQ}{\sqrt{x^2+a^2}}=\frac{k_eQ}{a}\left(\frac{2}{\sqrt{(x/a)^2+1}}\right)$$

$$\frac{V(x)}{(k_eQ/a)}=\boxed{\frac{2}{\sqrt{(x/a)^2+1}}}$$

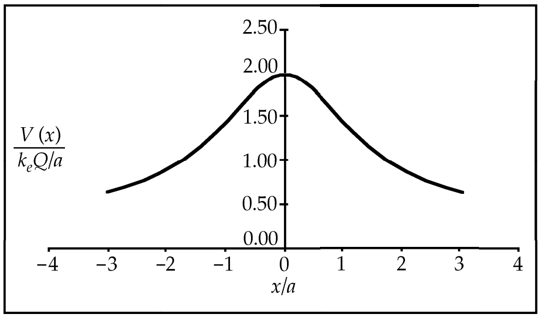

FIG. P25.30(a)

(b) $$V(y)=\frac{k_eQ_1}{r_1}+\frac{k_eQ_2}{r_2}=\frac{k_e(+Q)}{|y-a|}+\frac{k_e(-Q)}{|y+a|}$$

$$V(y)=\frac{k_eQ}{a}\left(\frac{1}{|y/a-1|}-\frac{1}{|y/a+1|}\right)$$

$$\frac{V(y)}{(k_eQ/a)}=\boxed{\left(\frac{1}{|y/a-1|}-\frac{1}{|y/a+1|}\right)}$$

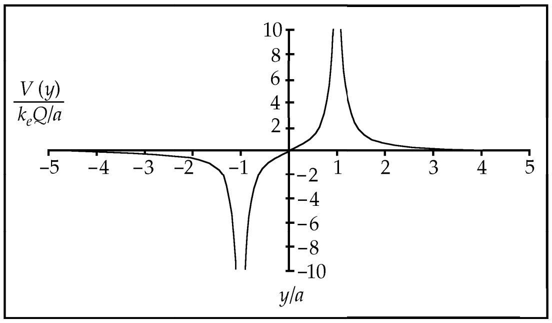

FIG. P25.30(b)

**P25.31** $V=\frac{k_eQ}{r}$ so $r=\frac{k_eQ}{V}=\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(8.00\times10^{-9}\ \text{C})}{V}=\frac{72.0\ \text{V}\cdot\text{m}}{V}$.

For $V=100$ V, 50.0 V, and 25.0 V, $\boxed{r=0.720\ \text{m, } 1.44\ \text{m, and } 2.88\ \text{m}}$.

The radii are $\boxed{\text{inversely proportional}}$ to the potential.

**P25.32** Using conservation of energy for the alpha particle-nucleus system,

we have $K_f+U_f=K_i+U_i$.

But $U_i=\frac{k_eq_\alpha q_{\text{gold}}}{r_i}$

and $r_i\approx\infty$.

Thus, $U_i=0$.

Also $K_f=0$ ($v_f=0$ at turning point),

so $U_f=K_i$

or $\frac{k_eq_\alpha q_{\text{gold}}}{r_{\min}}=\frac{1}{2}m_\alpha v_\alpha^2$

$$r_{\min}=\frac{2k_eq_\alpha q_{\text{gold}}}{m_\alpha v_\alpha^2}=\frac{2(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2)(79)(1.60\times10^{-19}\ \text{C})^2}{(6.64\times10^{-27}\ \text{kg})(2.00\times10^7\ \text{m/s})^2}=2.74\times10^{-14}\ \text{m}=\boxed{27.4\ \text{fm}}.$$

**P25.33** Using conservation of energy

we have: $$\frac{k_e eQ}{r_1} = \frac{k_e qQ}{r_2} + \frac{1}{2}mv^2$$

which gives: $$v = \sqrt{\frac{2k_e eQ}{m}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

or $$v = \sqrt{\frac{(2)\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(-1.60\times10^{-19}\ \text{C}\right)\left(10^{-9}\ \text{C}\right)}{9.11\times10^{-31}\ \text{kg}}\left(\frac{1}{0.030\,0\ \text{m}} - \frac{1}{0.020\,0\ \text{m}}\right)}.$$

Thus, $$v = \boxed{7.26\times10^{6}\ \text{m/s}}.$$

**P25.34** $U = \sum \frac{k_e q_i q_j}{r_{ij}}$, summed over all pairs of $(i,\ j)$ where $i \neq j$.

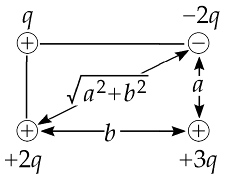

**FIG. P25.34**

$$U = k_e\left[\frac{q(-2q)}{b} + \frac{(-2q)(3q)}{a} + \frac{(2q)(3q)}{b} + \frac{q(2q)}{a} + \frac{q(3q)}{\sqrt{a^2+b^2}} + \frac{2q(-2q)}{\sqrt{a^2+b^2}}\right]$$

$$U = k_e q^2\left[\frac{-2}{0.400} - \frac{6}{0.200} + \frac{6}{0.400} + \frac{2}{0.200} + \frac{3}{0.447} - \frac{4}{0.447}\right]$$

$$U = \left(8.99\times10^{9}\right)\left(6.00\times10^{-6}\right)^2\left[\frac{4}{0.400} - \frac{4}{0.200} - \frac{1}{0.447}\right] = \boxed{-3.96\ \text{J}}$$

**P25.35** Each charge moves off on its diagonal line. All charges have equal speeds.

$$\sum(K+U)_i = \sum(K+U)_f$$

$$0 + \frac{4k_e q^2}{L} + \frac{2k_e q^2}{\sqrt{2}L} = 4\left(\frac{1}{2}mv^2\right) + \frac{4k_e q^2}{2L} + \frac{2k_e q^2}{2\sqrt{2}L}$$

$$\left(2 + \frac{1}{\sqrt{2}}\right)\frac{k_e q^2}{L} = 2mv^2$$

$$\boxed{v = \sqrt{\left(1 + \frac{1}{\sqrt{8}}\right)\frac{k_e q^2}{mL}}}$$

**P25.36** A cube has 12 edges and 6 faces. Consequently, there are 12 edge pairs separated by $s$, $2\times6 = 12$ face diagonal pairs separated by $\sqrt{2}s$ and 4 interior diagonal pairs separated $\sqrt{3}s$.

$$U = \frac{k_e q^2}{s}\left[12 + \frac{12}{\sqrt{2}} + \frac{4}{\sqrt{3}}\right] = \boxed{22.8\frac{k_e q^2}{s}}$$

---

## Section 25.4 **Obtaining the Value of the Electric Field from the Electric Potential**

**P25.37** $V = a + bx = 10.0\ \text{V} + (-7.00\ \text{V/m})x$

(a) At $x = 0$, $V = \boxed{10.0\ \text{V}}$

At $x = 3.00\ \text{m}$, $V = \boxed{-11.0\ \text{V}}$

At $x = 6.00\ \text{m}$, $V = \boxed{-32.0\ \text{V}}$

(b) $$E = -\frac{dV}{dx} = -b = -(-7.00\ \text{V/m}) = \boxed{7.00\ \text{N/C in the } +x \text{ direction}}$$

**P25.38** (a) For $r < R$ $$V = \frac{k_e Q}{R}$$ $$E_r = -\frac{dV}{dr} = \boxed{0}$$

(b) For $r \geq R$ $$V = \frac{k_e Q}{r}$$ $$E_r = -\frac{dV}{dr} = -\left(-\frac{k_e Q}{r^2}\right) = \boxed{\frac{k_e Q}{r^2}}$$

**P25.39** $V = 5x - 3x^2 y + 2yz^2$

Evaluate $E$ at $(1,\ 0,\ -2)$

$$E_x = -\frac{\partial V}{\partial x} = \boxed{-5 + 6xy} = -5 + 6(1)(0) = -5$$
$$E_y = -\frac{\partial V}{\partial y} = \boxed{+3x^2 - 2z^2} = 3(1)^2 - 2(-2)^2 = -5$$
$$E_z = -\frac{\partial V}{\partial z} = \boxed{-4yz} = -4(0)(-2) = 0$$
$$E = \sqrt{E_x^2 + E_y^2 + E_z^2} = \sqrt{(-5)^2 + (-5)^2 + 0^2} = \boxed{7.07\ \text{N/C}}$$

**P25.40** (a) $$\boxed{E_A > E_B \text{ since } E = \frac{\Delta V}{\Delta s}}$$

(b) $$E_B = -\frac{\Delta V}{\Delta s} = -\frac{(6-2)\ \text{V}}{2\ \text{cm}} = \boxed{200\ \text{N/C}}\ \text{down}$$

(c) The figure is shown to the right, with sample field lines sketched in.

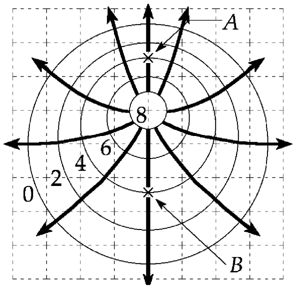

**FIG. P25.40**

**P25.41** $$E_y = -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}\left[\frac{k_e Q}{\ell} \ln\left(\frac{\ell + \sqrt{\ell^2 + y^2}}{y}\right)\right]$$

$$E_y = \frac{k_e Q}{\ell y}\left[1 - \frac{y^2}{\ell^2 + y^2 + \ell\sqrt{\ell^2 + y^2}}\right] = \boxed{\frac{k_e Q}{y\sqrt{\ell^2 + y^2}}}$$

## Section 25.5 **Electric Potential Due to Continuous Charge Distributions**

**P25.42** $$\Delta V = V_{2R} - V_0 = \frac{k_e Q}{\sqrt{R^2 + (2R)^2}} - \frac{k_e Q}{R} = \frac{k_e Q}{R}\left(\frac{1}{\sqrt{5}} - 1\right) = \boxed{-0.553\frac{k_e Q}{R}}$$

**P25.43** (a) $[\alpha]=\left[\frac{\lambda}{x}\right]=\frac{\text{C}}{\text{m}}\cdot\left(\frac{1}{\text{m}}\right)=\boxed{\frac{\text{C}}{\text{m}^2}}$

(b) $V=k_e\int\frac{dq}{r}=k_e\int\frac{\lambda dx}{r}=k_e\alpha\int_0^L\frac{xdx}{d+x}=\boxed{k_e\alpha\left[L-d\ln\left(1+\frac{L}{d}\right)\right]}$

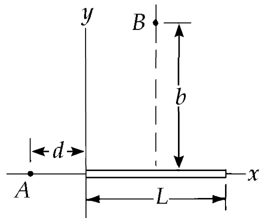

**FIG. P25.43**

**P25.44** $V=\int\frac{k_e dq}{r}=k_e\int\frac{\alpha x dx}{\sqrt{b^2+(L/2-x)^2}}$

Let $z=\frac{L}{2}-x$.

Then $x=\frac{L}{2}-z$, and $dx=-dz$

$$V=k_e\alpha\int\frac{(L/2-z)(-dz)}{\sqrt{b^2+z^2}}=-\frac{k_e\alpha L}{2}\int\frac{dz}{\sqrt{b^2+z^2}}+k_e\alpha\int\frac{zdz}{\sqrt{b^2+z^2}}=-\frac{k_e\alpha L}{2}\ln\left(z+\sqrt{z^2+b^2}\right)+k_e\alpha\sqrt{z^2+b^2}$$

$$V=-\frac{k_e\alpha L}{2}\ln\left[\left(\frac{L}{2}-x\right)+\sqrt{\left(\frac{L}{2}-x\right)^2+b^2}\right]\Bigg|_0^L+k_e\alpha\sqrt{\left(\frac{L}{2}-x\right)^2+b^2}\Bigg|_0^L$$

$$V=-\frac{k_e\alpha L}{2}\ln\left[\frac{L/2-L+\sqrt{(L/2)^2+b^2}}{L/2+\sqrt{(L/2)^2+b^2}}\right]+k_e\alpha\left[\sqrt{\left(\frac{L}{2}-L\right)^2+b^2}-\sqrt{\left(\frac{L}{2}\right)^2+b^2}\right]$$

$$V=\boxed{-\frac{k_e\alpha L}{2}\ln\left[\frac{\sqrt{b^2+\left(L^2/4\right)}-L/2}{\sqrt{b^2+\left(L^2/4\right)}+L/2}\right]}$$

**P25.45** $V=\int dV=\frac{1}{4\pi\in_0}\int\frac{dq}{r}$

All bits of charge are at the same distance from *O*.

So $V=\frac{1}{4\pi\in_0}\left(\frac{Q}{R}\right)=\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{-7.50\times10^{-6}\ \text{C}}{0.140\ \text{m}/\pi}\right)=\boxed{-1.51\ \text{MV}}$.

**P25.46** $dV=\frac{k_e dq}{\sqrt{r^2+x^2}}$ where $dq=\sigma dA=\sigma 2\pi r dr$

$V=2\pi\sigma k_e\int_a^b\frac{rdr}{\sqrt{r^2+x^2}}=\boxed{2\pi k_e\sigma\left[\sqrt{x^2+b^2}-\sqrt{x^2+a^2}\right]}$

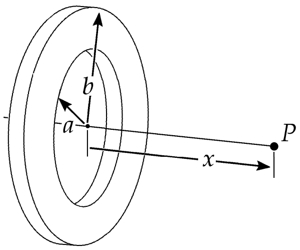

**FIG. P25.46**

**P25.47**

$$V = k_e \int_{\text{all charge}} \frac{dq}{r} = k_e \int_{-3R}^{-R} \frac{\lambda dx}{-x} + k_e \int_{\text{semicircle}} \frac{\lambda ds}{R} + k_e \int_{R}^{3R} \frac{\lambda dx}{x}$$

$$V = -k_e \lambda \ln(-x)\Big|_{-3R}^{-R} + \frac{k_e \lambda}{R} \pi R + k_e \lambda \ln x \Big|_{R}^{3R}$$

$$V = k_e \ln \frac{3R}{R} + k_e \lambda \pi + k_e \ln 3 = \boxed{k_e \lambda(\pi + 2\ln 3)}$$

## Section 25.6 **Electric Potential Due to a Charged Conductor**

**P25.48** Substituting given values into $V = \frac{k_e q}{r}$

$$7.50 \times 10^3 \text{ V} = \frac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right) q}{0.300 \text{ m}}.$$

Substituting $q = 2.50 \times 10^{-7}$ C,

$$N = \frac{2.50 \times 10^{-7} \text{ C}}{1.60 \times 10^{-19} \text{ C}/e^-} = \boxed{1.56 \times 10^{12} \text{ electrons}}.$$

**P25.49** (a) $E = \boxed{0}$;

$$V = \frac{k_e q}{R} = \frac{\left(8.99 \times 10^9\right)\left(26.0 \times 10^{-6}\right)}{0.140} = \boxed{1.67 \text{ MV}}$$

(b)

$$E = \frac{k_e q}{r^2} = \frac{\left(8.99 \times 10^9\right)\left(26.0 \times 10^{-6}\right)}{(0.200)^2} = \boxed{5.84 \text{ MN/C}} \text{ away}$$

$$V = \frac{k_e q}{R} = \frac{\left(8.99 \times 10^9\right)\left(26.0 \times 10^{-6}\right)}{0.200} = \boxed{1.17 \text{ MV}}$$

(c)

$$E = \frac{k_e q}{R^2} = \frac{\left(8.99 \times 10^9\right)\left(26.0 \times 10^{-6}\right)}{(0.140)^2} = \boxed{11.9 \text{ MN/C}} \text{ away}$$

$$V = \frac{k_e q}{R} = \boxed{1.67 \text{ MV}}$$

***P25.50** (a) Both spheres must be at the same potential according to $\frac{k_e q_1}{r_1} = \frac{k_e q_2}{r_2}$

where also $q_1 + q_2 = 1.20 \times 10^{-6}\ \text{C}$.

Then $q_1 = \frac{q_2 r_1}{r_2}$

$$\frac{q_2 r_1}{r_2} + q_2 = 1.20 \times 10^{-6}\ \text{C}$$

$$q_2 = \frac{1.20 \times 10^{-6}\ \text{C}}{1 + 6\ \text{cm}/2\ \text{cm}} = 0.300 \times 10^{-6}\ \text{C on the smaller sphere}$$

$$q_1 = 1.20 \times 10^{-6}\ \text{C} - 0.300 \times 10^{-6}\ \text{C} = 0.900 \times 10^{-6}\ \text{C}$$

$$V = \frac{k_e q_1}{r_1} = \frac{\left(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(0.900 \times 10^{-6}\ \text{C}\right)}{6 \times 10^{-2}\ \text{m}} = \boxed{1.35 \times 10^5\ \text{V}}$$

(b) Outside the larger sphere,

$$\mathbf{E}_1 = \frac{k_e q_1}{r_1^2}\hat{\mathbf{r}} = \frac{V_1}{r_1}\hat{\mathbf{r}} = \frac{1.35 \times 10^5\ \text{V}}{0.06\ \text{m}}\hat{\mathbf{r}} = \boxed{2.25 \times 10^6\ \text{V/m away}}.$$

Outside the smaller sphere,

$$\mathbf{E}_2 = \frac{1.35 \times 10^5\ \text{V}}{0.02\ \text{m}}\hat{\mathbf{r}} = \boxed{6.74 \times 10^6\ \text{V/m away}}.$$

The smaller sphere carries less charge but creates a much stronger electric field than the larger sphere.

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### Section 25.7 **The Milliken Oil Drop Experiment**

### Section 25.8 **Application of Electrostatistics**

**P25.51** (a) $E_{\text{max}} = 3.00 \times 10^6\ \text{V/m} = \frac{k_e Q}{r^2} = \frac{k_e Q}{r}\left(\frac{1}{r}\right) = V_{\text{max}}\left(\frac{1}{r}\right)$

$$V_{\text{max}} = E_{\text{max}} r = 3.00 \times 10^6 (0.150) = \boxed{450\ \text{kV}}$$

(b) $\frac{k_e Q_{\text{max}}}{r^2} = E_{\text{max}}$ $\left\{\text{or } \frac{k_e Q_{\text{max}}}{r} = V_{\text{max}}\right\}$ $Q_{\text{max}} = \frac{E_{\text{max}} r^2}{k_e} = \frac{3.00 \times 10^6 (0.150)^2}{8.99 \times 10^9} = \boxed{7.51\ \mu\text{C}}$

**P25.52** $V = \frac{k_e q}{r}$ and $E = \frac{k_e q}{r^2}$. Since $E = \frac{V}{r}$,

(b) $r = \frac{V}{E} = \frac{6.00 \times 10^5\ \text{V}}{3.00 \times 10^6\ \text{V/m}} = \boxed{0.200\ \text{m}}$ and

(a) $q = \frac{Vr}{k_e} = \boxed{13.3\ \mu\text{C}}$

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## Additional Problems

**P25.53** $U = qV = k_e \dfrac{q_1 q_2}{r_{12}} = \left(8.99\times10^9\right)\dfrac{(38)(54)\left(1.60\times10^{-19}\right)^2}{(5.50+6.20)\times10^{-15}} = 4.04\times10^{-11}\ \text{J} = \boxed{253\ \text{MeV}}$

**P25.54** (a) To make a spark 5 mm long in dry air between flat metal plates requires potential difference

$$\Delta V = Ed = \left(3\times10^6\ \text{V/m}\right)\left(5\times10^{-3}\ \text{m}\right) = 1.5\times10^4\ \text{V}\ \boxed{\sim 10^4\ \text{V}}.$$

(b) The area of your skin is perhaps $1.5\ \text{m}^2$, so model your body as a sphere with this surface area. Its radius is given by $1.5\ \text{m}^2 = 4\pi r^2$, $r = 0.35\ \text{m}$. We require that you are at the potential found in part (a):

$$V = \frac{k_e q}{r} \qquad q = \frac{Vr}{k_e} = \frac{1.5\times10^4\ \text{V}(0.35\ \text{m})}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}\left(\frac{\text{J}}{\text{V}\cdot\text{C}}\right)\left(\frac{\text{N}\cdot\text{m}}{\text{J}}\right)$$

$$q = 5.8\times10^{-7}\ \text{C}\ \boxed{\sim 10^{-6}\ \text{C}}.$$

**P25.55** (a) $U = \dfrac{k_e q_1 q_2}{r} = \dfrac{-\left(8.99\times10^9\right)\left(1.60\times10^{-19}\right)^2}{0.0529\times10^{-9}} = -4.35\times10^{-18}\ \text{J} = \boxed{-27.2\ \text{eV}}$

(b) $U = \dfrac{k_e q_1 q_2}{r} = \dfrac{-\left(8.99\times10^9\right)\left(1.60\times10^{-19}\right)^2}{2^2\left(0.0529\times10^{-9}\right)} = \boxed{-6.80\ \text{eV}}$

(c) $U = \dfrac{k_e q_1 q_2}{r} = \dfrac{-k_e e^2}{\infty} = \boxed{0}$

**P25.56** From Example 25.5, the potential created by the ring at the electron's starting point is

$$V_i = \frac{k_e Q}{\sqrt{x_i^2 + a^2}} = \frac{k_e(2\pi\lambda a)}{\sqrt{x_i^2 + a^2}}$$

while at the center, it is $V_f = 2\pi k_e \lambda$. From conservation of energy,

$$0 + (-eV_i) = \frac{1}{2} m_e v_f^2 + (-eV_f)$$

$$v_f^2 = \frac{2e}{m_e}(V_f - V_i) = \frac{4\pi e k_e \lambda}{m_e}\left(1 - \frac{a}{\sqrt{x_i^2 + a^2}}\right)$$

$$v_f^2 = \frac{4\pi\left(1.60\times10^{-19}\right)\left(8.99\times10^9\right)\left(1.00\times10^{-7}\right)}{9.11\times10^{-31}}\left(1 - \frac{0.200}{\sqrt{(0.100)^2 + (0.200)^2}}\right)$$

$$v_f = \boxed{1.45\times10^7\ \text{m/s}}$$

***P25.57** The plates create uniform electric field to the right in the picture, with magnitude $\frac{V_0-(-V_0)}{d}=\frac{2V_0}{d}$. Assume the ball swings a small distance $x$ to the right. It moves to a place where the voltage created by the plates is lower by $-Ex=-\frac{2V_0}{d}x$. Its ground connection maintains it at $V=0$ by allowing charge $q$ to flow from ground onto the ball, where $-\frac{2V_0x}{d}+\frac{k_eq}{R}=0 \qquad q=\frac{2V_0xR}{k_ed}$. Then the ball feels electric force $F=qE=\frac{4V_0^2xR}{k_ed^2}$ to the right. For equilibrium this must be balanced by the horizontal component of string tension according to $T\cos\theta=mg \qquad T\sin\theta=\frac{4V_0^2xR}{k_ed^2}$

$\tan\theta=\frac{4V_0^2xR}{k_ed^2mg}=\frac{x}{L}$ for small $x$. Then $V_0=\boxed{\left(\frac{k_ed^2mg}{4RL}\right)^{1/2}}$.

If $V_0$ is less than this value, the only equilibrium position of the ball is hanging straight down. If $V_0$ exceeds this value the ball will swing over to one plate or the other.

**P25.58** (a) Take the origin at the point where we will find the potential. One ring, of width $dx$, has charge $\frac{Qdx}{h}$ and, according to Example 25.5, creates potential

$$dV=\frac{k_eQdx}{h\sqrt{x^2+R^2}}.$$

The whole stack of rings creates potential

$$V=\int_{\text{all charge}}dV=\int_d^{d+h}\frac{k_eQdx}{h\sqrt{x^2+R^2}}=\frac{k_eQ}{h}\ln\left(x+\sqrt{x^2+R^2}\right)\Big|_d^{d+h}=\boxed{\frac{k_eQ}{h}\ln\left(\frac{d+h+\sqrt{(d+h)^2+R^2}}{d+\sqrt{d^2+R^2}}\right)}.$$

(b) A disk of thickness $dx$ has charge $\frac{Qdx}{h}$ and charge-per-area $\frac{Qdx}{\pi R^2h}$. According to Example 25.6, it creates potential

$$dV=2\pi k_e\frac{Qdx}{\pi R^2h}\left(\sqrt{x^2+R^2}-x\right).$$

Integrating,

$$V=\int_d^{d+h}\frac{2k_eQ}{R^2h}\left(\sqrt{x^2+R^2}dx-xdx\right)=\frac{2k_eQ}{R^2h}\left[\frac{1}{2}x\sqrt{x^2+R^2}+\frac{R^2}{2}\ln\left(x+\sqrt{x^2+R^2}\right)-\frac{x^2}{2}\right]_d^{d+h}$$

$$V=\boxed{\frac{k_eQ}{R^2h}\left[(d+h)\sqrt{(d+h)^2+R^2}-d\sqrt{d^2+R^2}-2dh-h^2+R^2\ln\left(\frac{d+h+\sqrt{(d+h)^2+R^2}}{d+\sqrt{d^2+R^2}}\right)\right]}$$

**P25.59** $W=\int_0^Q Vdq$

where $V=\frac{k_eq}{R}$.

Therefore, $W=\boxed{\frac{k_eQ^2}{2R}}$.

**P25.60** The positive plate by itself creates a field $E=\frac{\sigma}{2\in_0}=\frac{36.0\times10^{-9}\ \text{C/m}^2}{2\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)}=2.03\ \text{kN/C}$ away from the + plate. The negative plate by itself creates the same size field and between the plates it is in the same direction. Together the plates create a uniform field $4.07\ \text{kN/C}$ in the space between.

(a) Take $V=0$ at the negative plate. The potential at the positive plate is then

$$V-0=-\int_0^{12.0\ \text{cm}}(-4.07\ \text{kN/C})dx.$$

The potential difference between the plates is $V=\left(4.07\times10^3\ \text{N/C}\right)(0.120\ \text{m})=\boxed{488\ \text{V}}$.

(b) $$\left(\frac{1}{2}mv^2+qV\right)_i=\left(\frac{1}{2}mv^2+qV\right)_f$$

$$qV=\left(1.60\times10^{-19}\ \text{C}\right)(488\ \text{V})=\frac{1}{2}mv_f^2=\boxed{7.81\times10^{-17}\ \text{J}}$$

(c) $v_f=\boxed{306\ \text{km/s}}$

(d) $$v_f^2=v_i^2+2a\left(x_f-x_i\right)$$

$$\left(3.06\times10^5\ \text{m/s}\right)^2=0+2a(0.120\ \text{m})$$

$$a=\boxed{3.90\times10^{11}\ \text{m/s}^2}$$

(e) $\sum F=ma=\left(1.67\times10^{-27}\ \text{kg}\right)\left(3.90\times10^{11}\ \text{m/s}^2\right)=\boxed{6.51\times10^{-16}\ \text{N}}$

(f) $E=\frac{F}{q}=\frac{6.51\times10^{-16}\ \text{N}}{1.60\times10^{-19}\ \text{C}}=\boxed{4.07\ \text{kN/C}}$

**P25.61** (a) $V_B-V_A=-\int_A^B\mathbf{E}\cdot d\mathbf{s}$ and the field at distance $r$ from a uniformly charged rod (where $r>$ radius of charged rod) is

$$E=\frac{\lambda}{2\pi\in_0 r}=\frac{2k_e\lambda}{r}.$$

In this case, the field between the central wire and the coaxial cylinder is directed perpendicular to the line of charge so that

$$V_B-V_A=-\int_{r_a}^{r_b}\frac{2k_e\lambda}{r}dr=2k_e\lambda\ln\left(\frac{r_a}{r_b}\right),$$

or $$\boxed{\Delta V=2k_e\lambda\ln\left(\frac{r_a}{r_b}\right)}.$$

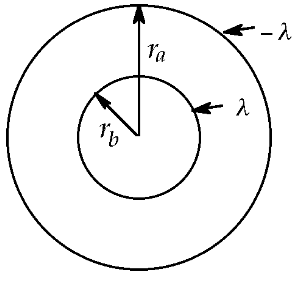

**FIG. P25.61**

*continued on next page*

(b) From part (a), when the outer cylinder is considered to be at zero potential, the potential at a distance $r$ from the axis is

$$V = 2k_e\lambda \ln\left(\frac{r_a}{r}\right).$$

The field at $r$ is given by

$$E = -\frac{\partial V}{\partial r} = -2k_e\lambda\left(\frac{r}{r_a}\right)\left(-\frac{r_a}{r^2}\right) = \frac{2k_e\lambda}{r}.$$

But, from part (a), $2k_e\lambda = \frac{\Delta V}{\ln(r_a/r_b)}$.

Therefore, $\boxed{E = \frac{\Delta V}{\ln(r_a/r_b)}\left(\frac{1}{r}\right)}$.

**P25.62** (a) From Problem 61,

$$E = \frac{\Delta V}{\ln(r_a/r_b)}\frac{1}{r}.$$

We require just outside the central wire

$$5.50\times10^6\ \text{V/m} = \frac{50.0\times10^3\ \text{V}}{\ln(0.850\ \text{m}/r_b)}\left(\frac{1}{r_b}\right)$$

or $\quad\left(110\ \text{m}^{-1}\right)r_b\ln\left(\frac{0.850\ \text{m}}{r_b}\right) = 1.$

We solve by homing in on the required value

| $r_b(\text{m})$ | 0.0100 | 0.00100 | 0.00150 | 0.00145 | 0.00143 | 0.00142 |
|---|---|---|---|---|---|---|
| $\left(110\ \text{m}^{-1}\right)r_b\ln\left(\frac{0.850\ \text{m}}{r_b}\right)$ | 4.89 | 0.740 | 1.05 | 1.017 | 1.005 | 0.999 |

Thus, to three significant figures,

$$r_b = \boxed{1.42\ \text{mm}}.$$

(b) At $r_a$,

$$E = \frac{50.0\ \text{kV}}{\ln(0.850\ \text{m}/0.001\,42\ \text{m})}\left(\frac{1}{0.850\ \text{m}}\right) = \boxed{9.20\ \text{kV/m}}.$$

**P25.63** $V_2 - V_1 = -\int_{r_1}^{r_2}\mathbf{E}\cdot d\mathbf{r} = -\int_{r_1}^{r_2}\frac{\lambda}{2\pi\epsilon_0 r}dr$

$$V_2 - V_1 = \boxed{\frac{-\lambda}{2\pi\epsilon_0}\ln\left(\frac{r_2}{r_1}\right)}$$

***P25.64** Take the illustration presented with the problem as an initial picture. No external horizontal forces act on the set of four balls, so its center of mass stays fixed at the location of the center of the square. As the charged balls 1 and 2 swing out and away from each other, balls 3 and 4 move up with equal $y$-components of velocity. The maximum-kinetic-energy point is illustrated. System energy is conserved:

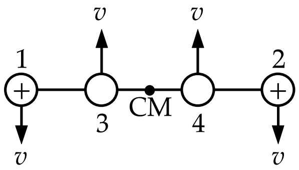

**FIG. P25.64**

$$\frac{k_e q^2}{a} = \frac{k_e q^2}{3a} + \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

$$\frac{2k_e q^2}{3a} = 2mv^2 \qquad \boxed{v = \sqrt{\frac{k_e q^2}{3am}}}$$

**P25.65** For the given charge distribution, $V(x,\, y,\, z) = \dfrac{k_e(q)}{r_1} + \dfrac{k_e(-2q)}{r_2}$

where $r_1 = \sqrt{(x+R)^2 + y^2 + z^2}$ and $r_2 = \sqrt{x^2 + y^2 + z^2}$.

The surface on which $V(x,\, y,\, z) = 0$

is given by $k_e q\left(\dfrac{1}{r_1} - \dfrac{2}{r_2}\right) = 0$, or $2r_1 = r_2$.

This gives: $4(x+R)^2 + 4y^2 + 4z^2 = x^2 + y^2 + z^2$

which may be written in the form: $x^2 + y^2 + z^2 + \left(\dfrac{8}{3}R\right)x + (0)y + (0)z + \left(\dfrac{4}{3}R^2\right) = 0$. [1]

The general equation for a sphere of radius $a$ centered at $(x_0,\, y_0,\, z_0)$ is:

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 - a^2 = 0$$

or $x^2 + y^2 + z^2 + (-2x_0)x + (-2y_0)y + (-2z_0)z + \left(x_0^2 + y_0^2 + z_0^2 - a^2\right) = 0$. [2]

Comparing equations [1] and [2], it is seen that the equipotential surface for which $V = 0$ is indeed a sphere and that:

$$-2x_0 = \frac{8}{3}R;\ -2y_0 = 0;\ -2z_0 = 0;\ x_0^2 + y_0^2 + z_0^2 - a^2 = \frac{4}{3}R^2.$$

Thus, $x_0 = -\dfrac{4}{3}R$, $y_0 = z_0 = 0$, and $a^2 = \left(\dfrac{16}{9} - \dfrac{4}{3}\right)R^2 = \dfrac{4}{9}R^2$.

The equipotential surface is therefore a sphere centered at $\boxed{\left(-\dfrac{4}{3}R,\, 0,\, 0\right)}$, having a radius $\boxed{\dfrac{2}{3}R}$.

**P25.66** (a) From Gauss's law, $\boxed{E_A = 0}$ (no charge within)

$$E_B = k_e \frac{q_A}{r^2} = \left(8.99 \times 10^9\right) \frac{\left(1.00 \times 10^{-8}\right)}{r^2} = \boxed{\left(\frac{89.9}{r^2}\right) \text{ V/m}}$$

$$E_C = k_e \frac{(q_A + q_B)}{r} = \left(8.99 \times 10^9\right) \frac{\left(-5.00 \times 10^{-9}\right)}{r^2} = \boxed{\left(-\frac{45.0}{r^2}\right) \text{ V/m}}$$

(b) $$V_C = k_e \frac{(q_A + q_B)}{r} = \left(8.99 \times 10^9\right) \frac{\left(-5.00 \times 10^{-9}\right)}{r} = \boxed{\left(-\frac{45.0}{r}\right) \text{ V}}$$

$$\therefore \text{At } r_2,\ V = -\frac{45.0}{0.300} = -150 \text{ V}$$

$$\text{Inside } r_2,\ V_B = -150 \text{ V} + \int_{r_2}^{r} \frac{89.9}{r^2} dr = -150 + 89.9\left(\frac{1}{r} - \frac{1}{0.300}\right) = \boxed{\left(-450 + \frac{89.9}{r}\right) \text{ V}}$$

$$\therefore \text{At } r_1,\ V = -450 + \frac{89.9}{0.150} = +150 \text{ V so } \boxed{V_A = +150 \text{ V}}.$$

**P25.67** From Example 25.5, the potential at the center of the ring is $V_i = \frac{k_e Q}{R}$ and the potential at an infinite distance from the ring is $V_f = 0$. Thus, the initial and final potential energies of the point charge-ring system are:

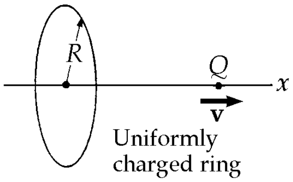

FIG. P25.67

$$U_i = QV_i = \frac{k_e Q^2}{R}$$

and $$U_f = QV_f = 0.$$

From conservation of energy,

$$K_f + U_f = K_i + U_i$$

or $$\frac{1}{2} M v_f^2 + 0 = 0 + \frac{k_e Q^2}{R}$$

giving $$v_f = \boxed{\sqrt{\frac{2k_e Q^2}{MR}}}.$$

**P25.68** $$V = k_e \int_a^{a+L} \frac{\lambda dx}{\sqrt{x^2 + b^2}} = k_e \lambda \ln\left[x + \sqrt{\left(x^2 + b^2\right)}\right]\Big|_a^{a+L} = \boxed{k_e \lambda \ln\left[\frac{a + L + \sqrt{(a+L)^2 + b^2}}{a + \sqrt{a^2 + b^2}}\right]}$$

***P25.69** (a) $V = \frac{k_e q}{r_1} - \frac{k_e q}{r_2} = \frac{k_e q}{r_1 r_2}(r_2 - r_1)$

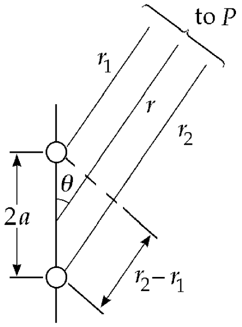

**FIG. P25.69**

From the figure, for $r >> a$, $r_2 - r_1 \cong 2a\cos\theta$.

Then $V \cong \frac{k_e q}{r_1 r_2} 2a\cos\theta \cong \frac{k_e p\cos\theta}{r^2}$.

(b) $E_r = -\frac{\partial V}{\partial r} = \boxed{\frac{2k_e p\cos\theta}{r^3}}$

In spherical coordinates, the $\theta$ component of the gradient is $\frac{1}{r}\left(\frac{\partial}{\partial\theta}\right)$.

Therefore, $E_\theta = -\frac{1}{r}\left(\frac{\partial V}{\partial\theta}\right) = \boxed{\frac{k_e p\sin\theta}{r^3}}$.

For $r >> a$ $E_r(0°) = \frac{2k_e p}{r^3}$

and $E_r(90°) = 0$,

$E_\theta(0°) = 0$

and $E_\theta(90°) = \frac{k_e p}{r^3}$.

These results are $\boxed{\text{reasonable for } r >> a}$. Their directions are as shown in Figure 25.13 (c). However, for $\boxed{r \to 0,\ E(0) \to \infty.\ \text{This is unreasonable,}}$ since $r$ is not much greater than $a$ if it is 0.

(c) $\boxed{V = \frac{k_e p y}{\left(x^2 + y^2\right)^{3/2}}}$

and $\boxed{E_x = -\frac{\partial V}{\partial x} = \frac{3k_e p x y}{\left(x^2 + y^2\right)^{5/2}}}$

$\boxed{E_y = -\frac{\partial V}{\partial y} = \frac{k_e p\left(2y^2 - x^2\right)}{\left(x^2 + y^2\right)^{5/2}}}$

**P25.70** Inside the sphere, $E_x = E_y = E_z = 0$.

Outside, $E_x = -\dfrac{\partial V}{\partial x} = -\dfrac{\partial}{\partial x}\left(V_0 - E_0 z + E_0 a^3 z\left(x^2+y^2+z^2\right)^{-3/2}\right)$

So $E_x = -\left[0+0+E_0 a^3 z\left(-\dfrac{3}{2}\right)\left(x^2+y^2+z^2\right)^{-5/2}(2x)\right] = \boxed{3E_0 a^3 xz\left(x^2+y^2+z^2\right)^{-5/2}}$

$$E_y = -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}\left(V_0 - E_0 z + E_0 a^3 z\left(x^2+y^2+z^2\right)^{-3/2}\right)$$

$$E_y = -E_0 a^3 z\left(-\frac{3}{2}\right)\left(x^2+y^2+z^2\right)^{-5/2} 2y = \boxed{3E_0 a^3 yz\left(x^2+y^2+z^2\right)^{-5/2}}$$

$$E_z = -\frac{\partial V}{\partial z} = E_0 - E_0 a^3 z\left(-\frac{3}{2}\right)\left(x^2+y^2+z^2\right)^{-5/2}(2z) - E_0 a^3\left(x^2+y^2+z^2\right)^{-3/2}$$

$$E_z = \boxed{E_0 + E_0 a^3\left(2z^2-x^2-y^2\right)\left(x^2+y^2+z^2\right)^{-5/2}}$$

**P25.71** For an element of area which is a ring of radius *r* and width *dr*, $dV = \dfrac{k_e dq}{\sqrt{r^2+x^2}}$.

$dq = \sigma dA = Cr(2\pi r dr)$ and

$$V = C(2\pi k_e)\int_0^R \frac{r^2 dr}{\sqrt{r^2+x^2}} = \boxed{C(\pi k_e)\left[R\sqrt{R^2+x^2} + x^2 \ln\left(\frac{x}{R+\sqrt{R^2+x^2}}\right)\right]}.$$

**P25.72** $dU = V dq$ where the potential $V = \dfrac{k_e q}{r}$.

The element of charge in a shell is $dq = \rho$ (volume element) or $dq = \rho\left(4\pi r^2 dr\right)$ and the charge *q* in a sphere of radius *r* is

$$q = 4\pi\rho\int_0^r r^2 dr = \rho\left(\frac{4\pi r^3}{3}\right).$$

Substituting this into the expression for *dU*, we have

$$dU = \left(\frac{k_e q}{r}\right)dq = k_e \rho\left(\frac{4\pi r^3}{3}\right)\left(\frac{1}{r}\right)\rho\left(4\pi r^2 dr\right) = k_e\left(\frac{16\pi^2}{3}\right)\rho^2 r^4 dr$$

$$U = \int dU = k_e\left(\frac{16\pi^2}{3}\right)\rho^2\int_0^R r^4 dr = k_e\left(\frac{16\pi^2}{15}\right)\rho^2 R^5$$

But the *total* charge, $Q = \rho\dfrac{4}{3}\pi R^3$. Therefore, $\boxed{U = \dfrac{3}{5}\dfrac{k_e Q^2}{R}}$.

***P25.73** (a) The whole charge on the cube is $q=\left(100\times10^{-6}\ \text{C/m}^3\right)(0.1\ \text{m})^3=10^{-7}\ \text{C}$. Divide up the cube into 64 or more elements. The little cube labeled *a* creates at *P* potential $\dfrac{k_eq}{64\sqrt{6.25^2+1.25^2+1.25^2}\,10^{-2}\ \text{m}}$. The others in the horizontal row behind it contribute

$$\frac{k_eq}{64\left(10^{-2}\ \text{m}\right)}\left(\frac{1}{\sqrt{8.75^2+3.125}}+\frac{1}{\sqrt{11.25^2+3.125}}+\frac{1}{\sqrt{13.75^2+3.125}}\right).$$

The little cubes in the rows containing *b* and *c* add

$$\frac{2k_eq}{64\left(10^{-2}\ \text{m}\right)}\Big[\left(6.25^2+1.25^2+3.75^2\right)^{-1/2}+\left(8.75^2+15.625\right)^{-1/2}+\left(11.25^2+15.625\right)^{-1/2}+\left(13.75^2+15.625\right)^{-1/2}\Big]$$

and the bits in row *d* make potential at *P*

$$\frac{k_eq}{64\left(10^{-2}\ \text{m}\right)}\left[\left(6.25^2+28.125\right)^{-1/2}+\ldots+\left(13.75^2+28.125\right)^{-1/2}\right].$$

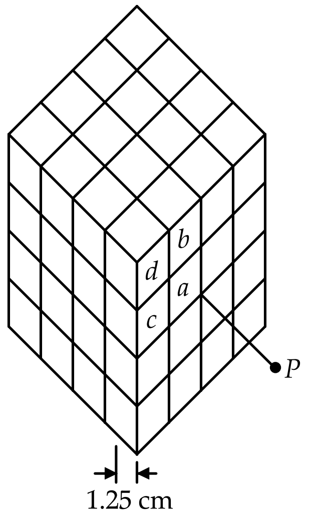

FIG. P25.73

The whole potential at *P* is $\dfrac{8.987\,6\times10^9\ \text{Nm}^2\times10^{-7}\ \text{C}}{\text{C}^2\,64\left(10^{-2}\ \text{m}\right)}(1.580\,190)4=\boxed{8\,876\ \text{V}}$. If we use more subdivisions of the large cube, we get the same answer to four digits.

(b) A sphere centered at the same point would create potential

$$\frac{k_eq}{r}=\frac{8.987\,6\times10^9\ \text{Nm}^2\times10^{-7}\ \text{C}}{\text{C}^2\quad10^{-1}\ \text{m}}=8\,988\ \text{V},\ \boxed{\text{larger by 112 V}}.$$

## ANSWERS TO EVEN PROBLEMS

| | |
|---|---|
| **P25.2** | $6.41\times10^{-19}$ C |
| **P25.4** | –0.502 V |
| **P25.6** | 1.67 MN/C |
| **P25.8** | (a) 59.0 V; (b) 4.55 Mm/s |
| **P25.10** | see the solution |
| **P25.12** | 40.2 kV |
| **P25.14** | 0.300 m/s |
| **P25.16** | (a) 0; (b) 0; (c) 45.0 kV |
| **P25.18** | (a) –4.83 m; (b) 0.667 m and –2.00 m |
| **P25.20** | (a) –386 nJ; (b) 103 V |
| **P25.22** | (a) 32.2 kV; (b) –96.5 mJ |
| **P25.24** | $-\dfrac{5k_eq}{R}$ |
| **P25.26** | (a) 10.8 m/s and 1.55 m/s; (b) greater |
| **P25.28** | (a) –45.0 $\mu$J; (b) 34.6 km/s |
| **P25.30** | see the solution |
| **P25.32** | 27.4 fm |
| **P25.34** | –3.96 J |
| **P25.36** | $22.8\dfrac{k_eq^2}{s}$ |

**P25.38** (a) 0; (b) $\frac{k_eQ}{r^2}$ radially outward

**P25.40** (a) larger at *A*; (b) $200\ \text{N/C}$ down; (c) see the solution

**P25.42** $-0.553\frac{k_eQ}{R}$

**P25.44** $-\frac{k_e\alpha L}{2}\ln\left[\frac{\sqrt{b^2+\left(L^2/4\right)}-L/2}{\sqrt{b^2+\left(L^2/4\right)}+L/2}\right]$

**P25.46** $2\pi k_e\sigma\left[\sqrt{x^2+b^2}-\sqrt{x^2+a^2}\right]$

**P25.48** $1.56\times10^{12}$ electrons

**P25.50** (a) $135\ \text{kV}$; (b) $2.25\ \text{MV/m}$ away from the large sphere and $6.74\ \text{MV/m}$ away from the small sphere

**P25.52** (a) $13.3\ \mu\text{C}$; (b) $0.200\ \text{m}$

**P25.54** (a) $\sim10^4\ \text{V}$; (b) $\sim10^{-6}\ \text{C}$

**P25.56** $14.5\ \text{Mm/s}$

**P25.58** (a) $\frac{k_eQ}{h}\ln\left(\frac{d+h+\sqrt{(d+h)^2+R^2}}{d+\sqrt{d^2+R^2}}\right)$;

(b) $$\frac{k_eQ}{R^2h}\left[\begin{array}{l}(d+h)\sqrt{(d+h)^2+R^2}-d\sqrt{d^2+R^2}\\ -2dh-h^2+R^2\ln\left(\frac{d+h+\sqrt{(d+h)^2+R^2}}{d+\sqrt{d^2+R^2}}\right)\end{array}\right]$$

**P25.60** (a) $488\ \text{V}$; (b) $7.81\times10^{-17}\ \text{J}$; (c) $306\ \text{km/s}$; (d) $390\ \text{Gm/s}^2$ toward the negative plate; (e) $6.51\times10^{-16}\ \text{N}$ toward the negative plate; (f) $4.07\ \text{kN/C}$ toward the negative plate

**P25.62** (a) $1.42\ \text{mm}$; (b) $9.20\ \text{kV/m}$

**P25.64** $\left(\frac{k_eq^2}{3am}\right)^{1/2}$

**P25.66** (a) $\mathbf{E}_A=0$; $\mathbf{E}_B=\left(\frac{89.9}{r^2}\right)$ V/m radially outward; $\mathbf{E}_C=\left(-\frac{45.0}{r^2}\right)$ V/m radially outward;

(b) $V_A=150\ \text{V}$; $V_B=\left(-450+\frac{89.9}{r}\right)\text{V}$; $V_C=\left(-\frac{45.0}{r}\right)\text{V}$

**P25.68** $k_e\lambda\ln\left[\frac{a+L+\sqrt{(a+L)^2+b^2}}{a+\sqrt{a^2+b^2}}\right]$

**P25.70** $E_x=3E_0a^3xz\left(x^2+y^2+z^2\right)^{-5/2}$;

$E_y=3E_0a^3yz\left(x^2+y^2+z^2\right)^{-5/2}$;

$E_z=E_0+\frac{E_0a^3\left(2z^2-x^2-y^2\right)}{\left(x^2+y^2+z^2\right)^{5/2}}$ outside and

$\mathbf{E}=0$ inside

**P25.72** $\frac{3}{5}\frac{k_eQ^2}{R}$

# 26

# Capacitance and Dielectrics

**CHAPTER OUTLINE**

26.1 Definition of Capacitance
26.2 Calculating Capacitance
26.3 Combinations of Capacitors
26.4 Energy Stored in a Charged Capacitor
26.5 Capacitors with Dielectrics
26.6 Electric Dipole in an Electric Field
26.7 An Atomic Description of Dielectrics

## ANSWERS TO QUESTIONS

**Q26.1** Nothing happens to the charge if the wires are disconnected. If the wires are connected to each other, charges in the single conductor which now exists move between the wires and the plates until the entire conductor is at a single potential and the capacitor is discharged.

**Q26.2** 336 km. The plate area would need to be $\frac{1}{\epsilon_0}\ \text{m}^2$.

**Q26.3** The parallel-connected capacitors store more energy, since they have higher equivalent capacitance.

**Q26.4** Seventeen combinations:

Individual $C_1,\ C_2,\ C_3$

Parallel $C_1+C_2+C_3,\ C_1+C_2,\ C_1+C_3,\ C_2+C_3$

Series-Parallel $\left(\frac{1}{C_1}+\frac{1}{C_2}\right)^{-1}+C_3,\ \left(\frac{1}{C_1}+\frac{1}{C_3}\right)^{-1}+C_2,\ \left(\frac{1}{C_2}+\frac{1}{C_3}\right)^{-1}+C_1$

$$\left(\frac{1}{C_1+C_2}+\frac{1}{C_3}\right)^{-1},\ \left(\frac{1}{C_1+C_3}+\frac{1}{C_2}\right)^{-1},\ \left(\frac{1}{C_2+C_3}+\frac{1}{C_1}\right)^{-1}$$

Series $\left(\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}\right)^{-1},\ \left(\frac{1}{C_1}+\frac{1}{C_2}\right)^{-1},\ \left(\frac{1}{C_2}+\frac{1}{C_3}\right)^{-1},\ \left(\frac{1}{C_1}+\frac{1}{C_3}\right)^{-1}$

**Q26.5** This arrangement would decrease the potential difference between the plates of any individual capacitor by a factor of 2, thus decreasing the possibility of dielectric breakdown. Depending on the application, this could be the difference between the life or death of some other (most likely more expensive) electrical component connected to the capacitors.

**Q26.6** No—not just using rules about capacitors in series or in parallel. See Problem 72 for an example. If connections can be made to a combination of capacitors at more than two points, the combination may be irreducible.

**Q26.7** A capacitor stores energy in the electric field between the plates. This is most easily seen when using a "dissectable" capacitor. If the capacitor is charged, carefully pull it apart into its component pieces. One will find that very little residual charge remains on each plate. When reassembled, the capacitor is suddenly "recharged"—by induction—due to the electric field set up and "stored" in the dielectric. This proves to be an instructive classroom demonstration, especially when you ask a student to reconstruct the capacitor without supplying him/her with any rubber gloves or other insulating material. (Of course, this is *after* they sign a liability waiver).

**Q26.8** The work you do to pull the plates apart becomes additional electric potential energy stored in the capacitor. The charge is constant and the capacitance decreases but the potential difference increases to drive up the potential energy $\frac{1}{2}Q\Delta V$. The electric field between the plates is constant in strength but fills more volume as you pull the plates apart.

**Q26.9** A capacitor stores energy in the electric field inside the dielectric. Once the external voltage source is removed—provided that there is no external resistance through which the capacitor can discharge—the capacitor can hold onto this energy for a very long time. To make the capacitor safe to handle, you can discharge the capacitor through a conductor, such as a screwdriver, provided that you only touch the insulating handle. If the capacitor is a large one, it is best to use an external resistor to discharge the capacitor more slowly to prevent damage to the dielectric, or welding of the screwdriver to the terminals of the capacitor.

**Q26.10** The work done, $W = Q\Delta V$, is the work done by an external agent, like a battery, to move a charge through a potential difference, $\Delta V$. To determine the energy in a charged capacitor, we must add the work done to move bits of charge from one plate to the other. Initially, there is no potential difference between the plates of an uncharged capacitor. As more charge is transferred from one plate to the other, the potential difference increases as shown in Figure 26.12, meaning that more work is needed to transfer each additional bit of charge. The total work is the area under the curve of Figure 26.12, and thus $W = \frac{1}{2}Q\Delta V$.

**Q26.11** Energy is proportional to voltage squared. It gets four times larger.

**Q26.12** Let $C$ = the capacitance of an individual capacitor, and $C_s$ represent the equivalent capacitance of the group in series. While being charged in parallel, each capacitor receives charge

$Q = C\Delta V_{\text{charge}} = \left(500 \times 10^{-4}\ \text{F}\right)(800\ \text{V}) = 0.400\ \text{C}$.

While being discharged in series, $\Delta V_{\text{discharge}} = \frac{Q}{C_s} = \frac{Q}{C/10} = \frac{0.400\ \text{C}}{5.00 \times 10^{-5}\ \text{F}} = 8.00\ \text{kV}$

(or 10 times the original voltage).

**Q26.13** Put a material with higher dielectric strength between the plates, or evacuate the space between the plates. At very high voltages, you may want to cool off the plates or choose to make them of a different chemically stable material, because atoms in the plates themselves can ionize, showing *thermionic emission* under high electric fields.

**Q26.14** The potential difference must decrease. Since there is no external power supply, the charge on the capacitor, $Q$, will remain constant—that is assuming that the resistance of the meter is sufficiently large. Adding a dielectric *increases* the capacitance, which must therefore *decrease* the potential difference between the plates.

**Q26.15** Each polar molecule acts like an electric "compass" needle, aligning itself with the external electric field set up by the charged plates. The contribution of these electric dipoles pointing in the same direction reduces the net electric field. As each dipole falls into a configuration of lower potential energy it can contribute to increasing the internal energy of the material.

**Q26.16** The material of the dielectric may be able to support a larger electric field than air, without breaking down to pass a spark between the capacitor plates.

**Q26.17** The dielectric strength is a measure of the potential difference per unit length that a dielectric can withstand without having individual molecules ionized, leaving in its wake a conducting path from plate to plate. For example, dry air has a dielectric strength of about 3 MV/m. The dielectric constant in effect describes the contribution of the electric dipoles of the polar molecules in the dielectric to the electric field once aligned.

**Q26.18** In water, the oxygen atom and one hydrogen atom considered alone have an electric dipole moment that points from the hydrogen to the oxygen. The other O-H pair has its own dipole moment that points again toward the oxygen. Due to the geometry of the molecule, these dipole moments add to have a non-zero component along the axis of symmetry and pointing toward the oxygen.

A non-polarized molecule could either have no intrinsic dipole moments, or have dipole moments that add to zero. An example of the latter case is $CO_2$. The molecule is structured so that each CO pair has a dipole moment, but since both dipole moments have the same magnitude and opposite direction—due to the linear geometry of the molecule—the entire molecule has no dipole moment.

**Q26.19** Heating a dielectric will decrease its dielectric constant, decreasing the capacitance of a capacitor. When you heat a material, the average kinetic energy per molecule increases. If you refer back to the answer to Question 26.15, each polar molecule will no longer be nicely aligned with the applied electric field, but will begin to "dither"—rock back and forth—effectively decreasing its contribution to the overall field.

**Q26.20** The primary choice would be the dielectric. You would want to chose a dielectric that has a large dielectric constant and dielectric strength, such as strontium titanate, where $\kappa \approx 233$ (Table 26.1). A convenient choice could be thick plastic or mylar. Secondly, geometry would be a factor. To maximize capacitance, one would want the individual plates as close as possible, since the capacitance is proportional to the inverse of the plate separation—hence the need for a dielectric with a high dielectric strength. Also, one would want to build, instead of a single parallel plate capacitor, several capacitors in parallel. This could be achieved through "stacking" the plates of the capacitor. For example, you can alternately lay down sheets of a conducting material, such as aluminum foil, sandwiched between your sheets of insulating dielectric. Making sure that none of the conducting sheets are in contact with their next neighbors, connect every other plate together. Figure Q26.20 illustrates this idea.

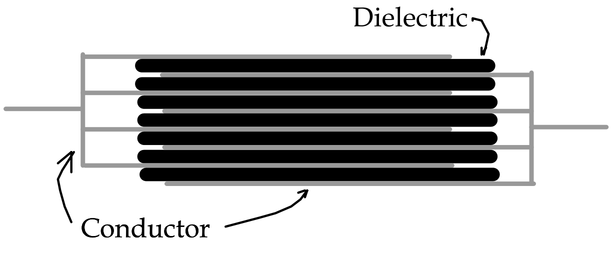

**FIG. Q26.20**

This technique is often used when "home-brewing" signal capacitors for radio applications, as they can withstand huge potential differences without flashover (without either discharge between plates around the dielectric or dielectric breakdown). One variation on this technique is to sandwich together flexible materials such as aluminum roof flashing and thick plastic, so the whole product can be rolled up into a "capacitor burrito" and placed in an insulating tube, such as a PVC pipe, and then filled with motor oil (again to prevent flashover).

## SOLUTIONS TO PROBLEMS

### Section 26.1 **Definition of Capacitance**

**P26.1** (a) $Q = C\Delta V = \left(4.00\times10^{-6}\text{ F}\right)(12.0\text{ V}) = 4.80\times10^{-5}\text{ C} = \boxed{48.0\ \mu\text{C}}$

(b) $Q = C\Delta V = \left(4.00\times10^{-6}\text{ F}\right)(1.50\text{ V}) = 6.00\times10^{-6}\text{ C} = \boxed{6.00\ \mu\text{C}}$

**P26.2** (a) $C = \dfrac{Q}{\Delta V} = \dfrac{10.0\times10^{-6}\text{ C}}{10.0\text{ V}} = 1.00\times10^{-6}\text{ F} = \boxed{1.00\ \mu\text{F}}$

(b) $\Delta V = \dfrac{Q}{C} = \dfrac{100\times10^{-6}\text{ C}}{1.00\times10^{-6}\text{ F}} = \boxed{100\text{ V}}$

### Section 26.2 **Calculating Capacitance**

**P26.3** $E = \dfrac{k_e q}{r^2}$: $\quad q = \dfrac{\left(4.90\times10^4\text{ N/C}\right)(0.210\text{ m})^2}{\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)} = 0.240\ \mu\text{C}$

(a) $\sigma = \dfrac{q}{A} = \dfrac{0.240\times10^{-6}}{4\pi(0.120)^2} = \boxed{1.33\ \mu\text{C/m}^2}$

(b) $C = 4\pi \in_0 r = 4\pi\left(8.85\times10^{-12}\right)(0.120) = \boxed{13.3\text{ pF}}$

**P26.4** (a) $C = 4\pi \in_0 R$

$R = \dfrac{C}{4\pi \in_0} = k_e C = \left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.00\times10^{-12}\text{ F}\right) = \boxed{8.99\text{ mm}}$

(b) $C = 4\pi \in_0 R = \dfrac{4\pi\left(8.85\times10^{-12}\text{ C}^2\right)\left(2.00\times10^{-3}\text{ m}\right)}{\text{N}\cdot\text{m}^2} = \boxed{0.222\text{ pF}}$

(c) $Q = CV = \left(2.22\times10^{-13}\text{ F}\right)(100\text{ V}) = \boxed{2.22\times10^{-11}\text{ C}}$

**P26.5** (a) $\dfrac{Q_1}{Q_2} = \dfrac{R_1}{R_2}$

$Q_1 + Q_2 = \left(1 + \dfrac{R_1}{R_2}\right)Q_2 = 3.50Q_2 = 7.00\ \mu\text{C}$

$\boxed{Q_2 = 2.00\ \mu\text{C}}$ $\boxed{Q_1 = 5.00\ \mu\text{C}}$

(b) $V_1 = V_2 = \dfrac{Q_1}{C_1} = \dfrac{Q_2}{C_2} = \dfrac{5.00\ \mu\text{C}}{\left(8.99\times10^9\text{ m/F}\right)^{-1}(0.500\text{ m})} = 8.99\times10^4\text{ V} = \boxed{89.9\text{ kV}}$

**P26.6** $$C=\frac{\kappa \in_0 A}{d}=\frac{(1.00)\left(8.85\times10^{-12}\ \text{C}^2\right)\left(1.00\times10^{3}\ \text{m}\right)^2}{\text{N}\cdot\text{m}^2(800\ \text{m})}=\boxed{11.1\ \text{nF}}$$

The potential between ground and cloud is

$$\Delta V=Ed=\left(3.00\times10^{6}\ \text{N/C}\right)(800\ \text{m})=2.40\times10^{9}\ \text{V}$$

$$Q=C(\Delta V)=\left(11.1\times10^{-9}\ \text{C/V}\right)\left(2.40\times10^{9}\ \text{V}\right)=\boxed{26.6\ \text{C}}$$

**P26.7** (a) $\Delta V=Ed$

$$E=\frac{20.0\ \text{V}}{1.80\times10^{-3}\ \text{m}}=\boxed{11.1\ \text{kV/m}}$$

(b) $E=\dfrac{\sigma}{\in_0}$

$$\sigma=\left(1.11\times10^{4}\ \text{N/C}\right)\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)=\boxed{98.3\ \text{nC/m}^2}$$

(c) $$C=\frac{\in_0 A}{d}=\frac{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(7.60\ \text{cm}^2\right)(1.00\ \text{m}/100\ \text{cm})^2}{1.80\times10^{-3}\ \text{m}}=\boxed{3.74\ \text{pF}}$$

(d) $\Delta V=\dfrac{Q}{C}$

$$Q=(20.0\ \text{V})\left(3.74\times10^{-12}\ \text{F}\right)=\boxed{74.7\ \text{pC}}$$

**P26.8** $$C=\frac{\kappa \in_0 A}{d}=60.0\times10^{-15}\ \text{F}$$

$$d=\frac{\kappa \in_0 A}{C}=\frac{(1)\left(8.85\times10^{-12}\right)\left(21.0\times10^{-12}\right)}{60.0\times10^{-15}}$$

$$d=3.10\times10^{-9}\ \text{m}=\boxed{3.10\ \text{nm}}$$

**P26.9** $$Q=\frac{\in_0 A}{d}(\Delta V)\qquad \frac{Q}{A}=\sigma=\frac{\in_0(\Delta V)}{d}$$

$$d=\frac{\in_0(\Delta V)}{\sigma}=\frac{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)(150\ \text{V})}{\left(30.0\times10^{-9}\ \text{C/cm}^2\right)\left(1.00\times10^{4}\ \text{cm}^2/\text{m}^2\right)}=\boxed{4.42\ \mu\text{m}}$$

**P26.10** With $\theta = \pi$, the plates are out of mesh and the overlap area is zero. With $\theta = 0$, the overlap area is that of a semi-circle, $\frac{\pi R^2}{2}$. By proportion, the effective area of a single sheet of charge is $\frac{(\pi - \theta)R^2}{2}$

When there are two plates in each comb, the number of adjoining sheets of positive and negative charge is 3, as shown in the sketch. When there are $N$ plates on each comb, the number of parallel capacitors is $2N-1$ and the total capacitance is

$$C = (2N-1)\frac{\in_0 A_{\text{effective}}}{\text{distance}} = \frac{(2N-1)\in_0 (\pi-\theta)R^2/2}{d/2} = \boxed{\frac{(2N-1)\in_0 (\pi-\theta)R^2}{d}}.$$

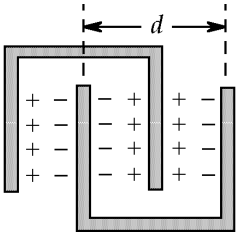

FIG. P26.10

**P26.11** (a) $C = \frac{\ell}{2k_e \ln\left(\frac{b}{a}\right)} = \frac{50.0}{2\left(8.99\times10^9\right)\ln\left(\frac{7.27}{2.58}\right)} = \boxed{2.68 \text{ nF}}$

(b) Method 1: $\Delta V = 2k_e \lambda \ln\left(\frac{b}{a}\right)$

$$\lambda = \frac{q}{\ell} = \frac{8.10\times10^{-6}\text{ C}}{50.0\text{ m}} = 1.62\times10^{-7}\text{ C/m}$$

$$\Delta V = 2\left(8.99\times10^9\right)\left(1.62\times10^{-7}\right)\ln\left(\frac{7.27}{2.58}\right) = \boxed{3.02 \text{ kV}}$$

Method 2: $\Delta V = \frac{Q}{C} = \frac{8.10\times10^{-6}}{2.68\times10^{-9}} = \boxed{3.02 \text{ kV}}$

**P26.12** Let the radii be $b$ and $a$ with $b = 2a$. Put charge $Q$ on the inner conductor and $-Q$ on the outer. Electric field exists only in the volume between them. The potential of the inner sphere is $V_a = \frac{k_e Q}{a}$; that of the outer is $V_b = \frac{k_e Q}{b}$. Then

$$V_a - V_b = \frac{k_e Q}{a} - \frac{k_e Q}{b} = \frac{Q}{4\pi \in_0}\left(\frac{b-a}{ab}\right) \text{ and } C = \frac{Q}{V_a - V_b} = \frac{4\pi \in_0 ab}{b-a}.$$

Here $C = \frac{4\pi \in_0 2a^2}{a} = 8\pi \in_0 a \qquad a = \frac{C}{8\pi \in_0}.$

The intervening volume is $\quad \text{Volume} = \frac{4}{3}\pi b^3 - \frac{4}{3}\pi a^3 = 7\left(\frac{4}{3}\pi a^3\right) = 7\left(\frac{4}{3}\pi\right)\frac{C^3}{8^3 \pi^3 \in_0^3} = \frac{7C^3}{384\pi^2 \in_0^3}$

$$\text{Volume} = \frac{7\left(20.0\times10^{-6}\text{ C}^2/\text{N}\cdot\text{m}\right)^3}{384\pi^2\left(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2\right)^3} = \boxed{2.13\times10^{16}\text{ m}^3}.$$

The outer sphere is 360 km in diameter.

**P26.13** (a) $C = \frac{ab}{k_e(b-a)} = \frac{(0.070\,0)(0.140)}{\left(8.99\times10^9\right)(0.140 - 0.070\,0)} = \boxed{15.6 \text{ pF}}$

(b) $C = \frac{Q}{\Delta V} \qquad \Delta V = \frac{Q}{C} = \frac{4.00\times10^{-6}\text{ C}}{15.6\times10^{-12}\text{ F}} = \boxed{256 \text{ kV}}$

**P26.14** $\sum F_y = 0:$ $T\cos\theta - mg = 0$

$\sum F_x = 0:$ $T\sin\theta - Eq = 0$

Dividing, $\tan\theta = \dfrac{Eq}{mg}$

so $E = \dfrac{mg}{q}\tan\theta$

and $\Delta V = Ed = \boxed{\dfrac{mgd\tan\theta}{q}}$.

**P26.15** $C = 4\pi \in_0 R = 4\pi\left(8.85\times10^{-12}\text{ C/N}\cdot\text{m}^2\right)\left(6.37\times10^{6}\text{ m}\right) = \boxed{7.08\times10^{-4}\text{ F}}$

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## Section 26.3 **Combinations of Capacitors**

**P26.16** (a) Capacitors in parallel add. Thus, the equivalent capacitor has a value of

$$C_{eq} = C_1 + C_2 = 5.00\ \mu\text{F} + 12.0\ \mu\text{F} = \boxed{17.0\ \mu\text{F}}.$$

(b) The potential difference across each branch is the same and equal to the voltage of the battery.

$$\Delta V = \boxed{9.00\text{ V}}$$

(c) $Q_5 = C\Delta V = (5.00\ \mu\text{F})(9.00\text{ V}) = \boxed{45.0\ \mu\text{C}}$

and $Q_{12} = C\Delta V = (12.0\ \mu\text{F})(9.00\text{ V}) = \boxed{108\ \mu\text{C}}$

**P26.17** (a) In series capacitors add as

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{5.00\ \mu\text{F}} + \frac{1}{12.0\ \mu\text{F}}$$

and $C_{eq} = \boxed{3.53\ \mu\text{F}}$.

(c) The charge on the equivalent capacitor is $Q_{eq} = C_{eq}\Delta V = (3.53\ \mu\text{F})(9.00\text{ V}) = 31.8\ \mu\text{C}$.

Each of the series capacitors has this same charge on it.

So $Q_1 = Q_2 = \boxed{31.8\ \mu\text{C}}$.

(b) The potential difference across each is $\Delta V_1 = \dfrac{Q_1}{C_1} = \dfrac{31.8\ \mu\text{C}}{5.00\ \mu\text{F}} = \boxed{6.35\text{ V}}$

and $\Delta V_2 = \dfrac{Q_2}{C_2} = \dfrac{31.8\ \mu\text{C}}{12.0\ \mu\text{F}} = \boxed{2.65\text{ V}}$.

**P26.18** The circuit reduces first according to the rule for capacitors in series, as shown in the figure, then according to the rule for capacitors in parallel, shown below.

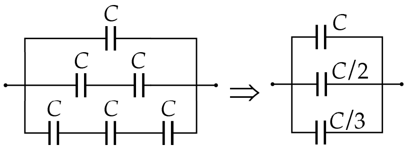


FIG. P26.18

$$C_{eq}=C\left(1+\frac{1}{2}+\frac{1}{3}\right)=\frac{11}{6}C=\boxed{1.83C}$$

**P26.19** $C_p=C_1+C_2$ $\qquad\frac{1}{C_s}=\frac{1}{C_1}+\frac{1}{C_2}$

Substitute $C_2=C_p-C_1$ $\qquad\frac{1}{C_s}=\frac{1}{C_1}+\frac{1}{C_p-C_1}=\frac{C_p-C_1+C_1}{C_1\left(C_p-C_1\right)}$.

Simplifying, $\qquad C_1^2-C_1C_p+C_pC_s=0$.

$$C_1=\frac{C_p\pm\sqrt{C_p^2-4C_pC_s}}{2}=\frac{1}{2}C_p\pm\sqrt{\frac{1}{4}C_p^2-C_pC_s}$$

We choose arbitrarily the + sign. (This choice can be arbitrary, since with the case of the minus sign, we would get the same two answers with their names interchanged.)

$$C_1=\frac{1}{2}C_p+\sqrt{\frac{1}{4}C_p^2-C_pC_s}=\frac{1}{2}(9.00\text{ pF})+\sqrt{\frac{1}{4}(9.00\text{ pF})^2-(9.00\text{ pF})(2.00\text{ pF})}=\boxed{6.00\text{ pF}}$$

$$C_2=C_p-C_1=\frac{1}{2}C_p-\sqrt{\frac{1}{4}C_p^2-C_pC_s}=\frac{1}{2}(9.00\text{ pF})-1.50\text{ pF}=\boxed{3.00\text{ pF}}$$

**P26.20** $C_p=C_1+C_2$

and $\qquad\frac{1}{C_s}=\frac{1}{C_1}+\frac{1}{C_2}$.

Substitute $\qquad C_2=C_p-C_1$: $\qquad\frac{1}{C_s}=\frac{1}{C_1}+\frac{1}{C_p-C_1}=\frac{C_p-C_1+C_1}{C_1\left(C_p-C_1\right)}$.

Simplifying, $\qquad C_1^2-C_1C_p+C_pC_s=0$

and $\qquad C_1=\frac{C_p\pm\sqrt{C_p^2-4C_pC_s}}{2}=\boxed{\frac{1}{2}C_p+\sqrt{\frac{1}{4}C_p^2-C_pC_s}}$

where the positive sign was arbitrarily chosen (choosing the negative sign gives the same values for the capacitances, with the names reversed).

Then, from $\qquad C_2=C_p-C_1$

$$C_2=\boxed{\frac{1}{2}C_p-\sqrt{\frac{1}{4}C_p^2-C_pC_s}}.$$

**P26.21** (a) $\frac{1}{C_s}=\frac{1}{15.0}+\frac{1}{3.00}$

$C_s = 2.50\ \mu\text{F}$

$C_p = 2.50 + 6.00 = 8.50\ \mu\text{F}$

$$C_{eq} = \left(\frac{1}{8.50\ \mu\text{F}}+\frac{1}{20.0\ \mu\text{F}}\right)^{-1} = \boxed{5.96\ \mu\text{F}}$$

(b) $Q = C\Delta V = (5.96\ \mu\text{F})(15.0\ \text{V}) = \boxed{89.5\ \mu\text{C}}$ on $20.0\ \mu\text{F}$

$\Delta V = \frac{Q}{C} = \frac{89.5\ \mu\text{C}}{20.0\ \mu\text{F}} = 4.47\ \text{V}$

$15.0 - 4.47 = 10.53\ \text{V}$

$Q = C\Delta V = (6.00\ \mu\text{F})(10.53\ \text{V}) = \boxed{63.2\ \mu\text{C}}$ on $6.00\ \mu\text{F}$

$89.5 - 63.2 = \boxed{26.3\ \mu\text{C}}$ on $15.0\ \mu\text{F}$ and $3.00\ \mu\text{F}$

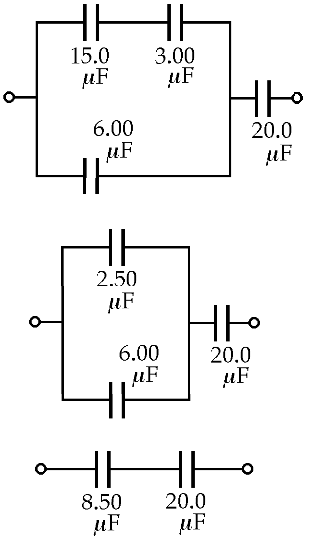


FIG. P26.21

***P26.22** (a) Capacitors 2 and 3 are in parallel and present equivalent capacitance 6*C*. This is in series with capacitor 1, so the battery sees capacitance $\left[\frac{1}{3C}+\frac{1}{6C}\right]^{-1} = \boxed{2C}$.

(b) If they were initially unchanged, $C_1$ stores the same charge as $C_2$ and $C_3$ together. With greater capacitance, $C_3$ stores more charge than $C_2$. Then $\boxed{Q_1 > Q_3 > Q_2}$.

(c) The $(C_2 || C_3)$ equivalent capacitor stores the same charge as $C_1$. Since it has greater capacitance, $\Delta V = \frac{Q}{C}$ implies that it has smaller potential difference across it than $C_1$. In parallel with each other, $C_2$ and $C_3$ have equal voltages: $\boxed{\Delta V_1 > \Delta V_2 = \Delta V_3}$.

(d) If $C_3$ is increased, the overall equivalent capacitance increases. More charge moves through the battery and *Q* increases. As $\Delta V_1$ increases, $\Delta V_2$ must decrease so $Q_2$ decreases. Then $Q_3$ must increase even more: $\boxed{Q_3 \text{ and } Q_1 \text{ increase; } Q_2 \text{ decreases}}$.

**P26.23** $C = \frac{Q}{\Delta V}$ so $6.00 \times 10^{-6} = \frac{Q}{20.0}$

and $Q = \boxed{120\ \mu\text{C}}$

$Q_1 = 120\ \mu\text{C} - Q_2$

and $\Delta V = \frac{Q}{C}$: $\frac{120 - Q_2}{C_1} = \frac{Q_2}{C_2}$

or $\frac{120 - Q_2}{6.00} = \frac{Q_2}{3.00}$

$(3.00)(120 - Q_2) = (6.00)Q_2$

$Q_2 = \frac{360}{9.00} = \boxed{40.0\ \mu\text{C}}$ $Q_1 = 120\ \mu\text{C} - 40.0\ \mu\text{C} = \boxed{80.0\ \mu\text{C}}$

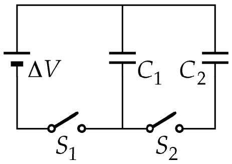


FIG. P26.23

**P26.24** (a) In $\boxed{\text{series}}$, to reduce the effective capacitance:

$$\frac{1}{32.0\ \mu\text{F}}=\frac{1}{34.8\ \mu\text{F}}+\frac{1}{C_s}$$

$$C_s=\frac{1}{2.51\times10^{-3}/\mu\text{F}}=\boxed{398\ \mu\text{F}}$$

(b) In $\boxed{\text{parallel}}$, to increase the total capacitance:

$$29.8\ \mu\text{F}+C_p=32.0\ \mu\text{F}$$

$$C_p=\boxed{2.20\ \mu\text{F}}$$

**P26.25**
$$nC=\frac{100}{\underbrace{\frac{1}{C}+\frac{1}{C}+\frac{1}{C}+\cdots}_{n\text{ capacitors}}}=\frac{100}{n/C}$$

$$nC=\frac{100C}{n}\ \text{ so }\ n^2=100\ \text{ and }\ n=\boxed{10}$$

***P26.26** For $C_1$ connected by itself, $C_1\Delta V=30.8\ \mu\text{C}$ where $\Delta V$ is the battery voltage: $\Delta V=\frac{30.8\ \mu\text{C}}{C_1}$.

For $C_1$ and $C_2$ in series:

$$\left(\frac{1}{1/C_1+1/C_2}\right)\Delta V=23.1\ \mu\text{C}$$

substituting, $\frac{30.8\ \mu\text{C}}{C_1}=\frac{23.1\ \mu\text{C}}{C_1}+\frac{23.1\ \mu\text{C}}{C_2}$ $\qquad C_1=0.333C_2$.

For $C_1$ and $C_3$ in series:

$$\left(\frac{1}{1/C_1+1/C_3}\right)\Delta V=25.2\ \mu\text{C}$$

$\frac{30.8\ \mu\text{C}}{C_1}=\frac{25.2\ \mu\text{C}}{C_1}+\frac{25.2\ \mu\text{C}}{C_3}$ $\qquad C_1=0.222C_3$.

For all three:

$$Q=\left(\frac{1}{1/C_1+1/C_2+1/C_3}\right)\Delta V=\frac{C_1\Delta V}{1+C_1/C_2+C_1/C_3}=\frac{30.8\ \mu\text{C}}{1+0.333+0.222}=\boxed{19.8\ \mu\text{C}}.$$

This is the charge on each one of the three.

**P26.27**
$$C_s=\left(\frac{1}{5.00}+\frac{1}{10.0}\right)^{-1}=3.33\ \mu\text{F}$$

$$C_{p1}=2(3.33)+2.00=8.66\ \mu\text{F}$$

$$C_{p2}=2(10.0)=20.0\ \mu\text{F}$$

$$C_{eq}=\left(\frac{1}{8.66}+\frac{1}{20.0}\right)^{-1}=\boxed{6.04\ \mu\text{F}}$$

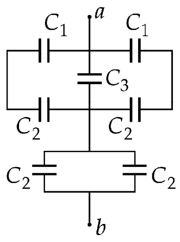


FIG. P26.27

**P26.28** $Q_{eq}=C_{eq}(\Delta V)=\left(6.04\times10^{-6}\text{ F}\right)(60.0\text{ V})=3.62\times10^{-4}\text{ C}$

$$Q_{p1}=Q_{eq}\text{, so }\Delta V_{p1}=\frac{Q_{eq}}{C_{p1}}=\frac{3.62\times10^{-4}\text{ C}}{8.66\times10^{-6}\text{ F}}=41.8\text{ V}$$

$$Q_3=C_3\left(\Delta V_{p1}\right)=\left(2.00\times10^{-6}\text{ F}\right)(41.8\text{ V})=\boxed{83.6\ \mu\text{C}}$$

**P26.29** $$C_s=\left(\frac{1}{5.00}+\frac{1}{7.00}\right)^{-1}=2.92\ \mu\text{F}$$

$$C_p=2.92+4.00+6.00=\boxed{12.9\ \mu\text{F}}$$

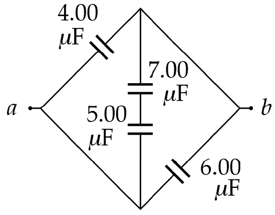
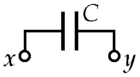


**FIG. P26.29**

***P26.30** According to the suggestion, the combination of capacitors shown is equivalent to 

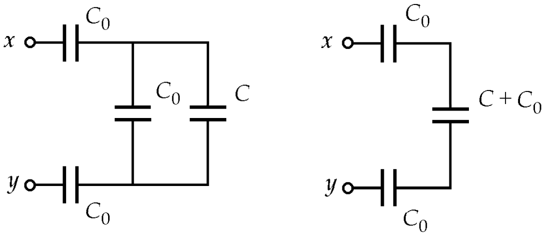


**FIG. P26.30**

Then $$\frac{1}{C}=\frac{1}{C_0}+\frac{1}{C+C_0}+\frac{1}{C_0}=\frac{C+C_0+C_0+C+C_0}{C_0(C+C_0)}$$

$$C_0C+C_0^2=2C^2+3C_0C$$

$$2C^2+2C_0C-C_0^2=0$$

$$C=\frac{-2C_0\pm\sqrt{4C_0^2+4\left(2C_0^2\right)}}{4}$$

Only the positive root is physical

$$\boxed{C=\frac{C_0}{2}\left(\sqrt{3}-1\right)}$$

## Section 26.4 **Energy Stored in a Charged Capacitor**

**P26.31** (a) $$U=\frac{1}{2}C(\Delta V)^2=\frac{1}{2}(3.00\ \mu\text{F})(12.0\text{ V})^2=\boxed{216\ \mu\text{J}}$$

(b) $$U=\frac{1}{2}C(\Delta V)^2=\frac{1}{2}(3.00\ \mu\text{F})(6.00\text{ V})^2=\boxed{54.0\ \mu\text{J}}$$

**P26.32** $$U=\frac{1}{2}C\Delta V^2$$

$$\Delta V=\sqrt{\frac{2U}{C}}=\sqrt{\frac{2(300\text{ J})}{30\times10^{-6}\text{ C/V}}}=\boxed{4.47\times10^3\text{ V}}$$

**P26.33** $U = \frac{1}{2}C(\Delta V)^2$

The circuit diagram is shown at the right.

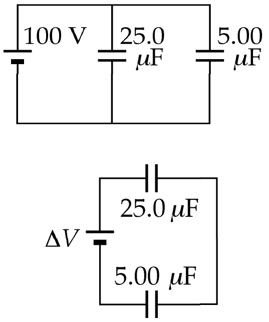


FIG. P26.33

(a) $C_p = C_1 + C_2 = 25.0\ \mu\text{F} + 5.00\ \mu\text{F} = 30.0\ \mu\text{F}$

$$U = \frac{1}{2}\left(30.0\times10^{-6}\right)(100)^2 = \boxed{0.150\ \text{J}}$$

(b) $C_s = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} = \left(\frac{1}{25.0\ \mu\text{F}} + \frac{1}{5.00\ \mu\text{F}}\right)^{-1} = 4.17\ \mu\text{F}$

$$U = \frac{1}{2}C(\Delta V)^2$$

$$\Delta V = \sqrt{\frac{2U}{C}} = \sqrt{\frac{2(0.150)}{4.17\times10^{-6}}} = \boxed{268\ \text{V}}$$

**P26.34** Use $U = \frac{1}{2}\frac{Q^2}{C}$ and $C = \frac{\in_0 A}{d}$.

If $d_2 = 2d_1$, $C_2 = \frac{1}{2}C_1$. Therefore, the $\boxed{\text{stored energy doubles}}$.

***P26.35** (a) $Q = C\Delta V = \left(150\times10^{-12}\ \text{F}\right)\left(10\times10^{3}\ \text{V}\right) = \boxed{1.50\times10^{-6}\ \text{C}}$

(b) $U = \frac{1}{2}C(\Delta V)^2$

$$\Delta V = \sqrt{\frac{2U}{C}} = \sqrt{\frac{2\left(250\times10^{-6}\ \text{J}\right)}{150\times10^{-12}\ \text{F}}} = \boxed{1.83\times10^{3}\ \text{V}}$$

**P26.36** $u = \frac{U}{V} = \frac{1}{2}\in_0 E^2$

$$\frac{1.00\times10^{-7}}{V} = \frac{1}{2}\left(8.85\times10^{-12}\right)(3\,000)^2$$

$$V = \boxed{2.51\times10^{-3}\ \text{m}^3} = \left(2.51\times10^{-3}\ \text{m}^3\right)\left(\frac{1\,000\ \text{L}}{\text{m}^3}\right) = \boxed{2.51\ \text{L}}$$

**P26.37** $W = U = \int F\,dx$

so $F = \frac{dU}{dx} = \frac{d}{dx}\left(\frac{Q^2}{2C}\right) = \frac{d}{dx}\left(\frac{Q^2 x}{2\in_0 A}\right) = \boxed{\frac{Q^2}{2\in_0 A}}$

**P26.38** With switch closed, distance $d' = 0.500d$ and capacitance $C' = \frac{\in_0 A}{d'} = \frac{2 \in_0 A}{d} = 2C$.

(a) $Q = C'(\Delta V) = 2C(\Delta V) = 2\left(2.00 \times 10^{-6}\ \text{F}\right)(100\ \text{V}) = \boxed{400\ \mu\text{C}}$

(b) The force stretching out one spring is

$$F = \frac{Q^2}{2 \in_0 A} = \frac{4C^2(\Delta V)^2}{2 \in_0 A} = \frac{2C^2(\Delta V)^2}{\left(\in_0 A/d\right)d} = \frac{2C(\Delta V)^2}{d}.$$

One spring stretches by distance $x = \frac{d}{4}$, so

$$k = \frac{F}{x} = \frac{2C(\Delta V)^2}{d}\left(\frac{4}{d}\right) = \frac{8C(\Delta V)^2}{d^2} = \frac{8\left(2.00 \times 10^{-6}\ \text{F}\right)(100\ \text{V})^2}{\left(8.00 \times 10^{-3}\ \text{m}\right)^2} = \boxed{2.50\ \text{kN/m}}.$$

**P26.39** The energy transferred is $H_{\text{ET}} = \frac{1}{2}Q\Delta V = \frac{1}{2}(50.0\ \text{C})\left(1.00 \times 10^8\ \text{V}\right) = 2.50 \times 10^9\ \text{J}$

and 1% of this (or $\Delta E_{\text{int}} = 2.50 \times 10^7\ \text{J}$) is absorbed by the tree. If *m* is the amount of water boiled away,

then $\Delta E_{\text{int}} = m(4\,186\ \text{J/kg}\cdot{}^\circ\text{C})(100^\circ\text{C} - 30.0^\circ\text{C}) + m\left(2.26 \times 10^6\ \text{J/kg}\right) = 2.50 \times 10^7\ \text{J}$

giving $m = \boxed{9.79\ \text{kg}}$.

***P26.40** (a) $U = \frac{1}{2}C(\Delta V)^2 + \frac{1}{2}C(\Delta V)^2 = \boxed{C(\Delta V)^2}$

(b) The altered capacitor has capacitance $C' = \frac{C}{2}$. The total charge is the same as before:

$$C(\Delta V) + C(\Delta V) = C(\Delta V') + \frac{C}{2}(\Delta V') \qquad \boxed{\Delta V' = \frac{4\Delta V}{3}}.$$

(c) $U' = \frac{1}{2}C\left(\frac{4\Delta V}{3}\right)^2 + \frac{1}{2}\frac{1}{2}C\left(\frac{4\Delta V}{3}\right)^2 = \boxed{4C\frac{(\Delta V)^2}{3}}$

(d) The extra energy comes from work put into the system by the agent pulling the capacitor plates apart.

**P26.41** $U = \frac{1}{2}C(\Delta V)^2$ where $C = 4\pi \in_0 R = \frac{R}{k_e}$ and $\Delta V = \frac{k_eQ}{R} - 0 = \frac{k_eQ}{R}$

$$U = \frac{1}{2}\left(\frac{R}{k_e}\right)\left(\frac{k_eQ}{R}\right)^2 = \boxed{\frac{k_eQ^2}{2R}}$$

***P26.42** (a) The total energy is $U = U_1 + U_2 = \frac{1}{2}\frac{q_1^2}{C_1} + \frac{1}{2}\frac{q_2^2}{C_2} = \frac{1}{2}\frac{q_1^2}{4\pi\epsilon_0 R_1} + \frac{1}{2}\frac{(Q-q_1)^2}{4\pi\epsilon_0 R^2}$.

For a minimum we set $\frac{dU}{dq_1} = 0$:

$$\frac{1}{2}\frac{2q_1}{4\pi\epsilon_0 R_1} + \frac{1}{2}\frac{2(Q-q_1)}{4\pi\epsilon_0 R_2}(-1) = 0$$

$$R_2 q_1 = R_1 Q - R_1 q_1 \qquad \boxed{q_1 = \frac{R_1 Q}{R_1 + R_2}}$$

Then $q_2 = Q - q_1 = \boxed{\frac{R_2 Q}{R_1 + R_2} = q_2}$.

(b)

$$V_1 = \frac{k_e q_1}{R_1} = \frac{k_e R_1 Q}{R_1(R_1 + R_2)} = \frac{k_e Q}{R_1 + R_2}$$

$$V_2 = \frac{k_e q_2}{R_2} = \frac{k_e R_2 Q}{R_2(R_1 + R_2)} = \frac{k_e Q}{R_1 + R_2}$$

and $V_1 - V_2 = 0$.

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## Section 26.5 **Capacitors with Dielectrics**

**P26.43** (a) $$C = \frac{\kappa \epsilon_0 A}{d} = \frac{2.10\left(8.85\times10^{-12}\text{ F/m}\right)\left(1.75\times10^{-4}\text{ m}^2\right)}{4.00\times10^{-5}\text{ m}} = 8.13\times10^{-11}\text{ F} = \boxed{81.3\text{ pF}}$$

(b) $$\Delta V_{\max} = E_{\max} d = \left(60.0\times10^{6}\text{ V/m}\right)\left(4.00\times10^{-5}\text{ m}\right) = \boxed{2.40\text{ kV}}$$

**P26.44** $Q_{\max} = C\Delta V_{\max}$,

but $\Delta V_{\max} = E_{\max} d$.

Also, $C = \frac{\kappa\epsilon_0 A}{d}$.

Thus, $Q_{\max} = \frac{\kappa\epsilon_0 A}{d}\left(E_{\max} d\right) = \kappa\epsilon_0 A E_{\max}$.

(a) With air between the plates, $\kappa = 1.00$

and $E_{\max} = 3.00\times10^{6}\text{ V/m}$.

Therefore,

$$Q_{\max} = \kappa\epsilon_0 A E_{\max} = \left(8.85\times10^{-12}\text{ F/m}\right)\left(5.00\times10^{-4}\text{ m}^2\right)\left(3.00\times10^{6}\text{ V/m}\right) = \boxed{13.3\text{ nC}}.$$

(b) With polystyrene between the plates, $\kappa = 2.56$ and $E_{\max} = 24.0\times10^{6}\text{ V/m}$.

$$Q_{\max} = \kappa\epsilon_0 A E_{\max} = 2.56\left(8.85\times10^{-12}\text{ F/m}\right)\left(5.00\times10^{-4}\text{ m}^2\right)\left(24.0\times10^{6}\text{ V/m}\right) = \boxed{272\text{ nC}}$$

**P26.45** $C = \dfrac{\kappa \in_0 A}{d}$

or $95.0 \times 10^{-9} = \dfrac{3.70\left(8.85 \times 10^{-12}\right)(0.070\,0)\ell}{0.025\,0 \times 10^{-3}}$

$\ell = \boxed{1.04 \text{ m}}$

**P26.46** Consider two sheets of aluminum foil, each 40 cm by 100 cm, with one sheet of plastic between them. Suppose the plastic has $\kappa \cong 3$, $E_{\max} \sim 10^7$ V/m and thickness 1 mil $= \dfrac{2.54 \text{ cm}}{1\,000}$. Then,

$$C = \frac{\kappa \in_0 A}{d} \sim \frac{3\left(8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2\right)\left(0.4 \text{ m}^2\right)}{2.54 \times 10^{-5} \text{ m}} \sim \boxed{10^{-6} \text{ F}}$$

$$\Delta V_{\max} = E_{\max} d \sim \left(10^7 \text{ V/m}\right)\left(2.54 \times 10^{-5} \text{ m}\right) \sim \boxed{10^2 \text{ V}}$$

**P26.47** Originally, $C = \dfrac{\in_0 A}{d} = \dfrac{Q}{(\Delta V)_i}$.

(a) The charge is the same before and after immersion, with value $Q = \dfrac{\in_0 A(\Delta V)_i}{d}$.

$$Q = \frac{\left(8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2\right)\left(25.0 \times 10^{-4} \text{ m}^2\right)(250 \text{ V})}{\left(1.50 \times 10^{-2} \text{ m}\right)} = \boxed{369 \text{ pC}}$$

(b) Finally,

$$C_f = \frac{\kappa \in_0 A}{d} = \frac{Q}{(\Delta V)_f} \qquad C_f = \frac{80.0\left(8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2\right)\left(25.0 \times 10^{-4} \text{ m}^2\right)}{\left(1.50 \times 10^{-2} \text{ m}\right)} = \boxed{118 \text{ pF}}$$

$$(\Delta V)_f = \frac{Qd}{\kappa \in_0 A} = \frac{\in_0 A(\Delta V)_i d}{\kappa \in_0 Ad} = \frac{(\Delta V)_i}{\kappa} = \frac{250 \text{ V}}{80.0} = \boxed{3.12 \text{ V}}.$$

(c) Originally, $U_i = \dfrac{1}{2}C(\Delta V)_i^2 = \dfrac{\in_0 A(\Delta V)_i^2}{2d}$.

Finally, $U_f = \dfrac{1}{2}C_f(\Delta V)_f^2 = \dfrac{\kappa \in_0 A(\Delta V)_i^2}{2d\kappa^2} = \dfrac{\in_0 A(\Delta V)_i^2}{2d\kappa}$.

So, $\Delta U = U_f - U_i = \dfrac{-\in_0 A(\Delta V)_i^2(\kappa - 1)}{2d\kappa}$

$$\Delta U = -\frac{\left(8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2\right)\left(25.0 \times 10^{-4} \text{ m}^2\right)(250 \text{ V})^2(79.0)}{2\left(1.50 \times 10^{-2} \text{ m}\right)(80.0)} = \boxed{-45.5 \text{ nJ}}.$$

**P26.48** (a) $$C=\kappa C_0=\frac{\kappa \in_0 A}{d}=\frac{(173)\left(8.85\times 10^{-12}\right)\left(1.00\times 10^{-4}\text{ m}^2\right)}{0.100\times 10^{-3}\text{ m}}=\boxed{1.53\text{ nF}}$$

(b) The battery delivers the free charge

$$Q=C(\Delta V)=\left(1.53\times 10^{-9}\text{ F}\right)(12.0\text{ V})=\boxed{18.4\text{ nC}}.$$

(c) The surface density of free charge is

$$\sigma=\frac{Q}{A}=\frac{18.4\times 10^{-9}\text{ C}}{1.00\times 10^{-4}\text{ m}^2}=\boxed{1.84\times 10^{-4}\text{ C/m}^2}.$$

The surface density of polarization charge is

$$\sigma_p=\sigma\left(1-\frac{1}{\kappa}\right)=\sigma\left(1-\frac{1}{173}\right)=\boxed{1.83\times 10^{-4}\text{ C/m}^2}.$$

(d) We have $E=\frac{E_0}{\kappa}$ and $E_0=\frac{\Delta V}{d}$; hence,

$$E=\frac{\Delta V}{\kappa d}=\frac{12.0\text{ V}}{(173)\left(1.00\times 10^{-4}\text{ m}\right)}=\boxed{694\text{ V/m}}.$$

**P26.49** The given combination of capacitors is equivalent to the circuit diagram shown to the right.

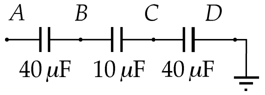

**FIG. P26.49**

Put charge $Q$ on point $A$. Then,

$$Q=(40.0\ \mu\text{F})\Delta V_{AB}=(10.0\ \mu\text{F})\Delta V_{BC}=(40.0\ \mu\text{F})\Delta V_{CD}.$$

So, $\Delta V_{BC}=4\Delta V_{AB}=4\Delta V_{CD}$, and the center capacitor will break down first, at $\Delta V_{BC}=15.0\text{ V}$. When this occurs,

$$\Delta V_{AB}=\Delta V_{CD}=\frac{1}{4}(\Delta V_{BC})=3.75\text{ V}$$

and $V_{AD}=V_{AB}+V_{BC}+V_{CD}=3.75\text{ V}+15.0\text{ V}+3.75\text{ V}=\boxed{22.5\text{ V}}$.

## Section 26.6 Electric Dipole in an Electric Field

**P26.50** (a) The displacement from negative to positive charge is

$$2a=\left(-1.20\hat{\mathbf{i}}+1.10\hat{\mathbf{j}}\right)\text{ mm}-\left(1.40\hat{\mathbf{i}}-1.30\hat{\mathbf{j}}\right)\text{ mm}=\left(-2.60\hat{\mathbf{i}}+2.40\hat{\mathbf{j}}\right)\times 10^{-3}\text{ m}.$$

The electric dipole moment is

$$\mathbf{p}=2\mathbf{a}q=\left(3.50\times 10^{-9}\text{ C}\right)\left(-2.60\hat{\mathbf{i}}+2.40\hat{\mathbf{j}}\right)\times 10^{-3}\text{ m}=\boxed{\left(-9.10\hat{\mathbf{i}}+8.40\hat{\mathbf{j}}\right)\times 10^{-12}\text{ C}\cdot\text{m}}.$$

(b) $$\boldsymbol{\tau}=\mathbf{p}\times\mathbf{E}=\left[\left(-9.10\hat{\mathbf{i}}+8.40\hat{\mathbf{j}}\right)\times 10^{-12}\text{ C}\cdot\text{m}\right]\times\left[\left(7.80\hat{\mathbf{i}}-4.90\hat{\mathbf{j}}\right)\times 10^{3}\text{ N/C}\right]$$

$$\boldsymbol{\tau}=\left(+44.6\hat{\mathbf{k}}-65.5\hat{\mathbf{k}}\right)\times 10^{-9}\text{ N}\cdot\text{m}=\boxed{-2.09\times 10^{-8}\text{ N}\cdot\text{m}\hat{\mathbf{k}}}$$

*continued on next page*

(c) $$U=-\mathbf{p}\cdot\mathbf{E}=-\left[\left(-9.10\hat{\mathbf{i}}+8.40\hat{\mathbf{j}}\right)\times10^{-12}\text{ C}\cdot\text{m}\right]\cdot\left[\left(7.80\hat{\mathbf{i}}-4.90\hat{\mathbf{j}}\right)\times10^{3}\text{ N/C}\right]$$

$$U=(71.0+41.2)\times10^{-9}\text{ J}=\boxed{112\text{ nJ}}$$

(d) $$|\mathbf{p}|=\sqrt{(9.10)^2+(8.40)^2}\times10^{-12}\text{ C}\cdot\text{m}=12.4\times10^{-12}\text{ C}\cdot\text{m}$$

$$|\mathbf{E}|=\sqrt{(7.80)^2+(4.90)^2}\times10^{3}\text{ N/C}=9.21\times10^{3}\text{ N/C}$$

$$U_{\max}=|\mathbf{p}||\mathbf{E}|=114\text{ nJ},\qquad U_{\min}=-114\text{ nJ}$$

$$U_{\max}-U_{\min}=\boxed{228\text{ nJ}}$$

**P26.51** (a) Let $x$ represent the coordinate of the negative charge. Then $x+2a\cos\theta$ is the coordinate of the positive charge. The force on the negative charge is $\mathbf{F}_-=-qE(x)\hat{\mathbf{i}}$. The force on the positive charge is

$$\mathbf{F}_+=+qE(x+2a\cos\theta)\hat{\mathbf{i}}\approx qE(x)\hat{\mathbf{i}}+q\frac{dE}{dx}(2a\cos\theta)\hat{\mathbf{i}}.$$

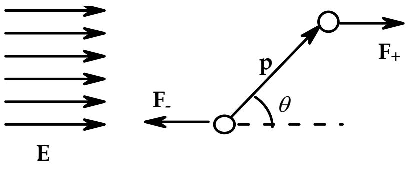

**FIG. P26.51(a)**

The force on the dipole is altogether $\mathbf{F}=\mathbf{F}_-+\mathbf{F}_+=q\dfrac{dE}{dx}(2a\cos\theta)\hat{\mathbf{i}}=\boxed{p\dfrac{dE}{dx}\cos\theta\,\hat{\mathbf{i}}}$.

(b) The balloon creates field along the $x$-axis of $\dfrac{k_eq}{x^2}\hat{\mathbf{i}}$.

Thus, $\dfrac{dE}{dx}=\dfrac{(-2)k_eq}{x^3}$.

At $x=16.0$ cm, $$\frac{dE}{dx}=\frac{(-2)\left(8.99\times10^9\right)\left(2.00\times10^{-6}\right)}{(0.160)^3}=\boxed{-8.78\text{ MN/C}\cdot\text{m}}$$

$$\mathbf{F}=\left(6.30\times10^{-9}\text{ C}\cdot\text{m}\right)\left(-8.78\times10^{6}\text{ N/C}\cdot\text{m}\right)\cos0°\,\hat{\mathbf{i}}=\boxed{-55.3\hat{\mathbf{i}}\text{ mN}}$$

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## Section 26.7 An Atomic Description of Dielectrics

**P26.52** $$2\pi r\ell E=\frac{q_{\text{in}}}{\epsilon_0}$$

so $$E=\frac{\lambda}{2\pi r\,\epsilon_0}$$

$$\Delta V=-\int_{r_1}^{r_2}\mathbf{E}\cdot d\mathbf{r}=\int_{r_1}^{r_2}\frac{\lambda}{2\pi r\,\epsilon_0}dr=\frac{\lambda}{2\pi\,\epsilon_0}\ln\left(\frac{r_1}{r_2}\right)$$

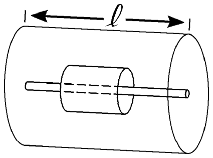

**FIG. P26.52**

$$\frac{\lambda_{\max}}{2\pi\,\epsilon_0}=E_{\max}r_{\text{inner}}$$

$$\Delta V=\left(1.20\times10^{6}\text{ V/m}\right)\left(0.100\times10^{-3}\text{ m}\right)\ln\left(\frac{25.0}{0.200}\right)$$

$$\Delta V_{\max}=\boxed{579\text{ V}}$$

**P26.53** (a) Consider a gaussian surface in the form of a cylindrical pillbox with ends of area $A' << A$ parallel to the sheet. The side wall of the cylinder passes no flux of electric field since this surface is everywhere parallel to the field. Gauss's law becomes

$$EA' + EA' = \frac{Q}{\in A} A', \text{ so } \boxed{E = \frac{Q}{2 \in A}} \text{ directed away from the positive sheet.}$$

(b) In the space between the sheets, each creates field $\frac{Q}{2 \in A}$ away from the positive and toward the negative sheet. Together, they create a field of

$$\boxed{E = \frac{Q}{\in A}}.$$

(c) Assume that the field is in the positive x-direction. Then, the potential of the positive plate relative to the negative plate is

$$\Delta V = -\int_{-\text{plate}}^{+\text{plate}} \mathbf{E} \cdot d\mathbf{s} = -\int_{-\text{plate}}^{+\text{plate}} \frac{Q}{\in A} \hat{\mathbf{i}} \cdot \left(-\hat{\mathbf{i}} dx\right) = \boxed{+\frac{Qd}{\in A}}.$$

(d) Capacitance is defined by: $C = \frac{Q}{\Delta V} = \frac{Q}{Qd/\in A} = \boxed{\frac{\in A}{d} = \frac{\kappa \in_0 A}{d}}$.

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## Additional Problems

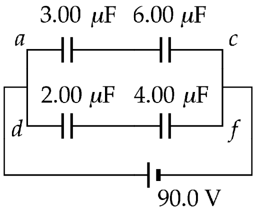


**FIG. P26.54**

**P26.54** (a) $C = \left[\frac{1}{3.00} + \frac{1}{6.00}\right]^{-1} + \left[\frac{1}{2.00} + \frac{1}{4.00}\right]^{-1} = \boxed{3.33\ \mu\text{F}}$

(c) $Q_{ac} = C_{ac}\left(\Delta V_{ac}\right) = (2.00\ \mu\text{F})(90.0\ \text{V}) = 180\ \mu\text{C}$

Therefore, $Q_3 = Q_6 = \boxed{180\ \mu\text{C}}$

$Q_{df} = C_{df}\left(\Delta V_{df}\right) = (1.33\ \mu\text{F})(90.0\ \text{V}) = \boxed{120\ \mu\text{C}}$

(b) $\Delta V_3 = \frac{Q_3}{C_3} = \frac{180\ \mu\text{C}}{3.00\ \mu\text{F}} = \boxed{60.0\ \text{V}}$

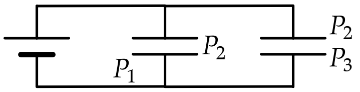
$\Delta V_6 = \frac{Q_6}{C_6} = \frac{180\ \mu\text{C}}{6.00\ \mu\text{F}} = \boxed{30.0\ \text{V}}$

$\Delta V_2 = \frac{Q_2}{C_2} = \frac{120\ \mu\text{C}}{2.00\ \mu\text{F}} = \boxed{60.0\ \text{V}}$

$\Delta V_4 = \frac{Q_4}{C_4} = \frac{120\ \mu\text{C}}{4.00\ \mu\text{F}} = \boxed{30.0\ \text{V}}$

(d) $U_T = \frac{1}{2} C_{eq} (\Delta V)^2 = \frac{1}{2}\left(3.33 \times 10^{-6}\right)(90.0\ \text{V})^2 = \boxed{13.4\ \text{mJ}}$

***P26.55** (a) Each face of $P_2$ carries charge, so the three-plate system is equivalent to



Each capacitor by itself has capacitance

$$C=\frac{\kappa \in_0 A}{d}=\frac{1\left(8.85\times10^{-12}\ \text{C}^2\right)7.5\times10^{-4}\ \text{m}^2}{\text{N}\cdot\text{m}^2\ \ 1.19\times10^{-3}\ \text{m}}=5.58\ \text{pF}.$$

Then equivalent capacitance $=5.58+5.58=\boxed{11.2\ \text{pF}}$.

(b) $Q=C\Delta V+C\Delta V=11.2\times10^{-12}\ \text{F}(12\ \text{V})=\boxed{134\ \text{pC}}$

(c) Now $P_3$ has charge on two surfaces and in effect three capacitors are in parallel:

$C=3(5.58\ \text{pF})=\boxed{16.7\ \text{pF}}$.

(d) Only one face of $P_4$ carries charge:

$Q=C\Delta V=5.58\times10^{-12}\ \text{F}(12\ \text{V})=\boxed{66.9\ \text{pC}}$.

***P26.56** From the example about a cylindrical capacitor,

$$V_b-V_a=-2k_e\lambda\ln\frac{b}{a}$$

$$V_b-345\ \text{kV}=-2\left(8.99\times10^9\ \text{Nm}^2/\text{C}^2\right)\left(1.40\times10^{-6}\ \text{C/m}\right)\ln\frac{12\ \text{m}}{0.024\ \text{m}}$$

$$=-2(8.99)\left(1.4\times10^3\ \text{J/C}\right)\ln 500$$

$$=-1.5643\times10^5\ \text{V}$$

$$V_b=3.45\times10^5\ \text{V}-1.56\times10^5\ \text{V}=\boxed{1.89\times10^5\ \text{V}}$$

***P26.57** Imagine the center plate is split along its midplane and pulled apart. We have two capacitors in parallel, supporting the same $\Delta V$ and carrying total charge $Q$. The upper has capacitance $C_1=\frac{\in_0 A}{d}$ and the lower $C_2=\frac{\in_0 A}{2d}$. Charge flows from ground onto each of the outside plates so that $Q_1+Q_2=Q \qquad \Delta V_1=\Delta V_2=\Delta V$.

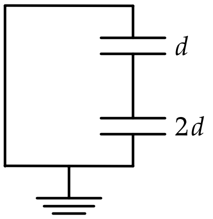


**FIG. P26.57**

Then $\frac{Q_1}{C_1}=\frac{Q_2}{C_2}=\frac{Q_1 d}{\in_0 A}=\frac{Q_2 2d}{\in_0 A} \qquad Q_1=2Q_2 \qquad 2Q_2+Q_2=Q$.

(a) $Q_2=\frac{Q}{3}$. $\boxed{\text{On the lower plate the charge is } -\frac{Q}{3}.}$

$Q_1=\frac{2Q}{3}$. $\boxed{\text{On the upper plate the charge is } -\frac{2Q}{3}.}$

(b) $\Delta V=\frac{Q_1}{C_1}=\boxed{\frac{2Qd}{3\in_0 A}}$

**P26.58** (a) We use Equation 26.11 to find the potential energy of the capacitor. As we will see, the potential difference $\Delta V$ changes as the dielectric is withdrawn. The initial and final energies are $U_i = \frac{1}{2}\left(\frac{Q^2}{C_i}\right)$ and $U_f = \frac{1}{2}\left(\frac{Q^2}{C_f}\right)$.

But the initial capacitance (with the dielectric) is $C_i = \kappa C_f$. Therefore, $U_f = \frac{1}{2}\kappa\left(\frac{Q^2}{C_i}\right)$.

Since the work done by the external force in removing the dielectric equals the change in potential energy, we have $W = U_f - U_i = \frac{1}{2}\kappa\left(\frac{Q^2}{C_i}\right) - \frac{1}{2}\left(\frac{Q^2}{C_i}\right) = \frac{1}{2}\left(\frac{Q^2}{C_i}\right)(\kappa - 1)$.

To express this relation in terms of potential difference $\Delta V_i$, we substitute $Q = C_i(\Delta V_i)$, and evaluate: $W = \frac{1}{2}C_i(\Delta V_i)^2(\kappa - 1) = \frac{1}{2}\left(2.00\times10^{-9}\text{ F}\right)(100\text{ V})^2(5.00 - 1.00) = \boxed{4.00\times10^{-5}\text{ J}}$.

The positive result confirms that the final energy of the capacitor is greater than the initial energy. The extra energy comes from the work done *on* the system by the external force that pulled out the dielectric.

(b) The final potential difference across the capacitor is $\Delta V_f = \frac{Q}{C_f}$.

Substituting $C_f = \frac{C_i}{\kappa}$ and $Q = C_i(\Delta V_i)$ gives $\Delta V_f = \kappa\Delta V_i = 5.00(100\text{ V}) = \boxed{500\text{ V}}$.

Even though the capacitor is isolated and its charge remains constant, the potential difference across the plates does increase in this case.

**P26.59** $\kappa = 3.00$, $E_{\max} = 2.00\times10^{8}\text{ V/m} = \frac{\Delta V_{\max}}{d}$

For $C = \frac{\kappa \in_0 A}{d} = 0.250\times10^{-6}\text{ F}$

$$A = \frac{Cd}{\kappa \in_0} = \frac{C\Delta V_{\max}}{\kappa \in_0 E_{\max}} = \frac{\left(0.250\times10^{-6}\right)(4\,000)}{3.00\left(8.85\times10^{-12}\right)\left(2.00\times10^{8}\right)} = \boxed{0.188\text{ m}^2}$$

***P26.60** The original kinetic energy of the particle is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\left(2\times10^{-16}\text{ kg}\right)\left(2\times10^{6}\text{ m/s}\right)^2 = 4.00\times10^{-4}\text{ J}.$$

The potential difference across the capacitor is $\Delta V = \frac{Q}{C} = \frac{1\,000\ \mu\text{C}}{10\ \mu\text{F}} = 100\text{ V}$.

For the particle to reach the negative plate, the particle-capacitor system would need energy

$$U = q\Delta V = \left(-3\times10^{6}\text{ C}\right)(-100\text{ V}) = 3.00\times10^{-4}\text{ J}.$$

Since its original kinetic energy is greater than this, $\boxed{\text{the particle will reach the negative plate}}$.

As the particle moves, the system keeps constant total energy

$$(K+U)_{\text{at +plate}} = (K+U)_{\text{at −plate}}: \qquad 4.00\times10^{-4}\text{ J} + \left(-3\times10^{-6}\text{ C}\right)(+100\text{ V}) = \frac{1}{2}\left(2\times10^{-16}\right)v_f^2 + 0$$

$$v_f = \sqrt{\frac{2\left(1.00\times10^{-4}\text{ J}\right)}{2\times10^{-16}\text{ kg}}} = \boxed{1.00\times10^{6}\text{ m/s}}.$$

**P26.61** (a) $$C_1=\frac{\kappa_1\in_0 A/2}{d};\ C_2=\frac{\kappa_2\in_0 A/2}{d/2};\ C_3=\frac{\kappa_3\in_0 A/2}{d/2}$$

$$\left(\frac{1}{C_2}+\frac{1}{C_3}\right)^{-1}=\frac{C_2C_3}{C_2+C_3}=\frac{\in_0 A}{d}\left(\frac{\kappa_2\kappa_3}{\kappa_2+\kappa_3}\right)$$

$$C=C_1+\left(\frac{1}{C_2}+\frac{1}{C_3}\right)^{-1}=\boxed{\frac{\in_0 A}{d}\left(\frac{\kappa_1}{2}+\frac{\kappa_2\kappa_3}{\kappa_2+\kappa_3}\right)}$$

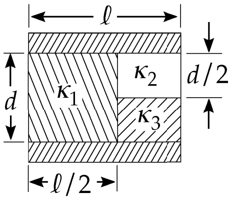


**FIG. P26.61**

(b) Using the given values we find: $C_{\text{total}}=1.76\times10^{-12}\text{ F}=\boxed{1.76\text{ pF}}$.

***P26.62** The initial charge on the larger capacitor is

$$Q=C\Delta V=10\ \mu\text{F}(15\text{ V})=150\ \mu\text{C}.$$

An additional charge $q$ is pushed through the 50-V battery, giving the smaller capacitor charge $q$ and the larger charge $150\ \mu\text{C}+q$.

Then $$50\text{ V}=\frac{q}{5\ \mu\text{F}}+\frac{150\ \mu\text{C}+q}{10\ \mu\text{F}}$$

$$500\ \mu\text{C}=2q+150\ \mu\text{C}+q$$

$$q=117\ \mu\text{C}$$

So across the $5\text{-}\mu\text{F}$ capacitor $\Delta V=\frac{q}{C}=\frac{117\ \mu\text{C}}{5\ \mu\text{F}}=\boxed{23.3\text{ V}}$.

Across the $10\text{-}\mu\text{F}$ capacitor $\Delta V=\frac{150\ \mu\text{C}+117\ \mu\text{C}}{10\ \mu\text{F}}=\boxed{26.7\text{ V}}$.

**P26.63** (a) Put charge $Q$ on the sphere of radius $a$ and $-Q$ on the other sphere. Relative to $V=0$ at infinity,

the potential at the surface of $a$ is $V_a=\frac{k_eQ}{a}-\frac{k_eQ}{d}$

and the potential of $b$ is $V_b=\frac{-k_eQ}{b}+\frac{k_eQ}{d}$.

The difference in potential is $V_a-V_b=\frac{k_eQ}{a}+\frac{k_eQ}{b}-\frac{k_eQ}{d}-\frac{k_eQ}{d}$

and $C=\frac{Q}{V_a-V_b}=\boxed{\left(\frac{4\pi\in_0}{(1/a)+(1/b)-(2/d)}\right)}$.

(b) As $d\to\infty$, $\frac{1}{d}$ becomes negligible compared to $\frac{1}{a}$. Then,

$C=\frac{4\pi\in_0}{1/a+1/b}$ and $\frac{1}{C}=\boxed{\frac{1}{4\pi\in_0 a}+\frac{1}{4\pi\in_0 b}}$

as for two spheres in series.

**P26.64** (a) $$C=\frac{\epsilon_0}{d}\left[(\ell-x)\ell+\kappa\ell x\right]=\boxed{\frac{\epsilon_0}{d}\left[\ell^2+\ell x(\kappa-1)\right]}$$

(b) $$U=\frac{1}{2}C(\Delta V)^2=\boxed{\frac{1}{2}\left(\frac{\epsilon_0(\Delta V)^2}{d}\right)\left[\ell^2+\ell x(\kappa-1)\right]}$$

(c) $$\mathbf{F}=-\left(\frac{dU}{dx}\right)\hat{\mathbf{i}}=\boxed{\frac{\epsilon_0(\Delta V)^2}{2d}\ell(\kappa-1)\text{ to the left}}\text{ (out of the capacitor)}$$

(d) $$F=\frac{(2\,000)^2\left(8.85\times10^{-12}\right)(0.050\,0)(4.50-1)}{2\left(2.00\times10^{-3}\right)}=\boxed{1.55\times10^{-3}\text{ N}}$$

**P26.65** The portion of the capacitor nearly filled by metal has

capacitance $\frac{\kappa\epsilon_0(\ell x)}{d}\to\infty$

and stored energy $\frac{Q^2}{2C}\to0$.

The unfilled portion has

capacitance $\frac{\epsilon_0\ell(\ell-x)}{d}$.

The charge on this portion is $Q=\frac{(\ell-x)Q_0}{\ell}$.

(a) The stored energy is

$$U=\frac{Q^2}{2C}=\frac{\left[(\ell-x)Q_0/\ell\right]^2}{2\epsilon_0\ell(\ell-x)/d}=\boxed{\frac{Q_0^2(\ell-x)d}{2\epsilon_0\ell^3}}.$$

(b) $$F=-\frac{dU}{dx}=-\frac{d}{dx}\left(\frac{Q_0^2(\ell-x)d}{2\epsilon_0\ell^3}\right)=+\frac{Q_0^2d}{2\epsilon_0\ell^3}$$

$$\mathbf{F}=\boxed{\frac{Q_0^2d}{2\epsilon_0\ell^3}\text{ to the right}}\text{ (into the capacitor)}$$

(c) $$\text{Stress }=\frac{F}{\ell d}=\boxed{\frac{Q_0^2}{2\epsilon_0\ell^4}}$$

(d) $$u=\frac{1}{2}\epsilon_0E^2=\frac{1}{2}\epsilon_0\left(\frac{\sigma}{\epsilon_0}\right)^2=\frac{1}{2}\epsilon_0\left(\frac{Q_0}{\epsilon_0\ell^2}\right)^2=\boxed{\frac{Q_0^2}{2\epsilon_0\ell^4}}$$

**P26.66** Gasoline: $$(126\,000\ \text{Btu/gal})(1\,054\ \text{J/Btu})\left(\frac{1.00\ \text{gal}}{3.786\times10^{-3}\ \text{m}^3}\right)\left(\frac{1.00\ \text{m}^3}{670\ \text{kg}}\right)=5.24\times10^{7}\ \text{J/kg}$$

Battery: $$\frac{(12.0\ \text{J/C})(100\ \text{C/s})(3\,600\ \text{s})}{16.0\ \text{kg}}=2.70\times10^{5}\ \text{J/kg}$$

Capacitor: $$\frac{\frac{1}{2}(0.100\ \text{F})(12.0\ \text{V})^2}{0.100\ \text{kg}}=72.0\ \text{J/kg}$$

$\boxed{\text{Gasoline has 194 times the specific energy content of the battery and 727 000 times that of the capacitor.}}$

**P26.67** Call the unknown capacitance $C_u$

$$Q=C_u(\Delta V_i)=(C_u+C)(\Delta V_f)$$

$$\boxed{C_u=\frac{C(\Delta V_f)}{(\Delta V_i)-(\Delta V_f)}}=\frac{(10.0\ \mu\text{F})(30.0\ \text{V})}{(100\ \text{V}-30.0\ \text{V})}=\boxed{4.29\ \mu\text{F}}$$

***P26.68** She can clip together a series combination of parallel combinations of two 100-$\mu$F capacitors. The equivalent capacitance is $\dfrac{1}{(200\ \mu\text{F})^{-1}+(200\ \mu\text{F})^{-1}}=100\ \mu\text{F}$. When 90 V is connected across the combination, only $\boxed{45\ \text{V}}$ appears across each individual capacitor.

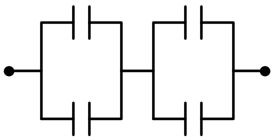


**FIG. P26.68**

**P26.69** (a) $$C_0=\frac{\epsilon_0 A}{d}=\frac{Q_0}{\Delta V_0}$$

When the dielectric is inserted at constant voltage,

$$C=\kappa C_0=\frac{Q}{\Delta V_0};$$

$$U_0=\frac{C_0(\Delta V_0)^2}{2}$$

$$U=\frac{C(\Delta V_0)^2}{2}=\frac{\kappa C_0(\Delta V_0^2)}{2}$$

and $\dfrac{U}{U_0}=\kappa$.

The extra energy comes from (part of the) electrical work done by the battery in separating the extra charge.

(b) $$Q_0=C_0\Delta V_0$$

and $Q=C\Delta V_0=\kappa C_0\Delta V_0$

so $\boxed{\dfrac{Q}{Q_0}=\kappa}$.

**P26.70** The vertical orientation sets up two capacitors in parallel, with equivalent capacitance

$$C_p = \frac{\epsilon_0 (A/2)}{d} + \frac{\kappa \epsilon_0 (A/2)}{d} = \left(\frac{\kappa+1}{2}\right)\frac{\epsilon_0 A}{d}$$

where $A$ is the area of either plate and $d$ is the separation of the plates. The horizontal orientation produces two capacitors in series. If $f$ is the fraction of the horizontal capacitor filled with dielectric, the equivalent capacitance is

$$\frac{1}{C_s} = \frac{fd}{\kappa \epsilon_0 A} + \frac{(1-f)d}{\epsilon_0 A} = \left[\frac{f+\kappa(1-f)}{\kappa}\right]\frac{d}{\epsilon_0 A}, \text{ or } C_s = \left[\frac{\kappa}{f+\kappa(1-f)}\right]\frac{\epsilon_0 A}{d}.$$

Requiring that $C_p = C_s$ gives $\frac{\kappa+1}{2} = \frac{\kappa}{f+\kappa(1-f)}$, or $(\kappa+1)[f+\kappa(1-f)] = 2\kappa$.

For $\kappa = 2.00$, this yields $3.00[2.00-(1.00)f] = 4.00$, with the solution $f = \boxed{\frac{2}{3}}$.

**P26.71** Initially (capacitors charged in parallel),

$$q_1 = C_1(\Delta V) = (6.00\ \mu\text{F})(250\ \text{V}) = 1\,500\ \mu\text{C}$$

$$q_2 = C_2(\Delta V) = (2.00\ \mu\text{F})(250\ \text{V}) = 500\ \mu\text{C}.$$

After reconnection (positive plate to negative plate),

$$q'_{\text{total}} = q_1 - q_2 = 1\,000\ \mu\text{C} \text{ and } \Delta V' = \frac{q'_{\text{total}}}{C_{\text{total}}} = \frac{1\,000\ \mu\text{C}}{8.00\ \mu\text{F}} = 125\ \text{V}.$$

Therefore,

$$q'_1 = C_1(\Delta V') = (6.00\ \mu\text{F})(125\ \text{V}) = \boxed{750\ \mu\text{C}}$$

$$q'_2 = C_2(\Delta V') = (2.00\ \mu\text{F})(125\ \text{V}) = \boxed{250\ \mu\text{C}}.$$

**P26.72** Assume a potential difference across $a$ and $b$, and notice that the potential difference across the $8.00\ \mu\text{F}$ capacitor must be zero by symmetry. Then the equivalent capacitance can be determined from the following circuit:

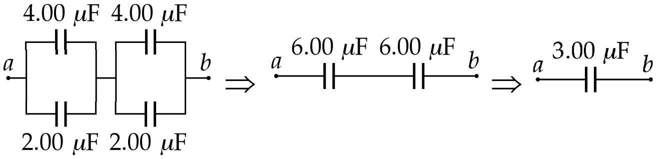


FIG. P26.72

$C_{ab} = \boxed{3.00\ \mu\text{F}}$.

**P26.73** $E_{\max}$ occurs at the inner conductor's surface.

$E_{\max} = \frac{2k_e\lambda}{a}$ from Equation 24.7.

$\Delta V = 2k_e\lambda\ln\left(\frac{b}{a}\right)$ from Example 26.2

$$E_{\max} = \frac{\Delta V}{a\ln(b/a)}$$

$$\Delta V_{\max} = E_{\max}a\ln\left(\frac{b}{a}\right) = \left(18.0\times10^{6}\text{ V/m}\right)\left(0.800\times10^{-3}\text{ m}\right)\ln\left(\frac{3.00}{0.800}\right) = \boxed{19.0\text{ kV}}.$$

**P26.74** $E = \frac{2\kappa\lambda}{a}$; $\Delta V = 2\kappa\lambda\ln\left(\frac{b}{a}\right)$

$$\Delta V_{\max} = E_{\max}a\ln\left(\frac{b}{a}\right)$$

$$\frac{dV_{\max}}{da} = E_{\max}\left[\ln\left(\frac{b}{a}\right) + a\left(\frac{1}{b/a}\right)\left(-\frac{b}{a^2}\right)\right] = 0$$

$$\ln\left(\frac{b}{a}\right) = 1 \text{ or } \frac{b}{a} = e^1 \text{ so } \boxed{a = \frac{b}{e}}$$

**P26.75** By symmetry, the potential difference across 3C is zero, so the circuit reduces to

$$C_{eq} = \left(\frac{1}{2C} + \frac{1}{4C}\right)^{-1} = \frac{8}{6}C = \boxed{\frac{4}{3}C}.$$

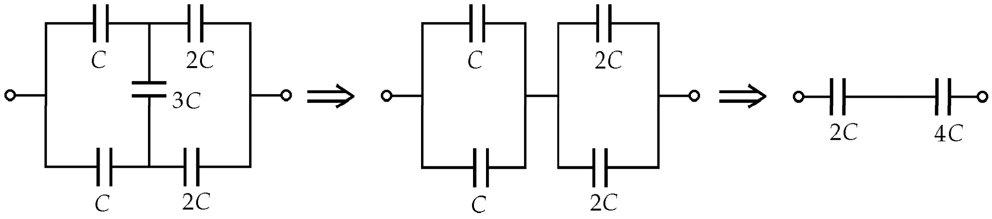


FIG. P26.75

**P26.76** The electric field due to the charge on the positive wire is perpendicular to the wire, radial, and of magnitude

$$E_+ = \frac{\lambda}{2\pi \in_0 r}.$$

The potential difference between wires due to the presence of this charge is

$$\Delta V_1 = -\int_{-\text{wire}}^{+\text{wire}} \mathbf{E}\cdot d\mathbf{r} = -\frac{\lambda}{2\pi \in_0}\int_{D-d}^{d}\frac{dr}{r} = \frac{\lambda}{2\pi \in_0}\ln\left(\frac{D-d}{d}\right).$$

The presence of the linear charge density $-\lambda$ on the negative wire makes an identical contribution to the potential difference between the wires. Therefore, the total potential difference is

$$\Delta V = 2(\Delta V_1) = \frac{\lambda}{\pi \in_0}\ln\left(\frac{D-d}{d}\right)$$

and the capacitance of this system of two wires, each of length $\ell$, is

$$C = \frac{Q}{\Delta V} = \frac{\lambda\ell}{\Delta V} = \frac{\lambda\ell}{(\lambda/\pi \in_0)\ln[(D-d)/d]} = \frac{\pi \in_0 \ell}{\ln[(D-d)/d]}.$$

The capacitance per unit length is: $\boxed{\dfrac{C}{\ell} = \dfrac{\pi \in_0}{\ln[(D-d)/d]}}$.

***P26.77** The condition that we are testing is that the capacitance increases by less than 10%, or,

$$\frac{C'}{C} < 1.10.$$

Substituting the expressions for $C$ and $C'$ from Example 26.2, we have,

$$\frac{C'}{C} = \frac{\dfrac{\ell}{2k_e\ln\left(\frac{b}{1.10a}\right)}}{\dfrac{\ell}{2k_e\ln\left(\frac{b}{a}\right)}} = \frac{\ln\left(\frac{b}{a}\right)}{\ln\left(\frac{b}{1.10a}\right)} < 1.10.$$

This becomes,

$$\ln\left(\frac{b}{a}\right) < 1.10\ln\left(\frac{b}{1.10a}\right) = 1.10\ln\left(\frac{b}{a}\right) + 1.10\ln\left(\frac{1}{1.10}\right) = 1.10\ln\left(\frac{b}{a}\right) - 1.10\ln(1.10).$$

We can rewrite this as,

$$-0.10\ln\left(\frac{b}{a}\right) < -1.10\ln(1.10)$$

$$\ln\left(\frac{b}{a}\right) > 11.0\ln(1.10) = \ln(1.10)^{11.0}$$

where we have reversed the direction of the inequality because we multiplied the whole expression by –1 to remove the negative signs. Comparing the arguments of the logarithms on both sides of the inequality, we see that,

$$\frac{b}{a} > (1.10)^{11.0} = 2.85.$$

Thus, if $b > 2.85a$, the increase in capacitance is less than 10% and it is more effective to increase $\ell$.

## ANSWERS TO EVEN PROBLEMS

| | |
|---|---|
| **P26.2** | (a) $1.00\ \mu\text{F}$; (b) $100\ \text{V}$ |
| **P26.4** | (a) $8.99\ \text{mm}$; (b) $0.222\ \text{pF}$; (c) $22.2\ \text{pC}$ |
| **P26.6** | $11.1\ \text{nF}$; $26.6\ \text{C}$ |
| **P26.8** | $3.10\ \text{nm}$ |
| **P26.10** | $\frac{(2N-1)\in_0(\pi-\theta)R^2}{d}$ |
| **P26.12** | $2.13\times10^{16}\ \text{m}^3$ |
| **P26.14** | $\frac{mgd\tan\theta}{q}$ |
| **P26.16** | (a) $17.0\ \mu\text{F}$; (b) $9.00\ \text{V}$;<br>(c) $45.0\ \mu\text{C}$ and $108\ \mu\text{C}$ |
| **P26.18** | $1.83C$ |
| **P26.20** | $\frac{C_p}{2}+\sqrt{\frac{C_p^2}{4}-C_pC_s}$ and $\frac{C_p}{2}-\sqrt{\frac{C_p^2}{4}-C_pC_s}$ |
| **P26.22** | (a) $2C$; (b) $Q_1>Q_3>Q_2$;<br>(c) $\Delta V_1>\Delta V_2=\Delta V_3$;<br>(d) $Q_3$ and $Q_1$ increase and $Q_2$ decreases |
| **P26.24** | (a) $398\ \mu\text{F}$ in series; (b) $2.20\ \mu\text{F}$ in parallel |
| **P26.26** | $19.8\ \mu\text{C}$ |
| **P26.28** | $83.6\ \mu\text{C}$ |
| **P26.30** | $\left(\sqrt{3}-1\right)\frac{C_0}{2}$ |
| **P26.32** | $4.47\ \text{kV}$ |
| **P26.34** | energy doubles |
| **P26.36** | $2.51\times10^{-3}\ \text{m}^3=2.51\ \text{L}$ |
| **P26.38** | (a) $400\ \mu\text{C}$; (b) $2.50\ \text{kN/m}$ |
| **P26.40** | (a) $C(\Delta V)^2$; (b) $\frac{4\Delta V}{3}$; (c) $4C\frac{(\Delta V)^2}{3}$;<br>(d) Positive work is done on the system by the agent pulling the plates apart. |
| **P26.42** | (a) $q_1=\frac{R_1Q}{R_1+R_2}$ and $q_2=\frac{R_2Q}{R_1+R_2}$;<br>(b) see the solution |
| **P26.44** | (a) $13.3\ \text{nC}$; (b) $272\ \text{nC}$ |
| **P26.46** | $\sim10^{-6}\ \text{F}$ and $\sim10^{2}\ \text{V}$ for two 40 cm by 100 cm sheets of aluminum foil sandwiching a thin sheet of plastic. |
| **P26.48** | (a) $1.53\ \text{nF}$; (b) $18.4\ \text{nC}$; (c) $184\ \mu\text{C/m}^2$ free; $183\ \mu\text{C/m}^2$ induced; (d) $694\ \text{V/m}$ |
| **P26.50** | (a) $\left(-9.10\hat{\mathbf{i}}+8.40\hat{\mathbf{j}}\right)\ \text{pC}\cdot\text{m}$;<br>(b) $-20.9\ \text{nN}\cdot\text{m}\hat{\mathbf{k}}$; (c) $112\ \text{nJ}$; (d) $228\ \text{nJ}$ |
| **P26.52** | $579\ \text{V}$ |
| **P26.54** | (a) $3.33\ \mu\text{F}$;<br>(b) $\Delta V_3=60.0\ \text{V}$; $\Delta V_6=30.0\ \text{V}$;<br>$\Delta V_2=60.0\ \text{V}$; $\Delta V_4=30.0\ \text{V}$;<br>(c) $Q_3=Q_6=180\ \mu\text{C}$; $Q_2=Q_4=120\ \mu\text{C}$;<br>(d) $13.4\ \text{mJ}$ |
| **P26.56** | $189\ \text{kV}$ |
| **P26.58** | (a) $40.0\ \mu\text{J}$; (b) $500\ \text{V}$ |
| **P26.60** | yes; $1.00\ \text{Mm/s}$ |
| **P26.62** | $23.3\ \text{V}$; $26.7\ \text{V}$ |
| **P26.64** | (a) $\frac{\in_0\left[\ell^2+\ell x(\kappa-1)\right]}{d}$;<br>(b) $\frac{\in_0(\Delta V)^2\left[\ell^2+\ell x(\kappa-1)\right]}{2d}$;<br>(c) $\frac{\in_0(\Delta V)^2\ell(\kappa-1)}{2d}$ to the left;<br>(d) $1.55\ \text{mN}$ left |

**P26.66** Gasoline has 194 times the specific energy content of the battery, and 727 000 times that of the capacitor.

**P26.68** see the solution; 45 V

**P26.70** $\frac{2}{3}$

**P26.72** 3.00 $\mu$F

**P26.74** see the solution

**P26.76** see the solution

# 27

# Current and Resistance

**CHAPTER OUTLINE**

27.1 Electric Current
27.2 Resistance
27.3 A Model for Electrical Conduction
27.4 Resistance and Temperature
27.5 Superconductors
27.6 Electric Power

## ANSWERS TO QUESTIONS

**Q27.1** Individual vehicles—cars, trucks and motorcycles—would correspond to charge. The number of vehicles that pass a certain point in a given time would correspond to the current.

**Q27.2** Voltage is a measure of potential difference, not of current. "Surge" implies a flow—and only charge, in coulombs, can flow through a system. It would also be correct to say that the victim carried a certain current, in amperes.

**Q27.3** Geometry and resistivity. In turn, the resistivity of the material depends on the temperature.

**Q27.4** Resistance is a physical property of the conductor based on the material of which it is made and its size and shape, including the locations where current is put in and taken out. Resistivity is a physical property only of the material of which the resistor is made.

**Q27.5** The radius of wire B is $\sqrt{3}$ times the radius of wire A, to make its cross–sectional area 3 times larger.

**Q27.6** Not all conductors obey Ohm's law at all times. For example, consider an experiment in which a variable potential difference is applied across an incandescent light bulb, and the current is measured. At very low voltages, the filament follows Ohm's law nicely. But then long before the filament begins to glow, the plot of $\frac{\Delta V}{I}$ becomes non-linear, because the resistivity is temperature-dependent.

**Q27.7** A conductor is not in electrostatic equilibrium when it is carrying a current, duh! If charges are placed on an isolated conductor, the electric fields established in the conductor by the charges will cause the charges to move until they are in positions such that there is zero electric field throughout the conductor. A conductor carrying a steady current is not an isolated conductor—its ends must be connected to a source of emf, such as a battery. The battery maintains a potential difference across the conductor and, therefore, an electric field in the conductor. The steady current is due to the response of the electrons in the conductor due to this constant electric field.

**Q27.8** The bottom of the rods on the *Jacob's Ladder* are close enough so that the supplied voltage is sufficient to produce dielectric breakdown of the air. The initial spark at the bottom includes a tube of ionized air molecules. Since this tube containing ions is warmer than the air around it, it is buoyed up by the surrounding air and begins to rise. The ions themselves significantly decrease the resistivity of the air. They significantly lower the dielectric strength of the air, marking longer sparks possible. Internal resistance in the power supply will typically make its terminal voltage drop, so that it cannot produce a spark across the bottom ends of the rods. A single "continuous" spark, therefore will rise up, becoming longer and longer, until the potential difference is not large enough to sustain dielectric breakdown of the air. Once the initial spark stops, another one will form at the bottom, where again, the supplied potential difference is sufficient to break down the air.

**Q27.9** The conductor does not follow Ohm's law, and must have a resistivity that is current-dependent, or more likely temperature-dependent.

**Q27.10** A power supply would correspond to a water pump; a resistor corresponds to a pipe of a certain diameter, and thus resistance to flow; charge corresponds to the water itself; potential difference corresponds to difference in height between the ends of a pipe or the ports of a water pump.

**Q27.11** The amplitude of atomic vibrations increases with temperature. Atoms can then scatter electrons more efficiently.

**Q27.12** In a metal, the conduction electrons are not strongly bound to individual ion cores. They can move in response to an applied electric field to constitute an electric current. Each metal ion in the lattice of a microcrystal exerts Coulomb forces on its neighbors. When one ion is vibrating rapidly, it can set its neighbors into vibration. This process represents energy moving though the material by heat.

**Q27.13** The resistance of copper *increases* with temperature, while the resistance of silicon *decreases* with increasing temperature. The conduction electrons are scattered more by vibrating atoms when copper heats up. Silicon's charge carrier density increases as temperature increases and more atomic electrons are promoted to become conduction electrons.

**Q27.14** A current will continue to exist in a superconductor without voltage because there is no resistance loss.

**Q27.15** Superconductors have no resistance when they are below a certain critical temperature. For most superconducting materials, this critical temperature is close to absolute zero. It requires expensive refrigeration, often using liquid helium. Liquid nitrogen at 77 K is much less expensive. Recent discoveries of materials that have higher critical temperatures suggest the possibility of developing superconductors that do not require expensive cooling systems.

**Q27.16** In a normal metal, suppose that we could proceed to a limit of zero resistance by lengthening the average time between collisions. The classical model of conduction then suggests that a constant applied voltage would cause constant acceleration of the free electrons, and a current steadily increasing in time.

On the other hand, we can actually switch to zero resistance by substituting a superconducting wire for the normal metal. In this case, the drift velocity of electrons is established by vibrations of atoms in the crystal lattice; the maximum current is limited; and it becomes impossible to establish a potential difference across the superconductor.

**Q27.17** Because there are so many electrons in a conductor (approximately $10^{28}$ electrons/m$^3$) the average velocity of charges is very slow. When you connect a wire to a potential difference, you establish an electric field everywhere in the wire nearly instantaneously, to make electrons start drifting everywhere all at once.

**Q27.18** Current moving through a wire is analogous to a longitudinal wave moving through the electrons of the atoms. The wave speed depends on the speed at which the disturbance in the electric field can be communicated between neighboring atoms, not on the drift velocities of the electrons themselves. If you leave a direct-current light bulb on for a reasonably short time, it is likely that no single electron will enter one end of the filament and leave at the other end.

**Q27.19** More power is delivered to the resistor with the smaller resistance, since $\mathcal{P}=\dfrac{\Delta V^2}{R}$.

**Q27.20** The 25 W bulb has a higher resistance. The 100 W bulb carries more current.

**Q27.21** One ampere–hour is 3 600 coulombs. The ampere–hour rating is the quantity of charge that the battery can lift though its nominal potential difference.

**Q27.22** Choose the voltage of the power supply you will use to drive the heater. Next calculate the required resistance $R$ as $\dfrac{\Delta V^2}{\mathcal{P}}$. Knowing the resistivity $\rho$ of the material, choose a combination of wire length and cross–sectional area to make $\left(\dfrac{\ell}{A}\right)=\left(\dfrac{R}{\rho}\right)$. You will have to pay for less material if you make both $\ell$ and $A$ smaller, but if you go too far the wire will have too little surface area to radiate away the energy; then the resistor will melt.

## SOLUTIONS TO PROBLEMS

### Section 27.1 **Electric Current**

**P27.1** $I=\dfrac{\Delta Q}{\Delta t}$ $\qquad \Delta Q=I\Delta t=\left(30.0\times10^{-6}\text{ A}\right)(40.0\text{ s})=1.20\times10^{-3}\text{ C}$

$$N=\frac{Q}{e}=\frac{1.20\times10^{-3}\text{ C}}{1.60\times10^{-19}\text{ C/electron}}=\boxed{7.50\times10^{15}\text{ electrons}}$$

**P27.2** The molar mass of silver $=107.9\text{ g/mole}$ and the volume $V$ is

$$V=(\text{area})(\text{thickness})=\left(700\times10^{-4}\text{ m}^2\right)\left(0.133\times10^{-3}\text{ m}\right)=9.31\times10^{-6}\text{ m}^3 .$$

The mass of silver deposited is $m_{\text{Ag}}=\rho V=\left(10.5\times10^3\text{ kg/m}^3\right)\left(9.31\times10^{-6}\text{ m}^3\right)=9.78\times10^{-2}\text{ kg}$.

And the number of silver atoms deposited is

$$N=\left(9.78\times10^{-2}\text{ kg}\right)\left(\frac{6.02\times10^{23}\text{ atoms}}{107.9\text{ g}}\right)\left(\frac{1\,000\text{ g}}{1\text{ kg}}\right)=5.45\times10^{23}\text{ atoms}$$

$$I=\frac{\Delta V}{R}=\frac{12.0\text{ V}}{1.80\ \Omega}=6.67\text{ A}=6.67\text{ C/s}$$

$$\Delta t=\frac{\Delta Q}{I}=\frac{Ne}{I}=\frac{\left(5.45\times10^{23}\right)\left(1.60\times10^{-19}\text{ C}\right)}{6.67\text{ C/s}}=1.31\times10^4\text{ s}=\boxed{3.64\text{ h}}$$

**P27.3** $Q(t)=\int_0^t I dt = I_0\tau\left(1-e^{-t/\tau}\right)$

(a) $Q(\tau)=I_0\tau\left(1-e^{-1}\right)=\boxed{(0.632)I_0\tau}$

(b) $Q(10\tau)=I_0\tau\left(1-e^{-10}\right)=\boxed{(0.999\,95)I_0\tau}$

(c) $Q(\infty)=I_0\tau\left(1-e^{-\infty}\right)=\boxed{I_0\tau}$

**P27.4** (a) Using $\frac{k_e e^2}{r^2}=\frac{mv^2}{r}$, we get: $v=\sqrt{\frac{k_e e^2}{mr}}=\boxed{2.19\times10^6\text{ m/s}}$.

(b) The time for the electron to revolve around the proton once is:

$$t=\frac{2\pi r}{v}=\frac{2\pi\left(5.29\times10^{-11}\text{ m}\right)}{\left(2.19\times10^6\text{ m/s}\right)}=1.52\times10^{-16}\text{ s}.$$

The total charge flow in this time is $1.60\times10^{-19}\text{ C}$, so the current is

$$I=\frac{1.60\times10^{-19}\text{ C}}{1.52\times10^{-16}\text{ s}}=1.05\times10^{-3}\text{ A}=\boxed{1.05\text{ mA}}.$$

**P27.5** The period of revolution for the sphere is $T=\frac{2\pi}{\omega}$, and the average current represented by this revolving charge is $I=\frac{q}{T}=\boxed{\frac{q\omega}{2\pi}}$.

**P27.6** $q=4t^3+5t+6$

$$A=\left(2.00\text{ cm}^2\right)\left(\frac{1.00\text{ m}}{100\text{ cm}}\right)^2=2.00\times10^{-4}\text{ m}^2$$

(a) $I(1.00\text{ s})=\left.\frac{dq}{dt}\right|_{t=1.00\text{ s}}=\left.\left(12t^2+5\right)\right|_{t=1.00\text{ s}}=\boxed{17.0\text{ A}}$

(b) $J=\frac{I}{A}=\frac{17.0\text{ A}}{2.00\times10^{-4}\text{ m}^2}=\boxed{85.0\text{ kA/m}^2}$

**P27.7** $I=\frac{dq}{dt}$

$$q=\int dq=\int I dt=\int_0^{1/240\text{ s}}(100\text{ A})\sin\left(\frac{120\pi t}{\text{s}}\right)dt$$

$$q=\frac{-100\text{ C}}{120\pi}\left[\cos\left(\frac{\pi}{2}\right)-\cos 0\right]=\frac{+100\text{ C}}{120\pi}=\boxed{0.265\text{ C}}$$

**P27.8** (a) $J = \frac{I}{A} = \frac{5.00\text{ A}}{\pi\left(4.00 \times 10^{-3}\text{ m}\right)^2} = \boxed{99.5\text{ kA/m}^2}$

(b) $J_2 = \frac{1}{4}J_1$; $\frac{I}{A_2} = \frac{1}{4}\frac{I}{A_1}$

$A_1 = \frac{1}{4}A_2$ so $\pi\left(4.00 \times 10^{-3}\right)^2 = \frac{1}{4}\pi r_2^2$

$r_2 = 2\left(4.00 \times 10^{-3}\right) = 8.00 \times 10^{-3}\text{ m} = \boxed{8.00\text{ mm}}$

**P27.9** (a) $J = \frac{I}{A} = \frac{8.00 \times 10^{-6}\text{ A}}{\pi\left(1.00 \times 10^{-3}\text{ m}\right)^2} = \boxed{2.55\text{ A/m}^2}$

(b) From $J = nev_d$, we have $n = \frac{J}{ev_d} = \frac{2.55\text{ A/m}^2}{\left(1.60 \times 10^{-19}\text{ C}\right)\left(3.00 \times 10^8\text{ m/s}\right)} = \boxed{5.31 \times 10^{10}\text{ m}^{-3}}$.

(c) From $I = \frac{\Delta Q}{\Delta t}$, we have $\Delta t = \frac{\Delta Q}{I} = \frac{N_A e}{I} = \frac{\left(6.02 \times 10^{23}\right)\left(1.60 \times 10^{-19}\text{ C}\right)}{8.00 \times 10^{-6}\text{ A}} = \boxed{1.20 \times 10^{10}\text{ s}}$.

(This is about 382 years!)

**P27.10** (a) The speed of each deuteron is given by $K = \frac{1}{2}mv^2$

$\left(2.00 \times 10^6\right)\left(1.60 \times 10^{-19}\text{ J}\right) = \frac{1}{2}\left(2 \times 1.67 \times 10^{-27}\text{ kg}\right)v^2$ and $v = 1.38 \times 10^7\text{ m/s}$.

The time between deuterons passing a stationary point is $t$ in $I = \frac{q}{t}$

$10.0 \times 10^{-6}\text{ C/s} = 1.60 \times 10^{-19}\text{ C}/t$ or $t = 1.60 \times 10^{-14}\text{ s}$.

So the distance between them is $vt = \left(1.38 \times 10^7\text{ m/s}\right)\left(1.60 \times 10^{-14}\text{ s}\right) = \boxed{2.21 \times 10^{-7}\text{ m}}$.

(b) One nucleus will put its nearest neighbor at potential

$$V = \frac{k_e q}{r} = \frac{\left(8.99 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60 \times 10^{-19}\text{ C}\right)}{2.21 \times 10^{-7}\text{ m}} = 6.49 \times 10^{-3}\text{ V}.$$

This is very small compared to the 2 MV accelerating potential, so repulsion within the beam is a small effect.

**P27.11** We use $I = nqAv_d$ $n$ is the number of charge carriers per unit volume, and is identical to the number of atoms per unit volume. We assume a contribution of 1 free electron per atom in the relationship above. For aluminum, which has a molar mass of 27, we know that Avogadro's number of atoms, $N_A$, has a mass of 27.0 g. Thus, the mass per atom is

$$\frac{27.0\text{ g}}{N_A} = \frac{27.0\text{ g}}{6.02\times10^{23}} = 4.49\times10^{-23}\text{ g/atom}.$$

Thus, $$n = \frac{\text{density of aluminum}}{\text{mass per atom}} = \frac{2.70\text{ g/cm}^3}{4.49\times10^{-23}\text{ g/atom}}$$

$$n = 6.02\times10^{22}\text{ atoms/cm}^3 = 6.02\times10^{28}\text{ atoms/m}^3 .$$

Therefore, $$v_d = \frac{I}{nqA} = \frac{5.00\text{ A}}{\left(6.02\times10^{28}\text{ m}^{-3}\right)\left(1.60\times10^{-19}\text{ C}\right)\left(4.00\times10^{-6}\text{ m}^2\right)} = 1.30\times10^{-4}\text{ m/s}$$

or, $$v_d = \boxed{0.130\text{ mm/s}}.$$

## Section 27.2 **Resistance**

***P27.12** $$J = \sigma E = \frac{E}{\rho} = \frac{0.740\text{ V/m}}{2.44\times10^{-8}\ \Omega\cdot\text{m}}\left(\frac{1\ \Omega\cdot\text{A}}{1\text{ V}}\right) = \boxed{3.03\times10^{7}\text{ A/m}^2}$$

**P27.13** $$I = \frac{\Delta V}{R} = \frac{120\text{ V}}{240\ \Omega} = 0.500\text{ A} = \boxed{500\text{ mA}}$$

**P27.14** (a) Applying its definition, we find the resistance of the rod,

$$R = \frac{\Delta V}{I} = \frac{15.0\text{ V}}{4.00\times10^{-3}\text{ A}} = 3\,750\ \Omega = \boxed{3.75\text{ k}\Omega}.$$

(b) The length of the rod is determined from the definition of resistivity: $R = \frac{\rho\ell}{A}$. Solving for $\ell$ and substituting numerical values for $R$, $A$, and the value of $\rho$ given for carbon in Table 27.1, we obtain

$$\ell = \frac{RA}{\rho} = \frac{\left(3.75\times10^{3}\ \Omega\right)\left(5.00\times10^{-6}\text{ m}^2\right)}{\left(3.50\times10^{-5}\ \Omega\cdot\text{m}\right)} = \boxed{536\text{ m}}.$$

**P27.15** $\Delta V = IR$

and $R = \frac{\rho\ell}{A}$: $$A = (0.600\text{ mm})^2\left(\frac{1.00\text{ m}}{1\,000\text{ mm}}\right)^2 = 6.00\times10^{-7}\text{ m}^2$$

$\Delta V = \frac{I\rho\ell}{A}$: $$I = \frac{\Delta VA}{\rho\ell} = \frac{(0.900\text{ V})\left(6.00\times10^{-7}\text{ m}^2\right)}{\left(5.60\times10^{-8}\ \Omega\cdot\text{m}\right)(1.50\text{ m})}$$

$$I = \boxed{6.43\text{ A}}$$

**P27.16** $J = \frac{I}{\pi r^2} = \sigma E = \frac{3.00\text{ A}}{\pi(0.012\,0\text{ m})^2} = \sigma(120\text{ N/C})$

$\sigma = 55.3(\Omega\cdot\text{m})^{-1}$ $\rho = \frac{1}{\sigma} = \boxed{0.018\,1\ \Omega\cdot\text{m}}$

**P27.17** (a) Given $M = \rho_d V = \rho_d A\ell$ where $\rho_d \equiv$ mass density,

we obtain: $A = \frac{M}{\rho_d \ell}$. Taking $\rho_r \equiv$ resistivity, $R = \frac{\rho_r \ell}{A} = \frac{\rho_r \ell}{M/\rho_d \ell} = \frac{\rho_r \rho_d \ell^2}{M}$.

Thus, $\ell = \sqrt{\frac{MR}{\rho_r \rho_d}} = \sqrt{\frac{(1.00\times10^{-3})(0.500)}{(1.70\times10^{-8})(8.92\times10^{3})}}$ $\ell = \boxed{1.82\text{ m}}$.

(b) $V = \frac{M}{\rho_d}$, or $\pi r^2 \ell = \frac{M}{\rho_d}$.

Thus, $r = \sqrt{\frac{M}{\pi \rho_d \ell}} = \sqrt{\frac{1.00\times10^{-3}}{\pi(8.92\times10^{3})(1.82)}}$ $r = 1.40\times10^{-4}\text{ m}$.

The diameter is twice this distance: diameter $= \boxed{280\ \mu\text{m}}$.

***P27.18** The volume of the gram of gold is given by $\rho = \frac{m}{V}$

$$V = \frac{m}{\rho} = \frac{10^{-3}\text{ kg}}{19.3\times10^{3}\text{ kg/m}^3} = 5.18\times10^{-8}\text{ m}^3 = A(2.40\times10^{3}\text{ m})$$

$$A = 2.16\times10^{-11}\text{ m}^2$$

$$R = \frac{\rho\ell}{A} = \frac{2.44\times10^{-8}\ \Omega\cdot\text{m}(2.4\times10^{3}\text{ m})}{2.16\times10^{-11}\text{ m}^2} = \boxed{2.71\times10^{6}\ \Omega}$$

**P27.19** (a) Suppose the rubber is 10 cm long and 1 mm in diameter.

$$R = \frac{\rho\ell}{A} = \frac{4\rho\ell}{\pi d^2} \sim \frac{4(10^{13}\ \Omega\cdot\text{m})(10^{-1}\text{ m})}{\pi(10^{-3}\text{ m})^2} = \boxed{\sim10^{18}\ \Omega}$$

(b) $$R = \frac{4\rho\ell}{\pi d^2} \sim \frac{4(1.7\times10^{-8}\ \Omega\cdot\text{m})(10^{-3}\text{ m})}{\pi(2\times10^{-2}\text{ m})^2}\boxed{\sim10^{-7}\ \Omega}$$

(c) $$I = \frac{\Delta V}{R} \sim \frac{10^{2}\text{ V}}{10^{18}\ \Omega}\boxed{\sim10^{-16}\text{ A}}$$

$$I \sim \frac{10^{2}\text{ V}}{10^{-7}\ \Omega}\boxed{\sim10^{9}\text{ A}}$$

**P27.20** The distance between opposite faces of the cube is $\ell = \left(\frac{90.0\text{ g}}{10.5\text{ g/cm}^3}\right)^{1/3} = 2.05\text{ cm}$.

(a) $$R = \frac{\rho\ell}{A} = \frac{\rho\ell}{\ell^2} = \frac{\rho}{\ell} = \frac{1.59\times10^{-8}\ \Omega\cdot\text{m}}{2.05\times10^{-2}\text{ m}} = 7.77\times10^{-7}\ \Omega = \boxed{777\text{ n}\Omega}$$

(b) $$I = \frac{\Delta V}{R} = \frac{1.00\times10^{-5}\text{ V}}{7.77\times10^{-7}\ \Omega} = 12.9\text{ A}$$

$$n = \frac{10.5\text{ g/cm}^3}{107.87\text{ g/mol}}\left(6.02\times10^{23}\text{ electrons/mol}\right)$$

$$n = \left(5.86\times10^{22}\text{ electrons/cm}^3\right)\left(\frac{1.00\times10^6\text{ cm}^3}{1.00\text{ m}^3}\right) = 5.86\times10^{28}/\text{m}^3$$

$$I = nqvA \text{ and } v = \frac{I}{nqA} = \frac{12.9\text{ C/s}}{\left(5.86\times10^{28}/\text{m}^3\right)\left(1.60\times10^{-19}\text{ C}\right)(0.020\,5\text{ m})^2} = \boxed{3.28\ \mu\text{m/s}}$$

**P27.21** Originally, $R = \frac{\rho\ell}{A}$. Finally, $R_f = \frac{\rho(\ell/3)}{3A} = \frac{\rho\ell}{9A} = \boxed{\frac{R}{9}}$.

**P27.22** $$\frac{\rho_{\text{Al}}\ell}{\pi(r_{\text{Al}})^2} = \frac{\rho_{\text{Cu}}\ell}{\pi(r_{\text{Cu}})^2}$$

$$\frac{r_{\text{Al}}}{r_{\text{Cu}}} = \sqrt{\frac{\rho_{\text{Al}}}{\rho_{\text{Cu}}}} = \sqrt{\frac{2.82\times10^{-8}}{1.70\times10^{-8}}} = \boxed{1.29}$$

**P27.23** $J = \sigma E$ so $$\sigma = \frac{J}{E} = \frac{6.00\times10^{-13}\text{ A/m}^2}{100\text{ V/m}} = \boxed{6.00\times10^{-15}\ (\Omega\cdot\text{m})^{-1}}$$

**P27.24** $$R = \frac{\rho_1\ell_1}{A_1} + \frac{\rho_2\ell_2}{A_2} = \frac{\rho_1\ell_1 + \rho_2\ell_2}{d^2}$$

$$R = \frac{\left(4.00\times10^{-3}\ \Omega\cdot\text{m}\right)(0.250\text{ m}) + \left(6.00\times10^{-3}\ \Omega\cdot\text{m}\right)(0.400\text{ m})}{\left(3.00\times10^{-3}\text{ m}\right)^2} = \boxed{378\ \Omega}$$

## Section 27.3 A Model for Electrical Conduction

**P27.25** $$\rho = \frac{m}{nq^2\tau}$$

so $$\tau = \frac{m}{\rho nq^2} = \frac{\left(9.11\times10^{-31}\right)}{\left(1.70\times10^{-8}\right)\left(8.49\times10^{28}\right)\left(1.60\times10^{-19}\right)} = 2.47\times10^{-14}\text{ s}$$

$$v_d = \frac{qE}{m}\tau$$

so $$7.84\times10^{-4} = \frac{\left(1.60\times10^{-19}\right)E\left(2.47\times10^{-14}\right)}{9.11\times10^{-31}}$$

Therefore, $E = \boxed{0.181\text{ V/m}}$.

**P27.26** (a) $n$ is $\boxed{\text{unaffected}}$

(b) $|J| = \dfrac{I}{A} \propto I$

so it $\boxed{\text{doubles}}$.

(c) $J = nev_d$

so $v_d$ $\boxed{\text{doubles}}$.

(d) $\tau = \dfrac{m\sigma}{nq^2}$ is $\boxed{\text{unchanged}}$ as long as $\sigma$ does not change due to a temperature change in the conductor.

**P27.27** From Equation 27.17,

$$\tau = \frac{m_e}{nq^2\rho} = \frac{9.11\times10^{-31}}{\left(8.49\times10^{28}\right)\left(1.60\times10^{-19}\right)^2\left(1.70\times10^{-8}\right)} = 2.47\times10^{-14}\text{ s}$$

$$\ell = v\tau = \left(8.60\times10^{5}\text{ m/s}\right)\left(2.47\times10^{-14}\text{ s}\right) = 2.12\times10^{-8}\text{ m} = \boxed{21.2\text{ nm}}$$

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## Section 27.4 **Resistance and Temperature**

**P27.28** At the low temperature $T_C$ we write $R_C = \dfrac{\Delta V}{I_C} = R_0\left[1+\alpha\left(T_C - T_0\right)\right]$ where $T_0 = 20.0°\text{C}$.

At the high temperature $T_h$, $R_h = \dfrac{\Delta V}{I_h} = \dfrac{\Delta V}{1\text{ A}} = R_0\left[1+\alpha\left(T_h - T_0\right)\right]$.

Then $$\frac{(\Delta V)/(1.00\text{ A})}{(\Delta V)/I_C} = \frac{1+\left(3.90\times10^{-3}\right)(38.0)}{1+\left(3.90\times10^{-3}\right)(-108)}$$

and $$I_C = (1.00\text{ A})\left(\frac{1.15}{0.579}\right) = \boxed{1.98\text{ A}}.$$

**P27.29** $R = R_0\left[1+\alpha(\Delta T)\right]$ gives $140\ \Omega = (19.0\ \Omega)\left[1+\left(4.50\times10^{-3}/°\text{C}\right)\Delta T\right]$.

Solving, $\Delta T = 1.42\times10^{3}\ °\text{C} = T - 20.0°\text{C}$.

And, the final temperature is $\boxed{T = 1.44\times10^{3}\ °\text{C}}$.

**P27.30** $R = R_c + R_n = R_c\left[1+\alpha_c(T-T_0)\right] + R_n\left[1+\alpha_n(T-T_0)\right]$

$$0 = R_c\alpha_c(T-T_0) + R_n\alpha_n(T-T_0) \text{ so } R_c = -R_n\frac{\alpha_n}{\alpha_c}$$

$$R = -R_n\frac{\alpha_n}{\alpha_c} + R_n$$

$$R_n = R\left(1-\frac{\alpha_n}{\alpha_c}\right)^{-1} \qquad R_c = R\left(1-\frac{\alpha_c}{\alpha_n}\right)^{-1}$$

$$R_n = 10.0\text{ k}\Omega\left[1-\frac{\left(0.400\times10^{-3}/{}^\circ\text{C}\right)}{\left(-0.500\times10^{-3}/{}^\circ\text{C}\right)}\right]^{-1}$$

$$\boxed{R_n = 5.56\text{ k}\Omega} \quad\text{and}\quad \boxed{R_c = 4.44\text{ k}\Omega}$$

**P27.31** (a) $\rho = \rho_0\left[1+\alpha(T-T_0)\right] = \left(2.82\times10^{-8}\ \Omega\cdot\text{m}\right)\left[1+3.90\times10^{-3}(30.0^\circ)\right] = \boxed{3.15\times10^{-8}\ \Omega\cdot\text{m}}$

(b) $J = \dfrac{E}{\rho} = \dfrac{0.200\ \text{V/m}}{3.15\times10^{-8}\ \Omega\cdot\text{m}} = \boxed{6.35\times10^{6}\ \text{A/m}^2}$

(c) $I = JA = J\left(\dfrac{\pi d^2}{4}\right) = \left(6.35\times10^{6}\ \text{A/m}^2\right)\left[\dfrac{\pi\left(1.00\times10^{-4}\ \text{m}\right)^2}{4}\right] = \boxed{49.9\text{ mA}}$

(d) $n = \dfrac{6.02\times10^{23}\text{ electrons}}{\left[26.98\text{ g}/\left(2.70\times10^{6}\ \text{g/m}^3\right)\right]} = 6.02\times10^{28}\ \text{electrons/m}^3$

$$v_d = \frac{J}{ne} = \frac{\left(6.35\times10^{6}\ \text{A/m}^2\right)}{\left(6.02\times10^{28}\ \text{electrons/m}^3\right)\left(1.60\times10^{-19}\ \text{C}\right)} = \boxed{659\ \mu\text{m/s}}$$

(e) $\Delta V = E\ell = (0.200\ \text{V/m})(2.00\text{ m}) = \boxed{0.400\text{ V}}$

**P27.32** For aluminum,

$$\alpha_E = 3.90\times10^{-3}\ {}^\circ\text{C}^{-1} \qquad \text{(Table 27.1)}$$

$$\alpha = 24.0\times10^{-6}\ {}^\circ\text{C}^{-1} \qquad \text{(Table 19.1)}$$

$$R = \frac{\rho\ell}{A} = \frac{\rho_0(1+\alpha_E\Delta T)\ell(1+\alpha\Delta T)}{A(1+\alpha\Delta T)^2} = R_0\frac{(1+\alpha_E\Delta T)}{(1+\alpha\Delta T)} = (1.234\ \Omega)\left(\frac{1.39}{1.002\,4}\right) = \boxed{1.71\ \Omega}$$

**P27.33** $R = R_0[1+\alpha T]$

$$R - R_0 = R_0 \alpha \Delta T$$

$$\frac{R-R_0}{R_0} = \alpha \Delta T = \left(5.00\times10^{-3}\right)25.0 = \boxed{0.125}$$

**P27.34** Assuming linear change of resistance with temperature, $R = R_0(1+\alpha\Delta T)$

$$R_{77\text{ K}} = (1.00\ \Omega)\left[1+\left(3.92\times10^{-3}\right)(-216\text{°C})\right] = \boxed{0.153\ \Omega}\ .$$

**P27.35** $\rho = \rho_0(1+\alpha\Delta T)$ or $\Delta T_W = \dfrac{1}{\alpha_W}\left(\dfrac{\rho_W}{\rho_{0W}} - 1\right)$

Require that $\rho_W = 4\rho_{0_{\text{Cu}}}$ so that $\Delta T_W = \left(\dfrac{1}{4.50\times10^{-3}/\text{°C}}\right)\left(\dfrac{4\left(1.70\times10^{-8}\right)}{5.60\times10^{-8}} - 1\right) = 47.6\text{°C}$ .

Therefore, $T_W = 47.6\text{°C} + T_0 = \boxed{67.6\text{°C}}$ .

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## Section 27.5 **Superconductors**

Problem 48 in Chapter 43 can be assigned with this section.

## Section 27.6 **Electric Power**

**P27.36** $I = \dfrac{\mathcal{P}}{\Delta V} = \dfrac{600\text{ W}}{120\text{ V}} = \boxed{5.00\text{ A}}$

and $R = \dfrac{\Delta V}{I} = \dfrac{120\text{ V}}{5.00\text{ A}} = \boxed{24.0\ \Omega}$ .

***P27.37** $\mathcal{P} = I\Delta V = 500\times10^{-6}\text{ A}\left(15\times10^{3}\text{ V}\right) = \boxed{7.50\text{ W}}$

**P27.38** $\mathcal{P} = 0.800(1\,500\text{ hp})(746\text{ W/hp}) = 8.95\times10^{5}\text{ W}$

$\mathcal{P} = I\Delta V$ $\qquad 8.95\times10^{5} = I(2\,000)$ $\qquad I = \boxed{448\text{ A}}$

**P27.39** The heat that must be added to the water is

$$Q = mc\Delta T = (1.50\text{ kg})(4\,186\text{ J/kg°C})(40.0\text{°C}) = 2.51\times10^{5}\text{ J}\ .$$

Thus, the power supplied by the heater is

$$\mathcal{P} = \frac{W}{\Delta t} = \frac{Q}{\Delta t} = \frac{2.51\times10^{5}\text{ J}}{600\text{ s}} = 419\text{ W}$$

and the resistance is $R = \dfrac{(\Delta V)^2}{\mathcal{P}} = \dfrac{(110\text{ V})^2}{419\text{ W}} = \boxed{28.9\ \Omega}$ .

***P27.40** The battery takes in energy by electric transmission

$$\mathcal{P}\Delta t=(\Delta V)I(\Delta t)=2.3\ \text{J/C}\left(13.5\times10^{-3}\ \text{C/s}\right)4.2\ \text{h}\left(\frac{3\,600\ \text{s}}{1\ \text{h}}\right)=469\ \text{J}.$$

It puts out energy by electric transmission

$$(\Delta V)I(\Delta t)=1.6\ \text{J/C}\left(18\times10^{-3}\ \text{C/s}\right)2.4\ \text{h}\left(\frac{3\,600\ \text{s}}{1\ \text{h}}\right)=249\ \text{J}.$$

(a) $$\text{efficiency}=\frac{\text{useful output}}{\text{total input}}=\frac{249\ \text{J}}{469\ \text{J}}=\boxed{0.530}$$

(b) The only place for the missing energy to go is into internal energy:

$$469\ \text{J}=249\ \text{J}+\Delta E_{\text{int}}$$

$$\Delta E_{\text{int}}=\boxed{221\ \text{J}}$$

(c) We imagine toasting the battery over a fire with 221 J of heat input:

$$Q=mc\Delta T$$

$$\Delta T=\frac{Q}{mc}=\frac{221\ \text{J}}{0.015\ \text{kg}}\,\frac{\text{kg}^\circ\text{C}}{975\ \text{J}}=\boxed{15.1^\circ\text{C}}$$

**P27.41** $$\frac{\mathcal{P}}{\mathcal{P}_0}=\frac{(\Delta V)^2/R}{(\Delta V_0)^2/R}=\left(\frac{\Delta V}{\Delta V_0}\right)^2=\left(\frac{140}{120}\right)^2=1.361$$

$$\Delta\%=\left(\frac{\mathcal{P}-\mathcal{P}_0}{\mathcal{P}_0}\right)(100\%)=\left(\frac{\mathcal{P}}{\mathcal{P}_0}-1\right)(100\%)=(1.361-1)100\%=\boxed{36.1\%}$$

**P27.42** $$\mathcal{P}=I(\Delta V)=\frac{(\Delta V)^2}{R}=500\ \text{W}$$

$$R=\frac{(110\ \text{V})^2}{(500\ \text{W})}=24.2\ \Omega$$

(a) $$R=\frac{\rho}{A}\ell \qquad \text{so} \qquad \ell=\frac{RA}{\rho}=\frac{(24.2\ \Omega)\pi\left(2.50\times10^{-4}\ \text{m}\right)^2}{1.50\times10^{-6}\ \Omega\cdot\text{m}}=\boxed{3.17\ \text{m}}$$

(b) $$R=R_0[1+\alpha\Delta T]=24.2\ \Omega\left[1+\left(0.400\times10^{-3}\right)(1\,180)\right]=35.6\ \Omega$$

$$\mathcal{P}=\frac{(\Delta V)^2}{R}=\frac{(110)^2}{35.6}=\boxed{340\ \text{W}}$$

**P27.43** $R = \dfrac{\rho\ell}{A} = \dfrac{\left(1.50\times10^{-6}\ \Omega\cdot\text{m}\right)25.0\ \text{m}}{\pi\left(0.200\times10^{-3}\ \text{m}\right)^2} = 298\ \Omega$

$\Delta V = IR = (0.500\ \text{A})(298\ \Omega) = 149\ \text{V}$

(a) $E = \dfrac{\Delta V}{\ell} = \dfrac{149\ \text{V}}{25.0\ \text{m}} = \boxed{5.97\ \text{V/m}}$

(b) $\mathcal{P} = (\Delta V)I = (149\ \text{V})(0.500\ \text{A}) = \boxed{74.6\ \text{W}}$

(c) $R = R_0\left[1+\alpha(T-T_0)\right] = 298\ \Omega\left[1+\left(0.400\times10^{-3}/^\circ\text{C}\right)320^\circ\text{C}\right] = 337\ \Omega$

$I = \dfrac{\Delta V}{R} = \dfrac{(149\ \text{V})}{(337\ \Omega)} = 0.443\ \text{A}$

$\mathcal{P} = (\Delta V)I = (149\ \text{V})(0.443\ \text{A}) = \boxed{66.1\ \text{W}}$

**P27.44** (a) $\Delta U = q(\Delta V) = It(\Delta V) = (55.0\ \text{A}\cdot\text{h})(12.0\ \text{V})\left(\dfrac{1\ \text{C}}{1\ \text{A}\cdot\text{s}}\right)\left(\dfrac{1\ \text{J}}{1\ \text{V}\cdot\text{C}}\right)\left(\dfrac{1\ \text{W}\cdot\text{s}}{1\ \text{J}}\right) = 660\ \text{W}\cdot\text{h} = \boxed{0.660\ \text{kWh}}$

(b) $\text{Cost } = 0.660\ \text{kWh}\left(\dfrac{\$0.060\,0}{1\ \text{kWh}}\right) = \boxed{3.96¢}$

**P27.45** $\mathcal{P} = I(\Delta V) \qquad \Delta V = IR$

$\mathcal{P} = \dfrac{(\Delta V)^2}{R} = \dfrac{(10.0)^2}{120} = \boxed{0.833\ \text{W}}$

***P27.46** (a) The resistance of 1 m of 12-gauge copper wire is

$R = \dfrac{\rho\ell}{A} = \dfrac{\rho\ell}{\pi(d/2)^2} = \dfrac{4\rho\ell}{\pi d^2} = \dfrac{4\left(1.7\times10^{-8}\ \Omega\cdot\text{m}\right)1\ \text{m}}{\pi\left(0.205\,3\times10^{-2}\ \text{m}\right)^2} = 5.14\times10^{-3}\ \Omega.$

The rate of internal energy production is $\mathcal{P} = I\Delta V = I^2R = (20\ \text{A})^2 5.14\times10^{-3}\ \Omega = \boxed{2.05\ \text{W}}$.

(b) $\mathcal{P}_{\text{Al}} = I^2R = \dfrac{I^2 4\rho_{\text{Al}}\ell}{\pi d^2}$

$\dfrac{\mathcal{P}_{\text{Al}}}{\mathcal{P}_{\text{Cu}}} = \dfrac{\rho_{\text{Al}}}{\rho_{\text{Cu}}} \qquad \mathcal{P}_{\text{Al}} = \dfrac{2.82\times10^{-8}\ \Omega\cdot\text{m}}{1.7\times10^{-8}\ \Omega\cdot\text{m}}2.05\ \text{W} = \boxed{3.41\ \text{W}}$

Aluminum of the same diameter will get hotter than copper.

***P27.47** The energy taken in by electric transmission for the fluorescent lamp is

$$\mathscr{P}\Delta t = 11\ \text{J/s}(100\ \text{h})\left(\frac{3\,600\ \text{s}}{1\ \text{h}}\right) = 3.96\times10^{6}\ \text{J}$$

$$\text{cost} = 3.96\times10^{6}\ \text{J}\left(\frac{\$0.08}{\text{kWh}}\right)\left(\frac{\text{k}}{1\,000}\right)\left(\frac{\text{W}\cdot\text{s}}{\text{J}}\right)\left(\frac{\text{h}}{3\,600\ \text{s}}\right) = \$0.088$$

For the incandescent bulb,

$$\mathscr{P}\Delta t = 40\ \text{W}(100\ \text{h})\left(\frac{3\,600\ \text{s}}{1\ \text{h}}\right) = 1.44\times10^{7}\ \text{J}$$

$$\text{cost} = 1.44\times10^{7}\ \text{J}\left(\frac{\$0.08}{3.6\times10^{6}\ \text{J}}\right) = \$0.32$$

$$\text{saving} = \$0.32 - \$0.088 = \boxed{\$0.232}$$

**P27.48** The total clock power is

$$\left(270\times10^{6}\ \text{clocks}\right)\left(2.50\ \frac{\text{J/s}}{\text{clock}}\right)\left(\frac{3\,600\ \text{s}}{1\ \text{h}}\right) = 2.43\times10^{12}\ \text{J/h}\,.$$

From $e = \dfrac{W_{\text{out}}}{Q_{\text{in}}}$, the power input to the generating plants must be:

$$\frac{Q_{\text{in}}}{\Delta t} = \frac{W_{\text{out}}/\Delta t}{e} = \frac{2.43\times10^{12}\ \text{J/h}}{0.250} = 9.72\times10^{12}\ \text{J/h}$$

and the rate of coal consumption is

$$\text{Rate} = \left(9.72\times10^{12}\ \text{J/h}\right)\left(\frac{1.00\ \text{kg coal}}{33.0\times10^{6}\ \text{J}}\right) = 2.95\times10^{5}\ \text{kg coal/h} = \boxed{295\ \text{metric ton/h}}\,.$$

**P27.49** $\mathscr{P} = I(\Delta V) = (1.70\ \text{A})(110\ \text{V}) = 187\ \text{W}$

Energy used in a 24-hour day $= (0.187\ \text{kW})(24.0\ \text{h}) = 4.49\ \text{kWh}$

$$\therefore \text{cost} = 4.49\ \text{kWh}\left(\frac{\$0.060\,0}{\text{kWh}}\right) = \$0.269 = \boxed{26.9¢}$$

**P27.50** $\mathscr{P} = I\Delta V = (2.00\ \text{A})(120\ \text{V}) = 240\ \text{W}$

$\Delta E_{\text{int}} = (0.500\ \text{kg})(4\,186\ \text{J/kg}\cdot{}^{\circ}\text{C})(77.0^{\circ}\text{C}) = 161\ \text{kJ}$

$$\Delta t = \frac{\Delta E_{\text{int}}}{\mathscr{P}} = \frac{1.61\times10^{5}\ \text{J}}{240\ \text{W}} = \boxed{672\ \text{s}}$$

**P27.51** At operating temperature,

(a) $\mathcal{P} = I\Delta V = (1.53\text{ A})(120\text{ V}) = \boxed{184\text{ W}}$

(b) Use the change in resistance to find the final operating temperature of the toaster.

$R = R_0(1+\alpha\Delta T)$ $\qquad \dfrac{120}{1.53} = \dfrac{120}{1.80}\left[1+\left(0.400\times10^{-3}\right)\Delta T\right]$

$\Delta T = 441°\text{C}$ $\qquad T = 20.0°\text{C} + 441°\text{C} = \boxed{461°\text{C}}$

***P27.52** You pay the electric company for energy transferred in the amount $E = \mathcal{P}\Delta t$

(a) $$\mathcal{P}\Delta t = 40\text{ W}(2\text{ weeks})\left(\frac{7\text{ d}}{1\text{ week}}\right)\left(\frac{86\,400\text{ s}}{1\text{ d}}\right)\left(\frac{1\text{ J}}{1\text{ W}\cdot\text{s}}\right) = 48.4\text{ MJ}$$

$$\mathcal{P}\Delta t = 40\text{ W}(2\text{ weeks})\left(\frac{7\text{ d}}{1\text{ week}}\right)\left(\frac{24\text{ h}}{1\text{ d}}\right)\left(\frac{\text{k}}{1\,000}\right) = 13.4\text{ kWh}$$

$$\mathcal{P}\Delta t = 40\text{ W}(2\text{ weeks})\left(\frac{7\text{ d}}{1\text{ week}}\right)\left(\frac{24\text{ h}}{1\text{ d}}\right)\left(\frac{\text{k}}{1\,000}\right)\left(\frac{0.12\text{ \$}}{\text{kWh}}\right) = \boxed{\$1.61}$$

(b) $$\mathcal{P}\Delta t = 970\text{ W}(3\text{ min})\left(\frac{1\text{ h}}{60\text{ min}}\right)\left(\frac{\text{k}}{1\,000}\right)\left(\frac{0.12\text{ \$}}{\text{kWh}}\right) = \boxed{\$0.005\,82} = 0.582¢$$

(c) $$\mathcal{P}\Delta t = 5\,200\text{ W}(40\text{ min})\left(\frac{1\text{ h}}{60\text{ min}}\right)\left(\frac{\text{k}}{1\,000}\right)\left(\frac{0.12\text{ \$}}{\text{kWh}}\right) = \boxed{\$0.416}$$

**P27.53** Consider a 400-W blow dryer used for ten minutes daily for a year. The energy transferred to the dryer is

$$\mathcal{P}\Delta t = (400\text{ J/s})(600\text{ s/d})(365\text{ d}) \approx 9\times10^{7}\text{ J}\left(\frac{1\text{ kWh}}{3.6\times10^{6}\text{ J}}\right) \approx 20\text{ kWh}.$$

We suppose that electrically transmitted energy costs on the order of ten cents per kilowatt-hour. Then the cost of using the dryer for a year is on the order of

$$\text{Cost} \cong (20\text{ kWh})(\$0.10/\text{kWh}) = \$2\boxed{\sim\$1}.$$

## Additional Problems

**P27.54** (a) $I = \dfrac{\Delta V}{R}$ so $\mathcal{P} = I\Delta V = \dfrac{(\Delta V)^2}{R}$

$R = \dfrac{(\Delta V)^2}{\mathcal{P}} = \dfrac{(120\ \text{V})^2}{25.0\ \text{W}} = \boxed{576\ \Omega}$ and $R = \dfrac{(\Delta V)^2}{\mathcal{P}} = \dfrac{(120\ \text{V})^2}{100\ \text{W}} = \boxed{144\ \Omega}$

(b) $I = \dfrac{\mathcal{P}}{\Delta V} = \dfrac{25.0\ \text{W}}{120\ \text{V}} = 0.208\ \text{A} = \dfrac{Q}{\Delta t} = \dfrac{1.00\ \text{C}}{\Delta t}$

$\Delta t = \dfrac{1.00\ \text{C}}{0.208\ \text{A}} = \boxed{4.80\ \text{s}}$

The bulb takes in charge at high potential and puts out the same amount of charge at low potential.

(c) $\mathcal{P} = 25.0\ \text{W} = \dfrac{\Delta U}{\Delta t} = \dfrac{1.00\ \text{J}}{\Delta t}$ $\Delta t = \dfrac{1.00\ \text{J}}{25.0\ \text{W}} = \boxed{0.040\,0\ \text{s}}$

The bulb takes in energy by electrical transmission and puts out the same amount of energy by heat and light.

(d) $\Delta U = \mathcal{P}\Delta t = (25.0\ \text{J/s})(86\,400\ \text{s/d})(30.0\ \text{d}) = 64.8 \times 10^8\ \text{J}$

The electric company sells $\boxed{\text{energy}}$.

$\text{Cost} = 64.8 \times 10^6\ \text{J}\left(\dfrac{\$0.070\,0}{\text{kWh}}\right)\left(\dfrac{\text{k}}{1\,000}\right)\left(\dfrac{\text{W}\cdot\text{s}}{\text{J}}\right)\left(\dfrac{\text{h}}{3\,600\ \text{s}}\right) = \boxed{\$1.26}$

$\text{Cost per joule} = \dfrac{\$0.070\,0}{\text{kWh}}\left(\dfrac{\text{kWh}}{3.60 \times 10^6\ \text{J}}\right) = \boxed{\$1.94 \times 10^{-8}/\text{J}}$

***P27.55** The original stored energy is $U_i = \dfrac{1}{2}Q\Delta V_i = \dfrac{1}{2}\dfrac{Q^2}{C}$.

(a) When the switch is closed, charge $Q$ distributes itself over the plates of $C$ and $3C$ in parallel, presenting equivalent capacitance $4C$. Then the final potential difference is $\boxed{\Delta V_f = \dfrac{Q}{4C}}$ for both.

(b) The smaller capacitor then carries charge $C\Delta V_f = \dfrac{Q}{4C}C = \boxed{\dfrac{Q}{4}}$. The larger capacitor carries charge $3C\dfrac{Q}{4C} = \boxed{\dfrac{3Q}{4}}$.

(c) The smaller capacitor stores final energy $\dfrac{1}{2}C(\Delta V_f)^2 = \dfrac{1}{2}C\left(\dfrac{Q}{4C}\right)^2 = \boxed{\dfrac{Q^2}{32C}}$. The larger capacitor possesses energy $\dfrac{1}{2}3C\left(\dfrac{Q}{4C}\right)^2 = \boxed{\dfrac{3Q^2}{32C}}$.

(d) The total final energy is $\dfrac{Q^2}{32C} + \dfrac{3Q^2}{32C} = \dfrac{Q^2}{8C}$. The loss of potential energy is the energy appearing as internal energy in the resistor: $\dfrac{Q^2}{2C} = \dfrac{Q^2}{8C} + \Delta E_{\text{int}}$ $\Delta E_{\text{int}} = \boxed{\dfrac{3Q^2}{8C}}$.

**P27.56** We find the drift velocity from $I = nqv_dA = nqv_d\pi r^2$

$$v_d = \frac{I}{nq\pi r^2} = \frac{1\,000\text{ A}}{8.49\times10^{28}\text{ m}^{-3}\left(1.60\times10^{-19}\text{ C}\right)\pi\left(10^{-2}\text{ m}\right)^2} = 2.34\times10^{-4}\text{ m/s}$$

$$v = \frac{x}{t} \qquad t = \frac{x}{v} = \frac{200\times10^3\text{ m}}{2.34\times10^{-4}\text{ m/s}} = 8.54\times10^8\text{ s} = \boxed{27.0\text{ yr}}$$

**P27.57** We begin with the differential equation $\alpha = \frac{1}{\rho}\frac{d\rho}{dT}$.

(a) Separating variables, $\int_{\rho_0}^{\rho}\frac{d\rho}{\rho} = \int_{T_0}^{T}\alpha dT$

$\ln\left(\frac{\rho}{\rho_0}\right) = \alpha(T - T_0)$ and $\boxed{\rho = \rho_0 e^{\alpha(T-T_0)}}$.

(b) From the series expansion $e^x \cong 1 + x$, $(x \ll 1)$,

$\boxed{\rho \cong \rho_0\left[1 + \alpha(T - T_0)\right]}$.

**P27.58** The resistance of one wire is $\left(\frac{0.500\ \Omega}{\text{mi}}\right)(100\text{ mi}) = 50.0\ \Omega$.

The whole wire is at nominal 700 kV away from ground potential, but the potential difference between its two ends is

$IR = (1\,000\text{ A})(50.0\ \Omega) = 50.0\text{ kV}$.

Then it radiates as heat power $\mathcal{P} = (\Delta V)I = \left(50.0\times10^3\text{ V}\right)(1\,000\text{ A}) = \boxed{50.0\text{ MW}}$.

**P27.59** $\rho = \frac{RA}{\ell} = \frac{(\Delta V)}{I}\frac{A}{\ell}$

| $\ell$ (m) | $R$ (Ω) | $\rho$ (Ω·m) |
|---|---|---|
| 0.540 | 10.4 | $1.41\times10^{-6}$ |
| 1.028 | 21.1 | $1.50\times10^{-6}$ |
| 1.543 | 31.8 | $1.50\times10^{-6}$ |

$\bar{\rho} = \boxed{1.47\times10^{-6}\ \Omega\cdot\text{m}}$ (in agreement with tabulated value of $1.50\times10^{-6}\ \Omega\cdot\text{m}$ in Table 27.1)

**P27.60** 2 wires → $\ell = 100$ m

$$R = \frac{0.108\ \Omega}{300\text{ m}}(100\text{ m}) = 0.036\,0\ \Omega$$

(a) $(\Delta V)_{\text{home}} = (\Delta V)_{\text{line}} - IR = 120 - (110)(0.036\,0) = \boxed{116\text{ V}}$

(b) $\mathcal{P} = I(\Delta V) = (110\text{ A})(116\text{ V}) = \boxed{12.8\text{ kW}}$

(c) $\mathcal{P}_{\text{wires}} = I^2R = (110\text{ A})^2(0.036\,0\ \Omega) = \boxed{436\text{ W}}$

**P27.61** (a) $$\mathbf{E} = -\frac{dV}{dx}\hat{\mathbf{i}} = -\frac{(0-4.00)\text{ V}}{(0.500-0)\text{ m}} = \boxed{8.00\hat{\mathbf{i}}\text{ V/m}}$$

(b) $$R = \frac{\rho\ell}{A} = \frac{\left(4.00\times10^{-8}\ \Omega\cdot\text{m}\right)(0.500\text{ m})}{\pi\left(1.00\times10^{-4}\text{ m}\right)^2} = \boxed{0.637\ \Omega}$$

(c) $$I = \frac{\Delta V}{R} = \frac{4.00\text{ V}}{0.637\ \Omega} = \boxed{6.28\text{ A}}$$

(d) $$\mathbf{J} = \frac{I}{A}\hat{\mathbf{i}} = \frac{6.28\text{ A}}{\pi\left(1.00\times10^{-4}\text{ m}\right)^2} = 2.00\times10^{8}\hat{\mathbf{i}}\text{ A/m}^2 = \boxed{200\hat{\mathbf{i}}\text{ MA/m}^2}$$

(e) $$\rho\mathbf{J} = \left(4.00\times10^{-8}\ \Omega\cdot\text{m}\right)\left(2.00\times10^{8}\hat{\mathbf{i}}\text{ A/m}^2\right) = 8.00\hat{\mathbf{i}}\text{ V/m} = \mathbf{E}$$

**P27.62** (a) $$\mathbf{E} = -\frac{dV(x)}{dx}\hat{\mathbf{i}} = \boxed{\frac{V}{L}\hat{\mathbf{i}}}$$

(b) $$R = \frac{\rho\ell}{A} = \boxed{\frac{4\rho L}{\pi d^2}}$$

(c) $$I = \frac{\Delta V}{R} = \boxed{\frac{V\pi d^2}{4\rho L}}$$

(d) $$\mathbf{J} = \frac{I}{A}\hat{\mathbf{i}} = \boxed{\frac{V}{\rho L}\hat{\mathbf{i}}}$$

(e) $$\rho\mathbf{J} = \frac{V}{L}\hat{\mathbf{i}} = \boxed{\mathbf{E}}$$

**P27.63** $R = R_0\left[1+\alpha\left(T-T_0\right)\right]$ so $T = T_0 + \frac{1}{\alpha}\left[\frac{R}{R_0}-1\right] = T_0 + \frac{1}{\alpha}\left[\frac{I_0}{I}-1\right]$.

In this case, $I = \frac{I_0}{10}$, so $T = T_0 + \frac{1}{\alpha}(9) = 20° + \frac{9}{0.004\,50/°\text{C}} = \boxed{2\,020°\text{C}}$.

**P27.64** $R = \frac{\Delta V}{I} = \frac{12.0}{I} = \frac{6.00}{(I-3.00)}$ thus $12.0I - 36.0 = 6.00I$ and $I = 6.00\text{ A}$.

Therefore, $R = \frac{12.0\text{ V}}{6.00\text{ A}} = \boxed{2.00\ \Omega}$.

**P27.65** (a) $\mathscr{P} = I\Delta V$

so $I = \dfrac{\mathscr{P}}{\Delta V} = \dfrac{8.00 \times 10^3 \text{ W}}{12.0 \text{ V}} = \boxed{667 \text{ A}}$.

(b) $\Delta t = \dfrac{\Delta U}{\mathscr{P}} = \dfrac{2.00 \times 10^7 \text{ J}}{8.00 \times 10^3 \text{ W}} = 2.50 \times 10^3 \text{ s}$

and $\Delta x = v\Delta t = (20.0 \text{ m/s})\left(2.50 \times 10^3 \text{ s}\right) = \boxed{50.0 \text{ km}}$.

**P27.66** (a) We begin with $R = \dfrac{\rho\ell}{A} = \dfrac{\rho_0\left[1+\alpha(T-T_0)\right]\ell_0\left[1+\alpha'(T-T_0)\right]}{A_0\left[1+2\alpha'(T-T_0)\right]}$,

which reduces to $R = \boxed{\dfrac{R_0\left[1+\alpha(T-T_0)\right]\left[1+\alpha'(T-T_0)\right]}{\left[1+2\alpha'(T-T_0)\right]}}$.

(b) For copper: $\rho_0 = 1.70 \times 10^{-8}\ \Omega\cdot\text{m}$, $\alpha = 3.90 \times 10^{-3}\ {}^\circ\text{C}^{-1}$, and $\alpha' = 17.0 \times 10^{-6}\ {}^\circ\text{C}^{-1}$

$$R_0 = \frac{\rho_0\ell_0}{A_0} = \frac{\left(1.70 \times 10^{-8}\right)(2.00)}{\pi\left(0.100 \times 10^{-3}\right)^2} = \boxed{1.08\ \Omega}.$$

The simple formula for $R$ gives:

$$R = (1.08\ \Omega)\left[1+\left(3.90 \times 10^{-3}\ {}^\circ\text{C}^{-1}\right)(100^\circ\text{C} - 20.0^\circ\text{C})\right] = \boxed{1.420\ \Omega}$$

while the more complicated formula gives:

$$R = \frac{(1.08\ \Omega)\left[1+\left(3.90 \times 10^{-3}\ {}^\circ\text{C}^{-1}\right)(80.0^\circ\text{C})\right]\left[1+\left(17.0 \times 10^{-6}\ {}^\circ\text{C}^{-1}\right)(80.0^\circ\text{C})\right]}{\left[1+2\left(17.0 \times 10^{-6}\ {}^\circ\text{C}^{-1}\right)(80.0^\circ\text{C})\right]} = \boxed{1.418\ \Omega}.$$

**P27.67** Let $\alpha$ be the temperature coefficient at 20.0°C, and $\alpha'$ be the temperature coefficient at 0 °C. Then $\rho = \rho_0\left[1+\alpha(T-20.0^\circ\text{C})\right]$, and $\rho = \rho'\left[1+\alpha'(T-0^\circ\text{C})\right]$ must both give the correct resistivity at any temperature $T$. That is, we must have:

$$\rho_0\left[1+\alpha(T-20.0^\circ\text{C})\right] = \rho'\left[1+\alpha'(T-0^\circ\text{C})\right]. \qquad (1)$$

Setting $T = 0$ in equation (1) yields: $\rho' = \rho_0\left[1-\alpha(20.0^\circ\text{C})\right]$,

and setting $T = 20.0^\circ\text{C}$ in equation (1) gives: $\rho_0 = \rho'\left[1+\alpha'(20.0^\circ\text{C})\right]$.

Put $\rho'$ from the first of these results into the second to obtain:

$$\rho_0 = \rho_0\left[1-\alpha(20.0^\circ\text{C})\right]\left[1+\alpha'(20.0^\circ\text{C})\right].$$

*continued on next page*

Therefore $$1+\alpha'(20.0°\text{C})=\frac{1}{1-\alpha(20.0°\text{C})}$$

which simplifies to $$\alpha'=\frac{\alpha}{\left[1-\alpha(20.0°\text{C})\right]}.$$

From this, the temperature coefficient, based on a reference temperature of 0°C, may be computed for any material. For example, using this, $\boxed{\text{Table 27.1 becomes at 0°C}}$:

| Material | Temp Coefficients at 0°C |
|---|---|
| Silver | $4.1\times10^{-3}/°\text{C}$ |
| Copper | $4.2\times10^{-3}/°\text{C}$ |
| Gold | $3.6\times10^{-3}/°\text{C}$ |
| Aluminum | $4.2\times10^{-3}/°\text{C}$ |
| Tungsten | $4.9\times10^{-3}/°\text{C}$ |
| Iron | $5.6\times10^{-3}/°\text{C}$ |
| Platinum | $4.25\times10^{-3}/°\text{C}$ |
| Lead | $4.2\times10^{-3}/°\text{C}$ |
| Nichrome | $0.4\times10^{-3}/°\text{C}$ |
| Carbon | $-0.5\times10^{-3}/°\text{C}$ |
| Germanium | $-24\times10^{-3}/°\text{C}$ |
| Silicon | $-30\times10^{-3}/°\text{C}$ |

**P27.68** (a) A thin cylindrical shell of radius *r*, thickness *dr*, and length *L* contributes resistance

$$dR=\frac{\rho d\ell}{A}=\frac{\rho dr}{(2\pi r)L}=\left(\frac{\rho}{2\pi L}\right)\frac{dr}{r}.$$

The resistance of the whole annulus is the series summation of the contributions of the thin shells:

$$R=\frac{\rho}{2\pi L}\int_{r_a}^{r_b}\frac{dr}{r}=\boxed{\frac{\rho}{2\pi L}\ln\left(\frac{r_b}{r_a}\right)}.$$

(b) In this equation $\frac{\Delta V}{I}=\frac{\rho}{2\pi L}\ln\left(\frac{r_b}{r_a}\right)$

we solve for $$\boxed{\rho=\frac{2\pi L\Delta V}{I\ln(r_b/r_a)}}.$$

**P27.69** Each speaker receives 60.0 W of power. Using $\mathcal{P} = I^2 R$, we then have

$$I = \sqrt{\frac{\mathcal{P}}{R}} = \sqrt{\frac{60.0\text{ W}}{4.00\ \Omega}} = 3.87\text{ A}.$$

The system is $\boxed{\text{not adequately protected}}$ since the $\boxed{\text{fuse should be set to melt at 3.87 A, or less}}$.

**P27.70** $\Delta V = -E\cdot\ell$ or $dV = -E\cdot dx$

$\Delta V = -IR = -E\cdot\ell$

$$I = \frac{dq}{dt} = \frac{E\cdot\ell}{R} = \frac{A}{\rho\ell}E\cdot\ell = \frac{A}{\rho}E = -\sigma A\frac{dV}{dx} = \boxed{\sigma A\left|\frac{dV}{dx}\right|}$$

Current flows in the direction of decreasing voltage. Energy flows as heat in the direction of decreasing temperature.

**P27.71** $R = \int\frac{\rho dx}{A} = \int\frac{\rho dx}{wy}$ where $y = y_1 + \frac{y_2 - y_1}{L}x$

$$R = \frac{\rho}{w}\int_0^L \frac{dx}{y_1 + \left[(y_2 - y_1)/L\right]x} = \frac{\rho L}{w(y_2 - y_1)}\ln\left[y_1 + \frac{y_2 - y_1}{L}x\right]_0^L$$

$$R = \boxed{\frac{\rho L}{w(y_2 - y_1)}\ln\left(\frac{y_2}{y_1}\right)}$$

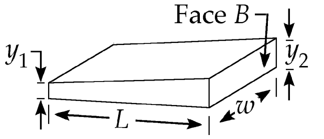


FIG. P27.71

**P27.72** From the geometry of the longitudinal section of the resistor shown in the figure, we see that

$$\frac{(b-r)}{y} = \frac{(b-a)}{h}.$$

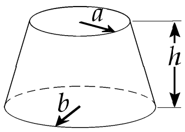


FIG. P27.72

From this, the radius at a distance $y$ from the base is $r = (a-b)\frac{y}{h} + b$.

For a disk-shaped element of volume $dR = \frac{\rho dy}{\pi r^2}$: $R = \frac{\rho}{\pi}\int_0^h \frac{dy}{\left[(a-b)(y/h)+b\right]^2}$.

Using the integral formula $\int\frac{du}{(au+b)^2} = -\frac{1}{a(au+b)}$, $\boxed{R = \frac{\rho}{\pi}\frac{h}{ab}}$.

***P27.73** (a) The resistance of the dielectric block is $R = \frac{\rho\ell}{A} = \frac{d}{\sigma A}$.

The capacitance of the capacitor is $C = \frac{\kappa \in_0 A}{d}$.

Then $RC = \frac{d}{\sigma A}\frac{\kappa \in_0 A}{d} = \frac{\kappa \in_0}{\sigma}$ is a characteristic of the material only.

(b) $$R = \frac{\kappa \in_0}{\sigma C} = \frac{\rho\kappa \in_0}{C} = \frac{75\times 10^{16}\ \Omega\cdot\text{m}(3.78)8.85\times 10^{-12}\ \text{C}^2}{14\times 10^{-9}\ \text{F}\qquad \text{N}\cdot\text{m}^2} = \boxed{1.79\times 10^{15}\ \Omega}$$

**P27.74** $I = I_0\left[\exp\left(\frac{e\Delta V}{k_BT}\right) - 1\right]$ and $R = \frac{\Delta V}{I}$

with $I_0 = 1.00\times10^{-9}$ A, $e = 1.60\times10^{-19}$ C, and $k_B = 1.38\times10^{-23}$ J/K.

The following includes a partial table of calculated values and a graph for each of the specified temperatures.

(i) For $T = 280$ K:

| $\Delta V$(V) | $I$(A) | $R(\Omega)$ |
|---|---|---|
| 0.400 | 0.015 6 | 25.6 |
| 0.440 | 0.081 8 | 5.38 |
| 0.480 | 0.429 | 1.12 |
| 0.520 | 2.25 | 0.232 |
| 0.560 | 11.8 | 0.047 6 |
| 0.600 | 61.6 | 0.009 7 |

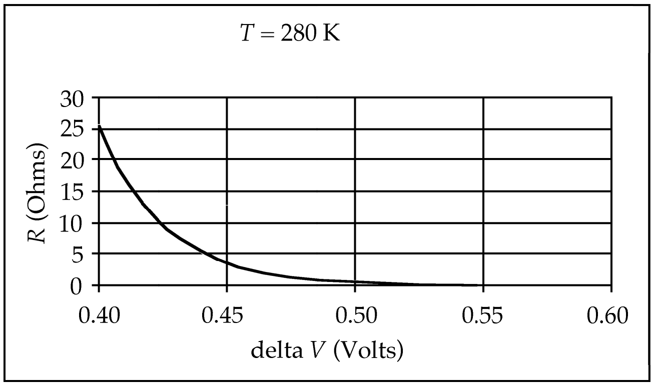


FIG. P27.74(i)

(ii) For $T = 300$ K:

| $\Delta V$(V) | $I$(A) | $R(\Omega)$ |
|---|---|---|
| 0.400 | 0.005 | 77.3 |
| 0.440 | 0.024 | 18.1 |
| 0.480 | 0.114 | 4.22 |
| 0.520 | 0.534 | 0.973 |
| 0.560 | 2.51 | 0.223 |
| 0.600 | 11.8 | 0.051 |

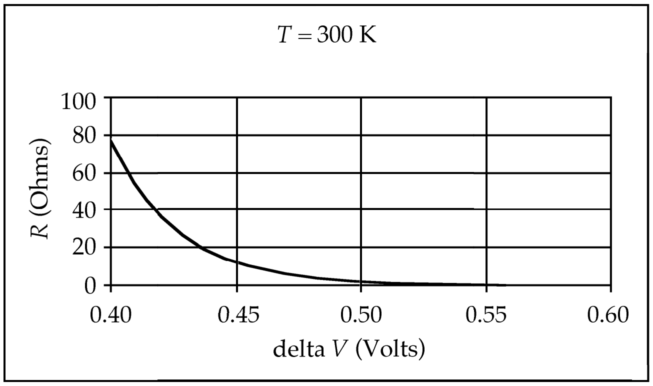


FIG. P27.74(ii)

(iii) For $T = 320$ K:

| $\Delta V$(V) | $I$(A) | $R(\Omega)$ |
|---|---|---|
| 0.400 | 0.002 0 | 203 |
| 0.440 | 0.008 4 | 52.5 |
| 0.480 | 0.035 7 | 13.4 |
| 0.520 | 0.152 | 3.42 |
| 0.560 | 0.648 | 0.864 |
| 0.600 | 2.76 | 0.217 |

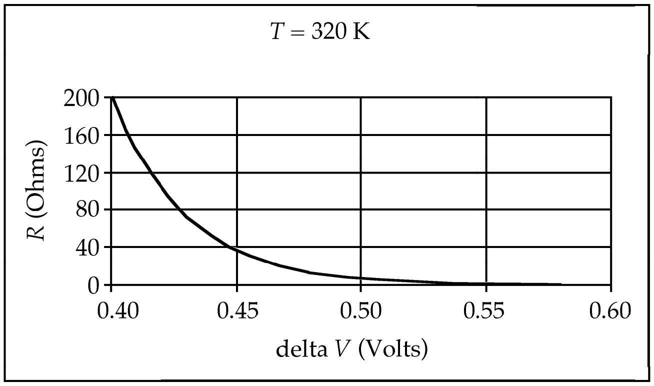


FIG. P27.74(iii)

***P27.75** (a) Think of the device as two capacitors in parallel. The one on the left has $\kappa_1 = 1$, $A_1 = \left(\frac{\ell}{2} + x\right)\ell$. The equivalent capacitance is

$$\frac{\kappa_1 \in_0 A_1}{d} + \frac{\kappa_2 \in_0 A_2}{d} = \frac{\in_0 \ell}{d}\left(\frac{\ell}{2} + x\right) + \frac{\kappa \in_0 \ell}{d}\left(\frac{\ell}{2} - x\right) = \boxed{\frac{\in_0 \ell}{2d}(\ell + 2x + \kappa\ell - 2\kappa x)}.$$

(b) The charge on the capacitor is $Q = C\Delta V$

$$Q = \frac{\in_0 \ell \Delta V}{2d}(\ell + 2x + \kappa\ell - 2\kappa x).$$

The current is

$$I = \frac{dQ}{dt} = \frac{dQ}{dx}\frac{dx}{dt} = \frac{\in_0 \ell \Delta V}{2d}(0 + 2 + 0 - 2\kappa)v = -\frac{\in_0 \ell \Delta V v}{d}(\kappa - 1).$$

The negative value indicates that the current drains charge from the capacitor. Positive current is $\boxed{\text{clockwise } \frac{\in_0 \ell \Delta V v}{d}(\kappa - 1)}$.

## ANSWERS TO EVEN PROBLEMS

| | |
|---|---|
| **P27.2** | 3.64 h |
| **P27.4** | (a) see the solution; (b) 1.05 mA |
| **P27.6** | (a) 17.0 A ; (b) 85.0 $\text{kA/m}^2$ |
| **P27.8** | (a) 99.5 $\text{kA/m}^2$ ; (b) 8.00 mm |
| **P27.10** | (a) 221 nm ; (b) no; see the solution |
| **P27.12** | 30.3 $\text{MA/m}^2$ |
| **P27.14** | (a) 3.75 kΩ ; (b) 536 m |
| **P27.16** | 0.018 1 Ω·m |
| **P27.18** | 2.71 MΩ |
| **P27.20** | (a) 777 nΩ ; (b) 3.28 $\mu$m/s |
| **P27.22** | $\frac{r_{\text{Al}}}{r_{\text{Cu}}} = 1.29$ |
| **P27.24** | 378 Ω |
| **P27.26** | (a) nothing; (b) doubles; (c) doubles;<br>(d) nothing |
| **P27.28** | 1.98 A |
| **P27.30** | carbon, 4.44 kΩ ; nichrome, 5.56 kΩ |
| **P27.32** | 1.71 Ω |
| **P27.34** | 0.153 Ω |
| **P27.36** | 5.00 A , 24.0 Ω |
| **P27.38** | 448 A |
| **P27.40** | (a) 0.530; (b) 221 J; (c) 15.1°C |
| **P27.42** | (a) 3.17 m ; (b) 340 W |
| **P27.44** | (a) 0.660 kWh ; (b) 3.96¢ |
| **P27.46** | (a) 2.05 W; (b) 3.41 W; no |
| **P27.48** | 295 metric ton/h |
| **P27.50** | 672 s |
| **P27.52** | (a) \$1.61; (b) \$0.005 82; (c) \$0.416 |
| **P27.54** | (a) 576 Ω and 144 Ω ;<br>(b) 4.80 s; The charge is the same. The charge-field system is in a lower-energy configuration.<br>(c) 0.040 0 s; The energy enters by electric transmission and exits by heat and electromagnetic radiation;<br>(d) \$1.26; energy; $1.94 \times 10^{-8}$ \$/J |

**P27.56** 27.0 yr

**P27.58** 50.0 MW

**P27.60** (a) 116 V ; (b) 12.8 kW ; (c) 436 W

**P27.62** (a) $\mathbf{E} = \dfrac{V\hat{\mathbf{i}}}{L}$ ; (b) $R = \dfrac{4\rho L}{\pi d^2}$ ; (c) $I = \dfrac{V\pi d^2}{4\rho L}$ ;
(d) $\mathbf{J} = \dfrac{V\hat{\mathbf{i}}}{\rho L}$ ; (e) see the solution

**P27.64** 2.00 Ω

**P27.66** (a) see the solution;
(b) 1.418 Ω nearly agrees with 1.420 Ω

**P27.68** (a) $R = \dfrac{\rho}{2\pi L}\ln\dfrac{r_b}{r_a}$; (b) $\rho = \dfrac{2\pi L\Delta V}{I\ln(r_b/r_a)}$

**P27.70** see the solution

**P27.72** see the solution

**P27.74** see the solution

# 28

# Direct Current Circuits

**CHAPTER OUTLINE**

28.1 Electromotive Force
28.2 Resistors in Series and Parallel
28.3 Kirchhoff's Rules
28.4 *RC* Circuits
28.5 Electrical Meters
28.6 Household Wiring and Electrical Safety

## ANSWERS TO QUESTIONS

**Q28.1** The load resistance in a circuit is the effective resistance of all of the circuit elements excluding the emf source. In energy terms, it can be used to determine the energy delivered to the load by electrical transmission and there appearing as internal energy to raise the temperature of the resistor. The internal resistance of a battery represents the limitation on the efficiency of the chemical reaction that takes place in the battery to supply current to the load. The emf of the battery represents its conversion of chemical energy into energy which it puts out by electric transmission; the battery also creates internal energy within itself, in an amount that can be computed from its internal resistance. We model the internal resistance as constant for a given battery, but it may increase greatly as the battery ages. It may increase somewhat with increasing current demand by the load. For a load described by Ohm's law, the load resistance is a precisely fixed value.

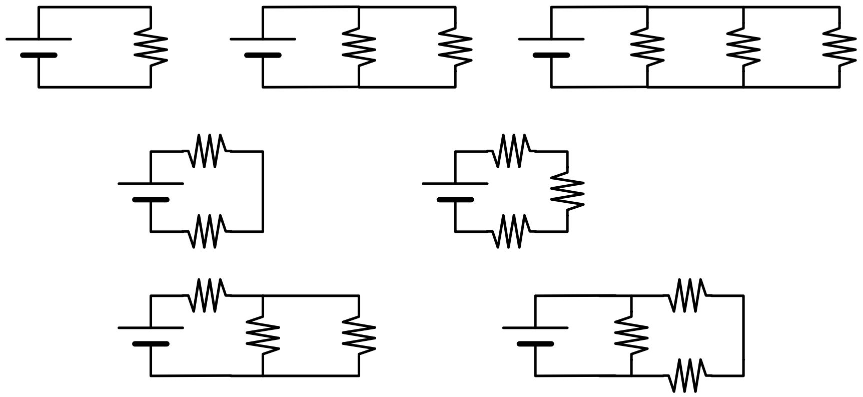
**Q28.2** The potential difference between the terminals of a battery will equal the emf of the battery when there is no current in the battery. At this time, the current though, and hence the potential drop across the internal resistance is zero. This only happens when there is no load placed on the battery—that includes measuring the potential difference with a voltmeter! The terminal voltage will exceed the emf of the battery when current is driven backward through the battery, in at its positive terminal and out at its negative terminal.

**Q28.3** No. If there is one battery in a circuit, the current inside it will be from its negative terminal to its positive terminal. Whenever a battery is delivering energy to a circuit, it will carry current in this direction. On the other hand, when another source of emf is charging the battery in question, it will have a current pushed through it from its positive terminal to its negative terminal.

**Q28.4** Connect the resistors in series. Resistors of 5.0 kΩ, 7.5 kΩ and 2.2 kΩ connected in series present equivalent resistance 14.7 kΩ.

**Q28.5** Connect the resistors in parallel. Resistors of 5.0 kΩ, 7.5 kΩ and 2.2 kΩ connected in parallel present equivalent resistance 1.3 kΩ.

**Q28.6**



**Q28.7** In series, the current is the same through each resistor. Without knowing individual resistances, nothing can be determined about potential differences or power.

**Q28.8** In parallel, the potential difference is the same across each resistor. Without knowing individual resistances, nothing can be determined about current or power.

**Q28.9** In this configuration, the power delivered to one individual resistor is significantly less than if only one equivalent resistor were used. This decreases the possibility of component failure, and possible electrical disaster to some more expensive circuit component than a resistor.

**Q28.10** Each of the two conductors in the extension cord itself has a small resistance. The longer the extension cord, the larger the resistance. Taken into account in the circuit, the extension cord will reduce the current from the power supply, and also will absorb energy itself in the form of internal energy, leaving less power available to the light bulb.

**Q28.11** The whole wire is very nearly at one uniform potential. There is essentially zero *difference* in potential between the bird's feet. Then negligible current goes through the bird. The resistance through the bird's body between its feet is much larger than the resistance through the wire between the same two points.

**Q28.12** The potential difference across a resistor is positive when it is measured *against* the direction of the current in the resistor.

**Q28.13** The bulb will light up for a while immediately after the switch is closed. As the capacitor charges, the bulb gets progressively dimmer. When the capacitor is fully charged the current in the circuit is zero and the bulb does not glow at all. If the value of $RC$ is small, this whole process might occupy a very short time interval.

**Q28.14** An ideal ammeter has zero resistance. An ideal voltmeter has infinite resistance. Real meters cannot attain these values, but do approach these values to the degree that they do not alter the current or potential difference that is being measured within the accuracy of the meter. Hooray for experimental uncertainty!

**Q28.15** A short circuit can develop when the last bit of insulation frays away between the two conductors in a lamp cord. Then the two conductors touch each other, opening a low-resistance branch in parallel with the lamp. The lamp will immediately go out, carrying no current and presenting no danger. A very large current exists in the power supply, the house wiring, and the rest of the lamp cord up to the contact point. Before it blows the fuse or pops the circuit breaker, the large current can quickly raise the temperature in the short-circuit path.

**Q28.16** A wire or cable in a transmission line is thick and made of material with very low resistivity. Only when its length is very large does its resistance become significant. To transmit power over a long distance it is most efficient to use low current at high voltage, minimizing the $I^2R$ power loss in the transmission line. Alternating current, as opposed to the direct current we study first, can be stepped up in voltage and then down again, with high-efficiency transformers at both ends of the power line.

**Q28.17** Car headlights are in parallel. If they were in series, both would go out when the filament of one failed. An important safety factor would be lost.

**Q28.18** Kirchhoff's junction rule expresses conservation of electric charge. If the total current into a point were different from the total current out, then charge would be continuously created or annihilated at that point.

Kirchhoff's loop rule expresses conservation of energy. For a single-loop circle with two resistors, the loop rule reads $+\varepsilon - IR_1 - IR_2 = 0$. This is algebraically equivalent to $q\varepsilon = qIR_1 + qIR_2$, where $q = I\Delta t$ is the charge passing a point in the loop in the time interval $\Delta t$. The equivalent equation states that the power supply injects energy into the circuit equal in amount to that which the resistors degrade into internal energy.

**Q28.19** At their normal operating temperatures, from $\mathcal{P} = \frac{\Delta V^2}{R}$, the bulbs present resistances $R = \frac{\Delta V^2}{\mathcal{P}} = \frac{(120\text{ V})^2}{60\text{ W}} = 240\ \Omega$, and $\frac{(120\text{ V})^2}{75\text{ W}} = 190\ \Omega$, and $\frac{(120\text{ V})^2}{200\text{ W}} = 72\ \Omega$. The nominal 60 W lamp has greatest resistance. When they are connected in series, they all carry the same small current. Here the highest-resistance bulb glows most brightly and the one with lowest resistance is faintest. This is just the reverse of their order of intensity if they were connected in parallel, as they are designed to be.

**Q28.20** Answer their question with a challenge. If the student is just looking at a diagram, provide the materials to build the circuit. If you are looking at a circuit where the second bulb really is fainter, get the student to unscrew them both and interchange them. But check that the student's understanding of potential has not been impaired: if you patch past the first bulb to short it out, the second gets brighter.

**Q28.21** Series, because the circuit breaker trips and opens the circuit when the current in that circuit loop exceeds a certain preset value. The circuit breaker must be in series to sense the appropriate current (see Fig. 28.30).

**Q28.22** The hospital maintenance worker is right. A hospital room is full of electrical grounds, including the bed frame. If your grandmother touched the faulty knob and the bed frame at the same time, she could receive quite a jolt, as there would be a potential difference of 120 V across her. If the 120 V is DC, the shock could send her into ventricular fibrillation, and the hospital staff could use the defibrillator you read about in Section 26.4. If the 120 V is AC, which is most likely, the current could produce external and internal burns along the path of conduction. Likely no one got a shock from the radio back at home because her bedroom contained no electrical grounds—no conductors connected to zero volts. Just like the bird in Question 28.11, granny could touch the "hot" knob without getting a shock so long as there was no path to ground to supply a potential difference across her. A new appliance in the bedroom or a flood could make the radio lethal. Repair it or discard it. Enjoy the news from Lake Wobegon on the new plastic radio.

**Q28.23** So long as you only grab one wire, and you do not touch anything that is grounded, you are safe (see Question 28.11). If the wire breaks, *let go!* If you continue to hold on to the wire, there will be a large—and rather lethal—potential difference between the wire and your feet when you hit the ground. Since your body can have a resistance of about 10 kΩ, the current in you would be sufficient to ruin your day.

**Q28.24** Both 120-V and 240-V lines can deliver injurious or lethal shocks, but there is a somewhat better safety factor with the lower voltage. To say it a different way, the insulation on a 120-V line can be thinner. On the other hand, a 240-V device carries less current to operate a device with the same power, so the conductor itself can be thinner. Finally, as we will see in Chapter 33, the last step-down transformer can also be somewhat smaller if it has to go down only to 240 volts from the high voltage of the main power line.

**Q28.25** As Luigi Galvani showed with his experiment with frogs' legs, muscles contract when electric current exists in them. If an electrician contacts a "live" wire, the muscles in his hands and fingers will contract, making his hand clench. If he touches the wire with the front of his hand, his hand will clench around the wire, and he may not be able to let go. Also, the back of his hand may be drier than his palm, so an actual shock may be much weaker.

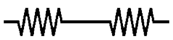
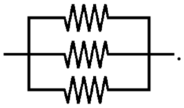
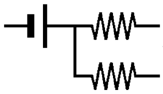
**Q28.26** Grab an insulator, like a stick or baseball bat, and bat for a home run. Hit the wire away from the person or hit them away from the wire. If you grab the person, you will learn very quickly about electrical circuits by becoming part of one.

**Q28.27** A high voltage can lead to a high current when placed in a circuit. A device cannot supply a high current—or any current—unless connected to a load. A more accurate sign saying *potentially high current* would just confuse the poor physics student who already has problems distinguishing between electrical potential and current.

**Q28.28** The two greatest factors are the potential difference between the wire and your feet, and the conductivity of the kite string. This is why Ben Franklin's experiment with lightning and flying a kite was so dangerous. Several scientists died trying to reproduce Franklin's results.

**Q28.29** Suppose $\varepsilon = 12\text{ V}$ and each lamp has $R = 2\ \Omega$. Before the switch is closed the current is $\frac{12\text{ V}}{6\ \Omega} = 2\text{ A}$. The potential difference across each lamp is $(2\text{ A})(2\ \Omega) = 4\text{ V}$. The power of each lamp is $(2\text{ A})(4\text{ V}) = 8\text{ W}$, totaling 24 W for the circuit. Closing the switch makes the switch and the wires connected to it a zero-resistance branch. All of the current through A and B will go through the switch and (b) lamp C goes out, with zero voltage across it. With less total resistance, the (c) current in the battery $\frac{12\text{ V}}{4\ \Omega} = 3\text{ A}$ becomes larger than before and (a) lamps A and B get brighter. (d) The voltage across each of A and B is $(3\text{ A})(2\ \Omega) = 6\text{ V}$, larger than before. Each converts power $(3\text{ A})(6\text{ V}) = 18\text{ W}$, totaling 36 W, which is (e) an increase.

**Q28.30** The starter motor draws a significant amount of current from the battery while it is starting the car. This, coupled with the internal resistance of the battery, decreases the output voltage of the battery below its the nominal 12 V emf. Then the current in the headlights decreases.

**Q28.31** Two runs in series:  . Three runs in parallel: . Junction of one lift and two runs: .

Gustav Robert Kirchhoff, Professor of Physics at Heidelberg and Berlin, was master of the obvious. A junction rule: The number of skiers coming into any junction must be equal to the number of skiers leaving. A loop rule: the total change in altitude must be zero for any skier completing a closed path.

## SOLUTIONS TO PROBLEMS

### Section 28.1 Electromotive Force

**P28.1** (a) $\mathcal{P}=\dfrac{(\Delta V)^2}{R}$

becomes $20.0\text{ W}=\dfrac{(11.6\text{ V})^2}{R}$

so $R=\boxed{6.73\ \Omega}$.

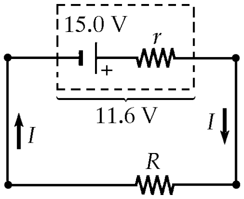


FIG. P28.1

(b) $\Delta V=IR$

so $11.6\text{ V}=I(6.73\ \Omega)$

and $I=1.72\text{ A}$

$\varepsilon=IR+Ir$

so $15.0\text{ V}=11.6\text{ V}+(1.72\text{ A})r$

$r=\boxed{1.97\ \Omega}$.

**P28.2** (a) $\Delta V_{\text{term}}=IR$

becomes $10.0\text{ V}=I(5.60\ \Omega)$

so $I=\boxed{1.79\text{ A}}$.

(b) $\Delta V_{\text{term}}=\varepsilon-Ir$

becomes $10.0\text{ V}=\varepsilon-(1.79\text{ A})(0.200\ \Omega)$

so $\varepsilon=\boxed{10.4\text{ V}}$.

**P28.3** The total resistance is $R=\dfrac{3.00\text{ V}}{0.600\text{ A}}=5.00\ \Omega$.

(a) $R_{\text{lamp}}=R-r_{\text{batteries}}=5.00\ \Omega-0.408\ \Omega=\boxed{4.59\ \Omega}$

(b) $\dfrac{\mathcal{P}_{\text{batteries}}}{\mathcal{P}_{\text{total}}}=\dfrac{(0.408\ \Omega)I^2}{(5.00\ \Omega)I^2}=0.081\,6=\boxed{8.16\%}$

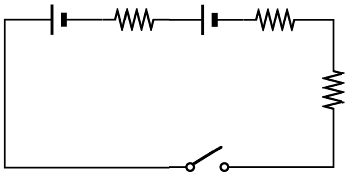


FIG. P28.3

**P28.4** (a) Here $\varepsilon = I(R+r)$, so $I = \frac{\varepsilon}{R+r} = \frac{12.6\text{ V}}{(5.00\ \Omega + 0.080\,0\ \Omega)} = 2.48\text{ A}$.

Then, $\Delta V = IR = (2.48\text{ A})(5.00\ \Omega) = \boxed{12.4\text{ V}}$.

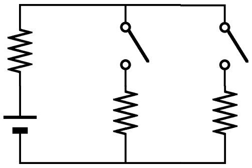


FIG. P28.4

(b) Let $I_1$ and $I_2$ be the currents flowing through the battery and the headlights, respectively.

Then, $I_1 = I_2 + 35.0\text{ A}$, and $\varepsilon - I_1 r - I_2 r = 0$

so $\varepsilon = (I_2 + 35.0\text{ A})(0.080\,0\ \Omega) + I_2(5.00\ \Omega) = 12.6\text{ V}$

giving $I_2 = 1.93\text{ A}$.

Thus, $\Delta V_2 = (1.93\text{ A})(5.00\ \Omega) = \boxed{9.65\text{ V}}$.

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## Section 28.2 **Resistors in Series and Parallel**

**P28.5** $\Delta V = I_1 R_1 = (2.00\text{ A})R_1$ and $\Delta V = I_2(R_1 + R_2) = (1.60\text{ A})(R_1 + 3.00\ \Omega)$

Therefore, $(2.00\text{ A})R_1 = (1.60\text{ A})(R_1 + 3.00\ \Omega)$ or $R_1 = \boxed{12.0\ \Omega}$.

**P28.6** (a) $R_p = \frac{1}{(1/7.00\ \Omega) + (1/10.0\ \Omega)} = 4.12\ \Omega$

$R_s = R_1 + R_2 + R_3 = 4.00 + 4.12 + 9.00 = \boxed{17.1\ \Omega}$

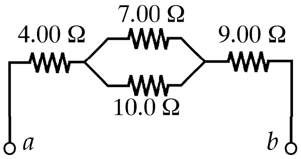


FIG. P28.6

(b) $\Delta V = IR$

$34.0\text{ V} = I(17.1\ \Omega)$

$I = \boxed{1.99\text{ A}}$ for 4.00 Ω, 9.00 Ω resistors.

Applying $\Delta V = IR$, $(1.99\text{ A})(4.12\ \Omega) = 8.18\text{ V}$

$8.18\text{ V} = I(7.00\ \Omega)$

so $I = \boxed{1.17\text{ A}}$ for 7.00 Ω resistor

$8.18\text{ V} = I(10.0\ \Omega)$

so $I = \boxed{0.818\text{ A}}$ for 10.0 Ω resistor.

**P28.7** For the bulb in use as intended,

$$I = \frac{\mathcal{P}}{\Delta V} = \frac{75.0\text{ W}}{120\text{ V}} = 0.625\text{ A}$$

and $$R = \frac{\Delta V}{I} = \frac{120\text{ V}}{0.625\text{ A}} = 192\ \Omega.$$

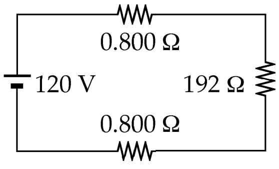


FIG. P28.7

Now, presuming the bulb resistance is unchanged,

$$I = \frac{120\text{ V}}{193.6\ \Omega} = 0.620\text{ A}.$$

Across the bulb is $\Delta V = IR = 192\ \Omega(0.620\text{ A}) = 119\text{ V}$

so its power is $\mathcal{P} = I\Delta V = 0.620\text{ A}(119\text{ V}) = \boxed{73.8\text{ W}}$.

**P28.8** $$120\text{ V} = IR_{\text{eq}} = I\left(\frac{\rho\ell}{A_1}+\frac{\rho\ell}{A_2}+\frac{\rho\ell}{A_3}+\frac{\rho\ell}{A_4}\right), \text{ or } I\rho\ell = \frac{(120\text{ V})}{\left(\frac{1}{A_1}+\frac{1}{A_2}+\frac{1}{A_3}+\frac{1}{A_4}\right)}$$

$$\Delta V_2 = \frac{I\rho\ell}{A_2} = \frac{(120\text{ V})}{A_2\left(\frac{1}{A_1}+\frac{1}{A_2}+\frac{1}{A_3}+\frac{1}{A_4}\right)} = \boxed{29.5\text{ V}}$$

**P28.9** If we turn the given diagram on its side, we find that it is the same as figure (a). The 20.0 Ω and 5.00 Ω resistors are in series, so the first reduction is shown in (b). In addition, since the 10.0 Ω, 5.00 Ω, and 25.0 Ω resistors are then in parallel, we can solve for their equivalent resistance as:

$$R_{\text{eq}} = \frac{1}{\left(\frac{1}{10.0\ \Omega}+\frac{1}{5.00\ \Omega}+\frac{1}{25.0\ \Omega}\right)} = 2.94\ \Omega.$$

This is shown in figure (c), which in turn reduces to the circuit shown in figure (d).

Next, we work backwards through the diagrams applying $I = \frac{\Delta V}{R}$ and $\Delta V = IR$ alternately to every resistor, real and equivalent. The 12.94 Ω resistor is connected across 25.0 V, so the current through the battery in every diagram is

$$I = \frac{\Delta V}{R} = \frac{25.0\text{ V}}{12.94\ \Omega} = 1.93\text{ A}.$$

In figure (c), this 1.93 A goes through the 2.94 Ω equivalent resistor to give a potential difference of:

$$\Delta V = IR = (1.93\text{ A})(2.94\ \Omega) = 5.68\text{ V}.$$

From figure (b), we see that this potential difference is the same across $\Delta V_{ab}$, the 10 Ω resistor, and the 5.00 Ω resistor.

(b) Therefore, $\Delta V_{ab} = \boxed{5.68\text{ V}}$.

(a) Since the current through the 20.0 Ω resistor is also the current through the 25.0 Ω line *ab*,

$$I = \frac{\Delta V_{ab}}{R_{ab}} = \frac{5.68\text{ V}}{25.0\ \Omega} = 0.227\text{ A} = \boxed{227\text{ mA}}.$$

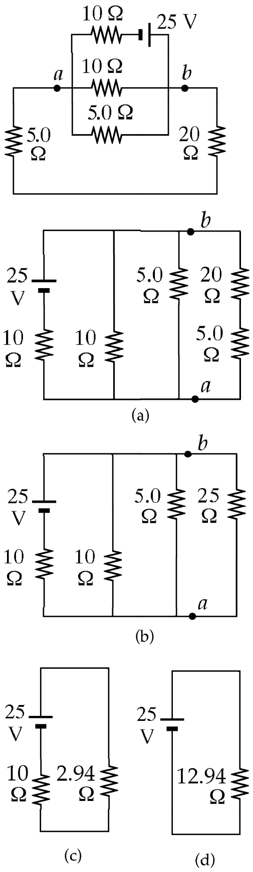


**FIG. P28.9**

***P28.10** We assume that the metal wand makes low-resistance contact with the person's hand and that the resistance through the person's body is negligible compared to the resistance $R_{\text{shoes}}$ of the shoe soles. The equivalent resistance seen by the power supply is $1.00\text{ M}\Omega + R_{\text{shoes}}$. The current through both resistors is $\frac{50.0\text{ V}}{1.00\text{ M}\Omega + R_{\text{shoes}}}$. The voltmeter displays

$$\Delta V = I(1.00\text{ M}\Omega) = \frac{50.0\text{ V}(1.00\text{ M}\Omega)}{1.00\text{ M}\Omega + R_{\text{shoes}}} = \Delta V.$$

(a) We solve to obtain $$50.0\text{ V}(1.00\text{ M}\Omega) = \Delta V(1.00\text{ M}\Omega) + \Delta V(R_{\text{shoes}})$$

$$R_{\text{shoes}} = \frac{1.00\text{ M}\Omega(50.0-\Delta V)}{\Delta V}.$$

(b) With $R_{\text{shoes}} \to 0$, the current through the person's body is

$$\frac{50.0\text{ V}}{1.00\text{ M}\Omega} = 50.0\ \mu\text{A} \qquad \boxed{\text{The current will never exceed } 50\ \mu\text{A}}.$$

**P28.11** (a) Since all the current in the circuit must pass through the series 100 Ω resistor, $\mathcal{P} = I^2R$

$$\mathcal{P}_{\max} = RI_{\max}^2$$

so $$I_{\max} = \sqrt{\frac{\mathcal{P}}{R}} = \sqrt{\frac{25.0\text{ W}}{100\ \Omega}} = 0.500\text{ A}$$

$$R_{eq} = 100\ \Omega + \left(\frac{1}{100} + \frac{1}{100}\right)^{-1}\ \Omega = 150\ \Omega$$

$$\Delta V_{\max} = R_{eq}I_{\max} = \boxed{75.0\text{ V}}$$

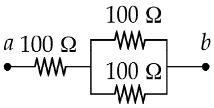


**FIG. P28.11**

(b) $$\mathcal{P} = I\Delta V = (0.500\text{ A})(75.0\text{ V}) = \boxed{37.5\text{ W}}\text{ total power}$$

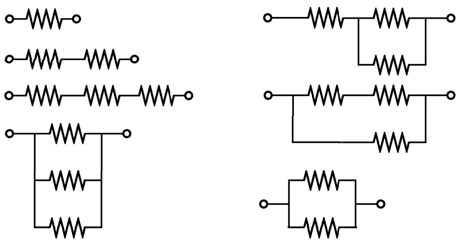
$$\mathcal{P}_1 = \boxed{25.0\text{ W}}$$

$$\mathcal{P}_2 = \mathcal{P}_3 = RI^2(100\ \Omega)(0.250\text{ A})^2 = \boxed{6.25\text{ W}}$$

**P28.12** Using 2.00-Ω, 3.00-Ω, 4.00-Ω resistors, there are 7 series, 4 parallel, and 6 mixed combinations:

| Series | | Parallel | Mixed |
|---|---|---|---|
| 2.00 Ω | 6.00 Ω | 0.923 Ω | 1.56 Ω |
| 3.00 Ω | 7.00 Ω | 1.20 Ω | 2.00 Ω |
| 4.00 Ω | 9.00 Ω | 1.33 Ω | 2.22 Ω |
| 5.00 Ω | | 1.71 Ω | 3.71 Ω |
| | | | 4.33 Ω |
| | | | 5.20 Ω |

The resistors may be arranged in patterns:



**P28.13** The potential difference is the same across either combination.

$$\Delta V = IR = 3I\frac{1}{\left(\frac{1}{R} + \frac{1}{500}\right)} \qquad \text{so} \qquad R\left(\frac{1}{R} + \frac{1}{500}\right) = 3$$

$$1 + \frac{R}{500} = 3 \qquad \text{and} \qquad R = 1\,000\ \Omega = \boxed{1.00\text{ k}\Omega}.$$

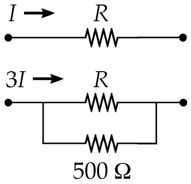


**FIG. P28.13**

***P28.14** *When S is open*, $R_1$, $R_2$, $R_3$ are in series with the battery. Thus:

$$R_1 + R_2 + R_3 = \frac{6\text{ V}}{10^{-3}\text{ A}} = 6\text{ k}\Omega. \qquad (1)$$

*When S is closed in position 1*, the parallel combination of the two $R_2$'s is in series with $R_1$, $R_3$, and the battery. Thus:

$$R_1 + \frac{1}{2}R_2 + R_3 = \frac{6\text{ V}}{1.2\times10^{-3}\text{ A}} = 5\text{ k}\Omega. \qquad (2)$$

*When S is closed in position 2*, $R_1$ and $R_2$ are in series with the battery. $R_3$ is shorted. Thus:

$$R_1 + R_2 = \frac{6\text{ V}}{2\times10^{-3}\text{ A}} = 3\text{ k}\Omega. \qquad (3)$$

From (1) and (3): $R_3 = 3\text{ k}\Omega$.

Subtract (2) from (1): $R_2 = 2\text{ k}\Omega$.

From (3): $R_1 = 1\text{ k}\Omega$.

Answers: $\boxed{R_1 = 1.00\text{ k}\Omega,\ R_2 = 2.00\text{ k}\Omega,\ R_3 = 3.00\text{ k}\Omega}$.

**P28.15** $R_p = \left(\dfrac{1}{3.00} + \dfrac{1}{1.00}\right)^{-1} = 0.750\ \Omega$

$R_s = (2.00 + 0.750 + 4.00)\ \Omega = 6.75\ \Omega$

$I_{\text{battery}} = \dfrac{\Delta V}{R_s} = \dfrac{18.0\text{ V}}{6.75\ \Omega} = 2.67\text{ A}$

$\mathcal{P} = I^2R$: $\quad \mathcal{P}_2 = (2.67\text{ A})^2(2.00\ \Omega)$

$\mathcal{P}_2 = \boxed{14.2\text{ W}}$ in $2.00\ \Omega$

$\mathcal{P}_4 = (2.67\text{ A})^2(4.00\text{ A}) = \boxed{28.4\text{ W}}$ in $4.00\ \Omega$

$\Delta V_2 = (2.67\text{ A})(2.00\ \Omega) = 5.33\text{ V},$

$\Delta V_4 = (2.67\text{ A})(4.00\ \Omega) = 10.67\text{ V}$

$\Delta V_p = 18.0\text{ V} - \Delta V_2 - \Delta V_4 = 2.00\text{ V}(= \Delta V_3 = \Delta V_1)$

$\mathcal{P}_3 = \dfrac{(\Delta V_3)^2}{R_3} = \dfrac{(2.00\text{ V})^2}{3.00\ \Omega} = \boxed{1.33\text{ W}}$ in $3.00\ \Omega$

$\mathcal{P}_1 = \dfrac{(\Delta V_1)}{R_1} = \dfrac{(2.00\text{ V})^2}{1.00\ \Omega} = \boxed{4.00\text{ W}}$ in $1.00\ \Omega$

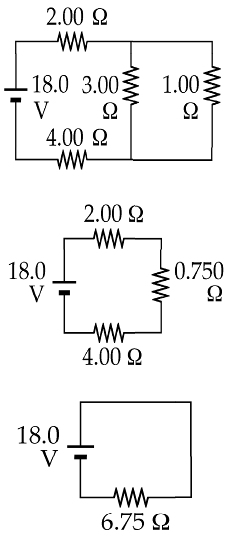


**FIG. P28.15**

**P28.16** Denoting the two resistors as $x$ and $y$,

$x + y = 690$, and $\dfrac{1}{150} = \dfrac{1}{x} + \dfrac{1}{y}$

$$\frac{1}{150} = \frac{1}{x} + \frac{1}{690 - x} = \frac{(690 - x) + x}{x(690 - x)}$$

$$x^2 - 690x + 103\,500 = 0$$

$$x = \frac{690 \pm \sqrt{(690)^2 - 414\,000}}{2}$$

$x = \boxed{470\ \Omega} \qquad y = \boxed{220\ \Omega}$

***P28.17** A certain quantity of energy $\Delta E_{\text{int}} = \mathcal{P}(\text{time})$ is required to raise the temperature of the water to 100°C. For the power delivered to the heaters we have $\mathcal{P} = I\Delta V = \dfrac{(\Delta V)^2}{R}$ where $(\Delta V)$ is a constant. Thus comparing coils 1 and 2, we have for the energy $\dfrac{(\Delta V)^2 \Delta t}{R_1} = \dfrac{(\Delta V)^2 2\Delta t}{R_2}$. Then $R_2 = 2R_1$.

(a) When connected in parallel, the coils present equivalent resistance

$R_p = \dfrac{1}{1/R_1 + 1/R_2} = \dfrac{1}{1/R_1 + 1/2R_1} = \dfrac{2R_1}{3}$. Now $\dfrac{(\Delta V)^2 \Delta t}{R_1} = \dfrac{(\Delta V)^2 \Delta t_p}{2R_1/3}$ $\qquad \Delta t_p = \boxed{\dfrac{2\Delta t}{3}}$.

(b) For the series connection, $R_s = R_1 + R_2 = R_1 + 2R_1 = 3R_1$ and $\dfrac{(\Delta V)^2 \Delta t}{R_1} = \dfrac{(\Delta V)^2 \Delta t_s}{3R_1}$

$\Delta t_s = \boxed{3\Delta t}$.

**P28.18** (a) $\Delta V = IR$: $33.0\text{ V} = I_1(11.0\ \Omega)$ $33.0\text{ V} = I_2(22.0\ \Omega)$

$I_3 = 3.00\text{ A}$ $I_2 = 1.50\text{ A}$

$\mathcal{P} = I^2R$: $\mathcal{P}_1 = (3.00\text{ A})^2(11.0\ \Omega)$ $\mathcal{P}_2 = (1.50\text{ A})^2(22.0\ \Omega)$

$\mathcal{P}_1 = 99.0\text{ W}$ $\mathcal{P}_2 = 49.5\text{ W}$

$\boxed{\text{The 11.0-}\Omega\text{ resistor uses more power.}}$

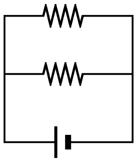


**FIG. P28.18(a)**

(b) $\mathcal{P}_1 + \mathcal{P}_2 = \boxed{148\text{ W}}$ $\mathcal{P} = I(\Delta V) = (4.50)(33.0) = \boxed{148\text{ W}}$

(c) $R_s = R_1 + R_2 = 11.0\ \Omega + 22.0\ \Omega = 33.0\ \Omega$

$\Delta V = IR$: $33.0\text{ V} = I(33.0\ \Omega)$, so $I = 1.00\text{ A}$

$\mathcal{P} = I^2R$: $\mathcal{P}_1 = (1.00\text{ A})^2(11.0\ \Omega)$ $\mathcal{P}_2 = (1.00\text{ A})^2(22.0\ \Omega)$

$\mathcal{P}_1 = 11.0\text{ W}$ $\mathcal{P}_2 = 22.0\text{ W}$

$\boxed{\text{The 22.0-}\Omega\text{ resistor uses more power.}}$

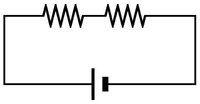


**FIG. P28.18(c)**

(d) $\mathcal{P}_1 + \mathcal{P}_2 = I^2(R_1 + R_2) = (1.00\text{ A})^2(33.0\ \Omega) = \boxed{33.0\text{ W}}$

$\mathcal{P} = I(\Delta V) = (1.00\text{ A})(33.0\text{ V}) = \boxed{33.0\text{ W}}$

(e) $\boxed{\text{The parallel configuration uses more power.}}$

***P28.19** (a) The resistors 2, 3, and 4 can be combined to a single 2*R* resistor. This is in series with resistor 1, with resistance *R*, so the equivalent resistance of the whole circuit is 3*R*. In series, potential difference is shared in proportion to the resistance, so resistor 1 gets $\frac{1}{3}$ of the battery voltage and the 2-3-4 parallel combination get $\frac{2}{3}$ of the battery voltage. This is the potential difference across resistor 4, but resistors 2 and 3 must share this voltage. $\frac{1}{3}$ goes to 2 and $\frac{2}{3}$ to 3. The ranking by potential difference is $\boxed{\Delta V_4 > \Delta V_3 > \Delta V_1 > \Delta V_2}$.

(b) Based on the reasoning above the potential differences are

$$\boxed{\Delta V_1 = \frac{\varepsilon}{3},\ \Delta V_2 = \frac{2\varepsilon}{9},\ \Delta V_3 = \frac{4\varepsilon}{9},\ \Delta V_4 = \frac{2\varepsilon}{3}}.$$

(c) All the current goes through resistor 1, so it gets the most. The current then splits at the parallel combination. Resistor 4 gets more than half, because the resistance in that branch is less than in the other branch. Resistors 2 and 3 have equal currents because they are in series. The ranking by current is $\boxed{I_1 > I_4 > I_2 = I_3}$.

(d) Resistor 1 has a current of *I*. Because the resistance of 2 and 3 in series is twice that of resistor 4, twice as much current goes through 4 as through 2 and 3. The current through the resistors are $\boxed{I_1 = I,\ I_2 = I_3 = \frac{I}{3},\ I_4 = \frac{2I}{3}}$.

*continued on next page*

(e) Increasing resistor 3 increases the equivalent resistance of the entire circuit. The current in the circuit, which is the current through resistor 1, decreases. This decreases the potential difference across resistor 1, increasing the potential difference across the parallel combination. With a larger potential difference the current through resistor 4 is increased. With more current through 4, and less in the circuit to start with, the current through resistors 2 and 3 must decrease. To summarize, $\boxed{I_4 \text{ increases and } I_1,\ I_2, \text{ and } I_3 \text{ decrease}}$.

(f) If resistor 3 has an infinite resistance it blocks any current from passing through that branch, and the circuit effectively is just resistor 1 and resistor 4 in series with the battery. The circuit now has an equivalent resistance of $4R$. The current in the circuit drops to $\frac{3}{4}$ of the original current because the resistance has increased by $\frac{4}{3}$. All this current passes through resistors 1 and 4, and none passes through 2 or 3. Therefore $\boxed{I_1=\frac{3I}{4},\ I_2=I_3=0,\ I_4=\frac{3I}{4}}$.

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## Section 28.3 **Kirchhoff's Rules**

**P28.20** $+15.0-(7.00)I_1-(2.00)(5.00)=0$

$5.00=7.00I_1$ so $\boxed{I_1=0.714\text{ A}}$

$I_3=I_1+I_2=2.00\text{ A}$

$0.714+I_2=2.00$ so $\boxed{I_2=1.29\text{ A}}$

$+\varepsilon-2.00(1.29)-5.00(2.00)=0$ $\boxed{\varepsilon=12.6\text{ V}}$

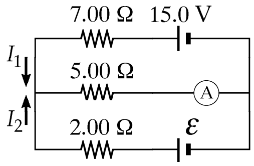


**FIG. P28.20**

**P28.21** We name currents $I_1$, $I_2$, and $I_3$ as shown.

From Kirchhoff's current rule, $I_3=I_1+I_2$.

Applying Kirchhoff's voltage rule to the loop containing $I_2$ and $I_3$,

$12.0\text{ V}-(4.00)I_3-(6.00)I_2-4.00\text{ V}=0$

$8.00=(4.00)I_3+(6.00)I_2$

Applying Kirchhoff's voltage rule to the loop containing $I_1$ and $I_2$,

$$-(6.00)I_2-4.00\text{ V}+(8.00)I_1=0 \qquad (8.00)I_1=4.00+(6.00)I_2.$$

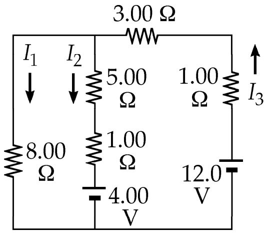


**FIG. P28.21**

Solving the above linear system, we proceed to the pair of simultaneous equations:

$$\begin{cases} 8=4I_1+4I_2+6I_2 \\ 8I_1=4+6I_2 \end{cases} \quad \text{or} \quad \begin{cases} 8=4I_1+10I_2 \\ I_2=1.33I_1-0.667 \end{cases}$$

and to the single equation $8=4I_1+13.3I_1-6.67$

$$I_1=\frac{14.7\text{ V}}{17.3\ \Omega}=0.846\text{ A}. \qquad \text{Then} \qquad I_2=1.33(0.846\text{ A})-0.667$$

and $I_3=I_1+I_2$ give $\boxed{I_1=846\text{ mA},\ I_2=462\text{ mA},\ I_3=1.31\text{ A}}$.

All currents are in the directions indicated by the arrows in the circuit diagram.

**P28.22** The solution figure is shown to the right.

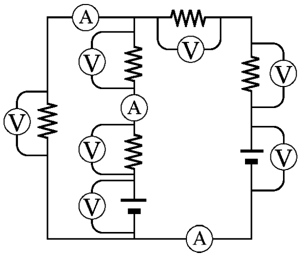


**FIG. P28.22**

**P28.23** We use the results of Problem 28.21.

(a) By the 4.00-V battery: $\Delta U = (\Delta V)I\Delta t = (4.00\text{ V})(-0.462\text{ A})120\text{ s} = \boxed{-222\text{ J}}$.

By the 12.0-V battery: $(12.0\text{ V})(1.31\text{ A})120\text{ s} = \boxed{1.88\text{ kJ}}$.

(b) By the 8.00-Ω resistor: $I^2R\Delta t = (0.846\text{ A})^2(8.00\ \Omega)120\text{ s} = \boxed{687\text{ J}}$.

By the 5.00-Ω resistor: $(0.462\text{ A})^2(5.00\ \Omega)120\text{ s} = \boxed{128\text{ J}}$.

By the 1.00-Ω resistor: $(0.462\text{ A})^2(1.00\ \Omega)120\text{ s} = \boxed{25.6\text{ J}}$.

By the 3.00-Ω resistor: $(1.31\text{ A})^2(3.00\ \Omega)120\text{ s} = \boxed{616\text{ J}}$.

By the 1.00-Ω resistor: $(1.31\text{ A})^2(1.00\ \Omega)120\text{ s} = \boxed{205\text{ J}}$.

(c) $-222\text{ J} + 1.88\text{ kJ} = \boxed{1.66\text{ kJ}}$ from chemical to electrical.

$687\text{ J} + 128\text{ J} + 25.6\text{ J} + 616\text{ J} + 205\text{ J} = 1.66\text{ kJ}$ from electrical to internal.

**P28.24** We name the currents $I_1$, $I_2$, and $I_3$ as shown.

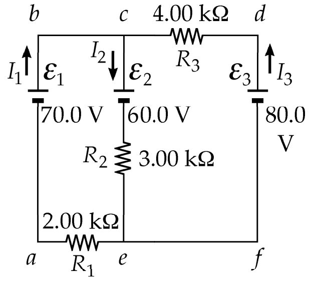


**FIG. P28.24**

[1] $70.0 - 60.0 - I_2(3.00\text{ k}\Omega) - I_1(2.00\text{ k}\Omega) = 0$

[2] $80.0 - I_3(4.00\text{ k}\Omega) - 60.0 - I_2(3.00\text{ k}\Omega) = 0$

[3] $I_2 = I_1 + I_3$

(a) Substituting for $I_2$ and solving the resulting simultaneous equations yields

$I_1 = \boxed{0.385\text{ mA}}$ (through $R_1$)

$I_3 = \boxed{2.69\text{ mA}}$ (through $R_3$)

$I_2 = \boxed{3.08\text{ mA}}$ (through $R_2$)

(b) $\Delta V_{cf} = -60.0\text{ V} - (3.08\text{ mA})(3.00\text{ k}\Omega) = \boxed{-69.2\text{ V}}$

$\boxed{\text{Point } c \text{ is at higher potential.}}$

**P28.25** Label the currents in the branches as shown in the first figure. Reduce the circuit by combining the two parallel resistors as shown in the second figure.

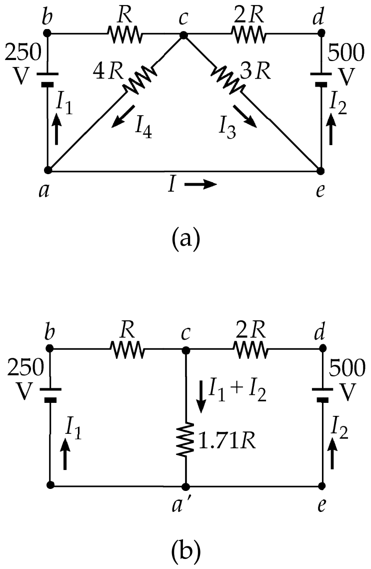


**FIG. P28.25**

Apply Kirchhoff's loop rule to both loops in Figure (b) to obtain:

$$(2.71R)I_1 + (1.71R)I_2 = 250$$

and $$(1.71R)I_1 + (3.71R)I_2 = 500.$$

With $R = 1\,000\ \Omega$, simultaneous solution of these equations yields:

$$I_1 = 10.0\text{ mA}$$

and $$I_2 = 130.0\text{ mA}.$$

From Figure (b), $$V_c - V_a = (I_1 + I_2)(1.71R) = 240\text{ V}.$$

Thus, from Figure (a), $$I_4 = \frac{V_c - V_a}{4R} = \frac{240\text{ V}}{4\,000\ \Omega} = 60.0\text{ mA}.$$

Finally, applying Kirchhoff's point rule at point *a* in Figure (a) gives:

$$I = I_4 - I_1 = 60.0\text{ mA} - 10.0\text{ mA} = +50.0\text{ mA},$$

or $$I = \boxed{50.0\text{ mA from point } a \text{ to point } e}.$$

**P28.26** Name the currents as shown in the figure to the right. Then $w + x + z = y$. Loop equations are

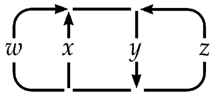


**FIG. P28.26**

$$-200w - 40.0 + 80.0x = 0$$
$$-80.0x + 40.0 + 360 - 20.0y = 0$$
$$+360 - 20.0y - 70.0z + 80.0 = 0$$

Eliminate $y$ by substitution. $$\begin{cases} x = 2.50w + 0.500 \\ 400 - 100x - 20.0w - 20.0z = 0 \\ 440 - 20.0w - 20.0x - 90.0z = 0 \end{cases}$$

Eliminate $x$. $$\begin{cases} 350 - 270w - 20.0z = 0 \\ 430 - 70.0w - 90.0z = 0 \end{cases}$$

Eliminate $z = 17.5 - 13.5w$ to obtain $$430 - 70.0w - 1\,575 + 1\,215w = 0$$

$$w = \frac{70.0}{70.0} = \boxed{1.00\text{ A upward in } 200\ \Omega}.$$

Now $$z = \boxed{4.00\text{ A upward in } 70.0\ \Omega}$$

$$x = \boxed{3.00\text{ A upward in } 80.0\ \Omega}$$

$$y = \boxed{8.00\text{ A downward in } 20.0\ \Omega}$$

and for the 200 Ω, $$\Delta V = IR = (1.00\text{ A})(200\ \Omega) = \boxed{200\text{ V}}.$$

**P28.27** Using Kirchhoff's rules,

$$12.0-(0.010\,0)I_1-(0.060\,0)I_3=0$$

$$10.0+(1.00)I_2-(0.060\,0)I_3=0$$

and $I_1=I_2+I_3$

$$12.0-(0.010\,0)I_2-(0.070\,0)I_3=0$$

$$10.0+(1.00)I_2-(0.060\,0)I_3=0$$

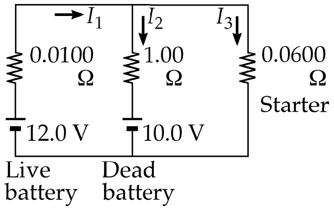


**FIG. P28.27**

Solving simultaneously,

$I_2=\boxed{0.283\text{ A downward}}$ in the dead battery

and $I_3=\boxed{171\text{ A downward}}$ in the starter.

The currents are forward in the live battery and in the starter, relative to normal starting operation. The current is backward in the dead battery, tending to charge it up.

**P28.28** $\Delta V_{ab}=(1.00)I_1+(1.00)(I_1-I_2)$

$$\Delta V_{ab}=(1.00)I_1+(1.00)I_2+(5.00)(I-I_1+I_2)$$

$$\Delta V_{ab}=(3.00)(I-I_1)+(5.00)(I-I_1+I_2)$$

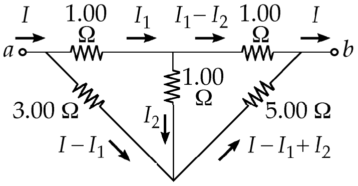


**FIG. P28.28**

Let $I=1.00$ A, $I_1=x$, and $I_2=y$.

Then, the three equations become:

$\Delta V_{ab}=2.00x-y$, or $y=2.00x-\Delta V_{ab}$

$$\Delta V_{ab}=-4.00x+6.00y+5.00$$

and $\Delta V_{ab}=8.00-8.00x+5.00y$.

Substituting the first into the last two gives:

$7.00\Delta V_{ab}=8.00x+5.00$ and $6.00\Delta V_{ab}=2.00x+8.00$.

Solving these simultaneously yields $\Delta V_{ab}=\frac{27}{17}$ V.

Then, $R_{ab}=\dfrac{\Delta V_{ab}}{I}=\dfrac{\frac{27}{17}\text{ V}}{1.00\text{ A}}$ or $\boxed{R_{ab}=\dfrac{27}{17}\ \Omega}$.

**P28.29** We name the currents $I_1$, $I_2$, and $I_3$ as shown.

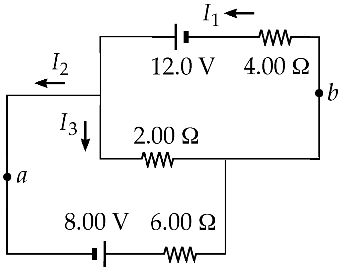


**FIG. P28.29**

(a) $I_1=I_2+I_3$

Counterclockwise around the top loop,

$$12.0\text{ V}-(2.00\ \Omega)I_3-(4.00\ \Omega)I_1=0.$$

Traversing the bottom loop,

$$8.00\text{ V}-(6.00\ \Omega)I_2+(2.00\ \Omega)I_3=0$$

$$I_1=3.00-\frac{1}{2}I_3,\ I_2=\frac{4}{3}+\frac{1}{3}I_3,\text{ and }\boxed{I_3=909\text{ mA}}.$$

(b) $V_a-(0.909\text{ A})(2.00\ \Omega)=V_b$

$$V_b-V_a=\boxed{-1.82\text{ V}}$$

**P28.30** We apply Kirchhoff's rules to the second diagram.

$$50.0-2.00I_1-2.00I_2=0 \qquad (1)$$

$$20.0-2.00I_3+2.00I_2=0 \qquad (2)$$

$$I_1=I_2+I_3 \qquad (3)$$

Substitute (3) into (1), and solve for $I_1$, $I_2$, and $I_3$

$I_1=20.0\text{ A}$; $I_2=5.00\text{ A}$; $I_3=15.0\text{ A}$.

Then apply $\mathcal{P}=I^2R$ to each resistor:

$(2.00\ \Omega)_1$: $\mathcal{P}=I_1^2(2.00\ \Omega)=(20.0\text{ A})^2(2.00\ \Omega)=\boxed{800\text{ W}}$

$(4.00\ \Omega)$: $\mathcal{P}=\left(\dfrac{5.00}{2}\text{ A}\right)^2(4.00\ \Omega)=\boxed{25.0\text{ W}}$
(Half of $I_2$ goes through each)

$(2.00\ \Omega)_3$: $\mathcal{P}=I_3^2(2.00\ \Omega)=(15.0\text{ A})^2(2.00\ \Omega)=\boxed{450\text{ W}}$.

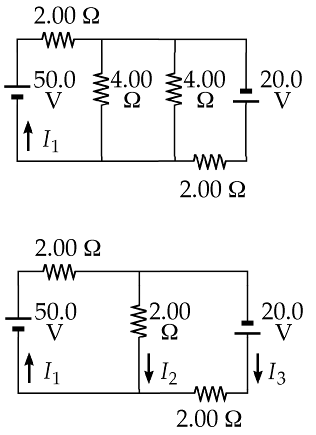


**FIG. P28.30**

## Section 28.4 *RC* Circuits

**P28.31** (a) $RC=\left(1.00\times10^6\ \Omega\right)\left(5.00\times10^{-6}\text{ F}\right)=\boxed{5.00\text{ s}}$

(b) $Q=C\varepsilon=\left(5.00\times10^{-6}\text{ C}\right)(30.0\text{ V})=\boxed{150\ \mu\text{C}}$

(c) $I(t)=\dfrac{\varepsilon}{R}e^{-t/RC}=\left(\dfrac{30.0}{1.00\times10^6}\right)\exp\left[\dfrac{-10.0}{\left(1.00\times10^6\right)\left(5.00\times10^{-6}\right)}\right]=\boxed{4.06\ \mu\text{A}}$

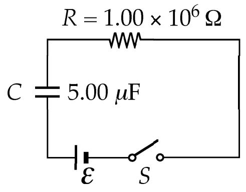


**FIG. P28.31**

**P28.32** (a) $I(t)=-I_0e^{-t/RC}$

$$I_0=\frac{Q}{RC}=\frac{5.10\times10^{-6}\text{ C}}{(1\,300\ \Omega)\left(2.00\times10^{-9}\text{ F}\right)}=1.96\text{ A}$$

$$I(t)=-(1.96\text{ A})\exp\left[\frac{-9.00\times10^{-6}\text{ s}}{(1\,300\ \Omega)\left(2.00\times10^{-9}\text{ F}\right)}\right]=\boxed{-61.6\text{ mA}}$$

(b) $q(t)=Qe^{-t/RC}=(5.10\ \mu\text{C})\exp\left[\dfrac{-8.00\times10^{-6}\text{ s}}{(1\,300\ \Omega)\left(2.00\times10^{-9}\text{ F}\right)}\right]=\boxed{0.235\ \mu\text{C}}$

(c) The magnitude of the maximum current is $I_0=\boxed{1.96\text{ A}}$.

**P28.33** $U=\dfrac{1}{2}C(\Delta V)^2$ and $\Delta V=\dfrac{Q}{C}$.

Therefore, $U=\dfrac{Q^2}{2C}$ and when the charge decreases to half its original value, the stored energy is one-quarter its original value: $\boxed{U_f=\dfrac{1}{4}U_0}$.

**P28.34** $q(t) = Q\left[1-e^{-t/RC}\right]$ so $\dfrac{q(t)}{Q} = 1-e^{-t/RC}$

$0.600 = 1-e^{-0.900/RC}$ or $e^{-0.900/RC} = 1-0.600 = 0.400$

$\dfrac{-0.900}{RC} = \ln(0.400)$ thus $RC = \dfrac{-0.900}{\ln(0.400)} = \boxed{0.982\text{ s}}$.

***P28.35** We are to calculate

$$\int_0^\infty e^{-2t/RC}dt = -\frac{RC}{2}\int_0^\infty e^{-2t/RC}\left(-\frac{2dt}{RC}\right) = -\frac{RC}{2}e^{-2t/RC}\Big|_0^\infty = -\frac{RC}{2}\left[e^{-\infty}-e^{0}\right] = -\frac{RC}{2}[0-1] = \boxed{+\frac{RC}{2}}.$$

**P28.36** (a) $\tau = RC = \left(1.50\times10^{5}\ \Omega\right)\left(10.0\times10^{-6}\ \text{F}\right) = \boxed{1.50\text{ s}}$

(b) $\tau = \left(1.00\times10^{5}\ \Omega\right)\left(10.0\times10^{-6}\ \text{F}\right) = \boxed{1.00\text{ s}}$

(c) The battery carries current $\dfrac{10.0\text{ V}}{50.0\times10^{3}\ \Omega} = 200\ \mu\text{A}$.

The 100 kΩ carries current of magnitude $I = I_0e^{-t/RC} = \left(\dfrac{10.0\text{ V}}{100\times10^{3}\ \Omega}\right)e^{-t/1.00\text{ s}}$.

So the switch carries downward current $\boxed{200\ \mu\text{A} + (100\ \mu\text{A})e^{-t/1.00\text{ s}}}$.

**P28.37** (a) Call the potential at the left junction $V_L$ and at the right $V_R$. After a "long" time, the capacitor is fully charged.

$V_L = 8.00$ V because of voltage divider:

$$I_L = \frac{10.0\text{ V}}{5.00\ \Omega} = 2.00\text{ A}$$

$$V_L = 10.0\text{ V} - (2.00\text{ A})(1.00\ \Omega) = 8.00\text{ V}$$

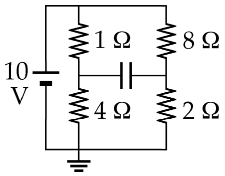


**FIG. P28.37(a)**

Likewise, $V_R = \left(\dfrac{2.00\ \Omega}{2.00\ \Omega + 8.00\ \Omega}\right)(10.0\text{ V}) = 2.00\text{ V}$

or $I_R = \dfrac{10.0\text{ V}}{10.0\ \Omega} = 1.00\text{ A}$

$$V_R = (10.0\text{ V}) - (8.00\ \Omega)(1.00\text{ A}) = 2.00\text{ V}.$$

Therefore, $\Delta V = V_L - V_R = 8.00 - 2.00 = \boxed{6.00\text{ V}}$.

(b) Redraw the circuit $R = \dfrac{1}{(1/9.00\ \Omega) + (1/6.00\ \Omega)} = 3.60\ \Omega$

$$RC = 3.60\times10^{-6}\text{ s}$$

and $e^{-t/RC} = \dfrac{1}{10}$

so $t = RC\ln10 = \boxed{8.29\ \mu\text{s}}$.

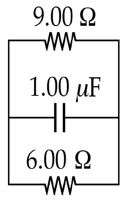


**FIG. P28.37(b)**

***P28.38** (a) We model the person's body and street shoes as shown. For the discharge to reach 100 V,

$$q(t)=Qe^{-t/RC}=C\Delta V(t)=C\Delta V_0e^{-t/RC}$$

$$\frac{\Delta V}{\Delta V_0}=e^{-t/RC}\qquad \frac{\Delta V_0}{\Delta V}=e^{+t/RC}\qquad \frac{t}{RC}=\ln\left(\frac{\Delta V_0}{\Delta V}\right)$$

$$t=RC\ln\left(\frac{\Delta V_0}{\Delta V}\right)=5\,000\times10^{6}\ \Omega\left(230\times10^{-12}\ \text{F}\right)\ln\left(\frac{3\,000}{100}\right)=\boxed{3.91\ \text{s}}$$

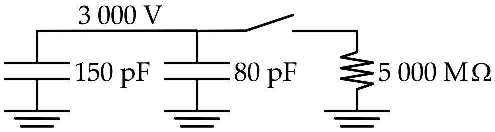


**FIG. P28.38(a)**

(b) $$t=1\times10^{6}\ \text{V/A}\left(230\times10^{-12}\ \text{C/V}\right)\ln 30=\boxed{782\ \mu\text{s}}$$

**P28.39** (a) $$\tau=RC=\left(4.00\times10^{6}\ \Omega\right)\left(3.00\times10^{-6}\ \text{F}\right)=\boxed{12.0\ \text{s}}$$

(b) $$I=\frac{\varepsilon}{R}e^{-t/RC}=\frac{12.0}{4.00\times10^{6}}e^{-t/12.0\ \text{s}}$$

$$q=C\varepsilon\left[1-e^{-t/RC}\right]=3.00\times10^{-6}(12.0)\left[1-e^{-t/12.0}\right]$$

$$\boxed{q=36.0\ \mu\text{C}\left[1-e^{-t/12.0}\right]}\quad \boxed{I=3.00\ \mu\text{A}e^{-t/12.0}}$$

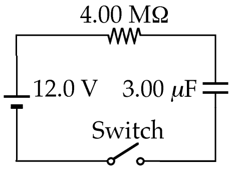


**FIG. P28.39**

**P28.40** $$\Delta V_0=\frac{Q}{C}$$

Then, if $q(t)=Qe^{-t/RC}$ $\qquad \Delta V(t)=(\Delta V_0)e^{-t/RC}$

and $\qquad \dfrac{\Delta V(t)}{(\Delta V_0)}=e^{-t/RC}.$

When $\Delta V(t)=\frac{1}{2}(\Delta V_0)$, then $\qquad e^{-t/RC}=\dfrac{1}{2}$

$$-\frac{t}{RC}=\ln\left(\frac{1}{2}\right)=-\ln 2.$$

Thus, $\qquad \boxed{R=\dfrac{t}{C(\ln 2)}}.$

## Section 28.5 **Electrical Meters**

**P28.41** $\Delta V=I_gr_g=\left(I-I_g\right)R_p$, or $R_p=\dfrac{I_gr_g}{\left(I-I_g\right)}=\dfrac{I_g(60.0\ \Omega)}{\left(I-I_g\right)}$

Therefore, to have $I=0.100\ \text{A}=100\ \text{mA}$ when $I_g=0.500\ \text{mA}$:

$$R_p=\frac{(0.500\ \text{mA})(60.0\ \Omega)}{99.5\ \text{mA}}=\boxed{0.302\ \Omega}.$$

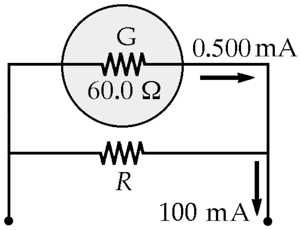


**FIG. P28.41**

**P28.42** Applying Kirchhoff's loop rule, $-I_g(75.0\ \Omega)+\left(I-I_g\right)R_p=0$.

Therefore, if $I=1.00\ \text{A}$ when $I_g=1.50\ \text{mA}$,

$$R_p=\frac{I_g(75.0\ \Omega)}{\left(I-I_g\right)}=\frac{\left(1.50\times10^{-3}\ \text{A}\right)(75.0\ \Omega)}{1.00\ \text{A}-1.50\times10^{-3}\ \text{A}}=\boxed{0.113\ \Omega}.$$

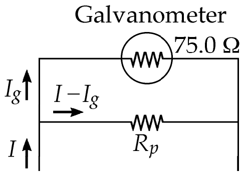


**FIG. P28.42**

**P28.43** Series Resistor → Voltmeter

$\Delta V=IR$: $\quad 25.0=1.50\times10^{-3}\left(R_s+75.0\right)$

Solving, $\quad \boxed{R_s=16.6\ \text{k}\Omega}$.

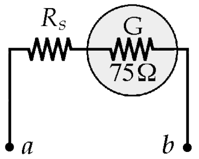
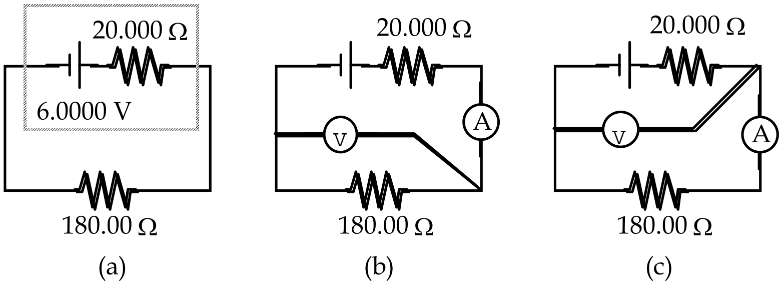


**FIG. P28.43**

**P28.44** (a) In Figure (a), the emf sees an equivalent resistance of $200.00\ \Omega$.

$$I=\frac{6.000\ 0\ \text{V}}{200.00\ \Omega}=\boxed{0.030\ 000\ \text{A}}$$



(a) (b) (c)

**FIG. P28.44**

The terminal potential difference is $\Delta V=IR=(0.030\ 000\ \text{A})(180.00\ \Omega)=\boxed{5.400\ 0\ \text{V}}$.

(b) In Figure (b), $R_{eq}=\left(\dfrac{1}{180.00\ \Omega}+\dfrac{1}{20\ 000\ \Omega}\right)^{-1}=178.39\ \Omega$.

The equivalent resistance across the emf is $178.39\ \Omega+0.500\ 00\ \Omega+20.000\ \Omega=198.89\ \Omega$.

The ammeter reads $I=\dfrac{\varepsilon}{R}=\dfrac{6.000\ 0\ \text{V}}{198.89\ \Omega}=\boxed{0.030\ 167\ \text{A}}$

and the voltmeter reads $\Delta V=IR=(0.030\ 167\ \text{A})(178.39\ \Omega)=\boxed{5.381\ 6\ \text{V}}$.

(c) In Figure (c), $\left(\dfrac{1}{180.50\ \Omega}+\dfrac{1}{20\ 000\ \Omega}\right)^{-1}=178.89\ \Omega$.

Therefore, the emf sends current through $R_{\text{tot}}=178.89\ \Omega+20.000\ \Omega=198.89\ \Omega$.

The current through the battery is $I=\dfrac{6.000\ 0\ \text{V}}{198.89\ \Omega}=0.030\ 168\ \text{A}$

but not all of this goes through the ammeter.

The voltmeter reads $\Delta V=IR=(0.030\ 168\ \text{A})(178.89\ \Omega)=\boxed{5.396\ 6\ \text{V}}$.

The ammeter measures current $I=\dfrac{\Delta V}{R}=\dfrac{5.396\ 6\ \text{V}}{180.50\ \Omega}=\boxed{0.029\ 898\ \text{A}}$.

The connection shown in Figure (c) is better than that shown in Figure (b) for accurate readings.

**P28.45** Consider the circuit diagram shown, realizing that $I_g = 1.00\text{ mA}$. For the 25.0 mA scale:

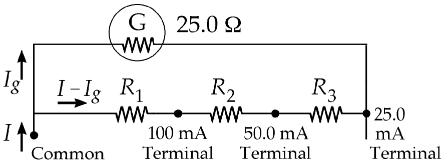


**FIG. P28.45**

$$(24.0\text{ mA})(R_1 + R_2 + R_3) = (1.00\text{ mA})(25.0\ \Omega)$$

or $$R_1 + R_2 + R_3 = \left(\frac{25.0}{24.0}\right)\Omega. \qquad (1)$$

For the 50.0 mA scale: $$(49.0\text{ mA})(R_1 + R_2) = (1.00\text{ mA})(25.0\ \Omega + R_3)$$

or $$49.0(R_1 + R_2) = 25.0\ \Omega + R_3. \qquad (2)$$

For the 100 mA scale: $$(99.0\text{ mA})R_1 = (1.00\text{ mA})(25.0\ \Omega + R_2 + R_3)$$

or $$99.0R_1 = 25.0\ \Omega + R_2 + R_3. \qquad (3)$$

Solving (1), (2), and (3) simultaneously yields

$$\boxed{R_1 = 0.260\ \Omega,\ R_2 = 0.261\ \Omega,\ R_3 = 0.521\ \Omega}.$$

**P28.46** $\Delta V = IR$

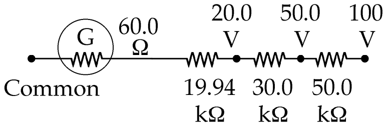


**FIG. P28.46**

(a) $$20.0\text{ V} = \left(1.00\times10^{-3}\text{ A}\right)(R_1 + 60.0\ \Omega)$$

$$R_1 = 1.994\times10^{4}\ \Omega = \boxed{19.94\text{ k}\Omega}$$

(b) $$50.0\text{ V} = \left(1.00\times10^{-3}\text{ A}\right)(R_2 + R_1 + 60.0\ \Omega) \qquad R_2 = \boxed{30.0\text{ k}\Omega}$$

(c) $$100\text{ V} = \left(1.00\times10^{-3}\text{ A}\right)(R_3 + R_1 + 60.0\ \Omega) \qquad R_3 = \boxed{50.0\text{ k}\Omega}$$

**P28.47** *Ammeter:* $$I_g r = \left(0.500\text{ A} - I_g\right)(0.220\ \Omega)$$

or $$I_g(r + 0.220\ \Omega) = 0.110\text{ V} \qquad (1)$$

*Voltmeter:* $$2.00\text{ V} = I_g(r + 2\,500\ \Omega) \qquad (2)$$

Solve (1) and (2) simultaneously to find:

$$I_g = \boxed{0.756\text{ mA}} \text{ and } r = \boxed{145\ \Omega}.$$

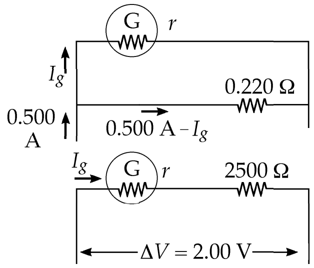


**FIG. P28.47**

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## Section 28.6 Household Wiring and Electrical Safety

**P28.48** (a) $$\mathcal{P} = I^2R = I^2\left(\frac{\rho\ell}{A}\right) = \frac{(1.00\text{ A})^2\left(1.70\times10^{-8}\ \Omega\cdot\text{m}\right)(16.0\text{ ft})(0.304\,8\text{ m/ft})}{\pi\left(0.512\times10^{-3}\text{ m}\right)^2} = \boxed{0.101\text{ W}}$$

(b) $$\mathcal{P} = I^2R = 100(0.101\ \Omega) = \boxed{10.1\text{ W}}$$

**P28.49** (a) $\mathcal{P} = I\Delta V$: So for the Heater, $I = \frac{\mathcal{P}}{\Delta V} = \frac{1\,500 \text{ W}}{120 \text{ V}} = \boxed{12.5 \text{ A}}$.

For the Toaster, $I = \frac{750 \text{ W}}{120 \text{ V}} = \boxed{6.25 \text{ A}}$.

And for the Grill, $I = \frac{1\,000 \text{ W}}{120 \text{ V}} = \boxed{8.33 \text{ A}}$.

(b) $12.5 + 6.25 + 8.33 = \boxed{27.1 \text{ A}}$

The current draw is greater than 25.0 amps, so this circuit breaker would not be sufficient.

**P28.50** $I_{\text{Al}}^2 R_{\text{Al}} = I_{\text{Cu}}^2 R_{\text{Cu}}$ so $I_{\text{Al}} = \sqrt{\frac{R_{\text{Cu}}}{R_{\text{Al}}}} I_{\text{Cu}} = \sqrt{\frac{\rho_{\text{Cu}}}{\rho_{\text{Al}}}} I_{\text{Cu}} = \sqrt{\frac{1.70}{2.82}}(20.0) = 0.776(20.0) = \boxed{15.5 \text{ A}}$

**P28.51** (a) Suppose that the insulation between either of your fingers and the conductor adjacent is a chunk of rubber with contact area 4 $\text{mm}^2$ and thickness 1 mm. Its resistance is

$$R = \frac{\rho \ell}{A} \approx \frac{\left(10^{13} \ \Omega \cdot \text{m}\right)\left(10^{-3} \text{ m}\right)}{4 \times 10^{-6} \text{ m}^2} \approx 2 \times 10^{15} \ \Omega .$$

The current will be driven by 120 V through total resistance (series)

$2 \times 10^{15} \ \Omega + 10^4 \ \Omega + 2 \times 10^{15} \ \Omega \approx 5 \times 10^{15} \ \Omega$.

It is: $I = \frac{\Delta V}{R} \sim \frac{120 \text{ V}}{5 \times 10^{15} \ \Omega} \boxed{\sim 10^{-14} \text{ A}}$.

(b) The resistors form a voltage divider, with the center of your hand at potential $\frac{V_h}{2}$, where $V_h$ is the potential of the "hot" wire. The potential difference between your finger and thumb is $\Delta V = IR \sim \left(10^{-14} \text{ A}\right)\left(10^4 \ \Omega\right) \sim 10^{-10}$ V. So the points where the rubber meets your fingers are at potentials of

$\boxed{\sim \frac{V_h}{2} + 10^{-10} \text{ V}}$ and $\boxed{\sim \frac{V_h}{2} - 10^{-10} \text{ V}}$.

## Additional Problems

**P28.52** The set of four batteries boosts the electric potential of each bit of charge that goes through them by $4 \times 1.50 \text{ V} = 6.00 \text{ V}$. The chemical energy they store is

$$\Delta U = q\Delta V = (240 \text{ C})(6.00 \text{ J/C}) = 1\,440 \text{ J}.$$

The radio draws current $I = \frac{\Delta V}{R} = \frac{6.00 \text{ V}}{200 \ \Omega} = 0.030\,0 \text{ A}$.

So, its power is $\mathcal{P} = (\Delta V)I = (6.00 \text{ V})(0.030\,0 \text{ A}) = 0.180 \text{ W} = 0.180 \text{ J/s}$.

Then for the time the energy lasts, we have $\mathcal{P} = \frac{E}{\Delta t}$: $\Delta t = \frac{E}{\mathcal{P}} = \frac{1\,440 \text{ J}}{0.180 \text{ J/s}} = 8.00 \times 10^3 \text{ s}$.

We could also compute this from $I = \frac{Q}{\Delta t}$: $\Delta t = \frac{Q}{I} = \frac{240 \text{ C}}{0.030\,0 \text{ A}} = 8.00 \times 10^3 \text{ s} = \boxed{2.22 \text{ h}}$.

**P28.53** $I = \dfrac{\varepsilon}{R+r}$, so $\mathcal{P} = I^2R = \dfrac{\varepsilon^2 R}{(R+r)^2}$ or $(R+r)^2 = \left(\dfrac{\varepsilon^2}{\mathcal{P}}\right)R$.

Let $x \equiv \dfrac{\varepsilon^2}{\mathcal{P}}$, then $(R+r)^2 = xR$ or $R^2 + (2r - x)R - r^2 = 0$.

With $r = 1.20\ \Omega$, this becomes $R^2 + (2.40 - x)R - 1.44 = 0$,

which has solutions of $R = \dfrac{-(2.40 - x) \pm \sqrt{(2.40 - x)^2 - 5.76}}{2}$.

(a) With $\varepsilon = 9.20\ \text{V}$ and $\mathcal{P} = 12.8\ \text{W}$, $x = 6.61$:

$$R = \frac{+4.21 \pm \sqrt{(4.21)^2 - 5.76}}{2} = \boxed{3.84\ \Omega} \text{ or } \boxed{0.375\ \Omega}.$$

(b) For $\varepsilon = 9.20\ \text{V}$ and $\mathcal{P} = 21.2\ \text{W}$, $x \equiv \dfrac{\varepsilon^2}{\mathcal{P}} = 3.99$

$$R = \frac{+1.59 \pm \sqrt{(1.59)^2 - 5.76}}{2} = \frac{1.59 \pm \sqrt{-3.22}}{2}.$$

The equation for the load resistance yields a complex number, so $\boxed{\text{there is no resistance}}$ that will extract 21.2 W from this battery. The maximum power output occurs when $R = r = 1.20\ \Omega$, and that maximum is: $\mathcal{P}_{\max} = \dfrac{\varepsilon^2}{4r} = 17.6\ \text{W}$.

**P28.54** Using Kirchhoff's loop rule for the closed loop, $+12.0 - 2.00I - 4.00I = 0$, so $I = 2.00\ \text{A}$

$V_b - V_a = +4.00\ \text{V} - (2.00\ \text{A})(4.00\ \Omega) - (0)(10.0\ \Omega) = -4.00\ \text{V}$.

Thus, $|\Delta V_{ab}| = \boxed{4.00\ \text{V}}$ and $\boxed{\text{point } a \text{ is at the higher potential}}$.

**P28.55** (a) $R_{\text{eq}} = 3R$ $\quad I = \dfrac{\varepsilon}{3R}$ $\quad \mathcal{P}_{\text{series}} = \varepsilon I = \boxed{\dfrac{\varepsilon^2}{3R}}$

(b) $R_{\text{eq}} = \dfrac{1}{(1/R) + (1/R) + (1/R)} = \dfrac{R}{3}$ $\quad I = \dfrac{3\varepsilon}{R}$ $\quad \mathcal{P}_{\text{parallel}} = \varepsilon I = \boxed{\dfrac{3\varepsilon^2}{R}}$

(c) Nine times more power is converted in the $\boxed{\text{parallel}}$ connection.

***P28.56** (a) We model the generator as a constant-voltage power supply. Connect two light bulbs across it in series. Each bulb is designed to carry current $I = \dfrac{\mathcal{P}}{\Delta V} = \dfrac{100\ \text{W}}{120\ \text{V}} = 0.833\ \text{A}$. Each has resistance $R = \dfrac{\Delta V}{I} = \dfrac{120\ \text{V}}{0.833\ \text{A}} = 144\ \Omega$. In the 240-V circuit the equivalent resistance is $144\ \Omega + 144\ \Omega = 288\ \Omega$. The current is $I = \dfrac{\Delta V}{R} = \dfrac{240\ \text{V}}{288\ \Omega} = \boxed{0.833\ \text{A}}$ and the generator delivers power $\mathcal{P} = I\Delta V = 0.833\ \text{A}(240\ \text{V}) = \boxed{200\ \text{W}}$.

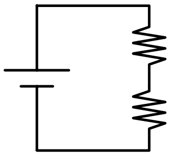


FIG. P28.56(a)

*continued on next page*

(b) The hot pot is designed to carry current

$$I=\frac{\mathcal{P}}{\Delta V}=\frac{500\text{ W}}{120\text{ V}}=4.17\text{ A}.$$

It has resistance

$$R=\frac{\Delta V}{I}=\frac{120\text{ V}}{4.17\text{ A}}=28.8\ \Omega.$$

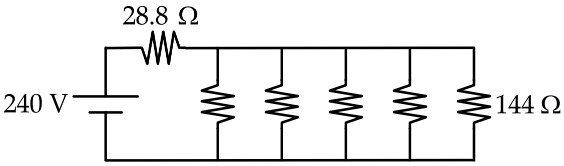


**FIG. P28.56(b)**

In terms of current, since $\frac{4.17\text{ A}}{0.833\text{ A}}=5$, we can place five light bulbs in parallel and the hot pot in series with their combination. The current in the generator is then $\boxed{4.17\text{ A}}$ and it delivers power $\mathcal{P}=I\Delta V=4.17\text{ A}(240\text{ V})=\boxed{1\,000\text{ W}}$.

**P28.57** The current in the simple loop circuit will be $I=\frac{\varepsilon}{R+r}$.

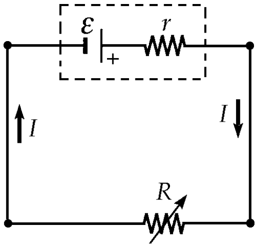


**FIG. P28.57**

(a) $\Delta V_{ter}=\varepsilon-Ir=\frac{\varepsilon R}{R+r}$ and $\Delta V_{ter}\to\varepsilon$ as $\boxed{R\to\infty}$.

(b) $I=\frac{\varepsilon}{R+r}$ and $I\to\frac{\varepsilon}{r}$ as $\boxed{R\to 0}$.

(c) $\mathcal{P}=I^2R=\varepsilon^2\frac{R}{(R+r)^2}$ $\qquad\frac{d\mathcal{P}}{dR}=\frac{-2\varepsilon^2R}{(R+r)^3}+\frac{\varepsilon^2}{(R+r)^2}=0$

Then $2R=R+r$ and $\boxed{R=r}$.

**P28.58** The potential difference across the capacitor $\Delta V(t)=\Delta V_{\max}\left(1-e^{-t/RC}\right)$.

Using 1 Farad $=1\ \text{s}/\Omega$, $\qquad 4.00\text{ V}=(10.0\text{ V})\left[1-e^{-(3.00\text{ s})/\left[R\left(10.0\times10^{-6}\text{ s}/\Omega\right)\right]}\right]$.

Therefore, $\qquad 0.400=1.00-e^{-\left(3.00\times10^{5}\ \Omega\right)/R}$.

Or $\qquad e^{-\left(3.00\times10^{5}\ \Omega\right)/R}=0.600$.

Taking the natural logarithm of both sides, $\qquad -\frac{3.00\times10^{5}\ \Omega}{R}=\ln(0.600)$

and $\qquad R=-\frac{3.00\times10^{5}\ \Omega}{\ln(0.600)}=+5.87\times10^{5}\ \Omega=\boxed{587\text{ k}\Omega}$.

**P28.59** Let the two resistances be $x$ and $y$.

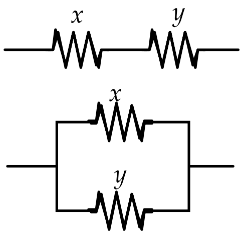


**FIG. P28.59**

Then, $R_s = x + y = \frac{\mathcal{P}_s}{I^2} = \frac{225\text{ W}}{(5.00\text{ A})^2} = 9.00\ \Omega \qquad y = 9.00\ \Omega - x$

and $R_p = \frac{xy}{x+y} = \frac{\mathcal{P}_p}{I^2} = \frac{50.0\text{ W}}{(5.00\text{ A})^2} = 2.00\ \Omega$

so $\frac{x(9.00\ \Omega - x)}{x + (9.00\ \Omega - x)} = 2.00\ \Omega \qquad x^2 - 9.00x + 18.0 = 0$.

Factoring the second equation, $(x - 6.00)(x - 3.00) = 0$

so $x = 6.00\ \Omega$ or $x = 3.00\ \Omega$.

Then, $y = 9.00\ \Omega - x$ gives $y = 3.00\ \Omega$ or $y = 6.00\ \Omega$.

The two resistances are found to be $\boxed{6.00\ \Omega}$ and $\boxed{3.00\ \Omega}$.

**P28.60** Let the two resistances be $x$ and $y$.

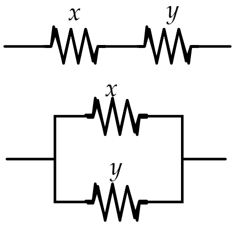


**FIG. P28.60**

Then, $R_s = x + y = \frac{\mathcal{P}_s}{I^2}$ and $R_p = \frac{xy}{x+y} = \frac{\mathcal{P}_p}{I^2}$.

From the first equation, $y = \frac{\mathcal{P}_s}{I^2} - x$, and the second

becomes $\frac{x\left(\mathcal{P}_s/I^2 - x\right)}{x + \left(\mathcal{P}_s/I^2 - x\right)} = \frac{\mathcal{P}_p}{I^2}$ or $x^2 - \left(\frac{\mathcal{P}_s}{I^2}\right)x + \frac{\mathcal{P}_s\mathcal{P}_p}{I^4} = 0$.

Using the quadratic formula, $x = \frac{\mathcal{P}_s \pm \sqrt{\mathcal{P}_s^2 - 4\mathcal{P}_s\mathcal{P}_p}}{2I^2}$.

Then, $y = \frac{\mathcal{P}_s}{I^2} - x$ gives $y = \frac{\mathcal{P}_s \mp \sqrt{\mathcal{P}_s^2 - 4\mathcal{P}_s\mathcal{P}_p}}{2I^2}$.

The two resistances are $\boxed{\frac{\mathcal{P}_s + \sqrt{\mathcal{P}_s^2 - 4\mathcal{P}_s\mathcal{P}_p}}{2I^2}}$ and $\boxed{\frac{\mathcal{P}_s - \sqrt{\mathcal{P}_s^2 - 4\mathcal{P}_s\mathcal{P}_p}}{2I^2}}$.

**P28.61** (a) $\varepsilon - I\left(\sum R\right) - \left(\varepsilon_1 + \varepsilon_2\right) = 0$

$40.0\text{ V} - (4.00\text{ A})\left[(2.00 + 0.300 + 0.300 + R)\Omega\right] - (6.00 + 6.00)\text{ V} = 0$; so $R = \boxed{4.40\ \Omega}$

(b) Inside the supply, $\mathcal{P} = I^2R = (4.00\text{ A})^2(2.00\ \Omega) = \boxed{32.0\text{ W}}$.

Inside both batteries together, $\mathcal{P} = I^2R = (4.00\text{ A})^2(0.600\ \Omega) = \boxed{9.60\text{ W}}$.

For the limiting resistor, $\mathcal{P} = (4.00\text{ A})^2(4.40\ \Omega) = \boxed{70.4\text{ W}}$.

(c) $\mathcal{P} = I\left(\varepsilon_1 + \varepsilon_2\right) = (4.00\text{ A})\left[(6.00 + 6.00)\text{ V}\right] = \boxed{48.0\text{ W}}$

***P28.62** (a) $\Delta V_1 = \Delta V_2 \qquad I_1R_1 = I_2R_2$

$$I = I_1 + I_2 = I_1 + \frac{I_1R_1}{R_2} = I_1\frac{R_2+R_1}{R_2}$$

$$\boxed{I_1 = \frac{IR_2}{R_1+R_2}}$$

$$I_2 = \frac{I_1R_1}{R_2} = \boxed{\frac{IR_1}{R_1+R_2} = I_2}$$

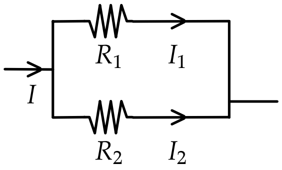


**FIG. P28.62(a)**

(b) The power delivered to the pair is $\mathcal{P} = I_1^2R_1 + I_2^2R_2 = I_1^2R_1 + (I-I_1)^2R_2$. For minimum power we want to find $I_1$ such that $\frac{d\mathcal{P}}{dI_1} = 0$.

$$\frac{d\mathcal{P}}{dI_1} = 2I_1R_1 + 2(I-I_1)(-1)R_2 = 0 \qquad I_1R_1 - IR_2 + I_1R_2 = 0$$

$$I_1 = \frac{IR_2}{R_1+R_2}$$

This is the same condition as that found in part (a).

**P28.63** Let $R_m$ = measured value, $R$ = actual value,

$I_R$ = current through the resistor $R$

$I$ = current measured by the ammeter.

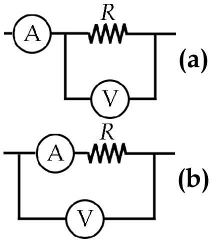


**FIG. P28.63**

(a) When using circuit (a), $I_RR = \Delta V = 20\,000(I - I_R)$ or $R = 20\,000\left[\frac{I}{I_R} - 1\right]$.

But since $I = \frac{\Delta V}{R_m}$ and $I_R = \frac{\Delta V}{R}$, we have $\frac{I}{I_R} = \frac{R}{R_m}$

and $R = 20\,000\frac{(R-R_m)}{R_m}$. (1)

When $R > R_m$, we require $\frac{(R-R_m)}{R} \le 0.050\,0$.

Therefore, $R_m \ge R(1 - 0.050\,0)$ and from (1) we find $\boxed{R \le 1\,050\ \Omega}$.

(b) When using circuit (b), $I_RR = \Delta V - I_R(0.5\ \Omega)$.

But since $I_R = \frac{\Delta V}{R_m}$, $R_m = (0.500 + R)$. (2)

When $R_m > R$, we require $\frac{(R_m - R)}{R} \le 0.050\,0$.

From (2) we find $\boxed{R \ge 10.0\ \Omega}$.

**P28.64** The battery supplies energy at a changing rate $\frac{dE}{dt}=\mathscr{P}=\varepsilon I=\varepsilon\left(\frac{\varepsilon}{R}e^{-1/RC}\right)$.

Then the total energy put out by the battery is $\int dE=\int\limits_{t=0}^{\infty}\frac{\varepsilon^2}{R}\exp\left(-\frac{t}{RC}\right)dt$

$$\int dE=\frac{\varepsilon^2}{R}(-RC)\int\limits_{0}^{\infty}\exp\left(-\frac{t}{RC}\right)\left(-\frac{dt}{RC}\right)=-\varepsilon^2C\exp\left(-\frac{t}{RC}\right)\Bigg|_0^{\infty}=-\varepsilon^2C[0-1]=\varepsilon^2C.$$

The power delivered to the resistor is $\frac{dE}{dt}=\mathscr{P}=\Delta V_RI=I^2R=R\frac{\varepsilon^2}{R^2}\exp\left(-\frac{2t}{RC}\right)$.

So the total internal energy appearing in the resistor is $\int dE=\int\limits_{0}^{\infty}\frac{\varepsilon^2}{R}\exp\left(-\frac{2t}{RC}\right)dt$

$$\int dE=\frac{\varepsilon^2}{R}\left(-\frac{RC}{2}\right)\int\limits_{0}^{\infty}\exp\left(-\frac{2t}{RC}\right)\left(-\frac{2dt}{RC}\right)=-\frac{\varepsilon^2C}{2}\exp\left(-\frac{2t}{RC}\right)\Bigg|_0^{\infty}=-\frac{\varepsilon^2C}{2}[0-1]=\frac{\varepsilon^2C}{2}.$$

The energy finally stored in the capacitor is $U=\frac{1}{2}C(\Delta V)^2=\frac{1}{2}C\varepsilon^2$. Thus, energy of the circuit is conserved $\varepsilon^2C=\frac{1}{2}\varepsilon^2C+\frac{1}{2}\varepsilon^2C$ and resistor and capacitor share equally in the energy from the battery.

**P28.65** (a) $q=C\Delta V\left(1-e^{-t/RC}\right)$

$$q=\left(1.00\times10^{-6}\text{ F}\right)(10.0\text{ V})\left[1-e^{-10.0/\left[\left(2.00\times10^{6}\right)\left(1.00\times10^{-6}\right)\right]}\right]=\boxed{9.93\ \mu\text{C}}$$

(b) $I=\frac{dq}{dt}=\left(\frac{\Delta V}{R}\right)e^{-t/RC}$

$$I=\left(\frac{10.0\text{ V}}{2.00\times10^{6}\ \Omega}\right)e^{-5.00}=3.37\times10^{-8}\text{ A}=\boxed{33.7\text{ nA}}$$

(c) $\frac{dU}{dt}=\frac{d}{dt}\left(\frac{1}{2}\frac{q^2}{C}\right)=\left(\frac{q}{C}\right)\frac{dq}{dt}=\left(\frac{q}{C}\right)I$

$$\frac{dU}{dt}=\left(\frac{9.93\times10^{-6}\text{ C}}{1.00\times10^{-6}\text{ C/V}}\right)\left(3.37\times10^{-8}\text{ A}\right)=3.34\times10^{-7}\text{ W}=\boxed{334\text{ nW}}$$

(d) $\mathscr{P}_{\text{battery}}=I\varepsilon=\left(3.37\times10^{-8}\text{ A}\right)(10.0\text{ V})=3.37\times10^{-7}\text{ W}=\boxed{337\text{ nW}}$

**P28.66** Start at the point when the voltage has just reached $\frac{2}{3}\Delta V$ and the switch has just closed. The voltage is $\frac{2}{3}\Delta V$ and is decaying towards 0 V with a time constant $R_2C$

$$\Delta V_C(t) = \left[\frac{2}{3}\Delta V\right]e^{-t/R_2C}.$$

We want to know when $\Delta V_C(t)$ will reach $\frac{1}{3}\Delta V$.

Therefore, $$\frac{1}{3}\Delta V = \left[\frac{2}{3}\Delta V\right]e^{-t/R_2C}$$

or $$e^{-t/R_2C} = \frac{1}{2}$$

or $$t_1 = R_2C\ln 2.$$

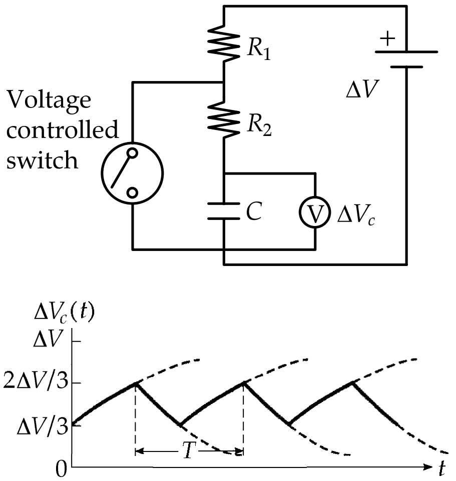


**FIG. P28.66**

After the switch opens, the voltage is $\frac{1}{3}\Delta V$, increasing toward $\Delta V$ with time constant $(R_1+R_2)C$:

$$\Delta V_C(t) = \Delta V - \left[\frac{2}{3}\Delta V\right]e^{-t/(R_1+R_2)C}.$$

When $$\Delta V_C(t) = \frac{2}{3}\Delta V$$

$$\frac{2}{3}\Delta V = \Delta V - \frac{2}{3}\Delta V e^{-t/(R_1+R_2)C} \quad \text{or} \quad e^{-t/(R_1+R_2)C} = \frac{1}{2}.$$

So $$t_2 = (R_1+R_2)C\ln 2 \quad \text{and} \quad T = t_1 + t_2 = \boxed{(R_1+2R_2)C\ln 2}.$$

**P28.67** (a) First determine the resistance of each light bulb: $\mathcal{P} = \frac{(\Delta V)^2}{R}$

$$R = \frac{(\Delta V)^2}{\mathcal{P}} = \frac{(120\text{ V})^2}{60.0\text{ W}} = 240\ \Omega.$$

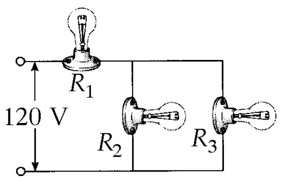


**FIG. P28.67**

We obtain the equivalent resistance $R_{\text{eq}}$ of the network of light bulbs by identifying series and parallel equivalent resistances:

$$R_{\text{eq}} = R_1 + \frac{1}{(1/R_2)+(1/R_3)} = 240\ \Omega + 120\ \Omega = 360\ \Omega.$$

The total power dissipated in the 360 Ω is $$\mathcal{P} = \frac{(\Delta V)^2}{R_{\text{eq}}} = \frac{(120\text{ V})^2}{360\ \Omega} = \boxed{40.0\text{ W}}.$$

(b) The current through the network is given by $\mathcal{P} = I^2R_{\text{eq}}$: $I = \sqrt{\frac{\mathcal{P}}{R_{\text{eq}}}} = \sqrt{\frac{40.0\text{ W}}{360\ \Omega}} = \frac{1}{3}\text{ A}.$

The potential difference across $R_1$ is $$\Delta V_1 = IR_1 = \left(\frac{1}{3}\text{ A}\right)(240\ \Omega) = \boxed{80.0\text{ V}}.$$

The potential difference $\Delta V_{23}$ across the parallel combination of $R_2$ and $R_3$ is

$$\Delta V_{23} = IR_{23} = \left(\frac{1}{3}\text{ A}\right)\left(\frac{1}{(1/240\ \Omega)+(1/240\ \Omega)}\right) = \boxed{40.0\text{ V}}.$$

***P28.68** (a) With the switch closed, current exists in a simple series circuit as shown. The capacitors carry no current. For $R_2$ we have

$$\mathcal{P} = I^2R_2 \qquad I = \sqrt{\frac{\mathcal{P}}{R_2}} = \sqrt{\frac{2.40\ \text{V}\cdot\text{A}}{7\,000\ \text{V/A}}} = 18.5\ \text{mA}.$$

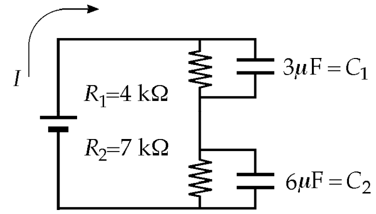


**FIG. P28.68(a)**

The potential difference across $R_1$ and $C_1$ is

$$\Delta V = IR_1 = \left(1.85\times10^{-2}\ \text{A}\right)(4\,000\ \text{V/A}) = 74.1\ \text{V}.$$

The charge on $C_1$

$$Q = C_1\Delta V = \left(3.00\times10^{-6}\ \text{C/V}\right)(74.1\ \text{V}) = \boxed{222\ \mu\text{C}}.$$

The potential difference across $R_2$ and $C_2$ is

$$\Delta V = IR_2 = \left(1.85\times10^{-2}\ \text{A}\right)(7\,000\ \Omega) = 130\ \text{V}.$$

The charge on $C_2$

$$Q = C_2\Delta V = \left(6.00\times10^{-6}\ \text{C/V}\right)(130\ \text{V}) = 778\ \mu\text{C}.$$

The battery emf is

$$IR_{eq} = I(R_1+R_2) = 1.85\times10^{-2}\ \text{A}(4\,000+7\,000)\ \text{V/A} = 204\ \text{V}.$$

(b) In equilibrium after the switch has been opened, no current exists. The potential difference across each resistor is zero. The full 204 V appears across both capacitors. The new charge $C_2$

$$Q = C_2\Delta V = \left(6.00\times10^{-6}\ \text{C/V}\right)(204\ \text{V}) = 1\,222\ \mu\text{C}$$

for a change of $1\,222\ \mu\text{C} - 778\ \mu\text{C} = \boxed{444\ \mu\text{C}}$.

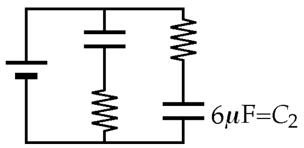


**FIG. P28.68(b)**

***P28.69** The battery current is

$$(150+45+14+4)\ \text{mA} = 213\ \text{mA}.$$

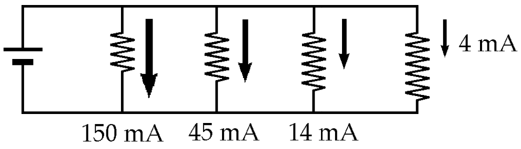


**FIG. P28.69**

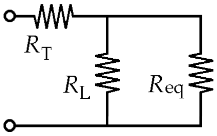
(a) The resistor with highest resistance is that carrying 4 mA. Doubling its resistance will reduce the current it carries to 2 mA. Then the total current is

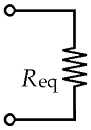
$(150+45+14+2)\ \text{mA} = 211\ \text{mA}$, nearly the same as before. The ratio is $\frac{211}{213} = \boxed{0.991}$.

(b) The resistor with least resistance carries 150 mA. Doubling its resistance changes this current to 75 mA and changes the total to

$(75+45+14+4)\ \text{mA} = 138\ \text{mA}$. The ratio is $\frac{138}{213} = \boxed{0.648}$, representing a much larger reduction (35.2% instead of 0.9%).

(c) This problem is precisely analogous. As a battery maintained a potential difference in parts (a) and (b), a furnace maintains a temperature difference here. Energy flow by heat is analogous to current and takes place through thermal resistances in parallel. Each resistance can have its "*R*-value" increased by adding insulation. Doubling the thermal resistance of the attic door will produce only a negligible (0.9%) saving in fuel. Doubling the thermal resistance of $\boxed{\text{the ceiling}}$ will produce a much larger saving. The ceiling originally has the smallest thermal resistance.

***P28.70** From the hint, the equivalent resistance of 

is just 

.

That is,

$$R_T + \frac{1}{1/R_L + 1/R_{eq}} = R_{eq}$$

$$R_T + \frac{R_L R_{eq}}{R_L + R_{eq}} = R_{eq}$$

$$R_T R_L + R_T R_{eq} + R_L R_{eq} = R_L R_{eq} + R_{eq}^2$$

$$R_{eq}^2 - R_T R_{eq} - R_T R_L = 0$$

$$R_{eq} = \frac{R_T \pm \sqrt{R_T^2 - 4(1)(-R_T R_L)}}{2(1)}$$

Only the + sign is physical:

$$R_{eq} = \frac{1}{2}\left(\sqrt{4R_T R_L + R_T^2} + R_T\right).$$

For example, if $R_T = 1\ \Omega$.

And $R_L = 20\ \Omega$, $R_{eq} = 5\ \Omega$.

**P28.71** (a) After steady-state conditions have been reached, there is no DC current through the capacitor.

Thus, for $R_3$: $\boxed{I_{R_3} = 0\ (\text{steady-state})}$.

For the other two resistors, the steady-state current is simply determined by the 9.00-V emf across the 12-kΩ and 15-kΩ resistors in series:

For $R_1$ and $R_2$: $$I_{(R_1+R_2)} = \frac{\varepsilon}{R_1 + R_2} = \frac{9.00\ \text{V}}{(12.0\ \text{k}\Omega + 15.0\ \text{k}\Omega)} = \boxed{333\ \mu\text{A}\ (\text{steady-state})}.$$

(b) After the transient currents have ceased, the potential difference across $C$ is the same as the potential difference across $R_2 (= IR_2)$ because there is no voltage drop across $R_3$. Therefore, the charge $Q$ on $C$ is

$$Q = C(\Delta V)_{R_2} = C(IR_2) = (10.0\ \mu\text{F})(333\ \mu\text{A})(15.0\ \text{k}\Omega)$$

$$= \boxed{50.0\ \mu\text{C}}.$$

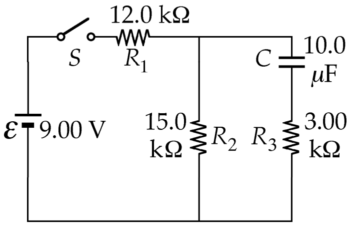


FIG. P28.71(b)

*continued on next page*

(c) When the switch is opened, the branch containing $R_1$ is no longer part of the circuit. The capacitor discharges through $\left(R_2+R_3\right)$ with a time constant of $\left(R_2+R_3\right)C=(15.0\text{ k}\Omega+3.00\text{ k}\Omega)(10.0\ \mu\text{F})=0.180\text{ s}$. The initial current $I_i$ in this discharge circuit is determined by the initial potential difference across the capacitor applied to $\left(R_2+R_3\right)$ in series:

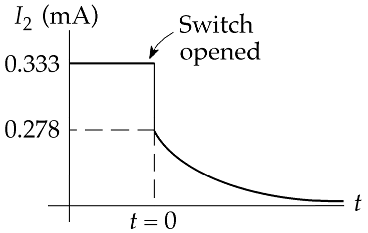


**FIG. P28.71(c)**

$$I_i=\frac{(\Delta V)_C}{\left(R_2+R_3\right)}=\frac{IR_2}{\left(R_2+R_3\right)}=\frac{(333\ \mu\text{A})(15.0\text{ k}\Omega)}{(15.0\text{ k}\Omega+3.00\text{ k}\Omega)}=278\ \mu\text{A}.$$

Thus, when the switch is opened, the current through $R_2$ changes instantaneously from 333 $\mu$A (downward) to 278 $\mu$A (downward) as shown in the graph. Thereafter, it decays according to

$$I_{R_2}=I_ie^{-t/\left(R_2+R_3\right)C}=\boxed{(278\ \mu\text{A})e^{-t/(0.180\text{ s})}\text{ (for }t>0)}.$$

(d) The charge $q$ on the capacitor decays from $Q_i$ to $\frac{Q_i}{5}$ according to

$$q=Q_ie^{-t/\left(R_2+R_3\right)C}$$
$$\frac{Q_i}{5}=Q_ie^{(-t/0.180\text{ s})}$$
$$5=e^{t/0.180\text{ s}}$$
$$\ln 5=\frac{t}{180\text{ ms}}$$
$$t=(0.180\text{ s})(\ln 5)=\boxed{290\text{ ms}}$$

***P28.72** (a) First let us flatten the circuit on a 2-D plane as shown; then reorganize it to a format easier to read. Notice that the five resistors on the top are in the same connection as those in Example 28.5; the same argument tells us that the middle resistor can be removed without affecting the circuit. The remaining resistors over the three parallel branches have equivalent resistance

$$R_{eq}=\left(\frac{1}{20}+\frac{1}{20}+\frac{1}{10}\right)^{-1}=\boxed{5.00\ \Omega}.$$

(b) So the current through the battery is

$$\frac{\Delta V}{R_{eq}}=\frac{12.0\text{ V}}{5.00\ \Omega}=\boxed{2.40\text{ A}}.$$

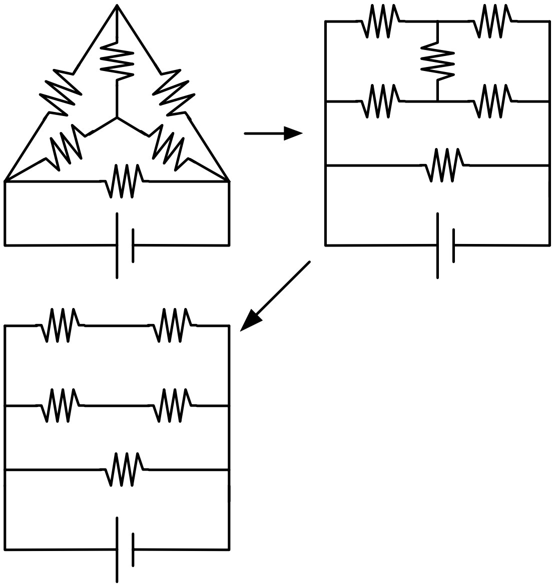


**FIG. P28.72(a)**

**P28.73** $\Delta V = \varepsilon e^{-t/RC}$

so $\ln\left(\frac{\varepsilon}{\Delta V}\right) = \left(\frac{1}{RC}\right)t$.

A plot of $\ln\left(\frac{\varepsilon}{\Delta V}\right)$ versus $t$ should be a straight line with slope equal to $\frac{1}{RC}$.

Using the given data values:

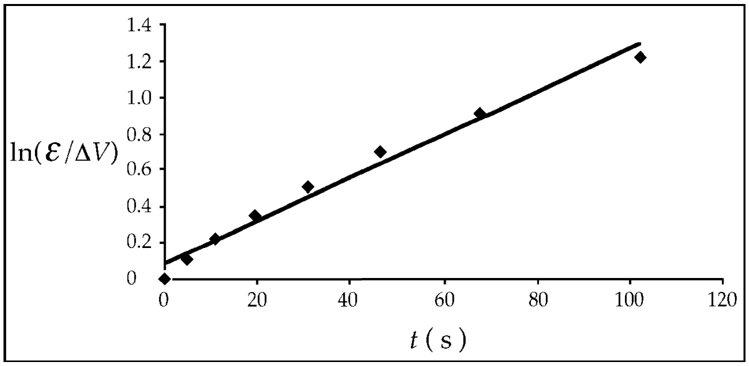


**FIG. P28.73**

(a) A least-square fit to this data yields the graph above.

$\sum x_i = 282$, $\sum x_i^2 = 1.86 \times 10^4$,

$\sum x_i y_i = 244$, $\sum y_i = 4.03$, $N = 8$

$$\text{Slope} = \frac{N\left(\sum x_i y_i\right) - \left(\sum x_i\right)\left(\sum y_i\right)}{N\left(\sum x_i^2\right) - \left(\sum x_i\right)^2} = 0.011\,8$$

$$\text{Intercept} = \frac{\left(\sum x_i^2\right)\left(\sum y_i\right) - \left(\sum x_i\right)\left(\sum x_i y_i\right)}{N\left(\sum x_i^2\right) - \left(\sum x_i\right)^2} = 0.088\,2$$

| $t(s)$ | $\Delta V(V)$ | $\ln(\varepsilon/\Delta V)$ |
|---|---|---|
| 0 | 6.19 | 0 |
| 4.87 | 5.55 | 0.109 |
| 11.1 | 4.93 | 0.228 |
| 19.4 | 4.34 | 0.355 |
| 30.8 | 3.72 | 0.509 |
| 46.6 | 3.09 | 0.695 |
| 67.3 | 2.47 | 0.919 |
| 102.2 | 1.83 | 1.219 |

The equation of the best fit line is: $\boxed{\ln\left(\frac{\varepsilon}{\Delta V}\right) = (0.011\,8)t + 0.088\,2}$.

(b) Thus, the time constant is $\tau = RC = \frac{1}{\text{slope}} = \frac{1}{0.011\,8} = \boxed{84.7\text{ s}}$

and the capacitance is $C = \frac{\tau}{R} = \frac{84.7\text{ s}}{10.0 \times 10^6\ \Omega} = \boxed{8.47\ \mu\text{F}}$.

**P28.74** (a) For the first measurement, the equivalent circuit is as shown in Figure 1.

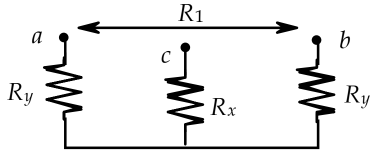


Figure 1

$$R_{ab} = R_1 = R_y + R_y = 2R_y$$

so $R_y = \frac{1}{2}R_1$. (1)

For the second measurement, the equivalent circuit is shown in Figure 2.

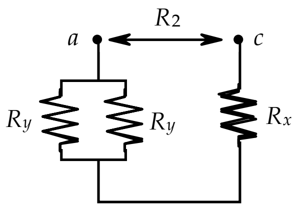


Figure 2

**FIG. P28.74**

Thus, $R_{ac} = R_2 = \frac{1}{2}R_y + R_x$. (2)

Substitute (1) into (2) to obtain:

$R_2 = \frac{1}{2}\left(\frac{1}{2}R_1\right) + R_x$, or $\boxed{R_x = R_2 - \frac{1}{4}R_1}$.

(b) If $R_1 = 13.0\ \Omega$ and $R_2 = 6.00\ \Omega$, then $\boxed{R_x = 2.75\ \Omega}$.

$\boxed{\text{The antenna is inadequately grounded}}$ since this exceeds the limit of $2.00\ \Omega$.

**P28.75** The total resistance between points $b$ and $c$ is:

$$R=\frac{(2.00\text{ k}\Omega)(3.00\text{ k}\Omega)}{2.00\text{ k}\Omega+3.00\text{ k}\Omega}=1.20\text{ k}\Omega.$$

The total capacitance between points $d$ and $e$ is:

$$C=2.00\ \mu\text{F}+3.00\ \mu\text{F}=5.00\ \mu\text{F}.$$

The potential difference between point $d$ and $e$ in this series $RC$ circuit at any time is:

$$\Delta V=\varepsilon\left[1-e^{-t/RC}\right]=(120.0\text{ V})\left[1-e^{-1\,000t/6}\right].$$

Therefore, the charge on each capacitor between points $d$ and $e$ is:

$$q_1=C_1\Delta V=(2.00\ \mu\text{F})(120.0\text{ V})\left[1-e^{-1\,000t/6}\right]=\boxed{(240\ \mu\text{C})\left[1-e^{-1\,000t/6}\right]}$$

and $q_2=C_2(\Delta V)=(3.00\ \mu\text{F})(120.0\text{ V})\left[1-e^{-1\,000t/6}\right]=\boxed{(360\ \mu\text{C})\left[1-e^{-1\,000t/6}\right]}$.

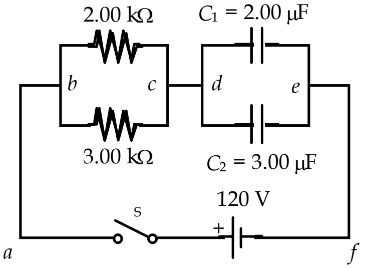


FIG. P28.75

***P28.76** (a) Let $i$ represent the current in the battery and $i_c$ the current charging the capacitor. Then $i-i_c$ is the current in the voltmeter. The loop rule applied to the inner loop is $+\varepsilon-iR-\frac{q}{C}=0$. The loop rule for the outer perimeter is $\varepsilon-iR-(i-i_c)r=0$. With $i_c=\frac{dq}{dt}$, this becomes $\varepsilon-iR-ir+\frac{dq}{dt}r=0$. Between the two loop equations we eliminate $i=\frac{\varepsilon}{R}-\frac{q}{RC}$ by substitution to obtain

$$\varepsilon-(R+r)\left(\frac{\varepsilon}{R}-\frac{q}{RC}\right)+\frac{dq}{dt}r=0$$

$$\varepsilon-\frac{R+r}{R}\varepsilon+\frac{R+r}{RC}q+\frac{dq}{dt}r=0$$

$$-\frac{r}{R+r}\varepsilon+\frac{q}{C}+\frac{Rr}{R+r}\frac{dq}{dt}=0$$

This is the differential equation required.

(b) To solve we follow the same steps as on page 875.

$$\frac{dq}{dt}=\frac{\varepsilon}{R}-\frac{R+r}{RrC}q=-\frac{R+r}{RrC}\left(q-\frac{\varepsilon rC}{R+r}\right)$$

$$\int_0^q\frac{dq}{q-\varepsilon rC/(R+r)}=-\frac{R+r}{RrC}\int_0^t dt \qquad \ln\left(q-\frac{\varepsilon rc}{R+r}\right)\Bigg|_0^q=-\frac{R+r}{RrC}t\Bigg|_0^t$$

$$\ln\left(\frac{q-\varepsilon rc/(R+r)}{-\varepsilon rc/(R+r)}\right)=-\frac{R+r}{RrC}t \qquad q-\frac{\varepsilon rc}{R+r}=-\frac{\varepsilon rc}{R+r}e^{[-(R+r)/RrC]t}$$

$$q=\frac{r}{r+R}C\varepsilon\left(1-e^{-t/R_{\text{eq}}C}\right)\text{ where }R_{\text{eq}}=\frac{Rr}{R+r}$$

The voltage across the capacitor is $V_C=\frac{q}{C}=\frac{r}{r+R}\varepsilon\left(1-e^{-t/R_{\text{eq}}C}\right)$.

(c) As $t\to\infty$ the capacitor voltage approaches $\frac{r}{r+R}\varepsilon(1-0)=\frac{r\varepsilon}{r+R}$. If the switch is then opened, the capacitor discharges through the voltmeter. Its voltage decays exponentially according to $\boxed{\frac{r\varepsilon}{r+R}e^{-t/rC}}$.

## ANSWERS TO EVEN PROBLEMS

| | |
|---|---|
| **P28.2** | (a) 1.79 A; (b) 10.4 V |
| **P28.4** | (a) 12.4 V; (b) 9.65 V |
| **P28.6** | (a) 17.1 Ω; (b) 1.99 A in 4 Ω and 9 Ω; 1.17 A in 7 Ω; 0.818 A in 10 Ω |
| **P28.8** | 29.5 V |
| **P28.10** | (a) see the solution; (b) no |
| **P28.12** | see the solution |
| **P28.14** | $R_1 = 1.00\text{ k}\Omega$; $R_2 = 2.00\text{ k}\Omega$; $R_3 = 3.00\text{ k}\Omega$ |
| **P28.16** | 470 Ω and 220 Ω |
| **P28.18** | (a) 11.0 Ω; (b) and (d) see the solution; (c) 220 Ω; (e) Parallel |
| **P28.20** | $I_1 = 714\text{ mA}$; $I_2 = 1.29\text{ A}$; $\varepsilon = 12.6\text{ V}$ |
| **P28.22** | see the solution |
| **P28.24** | (a) 0.385 mA in $R_1$; 2.69 mA in $R_3$; 3.08 mA in $R_2$; (b) *c* higher by 69.2 V |
| **P28.26** | 1.00 A up in 200 Ω; 4.00 A up in 70 Ω; 3.00 A up in 80 Ω; 8.00 A down in 20 Ω; 200 V |
| **P28.28** | see the solution |
| **P28.30** | 800 W to the left-hand resistor; 25.0 W to each 4 Ω; 450 W to the right-hand resistor |
| **P28.32** | (a) –61.6 mA; (b) 0.235 $\mu$C; (c) 1.96 A |
| **P28.34** | 0.982 s |
| **P28.36** | (a) 1.50 s; (b) 1.00 s; (c) $200\ \mu\text{A} + (100\ \mu\text{A})e^{-t/1.00\text{ s}}$ |
| **P28.38** | (a) 3.91 s; (b) 0.782 ms |
| **P28.40** | $\frac{t}{C\ln 2}$ |
| **P28.42** | 0.113 Ω |
| **P28.44** | (a) 30.000 mA, 5.400 0 V; (b) 30.167 mA, 5.381 6 V; (c) 29.898 mA; 5.396 6 V |
| **P28.46** | see the solution |
| **P28.48** | (a) 0.101 W; (b) 10.1 W |
| **P28.50** | 15.5 A |
| **P28.52** | 2.22 h |
| **P28.54** | *a* is 4.00 V higher |
| **P28.56** | (a) see the solution; 833 mA; 200 W; (b) see the solution; 4.17 A; 1.00 kW |
| **P28.58** | 587 kΩ |
| **P28.60** | $\frac{\mathcal{P}_s + \sqrt{\mathcal{P}_s^2 - 4\mathcal{P}_s\mathcal{P}_p}}{2I^2}$ and $\frac{\mathcal{P}_s - \sqrt{\mathcal{P}_s^2 - 4\mathcal{P}_s\mathcal{P}_p}}{2I^2}$ |
| **P28.62** | (a) $I_1 = \frac{IR_2}{(R_1 + R_2)}$; $I_2 = \frac{IR_1}{R_1 + R_2}$; (b) see the solution |
| **P28.64** | see the solution |
| **P28.66** | $(R_1 + 2R_2)C\ln 2$ |
| **P28.68** | (a) 222 $\mu$C; (b) increase by 444 $\mu$C |
| **P28.70** | see the solution |
| **P28.72** | (a) 5.00 Ω; (b) 2.40 A |
| **P28.74** | (a) $R_x = R_2 - \frac{R_1}{4}$; (b) no; $R_x = 2.75\ \Omega$ |
| **P28.76** | (a) and (b) see the solution; (c) $\frac{r\varepsilon}{r + R}e^{-t/rC}$ |

# 29

# Magnetic Fields

**CHAPTER OUTLINE**

29.1 Magnetic Fields and Forces
29.2 Magnetic Force Acting on a Current-Carrying Conductor
29.3 Torque on a Current Loop in a Uniform Magnetic Field
29.4 Motion of a Charged Particle in a Uniform Magnetic Field
29.5 Applications Involving Charged Particles Moving in a Magnetic Field
29.6 The Hall Effect

## ANSWERS TO QUESTIONS

**Q29.1** The force is in the $+y$ direction. No, the proton will not continue with constant velocity, but will move in a circular path in the $x$-$y$ plane. The magnetic force will always be perpendicular to the magnetic field and also to the velocity of the proton. As the velocity changes direction, the magnetic force on the proton does too.

**Q29.2** If they are projected in the same direction into the same magnetic field, the charges are of opposite sign.

**Q29.3** Not necessarily. If the magnetic field is parallel or antiparallel to the velocity of the charged particle, then the particle will experience no magnetic force.

**Q29.4** One particle veers in a circular path clockwise in the page, while the other veers in a counterclockwise circular path. If the magnetic field is into the page, the electron goes clockwise and the proton counterclockwise.

**Q29.5** Send the particle through the uniform field and look at its path. If the path of the particle is parabolic, then the field must be electric, as the electric field exerts a constant force on a charged particle. If you shoot a proton through an electric field, it will feel a constant force in the same direction as the electric field—it's similar to throwing a ball through a gravitational field. If the path of the particle is helical or circular, then the field is magnetic—see Question 29.1. If the path of the particle is straight, then observe the speed of the particle. If the particle accelerates, then the field is electric, as a constant force on a proton with or against its motion will make its speed change. If the speed remains constant, then the field is magnetic—see Question 29.3.

**Q29.6** Similarities: Both can alter the velocity of a charged particle moving through the field. Both exert forces directly proportional to the charge of the particle feeling the force. Positive and negative charges feel forces in opposite directions. Differences: The direction of the electric force is parallel or antiparallel to the direction of the electric field, but the direction of the magnetic force is perpendicular to the magnetic field and to the velocity of the charged particle. Electric forces can accelerate a charged particle from rest or stop a moving particle, but magnetic forces cannot.

**Q29.7** Since $\mathbf{F}_B = q(\mathbf{v} \times \mathbf{B})$, then the acceleration produced by a magnetic field on a particle of mass $m$ is $\mathbf{a}_B = \frac{q}{m}(\mathbf{v} \times \mathbf{B})$. For the acceleration to change the speed, a component of the acceleration must be in the direction of the velocity. The cross product tells us that the acceleration must be perpendicular to the velocity, and thus can only change the direction of the velocity.

**Q29.8** The magnetic field in a cyclotron essentially keeps the charged particle in the electric field for a longer period of time, and thus experiencing a larger change in speed from the electric field, by forcing it in a spiral path. Without the magnetic field, the particle would have to move in a straight line through an electric field over a distance that is very large compared to the size of the cyclotron.

**Q29.9** (a) The $q\mathbf{v} \times \mathbf{B}$ force on each electron is down. Since electrons are negative, $\mathbf{v} \times \mathbf{B}$ must be up. With $\mathbf{v}$ to the right, $\mathbf{B}$ must be into the page, away from you.

(b) Reversing the current in the coils would reverse the direction of **B**, making it toward you. Then $\mathbf{v} \times \mathbf{B}$ is in the direction **right** × **toward you** = **down**, and $q\mathbf{v} \times \mathbf{B}$ will make the electron beam curve up.

**Q29.10** If the current is in a direction *parallel* or *antiparallel* to the magnetic field, then there is no force.

**Q29.11** Yes. If the magnetic field is perpendicular to the plane of the loop, then it exerts no torque on the loop.

**Q29.12** If you can hook a spring balance to the particle and measure the force on it in a known electric field, then $q = \frac{F}{E}$ will tell you its charge. You cannot hook a spring balance to an electron. Measuring the acceleration of small particles by observing their deflection in known electric and magnetic fields can tell you the charge-to-mass ratio, but not separately the charge or mass. Both an acceleration produced by an electric field and an acceleration caused by a magnetic field depend on the properties of the particle only by being proportional to the ratio $\frac{q}{m}$.

**Q29.13** If the current loop feels a torque, it must be caused by a magnetic field. If the current loop feels no torque, try a different orientation—the torque is zero if the field is along the axis of the loop.

**Q29.14** The Earth's magnetic field exerts force on a charged incoming cosmic ray, tending to make it spiral around a magnetic field line. If the particle energy is low enough, the spiral will be tight enough that the particle will first hit some matter as it follows a field line down into the atmosphere or to the surface at a high geographic latitude.

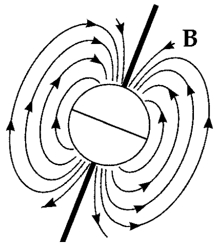


**FIG. Q29.14**

**Q29.15** The net force is zero, but not the net torque.

**Q29.16** Only a non-uniform field can exert a non-zero force on a magnetic dipole. If the dipole is aligned with the field, the direction of the resultant force is in the direction of increasing field strength.

**Q29.17** The proton will veer upward when it enters the field and move in a counter-clockwise semicircular arc. An electron would turn downward and move in a clockwise semicircular arc of smaller radius than that of the proton, due to its smaller mass.

**Q29.18** Particles of higher speeds will travel in semicircular paths of proportionately larger radius. They will take just the same time to travel farther with their higher speeds. As shown in Equation 29.15, the time it takes to follow the path is independent of particle's speed.

**Q29.19** The spiral tracks are left by charged particles gradually losing kinetic energy. A straight path might be left by an uncharged particle that managed to leave a trail of bubbles, or it might be the imperceptibly curving track of a very fast charged particle.

**Q29.20** No. Changing the velocity of a particle requires an accelerating force. The magnetic force is proportional to the speed of the particle. If the particle is not moving, there can be no magnetic force on it.

**Q29.21** Increase the current in the probe. If the material is a semiconductor, raising its temperature may increase the density of mobile charge carriers in it.

## SOLUTIONS TO PROBLEMS

### Section 29.1 Magnetic Fields and Forces

**P29.1** (a) up

(b) out of the page, since the charge is negative.

(c) no deflection

(d) into the page

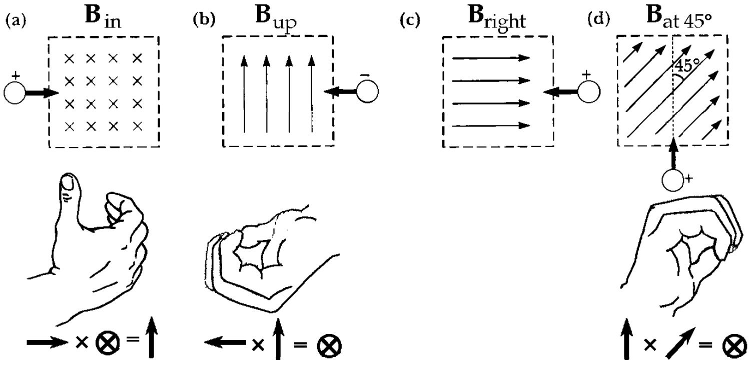
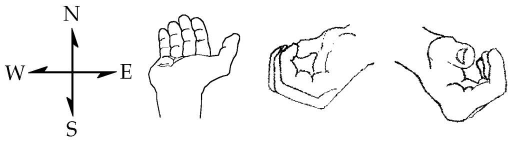


**FIG. P29.1**

**P29.2** At the equator, the Earth's magnetic field is horizontally north. Because an electron has negative charge, $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$ is opposite in direction to $\mathbf{v} \times \mathbf{B}$. Figures are drawn looking down.



(a) (c) (d)

**FIG. P29.2**

(a) Down × North = East, so the force is directed $\boxed{\text{West}}$.

(b) North × North $= \sin 0° = 0$: $\boxed{\text{Zero deflection}}$.

(c) West × North = Down, so the force is directed $\boxed{\text{Up}}$.

(d) Southeast × North = Up, so the force is $\boxed{\text{Down}}$.

**P29.3** $\mathbf{F}_B = q\mathbf{v}\times\mathbf{B}$; $|\mathbf{F}_B|(-\hat{\mathbf{j}}) = -e|\mathbf{v}|\hat{\mathbf{i}}\times\mathbf{B}$

Therefore, $B = |\mathbf{B}|(-\hat{\mathbf{k}})$ which indicates the $\boxed{\text{negative } z \text{ direction}}$.

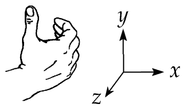


**FIG. P29.3**

**P29.4** (a) $F_B = qvB\sin\theta = (1.60\times10^{-19}\ \text{C})(3.00\times10^{6}\ \text{m/s})(3.00\times10^{-1}\ \text{T})\sin 37.0°$

$F_B = \boxed{8.67\times10^{-14}\ \text{N}}$

(b) $a = \dfrac{F}{m} = \dfrac{8.67\times10^{-14}\ \text{N}}{1.67\times10^{-27}\ \text{kg}} = \boxed{5.19\times10^{13}\ \text{m/s}^2}$

**P29.5** $F = ma = (1.67\times10^{-27}\ \text{kg})(2.00\times10^{13}\ \text{m/s}^2) = 3.34\times10^{-14}\ \text{N} = qvB\sin 90°$

$$B = \frac{F}{qv} = \frac{3.34\times10^{-14}\ \text{N}}{(1.60\times10^{-19}\ \text{C})(1.00\times10^{7}\ \text{m/s})} = \boxed{2.09\times10^{-2}\ \text{T}}$$

The right-hand rule shows that B must be in the $-y$ direction to yield a force in the $+x$ direction when $v$ is in the $z$ direction.

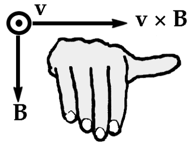


**FIG. P29.5**

**P29.6** First find the speed of the electron.

$$\Delta K = \frac{1}{2}mv^2 = e\Delta V = \Delta U: \qquad v = \sqrt{\frac{2e\Delta V}{m}} = \sqrt{\frac{2(1.60\times10^{-19}\ \text{C})(2\,400\ \text{J/C})}{(9.11\times10^{-31}\ \text{kg})}} = 2.90\times10^{7}\ \text{m/s}$$

(a) $F_{B,\,\max} = qvB = (1.60\times10^{-19}\ \text{C})(2.90\times10^{7}\ \text{m/s})(1.70\ \text{T}) = \boxed{7.90\times10^{-12}\ \text{N}}$

(b) $F_{B,\,\min} = \boxed{0}$ occurs when **v** is either parallel to or anti-parallel to **B**.

**P29.7** $F_B = qvB\sin\theta$ so $8.20\times10^{-13}\ \text{N} = (1.60\times10^{-19}\ \text{C})(4.00\times10^{6}\ \text{m/s})(1.70\ \text{T})\sin\theta$

$\sin\theta = 0.754$ and $\theta = \sin^{-1}(0.754) = \boxed{48.9° \text{ or } 131°}$.

**P29.8** Gravitational force: $F_g = mg = (9.11\times10^{-31}\ \text{kg})(9.80\ \text{m/s}^2) = \boxed{8.93\times10^{-30}\ \text{N down}}$.

Electric force: $F_e = qE = (-1.60\times10^{-19}\ \text{C})(100\ \text{N/C down}) = \boxed{1.60\times10^{-17}\ \text{N up}}$.

Magnetic force: $\mathbf{F}_B = q\mathbf{v}\times\mathbf{B} = (-1.60\times10^{-19}\ \text{C})(6.00\times10^{6}\ \text{m/s}\ \hat{\mathbf{E}})\times(50.0\times10^{-6}\ \text{N}\cdot\text{s/C}\cdot\text{m}\ \hat{\mathbf{N}})$.

$\mathbf{F}_B = -4.80\times10^{-17}\ \text{N up} = \boxed{4.80\times10^{-17}\ \text{N down}}$.

**P29.9** $\mathbf{F}_B = q\mathbf{v}\times\mathbf{B}$

$$\mathbf{v}\times\mathbf{B} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ +2 & -4 & +1 \\ +1 & +2 & -3 \end{vmatrix} = (12-2)\hat{\mathbf{i}} + (1+6)\hat{\mathbf{j}} + (4+4)\hat{\mathbf{k}} = 10\hat{\mathbf{i}} + 7\hat{\mathbf{j}} + 8\hat{\mathbf{k}}$$

$$|\mathbf{v}\times\mathbf{B}| = \sqrt{10^2 + 7^2 + 8^2} = 14.6\ \text{T}\cdot\text{m/s}$$

$$|\mathbf{F}_B| = q|\mathbf{v}\times\mathbf{B}| = \left(1.60\times10^{-19}\ \text{C}\right)(14.6\ \text{T}\cdot\text{m/s}) = \boxed{2.34\times10^{-18}\ \text{N}}$$

**P29.10** $q\mathbf{E} = \left(-1.60\times10^{-19}\ \text{C}\right)(20.0\ \text{N/C})\hat{\mathbf{k}} = \left(-3.20\times10^{-18}\ \text{N}\right)\hat{\mathbf{k}}$

$$\sum\mathbf{F} = q\mathbf{E} + q\mathbf{v}\times\mathbf{B} = m\mathbf{a}$$

$$\left(-3.20\times10^{-18}\ \text{N}\right)\hat{\mathbf{k}} - 1.60\times10^{-19}\ \text{C}\left(1.20\times10^{4}\ \text{m/s}\,\hat{\mathbf{i}}\right)\times\mathbf{B} = \left(9.11\times10^{-31}\right)\left(2.00\times10^{12}\ \text{m/s}^2\right)\hat{\mathbf{k}}$$

$$-\left(3.20\times10^{-18}\ \text{N}\right)\hat{\mathbf{k}} - \left(1.92\times10^{-15}\ \text{C}\cdot\text{m/s}\right)\hat{\mathbf{i}}\times\mathbf{B} = \left(1.82\times10^{-18}\ \text{N}\right)\hat{\mathbf{k}}$$

$$\left(1.92\times10^{-15}\ \text{C}\cdot\text{m/s}\right)\hat{\mathbf{i}}\times\mathbf{B} = -\left(5.02\times10^{-18}\ \text{N}\right)\hat{\mathbf{k}}$$

The magnetic field $\boxed{\text{may have any } x\text{-component}}$. $B_z = \boxed{0}$ and $B_y = \boxed{-2.62\ \text{mT}}$.

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## Section 29.2 **Magnetic Force Acting on a Current-Carrying Conductor**

**P29.11** $F_B = ILB\sin\theta$ with $F_B = F_g = mg$

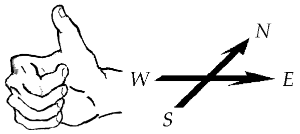

**FIG. P29.11**

$mg = ILB\sin\theta$ so $\dfrac{m}{L}g = IB\sin\theta$

$I = 2.00\ \text{A}$ and $\dfrac{m}{L} = (0.500\ \text{g/cm})\left(\dfrac{100\ \text{cm/m}}{1\,000\ \text{g/kg}}\right) = 5.00\times10^{-2}\ \text{kg/m}.$

Thus $\left(5.00\times10^{-2}\right)(9.80) = (2.00)B\sin90.0°$

$B = \boxed{0.245\ \text{Tesla}}$ with the direction given by right-hand rule: $\boxed{\text{eastward}}$.

**P29.12** $\mathbf{F}_B = I\boldsymbol{\ell}\times\mathbf{B} = (2.40\ \text{A})(0.750\ \text{m})\hat{\mathbf{i}}\times(1.60\ \text{T})\hat{\mathbf{k}} = \boxed{\left(-2.88\hat{\mathbf{j}}\right)\ \text{N}}$

**P29.13** (a) $F_B = ILB\sin\theta = (5.00\ \text{A})(2.80\ \text{m})(0.390\ \text{T})\sin60.0° = \boxed{4.73\ \text{N}}$

(b) $F_B = (5.00\ \text{A})(2.80\ \text{m})(0.390\ \text{T})\sin90.0° = \boxed{5.46\ \text{N}}$

(c) $F_B = (5.00\ \text{A})(2.80\ \text{m})(0.390\ \text{T})\sin120° = \boxed{4.73\ \text{N}}$

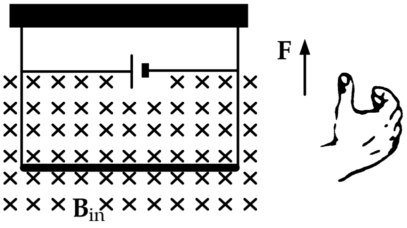

FIG. P29.14

**P29.14** $$\frac{|\mathbf{F}_B|}{\ell}=\frac{mg}{\ell}=\frac{I|\boldsymbol{\ell}\times\mathbf{B}|}{\ell}$$

$$I=\frac{mg}{B\ell}=\frac{(0.040\,0\text{ kg/m})(9.80\text{ m/s}^2)}{3.60\text{ T}}=\boxed{0.109\text{ A}}$$

The direction of $I$ in the bar is $\boxed{\text{to the right}}$.

**P29.15** The rod feels force $\mathbf{F}_B=I(\mathbf{d}\times\mathbf{B})=Id(\hat{\mathbf{k}})\times B(-\hat{\mathbf{j}})=IdB(\hat{\mathbf{i}})$.

The work-energy theorem is $(K_{\text{trasn}}+K_{\text{rot}})_i+\Delta E=(K_{\text{trans}}+K_{\text{rot}})_f$

$$0+0+F_s\cos\theta=\frac{1}{2}mv^2+\frac{1}{2}I\omega^2$$

$$IdBL\cos 0°=\frac{1}{2}mv^2+\frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{v}{R}\right)^2 \text{ and } IdBL=\frac{3}{4}mv^2$$

$$v=\sqrt{\frac{4IdBL}{3m}}=\sqrt{\frac{4(48.0\text{ A})(0.120\text{ m})(0.240\text{ T})(0.450\text{ m})}{3(0.720\text{ kg})}}=\boxed{1.07\text{ m/s}}.$$

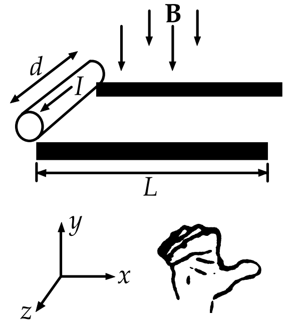

FIG. P29.15

**P29.16** The rod feels force $\mathbf{F}_B=I(\mathbf{d}\times\mathbf{B})=Id(\hat{\mathbf{k}})\times B(-\hat{\mathbf{j}})=IdB(\hat{\mathbf{i}})$.

The work-energy theorem is $(K_{\text{trans}}+K_{\text{rot}})_i+\Delta E=(K_{\text{trans}}+K_{\text{rot}})_f$

$$0+0+Fs\cos\theta=\frac{1}{2}mv^2+\frac{1}{2}I\omega^2$$

$$IdBL\cos 0°=\frac{1}{2}mv^2+\frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{v}{R}\right)^2 \text{ and } v=\boxed{\sqrt{\frac{4IdBL}{3m}}}.$$

**P29.17** The magnetic force on each bit of ring is $Id\mathbf{s}\times\mathbf{B}=IdsB$ radially inward and upward, at angle $\theta$ above the radial line. The radially inward components tend to squeeze the ring but all cancel out as forces. The upward components $IdsB\sin\theta$ all add to $\boxed{I2\pi rB\sin\theta\text{ up}}$.

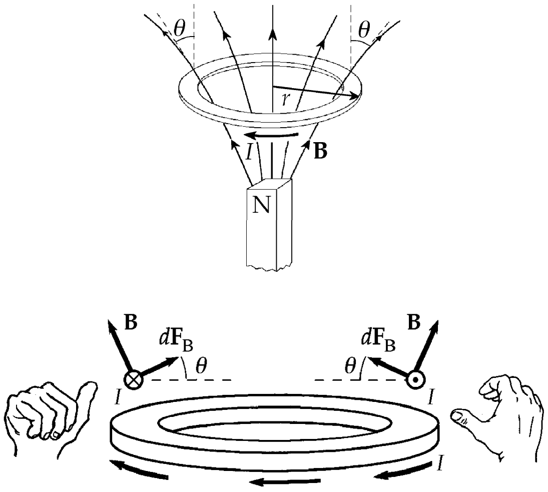

FIG. P29.17

**P29.18** For each segment, $I = 5.00\text{ A}$ and $\mathbf{B} = 0.020\,0\text{ N/A}\cdot\text{m}\,\hat{\mathbf{j}}$.

| Segment | $\ell$ | $\mathbf{F}_B = I(\ell \times \mathbf{B})$ |
|---|---|---|
| *ab* | $-0.400\text{ m}\,\hat{\mathbf{j}}$ | $\boxed{0}$ |
| *bc* | $0.400\text{ m}\,\hat{\mathbf{k}}$ | $\boxed{(40.0\text{ mN})(-\hat{\mathbf{i}})}$ |
| *cd* | $-0.400\text{ m}\,\hat{\mathbf{i}} + 0.400\text{ m}\,\hat{\mathbf{j}}$ | $\boxed{(40.0\text{ mN})(-\hat{\mathbf{k}})}$ |
| *da* | $0.400\text{ m}\,\hat{\mathbf{i}} - 0.400\text{ m}\,\hat{\mathbf{k}}$ | $\boxed{(40.0\text{ mN})(\hat{\mathbf{k}} + \hat{\mathbf{i}})}$ |

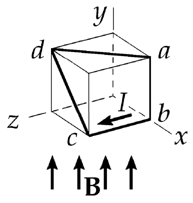

FIG. P29.18

**P29.19** Take the *x*-axis east, the *y*-axis up, and the *z*-axis south. The field is

$$\mathbf{B} = (52.0\ \mu\text{T})\cos 60.0°\left(-\hat{\mathbf{k}}\right) + (52.0\ \mu\text{T})\sin 60.0°\left(-\hat{\mathbf{j}}\right).$$

The current then has equivalent length: $\mathbf{L}' = 1.40\text{ m}\left(-\hat{\mathbf{k}}\right) + 0.850\text{ m}\left(\hat{\mathbf{j}}\right)$

$$\mathbf{F}_B = I\mathbf{L}' \times \mathbf{B} = (0.035\,0\text{ A})\left(0.850\hat{\mathbf{j}} - 1.40\hat{\mathbf{k}}\right)\text{m} \times \left(-45.0\hat{\mathbf{j}} - 26.0\hat{\mathbf{k}}\right)10^{-6}\text{ T}$$

$$\mathbf{F}_B = 3.50 \times 10^{-8}\text{ N}\left(-22.1\hat{\mathbf{i}} - 63.0\hat{\mathbf{i}}\right) = 2.98 \times 10^{-6}\text{ N}\left(-\hat{\mathbf{i}}\right) = \boxed{2.98\ \mu\text{N west}}.$$

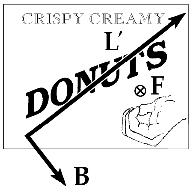

FIG. P29.19

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## Section 29.3 Torque on a Current Loop in a Uniform Magnetic Field

**P29.20** (a) $2\pi r = 2.00\text{ m}$

so $r = 0.318\text{ m}$

$$\mu = IA = \left(17.0 \times 10^{-3}\text{ A}\right)\left[\pi(0.318)^2\text{ m}^2\right] = \boxed{5.41\text{ mA}\cdot\text{m}^2}$$

(b) $\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$

so $\tau = \left(5.41 \times 10^{-3}\text{ A}\cdot\text{m}^2\right)(0.800\text{ T}) = \boxed{4.33\text{ mN}\cdot\text{m}}$

**P29.21** $\tau = \mu B \sin\theta$ so $4.60 \times 10^{-3}\text{ N}\cdot\text{m} = \mu(0.250)\sin 90.0°$

$\mu = 1.84 \times 10^{-2}\text{ A}\cdot\text{m}^2 = \boxed{18.4\text{ mA}\cdot\text{m}^2}$

**P29.22** (a) Let $\theta$ represent the unknown angle; $L$, the total length of the wire; and $d$, the length of one side of the square coil. Then, using the definition of magnetic moment and the right-hand rule in Figure 29.15, we find

$\mu = NAI$: $\quad \boldsymbol{\mu} = \left(\frac{L}{4d}\right)d^2 I$ at angle $\theta$ with the horizontal.

At equilibrium, $\quad \sum \boldsymbol{\tau} = (\boldsymbol{\mu} \times \mathbf{B}) - (\mathbf{r} \times m\mathbf{g}) = 0$

$$\left(\frac{ILBd}{4}\right)\sin(90.0° - \theta) - \left(\frac{mgd}{2}\right)\sin\theta = 0$$

and $\quad \left(\frac{mgd}{2}\right)\sin\theta = \left(\frac{ILBd}{4}\right)\cos\theta$

$$\theta = \tan^{-1}\left(\frac{ILB}{2mg}\right) = \tan^{-1}\left(\frac{(3.40\text{ A})(4.00\text{ m})(0.010\,0\text{ T})}{2(0.100\text{ kg})(9.80\text{ m/s}^2)}\right) = \boxed{3.97°}.$$

(b) $$\tau_m = \left(\frac{ILBd}{4}\right)\cos\theta = \frac{1}{4}(3.40\text{ A})(4.00\text{ m})(0.010\,0\text{ T})(0.100\text{ m})\cos 3.97° = \boxed{3.39\text{ mN}\cdot\text{m}}$$

**P29.23** $\tau = NBAI\sin\phi$

$\tau = 100(0.800\text{ T})(0.400 \times 0.300\text{ m}^2)(1.20\text{ A})\sin 60°$

$\tau = \boxed{9.98\text{ N}\cdot\text{m}}$

Note that $\phi$ is the angle between the magnetic moment and the **B** field. The loop will rotate so as to align the magnetic moment with the **B** field. Looking down along the $y$-axis, the loop will rotate in a $\boxed{\text{clockwise}}$ direction.

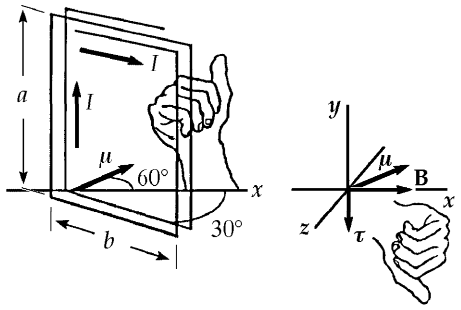

**FIG. P29.23**

**P29.24** From $\tau = \boldsymbol{\mu} \times \mathbf{B} = I\mathbf{A} \times \mathbf{B}$, the magnitude of the torque is $IAB\sin 90.0°$.

(a) Each side of the triangle is $\frac{40.0\text{ cm}}{3}$.

Its altitude is $\sqrt{13.3^2 - 6.67^2}\text{ cm} = 11.5\text{ cm}$ and its area is

$A = \frac{1}{2}(11.5\text{ cm})(13.3\text{ cm}) = 7.70 \times 10^{-3}\text{ m}^2$.

Then $\tau = (20.0\text{ A})(7.70 \times 10^{-3}\text{ m}^2)(0.520\text{ N}\cdot\text{s/C}\cdot\text{m}) = \boxed{80.1\text{ mN}\cdot\text{m}}$.

(b) Each side of the square is 10.0 cm and its area is $100\text{ cm}^2 = 10^{-2}\text{ m}^2$.

$\tau = (20.0\text{ A})(10^{-2}\text{ m}^2)(0.520\text{ T}) = \boxed{0.104\text{ N}\cdot\text{m}}$

(c) $r = \frac{0.400\text{ m}}{2\pi} = 0.063\,7\text{ m}$

$A = \pi r^2 = 1.27 \times 10^{-2}\text{ m}^2$

$\tau = (20.0\text{ A})(1.27 \times 10^{-2}\text{ m}^2)(0.520) = \boxed{0.132\text{ N}\cdot\text{m}}$

(d) The circular loop experiences the largest torque.

**P29.25** Choose $U = 0$ when the dipole moment is at $\theta = 90.0°$ to the field. The field exerts torque of magnitude $\mu B \sin\theta$ on the dipole, tending to turn the dipole moment in the direction of decreasing $\theta$. According to Equations 8.16 and 10.22, the potential energy of the dipole-field system is given by

$$U - 0 = \int_{90.0°}^{\theta} \mu B \sin\theta \, d\theta = \mu B(-\cos\theta)\Big|_{90.0°}^{\theta} = -\mu B\cos\theta + 0 \qquad \text{or} \qquad \boxed{U = -\boldsymbol{\mu}\cdot\mathbf{B}}.$$

**P29.26** (a) The field exerts torque on the needle tending to align it with the field, so the minimum energy orientation of the needle is:

$\boxed{\text{pointing north at 48.0° below the horizontal}}$

where its energy is $U_{\min} = -\mu B\cos 0° = -\left(9.70\times10^{-3}\ \text{A}\cdot\text{m}^2\right)\left(55.0\times10^{-6}\ \text{T}\right) = -5.34\times10^{-7}\ \text{J}$.

It has maximum energy when pointing in the opposite direction,

$\boxed{\text{south at 48.0° above the horizontal}}$

where its energy is $U_{\max} = -\mu B\cos 180° = +\left(9.70\times10^{-3}\ \text{A}\cdot\text{m}^2\right)\left(55.0\times10^{-6}\ \text{T}\right) = +5.34\times10^{-7}\ \text{J}$.

(b) $U_{\min} + W = U_{\max}:\ W = U_{\max} - U_{\min} = +5.34\times10^{-7}\ \text{J} - \left(-5.34\times10^{-7}\ \text{J}\right) = \boxed{1.07\ \mu\text{J}}$

**P29.27** (a) $\tau = \boldsymbol{\mu}\times\mathbf{B}$, so $\tau = |\boldsymbol{\mu}\times\mathbf{B}| = \mu B\sin\theta = NIAB\sin\theta$

$$\tau_{\max} = NIAB\sin 90.0° = 1(5.00\ \text{A})\left[\pi(0.050\,0\ \text{m})^2\right]\left(3.00\times10^{-3}\ \text{T}\right) = \boxed{118\ \mu\text{N}\cdot\text{m}}$$

(b) $U = -\mu\cdot\mathbf{B}$, so $-\mu B \le U \le +\mu B$

Since $\mu B = (NIA)B = 1(5.00\ \text{A})\left[\pi(0.050\,0\ \text{m})^2\right]\left(3.00\times10^{-3}\ \text{T}\right) = 118\ \mu\text{J}$,

the range of the potential energy is: $\boxed{-118\ \mu\text{J} \le U \le +118\ \mu\text{J}}$.

***P29.28** (a) $|\boldsymbol{\tau}| = |\boldsymbol{\mu}\times\mathbf{B}| = NIAB\sin\theta$

$$\tau_{\max} = 80\left(10^{-2}\ \text{A}\right)(0.025\ \text{m}\cdot 0.04\ \text{m})(0.8\ \text{N/A}\cdot\text{m})\sin 90° = \boxed{6.40\times10^{-4}\ \text{N}\cdot\text{m}}$$

(b) $$\mathcal{P}_{\max} = \tau_{\max}\omega = 6.40\times10^{-4}\ \text{N}\cdot\text{m}(3\,600\ \text{rev/min})\left(\frac{2\pi\,\text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = \boxed{0.241\ \text{W}}$$

(c) In one half revolution the work is

$$W = U_{\max} - U_{\min} = -\mu B\cos 180° - (-\mu B\cos 0°) = 2\mu B$$
$$= 2NIAB = 2\left(6.40\times10^{-4}\ \text{N}\cdot\text{m}\right) = 1.28\times10^{-3}\ \text{J}$$

In one full revolution, $W = 2\left(1.28\times10^{-3}\ \text{J}\right) = \boxed{2.56\times10^{-3}\ \text{J}}$.

(d) $$\mathcal{P}_{\text{avg}} = \frac{W}{\Delta t} = \frac{2.56\times10^{-3}\ \text{J}}{(1/60)\ \text{s}} = \boxed{0.154\ \text{W}}$$

The peak power in (b) is greater by the factor $\frac{\pi}{2}$.

## Section 29.4 **Motion of a Charged Particle in a Uniform Magnetic Field**

**P29.29** (a) $B = 50.0 \times 10^{-6}\ \text{T};\ v = 6.20 \times 10^{6}\ \text{m/s}$

Direction is given by the right-hand-rule: $\boxed{\text{southward}}$

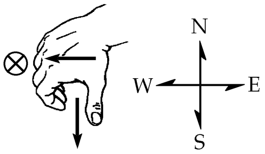

**FIG. P29.29**

$$F_B = qvB\sin\theta$$

$$F_B = \left(1.60 \times 10^{-19}\ \text{C}\right)\left(6.20 \times 10^{6}\ \text{m/s}\right)\left(50.0 \times 10^{-6}\ \text{T}\right)\sin 90.0°$$

$$= \boxed{4.96 \times 10^{-17}\ \text{N}}$$

(b) $F = \dfrac{mv^2}{r}$ so $r = \dfrac{mv^2}{F} = \dfrac{\left(1.67 \times 10^{-27}\ \text{kg}\right)\left(6.20 \times 10^{6}\ \text{m/s}\right)^2}{4.96 \times 10^{-17}\ \text{N}} = \boxed{1.29\ \text{km}}$

**P29.30** $\dfrac{1}{2}mv^2 = q(\Delta V)$ $\qquad \dfrac{1}{2}\left(3.20 \times 10^{-26}\ \text{kg}\right)v^2 = \left(1.60 \times 10^{-19}\ \text{C}\right)(833\ \text{V})$ $\qquad v = 91.3\ \text{km/s}$

The magnetic force provides the centripetal force: $qvB\sin\theta = \dfrac{mv^2}{r}$

$$r = \frac{mv}{qB\sin 90.0°} = \frac{\left(3.20 \times 10^{-26}\ \text{kg}\right)\left(9.13 \times 10^{4}\ \text{m/s}\right)}{\left(1.60 \times 10^{-19}\ \text{C}\right)\left(0.920\ \text{N}\cdot\text{s/C}\cdot\text{m}\right)} = \boxed{1.98\ \text{cm}}.$$

**P29.31** For each electron, $|q|vB\sin 90.0° = \dfrac{mv^2}{r}$ and $v = \dfrac{eBr}{m}$.

The electrons have no internal structure to absorb energy, so the collision must be perfectly elastic:

$$K = \frac{1}{2}mv_{1i}^2 + 0 = \frac{1}{2}mv_{1f}^2 + \frac{1}{2}mv_{2f}^2$$

$$K = \frac{1}{2}m\left(\frac{e^2B^2R_1^2}{m^2}\right) + \frac{1}{2}m\left(\frac{e^2B^2R_2^2}{m^2}\right) = \frac{e^2B^2}{2m}\left(R_1^2 + R_2^2\right)$$

$$K = \frac{e\left(1.60 \times 10^{-19}\ \text{C}\right)\left(0.044\,0\ \text{N}\cdot\text{s/C}\cdot\text{m}\right)^2}{2\left(9.11 \times 10^{-31}\ \text{kg}\right)}\left[(0.010\,0\ \text{m})^2 + (0.024\,0\ \text{m})^2\right] = \boxed{115\ \text{keV}}$$

**P29.32** We begin with $qvB = \dfrac{mv^2}{R}$, so $v = \dfrac{qRB}{m}$.

The time to complete one revolution is $T = \dfrac{2\pi R}{v} = \dfrac{2\pi R}{qRB/m} = \dfrac{2\pi m}{qB}$.

Solving for $B$, $B = \dfrac{2\pi m}{qT} = \boxed{6.56 \times 10^{-2}\ \text{T}}$.

**P29.33** $q(\Delta V)=\frac{1}{2}mv^2$ or $v=\sqrt{\frac{2q(\Delta V)}{m}}$.

Also, $qvB=\frac{mv^2}{r}$ so $r=\frac{mv}{qB}=\frac{m}{qB}\sqrt{\frac{2q(\Delta V)}{m}}=\sqrt{\frac{2m(\Delta V)}{qB^2}}$.

Therefore, $r_p^2=\frac{2m_p(\Delta V)}{eB^2}$

$$r_d^2=\frac{2m_d(\Delta V)}{q_dB^2}=\frac{2(2m_p)(\Delta V)}{eB^2}=2\left(\frac{2m_p(\Delta V)}{eB^2}\right)=2r_p^2$$

and $$r_\alpha^2=\frac{2m_\alpha(\Delta V)}{q_\alpha B^2}=\frac{2(4m_p)(\Delta V)}{(2e)B^2}=2\left(\frac{2m_p(\Delta V)}{eB^2}\right)=2r_p^2.$$

The conclusion is: $\boxed{r_\alpha=r_d=\sqrt{2}r_p}$.

**P29.34** (a) We begin with $qvB=\frac{mv^2}{R}$

or $qRB=mv$.

But $L=mvR=qR^2B$.

Therefore, $$R=\sqrt{\frac{L}{qB}}=\sqrt{\frac{4.00\times10^{-25}\text{ J}\cdot\text{s}}{(1.60\times10^{-19}\text{ C})(1.00\times10^{-3}\text{ T})}}=0.050\,0\text{ m}=\boxed{5.00\text{ cm}}.$$

(b) Thus, $$v=\frac{L}{mR}=\frac{4.00\times10^{-25}\text{ J}\cdot\text{s}}{(9.11\times10^{-31}\text{ kg})(0.050\,0\text{ m})}=\boxed{8.78\times10^{6}\text{ m/s}}.$$

**P29.35** $$\omega=\frac{qB}{m}=\frac{(1.60\times10^{-19}\text{ C})(5.20\text{ T})}{1.67\times10^{-27}\text{ kg}}=\boxed{4.98\times10^{8}\text{ rad/s}}$$

**P29.36** $\frac{1}{2}mv^2=q(\Delta V)$ so $v=\sqrt{\frac{2q(\Delta V)}{m}}$

$r=\frac{mv}{qB}$ so $r=\frac{m\sqrt{2q(\Delta V)/m}}{qB}$

$r^2=\frac{m}{q}\cdot\frac{2(\Delta V)}{B^2}$ and $(r')^2=\frac{m'}{q'}\cdot\frac{2(\Delta V)}{B^2}$

$m=\frac{qB^2r^2}{2(\Delta V)}$ and $(m')=\frac{(q')B^2(r')^2}{2(\Delta V)}$ so $\frac{m'}{m}=\frac{q'}{q}\cdot\frac{(r')^2}{r^2}=\left(\frac{2e}{e}\right)\left(\frac{2R}{R}\right)^2=\boxed{8}$

**P29.37** $E = \frac{1}{2}mv^2 = e\Delta V$

and $evB\sin 90° = \frac{mv^2}{R}$

$$B = \frac{mv}{eR} = \frac{m}{eR}\sqrt{\frac{2e\Delta V}{m}} = \frac{1}{R}\sqrt{\frac{2m\Delta V}{e}}$$

$$B = \frac{1}{5.80\times 10^{10}\text{ m}}\sqrt{\frac{2\left(1.67\times 10^{-27}\text{ kg}\right)\left(10.0\times 10^{6}\text{ V}\right)}{1.60\times 10^{-19}\text{ C}}} = \boxed{7.88\times 10^{-12}\text{ T}}$$

***P29.38** (a) At the moment shown in Figure 29.21, the particle must be moving upward in order for the magnetic force on it to be into the page, toward the center of this turn of its spiral path. Throughout its motion it circulates clockwise.

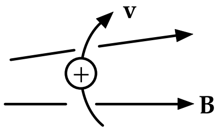

**FIG. P29.38(a)**

(b) After the particle has passed the middle of the bottle and moves into the region of increasing magnetic field, the magnetic force on it has a component to the left (as well as a radially inward component) as shown. This force in the $-x$ direction slows and reverses the particle's motion along the axis.

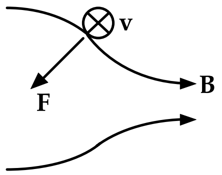

**FIG. P29.38(b)**

(c) The magnetic force is perpendicular to the velocity and does no work on the particle. The particle keeps constant kinetic energy. As its axial velocity component decreases, its tangential velocity component increases.

(d) The orbiting particle constitutes a loop of current in the $yz$ plane and therefore a magnetic dipole moment $I\text{A} = \frac{q}{T}\text{ A}$ in the $-x$ direction. It is like a little bar magnet with its N pole on the left.

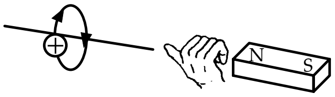

**FIG. P29.38(d)**

(e) Problem 17 showed that a nonuniform magnetic field exerts a net force on a magnetic dipole. When the dipole is aligned opposite to the external field, the force pushes it out of the region of stronger field. Here it is to the left, a force of repulsion of one magnetic south pole on another south pole.

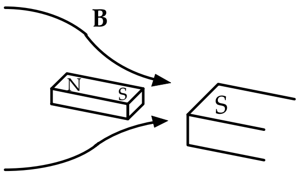

**FIG. P29.38(e)**

**P29.39** $r = \dfrac{mv}{qB}$ so $m = \dfrac{rqB}{v} = \dfrac{\left(7.94\times10^{-3}\text{ m}\right)\left(1.60\times10^{-19}\text{ C}\right)(1.80\text{ T})}{4.60\times10^{5}\text{ m/s}}$

$$m = 4.97\times10^{-27}\text{ kg}\left(\frac{1\text{ u}}{1.66\times10^{-27}\text{ kg}}\right) = \boxed{2.99\text{ u}}$$

The particle is singly ionized: either a tritium ion, $\boxed{{}^{3}_{1}\text{H}^{+}}$, or a helium ion, $\boxed{{}^{3}_{2}\text{He}^{+}}$.

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## Section 29.5 **Applications Involving Charged Particles Moving in a Magnetic Field**

**P29.40** $F_B = F_e$

so $qvB = qE$

where $v = \sqrt{\dfrac{2K}{m}}$ and $K$ is kinetic energy of the electron.

$$E = vB = \sqrt{\frac{2K}{m}}B = \sqrt{\frac{2(750)\left(1.60\times10^{-19}\right)}{9.11\times10^{-31}}}(0.015\,0) = \boxed{244\text{ kV/m}}$$

**P29.41** $K = \dfrac{1}{2}mv^2 = q(\Delta V)$ so $v = \sqrt{\dfrac{2q(\Delta V)}{m}}$

$|\mathbf{F}_B| = |q\mathbf{v}\times\mathbf{B}| = \dfrac{mv^2}{r}$ $r = \dfrac{mv}{qB} = \dfrac{m}{q}\dfrac{\sqrt{2q(\Delta V)/m}}{B} = \dfrac{1}{B}\sqrt{\dfrac{2m(\Delta V)}{q}}$

(a) $$r_{238} = \sqrt{\frac{2\left(238\times1.66\times10^{-27}\right)2\,000}{1.60\times10^{-19}}}\left(\frac{1}{1.20}\right) = 8.28\times10^{-2}\text{ m} = \boxed{8.28\text{ cm}}$$

(b) $r_{235} = \boxed{8.23\text{ cm}}$

$$\frac{r_{238}}{r_{235}} = \sqrt{\frac{m_{238}}{m_{235}}} = \sqrt{\frac{238.05}{235.04}} = 1.006\,4$$

The ratios of the orbit radius for different ions are independent of $\Delta V$ and $B$.

**P29.42** In the velocity selector: $v = \dfrac{E}{B} = \dfrac{2\,500\text{ V/m}}{0.035\,0\text{ T}} = 7.14\times10^{4}\text{ m/s}$.

In the deflection chamber: $r = \dfrac{mv}{qB} = \dfrac{\left(2.18\times10^{-26}\text{ kg}\right)\left(7.14\times10^{4}\text{ m/s}\right)}{\left(1.60\times10^{-19}\text{ C}\right)(0.035\,0\text{ T})} = \boxed{0.278\text{ m}}$.

**P29.43** (a) $F_B = qvB = \dfrac{mv^2}{R}$

$$\omega = \frac{v}{R} = \frac{qBR}{mR} = \frac{qB}{m} = \frac{\left(1.60\times10^{-19}\text{ C}\right)(0.450\text{ T})}{1.67\times10^{-27}\text{ kg}} = \boxed{4.31\times10^{7}\text{ rad/s}}$$

(b) $$v = \frac{qBR}{m} = \frac{\left(1.60\times10^{-19}\text{ C}\right)(0.450\text{ T})(1.20\text{ m})}{1.67\times10^{-27}\text{ kg}} = \boxed{5.17\times10^{7}\text{ m/s}}$$

**P29.44** $K = \dfrac{1}{2}mv^2$: $\left(34.0\times10^{6}\text{ eV}\right)\left(1.60\times10^{-19}\text{ J/eV}\right) = \dfrac{1}{2}\left(1.67\times10^{-27}\text{ kg}\right)v^2$

$v = 8.07\times10^{7}\text{ m/s}$ $\quad r = \dfrac{mv}{qB} = \dfrac{\left(1.67\times10^{-27}\text{ kg}\right)\left(8.07\times10^{7}\text{ m/s}\right)}{\left(1.60\times10^{-19}\text{ C}\right)(5.20\text{ T})} = \boxed{0.162\text{ m}}$

***P29.45** Note that the "cyclotron frequency" is an angular speed. The motion of the proton is described by $\sum F = ma$:

$$|q|vB\sin 90° = \frac{mv^2}{r}$$
$$|q|B = m\frac{v}{r} = m\omega$$

(a) $$\omega = \frac{|q|B}{m} = \frac{\left(1.60\times10^{-19}\text{ C}\right)(0.8\text{ N}\cdot\text{s/C}\cdot\text{m})}{\left(1.67\times10^{-27}\text{ kg}\right)}\left(\frac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}\right) = \boxed{7.66\times10^{7}\text{ rad/s}}$$

(b) $$v = \omega r = \left(7.66\times10^{7}\text{ rad/s}\right)(0.350\text{ m})\left(\frac{1}{1\text{ rad}}\right) = \boxed{2.68\times10^{7}\text{ m/s}}$$

(c) $$K = \frac{1}{2}mv^2 = \frac{1}{2}\left(1.67\times10^{-27}\text{ kg}\right)\left(2.68\times10^{7}\text{ m/s}\right)^2\left(\frac{1\text{ eV}}{1.6\times10^{-19}\text{ J}}\right) = \boxed{3.76\times10^{6}\text{ eV}}$$

(d) The proton gains 600 eV twice during each revolution, so the number of revolutions is

$$\frac{3.76\times10^{6}\text{ eV}}{2(600\text{ eV})} = \boxed{3.13\times10^{3}\text{ revolutions}}.$$

(e) $\theta = \omega t$ $\quad t = \dfrac{\theta}{\omega} = \dfrac{3.13\times10^{3}\text{ rev}}{7.66\times10^{7}\text{ rad/s}}\left(\dfrac{2\pi\text{ rad}}{1\text{ rev}}\right) = \boxed{2.57\times10^{-4}\text{ s}}$

**P29.46** $F_B = qvB = \dfrac{mv^2}{r}$

$$B = \frac{mv}{qr} = \frac{4.80\times10^{-16}\text{ kg}\cdot\text{m/s}}{\left(1.60\times10^{-19}\text{ C}\right)(1\,000\text{ m})} = \boxed{3.00\text{ T}}$$

**P29.47** $\theta = \tan^{-1}\left(\frac{25.0}{10.0}\right) = 68.2°$ and $R = \frac{1.00 \text{ cm}}{\sin 68.2°} = 1.08 \text{ cm}$.

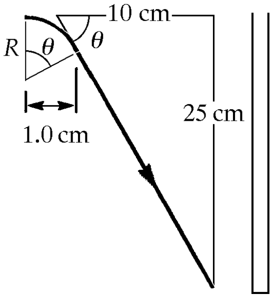

**FIG. P29.47**

Ignoring relativistic correction, the kinetic energy of the electrons is

$$\frac{1}{2}mv^2 = q\Delta V \qquad \text{so} \qquad v = \sqrt{\frac{2q\Delta V}{m}} = 1.33 \times 10^8 \text{ m/s}.$$

From Newton's second law $\frac{mv^2}{R} = qvB$, we find the magnetic field

$$B = \frac{mv}{|q|R} = \frac{(9.11 \times 10^{-31} \text{ kg})(1.33 \times 10^8 \text{ m/s})}{(1.60 \times 10^{-19} \text{ C})(1.08 \times 10^{-2} \text{ m})} = \boxed{70.1 \text{ mT}}.$$

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## Section 29.6 **The Hall Effect**

**P29.48** (a) $R_H \equiv \frac{1}{nq}$ so $n = \frac{1}{qR_H} = \frac{1}{(1.60 \times 10^{-19} \text{ C})(0.840 \times 10^{-10} \text{ m}^3/\text{C})} = \boxed{7.44 \times 10^{28} \text{ m}^{-3}}$

(b) $\Delta V_H = \frac{IB}{nqt}$

$$B = \frac{nqt(\Delta V_H)}{I} = \frac{(7.44 \times 10^{28} \text{ m}^{-3})(1.60 \times 10^{-19} \text{ C})(0.200 \times 10^{-3} \text{ m})(15.0 \times 10^{-6} \text{ V})}{20.0 \text{ A}} = \boxed{1.79 \text{ T}}$$

**P29.49** Since $\Delta V_H = \frac{IB}{nqt}$, and given that $I = 50.0 \text{ A}$, $B = 1.30 \text{ T}$, and $t = 0.330 \text{ mm}$, the number of charge carriers per unit volume is

$$n = \frac{IB}{e(\Delta V_H)t} = \boxed{1.28 \times 10^{29} \text{ m}^{-3}}$$

The number density of atoms we compute from the density:

$$n_0 = \frac{8.92 \text{ g}}{\text{cm}^3}\left(\frac{1 \text{ mole}}{63.5 \text{ g}}\right)\left(\frac{6.02 \times 10^{23} \text{ atoms}}{\text{mole}}\right)\left(\frac{10^6 \text{ cm}^3}{1 \text{ m}^3}\right) = 8.46 \times 10^{28} \text{ atom/m}^3$$

So the number of conduction electrons per atom is

$$\frac{n}{n_0} = \frac{1.28 \times 10^{29}}{8.46 \times 10^{28}} = \boxed{1.52}$$

**P29.50** (a) $\Delta V_H = \dfrac{IB}{nqt}$ so $\dfrac{nqt}{I} = \dfrac{B}{\Delta V_H} = \dfrac{0.080\,0\text{ T}}{0.700\times10^{-6}\text{ V}} = 1.14\times10^{5}\text{ T/V}$.

Then, the unknown field is $B = \left(\dfrac{nqt}{I}\right)(\Delta V_H)$

$$B = \left(1.14\times10^{5}\text{ T/V}\right)\left(0.330\times10^{-6}\text{ V}\right) = 0.037\,7\text{ T} = \boxed{37.7\text{ mT}}.$$

(b) $\dfrac{nqt}{I} = 1.14\times10^{5}\text{ T/V}$ so $n = \left(1.14\times10^{5}\text{ T/V}\right)\dfrac{I}{qt}$

$$n = \left(1.14\times10^{5}\text{ T/V}\right)\frac{0.120\text{ A}}{\left(1.60\times10^{-19}\text{ C}\right)\left(2.00\times10^{-3}\text{ m}\right)} = \boxed{4.29\times10^{25}\text{ m}^{-3}}.$$

**P29.51** $$B = \frac{nqt(\Delta V_H)}{I} = \frac{\left(8.49\times10^{28}\text{ m}^{-3}\right)\left(1.60\times10^{-19}\text{ C}\right)\left(5.00\times10^{-3}\text{ m}\right)\left(5.10\times10^{-12}\text{ V}\right)}{8.00\text{ A}}$$

$$B = 4.33\times10^{-5}\text{ T} = \boxed{43.3\ \mu\text{T}}$$

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## Additional Problems

**P29.52** (a) The boundary between a region of strong magnetic field and a region of zero field cannot be perfectly sharp, but we ignore the thickness of the transition zone. In the field the electron moves on an arc of a circle:

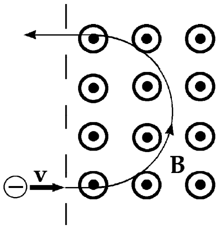


FIG. P29.52(a)

$\sum F = ma$:

$$|q|vB\sin 90° = \frac{mv^2}{r}$$

$$\frac{v}{r} = \omega = \frac{|q|B}{m} = \frac{\left(1.60\times10^{-19}\text{ C}\right)\left(10^{-3}\text{ N}\cdot\text{s/C}\cdot\text{m}\right)}{\left(9.11\times10^{-31}\text{ kg}\right)} = 1.76\times10^{8}\text{ rad/s}$$

The time for one half revolution is,

from $\Delta\theta = \omega\Delta t$

$$\Delta t = \frac{\Delta\theta}{\omega} = \frac{\pi\text{ rad}}{1.76\times10^{8}\text{ rad/s}} = \boxed{1.79\times10^{-8}\text{ s}}.$$

(b) The maximum depth of penetration is the radius of the path.

Then $v = \omega r = \left(1.76\times10^{8}\text{ s}^{-1}\right)(0.02\text{ m}) = 3.51\times10^{6}\text{ m/s}$

and

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\left(9.11\times10^{-31}\text{ kg}\right)\left(3.51\times10^{6}\text{ m/s}\right)^2 = 5.62\times10^{-18}\text{ J} = \frac{5.62\times10^{-18}\text{ J}\cdot\text{e}}{1.60\times10^{-19}\text{ C}}$$

$$= \boxed{35.1\text{ eV}}.$$

**P29.53** (a) Define vector **h** to have the downward direction of the current, and vector **L** to be along the pipe into the page as shown. The $\boxed{\text{electric current experiences a magnetic force}}$.

$I(\mathbf{h}\times\mathbf{B})$ in the direction of **L**.

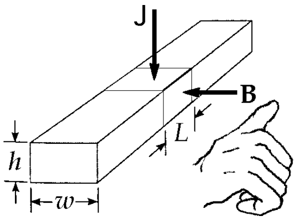


FIG. P29.53

(b) The sodium, consisting of ions and electrons, flows along the pipe transporting no net charge. But inside the section of length $L$, electrons drift upward to constitute downward electric current $J\times(\text{area})=JLw$.

The current then feels a magnetic force $I|\mathbf{h}\times\mathbf{B}|=JLwhB\sin 90°$.

This force along the pipe axis will make the fluid move, exerting pressure

$$\frac{F}{\text{area}}=\frac{JLwhB}{hw}=\boxed{JLB}.$$

**P29.54** $\sum F_y=0$: $+n-mg=0$

$\sum F_x=0$: $-\mu_k n+IB\sin 90.0°=0$

$$B=\frac{\mu_k mg}{Id}=\frac{0.100(0.200\text{ kg})\left(9.80\text{ m/s}^2\right)}{(10.0\text{ A})(0.500\text{ m})}=\boxed{39.2\text{ mT}}$$

**P29.55** The magnetic force on each proton, $\mathbf{F}_B=q\mathbf{v}\times\mathbf{B}=qvB\sin 90°$ downward perpendicular to velocity, causes centripetal acceleration, guiding it into a circular path of radius $r$, with

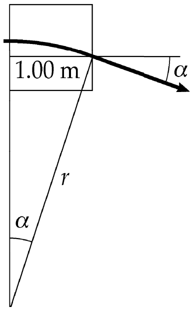


FIG. P29.55

$$qvB=\frac{mv^2}{r}$$

and $$r=\frac{mv}{qB}.$$

We compute this radius by first finding the proton's speed:

$$K=\frac{1}{2}mv^2$$

$$v=\sqrt{\frac{2K}{m}}=\sqrt{\frac{2\left(5.00\times10^{6}\text{ eV}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)}{1.67\times10^{-27}\text{ kg}}}=3.10\times10^{7}\text{ m/s}.$$

Now, $$r=\frac{mv}{qB}=\frac{\left(1.67\times10^{-27}\text{ kg}\right)\left(3.10\times10^{7}\text{ m/s}\right)}{\left(1.60\times10^{-19}\text{ C}\right)(0.050\,0\text{ N}\cdot\text{s/C}\cdot\text{m})}=6.46\text{ m}.$$

(b) From the figure, observe that

$$\sin\alpha=\frac{1.00\text{ m}}{r}=\frac{1\text{ m}}{6.46\text{ m}}$$

$$\boxed{\alpha=8.90°}$$

(a) The magnitude of the proton momentum stays constant, and its final $y$ component is

$$-\left(1.67\times10^{-27}\text{ kg}\right)\left(3.10\times10^{7}\text{ m/s}\right)\sin 8.90°=\boxed{-8.00\times10^{-21}\text{ kg}\cdot\text{m/s}}.$$

**P29.56** (a) If $\mathbf{B}=B_x\hat{\mathbf{i}}+B_y\hat{\mathbf{j}}+B_z\hat{\mathbf{k}}$, $\mathbf{F}_B=q\mathbf{v}\times\mathbf{B}=e\left(v_i\hat{\mathbf{i}}\right)\times\left(B_x\hat{\mathbf{i}}+B_y\hat{\mathbf{j}}+B_z\hat{\mathbf{k}}\right)=0+ev_iB_y\hat{\mathbf{k}}-ev_iB_z\hat{\mathbf{j}}$.

Since the force actually experienced is $\mathbf{F}_B=F_i\hat{\mathbf{j}}$, observe that

$\boxed{B_x \text{ could have any value}}$, $\boxed{B_y=0}$, and $\boxed{B_z=-\frac{F_i}{ev_i}}$.

(b) If $\mathbf{v}=-v_i\hat{\mathbf{i}}$, then $\mathbf{F}_B=q\mathbf{v}\times\mathbf{B}=e\left(-v_i\hat{\mathbf{i}}\right)\times\left(B_x\hat{\mathbf{i}}+0\hat{\mathbf{j}}-\frac{F_i}{ev_i}\hat{\mathbf{k}}\right)=\boxed{-F_i\hat{\mathbf{j}}}$.

(c) If $q=-e$ and $\mathbf{v}=v_i\hat{\mathbf{i}}$, then $\mathbf{F}_B=q\mathbf{v}\times\mathbf{B}=-e\left(v_i\hat{\mathbf{i}}\right)\times\left(B_x\hat{\mathbf{i}}+0\hat{\mathbf{j}}-\frac{F_i}{ev_i}\hat{\mathbf{k}}\right)=\boxed{-F_i\hat{\mathbf{j}}}$.

Reversing either the velocity or the sign of the charge reverses the force.

**P29.57** (a) The net force is the Lorentz force given by

$$\mathbf{F}=q\mathbf{E}+q\mathbf{v}\times\mathbf{B}=q(\mathbf{E}+\mathbf{v}\times\mathbf{B})$$

$$\mathbf{F}=\left(3.20\times10^{-19}\right)\left[\left(4\hat{\mathbf{i}}-1\hat{\mathbf{j}}-2\hat{\mathbf{k}}\right)+\left(2\hat{\mathbf{i}}+3\hat{\mathbf{j}}-1\hat{\mathbf{k}}\right)\times\left(2\hat{\mathbf{i}}+4\hat{\mathbf{j}}+1\hat{\mathbf{k}}\right)\right]\text{ N}$$

Carrying out the indicated operations, we find:

$$\mathbf{F}=\boxed{\left(3.52\hat{\mathbf{i}}-1.60\hat{\mathbf{j}}\right)\times10^{-18}\text{ N}}.$$

(b) $\theta=\cos^{-1}\left(\frac{F_x}{F}\right)=\cos^{-1}\left(\frac{3.52}{\sqrt{(3.52)^2+(1.60)^2}}\right)=\boxed{24.4°}$

**P29.58** A key to solving this problem is that reducing the normal force will reduce the friction force: $F_B=BIL$ or $B=\frac{F_B}{IL}$.

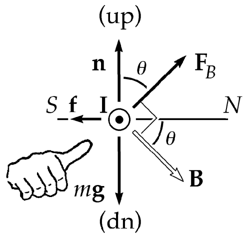


**FIG. P29.58**

When the wire is just able to move, $\sum F_y=n+F_B\cos\theta-mg=0$

so $n=mg-F_B\cos\theta$

and $f=\mu\left(mg-F_B\cos\theta\right)$.

Also, $\sum F_x=F_B\sin\theta-f=0$

so $F_B\sin\theta=f$: $F_B\sin\theta=\mu\left(mg-F_B\cos\theta\right)$ and $F_B=\frac{\mu mg}{\sin\theta+\mu\cos\theta}$.

We minimize $B$ by minimizing $F_B$: $\frac{dF_B}{d\theta}=(\mu mg)\frac{\cos\theta-\mu\sin\theta}{(\sin\theta+\mu\cos\theta)^2}=0\Rightarrow\mu\sin\theta=\cos\theta$.

Thus, $\theta=\tan^{-1}\left(\frac{1}{\mu}\right)=\tan^{-1}(5.00)=78.7°$ for the smallest field, and

$$B=\frac{F_B}{IL}=\left(\frac{\mu g}{I}\right)\frac{(m/L)}{\sin\theta+\mu\cos\theta}$$

$$B_{\min}=\left[\frac{(0.200)\left(9.80\text{ m/s}^2\right)}{1.50\text{ A}}\right]\frac{0.100\text{ kg/m}}{\sin78.7°+(0.200)\cos78.7°}=0.128\text{ T}$$

$\boxed{B_{\min}=0.128\text{ T pointing north at an angle of }78.7°\text{ below the horizontal}}$

***P29.59** The electrons are all fired from the electron gun with the same speed $v$ in

$$U_i = K_f \qquad qV = \frac{1}{2}mv^2 \qquad (-e)(-\Delta V) = \frac{1}{2}m_e v^2 \qquad v = \sqrt{\frac{2e\Delta V}{m_e}}$$

For $\phi$ small, $\cos\phi$ is nearly equal to 1. The time $T$ of passage of each electron in the chamber is given by

$$d = vT \qquad T = d\left(\frac{m_e}{2e\Delta V}\right)^{1/2}$$

Each electron moves in a different helix, around a different axis. If each completes just one revolution within the chamber, it will be in the right place to pass through the exit port. Its transverse velocity component $v_\perp = v\sin\phi$ swings around according to $F_\perp = ma_\perp$

$$qv_\perp B\sin 90° = \frac{mv_\perp^2}{r} \qquad eB = \frac{m_e v_\perp}{r} = m_e\omega = m_e\frac{2\pi}{T} \qquad T = \frac{m_e 2\pi}{eB} = d\left(\frac{m_e}{2e\Delta V}\right)^{1/2}$$

Then $\frac{2\pi}{B}\left(\frac{m_e}{e}\right)^{1/2} = \frac{d}{(2\Delta V)^{1/2}}$ $\boxed{B = \frac{2\pi}{d}\left(\frac{2m_e\Delta V}{e}\right)^{1/2}}$.

***P29.60** Let $v_i$ represent the original speed of the alpha particle. Let $v_\alpha$ and $v_p$ represent the particles' speeds after the collision. We have conservation of momentum $4m_p v_i = 4m_p v_\alpha + m_p v_p$ and the relative velocity equation $v_i - 0 = v_p - v_\alpha$. Eliminating $v_i$,

$$4v_p - 4v_\alpha = 4v_\alpha + v_p \qquad 3v_p = 8v_\alpha \qquad v_\alpha = \frac{3}{8}v_p.$$

For the proton's motion in the magnetic field,

$$\sum F = ma \qquad ev_p B\sin 90° = \frac{m_p v_p^2}{R} \qquad \frac{eBR}{m_p} = v_p.$$

For the alpha particle,

$$2ev_\alpha B\sin 90° = \frac{4m_p v_\alpha^2}{r_\alpha} \qquad r_\alpha = \frac{2m_p v_\alpha}{eB} \qquad r_\alpha = \frac{2m_p}{eB}\frac{3}{8}v_p = \frac{2m_p}{eB}\frac{3}{8}\frac{eBR}{m_p} = \boxed{\frac{3}{4}R}.$$

**P29.61** Let $\Delta x_1$ be the elongation due to the weight of the wire and let $\Delta x_2$ be the additional elongation of the springs when the magnetic field is turned on. Then $F_{\text{magnetic}} = 2k\Delta x_2$ where $k$ is the force constant of the spring and can be determined from $k = \frac{mg}{2\Delta x_1}$. (The factor 2 is included in the two previous equations since there are 2 springs in parallel.) Combining these two equations, we find

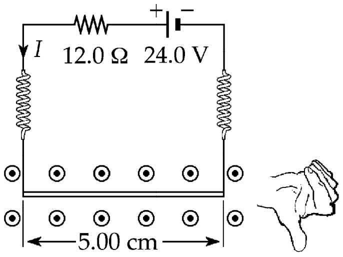


**FIG. P29.61**

$$F_{\text{magnetic}} = 2\left(\frac{mg}{2\Delta x_1}\right)\Delta x_2 = \frac{mg\Delta x_2}{\Delta x_1}; \text{ but } |\mathbf{F}_B| = I|\mathbf{L}\times\mathbf{B}| = ILB.$$

Therefore, where $I = \frac{24.0\text{ V}}{12.0\ \Omega} = 2.00\text{ A}$, $B = \frac{mg\Delta x_2}{IL\Delta x_1} = \frac{(0.100)(9.80)(3.00\times10^{-3})}{(2.00)(0.050\,0)(5.00\times10^{-3})} = \boxed{0.588\text{ T}}$.

**P29.62** Suppose the input power is

$120\ \text{W} = (120\ \text{V})I$: $\boxed{I \sim 1\ \text{A} = 10^0\ \text{A}}$.

Suppose $\omega = 2\,000\ \text{rev/min}\left(\dfrac{1\ \text{min}}{60\ \text{s}}\right)\left(\dfrac{2\pi\,\text{rad}}{1\ \text{rev}}\right) \sim 200\ \text{rad/s}$

and the output power is $20\ \text{W} = \tau\omega = \tau(200\ \text{rad/s})$ $\boxed{\tau \sim 10^{-1}\ \text{N}\cdot\text{m}}$.

Suppose the area is about $(3\ \text{cm}) \times (4\ \text{cm})$, or $\boxed{A \sim 10^{-3}\ \text{m}^2}$.

Suppose that the field is $\boxed{B \sim 10^{-1}\ \text{T}}$.

Then, the number of turns in the coil may be found from $\tau \cong NIAB$:

$$0.1\ \text{N}\cdot\text{m} \sim N(1\ \text{C/s})\left(10^{-3}\ \text{m}^2\right)\left(10^{-1}\ \text{N}\cdot\text{s/C}\cdot\text{m}\right)$$

giving $\boxed{N \sim 10^3}$.

***P29.63** The sphere is in translational equilibrium, thus

$f_s - Mg\sin\theta = 0$. (1)

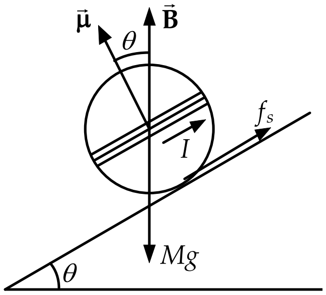


**FIG. P29.63**

The sphere is in rotational equilibrium. If torques are taken about the center of the sphere, the magnetic field produces a clockwise torque of magnitude $\mu B\sin\theta$, and the frictional force a counterclockwise torque of magnitude $f_sR$, where $R$ is the radius of the sphere. Thus:

$f_sR - \mu B\sin\theta = 0$. (2)

From (1): $f_s = Mg\sin\theta$. Substituting this in (2) and canceling out $\sin\theta$, one obtains

$\mu B = MgR$. (3)

Now $\mu = NI\pi R^2$. Thus (3) gives $I = \dfrac{Mg}{\pi NBR} = \dfrac{(0.08\ \text{kg})\left(9.80\ \text{m/s}^2\right)}{\pi(5)(0.350\ \text{T})(0.2\ \text{m})} = \boxed{0.713\ \text{A}}$. The current must be counterclockwise as seen from above.

**P29.64** Call the length of the rod $L$ and the tension in each wire alone $\dfrac{T}{2}$. Then, at equilibrium:

$\sum F_x = T\sin\theta - ILB\sin 90.0° = 0$ or $T\sin\theta = ILB$

$\sum F_y = T\cos\theta - mg = 0$, or $T\cos\theta = mg$

$\tan\theta = \dfrac{ILB}{mg} = \dfrac{IB}{(m/L)g}$ or $B = \dfrac{(m/L)g}{I}\tan\theta = \boxed{\dfrac{\lambda g}{I}\tan\theta}$

**P29.65** $\sum F = ma$ or $qvB\sin 90.0° = \dfrac{mv^2}{r}$

$\therefore$ the angular frequency for each ion is $\dfrac{v}{r} = \omega = \dfrac{qB}{m} = 2\pi f$ and

$$\Delta f = f_{12} - f_{14} = \frac{qB}{2\pi}\left(\frac{1}{m_{12}} - \frac{1}{m_{14}}\right) = \frac{\left(1.60 \times 10^{-19}\ \text{C}\right)(2.40\ \text{T})}{2\pi\left(1.66 \times 10^{-27}\ \text{kg/u}\right)}\left(\frac{1}{12.0\ \text{u}} - \frac{1}{14.0\ \text{u}}\right)$$

$\Delta f = f_{12} - f_{14} = 4.38 \times 10^5\ \text{s}^{-1} = \boxed{438\ \text{kHz}}$

**P29.66** Let $v_x$ and $v_\perp$ be the components of the velocity of the positron parallel to and perpendicular to the direction of the magnetic field.

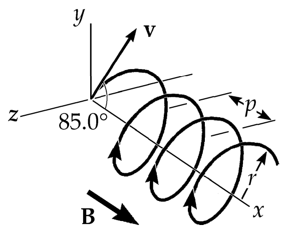


FIG. P29.66

(a) The pitch of trajectory is the distance moved along $x$ by the positron during each period, $T$ (see Equation 29.15)

$$p = v_x T = (v\cos 85.0°)\left(\frac{2\pi m}{Bq}\right)$$

$$p = \frac{\left(5.00\times 10^{6}\right)(\cos 85.0°)(2\pi)\left(9.11\times 10^{-31}\right)}{0.150\left(1.60\times 10^{-19}\right)} = \boxed{1.04\times 10^{-4}\text{ m}}$$

(b) From Equation 29.13, $r = \frac{mv_\perp}{Bq} = \frac{mv\sin 85.0°}{Bq}$

$$r = \frac{\left(9.11\times 10^{-31}\right)\left(5.00\times 10^{6}\right)(\sin 85.0°)}{(0.150)\left(1.60\times 10^{-19}\right)} = \boxed{1.89\times 10^{-4}\text{ m}}$$

**P29.67** $|\tau| = IAB$ where the effective current due to the orbiting electrons is $I = \frac{\Delta q}{\Delta t} = \frac{q}{T}$

and the period of the motion is $T = \frac{2\pi R}{v}$.

The electron's speed in its orbit is found by requiring $\frac{k_e q^2}{R^2} = \frac{mv^2}{R}$ or $v = q\sqrt{\frac{k_e}{mR}}$.

Substituting this expression for $v$ into the equation for $T$, we find $T = 2\pi\sqrt{\frac{mR^3}{q^2 k_e}}$

$$T = 2\pi\sqrt{\frac{\left(9.11\times 10^{-31}\right)\left(5.29\times 10^{-11}\right)^3}{\left(1.60\times 10^{-19}\right)^2\left(8.99\times 10^{9}\right)}} = 1.52\times 10^{-16}\text{ s}.$$

Therefore, $|\tau| = \left(\frac{q}{T}\right)AB = \frac{1.60\times 10^{-19}}{1.52\times 10^{-16}}\pi\left(5.29\times 10^{-11}\right)^2(0.400) = \boxed{3.70\times 10^{-24}\text{ N}\cdot\text{m}}$.

**P29.68** Use the equation for cyclotron frequency $\omega = \frac{qB}{m}$ or $m = \frac{qB}{\omega} = \frac{qB}{2\pi f}$

$$m = \frac{\left(1.60\times 10^{-19}\text{ C}\right)\left(5.00\times 10^{-2}\text{ T}\right)}{(2\pi)\left(5.00\text{ rev}/1.50\times 10^{-3}\text{ s}\right)} = \boxed{3.82\times 10^{-25}\text{ kg}}.$$

**P29.69** (a)

$$K=\frac{1}{2}mv^2=6.00\text{ MeV}=\left(6.00\times10^{6}\text{ eV}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)$$

$$K=9.60\times10^{-13}\text{ J}$$

$$v=\sqrt{\frac{2\left(9.60\times10^{-13}\text{ J}\right)}{1.67\times10^{-27}\text{ kg}}}=3.39\times10^{7}\text{ m/s}$$

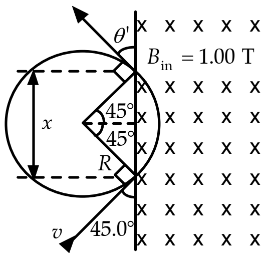


**FIG. P29.69**

$$F_B=qvB=\frac{mv^2}{R}\text{ so}$$

$$R=\frac{mv}{qB}=\frac{\left(1.67\times10^{-27}\text{ kg}\right)\left(3.39\times10^{7}\text{ m/s}\right)}{\left(1.60\times10^{-19}\text{ C}\right)(1.00\text{ T})}=0.354\text{ m}$$

Then, from the diagram, $x=2R\sin 45.0°=2(0.354\text{ m})\sin 45.0°=\boxed{0.501\text{ m}}$

(b) From the diagram, observe that $\theta'=\boxed{45.0°}$.

**P29.70** (a) See graph to the right. The Hall voltage is directly proportional to the magnetic field. A least-square fit to the data gives the equation of the best fitting line as:

$$\boxed{\Delta V_{\text{H}}=\left(1.00\times10^{-4}\text{ V/T}\right)B}.$$

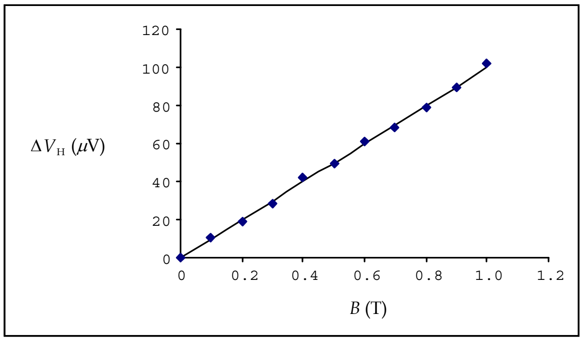


**FIG. P29.70**

(b) Comparing the equation of the line which fits the data best to

$$\Delta V_{\text{H}}=\left(\frac{1}{nqt}\right)B$$

observe that: $\frac{I}{nqt}=1.00\times10^{-4}\text{ V/T}$, or $t=\frac{I}{nq\left(1.00\times10^{-4}\text{ V/T}\right)}$.

Then, if $I=0.200\text{ A}$, $q=1.60\times10^{-19}\text{ C}$, and $n=1.00\times10^{26}\text{ m}^{-3}$, the thickness of the sample is

$$t=\frac{0.200\text{ A}}{\left(1.00\times10^{26}\text{ m}^{-3}\right)\left(1.60\times10^{-19}\text{ C}\right)\left(1.00\times10^{-4}\text{ V/T}\right)}=1.25\times10^{-4}\text{ m}=\boxed{0.125\text{ mm}}.$$

**P29.71** (a) The magnetic force acting on ions in the blood stream will deflect positive charges toward point *A* and negative charges toward point *B*. This separation of charges produces an electric field directed from *A* toward *B*. At equilibrium, the electric force caused by this field must balance the magnetic force, so

$$qvB = qE = q\left(\frac{\Delta V}{d}\right)$$

or $$v = \frac{\Delta V}{Bd} = \frac{\left(160 \times 10^{-6}\ \text{V}\right)}{(0.040\,0\ \text{T})\left(3.00 \times 10^{-3}\ \text{m}\right)} = \boxed{1.33\ \text{m/s}}.$$

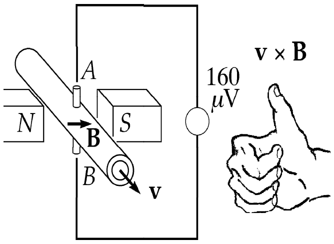


**FIG. P29.71**

(b) $\boxed{\text{No}}$. Negative ions moving in the direction of *v* would be deflected toward point *B*, giving *A* a higher potential than *B*. Positive ions moving in the direction of *v* would be deflected toward *A*, again giving *A* a higher potential than *B*. Therefore, the sign of the potential difference does not depend on whether the ions in the blood are positively or negatively charged.

**P29.72** When in the field, the particles follow a circular path according to $qvB = \frac{mv^2}{r}$, so the radius of the path is: $r = \frac{mv}{qB}$

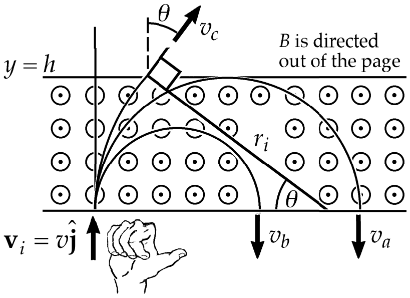

θ
$v_c$
B is directed out of the page
$y = h$
$r_i$
θ
$\mathbf{v}_i = v\hat{\mathbf{j}}$
$v_b$
$v_a$

**FIG. P29.72**

(a) When $r = h = \frac{mv}{qB}$, that is, when $\boxed{v = \frac{qBh}{m}}$, the particle will cross the band of field. It will move in a full semicircle of radius *h*, leaving the field at $(2h,\ 0,\ 0)$ with velocity $\boxed{\mathbf{v}_f = -v\hat{\mathbf{j}}}$.

(b) When $v < \frac{qBh}{m}$, the particle will move in a smaller $\boxed{\text{semicircle}}$ of radius $r = \frac{mv}{qB} < h$. It will leave the field at $(2r,\ 0,\ 0)$ with velocity $\boxed{\mathbf{v}_f = -v\hat{\mathbf{j}}}$.

(c) When $v > \frac{qBh}{m}$, the particle moves in a $\boxed{\text{circular arc}}$ of radius $r = \frac{mv}{qB} > h$, centered at $(r,\ 0,\ 0)$. The arc subtends an angle given by $\theta = \sin^{-1}\left(\frac{h}{r}\right)$. It will leave the field at the point with coordinates $\left[r(1-\cos\theta),\ h,\ 0\right]$ with velocity $\boxed{\mathbf{v}_f = v\sin\theta\,\hat{\mathbf{i}} + v\cos\theta\,\hat{\mathbf{j}}}$.

## ANSWERS TO EVEN PROBLEMS

**P29.2** (a) west; (b) no deflection; (c) up; (d) down

**P29.4** (a) 86.7 fN; (b) 51.9 $\text{Tm/s}^2$

**P29.6** (a) 7.90 pN; (b) 0

**P29.8** Gravitational force: $8.93 \times 10^{-30}$ N down;
Electric force: 16.0 aN up;
Magnetic force: 48.0 aN down

**P29.10** $B_y = -2.62$ mT; $B_z = 0$; $B_x$ may have any value

| | |
|---|---|
| **P29.12** | $\left(-2.88\hat{\mathbf{j}}\right)$ N |
| **P29.14** | 109 mA to the right |
| **P29.16** | $\left(\frac{4IdBL}{3m}\right)^{1/2}$ |
| **P29.18** | $\mathbf{F}_{ab} = 0$; $\mathbf{F}_{bc} = 40.0\ \text{mN}\left(-\hat{\mathbf{i}}\right)$;<br>$\mathbf{F}_{cd} = 40.0\ \text{mN}\left(-\hat{\mathbf{k}}\right)$; $\mathbf{F}_{da} = (40.0\ \text{mN})\left(\hat{\mathbf{i}} + \hat{\mathbf{k}}\right)$ |
| **P29.20** | (a) $5.41\ \text{mA}\cdot\text{m}^2$; (b) $4.33\ \text{mN}\cdot\text{m}$ |
| **P29.22** | (a) $3.97°$; (b) $3.39\ \text{mN}\cdot\text{m}$ |
| **P29.24** | (a) $80.1\ \text{mN}\cdot\text{m}$; (b) $104\ \text{mN}\cdot\text{m}$;<br>(c) $132\ \text{mN}\cdot\text{m}$;<br>(d) The torque on the circle. |
| **P29.26** | (a) minimum: pointing north at 48.0° below the horizontal; maximum: pointing south at 48.0° above the horizontal;<br>(b) 1.07 $\mu$J |
| **P29.28** | (a) 640 $\mu\text{N}\cdot\text{m}$; (b) 241 mW; (c) 2.56 mJ;<br>(d) 154 mW |
| **P29.30** | 1.98 cm |
| **P29.32** | 65.6 mT |
| **P29.34** | (a) 5.00 cm; (b) 8.78 Mm/s |
| **P29.36** | $\frac{m'}{m} = 8$ |
| **P29.38** | see the solution |
| **P29.40** | 244 kV/m |
| **P29.42** | 278 mm |
| **P29.44** | 162 mm |
| **P29.46** | 3.00 T |
| **P29.48** | (a) $7.44 \times 10^{28}/\text{m}^3$; (b) 1.79 T |
| **P29.50** | (a) 37.7 mT; (b) $4.29 \times 10^{25}/\text{m}^3$ |
| **P29.52** | (a) 17.9 ns; (b) 35.1 eV |
| **P29.54** | 39.2 mT |
| **P29.56** | (a) $B_x$ is indeterminate. $B_y = 0$; $B_z = \frac{-F_i}{ev_i}$;<br>(b) $-F_i\hat{\mathbf{j}}$; (c) $-F_i\hat{\mathbf{j}}$ |
| **P29.58** | 128 mT north at an angle of 78.7° below the horizontal |
| **P29.60** | $\frac{3R}{4}$ |
| **P29.62** | $B \sim 10^{-1}$ T; $\tau \sim 10^{-1}\ \text{N}\cdot\text{m}$; $I \sim 1$ A;<br>$A \sim 10^{-3}\ \text{m}^2$; $N \sim 10^3$ |
| **P29.64** | $\frac{\lambda g \tan\theta}{I}$ |
| **P29.66** | (a) 0.104 mm; (b) 0.189 mm |
| **P29.68** | $3.82 \times 10^{-25}$ kg |
| **P29.70** | (a) see the solution;<br>empirically, $\Delta V_{\text{H}} = (100\ \mu\text{V/T})B$;<br>(b) 0.125 mm |
| **P29.72** | (a) $v = \frac{qBh}{m}$; The particle moves in a semicircle of radius $h$ and leaves the field with velocity $-v\hat{\mathbf{j}}$;<br>(b) The particle moves in a smaller semicircle of radius $\frac{mv}{qB}$, attaining final velocity $-v\hat{\mathbf{j}}$;<br>(c) The particle moves in a circular arc of radius $r = \frac{mv}{qB}$, leaving the field with velocity $v\sin\theta\,\hat{\mathbf{i}} + v\cos\theta\,\hat{\mathbf{j}}$ where $\theta = \sin^{-1}\left(\frac{h}{r}\right)$ |

# 30

# Sources of the Magnetic Field

## CHAPTER OUTLINE

30.1 The Biot-Savart Law
30.2 The Magnetic Force Between Two Parallel Conductors
30.3 Ampère's Law
30.4 The Magnetic Field of a Solenoid
30.5 Magnetic Flux
30.6 Gauss's Law in Magnetism
30.7 Displacement Current and the General Form of Ampère's Law
30.8 Magnetism in Matter
30.9 The Magnetic Field of the Earth

## ANSWERS TO QUESTIONS

**Q30.1** It is not. The magnetic field created by a single loop of current resembles that of a bar magnet—strongest inside the loop, and decreasing in strength as you move away from the loop. Neither is it in a uniform direction—the magnetic field lines loop though the loop!

**Q30.2** No magnetic field is created by a stationary charge, as the rate of flow is zero. A moving charge creates a magnetic field.

**Q30.3** The magnetic field created by wire 1 at the position of wire 2 is into the paper. Hence, the magnetic force on wire 2 is in direction down × into the paper = to the right, away from wire 1. Now wire 2 creates a magnetic field into the page at the location of wire 1, so wire 1 feels force up ×into the paper = left, away from wire 2.

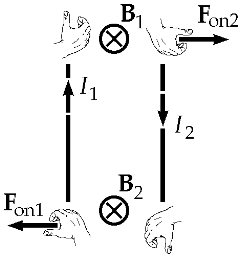


FIG. Q30.3

**Q30.4** No total force, but a torque. Let wire one carry current in the $y$ direction, toward the top of the page. Let wire two be a millimeter above the plane of the paper and carry current to the right, in the $x$ direction. On the left-hand side of wire one, wire one creates magnetic field in the $z$ direction, which exerts force in the $\hat{\mathbf{i}} \times \hat{\mathbf{k}} = -\hat{\mathbf{j}}$ direction on wire two. On the right-hand side, wire one produces magnetic field in the $-\hat{\mathbf{k}}$ direction and makes a $\hat{\mathbf{i}} \times \left(-\hat{\mathbf{k}}\right) = +\hat{\mathbf{j}}$ force of equal magnitude act on wire two. If wire two is free to move, its center section will twist counterclockwise and then be attracted to wire one.

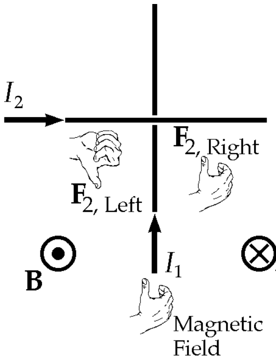


FIG. Q30.4

**Q30.5** Ampère's law is valid for all closed paths surrounding a conductor, but not always convenient. There are many paths along which the integral is cumbersome to calculate, although not impossible. Consider a circular path around but *not* coaxial with a long, straight current-carrying wire.

**Q30.6** The Biot-Savart law considers the contribution of each element of current in a conductor to determine the magnetic field, while for Ampère's law, one need only know the current passing through a given surface. Given situations of high degrees of symmetry, Ampère's law is more convenient to use, even though both laws are equally valid in all situations.

**Q30.7** If the radius of the toroid is very large compared to its cross-sectional area, then the field is nearly uniform. If not, as in many transformers, it is not.

**Q30.8** Both laws use the concept of flux—the "flow" of field lines through a surface to determine the field strength. They also both relate the integral of the field over a closed geometrical figure to a fundamental constant multiplied by the source of the appropriate field. The geometrical figure is a surface for Gauss's law and a line for Ampère's.

**Q30.9** Apply Ampère's law to the circular path labeled 1 in the picture. Since there is no current inside this path, the magnetic field inside the tube must be zero. On the other hand, the current through path 2 is the current carried by the conductor. Therefore the magnetic field outside the tube is nonzero.

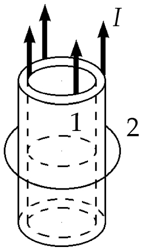


FIG. Q30.9

**Q30.10** The magnetic field inside a long solenoid is given by $B = \frac{\mu_0 N I}{\ell}$.

(a) If the length $\ell$ is doubled, the field is cut in half.

(b) If $N$ is doubled, the magnetic field is doubled.

**Q30.11** The magnetic flux is $\Phi_B = BA\cos\theta$. Therefore the flux is maximum when **B** is perpendicular to the loop of wire. The flux is zero when there is no component of magnetic field perpendicular to the loop—that is, when the plane of the loop contains the $x$ axis.

**Q30.12** Maxwell included a term in Ampère's law to account for the contributions to the magnetic field by changing electric fields, by treating those changing electric fields as "displacement currents."

**Q30.13** **M** measures the intrinsic magnetic field in the nail. Unless the nail was previously magnetized, then **M** starts out from zero. **H** is due to the current in the coil of wire around the nail. **B** is related to the sum of **M** and **H**. If the nail is aluminum or copper, **H** makes the dominant contribution to **B**, but **M** can add a little in the same or in the opposite direction. If the nail is iron, as it becomes magnetized **M** can become the dominant contributor to **B**.

**Q30.14** Magnetic domain alignment creates a stronger external magnetic field. The field of one piece of iron in turn can align domains in another iron sample. A nonuniform magnetic field exerts a net force of attraction on magnetic dipoles aligned with the field.

**Q30.15** The shock misaligns the domains. Heating will also decrease magnetism.

**Q30.16** Magnetic levitation is illustrated in Figure Q30.31. The Earth's magnetic field is so weak that the floor of his tomb should be magnetized as well as his coffin. Alternatively, the floor of his tomb could be made of superconducting material, which exerts a force of repulsion on any magnet.

**Q30.17** There is no magnetic material in a vacuum, so **M** must be zero. Therefore $\mathbf{B} = \mu_0 \mathbf{H}$ in a vacuum.

**Q30.18** Atoms that do not have a permanent magnetic dipole moment have electrons with spin and orbital magnetic moments that add to zero as vectors. Atoms with a permanent dipole moment have electrons with orbital and spin magnetic moments that show some net alignment.

**Q30.19** The magnetic dipole moment of an atom is the sum of the dipole moments due to the electrons' orbital motions and the dipole moments due to the spin of the electrons.

**Q30.20** **M** and **H** are in opposite directions. Section 30.8 argues that all atoms should be thought of as weakly diamagnetic due to the effect of an external magnetic field on the motions of atomic electrons. Paramagnetic and ferromagnetic effects dominate whenever they exist.

**Q30.21** The effects of diamagnetism are significantly smaller than those of paramagnetism.

**Q30.22** When the substance is above the Curie temperature, random thermal motion of the molecules prevents the formation of domains. It cannot be ferromagnetic, but only paramagnetic.

**Q30.23** A ferromagnetic substance is one in which the magnetic moments of the atoms are aligned within domains, and can be aligned macroscopically. A paramagnetic substance is one in which the magnetic moments are not naturally aligned, but when placed in an external magnetic field, the molecules line their magnetic moments up with the external field. A diamagnetic material is one in which the magnetic moments are also not naturally aligned, but when placed in an external magnetic field, the molecules line up to oppose the external magnetic field.

**Q30.24** (a) **B** increases slightly

(b) **B** decreases slightly

(c) **B** increases significantly

Equations 30.33 and 30.34 indicate that, when each metal is in the solenoid, the total field is $\mathbf{B} = \mu_0(1+\chi)\mathbf{H}$. Table 30.2 indicates that **B** is slightly greater than $\mu_0\mathbf{H}$ for aluminum and slightly less for copper. For iron, the field can be made thousands of times stronger, as seen in Example 30.10.

**Q30.25** A "hard" ferromagnetic material requires much more energy per molecule than a "soft" ferromagnetic material to change the orientation of the magnetic dipole moments. This way, a hard ferromagnetic material is more likely to retain its magnetization than a soft ferromagnetic material.

**Q30.26** The medium for any magnetic recording should be a hard ferromagnetic substance, so that thermal vibrations and stray magnetic fields will not rapidly erase the information.

**Q30.27** If a soft ferromagnetic substance were used, then the magnet would not be "permanent." Any significant shock, a heating/cooling cycle, or just rotating the magnet in the Earth's magnetic field would decrease the overall magnetization by randomly aligning some of the magnetic dipole moments.

**Q30.28** You can expect a magnetic tape to be weakly attracted to a magnet. Before you erase the information on the tape, the net magnetization of a macroscopic section of the tape would be nearly zero, as the different domains on the tape would have opposite magnetization, and be more or less equal in number and size. Once your external magnet aligns the magnetic moments on the tape, there would be a weak attraction, but not like that of picking up a paper clip with a magnet. A majority of the mass of the tape is non-magnetic, and so the gravitational force acting on the tape will likely be larger than the magnetic attraction.

**Q30.29** To magnetize the screwdriver, stroke one pole of the magnet along the blade of the screwdriver several or many times. To demagnetize the screwdriver, drop it on a hard surface a few times, or heat it to some high temperature.

**Q30.30** The north magnetic pole is near the south geographic pole. Straight up.

**Q30.31** (a) The magnets repel each other with a force equal to the weight of one of them.

(b) The pencil prevents motion to the side and prevents the magnets from rotating under their mutual torques. Its constraint changes unstable equilibrium into stable.

(c) Most likely, the disks are magnetized perpendicular to their flat faces, making one face a north pole and the other a south pole. One disk has its north pole on the top side and the other has its north pole on the bottom side.

(d) Then if either were inverted they would attract each other and stick firmly together.

## SOLUTIONS TO PROBLEMS

### Section 30.1 **The Biot-Savart Law**

**P30.1**

$$B=\frac{\mu_0 I}{2R}=\frac{\mu_0 q(v/2\pi R)}{2R}=\boxed{12.5\ \mathrm{T}}$$

**P30.2**

$$B=\frac{\mu_0 I}{2\pi R}=\frac{\left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)\left(1.00\times10^{4}\ \mathrm{A}\right)}{2\pi(100\ \mathrm{m})}=2.00\times10^{-5}\ \mathrm{T}=\boxed{20.0\ \mu\mathrm{T}}$$

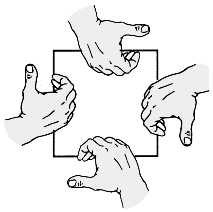


**FIG. P30.3**

**P30.3** (a) $$B = \frac{4\mu_0 I}{4\pi a}\left(\cos\frac{\pi}{4} - \cos\frac{3\pi}{4}\right) \text{ where } a = \frac{\ell}{2}$$

is the distance from any side to the center.

$$B = \frac{4.00\times10^{-6}}{0.200}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) = 2\sqrt{2}\times10^{-5}\text{ T} = \boxed{28.3\ \mu\text{T into the paper}}$$

(b) For a single circular turn with $4\ell = 2\pi R$,

$$B = \frac{\mu_0 I}{2R} = \frac{\mu_0 \pi I}{4\ell} = \frac{\left(4\pi^2\times10^{-7}\right)(10.0)}{4(0.400)} = \boxed{24.7\ \mu\text{T into the paper}}$$

**P30.4** $$B = \frac{\mu_0 I}{2\pi r} = \frac{\left(4\pi\times10^{-7}\right)(1.00\text{ A})}{2\pi(1.00\text{ m})} = \boxed{2.00\times10^{-7}\text{ T}}$$

**P30.5** For leg 1, $d\mathbf{s}\times\hat{\mathbf{r}} = 0$, so there is no contribution to the field from this segment. For leg 2, the wire is only semi-infinite; thus,

$$B = \frac{1}{2}\left(\frac{\mu_0 I}{2\pi x}\right) = \boxed{\frac{\mu_0 I}{4\pi x}\text{ into the paper}}.$$

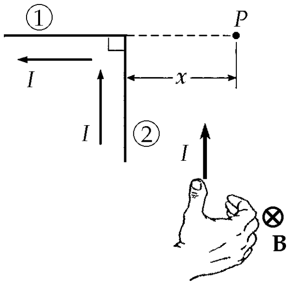


**FIG. P30.5**

**P30.6** We can think of the total magnetic field as the superposition of the field due to the long straight wire (having magnitude $\frac{\mu_0 I}{2\pi R}$ and directed into the page) and the field due to the circular loop (having magnitude $\frac{\mu_0 I}{2R}$ and directed into the page). The resultant magnetic field is:

$$\boxed{\mathbf{B} = \left(1 + \frac{1}{\pi}\right)\frac{\mu_0 I}{2R}\text{ (directed into the page)}}.$$

**P30.7** For the straight sections $d\mathbf{s}\times\hat{\mathbf{r}} = 0$. The quarter circle makes one-fourth the field of a full loop:

$$\mathbf{B} = \frac{1}{4}\frac{\mu_0 I}{2R} = \frac{\mu_0 I}{8R}\text{ into the paper} \qquad \mathbf{B} = \frac{\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)(5.00\text{ A})}{8(0.030\,0\text{ m})} = \boxed{26.2\ \mu\text{T into the paper}}$$

**P30.8** Along the axis of a circular loop of radius $R$,

$$B=\frac{\mu_0 IR^2}{2\left(x^2+R^2\right)^{3/2}}$$

or $$\frac{B}{B_0}=\left[\frac{1}{(x/R)^2+1}\right]^{3/2}$$

where $$B_0\equiv\frac{\mu_0 I}{2R}.$$

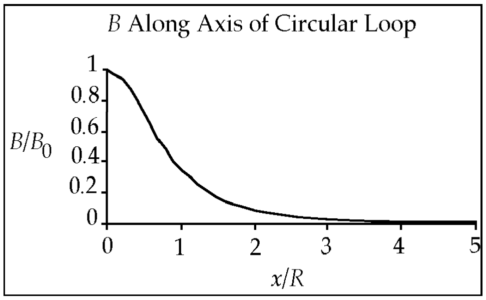


**FIG. P30.8**

| $x/R$ | $B/B_0$ |
|---|---|
| 0.00 | 1.00 |
| 1.00 | 0.354 |
| 2.00 | 0.0894 |
| 3.00 | 0.0316 |
| 4.00 | 0.0143 |
| 5.00 | 0.00754 |

***P30.9** Wire 1 creates at the origin magnetic field

$\mathbf{B}_1=\frac{\mu_0 I}{2\pi r}$ right hand rule $=\frac{\mu_0 I_1}{2\pi a}$ $=\frac{\mu_0 I_1}{2\pi a}\hat{\mathbf{j}}$

(a) If the total field at the origin is $\frac{2\mu_0 I_1}{2\pi a}\hat{\mathbf{j}}=\frac{\mu_0 I_1}{2\pi a}\hat{\mathbf{j}}+\mathbf{B}_2$ then the second wire must create field according to $\mathbf{B}_2=\frac{\mu_0 I_1}{2\pi a}\hat{\mathbf{j}}=\frac{\mu_0 I_2}{2\pi(2a)}$ .

Then $I_2=\boxed{2I_1 \text{ out of the paper}}=2I_1\hat{\mathbf{k}}$.

(b) The other possibility is $\mathbf{B}_1+\mathbf{B}_2=\frac{2\mu_0 I_1}{2\pi a}\left(-\hat{\mathbf{j}}\right)=\frac{\mu_0 I_1}{2\pi a}\hat{\mathbf{j}}+\mathbf{B}_2$. Then

$\mathbf{B}_2=\frac{3\mu_0 I_1}{2\pi a}\left(-\hat{\mathbf{j}}\right)=\frac{\mu_0 I_2}{2\pi(2a)}$ $\quad I_2=\boxed{6I_1 \text{ into the paper}}=6I_1\left(-\hat{\mathbf{k}}\right)$

***P30.10** Every element of current creates magnetic field in the same direction, into the page, at the center of the arc. The upper straight portion creates one-half of the field that an infinitely long straight wire would create. The curved portion creates one quarter of the field that a circular loop produces at its center. The lower straight segment also creates field $\frac{1}{2}\frac{\mu_0 I}{2\pi r}$.

The total field is

$$\mathbf{B}=\left(\frac{1}{2}\frac{\mu_0 I}{2\pi r}+\frac{1}{4}\frac{\mu_0 I}{2r}+\frac{1}{2}\frac{\mu_0 I}{2\pi r}\right)\text{ into the page}=\boxed{\frac{\mu_0 I}{2r}\left(\frac{1}{\pi}+\frac{1}{4}\right)\text{ into the plane of the paper}}$$

$$=\left(\frac{0.284\,15\mu_0 I}{r}\right)\text{ into the page.}$$

***P30.11** (a) Above the pair of wires, the field out of the page of the 50 A current will be stronger than the $\left(-\hat{\mathbf{k}}\right)$ field of the 30 A current, so they cannot add to zero. Between the wires, both produce fields into the page. They can only add to zero below the wires, at coordinate $y=-|y|$. Here the total field is

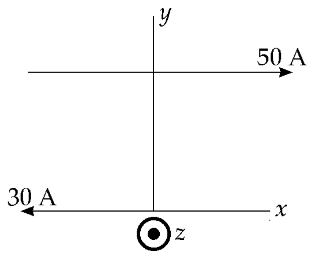


**FIG. P30.11**

$$\mathbf{B}=\frac{\mu_0 I}{2\pi r}\ \text{☞}\ +\frac{\mu_0 I}{2\pi r}\ \text{☜}\ :$$

$$0=\frac{\mu_0}{2\pi}\left[\frac{50\text{ A}}{\left(|y|+0.28\text{ m}\right)}\left(-\hat{\mathbf{k}}\right)+\frac{30\text{ A}}{|y|}\left(\hat{\mathbf{k}}\right)\right]$$

$$50|y|=30\left(|y|+0.28\text{ m}\right)$$

$$50(-y)=30(0.28\text{ m}-y)$$

$$-20y=30(0.28\text{ m})\ \boxed{\text{at } y=-0.420\text{ m}}$$

(b) At $y=0.1$ m the total field is $\mathbf{B}=\frac{\mu_0 I}{2\pi r}\ \text{☞}\ +\frac{\mu_0 I}{2\pi r}\ \text{☜}\ :$

$$\mathbf{B}=\frac{4\pi\times10^{-7}\text{ T}\cdot\text{m/A}}{2\pi}\left(\frac{50\text{ A}}{(0.28-0.10)\text{ m}}\left(-\hat{\mathbf{k}}\right)+\frac{30\text{ A}}{0.10\text{ m}}\left(-\hat{\mathbf{k}}\right)\right)=1.16\times10^{-4}\text{ T}\left(-\hat{\mathbf{k}}\right).$$

The force on the particle is

$$\mathbf{F}=q\mathbf{v}\times\mathbf{B}=\left(-2\times10^{-6}\text{ C}\right)\left(150\times10^{6}\text{ m/s}\right)\left(\hat{\mathbf{i}}\right)\times\left(1.16\times10^{-4}\text{ N}\cdot\text{s/C}\cdot\text{m}\right)\left(-\hat{\mathbf{k}}\right)=\boxed{3.47\times10^{-2}\text{ N}\left(-\hat{\mathbf{j}}\right)}.$$

(c) We require $\mathbf{F}_e=3.47\times10^{-2}\text{ N}\left(+\hat{\mathbf{j}}\right)=q\mathbf{E}=\left(-2\times10^{-6}\text{ C}\right)\mathbf{E}$.

So $\mathbf{E}=\boxed{-1.73\times10^{4}\hat{\mathbf{j}}\text{ N/C}}$.

**P30.12** $dB=\frac{\mu_0 I}{4\pi}\frac{|d\ell\times\hat{\mathbf{r}}|}{r^2}$

$$B=\frac{\mu_0 I}{4\pi}\left(\frac{\frac{1}{6}2\pi a}{a^2}-\frac{\frac{1}{6}2\pi b}{b^2}\right)$$

$$\mathbf{B}=\boxed{\frac{\mu_0 I}{12}\left(\frac{1}{a}-\frac{1}{b}\right)\text{ directed out of the paper}}$$

***P30.13** (a) We use equation 30.4. For the distance $a$ from the wire to the field point we have $\tan 30° = \frac{a}{L/2}$, $a = 0.288\,7L$. One wire contributes to the field at $P$

$$B = \frac{\mu_0 I}{4\pi a}(\cos\theta_1 - \cos\theta_2) = \frac{\mu_0 I}{4\pi(0.288\,7L)}(\cos 30° - \cos 150°)$$

$$= \frac{\mu_0 I(1.732)}{4\pi(0.288\,7L)} = \frac{1.50\mu_0 I}{\pi L}.$$

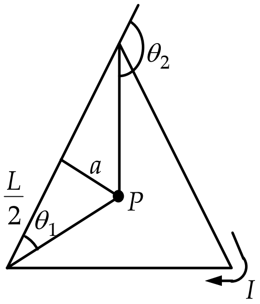


**FIG. P30.13(a)**

Each side contributes the same amount of field in the same direction, which is perpendicularly into the paper in the picture. So the total field is $3\left(\frac{1.50\mu_0 I}{\pi L}\right) = \boxed{\frac{4.50\mu_0 I}{\pi L}}$.

(b) As we showed in part (a), one whole side of the triangle creates field at the center $\frac{\mu_0 I(1.732)}{4\pi a}$. Now one-half of one nearby side of the triangle will be half as far away from point $P_b$ and have a geometrically similar situation. Then it creates at $P_b$ field $\frac{\mu_0 I(1.732)}{4\pi(a/2)} = \frac{2\mu_0 I(1.732)}{4\pi a}$. The two half-sides shown crosshatched in the picture create at $P_b$ field $2\left(\frac{2\mu_0 I(1.732)}{4\pi a}\right) = \frac{4\mu_0 I(1.732)}{4\pi(0.288\,7L)} = \frac{6\mu_0 I}{\pi L}$. The rest of the triangle will contribute somewhat more field in the same direction, so we already have a proof that the field at $P_b$ is $\boxed{\text{stronger}}$.

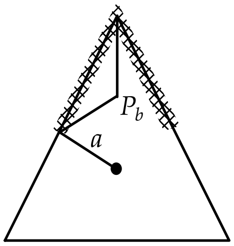


**FIG. P30.13(b)**

**P30.14** Apply Equation 30.4 three times:

$$\mathbf{B} = \frac{\mu_0 I}{4\pi a}\left(\cos 0 - \frac{d}{\sqrt{d^2+a^2}}\right)\text{ toward you} + \frac{\mu_0 I}{4\pi d}\left(\frac{a}{\sqrt{d^2+a^2}} + \frac{a}{\sqrt{d^2+a^2}}\right)\text{ away from you}$$

$$+\frac{\mu_0 I}{4\pi a}\left(\frac{-d}{\sqrt{d^2+a^2}} - \cos 180°\right)\text{ toward you}$$

$$\mathbf{B} = \boxed{\frac{\mu_0 I\left(a^2+d^2-d\sqrt{a^2+d^2}\right)}{2\pi a d\sqrt{a^2+d^2}}\text{ away from you}}$$

**P30.15** Take the $x$-direction to the right and the $y$-direction up in the plane of the paper. Current 1 creates at $P$ a field

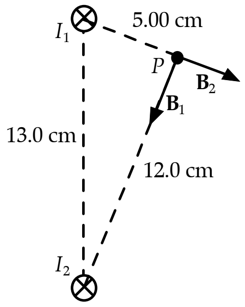


**FIG. P30.15**

$$B_1=\frac{\mu_0 I}{2\pi a}=\frac{\left(2.00\times10^{-7}\ \text{T}\cdot\text{m}\right)(3.00\ \text{A})}{\text{A}(0.050\,0\ \text{m})}$$

$\mathbf{B}_1=12.0\ \mu\text{T}$ downward and leftward, at angle 67.4° below the $-x$ axis.

Current 2 contributes

$$B_2=\frac{\left(2.00\times10^{-7}\ \text{T}\cdot\text{m}\right)(3.00\ \text{A})}{\text{A}(0.120\ \text{m})}\ \text{clockwise perpendicular to 12.0 cm}$$

$\mathbf{B}_2=5.00\ \mu\text{T}$ to the right and down, at angle $-22.6°$

Then, $\mathbf{B}=\mathbf{B}_1+\mathbf{B}_2=(12.0\ \mu\text{T})\left(-\hat{\mathbf{i}}\cos67.4°-\hat{\mathbf{j}}\sin67.4°\right)+(5.00\ \mu\text{T})\left(\hat{\mathbf{i}}\cos22.6°-\hat{\mathbf{j}}\sin22.6°\right)$

$$\mathbf{B}=(-11.1\ \mu\text{T})\hat{\mathbf{j}}-(1.92\ \mu\text{T})\hat{\mathbf{j}}=\boxed{(-13.0\ \mu\text{T})\hat{\mathbf{j}}}$$

## Section 30.2 **The Magnetic Force Between Two Parallel Conductors**

**P30.16** Let both wires carry current in the $x$ direction, the first at $y=0$ and the second at $y=10.0$ cm.

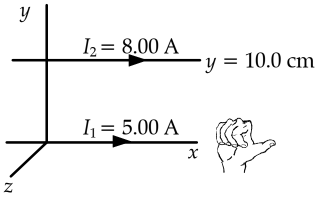


**FIG. P30.16(a)**

(a) $$\mathbf{B}=\frac{\mu_0 I}{2\pi r}\hat{\mathbf{k}}=\frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)(5.00\ \text{A})}{2\pi(0.100\ \text{m})}\hat{\mathbf{k}}$$

$$\mathbf{B}=\boxed{1.00\times10^{-5}\ \text{T out of the page}}$$

(b) $$\mathbf{F}_B=I_2\boldsymbol{\ell}\times\mathbf{B}=(8.00\ \text{A})\left[(1.00\ \text{m})\hat{\mathbf{i}}\times\left(1.00\times10^{-5}\ \text{T}\right)\hat{\mathbf{k}}\right]=\left(8.00\times10^{-5}\ \text{N}\right)\left(-\hat{\mathbf{j}}\right)$$

$$\boxed{\mathbf{F}_B=8.00\times10^{-5}\ \text{N toward the first wire}}$$

(c) $$\mathbf{B}=\frac{\mu_0 I}{2\pi r}\left(-\hat{\mathbf{k}}\right)=\frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)(8.00\ \text{A})}{2\pi(0.100\ \text{m})}\left(-\hat{\mathbf{k}}\right)=\left(1.60\times10^{-5}\ \text{T}\right)\left(-\hat{\mathbf{k}}\right)$$

$$\mathbf{B}=\boxed{1.60\times10^{-5}\ \text{T into the page}}$$

(d) $$\mathbf{F}_B=I_1\boldsymbol{\ell}\times\mathbf{B}=(5.00\ \text{A})\left[(1.00\ \text{m})\hat{\mathbf{i}}\times\left(1.60\times10^{-5}\ \text{T}\right)\left(-\hat{\mathbf{k}}\right)\right]=\left(8.00\times10^{-5}\ \text{N}\right)\left(+\hat{\mathbf{j}}\right)$$

$$\mathbf{F}_B=\boxed{8.00\times10^{-5}\ \text{N towards the second wire}}$$

**P30.17** By symmetry, we note that the magnetic forces on the top and bottom segments of the rectangle cancel. The net force on the vertical segments of the rectangle is (using Equation 30.11)

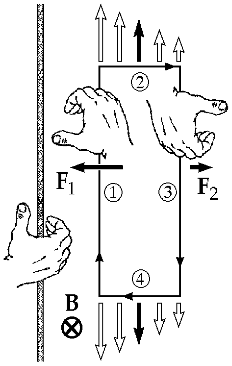


**FIG. P30.17**

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 = \frac{\mu_0 I_1 I_2 \ell}{2\pi}\left(\frac{1}{c+a} - \frac{1}{c}\right)\hat{\mathbf{i}} = \frac{\mu_0 I_1 I_2 \ell}{2\pi}\left(\frac{-a}{c(c+a)}\right)\hat{\mathbf{i}}$$

$$\mathbf{F} = \frac{\left(4\pi\times10^{-7}\ \mathrm{N/A^2}\right)(5.00\ \mathrm{A})(10.0\ \mathrm{A})(0.450\ \mathrm{m})}{2\pi}\left(\frac{-0.150\ \mathrm{m}}{(0.100\ \mathrm{m})(0.250\ \mathrm{m})}\right)\hat{\mathbf{i}}$$

$$\mathbf{F} = \left(-2.70\times10^{-5}\,\hat{\mathbf{i}}\right)\ \mathrm{N}$$

or $\mathbf{F} = \boxed{2.70\times10^{-5}\ \text{N toward the left}}$.

***P30.18** To attract, both currents must be to the right. The attraction is described by

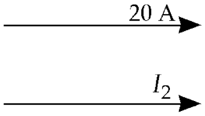


**FIG. P30.18**

$$F = I_2 \ell B \sin 90° = I_2 \ell \frac{\mu_0 I}{2\pi r}$$

So $$I_2 = \frac{F}{\ell}\frac{2\pi r}{\mu_0 I_1} = \left(320\times10^{-6}\ \mathrm{N/m}\right)\left(\frac{2\pi(0.5\ \mathrm{m})}{\left(4\pi\times10^{-7}\ \mathrm{N\cdot s/C\cdot m}\right)(20\ \mathrm{A})}\right) = 40.0\ \mathrm{A}$$

Let $y$ represent the distance of the zero-field point below the upper wire.

Then $\mathbf{B} = \frac{\mu_0 I}{2\pi r}$ $+ \frac{\mu_0 I}{2\pi r}$ $\quad 0 = \frac{\mu_0}{2\pi}\left(\frac{20\ \mathrm{A}}{y}(\text{away}) + \frac{40\ \mathrm{A}}{(0.5\ \mathrm{m} - y)}(\text{toward})\right)$

$20(0.5\ \mathrm{m} - y) = 40y \qquad 20(0.5\ \mathrm{m}) = 60y$

$y = \boxed{0.167\ \text{m below the upper wire}}$

***P30.19** Carrying oppositely directed currents, wires 1 and 2 repel each other. If wire 3 were between them, it would have to repel either 1 or 2, so the force on that wire could not be zero. If wire 3 were to the right of wire 2, it would feel a larger force exerted by 2 than that exerted by 1, so the total force on 3 could not be zero. Therefore wire 3 must be to the left of both other wires as shown. It must carry downward current so that it can attract wire 2.

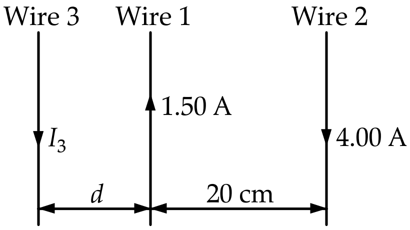


**FIG. P30.19**

(a) For the equilibrium of wire 3 we have

$F_{1\text{ on }3} = F_{2\text{ on }3} \qquad \frac{\mu_0(1.50\ \mathrm{A})I_3}{2\pi d} = \frac{\mu_0(4\ \mathrm{A})I_3}{2\pi(20\ \mathrm{cm} + d)}$

$1.5(20\ \mathrm{cm} + d) = 4d \qquad d = \frac{30\ \mathrm{cm}}{2.5} = \boxed{12.0\ \text{cm to the left of wire 1}}$

(b) For the equilibrium of wire 1,

$\frac{\mu_0 I_3(1.5\ \mathrm{A})}{2\pi(12\ \mathrm{cm})} = \frac{\mu_0(4\ \mathrm{A})(1.5\ \mathrm{A})}{2\pi(20\ \mathrm{cm})} \qquad I_3 = \frac{12}{20}4\ \mathrm{A} = \boxed{2.40\ \text{A down}}$

We know that wire 2 must be in equilibrium because the forces on it are equal in magnitude to the forces that it exerts on wires 1 and 3, which are equal because they both balance the equal-magnitude forces that 1 exerts on 3 and that 3 exerts on 1.

**P30.20** The separation between the wires is

$a = 2(6.00\text{ cm})\sin 8.00° = 1.67\text{ cm}.$

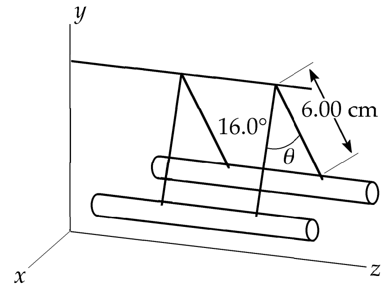


**FIG. P30.20**

(a) Because the wires repel, the currents are in $\boxed{\text{opposite directions}}$.

(b) Because the magnetic force acts horizontally,

$$\frac{F_B}{F_g} = \frac{\mu_0 I^2 \ell}{2\pi a m g} = \tan 8.00°$$

$$I^2 = \frac{mg 2\pi a}{\ell \mu_0}\tan 8.00° \text{ so } I = \boxed{67.8\text{ A}}.$$

---

Section 30.3 **Ampère's Law**

**P30.21** Each wire is distant from *P* by

$(0.200\text{ m})\cos 45.0° = 0.141\text{ m}.$

Each wire produces a field at *P* of equal magnitude:

$$B_A = \frac{\mu_0 I}{2\pi a} = \frac{\left(2.00\times10^{-7}\text{ T}\cdot\text{m/A}\right)(5.00\text{ A})}{(0.141\text{ m})} = 7.07\ \mu\text{T}.$$

Carrying currents into the page, *A* produces at *P* a field of 7.07 $\mu$T to the left and down at –135°, while *B* creates a field to the right and down at – 45°. Carrying currents toward you, *C* produces a field downward and to the right at – 45°, while *D*'s contribution is downward and to the left. The total field is then

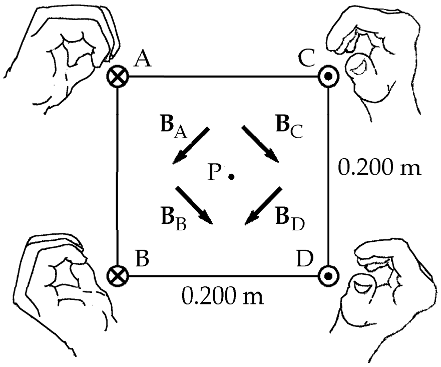


**FIG. P30.21**

$4(7.07\ \mu\text{T})\sin 45.0° = \boxed{20.0\ \mu\text{T}}$ toward the bottom of the page

**P30.22** Let the current *I* be to the right. It creates a field $B = \frac{\mu_0 I}{2\pi d}$ at the proton's location. And we have a balance between the weight of the proton and the magnetic force

$mg(-\hat{\mathbf{j}}) + qv(-\hat{\mathbf{i}}) \times \frac{\mu_0 I}{2\pi d}(\hat{\mathbf{k}}) = 0$ at a distance *d* from the wire

$$d = \frac{qv\mu_0 I}{2\pi mg} = \frac{\left(1.60\times10^{-19}\text{ C}\right)\left(2.30\times10^{4}\text{ m/s}\right)\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)\left(1.20\times10^{-6}\text{ A}\right)}{2\pi\left(1.67\times10^{-27}\text{ kg}\right)\left(9.80\text{ m/s}^2\right)} = \boxed{5.40\text{ cm}}$$

**P30.23** From Ampere's law, the magnetic field at point $a$ is given by $B_a = \frac{\mu_0 I_a}{2\pi r_a}$, where $I_a$ is the net current through the area of the circle of radius $r_a$. In this case, $I_a = 1.00\ \text{A}$ out of the page (the current in the inner conductor), so

$$B_a = \frac{\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)(1.00\ \text{A})}{2\pi\left(1.00 \times 10^{-3}\ \text{m}\right)} = \boxed{200\ \mu\text{T toward top of page}}.$$

Similarly at point $b$: $B_b = \frac{\mu_0 I_b}{2\pi r_b}$, where $I_b$ is the net current through the area of the circle having radius $r_b$.

Taking out of the page as positive, $I_b = 1.00\ \text{A} - 3.00\ \text{A} = -2.00\ \text{A}$, or $I_b = 2.00\ \text{A}$ into the page. Therefore,

$$B_b = \frac{\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)(2.00\ \text{A})}{2\pi\left(3.00 \times 10^{-3}\ \text{m}\right)} = \boxed{133\ \mu\text{T toward bottom of page}}.$$

**P30.24** (a) In $B = \frac{\mu_0 I}{2\pi r}$, the field will be one-tenth as large at a ten-times larger distance: $\boxed{400\ \text{cm}}$

(b) $$\mathbf{B} = \frac{\mu_0 I}{2\pi r_1}\hat{\mathbf{k}} + \frac{\mu_0 I}{2\pi r_2}\left(-\hat{\mathbf{k}}\right) \text{ so } B = \frac{4\pi \times 10^{-7}\ \text{T}\cdot\text{m}(2.00\ \text{A})}{2\pi\ \text{A}}\left(\frac{1}{0.398\,5\ \text{m}} - \frac{1}{0.401\,5\ \text{m}}\right) = \boxed{7.50\ \text{nT}}$$

(c) Call $r$ the distance from cord center to field point and $2d = 3.00\ \text{mm}$ the distance between conductors.

$$B = \frac{\mu_0 I}{2\pi}\left(\frac{1}{r-d} - \frac{1}{r+d}\right) = \frac{\mu_0 I}{2\pi}\frac{2d}{r^2 - d^2}$$

$$7.50 \times 10^{-10}\ \text{T} = \left(2.00 \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)(2.00\ \text{A})\frac{\left(3.00 \times 10^{-3}\ \text{m}\right)}{r^2 - 2.25 \times 10^{-6}\ \text{m}^2} \text{ so } r = \boxed{1.26\ \text{m}}$$

The field of the two-conductor cord is weak to start with and falls off rapidly with distance.

(d) The cable creates $\boxed{\text{zero}}$ field at exterior points, since a loop in Ampère's law encloses zero total current. Shall we sell coaxial-cable power cords to people who worry about biological damage from weak magnetic fields?

**P30.25** (a) One wire feels force due to the field of the other ninety-nine.

$$B = \frac{\mu_0 I_0 r}{2\pi R^2} = \frac{\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)(99)(2.00\ \text{A})\left(0.200 \times 10^{-2}\ \text{m}\right)}{2\pi\left(0.500 \times 10^{-2}\ \text{m}\right)^2} = 3.17 \times 10^{-3}\ \text{T}$$

This field points tangent to a circle of radius 0.200 cm and exerts force $\mathbf{F} = I\boldsymbol{\ell} \times \mathbf{B}$ toward the center of the bundle, on the single hundredth wire:

$$\frac{F}{\ell} = IB\sin\theta = (2.00\ \text{A})\left(3.17 \times 10^{-3}\ \text{T}\right)\sin 90° = 6.34\ \text{mN/m}$$

$$\frac{F_B}{\ell} = \boxed{6.34 \times 10^{-3}\ \text{N/m inward}}$$

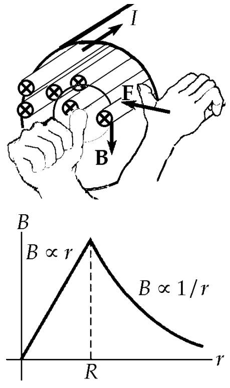


FIG. P30.25

(b) $B \propto r$, so $B$ is greatest at the outside of the bundle. Since each wire carries the same current, $F$ is $\boxed{\text{greatest at the outer surface}}$.

**P30.26** (a) $B_{\text{inner}} = \dfrac{\mu_0 NI}{2\pi r} = \dfrac{\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)(900)\left(14.0\times 10^{3}\ \text{A}\right)}{2\pi(0.700\ \text{m})} = \boxed{3.60\ \text{T}}$

(b) $B_{\text{outer}} = \dfrac{\mu_0 NI}{2\pi r} = \dfrac{\left(2\times 10^{-7}\ \text{T}\cdot\text{m/A}\right)(900)\left(14.0\times 10^{3}\ \text{A}\right)}{1.30\ \text{m}} = \boxed{1.94\ \text{T}}$

***P30.27** We assume the current is vertically upward.

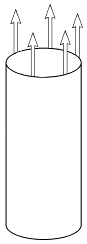


FIG. P30.27(a)

(a) Consider a circle of radius $r$ slightly less than $R$. It encloses no current so from

$\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0 I_{\text{inside}}$ $\qquad B(2\pi r) = 0$

we conclude that the magnetic field is $\boxed{\text{zero}}$.

(b) Now let the $r$ be barely larger than $R$. Ampere's law becomes $B(2\pi R) = \mu_0 I$,

so $\boxed{B = \dfrac{\mu_0 I}{2\pi R}}$.

The field's direction is $\boxed{\text{tangent to the wall of the cylinder in a counterclockwise sense}}$.

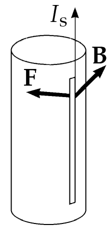


FIG. P30.27(c)

(c) Consider a strip of the wall of width $dx$ and length $\ell$. Its width is so small compared to $2\pi R$ that the field at its location would be essentially unchanged if the current in the strip were turned off.

The current it carries is $I_s = \dfrac{I\,dx}{2\pi R}$ up.

The force on it is

$\mathbf{F} = I_s \ell \times \mathbf{B} = \dfrac{I\,dx}{2\pi R}\left(\ell \dfrac{\mu_0 I}{2\pi R}\right)\ \textbf{up} \times \textbf{into page} = \dfrac{\mu_0 I^2 \ell\,dx}{4\pi^2 R^2}\ \textbf{radially inward}$.

The pressure on the strip and everywhere on the cylinder is

$P = \dfrac{F}{A} = \dfrac{\mu_0 I^2 \ell\,dx}{4\pi^2 R^2 \ell\,dx} = \boxed{\dfrac{\mu_0 I^2}{(2\pi R)^2}\ \text{inward}}$.

The pinch effect makes an effective demonstration when an aluminum can crushes itself as it carries a large current along its length.

**P30.28** From $\oint \mathbf{B}\cdot d\ell = \mu_0 I$, $\qquad I = \dfrac{2\pi r B}{\mu_0} = \dfrac{2\pi\left(1.00\times 10^{-3}\right)(0.100)}{4\pi\times 10^{-7}} = \boxed{500\ \text{A}}$.

**P30.29** Use Ampère's law, $\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0 I$. For current density $\mathbf{J}$, this becomes $\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0 \int \mathbf{J}\cdot d\mathbf{A}$.

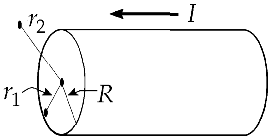


FIG. P30.29

(a) For $r_1 < R$, this gives $\qquad B2\pi r_1 = \mu_0 \displaystyle\int_0^{r_1} (br)(2\pi r\,dr)$ and

$B = \boxed{\dfrac{\mu_0 b r_1^2}{3}\ (\text{for } r_1 < R \text{ or inside the cylinder})}$.

(b) When $r_2 > R$, Ampère's law yields $\qquad (2\pi r_2)B = \mu_0 \displaystyle\int_0^{R} (br)(2\pi r\,dr) = \dfrac{2\pi\mu_0 b R^3}{3}$,

or $B = \boxed{\dfrac{\mu_0 b R^3}{3 r_2}\ (\text{for } r_2 > R \text{ or outside the cylinder})}$.

**P30.30** (a) See Figure (a) to the right.

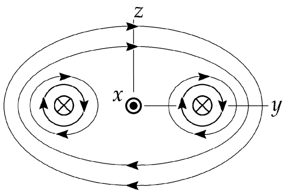
(b) At a point on the $z$ axis, the contribution from each wire has magnitude $B = \frac{\mu_0 I}{2\pi\sqrt{a^2+z^2}}$ and is perpendicular to the line from this point to the wire as shown in Figure (b). Combining fields, the vertical components cancel while the horizontal components add, yielding

$$B_y = 2\left(\frac{\mu_0 I}{2\pi\sqrt{a^2+z^2}}\sin\theta\right) = \frac{\mu_0 I}{\pi\sqrt{a^2+z^2}}\left(\frac{z}{\sqrt{a^2+z^2}}\right) = \frac{\mu_0 I z}{\pi\left(a^2+z^2\right)}$$

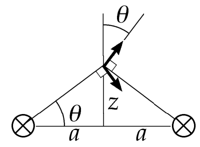
The condition for a maximum is:

$$\frac{dB_y}{dz} = \frac{-\mu_0 I z(2z)}{\pi\left(a^2+z^2\right)^2} + \frac{\mu_0 I}{\pi\left(a^2+z^2\right)} = 0\text{, or } \frac{\mu_0 I}{\pi}\frac{\left(a^2-z^2\right)}{\left(a^2+z^2\right)} = 0$$

Thus, along the $z$ axis, the field is a maximum at $\boxed{d = a}$.



(Currents are into the paper)
Figure (a)



At a distance $z$ above the plane of the conductors
Figure (b)

**FIG. P30.30**

## Section 30.4 The Magnetic Field of a Solenoid

**P30.31** $B = \mu_0 \frac{N}{\ell} I$ so $I = \frac{B}{\mu_0 n} = \frac{\left(1.00\times10^{-4}\text{ T}\right)0.400\text{ m}}{\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)1\,000} = \boxed{31.8\text{ mA}}$

***P30.32** Let the axis of the solenoid lie along the $y$–axis from $y = 0$ to $y = \ell$. We will determine the field at $y = a$. This point will be inside the solenoid if $0 < a < \ell$ and outside if $a < 0$ or $a > \ell$. We think of solenoid as formed of rings, each of thickness $dy$. Now $I$ is the symbol for the current in each turn of wire and the number of turns per length is $\left(\frac{N}{\ell}\right)$. So the number of turns in the ring is $\left(\frac{N}{\ell}\right)dy$ and the current in the ring is $I_{\text{ring}} = I\left(\frac{N}{\ell}\right)dy$. Now we use the result of Example 30.3 for the field created by one ring:

$$B_{\text{ring}} = \frac{\mu_0 I_{\text{ring}} R^2}{2\left(x^2+R^2\right)^{3/2}}$$

where $x$ is the name of the distance from the center of the ring, at location $y$, to the field point $x = a - y$. Each ring creates field in the same direction, along our $y$–axis, so the whole field of the solenoid is

$$B = \sum_{\text{all rings}} B_{\text{ring}} = \sum \frac{\mu_0 I_{\text{ring}} R^2}{2\left(x^2+R^2\right)^{3/2}} = \int_0^\ell \frac{\mu_0 I(N/\ell)dyR^2}{2\left((a-y)^2+R^2\right)^{3/2}} = \frac{\mu_0 INR^2}{2\ell}\int_0^\ell \frac{dy}{2\left((a-y)^2+R^2\right)^{3/2}}.$$

To perform the integral we change variables to $u = a - y$.

$$B = \frac{\mu_0 INR^2}{2\ell}\int_a^{a-\ell} \frac{-du}{\left(u^2+R^2\right)^{3/2}}$$

and then use the table of integrals in the appendix:

*continued on next page*

(a) $$B = \frac{\mu_0 INR^2}{2\ell}\frac{-u}{R^2\sqrt{u^2+R^2}}\Bigg|_a^{a-\ell} = \boxed{\frac{\mu_0 IN}{2\ell}\left[\frac{a}{\sqrt{a^2+R^2}} - \frac{a-\ell}{\sqrt{(a-\ell)^2+R^2}}\right]}$$

(b) If $\ell$ is much larger than $R$ and $a = 0$,

we have $B \cong \frac{\mu_0 IN}{2\ell}\left[0 - \frac{-\ell}{\sqrt{\ell^2}}\right] = \frac{\mu_0 IN}{2\ell}$.

This is just half the magnitude of the field deep within the solenoid. We would get the same result by substituting $a = \ell$ to describe the other end.

**P30.33** The field produced by the solenoid in its interior is given by

$$\mathbf{B} = \mu_0 nI(-\hat{\mathbf{i}}) = \left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(\frac{30.0}{10^{-2}\ \text{m}}\right)(15.0\ \text{A})(-\hat{\mathbf{i}})$$

$$\mathbf{B} = -\left(5.65\times10^{-2}\ \text{T}\right)\hat{\mathbf{i}}$$

The force exerted on side $AB$ of the square current loop is

$$(\mathbf{F}_B)_{AB} = I\mathbf{L}\times\mathbf{B} = (0.200\ \text{A})\left[\left(2.00\times10^{-2}\ \text{m}\right)\hat{\mathbf{j}}\times\left(5.65\times10^{-2}\ \text{T}\right)(-\hat{\mathbf{i}})\right]$$

$$(\mathbf{F}_B)_{AB} = \left(2.26\times10^{-4}\ \text{N}\right)\hat{\mathbf{k}}$$

Similarly, each side of the square loop experiences a force, lying in the plane of the loop, of $\boxed{226\ \mu\text{N directed away from the center}}$. From the above result, it is seen that the net torque exerted on the square loop by the field of the solenoid should be zero. More formally, the magnetic dipole moment of the square loop is given by

$$\boldsymbol{\mu} = I\mathbf{A} = (0.200\ \text{A})\left(2.00\times10^{-2}\ \text{m}\right)^2(-\hat{\mathbf{i}}) = -80.0\ \mu\text{A}\cdot\text{m}^2\hat{\mathbf{i}}$$

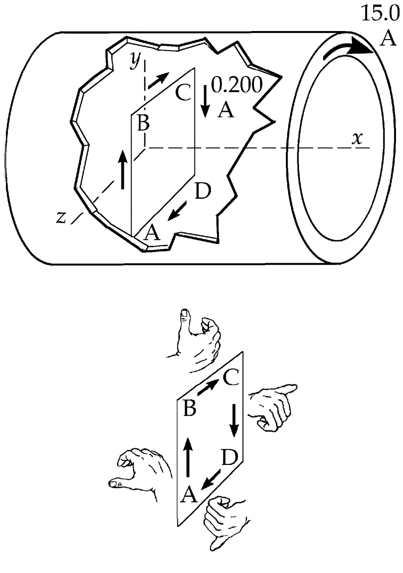


**FIG. P30.33**

The torque exerted on the loop is then $\boldsymbol{\tau} = \boldsymbol{\mu}\times\mathbf{B} = \left(-80.0\ \mu\text{A}\cdot\text{m}^2\hat{\mathbf{i}}\right)\times\left(-5.65\times10^{-2}\ \text{T}\hat{\mathbf{i}}\right) = \boxed{0}$

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## Section 30.5 Magnetic Flux

**P30.34** (a) $(\Phi_B)_{\text{flat}} = \mathbf{B}\cdot\mathbf{A} = B\pi R^2\cos(180-\theta) = \boxed{-B\pi R^2\cos\theta}$

(b) The net flux out of the closed surface is zero: $(\Phi_B)_{\text{flat}} + (\Phi_B)_{\text{curved}} = 0$.

$(\Phi_B)_{\text{curved}} = \boxed{B\pi R^2\cos\theta}$

**P30.35** (a) $\Phi_B = \int \mathbf{B}\cdot d\mathbf{A} = \mathbf{B}\cdot\mathbf{A} = \left(5\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 3\hat{\mathbf{k}}\right)\text{T}\cdot\left(2.50\times10^{-2}\ \text{m}\right)^2\hat{\mathbf{i}}$

$\Phi_B = 3.12\times10^{-3}\ \text{T}\cdot\text{m}^2 = 3.12\times10^{-3}\ \text{Wb} = \boxed{3.12\ \text{mWb}}$

(b) $(\Phi_B)_{\text{total}} = \oint \mathbf{B}\cdot d\mathbf{A} = \boxed{0}$ for *any* closed surface (Gauss's law for magnetism)

**P30.36** (a) $\Phi_B = \mathbf{B}\cdot\mathbf{A} = BA$ where $A$ is the cross-sectional area of the solenoid.

$$\Phi_B = \left(\frac{\mu_0 NI}{\ell}\right)\left(\pi r^2\right) = \boxed{7.40\ \mu\text{Wb}}$$

(b) $$\Phi_B = \mathbf{B}\cdot\mathbf{A} = BA = \left(\frac{\mu_0 NI}{\ell}\right)\left[\pi\left(r_2^2 - r_1^2\right)\right]$$

$$\Phi_B = \left[\frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)(300)(12.0\ \text{A})}{(0.300\ \text{m})}\right]\pi\left[(8.00)^2 - (4.00)^2\right]\left(10^{-3}\ \text{m}\right)^2 = \boxed{2.27\ \mu\text{Wb}}$$

## Section 30.6 **Gauss's Law in Magnetism**

No problems in this section

## Section 30.7 **Displacement Current and the General Form of Ampère's Law**

**P30.37** (a) $$\frac{d\Phi_E}{dt} = \frac{dQ/dt}{\epsilon_0} = \frac{I}{\epsilon_0} = \frac{(0.100\ \text{A})}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = \boxed{11.3\times10^9\ \text{V}\cdot\text{m/s}}$$

(b) $$I_d = \epsilon_0\frac{d\Phi_E}{dt} = I = \boxed{0.100\ \text{A}}$$

**P30.38** $$\frac{d\Phi_E}{dt} = \frac{d}{dt}(EA) = \frac{dQ/dt}{\epsilon_0} = \frac{I}{\epsilon_0}$$

(a) $$\frac{dE}{dt} = \frac{I}{\epsilon_0 A} = \boxed{7.19\times10^{11}\ \text{V/m}\cdot\text{s}}$$

(b) $$\oint B\cdot ds = \epsilon_0\mu_0\frac{d\Phi_E}{dt} \text{ so } 2\pi rB = \epsilon_0\mu_0\frac{d}{dt}\left[\frac{Q}{\epsilon_0 A}\cdot\pi r^2\right]$$

$$B = \frac{\mu_0 Ir}{2A} = \frac{\mu_0(0.200)\left(5.00\times10^{-2}\right)}{2\pi(0.100)^2} = \boxed{2.00\times10^{-7}\ \text{T}}$$

## Section 30.8 **Magnetism in Matter**

**P30.39** (a) $I = \dfrac{ev}{2\pi r}$ $\qquad \mu = IA = \left(\dfrac{ev}{2\pi r}\right)\pi r^2 = \boxed{9.27\times10^{-24}\ \text{A}\cdot\text{m}^2}$

The Bohr model predicts the correct magnetic moment. However, the "planetary model" is seriously deficient in other regards.

(b) Because the electron is (–), its [conventional] current is clockwise, as seen from above, and $\mu$ points $\boxed{\text{downward}}$.

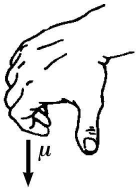
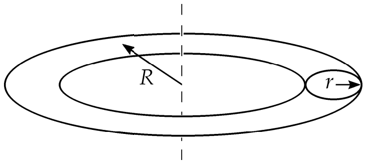


FIG. P30.39

**P30.40** $B=\mu nI=\mu\left(\frac{N}{2\pi r}\right)I$ so $I=\frac{(2\pi r)B}{\mu N}=\frac{2\pi(0.100\text{ m})(1.30\text{ T})}{5\,000\left(4\pi\times10^{-7}\text{ Wb/A}\cdot\text{m}\right)(470)}=\boxed{277\text{ mA}}$

**P30.41** Assuming a uniform $B$ inside the toroid is equivalent to assuming $r \ll R$; then $B_0\approx\mu_0\frac{NI}{2\pi R}$ as for a *tightly* wound solenoid.



$$B_0=\mu_0\frac{(630)(3.00)}{2\pi(0.200)}=0.001\,89\text{ T}$$

With the steel, $B=\kappa_m B_0=(1+\chi)B_0=(101)(0.001\,89\text{ T})$ $\boxed{B=0.191\text{ T}}$

**FIG. P30.41**

**P30.42** $C=\frac{TM}{B}=\frac{(4.00\text{ K})(10.0\%)\left(8.00\times10^{27}\text{ atoms/m}^3\right)(5.00)\left(9.27\times10^{-24}\text{ J/T}^2\right)}{5.00\text{ T}}=\boxed{2.97\times10^{4}\ \frac{\text{K}\cdot\text{J}}{\text{T}^2\cdot\text{m}^3}}$

**P30.43** $B=\mu_0(H+M)$ so $H=\frac{B}{\mu_0}-M=\boxed{2.62\times10^{6}\text{ A/m}}$

**P30.44** In $B=\mu_0(H+M)$ we have $2.00\text{ T}=\mu_0 M$. But also $M=xn\mu_B$. Then $B=\mu_0\mu_B xn$ where $n$ is the number of atoms per volume and $x$ is the number of electrons per atom contributing.

Then $x=\frac{B}{\mu_0\mu_B n}=\frac{2.00\text{ T}}{\left(8.50\times10^{28}\text{ m}^{-3}\right)\left(9.27\times10^{-24}\text{ N}\cdot\text{m/T}\right)\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)}=\boxed{2.02}$.

**P30.45** (a) Comparing Equations 30.29 and 30.30, we see that the applied field is described by $\mathbf{B}_0=\mu_0\mathbf{H}$. Then Eq. 30.35 becomes $M=C\frac{B_0}{T}=\frac{C}{T}\mu_0 H$, and the definition of susceptibility (Eq. 30.32) is $\boxed{\chi=\frac{M}{H}=\frac{C}{T}\mu_0}$.

(b) $C=\frac{\chi T}{\mu_0}=\frac{\left(2.70\times10^{-4}\right)(300\text{ K})}{4\pi\times10^{-7}\text{ T}\cdot\text{m/A}}=\boxed{6.45\times10^{4}\ \frac{\text{K}\cdot\text{A}}{\text{T}\cdot\text{m}}}$

Section 30.9 **The Magnetic Field of the Earth**

**P30.46** (a) $B_h=B_{\text{coil}}=\frac{\mu_0 NI}{2R}=\frac{\left(4\pi\times10^{-7}\right)(5.00)(0.600)}{0.300}=\boxed{12.6\ \mu\text{T}}$

(b) $B_h=B\sin\phi\rightarrow B=\frac{B_h}{\sin\phi}=\frac{12.6\ \mu\text{T}}{\sin 13.0°}=\boxed{56.0\ \mu\text{T}}$

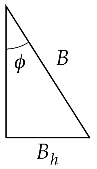


**FIG. P30.46**

**P30.47** (a) Number of unpaired electrons $=\frac{8.00\times10^{22}\text{ A}\cdot\text{m}^2}{9.27\times10^{-24}\text{ A}\cdot\text{m}^2}=\boxed{8.63\times10^{45}}$.

Each iron atom has two unpaired electrons, so the number of iron atoms required is $\frac{1}{2}\left(8.63\times10^{45}\right)$.

(b) $\text{Mass}=\frac{\left(4.31\times10^{45}\text{ atoms}\right)\left(7\,900\text{ kg/m}^3\right)}{\left(8.50\times10^{28}\text{ atoms/m}^3\right)}=\boxed{4.01\times10^{20}\text{ kg}}$

## Additional Problems

**P30.48** $B=\frac{\mu_0 IR^2}{2\left(R^2+R^2\right)^{3/2}}=\frac{\mu_0 I}{2^{5/2}R}$ $\qquad I=\frac{2^{5/2}BR}{\mu_0}=\frac{2^{5/2}\left(7.00\times10^{-5}\text{ T}\right)\left(6.37\times10^{6}\text{ m}\right)}{\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)}$

so $\boxed{I=2.01\times10^{9}\text{ A}}$ toward the west

**P30.49** Consider a longitudinal filament of the strip of width *dr* as shown in the sketch. The contribution to the field at point *P* due to the current *dI* in the element *dr* is

$$dB=\frac{\mu_0 dI}{2\pi r}$$

where $\quad dI=I\left(\frac{dr}{w}\right)$

$$\mathbf{B}=\int d\mathbf{B}=\int_b^{b+w}\frac{\mu_0 I dr}{2\pi wr}\hat{\mathbf{k}}=\boxed{\frac{\mu_0 I}{2\pi w}\ln\left(1+\frac{w}{b}\right)\hat{\mathbf{k}}}.$$

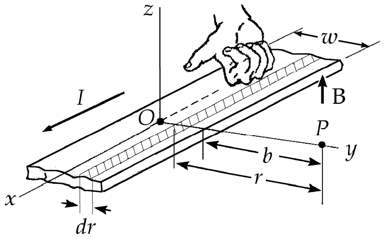


**FIG. P30.49**

**P30.50** Suppose you have two 100-W headlights running from a 12-V battery, with the whole $\frac{200\text{ W}}{12\text{ V}}=17\text{ A}$ current going through the switch 60 cm from the compass. Suppose the dashboard contains little iron, so $\mu\approx\mu_0$. Model the current as straight. Then,

$$B=\frac{\mu_0 I}{2\pi r}=\frac{\left(4\pi\times10^{-7}\right)17}{2\pi(0.6)}\boxed{\sim10^{-5}\text{ T}}.$$

If the local geomagnetic field is $5\times10^{-5}$ T, this is $\boxed{\sim10^{-1}\text{ times as large,}}$ enough to affect the compass noticeably.

**P30.51** We find the total number of turns: $\quad B=\frac{\mu_0 NI}{\ell}$

$$N=\frac{B\ell}{\mu_0 I}=\frac{(0.030\,0\text{ T})(0.100\text{ m})\text{ A}}{\left(4\pi\times10^{-7}\text{ T}\cdot\text{m}\right)(1.00\text{ A})}=2.39\times10^{3}$$

Each layer contains $\left(\frac{10.0\text{ cm}}{0.050\,0\text{ cm}}\right)=200$ closely wound turns

so she needs $\left(\frac{2.39\times10^{3}}{200}\right)=\boxed{12\text{ layers}}.$

The inner diameter of the innermost layer is 10.0 mm. The outer diameter of the outermost layer is $10.0\text{ mm}+2\times12\times0.500\text{ mm}=22.0\text{ mm}$. The average diameter is 16.0 mm, so the total length of wire is

$$\left(2.39\times10^{3}\right)\pi\left(16.0\times10^{-3}\text{ m}\right)=\boxed{120\text{ m}}.$$

***P30.52** At a point at distance $x$ from the left end of the bar, current $I_2$ creates magnetic field $\mathbf{B} = \frac{\mu_0 I_2}{2\pi\sqrt{h^2+x^2}}$ to the left and above the horizontal at angle $\theta$ where $\tan\theta = \frac{x}{h}$. This field exerts force on an element of the rod of length $dx$

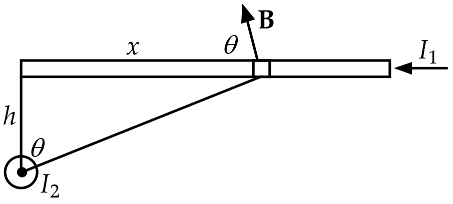


FIG. P30.52

$$d\mathbf{F} = I_1 \boldsymbol{\ell} \times \mathbf{B} = I_1 \frac{\mu_0 I_2 dx}{2\pi\sqrt{h^2+x^2}} \underset{\text{right hand rule}}{\sin\theta}$$

$$= \frac{\mu_0 I_1 I_2 dx}{2\pi\sqrt{h^2+x^2}} \frac{x}{\sqrt{h^2+x^2}} \text{ into the page}$$

$$d\mathbf{F} = \frac{\mu_0 I_1 I_2 x dx}{2\pi\left(h^2+x^2\right)}\left(-\hat{\mathbf{k}}\right)$$

The whole force is the sum of the forces on all of the elements of the bar:

$$\mathbf{F} = \int_{x=0}^{\ell} \frac{\mu_0 I_1 I_2 x dx}{2\pi\left(h^2+x^2\right)}\left(-\hat{\mathbf{k}}\right) = \frac{\mu_0 I_1 I_2\left(-\hat{\mathbf{k}}\right)}{4\pi}\int_0^{\ell} \frac{2x dx}{h^2+x^2} = \frac{\mu_0 I_1 I_2\left(-\hat{\mathbf{k}}\right)}{4\pi}\ln\left(h^2+x^2\right)\Big|_0^{\ell}$$

$$= \frac{\mu_0 I_1 I_2\left(-\hat{\mathbf{k}}\right)}{4\pi}\left[\ln\left(h^2+\ell^2\right) - \ln h^2\right] = \frac{10^{-7}\text{ N}(100\text{ A})(200\text{ A})\left(-\hat{\mathbf{k}}\right)}{\text{A}^2}\ln\left[\frac{(0.5\text{ cm})^2+(10\text{ cm})^2}{(0.5\text{ cm})^2}\right]$$

$$= 2\times10^{-3}\text{ N}\left(-\hat{\mathbf{k}}\right)\ln 401 = \boxed{1.20\times10^{-2}\text{ N}\left(-\hat{\mathbf{k}}\right)}$$

**P30.53** On the axis of a current loop, the magnetic field is given by $B = \frac{\mu_0 I R^2}{2\left(x^2+R^2\right)^{3/2}}$

where in this case $I = \frac{q}{(2\pi/\omega)}$. The magnetic field is directed away from the center, with a magnitude of

$$B = \frac{\mu_0 \omega R^2 q}{4\pi\left(x^2+R^2\right)^{3/2}} = \frac{\mu_0(20.0)(0.100)^2\left(10.0\times10^{-6}\right)}{4\pi\left[(0.050\,0)^2+(0.100)^2\right]^{3/2}} = \boxed{1.43\times10^{-10}\text{ T}}.$$

**P30.54** On the axis of a current loop, the magnetic field is given by $B = \frac{\mu_0 I R^2}{2\left(x^2+R^2\right)^{3/2}}$

where in this case $I = \frac{q}{(2\pi/\omega)}$. Therefore, $B = \frac{\mu_0 \omega R^2 q}{4\pi\left(x^2+R^2\right)^{3/2}}$

when $x = \frac{R}{2}$. then $B = \frac{\mu_0 \omega R^2 q}{4\pi\left(\frac{5}{4}R^2\right)^{3/2}} = \boxed{\frac{\mu_0 q\omega}{2.5\sqrt{5}\pi R}}$.

**P30.55** (a) Use equation 30.7 twice: $B_x = \dfrac{\mu_0 I R^2}{2\left(x^2+R^2\right)^{3/2}}$

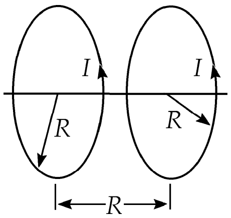


**FIG. P30.55**

If each coil has $N$ turns, the field is just $N$ times larger.

$$B = B_{x1} + B_{x2} = \frac{N\mu_0 I R^2}{2}\left[\frac{1}{\left(x^2+R^2\right)^{3/2}} + \frac{1}{\left[(R-x)^2+R^2\right]^{3/2}}\right]$$

$$\boxed{B = \frac{N\mu_0 I R^2}{2}\left[\frac{1}{\left(x^2+R^2\right)^{3/2}} + \frac{1}{\left(2R^2+x^2-2xR\right)^{3/2}}\right]}$$

(b) $$\frac{dB}{dx} = \frac{N\mu_0 I R^2}{2}\left[-\frac{3}{2}(2x)\left(x^2+R^2\right)^{-5/2} - \frac{3}{2}\left(2R^2+x^2-2xR\right)^{-5/2}(2x-2R)\right]$$

Substituting $x = \dfrac{R}{2}$ and canceling terms, $\boxed{\dfrac{dB}{dx} = 0}$.

$$\frac{d^2B}{dx^2} = \frac{-3N\mu_0 I R^2}{2}\left[\left(x^2+R^2\right)^{-5/2} - 5x^2\left(x^2+R^2\right)^{-7/2} + \left(2R^2+x^2-2xR\right)^{-5/2} - 5(x-R)^2\left(2R^2+x^2-2xR\right)^{-7/2}\right]$$

Again substituting $x = \dfrac{R}{2}$ and canceling terms, $\boxed{\dfrac{d^2B}{dx^2} = 0}$.

**P30.56** "Helmholtz pair" → separation distance = radius

$$B = \frac{2\mu_0 I R^2}{2\left[(R/2)^2+R^2\right]^{3/2}} = \frac{\mu_0 I R^2}{\left[\frac{1}{4}+1\right]^{3/2} R^3} = \frac{\mu_0 I}{1.40R} \text{ for 1 turn.}$$

For $N$ turns in each coil, $B = \dfrac{\mu_0 N I}{1.40R} = \dfrac{\left(4\pi\times10^{-7}\right)100(10.0)}{1.40(0.500)} = \boxed{1.80\times10^{-3}\text{ T}}$.

***P30.57** Consider first a solid cylindrical rod of radius $R$ carrying current toward you, uniformly distributed over its cross-sectional area. To find the field at distance $r$ from its center we consider a circular loop of radius $r$:

$$\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0 I_{\text{inside}}$$

$$B2\pi r = \mu_0 \pi r^2 J \qquad B = \frac{\mu_0 J r}{2} \qquad \mathbf{B} = \frac{\mu_0 J}{2}\hat{\mathbf{k}}\times\mathbf{r}$$

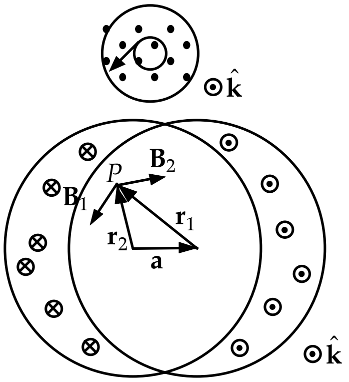


**FIG. P30.57**

Now the total field at $P$ inside the saddle coils is the field due to a solid rod carrying current toward you, centered at the head of vector **a**, plus the field of a solid rod centered at the tail of vector **a** carrying current away from you.

$$\mathbf{B}_1 + \mathbf{B}_2 = \frac{\mu_0 J}{2}\hat{\mathbf{k}}\times\mathbf{r}_1 + \frac{\mu_0 J}{2}\left(-\hat{\mathbf{k}}\right)\times\mathbf{r}_2$$

Now note $\mathbf{a} + \mathbf{r}_1 = \mathbf{r}_2$

$$\mathbf{B}_1 + \mathbf{B}_2 = \frac{\mu_0 J}{2}\hat{\mathbf{k}}\times\mathbf{r}_1 - \frac{\mu_0 J}{2}\hat{\mathbf{k}}\times\left(\mathbf{a}+\mathbf{r}_1\right) = \frac{\mu_0 J}{2}\mathbf{a}\times\hat{\mathbf{k}} = \boxed{\frac{\mu_0 J a}{2}\text{ down in the diagram}}.$$

***P30.58** From example 30.6, the upper sheet creates field $\mathbf{B} = \frac{\mu_0 J_s}{2}\hat{\mathbf{k}}$ above it and $\frac{\mu_0 J_s}{2}\left(-\hat{\mathbf{k}}\right)$ below it. Consider a patch of the sheet of width $w$ parallel to the $z$ axis and length $d$ parallel to the $x$ axis. The charge on it $\sigma wd$ passes a point in time $\frac{d}{v}$, so the current it constitutes is $\frac{q}{t} = \frac{\sigma wdv}{d}$ and the linear current density is $J_s = \frac{\sigma wv}{w} = \sigma v$. Then the magnitude of the magnetic field created by the upper sheet is $\frac{1}{2}\mu_0 \sigma v$. Similarly, the lower sheet in its motion toward the right constitutes current toward the left. It creates magnetic field $\frac{1}{2}\mu_0 \sigma v\left(-\hat{\mathbf{k}}\right)$ above it and $\frac{1}{2}\mu_0 \sigma v\hat{\mathbf{k}}$ below it.

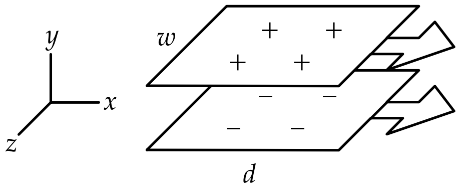


FIG. P30.58

(b) Above both sheets and below both, their equal-magnitude fields add to $\boxed{\text{zero}}$.

(a) Between the plates, their fields add to $\mu_0 \sigma v\left(-\hat{\mathbf{k}}\right) = \boxed{\mu_0 \sigma v \text{ away from you horizontally.}}$

(c) The upper plate exerts no force on itself. The field of the lower plate, $\frac{1}{2}\mu_0 \sigma v\left(-\hat{\mathbf{k}}\right)$ will exert a force on the current in the $w$- by $d$-section, given by

$$I\ell \times \mathbf{B} = \sigma wvd\hat{\mathbf{i}} \times \frac{1}{2}\mu_0 \sigma v\left(-\hat{\mathbf{k}}\right) = \frac{1}{2}\mu_0 \sigma^2 v^2 wd\hat{\mathbf{j}}.$$

The force per area is $\frac{1}{2}\frac{\mu_0 \sigma^2 v^2 wd}{wd}\hat{\mathbf{j}} = \boxed{\frac{1}{2}\mu_0 \sigma^2 v^2 \text{ up}}$.

(d) The electrical force on our section of the upper plate is $q\mathbf{E}_{\text{lower}} = \sigma \ell w \frac{\sigma}{2\in_0}\left(-\hat{\mathbf{j}}\right) = \frac{\ell w\sigma^2}{2\in_0}\left(-\hat{\mathbf{j}}\right)$.

The electrical force per area is $\frac{\ell w \sigma^2}{2\in_0 \ell w}$ down $= \frac{\sigma^2}{2\in_0}$ down. To have $\frac{1}{2}\mu_0 \sigma^2 v^2 = \frac{\sigma^2}{2\in_0}$ we require

$$v = \frac{1}{\sqrt{\mu_0 \in_0}} = \frac{1}{\sqrt{4\pi \times 10^{-7}(\text{Tm/A})(\text{N/TAm})8.85 \times 10^{-12}\left(\text{C}^2/\text{Nm}^2\right)(\text{As/C})^2}} = \boxed{3.00 \times 10^8 \text{ m/s}}.$$

This is the speed of light, not a possible speed for a metal plate.

**P30.59** Model the two wires as straight parallel wires (!)

(a) $F_B = \dfrac{\mu_0 I^2 \ell}{2\pi a}$ (Equation 30.12)

$$F_B = \frac{\left(4\pi\times10^{-7}\right)(140)^2(2\pi)(0.100)}{2\pi\left(1.00\times10^{-3}\right)}$$

$$= \boxed{2.46\text{ N}}\text{ upward}$$

(b) $a_{\text{loop}} = \dfrac{2.46\text{ N} - m_{\text{loop}}g}{m_{\text{loop}}} = \boxed{107\text{ m/s}^2}$ upward

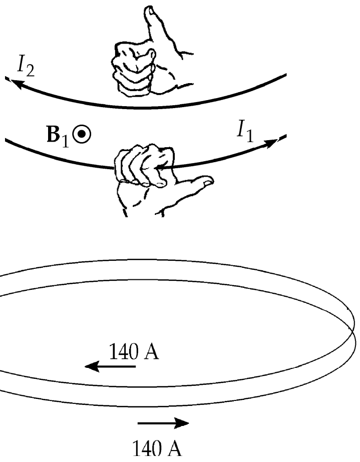


**FIG. P30.59**

**P30.60** (a) In $d\mathbf{B} = \dfrac{\mu_0}{4\pi r^2} I d\mathbf{s}\times\hat{\mathbf{r}}$, the moving charge constitutes a bit of current as in $I = nqvA$. For a positive charge the direction of $d\mathbf{s}$ is the direction of $\mathbf{v}$, so $d\mathbf{B} = \dfrac{\mu_0}{4\pi r^2} nqA(ds)\mathbf{v}\times\hat{\mathbf{r}}$. Next, $A(ds)$ is the volume occupied by the moving charge, and $nA(ds) = 1$ for just one charge. Then,

$$\boxed{\mathbf{B} = \frac{\mu_0}{4\pi r^2} q\mathbf{v}\times\hat{\mathbf{r}}}.$$

(b) $$B = \frac{\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)\left(1.60\times10^{-19}\text{ C}\right)\left(2.00\times10^{7}\text{ m/s}\right)}{4\pi\left(1.00\times10^{-3}\right)^2}\sin 90.0° = \boxed{3.20\times10^{-13}\text{ T}}$$

(c) $$F_B = q\left|\mathbf{v}\times\mathbf{B}\right| = \left(1.60\times10^{-19}\text{ C}\right)\left(2.00\times10^{7}\text{ m/s}\right)\left(3.20\times10^{-13}\text{ T}\right)\sin 90.0°$$

$$F_B = \boxed{1.02\times10^{-24}\text{ N directed away from the first proton}}$$

(d) $$F_e = qE = \frac{k_e q_1 q_2}{r^2} = \frac{\left(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\text{ C}\right)^2}{\left(1.00\times10^{-3}\right)^2}$$

$$F_e = \boxed{2.30\times10^{-22}\text{ N directed away from the first proton}}$$

Both forces act together. The electrical force is stronger by two orders of magnitude. It is productive to think about how it would look to an observer in a reference frame moving along with one proton or the other.

**P30.61** (a) $$B=\frac{\mu_0 I}{2\pi r}=\frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)(24.0\ \text{A})}{2\pi(0.0175\ \text{m})}=\boxed{2.74\times10^{-4}\ \text{T}}$$

(b) At point $C$, conductor $AB$ produces a field $\frac{1}{2}\left(2.74\times10^{-4}\ \text{T}\right)\left(-\hat{\mathbf{j}}\right)$, conductor $DE$ produces a field of $\frac{1}{2}\left(2.74\times10^{-4}\ \text{T}\right)\left(-\hat{\mathbf{j}}\right)$, $BD$ produces no field, and $AE$ produces negligible field. The total field at $C$ is $\boxed{2.74\times10^{-4}\ \text{T}\left(-\hat{\mathbf{j}}\right)}$.

(c) $$\mathbf{F}_B=I\boldsymbol{\ell}\times\mathbf{B}=(24.0\ \text{A})\left(0.0350\ \text{m}\hat{\mathbf{k}}\right)\times\left[5\left(2.74\times10^{-4}\ \text{T}\right)\left(-\hat{\mathbf{j}}\right)\right]=\boxed{\left(1.15\times10^{-3}\ \text{N}\right)\hat{\mathbf{i}}}$$

(d) $$\mathbf{a}=\frac{\sum\mathbf{F}}{m}=\frac{\left(1.15\times10^{-3}\ \text{N}\right)\hat{\mathbf{i}}}{3.0\times10^{-3}\ \text{kg}}=\boxed{\left(0.384\ \text{m/s}^2\right)\hat{\mathbf{i}}}$$

(e) The bar is already so far from $AE$ that it moves through nearly constant magnetic field. The force acting on the bar is constant, and therefore the bar's $\boxed{\text{acceleration is constant}}$.

(f) $$v_f^2=v_i^2+2ax=0+2\left(0.384\ \text{m/s}^2\right)(1.30\ \text{m}),\ \text{so}\ \mathbf{v}_f=\boxed{(0.999\ \text{m/s})\hat{\mathbf{i}}}$$

***P30.62** Each turn creates field at the center $\frac{\mu_0 I}{2R}$. Together they create field

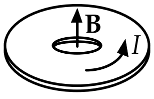


FIG. P30.62

$$\frac{\mu_0 I}{2}\left(\frac{1}{R_1}+\frac{1}{R_2}+\ldots+\frac{1}{R_{50}}\right)=\frac{4\pi\times10^{-7}\ \text{Tm}I}{2\ \text{A}}\left(\frac{1}{5.05}+\frac{1}{5.15}+\ldots+\frac{1}{9.95}\right)\frac{1}{10^{-2}\ \text{m}}$$
$$=\mu_0 I(50/\text{m})6.93=\boxed{347\mu_0 I/\text{m}}$$

**P30.63** At equilibrium, $\frac{F_B}{\ell}=\frac{\mu_0 I_A I_B}{2\pi a}=\frac{mg}{\ell}$ or $I_B=\frac{2\pi a(m/\ell)g}{\mu_0 I_A}$

$$I_B=\frac{2\pi(0.0250\ \text{m})(0.0100\ \text{kg/m})\left(9.80\ \text{m/s}^2\right)}{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)(150\ \text{A})}=\boxed{81.7\ \text{A}}$$

**P30.64** (a) The magnetic field due to an infinite sheet of current (or the magnetic field at points near a large sheet of current) is given by $B=\frac{\mu_0 J_s}{2}$. The current density $J_s=\frac{I}{\ell}$ and in this case the equivalent current of the moving charged belt is

$$I=\frac{dq}{dt}=\frac{d}{dt}(\sigma\ell x)=\sigma\ell v;\ v=\frac{dx}{dt}.$$

Therefore, $J_s=\sigma v$ and $\boxed{B=\frac{\mu_0\sigma v}{2}}$.

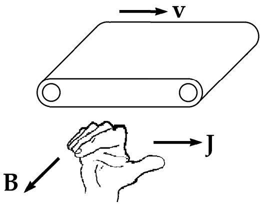


FIG. P30.64

(b) If the sheet is positively charged and moving in the direction shown, the magnetic field is $\boxed{\text{out of the page, parallel to the roller axes}}$.

**P30.65** The central wire creates field $\mathbf{B}=\dfrac{\mu_0 I_1}{2\pi R}$ counterclockwise. The curved portions of the loop feels no force since $\ell\times\mathbf{B}=0$ there. The straight portions both feel $I\ell\times\mathbf{B}$ forces to the right, amounting to

$$\mathbf{F}_B=I_2 2L\frac{\mu_0 I_1}{2\pi R}=\boxed{\frac{\mu_0 I_1 I_2 L}{\pi R}\text{ to the right}}.$$

**P30.66** $I=\dfrac{2\pi rB}{\mu_0}=\dfrac{2\pi\left(9.00\times10^{3}\right)\left(1.50\times10^{-8}\right)}{4\pi\times10^{-7}}=\boxed{675\text{ A}}$

Flow of $\boxed{\text{positive current is downward } \textit{or} \text{ negative charge flows upward}}$.

**P30.67** By symmetry of the arrangement, the magnitude of the net magnetic field at point *P* is $B=8B_{0x}$ where $B_0$ is the contribution to the field due to current in an edge length equal to $\dfrac{L}{2}$. In order to calculate $B_0$, we use the Biot-Savart law and consider the plane of the square to be the *yz*-plane with point *P* on the *x*-axis. The contribution to the magnetic field at point *P* due to a current element of length *dz* and located a distance *z* along the axis is given by Equation 30.3.

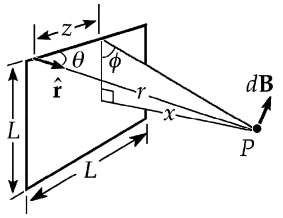


**FIG. P30.67**

$$\mathbf{B}_0=\frac{\mu_0 I}{4\pi}\int\frac{d\boldsymbol{\ell}\times\hat{\mathbf{r}}}{r^2}.$$

From the figure we see that

$$r=\sqrt{x^2+\left(L^2/4\right)+z^2}\text{ and }\left|d\boldsymbol{\ell}\times\hat{\mathbf{r}}\right|=dz\sin\theta=dz\sqrt{\frac{\left(L^2/4\right)+x^2}{\left(L^2/4\right)+x^2+z^2}}.$$

By symmetry all components of the field **B** at *P* cancel except the components along *x* (perpendicular to the plane of the square); and

$$B_{0x}=B_0\cos\phi\text{ where }\cos\phi=\frac{L/2}{\sqrt{\left(L^2/4\right)+x^2}}.$$

Therefore, $\mathbf{B}_{0x}=\dfrac{\mu_0 I}{4\pi}\displaystyle\int_0^{L/2}\frac{\sin\theta\cos\phi dz}{r^2}$ and $B=8B_{0x}$.

Using the expressions given above for $\sin\theta\cos\phi$, and *r*, we find

$$B=\frac{\mu_0 I L^2}{2\pi\left(x^2+\left(L^2/4\right)\right)\sqrt{x^2+\left(L^2/2\right)}}.$$

**P30.68** (a) From Equation 30.9, the magnetic field produced by one loop at the center of the second loop is given by $B=\dfrac{\mu_0 IR^2}{2x^3}=\dfrac{\mu_0 I\left(\pi R^2\right)}{2\pi x^3}=\dfrac{\mu_0\mu}{2\pi x^3}$ where the magnetic moment of either loop is $\mu=I\left(\pi R^2\right)$. Therefore,

$$\left|F_x\right|=\mu\frac{dB}{dx}=\mu\left(\frac{\mu_0\mu}{2\pi}\right)\left(\frac{3}{x^4}\right)=\frac{3\mu_0\left(\pi R^2 I\right)^2}{2\pi x^4}=\boxed{\frac{3\pi}{2}\frac{\mu_0 I^2R^4}{x^4}}.$$

(b) $$\left|F_x\right|=\frac{3\pi}{2}\frac{\mu_0 I^2R^4}{x^4}=\frac{3\pi}{2}\frac{\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)\left(10.0\text{ A}\right)^2\left(5.00\times10^{-3}\text{ m}\right)^4}{\left(5.00\times10^{-2}\text{ m}\right)^4}=\boxed{5.92\times10^{-8}\text{ N}}$$

**P30.69** There is no contribution from the straight portion of the wire since $d\mathbf{s}\times\hat{\mathbf{r}}=0$. For the field of the spiral,

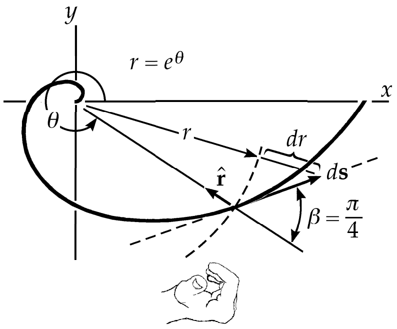


FIG. P30.69

$$d\mathbf{B}=\frac{\mu_0 I}{(4\pi)}\frac{(d\mathbf{s}\times\hat{\mathbf{r}})}{r^2}$$

$$B=\frac{\mu_0 I}{4\pi}\int_{\theta=0}^{2\pi}\frac{|d\mathbf{s}|\sin\theta|\hat{\mathbf{r}}|}{r^2}=\frac{\mu_0 I}{4\pi}\int_{\theta=0}^{2\pi}\left(\sqrt{2}dr\right)\left[\sin\left(\frac{3\pi}{4}\right)\right]\frac{1}{r^2}$$

$$B=\frac{\mu_0 I}{4\pi}\int_{\theta=0}^{2\pi}r^{-2}dr=-\frac{\mu_0 I}{4\pi}\left(r^{-1}\right)\Big|_{\theta=0}^{2\theta}$$

Substitute $r=e^{\theta}$: $B=-\frac{\mu_0 I}{4\pi}\left[e^{-\theta}\right]_0^{2\pi}=-\frac{\mu_0 I}{4\pi}\left[e^{-2\pi}-e^0\right]=\boxed{\frac{\mu_0 I}{4\pi}\left(1-e^{-2\pi}\right)}$ out of the page.

**P30.70** (a) $\mathbf{B}=\mathbf{B}_0+\mu_0\mathbf{M}$

$$\mathbf{M}=\frac{\mathbf{B}-\mathbf{B}_0}{\mu_0}\text{ and }M=\frac{|\mathbf{B}-\mathbf{B}_0|}{\mu_0}$$

Assuming that $\mathbf{B}$ and $\mathbf{B}_0$ are parallel, this becomes $M=\frac{B-B_0}{\mu_0}$.

The magnetization curve gives a plot of $M$ versus $B_0$.

(b) The second graph is a plot of the relative permeability $\left(\frac{B}{B_0}\right)$ as a function of the applied field $B_0$.

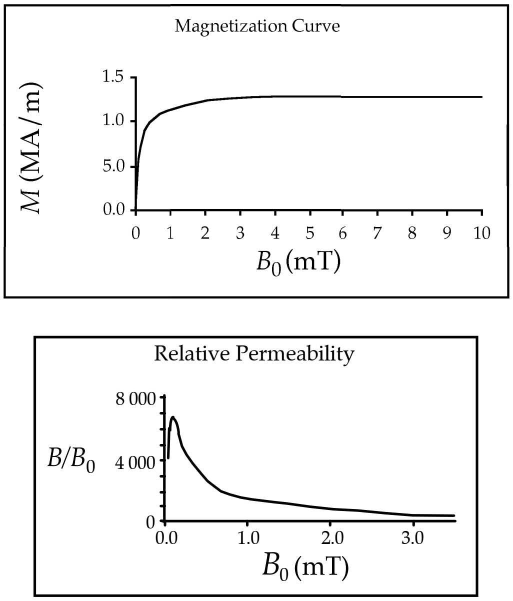


FIG. P30.70

**P30.71** Consider the sphere as being built up of little rings of radius *r*, centered on the rotation axis. The contribution to the field from each ring is

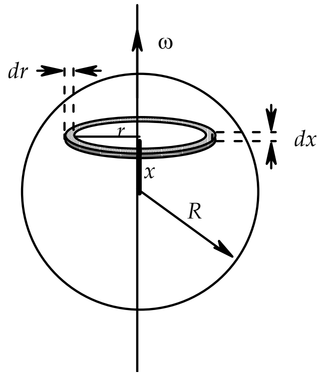


FIG. P30.71

$$dB = \frac{\mu_0 r^2 dI}{2\left(x^2 + r^2\right)^{3/2}} \text{ where } dI = \frac{dQ}{t} = \frac{\omega dQ}{2\pi}$$

$$dQ = \rho dV = \rho(2\pi r dr)(dx)$$

$$dB = \frac{\mu_0 \rho \omega r^3 dr dx}{2\left(x^2 + r^2\right)^{3/2}} \text{ where } \rho = \frac{Q}{(4/3)\pi R^3}$$

$$B = \int_{x=-R}^{+R} \int_{r=0}^{\sqrt{R^2 - x^2}} \frac{\mu_0 \rho \omega}{2} \frac{r^3 dr dx}{\left(x^2 + r^2\right)^{3/2}}$$

Let $v = r^2 + x^2$, $dv = 2r dr$, and $r^2 = v - x^2$

$$B = \int_{x=-R}^{+R} \int_{v=x^2}^{R^2} \frac{\mu_0 \rho \omega}{2} \frac{\left(v - x^2\right) dv}{2 v^{3/2}} dx = \frac{\mu_0 \rho \omega}{4} \int_{x=-R}^{R} \left[ \int_{v=x^2}^{R^2} v^{-1/2} dv - x^2 \int_{v=x^2}^{R^2} v^{-3/2} dv \right] dx$$

$$B = \frac{\mu_0 \rho \omega}{4} \int_{x=-R}^{R} \left[ 2 v^{1/2} \Big|_{x^2}^{R^2} + \left(2x^2\right) v^{-1/2} \Big|_{x^2}^{R^2} \right] dx = \frac{\mu_0 \rho \omega}{4} \int_{x=-R}^{R} \left[ 2\left(R - |x|\right) + 2x^2 \left( \frac{1}{R} - \frac{1}{|x|} \right) \right] dx$$

$$B = \frac{\mu_0 \rho \omega}{4} \int_{-R}^{R} \left[ 2\frac{x^2}{R} - 4|x| + 2R \right] dx = \frac{2\mu_0 \rho \omega}{4} \int_{0}^{R} \left[ 2\frac{x^2}{R} - 4x + 2R \right] dx$$

$$B = \frac{2\mu_0 \rho \omega}{4} \left( \frac{2R^3}{3R} - \frac{4R^2}{2} + 2R^2 \right) = \boxed{\frac{\mu_0 \rho \omega R^2}{3}}$$

**P30.72** Consider the sphere as being built up of little rings of radius *r*, centered on the rotation axis. The current associated with each rotating ring of charge is

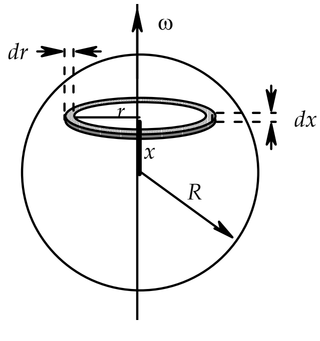


FIG. P30.72

$$dI = \frac{dQ}{t} = \frac{\omega}{2\pi}\left[\rho(2\pi r dr)(dx)\right].$$

The magnetic moment contributed by this ring is

$$d\mu = A(dI) = \pi r^2 \frac{\omega}{2\pi}\left[\rho(2\pi r dr)(dx)\right] = \pi \omega \rho r^3 dr dx$$

$$\mu = \pi \omega \rho \int_{x=-R}^{+R} \left[ \int_{r=0}^{\sqrt{R^2 - x^2}} r^3 dr \right] dx = \pi \omega \rho \int_{x=-R}^{+R} \frac{\left(\sqrt{R^2 - x^2}\right)^4}{4} dx = \pi \omega \rho \int_{x=-R}^{+R} \frac{\left(R^2 - x^2\right)^2}{4} dx$$

$$\mu = \frac{\pi \omega \rho}{4} \int_{x=-R}^{+R} \left(R^4 - 2R^2 x^2 + x^4\right) dx = \frac{\pi \omega \rho}{4} \left[ R^4 (2R) - 2R^2 \left( \frac{2R^3}{3} \right) + \frac{2R^5}{5} \right]$$

$$\mu = \frac{\pi \omega \rho}{4} R^5 \left( 2 - \frac{4}{3} + \frac{2}{5} \right) = \frac{\pi \omega \rho R^5}{4} \left( \frac{16}{15} \right) = \boxed{\frac{4\pi \omega \rho R^5}{15}} \text{ up}$$

**P30.73** Note that the current $I$ exists in the conductor with a current density $J=\frac{I}{A}$, where

$$A=\pi\left[a^2-\frac{a^2}{4}-\frac{a^2}{4}\right]=\frac{\pi a^2}{2}.$$

Therefore, $J=\frac{2I}{\pi a^2}$.

To find the field at either point $P_1$ or $P_2$, find $B_s$ which would exist if the conductor were solid, using Ampère's law. Next, find $B_1$ and $B_2$ that *would* be due to the conductors of radius $\frac{a}{2}$ that *could* occupy the void where the holes exist. Then use the superposition principle and subtract the field that would be due to the part of the conductor where the holes exist from the field of the solid conductor.

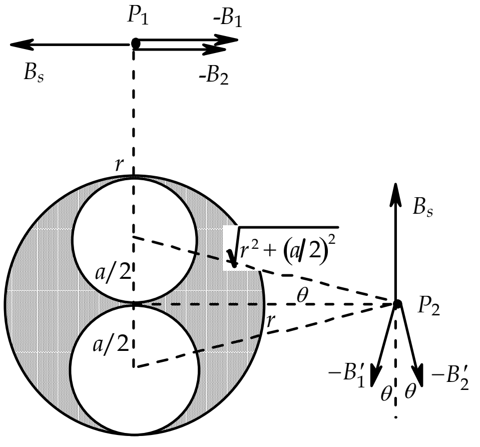


**FIG. P30.73**

(a) At point $P_1$, $B_s=\frac{\mu_0 J\left(\pi a^2\right)}{2\pi r}$, $B_1=\frac{\mu_0 J\pi(a/2)^2}{2\pi\left(r-(a/2)\right)}$, and $B_2=\frac{\mu_0 J\pi(a/2)^2}{2\pi\left(r+(a/2)\right)}$.

$$B=B_s-B_1-B_2=\frac{\mu J\pi a^2}{2\pi}\left[\frac{1}{r}-\frac{1}{4\left(r-(a/2)\right)}-\frac{1}{4\left(r+(a/2)\right)}\right]$$

$$B=\frac{\mu_0(2I)}{2\pi}\left[\frac{4r^2-a^2-2r^2}{4r\left(r^2-\left(a^2/4\right)\right)}\right]=\boxed{\frac{\mu_0 I}{\pi r}\left[\frac{2r^2-a^2}{4r^2-a^2}\right]\text{ directed to the left}}$$

(b) At point $P_2$, $B_s=\frac{\mu_0 J\left(\pi a^2\right)}{2\pi r}$ and $B_1'=B_2'=\frac{\mu_0 J\,\pi(a/2)^2}{2\pi\sqrt{r^2+(a/2)^2}}$.

The horizontal components of $B_1'$ and $B_2'$ cancel while their vertical components add.

$$B=B_s-B_1'\cos\theta-B_2'\cos\theta=\frac{\mu_0 J\left(\pi a^2\right)}{2\pi r}-2\left(\frac{\mu_0 J\,\pi a^2/4}{2\pi\sqrt{r^2+\left(a^2/4\right)}}\right)\frac{r}{\sqrt{r^2+\left(a^2/4\right)}}$$

$$B=\frac{\mu_0 J\left(\pi a^2\right)}{2\pi r}\left[1-\frac{r^2}{2\left(r^2+\left(a^2/4\right)\right)}\right]=\frac{\mu_0(2I)}{2\pi r}\left[1-\frac{2r^2}{4r^2+a^2}\right]$$

$$=\boxed{\frac{\mu_0 I}{\pi r}\left[\frac{2r^2+a^2}{4r^2+a^2}\right]\text{ directed toward the top of the page}}$$

## ANSWERS TO EVEN PROBLEMS

**P30.2** 20.0 $\mu$T

**P30.4** 200 nT

**P30.6** $\left(1+\frac{1}{\pi}\right)\frac{\mu_0 I}{2R}$ into the page

**P30.8** see the solution

**P30.10** $\left(\frac{1}{\pi}+\frac{1}{4}\right)\frac{\mu_0 I}{2r}$ into the page

| | |
|---|---|
| **P30.12** | $\frac{\mu_0 I}{12}\left(\frac{1}{a}-\frac{1}{b}\right)$ out of the page |
| **P30.14** | $\frac{\mu_0 I\left(a^2+d^2-d\sqrt{a^2+d^2}\right)}{2\pi ad\sqrt{a^2+d^2}}$ into the page |
| **P30.16** | (a) 10.0 $\mu$T; (b) 80.0 $\mu$N toward wire 1;<br>(c) 16.0 $\mu$T; (d) 80.0 $\mu$N toward wire 2 |
| **P30.18** | Parallel to the wires and 0.167 m below the upper wire |
| **P30.20** | (a) opposite; (b) 67.8 A |
| **P30.22** | 5.40 cm |
| **P30.24** | (a) 400 cm; (b) 7.50 nT; (c) 1.26 m; (d) zero |
| **P30.26** | (a) 3.60 T; (b) 1.94 T |
| **P30.28** | 500 A |
| **P30.30** | (a) see the solution; (b) $d = a$ |
| **P30.32** | (a) $\frac{\mu_0 IN}{2\ell}\left[\frac{a}{\sqrt{a^2+R^2}}-\frac{a-\ell}{\sqrt{(a-\ell)^2+R^2}}\right]$;<br>(b) see the solution |
| **P30.34** | (a) $-B\pi R^2\cos\theta$; (b) $B\pi R^2\cos\theta$ |
| **P30.36** | (a) 7.40 $\mu$Wb; (b) 2.27 $\mu$Wb |
| **P30.38** | (a) $7.19\times10^{11}$ V/m·s; (b) 200 nT |
| **P30.40** | 277 mA |
| **P30.42** | $2.97\times10^{4}\ \frac{\mathrm{K}\cdot\mathrm{J}}{\mathrm{T}^2\cdot\mathrm{m}^3}$ |
| **P30.44** | 2.02 |
| **P30.46** | (a) 12.6 $\mu$T; (b) 56.0 $\mu$T |
| **P30.48** | 2.01 GA west |
| **P30.50** | $\sim10^{-5}$ T, enough to affect the compass noticeably |
| **P30.52** | 12.0 mN$\left(-\hat{\mathbf{k}}\right)$ |
| **P30.54** | $\frac{\mu_0 q\omega}{2.5\sqrt{5}\pi R}$ |
| **P30.56** | 1.80 mT |
| **P30.58** | (a) $\mu_0\sigma v$ horizontally away from you;<br>(b) 0; (c) $\frac{1}{2}\mu_0\sigma^2 v^2$ up; (d) $3.00\times10^{8}$ m/s |
| **P30.60** | (a) see the solution; (b) $3.20\times10^{-13}$ T;<br>(c) $1.02\times10^{-24}$ N away from the first proton;<br>(d) $2.30\times10^{-22}$ N away from the first proton |
| **P30.62** | $347\mu_0 I$/m perpendicular to the coil |
| **P30.64** | (a) $\frac{1}{2}\mu_0\sigma v$; (b) out of the page, parallel to the roller axes |
| **P30.66** | 675 A downward |
| **P30.68** | (a) see the solution; (b) 59.2 nN |
| **P30.70** | see the solution |
| **P30.72** | $\frac{4}{15}\pi\omega\rho R^5$ upward |

# 31

## Faraday's Law

**CHAPTER OUTLINE**

31.1 Faraday's Law of Induction
31.2 Motional emf
31.3 Lenz's Law
31.4 Induced emf and Electric Fields
31.5 Generators and Motors
31.6 Eddy Currents
31.7 Maxwell's Equations

### ANSWERS TO QUESTIONS

**Q31.1** Magnetic flux measures the "flow" of the magnetic field through a given area of a loop—even though the field does not actually flow. By changing the size of the loop, or the orientation of the loop and the field, one can change the magnetic flux through the loop, but the magnetic field will not change.

**Q31.2** The magnetic flux is $\Phi_B = BA\cos\theta$. Therefore the flux is maximum when **B** is perpendicular to the loop of wire and zero when there is no component of magnetic field perpendicular to the loop. The flux is zero when the loop is turned so that the field lies in the plane of its area.

**Q31.3** The force on positive charges in the bar is $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$. If the bar is moving to the left, positive charge will move downward and accumulate at the bottom end of the bar, so that an electric field will be established upward.

**Q31.4** No. The magnetic force acts within the bar, but has no influence on the forward motion of the bar.

**Q31.5** By the magnetic force law $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$: the positive charges in the moving bar will flow downward and therefore clockwise in the circuit. If the bar is moving to the left, the positive charge in the bar will flow upward and therefore counterclockwise in the circuit.

**Q31.6** We ignore mechanical friction between the bar and the rails. Moving the conducting bar through the magnetic field will force charges to move around the circuit to constitute clockwise current. The downward current in the bar feels a magnetic force to the left. Then a counterbalancing applied force to the right is required to maintain the motion.

**Q31.7** A current could be set up in the bracelet by moving the bracelet through the magnetic field, or if the field rapidly changed.

**Q31.8** Moving a magnet inside the hole of the doughnut-shaped toroid will not change the magnetic flux through any turn of wire in the toroid, and thus not induce any current.

**Q31.9** As water falls, it gains speed and kinetic energy. It then pushes against turbine blades, transferring its energy to the rotor coils of a large AC generator. The rotor of the generator turns within a strong magnetic field. Because the rotor is spinning, the magnetic flux through its turns changes in time as $\Phi_B = BA\cos\omega t$. Generated in the rotor is an induced emf of $\varepsilon = \frac{-Nd\Phi_B}{dt}$. This induced emf is the voltage driving the current in our electric power lines.

**Q31.10** Yes. Eddy currents will be induced around the circumference of the copper tube so as to fight the changing magnetic flux by the falling magnet. If a bar magnet is dropped with its north pole downwards, a ring of counterclockwise current will surround its approaching bottom end and a ring of clockwise current will surround the receding south pole at its top end. The magnetic fields created by these loops of current will exert forces on the magnet to slow the fall of the magnet quite significantly. Some of the original gravitational energy of the magnet will appear as internal energy in the walls of the tube.

**Q31.11** Yes. The induced eddy currents on the surface of the aluminum will slow the descent of the aluminum. It may fall very slowly.

**Q31.12** The maximum induced emf will increase, increasing the terminal voltage of the generator.

**Q31.13** The increasing counterclockwise current in the solenoid coil produces an upward magnetic field that increases rapidly. The increasing upward flux of this field through the ring induces an emf to produce clockwise current in the ring. The magnetic field of the solenoid has a radially outward component at each point on the ring. This field component exerts upward force on the current in the ring there. The whole ring feels a total upward force larger than its weight.

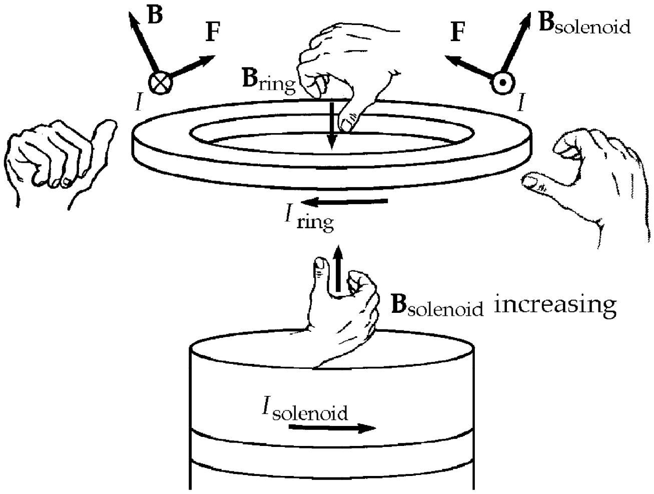


**FIG. Q31.13**

**Q31.14** Oscillating current in the solenoid produces an always-changing magnetic field. Vertical flux through the ring, alternately increasing and decreasing, produces current in it with a direction that is alternately clockwise and counterclockwise. The current through the ring's resistance produces internal energy at the rate $I^2R$.

**Q31.15** (a) The south pole of the magnet produces an upward magnetic field that increases as the magnet approaches. The loop opposes change by making its own downward magnetic field; it carries current clockwise, which goes to the left through the resistor.

(b) The north pole of the magnet produces an upward magnetic field. The loop sees decreasing upward flux as the magnet falls away, and tries to make an upward magnetic field of its own by carrying current counterclockwise, to the right in the resistor.

**Q31.16** (a) The battery makes counterclockwise current $I_1$ in the primary coil, so its magnetic field $\mathbf{B}_1$ is to the right and increasing just after the switch is closed. The secondary coil will oppose the change with a leftward field $\mathbf{B}_2$, which comes from an induced clockwise current $I_2$ that goes to the right in the resistor.

(b) At steady state the primary magnetic field is unchanging, so no emf is induced in the secondary.

(c) The primary's field is to the right and decreasing as the switch is opened. The secondary coil opposes this decrease by making its own field to the right, carrying counterclockwise current to the left in the resistor.

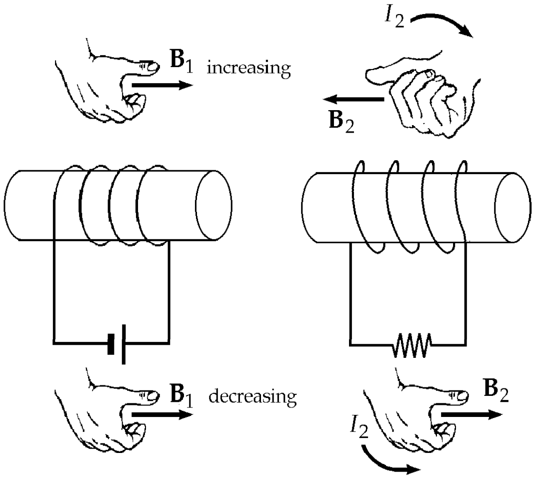


**FIG. Q31.16**

**Q31.17** The motional emf between the wingtips cannot be used to run a light bulb. To connect the light, an extra insulated wire would have to be run out along each wing, making contact with the wing tip. The wings with the extra wires and the bulb constitute a single-loop circuit. As the plane flies through a uniform magnetic field, the magnetic flux through this loop is constant and zero emf is generated. On the other hand, if the magnetic field is not uniform, a large loop towed through it will generate pulses of positive and negative emf. This phenomenon has been demonstrated with a cable unreeled from the Space Shuttle.

**Q31.18** No, they do not. Specifically, Gauss's law in magnetism prohibits magnetic monopoles. If magnetic monopoles existed, then the magnetic field lines would not have to be closed loops, but could begin or terminate on a magnetic monopole, as they can in Gauss's law in electrostatics.

**Q31.19** (a) A current is induced by the changing magnetic flux through the a ring of the tube, produced by the high frequency alternating current in the coil.

(b) The higher frequency implies a greater rate of change in the magnetic field, for a larger induced voltage.

(c) The resistance of one cubic centimeter in the bulk sheet metal is low, so the $I^2R$ rate of production of internal energy is low. At the seam, the current starts out crowded into a small area with high resistance, so the temperature rises rapidly, and the edges melt together.

(d) The edges must be in contact to allow the induced emf to create an electric current around the circumference of the tube. Additionally, (duh) the two edges must be in contact to be welded at all, just as you can't glue two pieces of paper together without putting them in contact with each other.

## SOLUTIONS TO PROBLEMS

### Section 31.1 Faraday's Law of Induction

### Section 31.3 Lenz's Law

**P31.1** $\varepsilon = \left|\dfrac{\Delta\Phi_B}{\Delta t}\right| = \dfrac{\Delta(NBA)}{\Delta t} = \boxed{500\text{ mV}}$

**P31.2** $|\varepsilon| = \left|\dfrac{\Delta\Phi_B}{\Delta t}\right| = \dfrac{\Delta(\mathbf{B}\cdot\mathbf{A})}{\Delta t} = \dfrac{(2.50\text{ T} - 0.500\text{ T})\left(8.00\times10^{-4}\text{ m}^2\right)}{1.00\text{ s}}\left(\dfrac{1\text{ N}\cdot\text{s}}{1\text{ T}\cdot\text{C}\cdot\text{m}}\right)\left(\dfrac{1\text{ V}\cdot\text{C}}{1\text{ N}\cdot\text{m}}\right)$

$|\varepsilon| = 1.60\text{ mV}$ and $I_{\text{loop}} = \dfrac{\varepsilon}{R} = \dfrac{1.60\text{ mV}}{2.00\ \Omega} = \boxed{0.800\text{ mA}}$

**P31.3** $\varepsilon = -N\dfrac{\Delta BA\cos\theta}{\Delta t} = -NB\pi r^2\left(\dfrac{\cos\theta_f - \cos\theta_i}{\Delta t}\right) = -25.0\left(50.0\times10^{-6}\text{ T}\right)\left[\pi(0.500\text{ m})^2\right]\left(\dfrac{\cos180° - \cos0°}{0.200\text{ s}}\right)$

$\varepsilon = \boxed{+9.82\text{ mV}}$

**P31.4** (a) $\varepsilon = -\dfrac{d\Phi_B}{dt} = -A\dfrac{dB}{dt} = \boxed{\dfrac{AB_{\max}}{\tau}e^{-t/\tau}}$

(b) $\varepsilon = \dfrac{\left(0.160\text{ m}^2\right)(0.350\text{ T})}{2.00\text{ s}}e^{-4.00/2.00} = \boxed{3.79\text{ mV}}$

(c) At $t = 0$ $\varepsilon = \boxed{28.0\text{ mV}}$

**P31.5** Noting unit conversions from $\mathbf{F} = q\mathbf{v}\times\mathbf{B}$ and $U = qV$, the induced voltage is

$$\varepsilon = -N\frac{d(\mathbf{B}\cdot\mathbf{A})}{dt} = -N\left(\frac{0 - B_iA\cos\theta}{\Delta t}\right) = \frac{+200(1.60\text{ T})\left(0.200\text{ m}^2\right)\cos0°}{20.0\times10^{-3}\text{ s}}\left(\frac{1\text{ N}\cdot\text{s}}{1\text{ T}\cdot\text{C}\cdot\text{m}}\right)\left(\frac{1\text{ V}\cdot\text{C}}{\text{N}\cdot\text{m}}\right) = 3\,200\text{ V}$$

$I = \dfrac{\varepsilon}{R} = \dfrac{3\,200\text{ V}}{20.0\ \Omega} = \boxed{160\text{ A}}$

**P31.6** $\varepsilon = -N\dfrac{d\Phi_B}{dt} = -\dfrac{N(BA - 0)}{\Delta t}$

$\Delta t = \dfrac{NBA}{|\varepsilon|} = \dfrac{NB\left(\pi r^2\right)}{\varepsilon} = \dfrac{500(0.200)\pi\left(5.00\times10^{-2}\right)^2}{10.0\times10^3} = \boxed{7.85\times10^{-5}\text{ s}}$

**P31.7** $$|\varepsilon| = \frac{d(BA)}{dt} = 0.500\mu_0 nA\frac{dI}{dt} = 0.480\times10^{-3}\ \text{V}$$

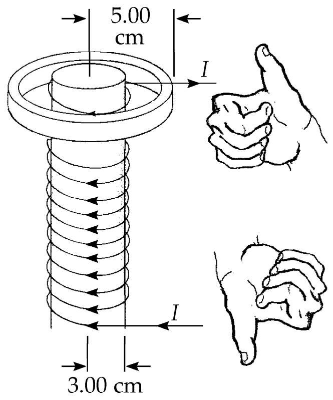


FIG. P31.7

(a) $$I_{\text{ring}} = \frac{\varepsilon}{R} = \frac{4.80\times10^{-4}}{3.00\times10^{-4}} = \boxed{1.60\ \text{A}}$$

(b) $$B_{\text{ring}} = \frac{\mu_0 I}{2r_{\text{ring}}} = \boxed{20.1\ \mu\text{T}}$$

(c) Coil's field points downward, and is increasing, so $\boxed{B_{\text{ring}}\ \text{points upward}}$.

**P31.8** $$|\varepsilon| = \frac{d(BA)}{dt} = 0.500\mu_0 nA\frac{dI}{dt} = 0.500\mu_0 n\pi r_2^2\frac{\Delta I}{\Delta t}$$

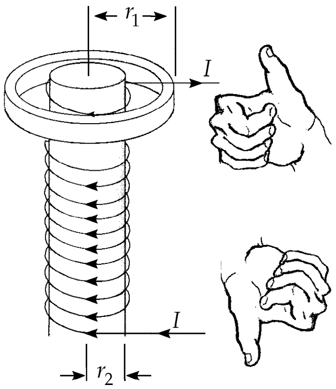


FIG. P31.8

(a) $$I_{\text{ring}} = \frac{\varepsilon}{R} = \boxed{\frac{\mu_0 n\pi r_2^2}{2R}\frac{\Delta I}{\Delta t}}$$

(b) $$B = \frac{\mu_0 I}{2r_1} = \boxed{\frac{\mu_0^2 n\pi r_2^2}{4r_1 R}\frac{\Delta I}{\Delta t}}$$

(c) The coil's field points downward, and is increasing, so $\boxed{B_{\text{ring}}\ \text{points upward}}$.

**P31.9** (a) $$d\Phi_B = \mathbf{B}\cdot d\mathbf{A} = \frac{\mu_0 I}{2\pi x}L\,dx:\ \Phi_B = \int_h^{h+w}\frac{\mu_0 IL}{2\pi}\frac{dx}{x} = \boxed{\frac{\mu_0 IL}{2\pi}\ln\left(\frac{h+w}{h}\right)}$$

(b) $$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left[\frac{\mu_0 IL}{2\pi}\ln\left(\frac{h+w}{h}\right)\right] = -\left[\frac{\mu_0 L}{2\pi}\ln\left(\frac{h+w}{h}\right)\right]\frac{dI}{dt}$$

$$\varepsilon = -\frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)(1.00\ \text{m})}{2\pi}\ln\left(\frac{1.00+10.0}{1.00}\right)(10.0\ \text{A/s}) = \boxed{-4.80\ \mu\text{V}}$$

The long wire produces magnetic flux into the page through the rectangle, shown by the first hand in the figure to the right.

As the magnetic flux increases, the rectangle produces its own magnetic field out of the page, which it does by carrying $\boxed{\text{counterclockwise}}$ current (second hand in the figure).

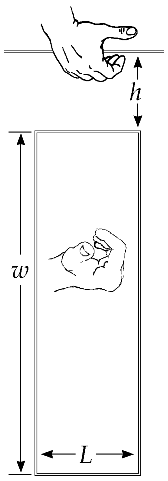


FIG. P31.9

**P31.10** $\Phi_B = (\mu_0 nI)A_{\text{solenoid}}$

$$\varepsilon = -N\frac{d\Phi_B}{dt} = -N\mu_0 n\left(\pi r_{\text{solenoid}}^2\right)\frac{dI}{dt}$$

$$\varepsilon = -15.0\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)\left(1.00\times10^{3}\text{ m}^{-1}\right)\pi(0.020\,0\text{ m})^2(600\text{ A/s})\cos(120t)$$

$$\boxed{\varepsilon = -14.2\cos(120t)\text{ mV}}$$

**P31.11** For a counterclockwise trip around the left-hand loop, with $B = At$

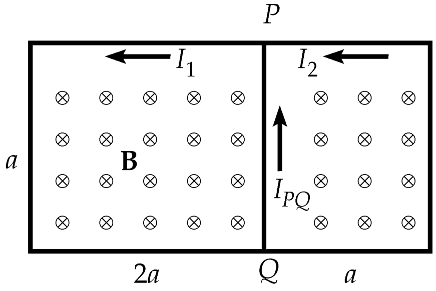


**FIG. P31.11**

$$\frac{d}{dt}\left[At\left(2a^2\right)\cos0°\right] - I_1(5R) - I_{PQ}R = 0$$

and for the right-hand loop,

$$\frac{d}{dt}\left[Ata^2\right] + I_{PQ}R - I_2(3R) = 0$$

where $I_{PQ} = I_1 - I_2$ is the upward current in $QP$.

Thus, $2Aa^2 - 5R\left(I_{PQ} + I_2\right) - I_{PQ}R = 0$

and $Aa^2 + I_{PQ}R = I_2(3R)$

$$2Aa^2 - 6RI_{PQ} - \frac{5}{3}\left(Aa^2 + I_{PQ}R\right) = 0$$

$I_{PQ} = \dfrac{Aa^2}{23R}$ upward, and since $R = (0.100\ \Omega/\text{m})(0.650\text{ m}) = 0.065\,0\ \Omega$

$$I_{PQ} = \frac{\left(1.00\times10^{-3}\text{ T/s}\right)(0.650\text{ m})^2}{23(0.065\,0\ \Omega)} = \boxed{283\ \mu\text{A upward}}.$$

**P31.12** $|\varepsilon| = \left|\dfrac{\Delta\Phi_B}{\Delta t}\right| = N\left(\dfrac{dB}{dt}\right)A = N(0.010\,0 + 0.080\,0t)A$

At $t = 5.00\text{ s}$, $|\varepsilon| = 30.0(0.410\text{ T/s})\left[\pi(0.040\,0\text{ m})^2\right] = \boxed{61.8\text{ mV}}$

**P31.13** $B = \mu_0 nI = \mu_0 n(30.0\text{ A})\left(1 - e^{-1.60t}\right)$

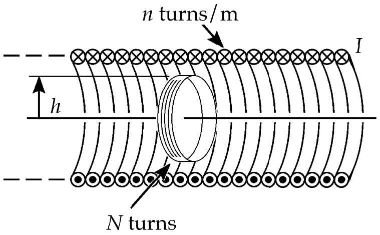


**FIG. P31.13**

$$\Phi_B = \int B\,dA = \mu_0 n(30.0\text{ A})\left(1 - e^{-1.60t}\right)\int dA$$

$$\Phi_B = \mu_0 n(30.0\text{ A})\left(1 - e^{-1.60t}\right)\pi R^2$$

$$\varepsilon = -N\frac{d\Phi_B}{dt} = -N\mu_0 n(30.0\text{ A})\pi R^2(1.60)e^{-1.60t}$$

$$\varepsilon = -(250)\left(4\pi\times10^{-7}\text{ N/A}^2\right)\left(400\text{ m}^{-1}\right)(30.0\text{ A})\left[\pi(0.060\,0\text{ m})^2\right]1.60\text{ s}^{-1}e^{-1.60t}$$

$$\varepsilon = \boxed{(68.2\text{ mV})e^{-1.60t}\text{ counterclockwise}}$$

***P31.14** (a) Each coil has a pulse of voltage tending to produce counterclockwise current as the projectile approaches, and then a pulse of clockwise voltage as the projectile recedes.

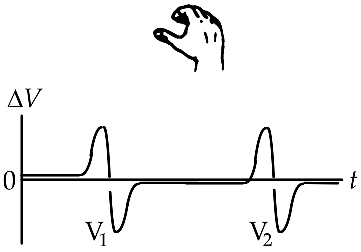


FIG. P31.14

(b) $$v = \frac{d}{t} = \frac{1.50\text{ m}}{2.40 \times 10^{-3}\text{ s}} = \boxed{625\text{ m/s}}$$

**P31.15** $$\varepsilon = \frac{d}{dt}\left(NB\ell^2 \cos\theta\right) = \frac{N\ell^2 \Delta B \cos\theta}{\Delta t}$$

$$\ell = \sqrt{\frac{\varepsilon \Delta t}{N \Delta B \cos\theta}} = \sqrt{\frac{\left(80.0 \times 10^{-3}\text{ V}\right)(0.400\text{ s})}{(50)\left(600 \times 10^{-6}\text{ T} - 200 \times 10^{-6}\text{ T}\right)\cos(30.0°)}} = 1.36\text{ m}$$

$$\text{Length } = 4\ell N = 4(1.36\text{ m})(50) = \boxed{272\text{ m}}$$

***P31.16** (a) Suppose, first, that the central wire is long and straight. The enclosed current of unknown amplitude creates a circular magnetic field around it, with the magnitude of the field given by Ampere's Law.

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I: \quad B = \frac{\mu_0 I_{\max} \sin\omega t}{2\pi R}$$

at the location of the Rogowski coil, which we assume is centered on the wire. This field passes perpendicularly through each turn of the toroid, producing flux

$$\mathbf{B} \cdot \mathbf{A} = \frac{\mu_0 I_{\max} A \sin\omega t}{2\pi R}.$$

The toroid has $2\pi Rn$ turns. As the magnetic field varies, the emf induced in it is

$$\varepsilon = -N \frac{d}{dt} \mathbf{B} \cdot \mathbf{A} = -2\pi Rn \frac{\mu_0 I_{\max} A}{2\pi R} \frac{d}{dt} \sin\omega t = -\mu_0 I_{\max} nA\omega \cos\omega t.$$

This is an alternating voltage with amplitude $\varepsilon_{\max} = \mu_0 nA\omega I_{\max}$. Measuring the amplitude determines the size $I_{\max}$ of the central current. Our assumptions that the central wire is long and straight and passes perpendicularly through the center of the Rogowski coil are all unnecessary.

(b) If the wire is not centered, the coil will respond to stronger magnetic fields on one side, but to correspondingly weaker fields on the opposite side. The emf induced in the coil is proportional to the line integral of the magnetic field around the circular axis of the toroid. Ampere's Law says that this line integral depends only on the amount of current the coil encloses. It does not depend on the shape or location of the current within the coil, or on any currents outside the coil.

**P31.17** In a toroid, all the flux is confined to the inside of the toroid.

$$B=\frac{\mu_0 NI}{2\pi r}=\frac{500\mu_0 I}{2\pi r}$$

$$\Phi_B=\int B\,dA=\frac{500\mu_0 I_{\max}}{2\pi}\sin\omega t\int\frac{a\,dr}{r}$$

$$\Phi_B=\frac{500\mu_0 I_{\max}}{2\pi}a\sin\omega t\ln\left(\frac{b+R}{R}\right)$$

$$\varepsilon=N'\frac{d\Phi_B}{dt}=20\left(\frac{500\mu_0 I_{\max}}{2\pi}\right)\omega a\ln\left(\frac{b+R}{R}\right)\cos\omega t$$

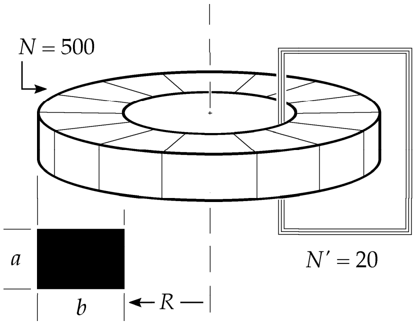


**FIG. P31.17**

$$\varepsilon=\frac{10^4}{2\pi}\left(4\pi\times10^{-7}\ \text{N/A}^2\right)(50.0\ \text{A})(377\ \text{rad/s})(0.020\,0\ \text{m})\ln\left(\frac{(3.00+4.00)\ \text{cm}}{4.00\ \text{cm}}\right)\cos\omega t$$

$$=\boxed{(0.422\ \text{V})\cos\omega t}$$

***P31.18** The upper loop has area $\pi(0.05\ \text{m})^2=7.85\times10^{-3}\ \text{m}^2$. The induced emf in it is

$$\varepsilon=-N\frac{d}{dt}BA\cos\theta=-1A\cos0°\frac{dB}{dt}=-7.85\times10^{-3}\ \text{m}^2(2\ \text{T/s})=-1.57\times10^{-2}\ \text{V}.$$

The minus sign indicates that it tends to produce counterclockwise current, to make its own magnetic field out of the page. Similarly, the induced emf in the lower loop is

$\varepsilon=-NA\cos\theta\frac{dB}{dt}=-\pi(0.09\ \text{m})^2 2\ \text{T/s}=-5.09\times10^{-2}\ \text{V}=+5.09\times10^{-2}\ \text{V}$ to produce

$\boxed{\text{counterclockwise current in the lower loop, which becomes clockwise current in the upper loop}}$.

The net emf for current in this sense around the figure 8 is

$5.09\times10^{-2}\ \text{V}-1.57\times10^{-2}\ \text{V}=3.52\times10^{-2}\ \text{V}$.

It pushes current in this sense through series resistance $[2\pi(0.05\ \text{m})+2\pi(0.09\ \text{m})]3\ \Omega/\text{m}=2.64\ \Omega$.

The current is $I=\frac{\varepsilon}{R}=\frac{3.52\times10^{-2}\ \text{V}}{2.64\ \Omega}=\boxed{13.3\ \text{mA}}$.

## Section 31.2 **Motional emf**

## Section 31.3 **Lenz's Law**

**P31.19** (a) For maximum induced emf, with positive charge at the top of the antenna, $\mathbf{F}_+=q_+(\mathbf{v}\times\mathbf{B})$, so the auto must move $\boxed{\textit{east}}$.

(b) $\varepsilon=B\ell v=\left(5.00\times10^{-5}\ \text{T}\right)(1.20\ \text{m})\left(\frac{65.0\times10^3\ \text{m}}{3\ 600\ \text{s}}\right)\cos65.0°=\boxed{4.58\times10^{-4}\ \text{V}}$

**P31.20** $I=\frac{\varepsilon}{R}=\frac{B\ell v}{R}$

$\boxed{v=1.00\ \text{m/s}}$

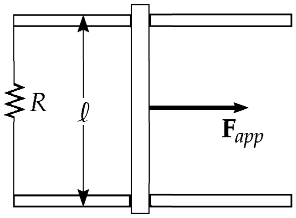


FIG. P31.20

**P31.21** (a) $|\mathbf{F}_B|=I|\ell\times\mathbf{B}|=I\ell B$

When $I=\frac{\varepsilon}{R}$

and $\varepsilon=B\ell v$

we get $F_B=\frac{B\ell v}{R}(\ell B)=\frac{B^2\ell^2 v}{R}=\frac{(2.50)^2(1.20)^2(2.00)}{6.00}=3.00\ \text{N}$.

The applied force is $\boxed{3.00\ \text{N to the right}}$.

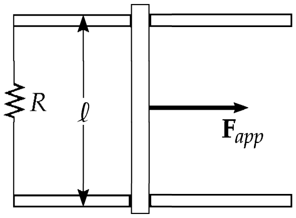


FIG. P31.21

(b) $\mathcal{P}=I^2R=\frac{B^2\ell^2v^2}{R}=6.00\ \text{W}$ or $\mathcal{P}=Fv=\boxed{6.00\ \text{W}}$

**P31.22** $F_B=I\ell B$ and $\varepsilon=B\ell v$

$I=\frac{\varepsilon}{R}=\frac{B\ell v}{R}$ so $B=\frac{IR}{\ell v}$

(a) $F_B=\frac{I^2\ell R}{\ell v}$ and $I=\sqrt{\frac{F_B v}{R}}=\boxed{0.500\ \text{A}}$

(b) $I^2R=\boxed{2.00\ \text{W}}$

(c) For constant force, $\mathcal{P}=\mathbf{F}\cdot\mathbf{v}=(1.00\ \text{N})(2.00\ \text{m/s})=\boxed{2.00\ \text{W}}$.

***P31.23** Model the magnetic flux inside the metallic tube as constant as it shrinks form radius $R$ to radius $r$:

$2.50\ \text{T}\left(\pi R^2\right)=B_f\pi r^2$

$B_f=2.50\ \text{T}\left(\frac{R}{r}\right)^2=2.50\ \text{T}(12)^2=\boxed{360\ \text{T}}$

***P31.24** Observe that the homopolar generator has no commutator and produces a voltage constant in time: DC with no ripple. In time $dt$, the disk turns by angle $d\theta = \omega dt$. The outer brush slides over distance $rd\theta$. The radial line to the outer brush sweeps over area

$$dA = \frac{1}{2} r r d\theta = \frac{1}{2} r^2 \omega dt.$$

The emf generated is $\varepsilon = -N \frac{d}{dt} \mathbf{B} \cdot \mathbf{A}$

$$\varepsilon = -(1) B \cos 0° \frac{dA}{dt} = -B\left(\frac{1}{2} r^2 \omega\right)$$

(We could think of this as following from the result of Example 31.4.)
The magnitude of the emf is

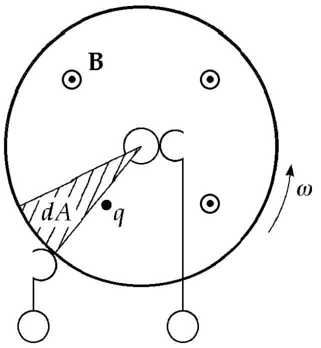


**FIG. P31.24**

$$|\varepsilon| = B\left(\frac{1}{2} r^2 \omega\right) = (0.9 \text{ N} \cdot \text{s/C} \cdot \text{m})\left[\frac{1}{2}(0.4 \text{ m})^2 (3\,200 \text{ rev/min})\right]\left(\frac{2\pi \text{ rad/rev}}{60 \text{ s/min}}\right)$$

$$|\varepsilon| = \boxed{24.1 \text{ V}}$$

A free positive charge $q$ shown, turning with the disk, feels a magnetic force $q\mathbf{v} \times \mathbf{B}$ radially outward. Thus the $\boxed{\text{outer contact is positive}}$.

***P31.25** The speed of waves on the wire is

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{267 \text{ N} \cdot \text{m}}{3 \times 10^{-3} \text{ kg}}} = 298 \text{ m/s}.$$

In the simplest standing-wave vibration state,

$$d_{NN} = 0.64 \text{ m} = \frac{\lambda}{2} \qquad \lambda = 1.28 \text{ m}$$

and $f = \frac{v}{\lambda} = \frac{298 \text{ m/s}}{1.28 \text{ m}} = 233 \text{ Hz}$.

(a) The changing flux of magnetic field through the circuit containing the wire will drive current to the left in the wire as it moves up and to the right as it moves down. The emf will have this same frequency of $\boxed{233 \text{ Hz}}$.

(b) The vertical coordinate of the center of the wire is described by

$x = A \cos \omega t = (1.5 \text{ cm}) \cos(2\pi 233\, t/\text{s})$.

Its velocity is $v = \frac{dx}{dt} = -(1.5 \text{ cm})(2\pi\, 233/\text{s}) \sin(2\pi 233\ t/\text{s})$.

Its maximum speed is $1.5 \text{ cm}(2\pi)233/\text{s} = 22.0 \text{ m/s}$.

The induced emf is $\varepsilon = -B\ell v$, with amplitude

$$\varepsilon_{\max} = B\ell v_{\max} = 4.50 \times 10^{-3} \text{ T}(0.02 \text{ m})22 \text{ m/s} = \boxed{1.98 \times 10^{-3} \text{ V}}.$$

**P31.26** $$\varepsilon = -N\frac{d}{dt}BA\cos\theta = -NB\cos\theta\left(\frac{\Delta A}{\Delta t}\right)$$

$$\varepsilon = -1(0.100\text{ T})\cos 0°\,\frac{(3.00\text{ m}\times 3.00\text{ m}\sin 60.0°)-(3.00\text{ m})^2}{0.100\text{ s}} = 1.21\text{ V}$$

$$I = \frac{1.21\text{ V}}{10.0\ \Omega} = \boxed{0.121\text{ A}}$$

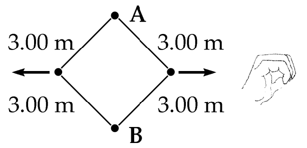


FIG. P31.26

The flux is into the page and decreasing. The loop makes its own magnetic field into the page by carrying $\boxed{\text{clockwise}}$ current.

**P31.27** $$\omega = (2.00\text{ rev/s})(2\pi\text{ rad/rev}) = 4.00\pi\text{ rad/s}$$

$$\varepsilon = \frac{1}{2}B\omega\ell^2 = \boxed{2.83\text{ mV}}$$

**P31.28** (a) $\mathbf{B}_{ext} = B_{ext}\hat{\mathbf{i}}$ and $B_{ext}$ decreases; therefore, the induced field is $\mathbf{B}_0 = B_0\hat{\mathbf{i}}$ (to the right) and the current in the resistor is directed $\boxed{\text{to the right}}$.

(b) $\mathbf{B}_{ext} = B_{ext}\left(-\hat{\mathbf{i}}\right)$ increases; therefore, the induced field $\mathbf{B}_0 = B_0\left(+\hat{\mathbf{i}}\right)$ is to the right, and the current in the resistor is directed $\boxed{\text{to the right}}$.

(c) $\mathbf{B}_{ext} = B_{ext}\left(-\hat{\mathbf{k}}\right)$ into the paper and $B_{ext}$ decreases; therefore, the induced field is $\mathbf{B}_0 = B_0\left(-\hat{\mathbf{k}}\right)$ into the paper, and the current in the resistor is directed $\boxed{\text{to the right}}$.

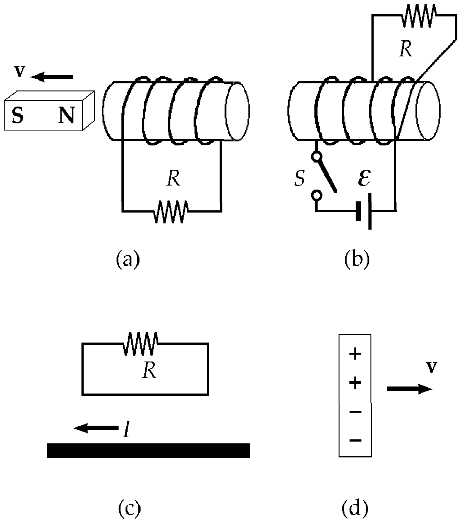


FIG. P31.28

(d) By the magnetic force law, $F_B = q(\mathbf{v}\times\mathbf{B})$. Therefore, a positive charge will move to the top of the bar if $\mathbf{B}$ is $\boxed{\text{into the paper}}$.

**P31.29** (a) The force on the side of the coil entering the field (consisting of $N$ wires) is

$$F = N(ILB) = N(IwB).$$

The induced emf in the coil is

$$|\varepsilon| = N\frac{d\Phi_B}{dt} = N\frac{d(Bwx)}{dt} = NBwv.$$

so the current is $I = \frac{|\varepsilon|}{R} = \frac{NBwv}{R}$ counterclockwise.

The force on the leading side of the coil is then:

$$F = N\left(\frac{NBwv}{R}\right)wB = \boxed{\frac{N^2B^2w^2v}{R}\text{ to the left}}.$$

(b) Once the coil is entirely inside the field, $\Phi_B = NBA = \text{constant}$,

so $\varepsilon = 0$, $I = 0$, and $F = \boxed{0}$.

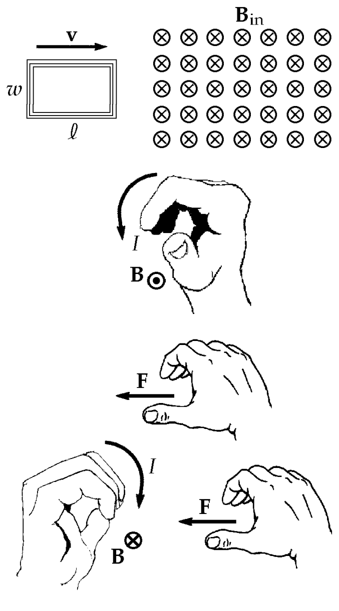


**FIG. P31.29**

(c) As the coil starts to leave the field, the flux *decreases* at the rate $Bwv$, so the magnitude of the current is the same as in part (a), but now the current is clockwise. Thus, the force exerted on the trailing side of the coil is:

$$F = \boxed{\frac{N^2B^2w^2v}{R}\text{ to the left again}}.$$

**P31.30** Look in the direction of *ba*. The bar magnet creates a field into the page, and the field increases. The loop will create a field out of the page by carrying a counterclockwise current. Therefore, current must flow from $b$ to $a$ through the resistor. Hence, $V_a - V_b$ will be $\boxed{\text{negative}}$.

**P31.31** Name the currents as shown in the diagram:

Left loop: $+Bdv_2 - I_2R_2 - I_1R_1 = 0$

Right loop: $+Bdv_3 - I_3R_3 + I_1R_1 = 0$

At the junction: $I_2 = I_1 + I_3$

Then, $Bdv_2 - I_1R_2 - I_3R_2 - I_1R_1 = 0$

$$I_3 = \frac{Bdv_3}{R_3} + \frac{I_1R_1}{R_3}.$$

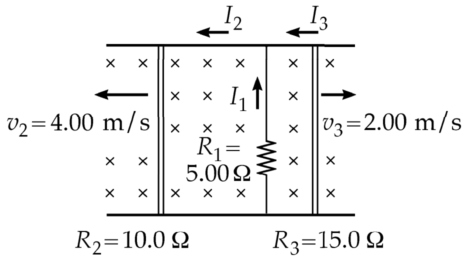


**FIG. P31.31**

So, $$Bdv_2 - I_1(R_1 + R_2) - \frac{Bdv_3R_2}{R_3} - \frac{I_1R_1R_2}{R_3} = 0$$

$$I_1 = Bd\left(\frac{v_2R_3 - v_3R_2}{R_1R_2 + R_1R_3 + R_2R_3}\right)\text{ upward}$$

$$I_1 = (0.010\,0\text{ T})(0.100\text{ m})\left[\frac{(4.00\text{ m/s})(15.0\ \Omega) - (2.00\text{ m/s})(10.0\ \Omega)}{(5.00\ \Omega)(10.0\ \Omega) + (5.00\ \Omega)(15.0\ \Omega) + (10.0\ \Omega)(15.0\ \Omega)}\right] = \boxed{145\ \mu\text{A}}\text{ upward.}$$

## Section 31.4 Induced emf and Electric Fields

**P31.32** (a) $\dfrac{dB}{dt} = 6.00t^2 - 8.00t$ $\qquad |\varepsilon| = \dfrac{d\Phi_B}{dt}$

At $t = 2.00$ s, $\qquad E = \dfrac{\pi R^2 (dB/dt)}{2\pi r_2} = \dfrac{8.00\pi(0.025\,0)^2}{2\pi(0.050\,0)}$

$$F = qE = \boxed{8.00 \times 10^{-21}\ \text{N}}$$

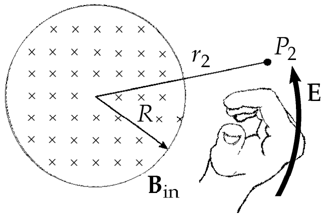


FIG. P31.32

(b) When $6.00t^2 - 8.00t = 0$, $\quad t = \boxed{1.33\ \text{s}}$

**P31.33** $\dfrac{dB}{dt} = 0.060\,0t$ $\qquad |\varepsilon| = \dfrac{d\Phi_B}{dt} = \pi r_1^2 \dfrac{dB}{dt} = 2\pi r_1 E$

At $t = 3.00$ s,

$$E = \left(\frac{\pi r_1^2}{2\pi r_1}\right)\frac{dB}{dt} = \frac{0.020\,0\ \text{m}}{2}\left(0.060\,0\ \text{T/s}^2\right)(3.00\ \text{s})\left(\frac{1\ \text{N}\cdot\text{s}}{1\ \text{T}\cdot\text{C}\cdot\text{m}}\right)$$

$$\mathbf{E} = \boxed{1.80 \times 10^{-3}\ \text{N/C perpendicular to } r_1 \text{ and counterclockwise}}$$

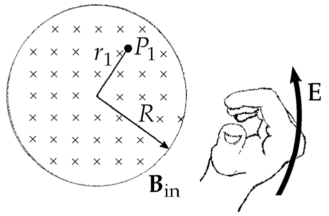


FIG. P31.33

**P31.34** (a) $\displaystyle\oint \mathbf{E} \cdot d\ell = \left|\frac{d\Phi_B}{dt}\right|$

$2\pi r E = \left(\pi r^2\right)\dfrac{dB}{dt}$ $\qquad$ so $\qquad E = \boxed{(9.87\ \text{mV/m})\cos(100\pi t)}$

(b) The $E$ field is always opposite to increasing $B$. $\therefore$ $\boxed{\text{clockwise}}$.

## Section 31.5 Generators and Motors

**P31.35** (a) $\varepsilon_{\max} = NAB\omega = (1\,000)(0.100)(0.200)(120\pi) = \boxed{7.54\ \text{kV}}$

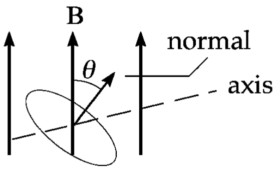


FIG. P31.35

(b) $\varepsilon(t) = NBA\omega \sin \omega t = NBA\omega \sin\theta$

$|\varepsilon|$ is maximal when $|\sin\theta| = 1$

or $\theta = \pm\dfrac{\pi}{2}$

so the $\boxed{\text{plane of coil is parallel to } \mathbf{B}}$.

**P31.36** For the alternator, $\omega = (3\,000\ \text{rev/min})\left(\dfrac{2\pi\,\text{rad}}{1\ \text{rev}}\right)\left(\dfrac{1\ \text{min}}{60\ \text{s}}\right) = 314\ \text{rad/s}$

$$\varepsilon = -N\frac{d\Phi_B}{dt} = -250\frac{d}{dt}\left[\left(2.50\times10^{-4}\ \text{T}\cdot\text{m}^2\right)\cos(314t/\text{s})\right] = +250\left(2.50\times10^{-4}\ \text{T}\cdot\text{m}^2\right)(314/\text{s})\sin(314t)$$

(a) $\boxed{\varepsilon = (19.6\ \text{V})\sin(314t)}$

(b) $\boxed{\varepsilon_{\text{max}} = 19.6\ \text{V}}$

**P31.37** $B = \mu_0 nI = \left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(200\ \text{m}^{-1}\right)(15.0\ \text{A}) = 3.77\times10^{-3}\ \text{T}$

For the small coil, $\Phi_B = N\mathbf{B}\cdot\mathbf{A} = NBA\cos\omega t = NB\left(\pi r^2\right)\cos\omega t$.

Thus, $\varepsilon = -\dfrac{d\Phi_B}{dt} = NB\pi r^2\omega\sin\omega t$

$$\varepsilon = (30.0)\left(3.77\times10^{-3}\ \text{T}\right)\pi(0.080\,0\ \text{m})^2\left(4.00\pi\,\text{s}^{-1}\right)\sin(4.00\pi t) = \boxed{(28.6\ \text{mV})\sin(4.00\pi t)}\,.$$

**P31.38** As the magnet rotates, the flux through the coil varies sinusoidally in time with $\Phi_B = 0$ at $t = 0$. Choosing the flux as positive when the field passes from left to right through the area of the coil, the flux at any time may be written as $\Phi_B = -\Phi_{\text{max}}\sin\omega t$ so the induced emf is given by

$$\varepsilon = -\frac{d\Phi_B}{dt} = \omega\Phi_{\text{max}}\cos\omega t\,.$$

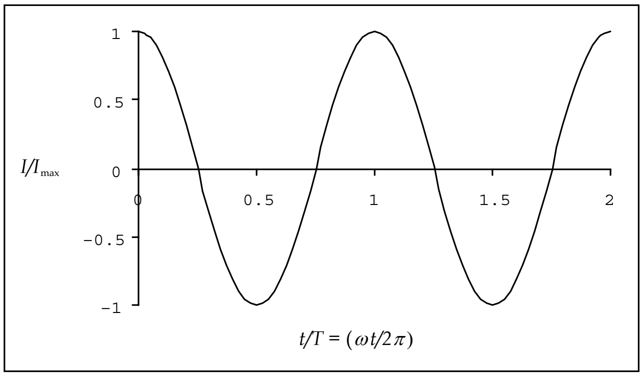
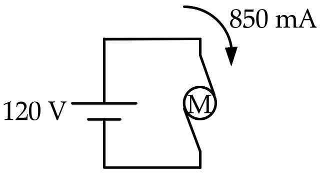


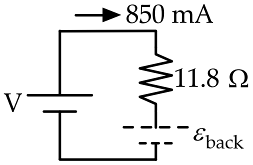
FIG. P31.38

The current in the coil is then $I = \dfrac{\varepsilon}{R} = \dfrac{\omega\Phi_{\text{max}}}{R}\cos\omega t = \boxed{I_{\text{max}}\cos\omega t}\,.$

***P31.39**



To analyze the actual circuit, we model it as



.

(a) The loop rule gives $+120\ \text{V} - 0.85\ \text{A}(11.8\ \Omega) - \varepsilon_{\text{back}} = 0$ $\qquad \varepsilon_{\text{back}} = \boxed{110\ \text{V}}$.

(b) The resistor is the device changing electrical work input into internal energy: $\mathcal{P} = I^2R = (0.85\ \text{A})^2(11.8\ \Omega) = \boxed{8.53\ \text{W}}$.

(c) With no motion, the motor does not function as a generator, and $\varepsilon_{\text{back}} = 0$. Then

$120\ \text{V} - I_c(11.8\ \Omega) = 0 \qquad I_c = 10.2\ \text{A}$

$\mathcal{P}_c = I_c^2R = (10.2\ \text{A})^2(11.8\ \Omega) = \boxed{1.22\ \text{kW}}$

**P31.40** (a) $\varepsilon_{\max} = BA\omega = B\left(\frac{1}{2}\pi R^2\right)\omega$

$$\varepsilon_{\max} = (1.30\text{ T})\frac{\pi}{2}(0.250\text{ m})^2(4.00\pi\text{ rad/s})$$

$$\varepsilon_{\max} = \boxed{1.60\text{ V}}$$

(b) $\bar{\varepsilon} = \int_0^{2\pi} \frac{\varepsilon}{2\pi} d\theta = \frac{BA\omega}{2\pi}\int_0^{2\pi} \sin\theta d\theta = \boxed{0}$

(c) The maximum and average $\varepsilon$ would remain unchanged.

(d) See Figure 1 at the right.

(e) See Figure 2 at the right.

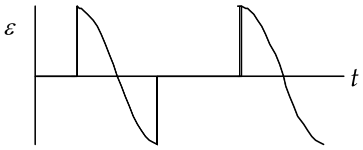


Figure 1

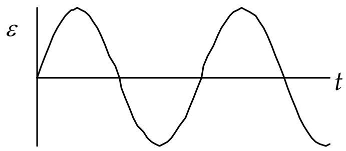


Figure 2

**FIG. P31.40**

**P31.41** (a) $\Phi_B = BA\cos\theta = BA\cos\omega t = (0.800\text{ T})\left(0.010\,0\text{ m}^2\right)\cos 2\pi(60.0)t = \boxed{\left(8.00\text{ mT}\cdot\text{m}^2\right)\cos(377t)}$

(b) $\varepsilon = -\frac{d\Phi_B}{dt} = \boxed{(3.02\text{ V})\sin(377t)}$

(c) $I = \frac{\varepsilon}{R} = \boxed{(3.02\text{ A})\sin(377t)}$

(d) $\mathscr{P} = I^2 R = \boxed{(9.10\text{ W})\sin^2(377t)}$

(e) $\mathscr{P} = Fv = \tau\omega$ so $\tau = \frac{\mathscr{P}}{\omega} = \boxed{(24.1\text{ mN}\cdot\text{m})\sin^2(377t)}$

---

## Section 31.6 **Eddy Currents**

**P31.42** The current in the magnet creates an upward magnetic field, so the N and S poles on the solenoid core are shown correctly. On the rail in front of the brake, the upward flux of **B** increases as the coil approaches, so a current is induced here to create a downward magnetic field. This is clockwise current, so the S pole on the rail is shown correctly. On the rail behind the brake, the upward magnetic flux is decreasing. The induced current in the rail will produce upward magnetic field by being counterclockwise as the picture correctly shows.

**P31.43** (a) At terminal speed,

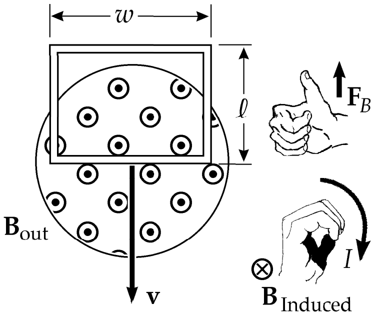


**FIG. P31.43**

$$Mg = F_B = IwB = \left(\frac{\varepsilon}{R}\right)wB = \left(\frac{Bwv_T}{R}\right)wB = \frac{B^2w^2v_T}{R}$$

or $$\boxed{v_T = \frac{MgR}{B^2\omega^2}}.$$

(b) The emf is directly proportional to $v_T$, but the current is inversely proportional to $R$. A large $R$ means a small current at a given speed, so the loop must travel faster to get $F_B = mg$.

(c) At a given speed, the current is directly proportional to the magnetic field. But the force is proportional to the product of the current and the field. For a small $B$, the speed must increase to compensate for both the small $B$ and also the current, so $v_T \propto B^2$.

## Section 31.7 **Maxwell's Equations**

**P31.44** $\mathbf{F} = m\mathbf{a} = q\mathbf{E} + q\mathbf{v}\times\mathbf{B}$ so $\mathbf{a} = \frac{-e}{m}[\mathbf{E} + \mathbf{v}\times\mathbf{B}]$ where $\mathbf{v}\times\mathbf{B} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 10.0 & 0 & 0 \\ 0 & 0 & 0.400 \end{vmatrix} = -4.00\hat{\mathbf{j}}$

$$\mathbf{a} = \frac{\left(-1.60\times10^{-19}\right)}{9.11\times10^{-31}}\left[2.50\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}} - 4.00\hat{\mathbf{j}}\right] = \left(-1.76\times10^{11}\right)\left[2.50\hat{\mathbf{i}} + 1.00\hat{\mathbf{j}}\right]$$

$$\mathbf{a} = \boxed{\left(-4.39\times10^{11}\,\hat{\mathbf{i}} - 1.76\times10^{11}\,\hat{\mathbf{j}}\right)\ \text{m/s}^2}$$

**P31.45** $\mathbf{F} = m\mathbf{a} = q\mathbf{E} + q\mathbf{v}\times\mathbf{B}$

$$\mathbf{a} = \frac{e}{m}[\mathbf{E} + \mathbf{v}\times\mathbf{B}] \text{ where } \mathbf{v}\times\mathbf{B} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 200 & 0 & 0 \\ 0.200 & 0.300 & 0.400 \end{vmatrix} = -200(0.400)\hat{\mathbf{j}} + 200(0.300)\hat{\mathbf{k}}$$

$$\mathbf{a} = \frac{1.60\times10^{-19}}{1.67\times10^{-27}}\left[50.0\hat{\mathbf{j}} - 80.0\hat{\mathbf{j}} + 60.0\hat{\mathbf{k}}\right] = 9.58\times10^{7}\left[-30.0\hat{\mathbf{j}} + 60.0\hat{\mathbf{k}}\right]$$

$$\mathbf{a} = 2.87\times10^{9}\left[-\hat{\mathbf{j}} + 2\hat{\mathbf{k}}\right]\ \text{m/s}^2 = \boxed{\left(-2.87\times10^{9}\,\hat{\mathbf{j}} + 5.75\times10^{9}\,\hat{\mathbf{k}}\right)\ \text{m/s}^2}$$

## Additional Problems

**P31.46** $$\varepsilon = -N\frac{d}{dt}(BA\cos\theta) = -N\left(\pi r^2\right)\cos 0^\circ\left(\frac{dB}{dt}\right)$$

$$\varepsilon = -(30.0)\left[\pi\left(2.70\times10^{-3}\ \text{m}\right)^2\right](1)\frac{d}{dt}\left[50.0\ \text{mT} + (3.20\ \text{mT})\sin\left(2\pi\left[523t\ \text{s}^{-1}\right]\right)\right]$$

$$\varepsilon = -(30.0)\left[\pi\left(2.70\times10^{-3}\ \text{m}\right)^2\right]\left(3.20\times10^{-3}\ \text{T}\right)\left[2\pi\left(523\ \text{s}^{-1}\right)\cos\left(2\pi\left[523t\ \text{s}^{-1}\right]\right)\right]$$

$$\varepsilon = \boxed{-\left(7.22\times10^{-3}\ \text{V}\right)\cos\left[2\pi\left(523t\ \text{s}^{-1}\right)\right]}$$

**P31.47** (a) Doubling the number of turns.

$\boxed{\text{Amplitude doubles: period unchanged}}$

(b) Doubling the angular velocity.

$\boxed{\text{doubles the amplitude: cuts the period in half}}$

(c) Doubling the angular velocity while reducing the number of turns to one half the original value.

$\boxed{\text{Amplitude unchanged: cuts the period in half}}$

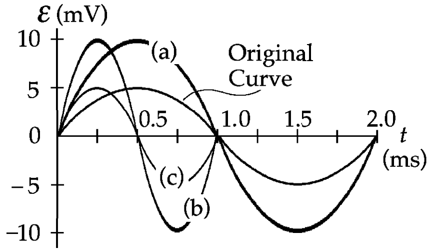


FIG. P31.47

**P31.48** $$\varepsilon = -N\frac{\Delta}{\Delta t}(BA\cos\theta) = -N\left(\pi r^2\right)\cos 0°\frac{\Delta B}{\Delta t} = -1\left(0.005\,00\text{ m}^2\right)(1)\left(\frac{1.50\text{ T} - 5.00\text{ T}}{20.0\times 10^{-3}\text{ s}}\right) = 0.875\text{ V}$$

(a) $I = \dfrac{\varepsilon}{R} = \dfrac{0.875\text{ V}}{0.020\,0\ \Omega} = \boxed{43.8\text{ A}}$

(b) $\mathcal{P} = \varepsilon I = (0.875\text{ V})(43.8\text{ A}) = \boxed{38.3\text{ W}}$

**P31.49** In the loop on the left, the induced emf is

$$|\varepsilon| = \frac{d\Phi_B}{dt} = A\frac{dB}{dt} = \pi(0.100\text{ m})^2(100\text{ T/s}) = \pi\text{ V}$$

and it attempts to produce a counterclockwise current in this loop.

In the loop on the right, the induced emf is

$$|\varepsilon| = \frac{d\Phi_B}{dt} = \pi(0.150\text{ m})^2(100\text{ T/s}) = 2.25\pi\text{ V}$$

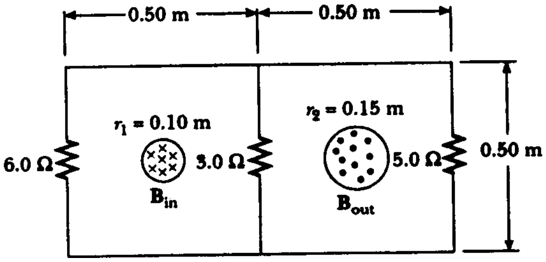


FIG. P31.49

and it attempts to produce a clockwise current. Assume that $I_1$ flows down through the 6.00-Ω resistor, $I_2$ flows down through the 5.00-Ω resistor, and that $I_3$ flows up through the 3.00-Ω resistor.

From Kirchhoff's junction rule: $I_3 = I_1 + I_2$ (1)

Using the loop rule on the left loop: $6.00I_1 + 3.00I_3 = \pi$ (2)

Using the loop rule on the right loop: $5.00I_2 + 3.00I_3 = 2.25\pi$ (3)

Solving these three equations simultaneously,

$I_1 = \boxed{0.062\,3\text{ A}}$, $I_2 = \boxed{0.860\text{ A}}$, and $I_3 = \boxed{0.923\text{ A}}$.

**P31.50** The emf induced between the ends of the moving bar is

$$\varepsilon = B\ell v = (2.50\text{ T})(0.350\text{ m})(8.00\text{ m/s}) = 7.00\text{ V}.$$

The left-hand loop contains decreasing flux away from you, so the induced current in it will be clockwise, to produce its own field directed away from you. Let $I_1$ represent the current flowing upward through the 2.00-Ω resistor. The right-hand loop will carry counterclockwise current. Let $I_3$ be the upward current in the 5.00-Ω resistor.

(a) Kirchhoff's loop rule then gives: $+7.00\text{ V} - I_1(2.00\ \Omega) = 0$ $\quad I_1 = \boxed{3.50\text{ A}}$

and $+7.00\text{ V} - I_3(5.00\ \Omega) = 0$ $\quad I_3 = \boxed{1.40\text{ A}}$.

(b) The total power dissipated in the resistors of the circuit is

$$P = \varepsilon I_1 + \varepsilon I_3 = \varepsilon(I_1 + I_3) = (7.00\text{ V})(3.50\text{ A} + 1.40\text{ A}) = \boxed{34.3\text{ W}}.$$

(c) *Method 1*: The current in the sliding conductor is downward with value $I_2 = 3.50\text{ A} + 1.40\text{ A} = 4.90\text{ A}$. The magnetic field exerts a force of $F_m = I\ell B = (4.90\text{ A})(0.350\text{ m})(2.50\text{ T}) = 4.29\text{ N}$ directed toward the right on this conductor. An outside agent must then exert a force of $\boxed{4.29\text{ N}}$ to the left to keep the bar moving.

*Method 2*: The agent moving the bar must supply the power according to $\mathcal{P} = \mathbf{F}\cdot\mathbf{v} = Fv\cos 0°$. The force required is then:

$$F = \frac{\mathcal{P}}{v} = \frac{34.3\text{ W}}{8.00\text{ m/s}} = \boxed{4.29\text{ N}}.$$

**P31.51** Suppose we wrap twenty turns of wire into a flat compact circular coil of diameter 3 cm. Suppose we use a bar magnet to produce field $10^{-3}$ T through the coil in one direction along its axis. Suppose we then flip the magnet to reverse the flux in $10^{-1}$ s. The average induced emf is then

$$\bar{\varepsilon} = -N\frac{\Delta\Phi_B}{\Delta t} = -N\frac{\Delta[BA\cos\theta]}{\Delta t} = -NB(\pi r^2)\left(\frac{\cos 180° - \cos 0°}{\Delta t}\right)$$

$$\bar{\varepsilon} = -(20)(10^{-3}\text{ T})\pi(0.015\,0\text{ m})^2\left(\frac{-2}{10^{-1}\text{ s}}\right)\boxed{\sim 10^{-4}\text{ V}}$$

**P31.52** $I = \dfrac{\varepsilon + \varepsilon_{\text{induced}}}{R}$ and $\varepsilon_{\text{induced}} = -\dfrac{d}{dt}(BA)$

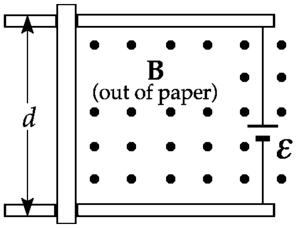


FIG. P31.52

$$F = m\frac{dv}{dt} = IBd$$

$$\frac{dv}{dt} = \frac{IBd}{m} = \frac{Bd}{mR}\left(\varepsilon + \varepsilon_{\text{induced}}\right)$$

$$\frac{dv}{dt} = \frac{Bd}{mR}(\varepsilon - Bvd)$$

To solve the differential equation, let $u = \varepsilon - Bvd$

$$\frac{du}{dt} = -Bd\frac{dv}{dt}$$

$$-\frac{1}{Bd}\frac{du}{dt} = \frac{Bd}{mR}u$$

so $\displaystyle\int_{u_0}^{u}\frac{du}{u} = -\int_0^t \frac{(Bd)^2}{mR}dt$.

Integrating from $t = 0$ to $t = t$, $\ln\dfrac{u}{u_0} = -\dfrac{(Bd)^2}{mR}t$

or $\dfrac{u}{u_0} = e^{-B^2d^2t/mR}$.

Since $v = 0$ when $t = 0$, $u_0 = \varepsilon$

and $u = \varepsilon - Bvd$

$$\varepsilon - Bvd = \varepsilon e^{-B^2d^2t/mR}.$$

Therefore, $\boxed{v = \dfrac{\varepsilon}{Bd}\left(1 - e^{-B^2d^2t/mR}\right)}$.

***P31.53** The enclosed flux is $\Phi_B = BA = B\pi r^2$.

The particle moves according to $\sum \mathbf{F} = m\mathbf{a}$: $qvB\sin 90° = \dfrac{mv^2}{r}$

$$r = \frac{mv}{qB}.$$

Then $\Phi_B = \dfrac{B\pi m^2v^2}{q^2B^2}$.

(a) $$v = \sqrt{\frac{\Phi_B q^2 B}{\pi m^2}} = \sqrt{\frac{\left(15\times10^{-6}\ \text{T}\cdot\text{m}^2\right)\left(30\times10^{-9}\ \text{C}\right)^2(0.6\ \text{T})}{\pi\left(2\times10^{-16}\ \text{kg}\right)^2}} = \boxed{2.54\times10^5\ \text{m/s}}$$

(b) Energy for the particle-electric field system is conserved in the firing process:

$U_i = K_f$: $q\Delta V = \dfrac{1}{2}mv^2$

$$\Delta V = \frac{mv^2}{2q} = \frac{\left(2\times10^{-16}\ \text{kg}\right)\left(2.54\times10^5\ \text{m/s}\right)^2}{2\left(30\times10^{-9}\ \text{C}\right)} = \boxed{215\ \text{V}}.$$

***P31.54** (a) Consider an annulus of radius $r$, width $dr$, height $b$, and resistivity $\rho$. Around its circumference, a voltage is induced according to

$$\varepsilon = -N\frac{d}{dt}\mathbf{B}\cdot\mathbf{A} = -1\frac{d}{dt}B_{\max}(\cos\omega t)\pi r^2 = +B_{\max}\pi r^2\omega\sin\omega t.$$

The resistance around the loop is $\frac{\rho\ell}{A_x} = \frac{\rho(2\pi r)}{bdr}$.

The eddy current in the ring is $dI = \frac{\varepsilon}{\text{resistance}} = \frac{B_{\max}\pi r^2\omega(\sin\omega t)bdr}{\rho(2\pi r)} = \frac{B_{\max}rb\omega dr\sin\omega t}{2\rho}$.

The instantaneous power is $d\mathcal{P}_i = \varepsilon\, dI = \frac{B_{\max}^2\pi r^3 b\omega^2 dr\sin^2\omega t}{2\rho}$.

The time average of the function $\sin^2\omega t = \frac{1}{2} - \frac{1}{2}\cos 2\omega t$ is $\frac{1}{2} - 0 = \frac{1}{2}$

so the time-averaged power delivered to the annulus is

$$d\mathcal{P} = \frac{B_{\max}^2\pi r^3 b\omega^2 dr}{4\rho}.$$

The power delivered to the disk is $\mathcal{P} = \int d\mathcal{P} = \int_0^R \frac{B_{\max}^2\pi b\omega^2}{4\rho}r^3 dr$

$$\mathcal{P} = \frac{B_{\max}^2\pi b\omega^2}{4\rho}\left(\frac{R^4}{4} - 0\right) = \boxed{\frac{\pi B_{\max}^2 R^4 b\omega^2}{16\rho}}.$$

(b) When $B_{\max}$ gets two times larger, $B_{\max}^2$ and $\mathcal{P}$ get $\boxed{4}$ times larger.

(c) When $f$ and $\omega = 2\pi f$ double, $\omega^2$ and $\mathcal{P}$ get $\boxed{4}$ times larger.

(d) When $R$ doubles, $R^4$ and $\mathcal{P}$ become $2^4 = \boxed{16}$ times larger.

**P31.55** $I = \frac{\varepsilon}{R} = \frac{B}{R}\frac{|A|}{\Delta t}$

so $q = I\Delta t = \frac{(15.0\ \mu\text{T})(0.200\ \text{m})^2}{0.500\ \Omega} = \boxed{1.20\ \mu\text{C}}$

**P31.56** (a) $I = \frac{dq}{dt} = \frac{\varepsilon}{R}$ where $\varepsilon = -N\frac{d\Phi_B}{dt}$ so $\int dq = \frac{N}{R}\int_{\Phi_1}^{\Phi_2} d\Phi_B$

and the charge through the circuit will be $|Q| = \frac{N}{R}(\Phi_2 - \Phi_1)$.

(b) $Q = \frac{N}{R}\left[BA\cos 0 - BA\cos\left(\frac{\pi}{2}\right)\right] = \frac{BAN}{R}$

so $B = \frac{RQ}{NA} = \frac{(200\ \Omega)(5.00\times10^{-4}\ \text{C})}{(100)(40.0\times10^{-4}\ \text{m}^2)} = \boxed{0.250\ \text{T}}$.

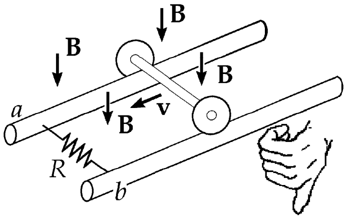


FIG. P31.57

**P31.57** (a) $\varepsilon = B\ell v = 0.360\text{ V}$ $\quad I = \frac{\varepsilon}{R} = \boxed{0.900\text{ A}}$

(b) $F_B = I\ell B = \boxed{0.108\text{ N}}$

(c) Since the magnetic flux $\mathbf{B}\cdot\mathbf{A}$ is in effect decreasing, the induced current flow through *R* is from *b* to *a*. $\boxed{\text{Point } b}$ is at higher potential.

(d) $\boxed{\text{No}}$. Magnetic flux will increase through a loop to the left of *ab*. Here counterclockwise current will flow to produce upward magnetic field. The current in *R* is still from *b* to *a*.

**P31.58** $\varepsilon = B\ell v$ at a distance *r* from wire

$$|\varepsilon| = \left(\frac{\mu_0 I}{2\pi r}\right)\ell v$$

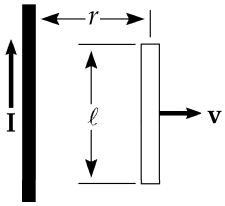


FIG. P31.58

**P31.59** (a) At time $t$, the flux through the loop is $\Phi_B = BA\cos\theta = (a+bt)(\pi r^2)\cos 0° = \pi(a+bt)r^2$.

At $t = 0$, $\Phi_B = \boxed{\pi a r^2}$.

(b) $\varepsilon = -\frac{d\Phi_B}{dt} = -\pi r^2 \frac{d(a+bt)}{dt} = \boxed{-\pi b r^2}$

(c) $I = \frac{\varepsilon}{R} = \boxed{-\frac{\pi b r^2}{R}}$

(d) $\mathscr{P} = \varepsilon I = \left(-\frac{\pi b r^2}{R}\right)\left(-\pi b r^2\right) = \boxed{\frac{\pi^2 b^2 r^4}{R}}$

**P31.60** $\varepsilon = -\frac{d}{dt}(NBA) = -1\left(\frac{dB}{dt}\right)\pi a^2 = \pi a^2 K$

(a) $Q = C\varepsilon = \boxed{C\pi a^2 K}$

(b) **B** into the paper is decreasing; therefore, current will attempt to counteract this. Positive charge will go to $\boxed{\text{upper plate}}$.

(c) The changing magnetic field through the enclosed area $\boxed{\text{induces an electric field}}$, surrounding the **B**-field, and this pushes on charges in the wire.

**P31.61** The flux through the coil is $\Phi_B = \mathbf{B}\cdot\mathbf{A} = BA\cos\theta = BA\cos\omega t$. The induced emf is

$$\varepsilon = -N\frac{d\Phi_B}{dt} = -NBA\frac{d(\cos\omega t)}{dt} = NBA\omega\sin\omega t.$$

(a) $\varepsilon_{\max} = NBA\omega = 60.0(1.00\ \text{T})\left(0.100\times 0.200\ \text{m}^2\right)(30.0\ \text{rad/s}) = \boxed{36.0\ \text{V}}$

(b) $\dfrac{d\Phi_B}{dt} = \dfrac{\varepsilon}{N}$, thus $\left|\dfrac{d\Phi_B}{dt}\right|_{\max} = \dfrac{\varepsilon_{\max}}{N} = \dfrac{36.0\ \text{V}}{60.0} = 0.600\ \text{V} = \boxed{0.600\ \text{Wb/s}}$

(c) At $t = 0.050\,0\ \text{s}$, $\omega t = 1.50\ \text{rad}$ and
$\varepsilon = \varepsilon_{\max}\sin(1.50\ \text{rad}) = (36.0\ \text{V})\sin(1.50\ \text{rad}) = \boxed{35.9\ \text{V}}$.

(d) The torque on the coil at any time is
$\tau = |\mu\times\mathbf{B}| = |NI\mathbf{A}\times\mathbf{B}| = (NAB)I|\sin\omega t| = \left(\dfrac{\varepsilon_{\max}}{\omega}\right)\left(\dfrac{\varepsilon}{R}\right)|\sin\omega t|$.
When $\varepsilon = \varepsilon_{\max}$, $\sin\omega t = 1.00$ and $\tau = \dfrac{\varepsilon_{\max}^2}{\omega R} = \dfrac{(36.0\ \text{V})^2}{(30.0\ \text{rad/s})(10.0\ \Omega)} = \boxed{4.32\ \text{N}\cdot\text{m}}$.

**P31.62** (a) We use $\varepsilon = -N\dfrac{\Delta\Phi_B}{\Delta t}$, with $N = 1$.
Taking $a = 5.00\times 10^{-3}\ \text{m}$ to be the radius of the washer, and $h = 0.500\ \text{m}$,

$$\Delta\Phi_B = B_2A - B_1A = A(B_2 - B_1) = \pi a^2\left(\frac{\mu_0 I}{2\pi(h+a)} - \frac{\mu_0 I}{2\pi a}\right) = \frac{a^2\mu_0 I}{2}\left(\frac{1}{h+a} - \frac{1}{a}\right) = \frac{-\mu_0 ahI}{2(h+a)}.$$

The time for the washer to drop a distance *h* (from rest) is: $\Delta t = \sqrt{\dfrac{2h}{g}}$.

Therefore, $\varepsilon = \dfrac{\mu_0 ahI}{2(h+a)\Delta t} = \dfrac{\mu_0 ahI}{2(h+a)}\sqrt{\dfrac{g}{2h}} = \dfrac{\mu_0 aI}{2(h+a)}\sqrt{\dfrac{gh}{2}}$

and $$\varepsilon = \frac{\left(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(5.00\times 10^{-3}\ \text{m}\right)(10.0\ \text{A})}{2(0.500\ \text{m} + 0.005\,00\ \text{m})}\sqrt{\frac{\left(9.80\ \text{m/s}^2\right)(0.500\ \text{m})}{2}} = \boxed{97.4\ \text{nV}}.$$

(b) Since the magnetic flux going through the washer (into the plane of the paper) is decreasing in time, a current will form in the washer so as to oppose that decrease. Therefore, the current will flow in a $\boxed{\text{clockwise direction}}$.

**P31.63** Find an expression for the flux through a rectangular area "swept out" by the bar in time *t*. The magnetic field at a distance *x* from wire is

$B = \dfrac{\mu_0 I}{2\pi x}$ and $\Phi_B = \int B\,dA$. Therefore,

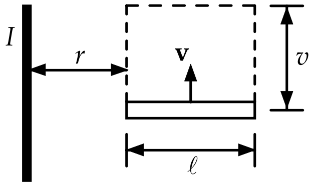


FIG. P31.63

$\Phi_B = \dfrac{\mu_0 Ivt}{2\pi}\displaystyle\int_r^{r+\ell}\dfrac{dx}{x}$ where *vt* is the distance the bar has moved in time *t*.

Then, $|\varepsilon| = \dfrac{d\Phi_B}{dt} = \boxed{\dfrac{\mu_0 Iv}{2\pi}\ln\left(1 + \dfrac{\ell}{r}\right)}$.

**P31.64** The magnetic field at a distance $x$ from a long wire is $B=\frac{\mu_0 I}{2\pi x}$. Find an expression for the flux through the loop.

$$d\Phi_B=\frac{\mu_0 I}{2\pi x}(\ell dx) \text{ so } \Phi_B=\frac{\mu_0 I\ell}{2\pi}\int_r^{r+w}\frac{dx}{x}=\frac{\mu_0 I\ell}{2\pi}\ln\left(1+\frac{w}{r}\right)$$

Therefore, $\varepsilon=-\frac{d\Phi_B}{dt}=\frac{\mu_0 I\ell v}{2\pi r}\frac{w}{(r+w)}$ and $I=\frac{\varepsilon}{R}=\boxed{\frac{\mu_0 I\ell v}{2\pi Rr}\frac{w}{(r+w)}}$.

**P31.65** We are given $\Phi_B=\left(6.00t^3-18.0t^2\right)\text{ T}\cdot\text{m}^2$

and $\varepsilon=-\frac{d\Phi_B}{dt}=-18.0t^2+36.0t$.

Maximum E occurs when $\frac{d\varepsilon}{dt}=-36.0t+36.0=0$

which gives $t=1.00\text{ s}$.

Therefore, the maximum current (at $t=1.00\text{ s}$) is $I=\frac{\varepsilon}{R}=\frac{(-18.0+36.0)\text{ V}}{3.00\ \Omega}=\boxed{6.00\text{ A}}$.

**P31.66** For the suspended mass, $M$: $\sum F=Mg-T=Ma$.

For the sliding bar, $m$: $\sum F=T-I\ell B=ma$, where $I=\frac{\varepsilon}{R}=\frac{B\ell v}{R}$

$Mg-\frac{B^2\ell^2 v}{R}=(m+M)a$ or $a=\frac{dv}{dt}=\frac{Mg}{m+M}-\frac{B^2\ell^2 v}{R(M+m)}$

$\int_0^v\frac{dv}{(\alpha-\beta v)}=\int_0^t dt$ where $\alpha=\frac{Mg}{M+m}$ and $\beta=\frac{B^2\ell^2}{R(M+m)}$.

Therefore, the velocity varies with time as $v=\frac{\alpha}{\beta}\left(1-e^{-\beta t}\right)=\boxed{\frac{MgR}{B^2\ell^2}\left[1-e^{-B^2\ell^2 t/R(M+m)}\right]}$.

**P31.67** (a) $\varepsilon=-N\frac{d\Phi_B}{dt}=-NA\frac{dB}{dt}=-NA\frac{d}{dt}(\mu_0 nI)$ where $A=$ area of coil

$N=$ number of turns in coil

and $n=$ number of turns per unit length in solenoid.

Therefore,

$$|\varepsilon|=N\mu_0 An\frac{d}{dt}\left[4\sin(120\pi t)\right]=N\mu_0 An(480\pi)\cos(120\pi t)$$

$$|\varepsilon|=40\left(4\pi\times10^{-7}\right)\left[\pi(0.050\,0\text{ m})^2\right]\left(2.00\times10^3\right)(480\pi)\cos(120\pi t)$$

$$|\varepsilon|=\boxed{(1.19\text{ V})\cos(120\pi t)}$$

(b) $I=\frac{\Delta V}{R}$ and $\mathcal{P}=\Delta VI=\frac{(1.19\text{ V})^2\cos^2(120\pi t)}{8.00\ \Omega}$

From $\cos^2\theta=\frac{1}{2}+\frac{1}{2}\cos 2\theta$

the average value of $\cos^2\theta$ is $\frac{1}{2}$, so $\overline{\mathcal{P}}=\frac{1}{2}\frac{(1.19\text{ V})^2}{(8.00\ \Omega)}=\boxed{88.5\text{ mW}}$.

***P31.68** (a) $$\varepsilon = -N\frac{d}{dt}BA\cos\theta = -1\frac{d}{dt}B\frac{\theta a^2}{2}\cos 0° = -\frac{Ba^2}{2}\frac{d\theta}{dt} = -\frac{1}{2}Ba^2\omega = -\frac{1}{2}(0.5\text{ T})(0.5\text{ m})^2 2\text{ rad/s}$$

$$= -0.125\text{ V} = \boxed{0.125\text{ V clockwise}}$$

The – sign indicates that the induced emf produces clockwise current, to make its own magnetic field into the page.

(b) At this instant $\theta = \omega t = 2\text{ rad/s}(0.25\text{ s}) = 0.5\text{ rad}$. The arc $PQ$ has length $r\theta == (0.5\text{ rad})(0.5\text{ m}) = 0.25\text{ m}$. The length of the circuit is $0.5\text{ m} + 0.5\text{ m} + 0.25\text{ m} = 1.25\text{ m}$ its resistance is $1.25\text{ m}(5\ \Omega/\text{m}) = 6.25\ \Omega$. The current is $\dfrac{0.125\text{ V}}{6.25\ \Omega} = \boxed{0.020\,0\text{ A clockwise}}$.

***P31.69** Suppose the field is vertically down. When an electron is moving away from you the force on it is in the direction given by

$q\mathbf{v}\times\mathbf{B}_c$ as $-(\text{away})\times\text{down} = -$ $= -\text{left} = \text{right}$.

Therefore, the electrons circulate clockwise.

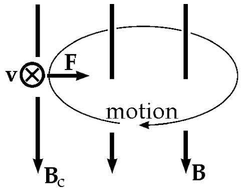


**FIG. P31.69**

(a) As the downward field increases, an emf is induced to produce some current that in turn produces an upward field. This current is directed counterclockwise, carried by negative electrons moving clockwise. Therefore the original electron motion speeds up.

(b) At the circumference, we have $\sum F_c = ma_c$: $$|q|vB_c\sin 90° = \frac{mv^2}{r}$$

$$mv = |q|rB_c.$$

The increasing magnetic field $\mathbf{B}_{\text{av}}$ in the area enclosed by the orbit produces a tangential electric field according to

$$\left|\oint \mathbf{E}\cdot d\mathbf{s}\right| = \left|-\frac{d}{dt}\mathbf{B}_{av}\cdot\mathbf{A}\right| \qquad E(2\pi r) = \pi r^2\frac{dB_{av}}{dt} \qquad E = \frac{r}{2}\frac{dB_{av}}{dt}.$$

An electron feels a tangential force according to $\sum F_t = ma_t$: $\quad |q|E = m\dfrac{dv}{dt}$.

Then $$|q|\frac{r}{2}\frac{dB_{av}}{dt} = m\frac{dv}{dt} \qquad |q|\frac{r}{2}B_{av} = mv = |q|rB_c$$

and $$B_{av} = 2B_c.$$

**P31.70** The induced emf is $\varepsilon = B\ell v$ where $B = \dfrac{\mu_0 I}{2\pi y}$, $v_f = v_i + gt = (9.80\text{ m/s}^2)t$, and

$$y_f = y_i - \frac{1}{2}gt^2 = 0.800\text{ m} - (4.90\text{ m/s}^2)t^2.$$

$$\varepsilon = \frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(200\text{ A})}{2\pi\left[0.800\text{ m} - (4.90\text{ m/s}^2)t^2\right]}(0.300\text{ m})(9.80\text{ m/s}^2)t = \boxed{\frac{(1.18\times10^{-4})t}{\left[0.800 - 4.90t^2\right]}\text{ V}}$$

At $t = 0.300\text{ s}$, $\varepsilon = \dfrac{(1.18\times10^{-4})(0.300)}{\left[0.800 - 4.90(0.300)^2\right]}\text{ V} = \boxed{98.3\ \mu\text{V}}$.

**P31.71** The magnetic field produced by the current in the straight wire is perpendicular to the plane of the coil at all points within the coil. The magnitude of the field is $B = \frac{\mu_0 I}{2\pi r}$. Thus, the flux linkage is

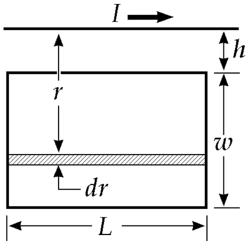


FIG. P31.71

$$N\Phi_B = \frac{\mu_0 NIL}{2\pi}\int_h^{h+w}\frac{dr}{r} = \frac{\mu_0 NI_{\max}L}{2\pi}\ln\left(\frac{h+w}{h}\right)\sin(\omega t + \phi).$$

Finally, the induced emf is

$$\varepsilon = -\frac{\mu_0 NI_{\max}L\omega}{2\pi}\ln\left(1+\frac{w}{h}\right)\cos(\omega t+\phi)$$

$$\varepsilon = -\frac{\left(4\pi\times10^{-7}\right)(100)(50.0)(0.200\text{ m})\left(200\pi\text{ s}^{-1}\right)}{2\pi}\ln\left(1+\frac{5.00\text{ cm}}{5.00\text{ cm}}\right)\cos(\omega t+\phi)$$

$$\varepsilon = \boxed{-(87.1\text{ mV})\cos(200\pi t+\phi)}$$

The term $\sin(\omega t+\phi)$ in the expression for the current in the straight wire does not change appreciably when $\omega t$ changes by 0.10 rad or less. Thus, the current does not change appreciably during a time interval

$$\Delta t < \frac{0.10}{\left(200\pi\text{ s}^{-1}\right)} = 1.6\times10^{-4}\text{ s}.$$

We define a critical length, $c\Delta t = \left(3.00\times10^{8}\text{ m/s}\right)\left(1.6\times10^{-4}\text{ s}\right) = 4.8\times10^{4}\text{ m}$ equal to the distance to which field changes could be propagated during an interval of $1.6\times10^{-4}$ s. This length is so much larger than any dimension of the coil or its distance from the wire that, although we consider the straight wire to be infinitely long, we can also safely ignore the field propagation effects in the vicinity of the coil. Moreover, the phase angle can be considered to be constant along the wire in the vicinity of the coil.

If the frequency $w$ were much larger, say, $200\pi\times10^{5}\text{ s}^{-1}$, the corresponding critical length would be only 48 cm. In this situation propagation effects would be important and the above expression for $\varepsilon$ would require modification. As a "rule of thumb" we can consider field propagation effects for circuits of laboratory size to be negligible for frequencies, $f = \frac{\omega}{2\pi}$, that are less than about $10^{6}$ Hz.

**P31.72** $\Phi_B = BA\cos\theta \quad \frac{d\Phi_B}{dt} = -\omega BA\sin\theta;$

$I \propto -\sin\theta$

$\tau \propto IB\sin\theta \boxed{\propto -\sin^2\theta}$

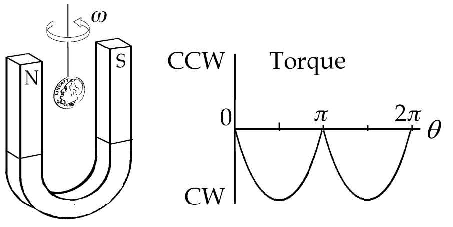


FIG. P31.72

## ANSWERS TO EVEN PROBLEMS

| | |
|---|---|
| **P31.2** | 0.800 mA |
| **P31.4** | (a) see the solution; (b) 3.79 mV; (c) 28.0 mV |
| **P31.6** | 78.5 $\mu$s |
| **P31.8** | (a) $\frac{\mu_0 n\pi r_2^2}{2R}\frac{\Delta I}{\Delta t}$ counterclockwise;<br>(b) $\frac{\mu_0^2 n\pi r_2^2}{4r_1 R}\frac{\Delta I}{\Delta t}$; (c) upward |
| **P31.10** | $-14.2\text{ mV}\cos(120t)$ |
| **P31.12** | 61.8 mV |
| **P31.14** | (a) see the solution; (b) 625 m/s |
| **P31.16** | see the solution |
| **P31.18** | 13.3 mA counterclockwise in the lower loop and clockwise in the upper loop. |
| **P31.20** | 1.00 m/s |
| **P31.22** | (a) 500 mA; (b) 2.00 W; (c) 2.00 W |
| **P31.24** | 24.1 V with the outer contact positive |
| **P31.26** | 121 mA clockwise |
| **P31.28** | (a) to the right; (b) to the right; (c) to the right; (d) into the paper |
| **P31.30** | negative; see the solution |
| **P31.32** | (a) $8.00\times10^{-21}$ N downward perpendicular to $r_1$; (b) 1.33 s |
| **P31.34** | (a) $(9.87\text{ mV/m})\cos(100\pi t)$; (b) clockwise |
| **P31.36** | (a) $(19.6\text{ V})\sin(314t)$; (b) 19.6 V |
| **P31.38** | see the solution |
| **P31.40** | (a) 1.60 V; (b) 0; (c) no change; (d) and (e) see the solution |
| **P31.42** | both are correct; see the solution |
| **P31.44** | $\left(-4.39\hat{\mathbf{i}}-1.76\hat{\mathbf{j}}\right)10^{11}\ \text{m/s}^2$ |
| **P31.46** | $-(7.22\text{ mV})\cos(2\pi 523\,t/\text{s})$ |
| **P31.48** | (a) 43.8 A; (b) 38.3 W |
| **P31.50** | (a) 3.50 A up in 2 Ω and 1.40 A up in 5 Ω; (b) 34.3 W; (c) 4.29 N |
| **P31.52** | see the solution |
| **P31.54** | (a) $\frac{\pi B_{\text{max}}^2 R^4 b\omega^2}{16\rho}$; (b) 4 times larger;<br>(c) 4 times larger; (d) 16 times larger |
| **P31.56** | (a) see the solution; (b) 0.250 T |
| **P31.58** | see the solution |
| **P31.60** | (a) $C\pi a^2 K$; (b) the upper plate; (c) see the solution |
| **P31.62** | (a) 97.4 nV; (b) clockwise |
| **P31.64** | $\frac{\mu_0 I\ell v}{2\pi Rr}\frac{w}{(r+w)}$ |
| **P31.66** | $\frac{MgR}{B^2\ell^2}\left[1-e^{-B^2\ell^2 t/R(M+m)}\right]$ |
| **P31.68** | (a) 0.125 V to produce clockwise current; (b) 20.0 mA clockwise |
| **P31.70** | $\frac{1.18\times10^{-4}}{0.800-4.90t^2}$; 98.3 $\mu$V |
| **P31.72** | see the solution |

# 32

# Inductance

**CHAPTER OUTLINE**

32.1 Self-Inductance
32.2 *RL* Circuits
32.3 Energy in a Magnetic Field
32.4 Mutual Inductance
32.5 Oscillations in an LC Circuit
32.6 The *RLC* Circuit

## ANSWERS TO QUESTIONS

**Q32.1** The emf induced in an inductor is opposite to the direction of the changing current. For example, in a simple *RL* circuit with current flowing clockwise, if the current in the circuit increases, the inductor will generate an emf to oppose the increasing current.

**Q32.2** The coil has an inductance regardless of the nature of the current in the circuit. Inductance depends only on the coil geometry and its construction. Since the current is constant, the self-induced emf in the coil is zero, and the coil does not affect the steady-state current. (We assume the resistance of the coil is negligible.)

**Q32.3** The inductance of a coil is determined by (a) the geometry of the coil and (b) the "contents" of the coil. This is similar to the parameters that determine the capacitance of a capacitor and the resistance of a resistor. With an inductor, the most important factor in the geometry is the number of turns of wire, or turns per unit length. By the "contents" we refer to the material in which the inductor establishes a magnetic field, notably the magnetic properties of the core around which the wire is wrapped.

**Q32.4** If the first set of turns is wrapped clockwise around a spool, wrap the second set counter-clockwise, so that the coil produces negligible magnetic field. Then the inductance of each set of turns effectively negates the inductive effects of the other set.

**Q32.5** After the switch is closed, the back emf will not exceed that of the battery. If this were the case, then the current in the circuit would change direction to counterclockwise. Just after the switch is opened, the back emf can be much larger than the battery emf, to temporarily maintain the clockwise current in a spark.

**Q32.6** The current decreases not instantaneously but over some span of time. The faster the decrease in the current, the larger will be the emf generated in the inductor. A spark can appear at the switch as it is opened because the self-induced voltage is a maximum at this instant. The voltage can therefore briefly cause dielectric breakdown of the air between the contacts.

**Q32.7** When it is being opened. When the switch is initially standing open, there is no current in the circuit. Just after the switch is then closed, the inductor tends to maintain the zero-current condition, and there is very little chance of sparking. When the switch is standing closed, there is current in the circuit. When the switch is then opened, the current rapidly decreases. The induced emf is created in the inductor, and this emf tends to maintain the original current. Sparking occurs as the current bridges the air gap between the contacts of the switch.

**Q32.8** A physicist's list of constituents of the universe in 1829 might include matter, light, heat, the stuff of stars, charge, momentum, and several other entries. Our list today might include the quarks, electrons, muons, tauons, and neutrinos of matter; gravitons of gravitational fields; photons of electric and magnetic fields; W and Z particles; gluons; energy; momentum; angular momentum; charge; baryon number; three different lepton numbers; upness; downness; strangeness; charm; topness; and bottomness. Alternatively, the relativistic interconvertability of mass and energy, and of electric and magnetic fields, can be used to make the list look shorter. Some might think of the conserved quantities energy, momentum, … bottomness as properties of matter, rather than as things with their own existence. The idea of a field is not due to Henry, but rather to Faraday, to whom Henry personally demonstrated self-induction. Still the thesis stated in the question has an important germ of truth. Henry precipitated a basic change if he did not cause it. The biggest difference between the two lists is that the 1829 list does not include fields and today's list does.

**Q32.9** The energy stored in the magnetic field of an inductor is proportional to the square of the current. Doubling $I$ makes $U=\frac{1}{2}LI^2$ get four times larger.

**Q32.10** The energy stored in a capacitor is proportional to the square of the electric field, and the energy stored in an induction coil is proportional to the square of the magnetic field. The capacitor's energy is proportional to its capacitance, which depends on its geometry and the dielectric material inside. The coil's energy is proportional to its inductance, which depends on its geometry and the core material. On the other hand, we can think of Henry's discovery of self-inductance as fundamentally new. Before a certain school vacation at the Albany Academy about 1830, one could visualize the universe as consisting of only one thing, matter. All the forms of energy then known (kinetic, gravitational, elastic, internal, electrical) belonged to chunks of matter. But the energy that temporarily maintains a current in a coil after the battery is removed is not energy that belongs to any bit of matter. This energy is vastly larger than the kinetic energy of the drifting electrons in the wires. This energy belongs to the magnetic field around the coil. Beginning in 1830, Nature has forced us to admit that the universe consists of matter and also of fields, massless and invisible, known only by their effects.

**Q32.11** The inductance of the series combination of inductor $L_1$ and inductor $L_2$ is $L_1+L_2+M_{12}$, where $M_{12}$ is the mutual inductance of the two coils. It can be defined as the emf induced in coil two when the current in coil one changes at one ampere per second, due to the magnetic field of coil one producing flux through coil two. The coils can be arranged to have large mutual inductance, as by winding them onto the same core. The coils can be arranged to have negligible mutual inductance, as separate toroids do.

**Q32.12** The mutual inductance of two loops in free space—that is, ignoring the use of cores—is a maximum if the loops are coaxial. In this way, the maximum flux of the primary loop will pass through the secondary loop, generating the largest possible emf given the changing magnetic field due to the first. The mutual inductance is a minimum if the magnetic field of the first coil lies in the plane of the second coil, producing no flux through the area the second coil encloses.

**Q32.13** The answer depends on the orientation of the solenoids. If they are coaxial, such as two solenoids end-to-end, then there certainty will be mutual induction. If, however, they are oriented in such a way that the magnetic field of one coil does not go through turns of the second coil, then there will be no mutual induction. Consider the case of two solenoids physically arranged in a "T" formation, but still connected electrically in series. The magnetic field lines of the first coil will not produce any net flux in the second coil, and thus no mutual induction will be present.

**Q32.14** When the capacitor is fully discharged, the current in the circuit is a maximum. The inductance of the coil is making the current continue to flow. At this time the magnetic field of the coil contains all the energy that was originally stored in the charged capacitor. The current has just finished discharging the capacitor and is proceeding to charge it up again with the opposite polarity.

**Q32.15** The oscillations would eventually decrease, but perhaps with very small damping. The original potential energy would be converted to internal energy within the wires. Such a situation constitutes an *RLC* circuit. Remember that a real battery generally contains an internal resistance.

**Q32.16** If $R > \sqrt{\frac{4L}{C}}$, then the oscillator is overdamped—it will not oscillate. If $R < \sqrt{\frac{4L}{C}}$, then the oscillator is underdamped and can go through several cycles of oscillation before the radiated signal falls below background noise.

**Q32.17** The condition for critical damping must be investigated to design a circuit for a particular purpose. For example, in building a radio receiver, one would want to construct the receiving circuit so that it is underdamped. Then it can oscillate in resonance and detect the desired signal. Conversely, when designing a probe to measure a changing signal, such free oscillations are undesirable. An electrical vibration in the probe would constitute "ringing" of the system, where the probe would measure an additional signal—that of the probe itself! In this case, one would want to design a probe that is critically damped or overdamped, so that the only signal measured is the one under study. Critical damping represents the threshold between underdamping and overdamping. One must know the condition for it to meet the design criteria for a project.

**Q32.18** An object cannot exert a net force on itself. An object cannot create momentum out of nothing. A coil can induce an emf in itself. When it does so, the actual forces acting on charges in different parts of the loop add as vectors to zero. The term electromotive force does not refer to a force, but to a voltage.

## SOLUTIONS TO PROBLEMS

### Section 32.1 **Self-Inductance**

**P32.1** $|\bar{\varepsilon}| = L\frac{\Delta I}{\Delta t} = \left(3.00\times 10^{-3}\text{ H}\right)\left(\frac{1.50\text{ A} - 0.200\text{ A}}{0.200\text{ s}}\right) = 1.95\times 10^{-2}\text{ V} = \boxed{19.5\text{ mV}}$

**P32.2** Treating the telephone cord as a solenoid, we have:

$$L = \frac{\mu_0 N^2 A}{\ell} = \frac{\left(4\pi\times 10^{-7}\text{ T}\cdot\text{m/A}\right)(70.0)^2\pi\left(6.50\times 10^{-3}\text{ m}\right)^2}{0.600\text{ m}} = \boxed{1.36\ \mu\text{H}}.$$

**P32.3** $\bar{\varepsilon} = -L\frac{\Delta I}{\Delta t} = (-2.00\text{ H})\left(\frac{0 - 0.500\text{ A}}{0.010\,0\text{ s}}\right)\left(\frac{1\text{ V}\cdot\text{s}}{1\text{ H}\cdot\text{A}}\right) = \boxed{100\text{ V}}$

**P32.4** $L = \frac{N\Phi_B}{I} \rightarrow \Phi_B = \frac{LI}{N} = \boxed{240\text{ nT}\cdot\text{m}^2}$ through each turn

**P32.5** $\varepsilon_{\text{back}} = -\varepsilon = L\frac{dI}{dt} = L\frac{d}{dt}\left(I_{\max}\sin\omega t\right) = L\omega I_{\max}\cos\omega t = \left(10.0\times 10^{-3}\right)(120\pi)(5.00)\cos\omega t$

$\varepsilon_{\text{back}} = (6.00\pi)\cos(120\pi t) = \boxed{(18.8\text{ V})\cos(377t)}$

**P32.6** From $|\varepsilon| = L\left(\frac{\Delta I}{\Delta t}\right)$, we have $L = \frac{\varepsilon}{(\Delta I/\Delta t)} = \frac{24.0\times10^{-3}\ \text{V}}{10.0\ \text{A/s}} = 2.40\times10^{-3}\ \text{H}$.

From $L = \frac{N\Phi_B}{I}$, we have $\Phi_B = \frac{LI}{N} = \frac{(2.40\times10^{-3}\ \text{H})(4.00\ \text{A})}{500} = \boxed{19.2\ \mu\text{T}\cdot\text{m}^2}$.

**P32.7** $$L = \frac{\mu_0 N^2 A}{\ell} = \frac{\mu_0(420)^2(3.00\times10^{-4})}{0.160} = 4.16\times10^{-4}\ \text{H}$$

$$\varepsilon = -L\frac{dI}{dt} \to \frac{dI}{dt} = \frac{-\varepsilon}{L} = \frac{-175\times10^{-6}\ \text{V}}{4.16\times10^{-4}\ \text{H}} = \boxed{-0.421\ \text{A/s}}$$

**P32.8** $$|\varepsilon| = L\frac{dI}{dt} = (90.0\times10^{-3})\frac{d}{dt}(t^2-6t)\ \text{V}$$

(a) At $t = 1.00$ s, $\varepsilon = \boxed{360\ \text{mV}}$

(b) At $t = 4.00$ s, $\varepsilon = \boxed{180\ \text{mV}}$

(c) $\varepsilon = (90.0\times10^{-3})(2t-6) = 0$

when $\boxed{t = 3.00\ \text{s}}$.

**P32.9** (a) $B = \mu_0 nI = \mu_0\left(\frac{450}{0.120}\right)(0.040\,0\ \text{A}) = \boxed{188\ \mu\text{T}}$

(b) $\Phi_B = BA = \boxed{3.33\times10^{-8}\ \text{T}\cdot\text{m}^2}$

(c) $L = \frac{N\Phi_B}{I} = \boxed{0.375\ \text{mH}}$

(d) $\boxed{B \text{ and } \Phi_B \text{ are proportional to current; } L \text{ is independent of current}}$

**P32.10** (a) $L = \frac{\mu_0 N^2 A}{\ell} = \frac{\mu_0(120)^2\pi(5.00\times10^{-3})^2}{0.090\,0} = \boxed{15.8\ \mu\text{H}}$

(b) $\Phi_B' = \frac{\mu_m}{\mu_0}\Phi_B \to L = \frac{\mu_m N^2 A}{\ell} = 800(1.58\times10^{-5}\ \text{H}) = \boxed{12.6\ \text{mH}}$

***P32.11** We can directly find the self inductance of the solenoid:

$$\varepsilon = -L\frac{dI}{dt} \qquad +0.08\ \text{V} = -L\frac{0-1.8\ \text{A}}{0.12\ \text{s}} \qquad L = 5.33\times10^{-3}\ \text{Vs/A} = \frac{\mu_0 N^2 A}{\ell}.$$

Here $A = \pi r^2$, $200\ \text{m} = N2\pi r$, and $\ell = N(10^{-3}\ \text{m})$. Eliminating extra unknowns step by step, we have

$$5.33\times10^{-3}\ \text{Vs/A} = \frac{\mu_0 N^2\pi r^2}{\ell} = \frac{\mu_0 N^2\pi}{\ell}\left(\frac{200\ \text{m}}{2\pi N}\right)^2 = \frac{\mu_0\, 40\,000\ \text{m}^2}{4\pi\ell} = \frac{10^{-7}(40\,000\ \text{m}^2)\text{Tm}}{\ell\qquad \text{A}}$$

$$\ell = \frac{4\times10^{-3}\ \text{WbmA}}{5.33\times10^{-3}\ \text{AVs}} = \boxed{0.750\ \text{m}}$$

**P32.12** $$L=\frac{N\Phi_B}{I}=\frac{NBA}{I}\approx\frac{NA}{I}\cdot\frac{\mu_0 NI}{2\pi R}=\boxed{\frac{\mu_0 N^2 A}{2\pi R}}$$

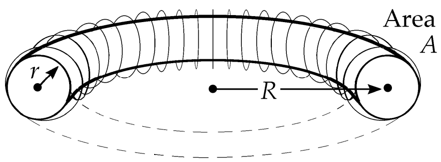


**FIG. P32.12**

**P32.13** $$\varepsilon=\varepsilon_0 e^{-kt}=-L\frac{dI}{dt}$$

$$dI=-\frac{\varepsilon_0}{L}e^{-kt}dt$$

If we require $I\to 0$ as $t\to\infty$, the solution is $I=\frac{\varepsilon_0}{kL}e^{-kt}=\frac{dq}{dt}$

$$Q=\int Idt=\int_0^\infty\frac{\varepsilon_0}{kL}e^{-kt}dt=-\frac{\varepsilon_0}{k^2L} \qquad \boxed{|Q|=\frac{\varepsilon_0}{k^2L}}\,.$$

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## Section 32.2 ***RL* Circuits**

**P32.14** $I=\frac{\varepsilon}{R}\left(1-e^{-Rt/L}\right)$:

$$0.900\frac{\varepsilon}{R}=\frac{\varepsilon}{R}\left[1-e^{-R(3.00\text{ s})/2.50\text{ H}}\right]$$

$$\exp\left(-\frac{R(3.00\text{ s})}{2.50\text{ H}}\right)=0.100$$

$$R=\frac{2.50\text{ H}}{3.00\text{ s}}\ln 10.0=\boxed{1.92\ \Omega}$$

**P32.15** (a)

At time $t$, $I(t)=\frac{\varepsilon\left(1-e^{-t/\tau}\right)}{R}$

where $\tau=\frac{L}{R}=0.200\text{ s}$.

After a long time, $I_{\max}=\frac{\varepsilon\left(1-e^{-\infty}\right)}{R}=\frac{\varepsilon}{R}$.

At $I(t)=0.500I_{\max}$ $(0.500)\frac{\varepsilon}{R}=\frac{\varepsilon\left(1-e^{-t/0.200\text{ s}}\right)}{R}$

so $0.500=1-e^{-t/0.200\text{ s}}$.

Isolating the constants on the right, $\ln\left(e^{-t/0.200\text{ s}}\right)=\ln(0.500)$

and solving for $t$, $-\frac{t}{0.200\text{ s}}=-0.693$

or $t=\boxed{0.139\text{ s}}$.

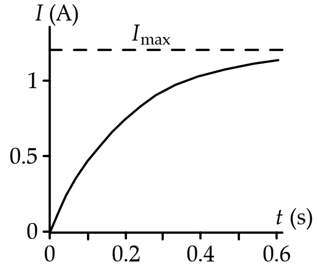


**FIG. P32.15**

(b)

Similarly, to reach 90% of $I_{\max}$, $0.900=1-e^{-t/\tau}$

and $t=-\tau\ln(1-0.900)$.

Thus, $t=-(0.200\text{ s})\ln(0.100)=\boxed{0.461\text{ s}}$.

**P32.16** Taking $\tau = \dfrac{L}{R}$, $\quad I = I_0 e^{-t/\tau}$: $\quad \dfrac{dI}{dt} = I_0 e^{-t/\tau}\left(-\dfrac{1}{\tau}\right)$

$IR + L\dfrac{dI}{dt} = 0$ will be true if $\quad I_0 R e^{-t/\tau} + L\left(I_0 e^{-t/\tau}\right)\left(-\dfrac{1}{\tau}\right) = 0$.

Because $\tau = \dfrac{L}{R}$, we have agreement with $0 = 0$.

**P32.17** (a) $\tau = \dfrac{L}{R} = 2.00 \times 10^{-3}\ \text{s} = \boxed{2.00\ \text{ms}}$

(b) $I = I_{\max}\left(1 - e^{-t/\tau}\right) = \left(\dfrac{6.00\ \text{V}}{4.00\ \Omega}\right)\left(1 - e^{-0.250/2.00}\right) = \boxed{0.176\ \text{A}}$

(c) $I_{\max} = \dfrac{\varepsilon}{R} = \dfrac{6.00\ \text{V}}{4.00\ \Omega} = \boxed{1.50\ \text{A}}$

(d) $0.800 = 1 - e^{-t/2.00\ \text{ms}} \rightarrow t = -(2.00\ \text{ms})\ln(0.200) = \boxed{3.22\ \text{ms}}$

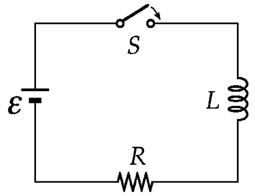


**FIG. P32.17**

**P32.18** $I = \dfrac{\varepsilon}{R}\left(1 - e^{-t/\tau}\right) = \dfrac{120}{9.00}\left(1 - e^{-1.80/7.00}\right) = 3.02\ \text{A}$

$\Delta V_R = IR = (3.02)(9.00) = 27.2\ \text{V}$

$\Delta V_L = \varepsilon - \Delta V_R = 120 - 27.2 = \boxed{92.8\ \text{V}}$

**P32.19** *Note*: It may not be correct to call the voltage or emf across a coil a "potential difference." Electric potential can only be defined for a conservative electric field, and not for the electric field around an inductor.

(a) $\Delta V_R = IR = (8.00\ \Omega)(2.00\ \text{A}) = 16.0\ \text{V}$

and $\quad \Delta V_L = \varepsilon - \Delta V_R = 36.0\ \text{V} - 16.0\ \text{V} = 20.0\ \text{V}$.

Therefore, $\quad \dfrac{\Delta V_R}{\Delta V_L} = \dfrac{16.0\ \text{V}}{20.0\ \text{V}} = \boxed{0.800}$.

(b) $\Delta V_R = IR = (4.50\ \text{A})(8.00\ \Omega) = 36.0\ \text{V}$

$\Delta V_L = \varepsilon - \Delta V_R = \boxed{0}$

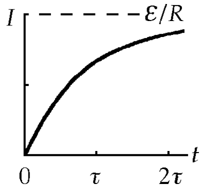


**FIG. P32.19**

**P32.20** After a long time, $12.0\ \text{V} = (0.200\ \text{A})R$. Thus, $R = 60.0\ \Omega$. Now, $\tau = \dfrac{L}{R}$ gives $L = \tau R = \left(5.00 \times 10^{-4}\ \text{s}\right)(60.0\ \text{V/A}) = \boxed{30.0\ \text{mH}}$.

**P32.21** $I = I_{\max}\left(1 - e^{-t/\tau}\right)$: $\quad \dfrac{dI}{dt} = -I_{\max}\left(e^{-t/\tau}\right)\left(-\dfrac{1}{\tau}\right)$

$\tau = \dfrac{L}{R} = \dfrac{15.0\ \text{H}}{30.0\ \Omega} = 0.500\ \text{s}$: $\quad \dfrac{dI}{dt} = \dfrac{R}{L} I_{\max} e^{-t/\tau}$ and $I_{\max} = \dfrac{\varepsilon}{R}$

(a) $t = 0$: $\dfrac{dI}{dt} = \dfrac{R}{L} I_{\max} e^{0} = \dfrac{\varepsilon}{L} = \dfrac{100\ \text{V}}{15.0\ \text{H}} = \boxed{6.67\ \text{A/s}}$

(b) $t = 1.50\ \text{s}$: $\dfrac{dI}{dt} = \dfrac{\varepsilon}{L} e^{-t/\tau} = (6.67\ \text{A/s})e^{-1.50/(0.500)} = (6.67\ \text{A/s})e^{-3.00} = \boxed{0.332\ \text{A/s}}$

**P32.22** $I = I_{\max}\left(1 - e^{-t/\tau}\right)$: $\quad 0.980 = 1 - e^{-3.00\times10^{-3}/\tau}$

$$0.020\,0 = e^{-3.00\times10^{-3}/\tau}$$

$$\tau = -\frac{3.00\times10^{-3}}{\ln(0.020\,0)} = 7.67\times10^{-4}\text{ s}$$

$\tau = \dfrac{L}{R}$, so $\quad L = \tau R = \left(7.67\times10^{-4}\right)(10.0) = \boxed{7.67\text{ mH}}$

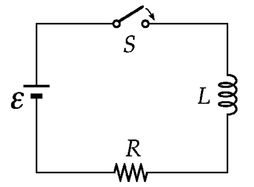


FIG. P32.22

**P32.23** Name the currents as shown. By Kirchhoff's laws:

$$I_1 = I_2 + I_3 \qquad (1)$$

$$+10.0\text{ V} - 4.00I_1 - 4.00I_2 = 0 \qquad (2)$$

$$+10.0\text{ V} - 4.00I_1 - 8.00I_3 - (1.00)\frac{dI_3}{dt} = 0 \qquad (3)$$

From (1) and (2), $\quad +10.0 - 4.00I_1 - 4.00I_1 + 4.00I_3 = 0$

and $\quad I_1 = 0.500I_3 + 1.25\text{ A}$.

Then (3) becomes $\quad 10.0\text{ V} - 4.00(0.500I_3 + 1.25\text{ A}) - 8.00I_3 - (1.00)\dfrac{dI_3}{dt} = 0$

$$(1.00\text{ H})\left(\frac{dI_3}{dt}\right) + (10.0\ \Omega)I_3 = 5.00\text{ V}.$$

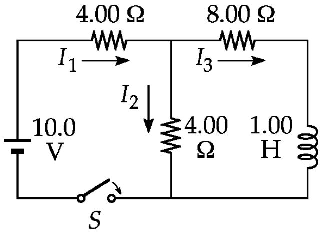


FIG. P32.23

We solve the differential equation using Equations 32.6 and 32.7:

$$I_3(t) = \left(\frac{5.00\text{ V}}{10.0\ \Omega}\right)\left[1 - e^{-(10.0\ \Omega)t/1.00\text{ H}}\right] = \boxed{(0.500\text{ A})\left[1 - e^{-10t/\text{s}}\right]}$$

$$I_1 = 1.25 + 0.500I_3 = \boxed{1.50\text{ A} - (0.250\text{ A})e^{-10t/\text{s}}}$$

**P32.24** (a) Using $\tau = RC = \dfrac{L}{R}$, we get $R = \sqrt{\dfrac{L}{C}} = \sqrt{\dfrac{3.00\text{ H}}{3.00\times10^{-6}\text{ F}}} = 1.00\times10^3\ \Omega = \boxed{1.00\text{ k}\Omega}$.

(b) $\tau = RC = \left(1.00\times10^3\ \Omega\right)\left(3.00\times10^{-6}\text{ F}\right) = 3.00\times10^{-3}\text{ s} = \boxed{3.00\text{ ms}}$

**P32.25** For $t \le 0$, the current in the inductor is $\boxed{\text{zero}}$. At $t = 0$, it starts to grow from zero toward 10.0 A with time constant

$$\tau = \frac{L}{R} = \frac{(10.0\text{ mH})}{(100\ \Omega)} = 1.00\times10^{-4}\text{ s}.$$

For $0 \le t \le 200\ \mu\text{s}$, $I = I_{\max}\left(1 - e^{-t/\tau}\right) = \boxed{(10.0\text{ A})\left(1 - e^{-10\,000t/\text{s}}\right)}$.

At $t = 200\ \mu\text{s}$, $I = (10.00\text{ A})\left(1 - e^{-2.00}\right) = 8.65\text{ A}$.

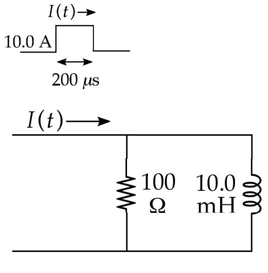


FIG. P32.25

Thereafter, it decays exponentially as $I = I_0 e^{-t'/\tau}$, so for $t \ge 200\ \mu\text{s}$,

$$I = (8.65\text{ A})e^{-10\,000(t-200\ \mu\text{s})/\text{s}} = (8.65\text{ A})e^{-10\,000t/\text{s}+2.00} = \left(8.65e^{2.00}\text{ A}\right)e^{-10\,000t/\text{s}} = \boxed{(63.9\text{ A})e^{-10\,000t/\text{s}}}.$$

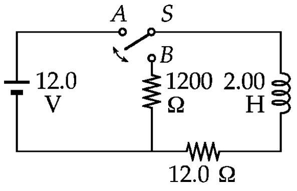


FIG. P32.26

**P32.26** (a) $I = \frac{\varepsilon}{R} = \frac{12.0\text{ V}}{12.0\ \Omega} = \boxed{1.00\text{ A}}$

(b) Initial current is 1.00 A: $\Delta V_{12} = (1.00\text{ A})(12.00\ \Omega) = \boxed{12.0\text{ V}}$

$$\Delta V_{1\,200} = (1.00\text{ A})(1\,200\ \Omega) = \boxed{1.20\text{ kV}}$$

$$\Delta V_L = \boxed{1.21\text{ kV}}\,.$$

(c) $I = I_{\max} e^{-Rt/L}$: $\qquad \frac{dI}{dt} = -I_{\max}\frac{R}{L}e^{-Rt/L}$

and $\qquad -L\frac{dI}{dt} = \Delta V_L = I_{\max} R\, e^{-Rt/L}\,.$

Solving $\qquad 12.0\text{ V} = (1\,212\text{ V})e^{-1\,212t/2.00}$

so $\qquad 9.90\times 10^{-3} = e^{-606t}\,.$

Thus, $\qquad \boxed{t = 7.62\text{ ms}}\,.$

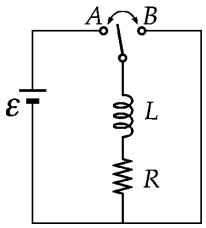


FIG. P32.27

**P32.27** $\tau = \frac{L}{R} = \frac{0.140}{4.90} = 28.6\text{ ms}$

$I_{\max} = \frac{\varepsilon}{R} = \frac{6.00\text{ V}}{4.90\ \Omega} = 1.22\text{ A}$

(a) $I = I_{\max}\left(1 - e^{-t/\tau}\right)$ so $\qquad 0.220 = 1.22\left(1 - e^{-t/\tau}\right)$

$e^{-t/\tau} = 0.820$: $\qquad t = -\tau\ln(0.820) = \boxed{5.66\text{ ms}}$

(b) $I = I_{\max}\left(1 - e^{-10.0/0.028\,6}\right) = (1.22\text{ A})\left(1 - e^{-350}\right) = \boxed{1.22\text{ A}}$

(c) $I = I_{\max}e^{-t/\tau}$ and $\qquad 0.160 = 1.22e^{-t/\tau}$

so $\qquad t = -\tau\ln(0.131) = \boxed{58.1\text{ ms}}\,.$

**P32.28** (a) For a series connection, both inductors carry equal currents at every instant, so $\frac{dI}{dt}$ is the same for both. The voltage across the pair is

$L_{\text{eq}}\frac{dI}{dt} = L_1\frac{dI}{dt} + L_2\frac{dI}{dt}$ so $\qquad \boxed{L_{\text{eq}} = L_1 + L_2}\,.$

(b) $L_{\text{eq}}\frac{dI}{dt} = L_1\frac{dI_1}{dt} = L_2\frac{dI_2}{dt} = \Delta V_L$ where $\qquad I = I_1 + I_2$ and $\frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt}\,.$

Thus, $\frac{\Delta V_L}{L_{\text{eq}}} = \frac{\Delta V_L}{L_1} + \frac{\Delta V_L}{L_2}$ and $\qquad \boxed{\frac{1}{L_{\text{eq}}} = \frac{1}{L_1} + \frac{1}{L_2}}\,.$

(c) $L_{\text{eq}}\frac{dI}{dt} + R_{\text{eq}}I = L_1\frac{dI}{dt} + IR_1 + L_2\frac{dI}{dt} + IR_2$

Now $I$ and $\frac{dI}{dt}$ are separate quantities under our control, so functional equality requires both $\boxed{L_{\text{eq}} = L_1 + L_2 \qquad \text{and} \qquad R_{\text{eq}} = R_1 + R_2}\,.$

*continued on next page*

(d) $\Delta V = L_{\text{eq}}\frac{dI}{dt} + R_{\text{eq}}I = L_1\frac{dI_1}{dt} + R_1I_1 = L_2\frac{dI_2}{dt} + R_2I_2$ where $I = I_1 + I_2$ and $\frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt}$.

We may choose to keep the currents constant in time. Then, $\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2}$.

We may choose to make the current swing through 0. Then, $\frac{1}{L_{\text{eq}}} = \frac{1}{L_1} + \frac{1}{L_2}$.

This equivalent coil with resistance will be equivalent to the pair of real inductors for all other currents as well.

## Section 32.3 **Energy in a Magnetic Field**

**P32.29** $L = \frac{N\Phi_B}{I} = \frac{200\left(3.70\times10^{-4}\right)}{1.75} = 42.3\text{ mH}$ so $U = \frac{1}{2}LI^2 = \frac{1}{2}(0.423\text{ H})(1.75\text{ A})^2 = \boxed{0.064\,8\text{ J}}$.

**P32.30** (a) The magnetic energy density is given by

$$\mu = \frac{B^2}{2\mu_0} = \frac{(4.50\text{ T})^2}{2\left(1.26\times10^{-6}\text{ T}\cdot\text{m/A}\right)} = \boxed{8.06\times10^6\text{ J/m}^3}.$$

(b) The magnetic energy stored in the field equals $u$ times the volume of the solenoid (the volume in which $B$ is non-zero).

$$U = uV = \left(8.06\times10^6\text{ J/m}^3\right)\left[(0.260\text{ m})\pi(0.031\,0\text{ m})^2\right] = \boxed{6.32\text{ kJ}}$$

**P32.31** $L = \mu_0\frac{N^2A}{\ell} = \mu_0\frac{(68.0)^2\left[\pi\left(0.600\times10^{-2}\right)^2\right]}{0.080\,0} = 8.21\ \mu\text{H}$

$$U = \frac{1}{2}LI^2 = \frac{1}{2}\left(8.21\times10^{-6}\text{ H}\right)(0.770\text{ A})^2 = \boxed{2.44\ \mu\text{J}}$$

**P32.32** (a) $U = \frac{1}{2}LI^2 = \frac{1}{2}L\left(\frac{\varepsilon}{2R}\right)^2 = \frac{L\varepsilon^2}{8R^2} = \frac{(0.800)(500)^2}{8(30.0)^2} = \boxed{27.8\text{ J}}$

(b) $I = \left(\frac{\varepsilon}{R}\right)\left[1 - e^{-(R/L)t}\right]$ so $\frac{\varepsilon}{2R} = \left(\frac{\varepsilon}{R}\right)\left[1 - e^{-(R/L)t}\right] \to e^{-(R/L)t} = \frac{1}{2}$

$\frac{R}{L}t = \ln 2$ so $t = \frac{L}{R}\ln 2 = \frac{0.800}{30.0}\ln 2 = \boxed{18.5\text{ ms}}$

**P32.33** $u = \in_0\frac{E^2}{2} = \boxed{44.2\text{ nJ/m}^3}$ $u = \frac{B^2}{2\mu_0} = \boxed{995\ \mu\text{J/m}^3}$

***P32.34** $\int_0^\infty e^{-2Rt/L}dt = -\frac{L}{2R}\int_0^\infty e^{-2Rt/L}\left(\frac{-2Rdt}{L}\right) = -\frac{L}{2R}e^{-2Rt/L}\Big|_0^\infty = -\frac{L}{2R}\left(e^{-\infty} - e^0\right) = \frac{L}{2R}(0-1) = \boxed{\frac{L}{2R}}$

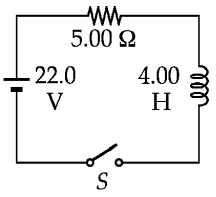


**FIG. P32.35**

**P32.35** (a) $U=\frac{1}{2}LI^2=\frac{1}{2}(4.00\text{ H})(0.500\text{ A})^2$ $\quad U=\boxed{0.500\text{ J}}$

(b) When the current is 1.00 A,

Kirchhoff's loop rule reads $+22.0\text{ V}-(1.00\text{ A})(5.00\ \Omega)-\Delta V_L=0$.

Then $\Delta V_L=17.0\text{ V}$.

The power being stored in the inductor is

$$I\Delta V_L=(1.00\text{ A})(17.0\text{ V})=\boxed{17.0\text{ W}}.$$

(c) $\mathcal{P}=I\Delta V=(0.500\text{ A})(22.0\text{ V})$ $\quad \mathcal{P}=\boxed{11.0\text{ W}}$

**P32.36** From Equation 32.7, $I=\frac{\varepsilon}{R}\left(1-e^{-Rt/L}\right)$.

(a) The maximum current, after a long time $t$, is $I=\frac{\varepsilon}{R}=2.00\text{ A}$.

At that time, the inductor is fully energized and $\mathcal{P}=I(\Delta V)=(2.00\text{ A})(10.0\text{ V})=\boxed{20.0\text{ W}}$.

(b) $\mathcal{P}_{\text{lost}}=I^2R=(2.00\text{ A})^2(5.00\ \Omega)=\boxed{20.0\text{ W}}$

(c) $\mathcal{P}_{\text{inductor}}=I\left(\Delta V_{\text{drop}}\right)=\boxed{0}$

(d) $U=\frac{LI^2}{2}=\frac{(10.0\text{ H})(2.00\text{ A})^2}{2}=\boxed{20.0\text{ J}}$

**P32.37** We have $u=\epsilon_0\frac{E^2}{2}$ and $u=\frac{B^2}{2\mu_0}$.

Therefore $\epsilon_0\frac{E^2}{2}=\frac{B^2}{2\mu_0}$ so $B^2=\epsilon_0\mu_0E^2$

$$B=E\sqrt{\epsilon_0\mu_0}=\frac{6.80\times10^5\text{ V/m}}{3.00\times10^8\text{ m/s}}=\boxed{2.27\times10^{-3}\text{ T}}.$$

**P32.38** The total magnetic energy is the volume integral of the energy density, $u=\frac{B^2}{2\mu_0}$.

Because $B$ changes with position, $u$ is not constant. For $B=B_0\left(\frac{R}{r}\right)^2$, $\quad u=\left(\frac{B_0^2}{2\mu_0}\right)\left(\frac{R}{r}\right)^4$.

Next, we set up an expression for the magnetic energy in a spherical shell of radius $r$ and thickness $dr$. Such a shell has a volume $4\pi r^2dr$, so the energy stored in it is

$$dU=u\left(4\pi r^2dr\right)=\left(\frac{2\pi B_0^2R^4}{\mu_0}\right)\frac{dr}{r^2}.$$

We integrate this expression for $r=R$ to $r=\infty$ to obtain the total magnetic energy outside the sphere. This gives

$$U=\boxed{\frac{2\pi B_0^2R^3}{\mu_0}}=\frac{2\pi\left(5.00\times10^{-5}\text{ T}\right)^2\left(6.00\times10^6\text{ m}\right)^3}{\left(1.26\times10^{-6}\text{ T}\cdot\text{m/A}\right)}=\boxed{2.70\times10^{18}\text{ J}}.$$

## Section 32.4 **Mutual Inductance**

**P32.39** $I_1(t) = I_{\max}e^{-\alpha t}\sin\omega t$ with $I_{\max} = 5.00\text{ A}$, $\alpha = 0.025\,0\text{ s}^{-1}$, and $\omega = 377\text{ rad/s}$

$$\frac{dI_1}{dt} = I_{\max}e^{-\alpha t}\left(-\alpha\sin\omega t + \omega\cos\omega t\right).$$

At $t = 0.800\text{ s}$,
$$\frac{dI_1}{dt} = (5.00\text{ A/s})e^{-0.020\,0}\left[-(0.025\,0)\sin\left(0.800(377)\right) + 377\cos\left(0.800(377)\right)\right]$$

$$\frac{dI_1}{dt} = 1.85\times10^{3}\text{ A/s}.$$

Thus, $\varepsilon_2 = -M\dfrac{dI_1}{dt}$: $\quad M = \dfrac{-\varepsilon_2}{dI_1/dt} = \dfrac{+3.20\text{ V}}{1.85\times10^{3}\text{ A/s}} = \boxed{1.73\text{ mH}}$.

**P32.40** $\varepsilon_2 = -M\dfrac{dI_1}{dt} = -\left(1.00\times10^{-4}\text{ H}\right)\left(1.00\times10^{4}\text{ A/s}\right)\cos(1\,000t)$

$(\varepsilon_2)_{\max} = \boxed{1.00\text{ V}}$

**P32.41** $M = \left|\dfrac{\varepsilon_2}{dI_1/dt}\right| = \dfrac{96.0\text{ mV}}{1.20\text{ A/s}} = \boxed{80.0\text{ mH}}$

**P32.42** Assume the long wire carries current *I*. Then the magnitude of the magnetic field it generates at distance *x* from the wire is $B = \dfrac{\mu_0 I}{2\pi x}$, and this field passes perpendicularly through the plane of the loop. The flux through the loop is

$$\Phi_B = \int \mathbf{B}\cdot d\mathbf{A} = \int B\,dA = \int B(\ell dx) = \frac{\mu_0 I\ell}{2\pi}\int_{0.400\text{ mm}}^{1.70\text{ mm}}\frac{dx}{x} = \frac{\mu_0 I\ell}{2\pi}\ln\left(\frac{1.70}{0.400}\right).$$

The mutual inductance between the wire and the loop is then

$$M = \frac{N_2\Phi_{12}}{I_1} = \frac{N_2\mu_0 I\ell}{2\pi I}\ln\left(\frac{1.70}{0.400}\right) = \frac{N_2\mu_0\ell}{2\pi}(1.45) = \frac{1\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)\left(2.70\times10^{-3}\text{ m}\right)}{2\pi}(1.45)$$

$$M = 7.81\times10^{-10}\text{ H} = \boxed{781\text{ pH}}$$

**P32.43** (a) $\quad M = \dfrac{N_B\Phi_{BA}}{I_A} = \dfrac{700\left(90.0\times10^{-6}\right)}{3.50} = \boxed{18.0\text{ mH}}$

(b) $\quad L_A = \dfrac{\Phi_A}{I_A} = \dfrac{400\left(300\times10^{-6}\right)}{3.50} = \boxed{34.3\text{ mH}}$

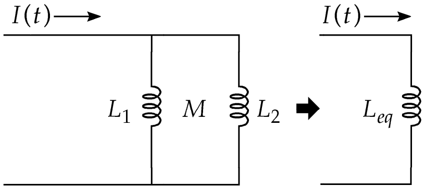
(c) $\quad \varepsilon_B = -M\dfrac{dI_A}{dt} = -(18.0\text{ mH})(0.500\text{ A/s}) = \boxed{-9.00\text{ mV}}$

***P32.44** The large coil produces this field at the center of the small coil: $\dfrac{N_1\mu_0 I_1 R_1^2}{2\left(x^2+R_1^2\right)^{3/2}}$. The field is normal to the area of the small coil and nearly uniform over this area, so it produces flux

$\Phi_{12}=\dfrac{N_1\mu_0 I_1 R_1^2}{2\left(x^2+R_1^2\right)^{3/2}}\pi R_2^2$ through the face area of the small coil. When current $I_1$ varies, this is the emf induced in the small coil:

$$\varepsilon_2=-N_2\frac{d}{dt}\frac{N_1\mu_0 R_1^2\pi R_2^2}{2\left(x^2+R_1^2\right)^{3/2}}I_1=-\frac{N_1N_2\pi\mu_0 R_1^2R_2^2}{2\left(x^2+R_1^2\right)^{3/2}}\frac{dI_1}{dt}=-M\frac{dI_1}{dt}\text{ so }\boxed{M=\frac{N_1N_2\pi\mu_0 R_1^2R_2^2}{2\left(x^2+R_1^2\right)^{3/2}}}.$$

**P32.45** With $I=I_1+I_2$, the voltage across the pair is:



(a) (b)

**FIG. P32.45**

$$\Delta V=-L_1\frac{dI_1}{dt}-M\frac{dI_2}{dt}=-L_2\frac{dI_2}{dt}-M\frac{dI_1}{dt}=-L_{\text{eq}}\frac{dI}{dt}.$$

So, $$-\frac{dI_1}{dt}=\frac{\Delta V}{L_1}+\frac{M}{L_1}\frac{dI_2}{dt}$$

and $$-L_2\frac{dI_2}{dt}+\frac{M(\Delta V)}{L_1}+\frac{M^2}{L_1}\frac{dI_2}{dt}=\Delta V$$

$$\left(-L_1L_2+M^2\right)\frac{dI_2}{dt}=\Delta V\left(L_1-M\right). \qquad [1]$$

By substitution, $$-\frac{dI_2}{dt}=\frac{\Delta V}{L_2}+\frac{M}{L_2}\frac{dI_1}{dt}$$

leads to $$\left(-L_1L_2+M^2\right)\frac{dI_1}{dt}=\Delta V\left(L_2-M\right). \qquad [2]$$

Adding [1] to [2], $$\left(-L_1L_2+M^2\right)\frac{dI}{dt}=\Delta V\left(L_1+L_2-2M\right).$$

So, $$L_{\text{eq}}=-\frac{\Delta V}{dI/dt}=\boxed{\frac{L_1L_2-M^2}{L_1+L_2-2M}}.$$

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## Section 32.5 **Oscillations in an LC Circuit**

**P32.46** At different times, $\left(U_C\right)_{\max}=\left(U_L\right)_{\max}$ so $\left[\frac{1}{2}C(\Delta V)^2\right]_{\max}=\left(\frac{1}{2}LI^2\right)_{\max}$

$$I_{\max}=\sqrt{\frac{C}{L}}(\Delta V)_{\max}=\sqrt{\frac{1.00\times10^{-6}\text{ F}}{10.0\times10^{-3}\text{ H}}}(40.0\text{ V})=\boxed{0.400\text{ A}}.$$

**P32.47** $\left[\frac{1}{2}C(\Delta V)^2\right]_{\max} = \left(\frac{1}{2}LI^2\right)_{\max}$ so $(\Delta V_C)_{\max} = \sqrt{\frac{L}{C}}I_{\max} = \sqrt{\frac{20.0\times10^{-3}\text{ H}}{0.500\times10^{-6}\text{ F}}}(0.100\text{ A}) = \boxed{20.0\text{ V}}$

**P32.48** When the switch has been closed for a long time, battery, resistor, and coil carry constant current $I_{\max} = \frac{\varepsilon}{R}$. When the switch is opened, current in battery and resistor drops to zero, but the coil carries this same current for a moment as oscillations begin in the $LC$ loop.

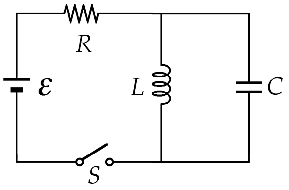

**FIG. P32.50**

We interpret the problem to mean that the voltage amplitude of these oscillations is $\Delta V$, in $\frac{1}{2}C(\Delta V)^2 = \frac{1}{2}LI_{\max}^2$.

Then, $L = \frac{C(\Delta V)^2}{I_{\max}^2} = \frac{C(\Delta V)^2R^2}{\varepsilon^2} = \frac{(0.500\times10^{-6}\text{ F})(150\text{ V})^2(250\ \Omega)^2}{(50.0\text{ V})^2} = \boxed{0.281\text{ H}}$.

**P32.49** This radio is a radiotelephone on a ship, according to frequency assignments made by international treaties, laws, and decisions of the National Telecommunications and Information Administration.

The resonance frequency is $f_0 = \frac{1}{2\pi\sqrt{LC}}$.

Thus, $C = \frac{1}{(2\pi f_0)^2L} = \frac{1}{\left[2\pi(6.30\times10^6\text{ Hz})\right]^2(1.05\times10^{-6}\text{ H})} = \boxed{608\text{ pF}}$.

**P32.50** $f = \frac{1}{2\pi\sqrt{LC}}$: $L = \frac{1}{(2\pi f)^2C} = \frac{1}{[2\pi(120)]^2(8.00\times10^{-6})} = \boxed{0.220\text{ H}}$

**P32.51** (a) $f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(0.082\,0\text{ H})(17.0\times10^{-6}\text{ F})}} = \boxed{135\text{ Hz}}$

(b) $Q = Q_{\max}\cos\omega t = (180\ \mu\text{C})\cos(847\times0.001\,00) = \boxed{119\ \mu\text{C}}$

(c) $I = \frac{dQ}{dt} = -\omega Q_{\max}\sin\omega t = -(847)(180)\sin(0.847) = \boxed{-114\text{ mA}}$

**P32.52** (a) $f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(0.100\text{ H})(1.00\times10^{-6}\text{ F})}} = \boxed{503\text{ Hz}}$

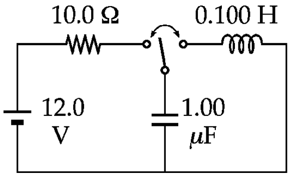

**FIG. P32.52**

(b) $Q = C\varepsilon = (1.00\times10^{-6}\text{ F})(12.0\text{ V}) = \boxed{12.0\ \mu\text{C}}$

(c) $\frac{1}{2}C\varepsilon^2 = \frac{1}{2}LI_{\max}^2$

$I_{\max} = \varepsilon\sqrt{\frac{C}{L}} = 12\text{ V}\sqrt{\frac{1.00\times10^{-6}\text{ F}}{0.100\text{ H}}} = \boxed{37.9\text{ mA}}$

(d) At all times $U = \frac{1}{2}C\varepsilon^2 = \frac{1}{2}(1.00\times10^{-6}\text{ F})(12.0\text{ V})^2 = \boxed{72.0\ \mu\text{J}}$.

**P32.53** $\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(3.30\text{ H})(840\times10^{-12}\text{ F})}} = 1.899\times10^{4}\text{ rad/s}$

$Q = Q_{\max}\cos\omega t$, $I = \frac{dQ}{dt} = -\omega Q_{\max}\sin\omega t$

(a) $U_C = \frac{Q^2}{2C} = \frac{\left(\left[105\times10^{-6}\right]\cos\left[(1.899\times10^{4}\text{ rad/s})(2.00\times10^{-3}\text{ s})\right]\right)^2}{2(840\times10^{-12})} = \boxed{6.03\text{ J}}$

(b) $U_L = \frac{1}{2}LI^2 = \frac{1}{2}L\omega^2Q_{\max}^2\sin^2(\omega t) = \frac{Q_{\max}^2\sin^2(\omega t)}{2C}$

$U_L = \frac{(105\times10^{-6}\text{ C})^2\sin^2\left[(1.899\times10^{4}\text{ rad/s})(2.00\times10^{-3}\text{ s})\right]}{2(840\times10^{-12}\text{ F})} = \boxed{0.529\text{ J}}$

(c) $U_{\text{total}} = U_C + U_L = \boxed{6.56\text{ J}}$

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## Section 32.6 **The *RLC* Circuit**

**P32.54** (a) $\omega_d = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} = \sqrt{\frac{1}{(2.20\times10^{-3})(1.80\times10^{-6})} - \left(\frac{7.60}{2(2.20\times10^{-3})}\right)^2} = 1.58\times10^{4}\text{ rad/s}$

Therefore, $f_d = \frac{\omega_d}{2\pi} = \boxed{2.51\text{ kHz}}$.

(b) $R_c = \sqrt{\frac{4L}{C}} = \boxed{69.9\ \Omega}$

**P32.55** (a) $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.500)(0.100\times10^{-6})}} = \boxed{4.47\text{ krad/s}}$

(b) $\omega_d = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} = \boxed{4.36\text{ krad/s}}$

(c) $\frac{\Delta\omega}{\omega_0} = \boxed{2.53\%\text{ lower}}$

**P32.56** Choose to call positive current clockwise in Figure 32.21. It drains charge from the capacitor according to $I = -\frac{dQ}{dt}$. A clockwise trip around the circuit then gives

$+\frac{Q}{C} - IR - L\frac{dI}{dt} = 0$

$+\frac{Q}{C} + \frac{dQ}{dt}R + L\frac{d}{dt}\frac{dQ}{dt} = 0$, identical with Equation 32.28.

***P32.57** The period of damped oscillation is $T=\frac{2\pi}{\omega_d}$. After one oscillation the charge returning to the capacitor is $Q=Q_{\max}e^{-RT/2L}=Q_{\max}e^{-2\pi R/2L\omega_d}$. The energy is proportional to the charge squared, so after one oscillation it is $U=U_0e^{-2\pi R/L\omega_d}=0.99U_0$. Then

$$e^{2\pi R/L\omega_d}=\frac{1}{0.99}$$

$$\frac{2\pi 2\ \Omega}{L\omega_d}=\ln(1.010\,1)=0.001\,005$$

$$L\omega_d=\frac{2\pi 2\ \Omega}{0.001\,005}=1\,250\ \Omega=L\left(\frac{1}{LC}-\frac{R^2}{4L^2}\right)^{1/2}$$

$$1.563\times10^6\ \Omega^2=\frac{L}{C}-\frac{(2\ \Omega)^2}{4}$$

$$\frac{L}{C}=1.563\times10^6\ \Omega^2$$

We are also given

$$\omega=2\pi\times10^3/\text{s}=\frac{1}{\sqrt{LC}}$$

$$LC=\frac{1}{\left(2\pi\times10^3/\text{s}\right)^2}=2.533\times10^{-8}\ \text{s}^2$$

Solving simultaneously,

$$C=2.533\times10^{-8}\ \text{s}^2/L$$

$$\frac{L^2}{2.533\times10^{-8}\ \text{s}^2}=1.563\times10^6\ \Omega^2 \quad \boxed{L=0.199\ \text{H}}$$

$$C=\frac{2.533\times10^{-8}\ \text{s}^2}{0.199\ \text{H}}=\boxed{127\ \text{nF}=C}$$

**P32.58** (a) $Q=Q_{\max}e^{-Rt/2L}\cos\omega_d t$ so $I_{\max}\propto e^{-Rt/2L}$

$0.500=e^{-Rt/2L}$ and $\frac{Rt}{2L}=-\ln(0.500)$

$$t=-\frac{2L}{R}\ln(0.500)=\boxed{0.693\left(\frac{2L}{R}\right)}$$

(b) $U_0\propto Q_{\max}^2$ and $U=0.500U_0$ so $Q=\sqrt{0.500}Q_{\max}=0.707Q_{\max}$

$$t=-\frac{2L}{R}\ln(0.707)=\boxed{0.347\left(\frac{2L}{R}\right)}$$ (half as long)

## Additional Problems

***P32.59** (a) Let $Q$ represent the magnitude of the opposite charges on the plates of a parallel plate capacitor, the two plates having area $A$ and separation $d$. The negative plate creates electric field $\mathbf{E} = \frac{Q}{2\epsilon_0 A}$ toward itself. It exerts on the positive plate force $\mathbf{F} = \frac{Q^2}{2\epsilon_0 A}$ toward the negative plate. The total field between the plates is $\frac{Q}{\epsilon_0 A}$. The energy density is $u_E = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\epsilon_0 \frac{Q^2}{\epsilon_0^2 A^2} = \frac{Q^2}{2\epsilon_0 A^2}$. Modeling this as a negative or inward pressure, we have for the force on one plate $F = PA = \frac{Q^2}{2\epsilon_0 A^2}$, in agreement with our first analysis.

(b) The lower of the two current sheets shown creates above it magnetic field $\mathbf{B} = \frac{\mu_0 J_s}{2}\left(-\hat{\mathbf{k}}\right)$. Let $\ell$ and $w$ represent the length and width of each sheet. The upper sheet carries current $J_s w$ and feels force

$$\mathbf{F} = I\boldsymbol{\ell} \times \mathbf{B} = J_s w\ell \frac{\mu_0 J_s}{2}\hat{\mathbf{i}} \times \left(-\hat{\mathbf{k}}\right) = \frac{\mu_0 w\ell J_s^2}{2}\hat{\mathbf{j}}.$$

The force per area is $P = \frac{F}{\ell w} = \boxed{\frac{\mu_0 J_s^2}{2}}$.

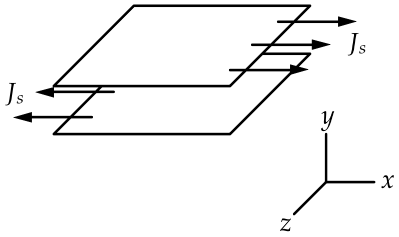


**FIG. P32.59(b)**

(c) Between the two sheets the total magnetic field is $\frac{\mu_0 J_s}{2}\left(-\hat{\mathbf{k}}\right) + \frac{\mu_0 J_s}{2}\left(-\hat{\mathbf{k}}\right) = \mu_0 J_s \hat{\mathbf{k}}$, with magnitude $\boxed{B = \mu_0 J_s}$. Outside the space they enclose, the fields of the separate sheets are in opposite directions and add to $\boxed{\text{zero}}$.

(d) $$u_B = \frac{1}{2\mu_0}B^2 = \frac{\mu_0^2 J_s^2}{2\mu_0} = \boxed{\frac{\mu_0 J_s^2}{2}}$$

(e) This energy density agrees with the magnetic pressure found in part (b).

**P32.60** With $Q = Q_{\max}$ at $t = 0$, the charge on the capacitor at any time is $Q = Q_{\max}\cos\omega t$ where $\omega = \frac{1}{\sqrt{LC}}$.

The energy stored in the capacitor at time $t$ is then

$$U = \frac{Q^2}{2C} = \frac{Q_{\max}^2}{2C}\cos^2\omega t = U_0 \cos^2\omega t.$$

When $U = \frac{1}{4}U_0$, $\quad \cos\omega t = \frac{1}{2} \quad$ and $\quad \omega t = \frac{1}{3}\pi \text{ rad}$.

Therefore, $\quad \frac{t}{\sqrt{LC}} = \frac{\pi}{3} \quad$ or $\quad \frac{t^2}{LC} = \frac{\pi^2}{9}$.

The inductance is then: $\quad L = \boxed{\frac{9t^2}{\pi^2 C}}$.

**P32.61** (a) $$\varepsilon_L = -L\frac{dI}{dt} = -(1.00\text{ mH})\frac{d(20.0t)}{dt} = \boxed{-20.0\text{ mV}}$$

(b) $$Q = \int_0^t I dt = \int_0^t (20.0t)dt = 10.0t^2$$

$$\Delta V_C = \frac{-Q}{C} = \frac{-10.0t^2}{1.00\times 10^{-6}\text{ F}} = \boxed{-\left(10.0\text{ MV/s}^2\right)t^2}$$

(c) When $\frac{Q^2}{2C} \ge \frac{1}{2}LI^2$, or $\frac{\left(-10.0t^2\right)^2}{2\left(1.00\times 10^{-6}\right)} \ge \frac{1}{2}\left(1.00\times 10^{-3}\right)(20.0t)^2$,

then $100t^4 \ge \left(400\times 10^{-9}\right)t^2$. The earliest time this is true is at $t = \sqrt{4.00\times 10^{-9}}\text{ s} = \boxed{63.2\ \mu\text{s}}$.

**P32.62** (a) $$\varepsilon_L = -L\frac{dI}{dt} = -L\frac{d}{dt}(Kt) = \boxed{-LK}$$

(b) $I = \frac{dQ}{dt}$, so $Q = \int_0^t I dt = \int_0^t Kt dt = \frac{1}{2}Kt^2$

and $\Delta V_C = \frac{-Q}{C} = \boxed{-\frac{Kt^2}{2C}}$

(c) When $\frac{1}{2}C(\Delta V_C)^2 = \frac{1}{2}LI^2$, $\frac{1}{2}C\left(\frac{K^2t^4}{4C^2}\right) = \frac{1}{2}L\left(K^2t^2\right)$

Thus $t = \boxed{2\sqrt{LC}}$

**P32.63** $\frac{1}{2}\frac{Q^2}{C} = \frac{1}{2C}\left(\frac{Q}{2}\right)^2 + \frac{1}{2}LI^2$ so $I = \sqrt{\frac{3Q^2}{4CL}}$.

The flux through each turn of the coil is $\Phi_B = \frac{LI}{N} = \boxed{\frac{Q}{2N}\sqrt{\frac{3L}{C}}}$

where $N$ is the number of turns.

**P32.64** $$B = \frac{\mu_0 NI}{2\pi r}$$

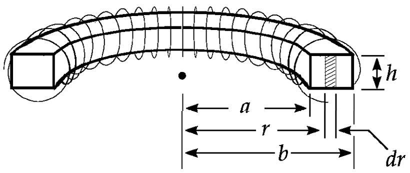


**FIG. P32.64**

(a) $$\Phi_B = \int B dA = \int_a^b \frac{\mu_0 NI}{2\pi r}h dr = \frac{\mu_0 NIh}{2\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 NIh}{2\pi}\ln\left(\frac{b}{a}\right)$$

$$L = \frac{N\Phi_B}{I} = \boxed{\frac{\mu_0 N^2 h}{2\pi}\ln\left(\frac{b}{a}\right)}$$

(b) $$L = \frac{\mu_0(500)^2(0.010\,0)}{2\pi}\ln\left(\frac{12.0}{10.0}\right) = \boxed{91.2\ \mu\text{H}}$$

(c) $L_{\text{appx}} = \frac{\mu_0 N^2}{2\pi}\left(\frac{A}{R}\right) = \frac{\mu_0(500)^2}{2\pi}\left(\frac{2.00\times 10^{-4}\text{ m}^2}{0.110}\right) = \boxed{90.9\ \mu\text{H}}$, only 0.3% different.

**P32.65** (a) At the center, $B = \dfrac{N\mu_0 I R^2}{2\left(R^2 + 0^2\right)^{3/2}} = \dfrac{N\mu_0 I}{2R}$.

So the coil creates flux through itself $\Phi_B = BA\cos\theta = \dfrac{N\mu_0 I}{2R}\pi R^2 \cos 0° = \dfrac{\pi}{2} N\mu_0 IR$.

When the current it carries changes, $\varepsilon_L = -N\dfrac{d\Phi_B}{dt} \approx -N\left(\dfrac{\pi}{2}\right)N\mu_0 R\dfrac{dI}{dt} = -L\dfrac{dI}{dt}$

so $\boxed{L \approx \dfrac{\pi}{2}N^2\mu_0 R}$.

(b) $2\pi r = 3(0.3 \text{ m})$ so $r \approx 0.14 \text{ m}$

$$L \approx \frac{\pi}{2}\left(1^2\right)\left(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}\right)(0.14 \text{ m}) = 2.8 \times 10^{-7} \text{ H}$$

$$\boxed{L \sim 100 \text{ nH}}$$

(c) $\dfrac{L}{R} = \dfrac{2.8 \times 10^{-7} \text{ V}\cdot\text{s/A}}{270 \text{ V/A}} = 1.0 \times 10^{-9} \text{ s}$ $\boxed{\dfrac{L}{R} \sim 1 \text{ ns}}$

**P32.66** (a) If unrolled, the wire forms the diagonal of a 0.100 m (10.0 cm) rectangle as shown. The length of this rectangle is

$$L' = \sqrt{(9.80 \text{ m})^2 - (0.100 \text{ m})^2}.$$

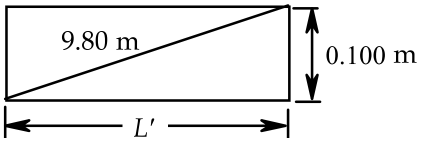


**FIG. P32.66(a)**

The mean circumference of each turn is $C = 2\pi r'$, where $r' = \dfrac{24.0 + 0.644}{2}$ mm is the mean radius of each turn. The number of turns is then:

$$N = \frac{L'}{C} = \frac{\sqrt{(9.80 \text{ m})^2 - (0.100 \text{ m})^2}}{2\pi\left[(24.0 + 0.644)/2\right] \times 10^{-3} \text{ m}} = \boxed{127}.$$

(b) $$R = \frac{\rho\ell}{A} = \frac{\left(1.70 \times 10^{-8} \ \Omega\cdot\text{m}\right)(10.0 \text{ m})}{\pi\left(0.322 \times 10^{-3} \text{ m}\right)^2} = \boxed{0.522 \ \Omega}$$

(c) $$L = \frac{\mu N^2 A}{\ell'} = \frac{800\mu_0}{\ell'}\left(\frac{L'}{C}\right)^2 \pi(r')^2$$

$$L = \frac{800\left(4\pi \times 10^{-7}\right)}{0.100 \text{ m}}\left(\frac{\sqrt{(9.80 \text{ m})^2 - (0.100 \text{ m})^2}}{\pi(24.0 + 0.644) \times 10^{-3} \text{ m}}\right)^2 \pi\left[\left(\frac{24.0 + 0.644}{2}\right) \times 10^{-3} \text{ m}\right]^2$$

$$L = 7.68 \times 10^{-2} \text{ H} = \boxed{76.8 \text{ mH}}$$

**P32.67** From Ampere's law, the magnetic field at distance $r \le R$ is found as:

$$B(2\pi r) = \mu_0 J(\pi r^2) = \mu_0\left(\frac{I}{\pi R^2}\right)(\pi r^2), \text{ or } B = \frac{\mu_0 I r}{2\pi R^2}.$$

The magnetic energy per unit length within the wire is then

$$\frac{U}{\ell} = \int_0^R \frac{B^2}{2\mu_0}(2\pi r\,dr) = \frac{\mu_0 I^2}{4\pi R^4}\int_0^R r^3 dr = \frac{\mu_0 I^2}{4\pi R^4}\left(\frac{R^4}{4}\right) = \boxed{\frac{\mu_0 I^2}{16\pi}}.$$

This is independent of the radius of the wire.

**P32.68** The primary circuit (containing the battery and solenoid) is an *RL* circuit with $R = 14.0\ \Omega$, and

$$L = \frac{\mu_0 N^2 A}{\ell} = \frac{(4\pi\times10^{-7})(12\,500)^2(1.00\times10^{-4})}{0.070\,0} = 0.280\text{ H}.$$

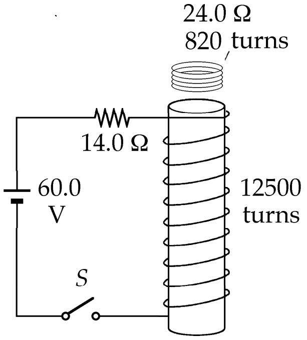


**FIG. P32.68**

(a) The time for the current to reach 63.2% of the maximum value is the time constant of the circuit:

$$\tau = \frac{L}{R} = \frac{0.280\text{ H}}{14.0\ \Omega} = 0.020\,0\text{ s} = \boxed{20.0\text{ ms}}.$$

(b) The solenoid's average back emf is $|\bar{\varepsilon}_L| = L\left(\frac{\Delta I}{\Delta t}\right) = L\left(\frac{I_f - 0}{\Delta t}\right)$

where $I_f = 0.632 I_{\max} = 0.632\left(\frac{\Delta V}{R}\right) = 0.632\left(\frac{60.0\text{ V}}{14.0\ \Omega}\right) = 2.71\text{ A}$.

Thus, $|\bar{\varepsilon}_L| = (0.280\text{ H})\left(\frac{2.71\text{ A}}{0.020\,0\text{ s}}\right) = \boxed{37.9\text{ V}}$.

(c) The average rate of change of flux through each turn of the overwrapped concentric coil is the same as that through a turn on the solenoid:

$$\frac{\Delta\Phi_B}{\Delta t} = \frac{\mu_0 n(\Delta I)A}{\Delta t} = \frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(12\,500/0.070\,0\text{ m})(2.71\text{ A})(1.00\times10^{-4}\text{ m}^2)}{0.020\,0\text{ s}}$$

$$= \boxed{3.04\text{ mV}}$$

(d) The magnitude of the average induced emf in the coil is $|\varepsilon_L| = N\left(\frac{\Delta\Phi_B}{\Delta t}\right)$ and magnitude of the average induced current is

$$I = \frac{|\varepsilon_L|}{R} = \frac{N}{R}\left(\frac{\Delta\Phi_B}{\Delta t}\right) = \frac{820}{24.0\ \Omega}(3.04\times10^{-3}\text{ V}) = 0.104\text{ A} = \boxed{104\text{ mA}}.$$

**P32.69** Left-hand loop: $\varepsilon-(I+I_2)R_1-I_2R_2=0$.

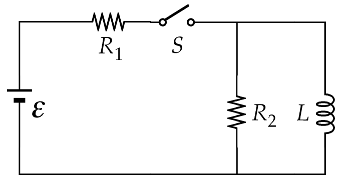


**FIG. P32.69**

Outside loop: $\varepsilon-(I+I_2)R_1-L\dfrac{dI}{dt}=0$.

Eliminating $I_2$ gives $\varepsilon'-IR'-L\dfrac{dI}{dt}=0$.

This is of the same form as Equation 32.6, so its solution is of the same form as Equation 32.7:

$$I(t)=\frac{\varepsilon'}{R'}\left(1-e^{-R't/L}\right).$$

But $R'=\dfrac{R_1R_2}{R_1+R_2}$ and $\varepsilon'=\dfrac{R_2\varepsilon}{R_1+R_2}$, so $\dfrac{\varepsilon'}{R'}=\dfrac{\varepsilon R_2/(R_1+R_2)}{R_1R_2/(R_1+R_2)}=\dfrac{\varepsilon}{R_1}$.

Thus $$I(t)=\frac{\varepsilon}{R_1}\left(1-e^{-R't/L}\right).$$

**P32.70** When switch is closed, steady current $I_0=1.20\text{ A}$. When the switch is opened after being closed a long time, the current in the right loop is

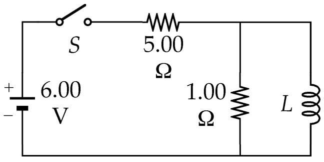


**FIG. P32.70**

$$I=I_0e^{-R_2t/L}$$

so $e^{Rt/L}=\dfrac{I_0}{I}$ and $\dfrac{Rt}{L}=\ln\left(\dfrac{I_0}{I}\right)$.

Therefore, $L=\dfrac{R_2t}{\ln(I_0/I)}=\dfrac{(1.00\ \Omega)(0.150\text{ s})}{\ln(1.20\text{ A}/0.250\text{ A})}=0.095\,6\text{ H}=\boxed{95.6\text{ mH}}$.

**P32.71** (a) While steady-state conditions exist, a 9.00 mA flows clockwise around the right loop of the circuit. Immediately after the switch is opened, a 9.00 mA current will flow around the outer loop of the circuit. Applying Kirchhoff's loop rule to this loop gives:

$$+\varepsilon_0-\left[(2.00+6.00)\times10^3\ \Omega\right]\left(9.00\times10^{-3}\text{ A}\right)=0$$

$$+\varepsilon_0=\boxed{72.0\text{ V with end } b \text{ at the higher potential}}$$

(b)

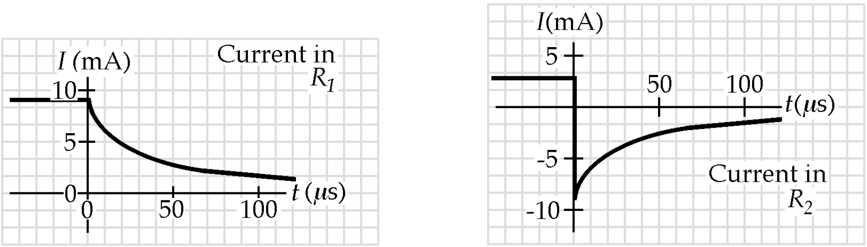


**FIG. P32.71(b)**

(c) After the switch is opened, the current around the outer loop decays as

$I=I_{\max}e^{-Rt/L}$ with $I_{\max}=9.00\text{ mA}$, $R=8.00\text{ k}\Omega$, and $L=0.400\text{ H}$.

Thus, when the current has reached a value $I=2.00\text{ mA}$, the elapsed time is:

$$t=\left(\frac{L}{R}\right)\ln\left(\frac{I_{\max}}{I}\right)=\left(\frac{0.400\text{ H}}{8.00\times10^3\ \Omega}\right)\ln\left(\frac{9.00}{2.00}\right)=7.52\times10^{-5}\text{ s}=\boxed{75.2\ \mu\text{s}}.$$

**P32.72** (a) The instant after the switch is closed, the situation is as shown in the circuit diagram of Figure (a). The requested quantities are:

$$\boxed{I_L = 0,\ I_C = \frac{\varepsilon_0}{R},\ I_R = \frac{\varepsilon_0}{R}}$$

$$\boxed{\Delta V_L = \varepsilon_0,\ \Delta V_C = 0,\ \Delta V_R = \varepsilon_0}$$

(b) After the switch has been closed a long time, the steady-state conditions shown in Figure (b) will exist. The currents and voltages are:

$$\boxed{I_L = 0,\ I_C = 0,\ I_R = 0}$$

$$\boxed{\Delta V_L = 0,\ \Delta V_C = \varepsilon_0,\ \Delta V_R = 0}$$

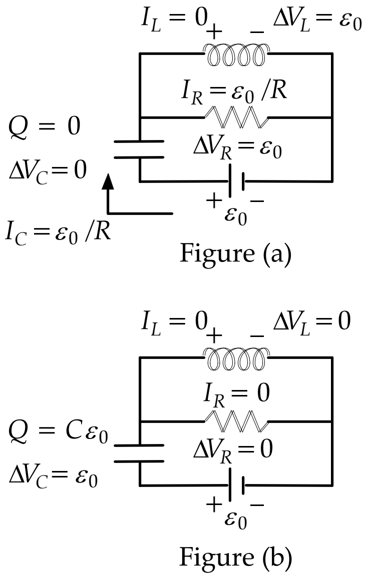


Figure (a)

Figure (b)

**FIG. P32.72**

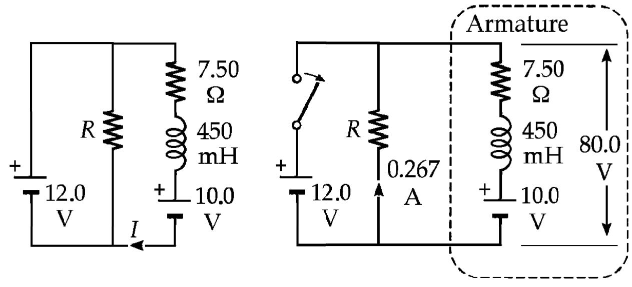
**P32.73** When the switch is closed, as shown in figure (a), the current in the inductor is $I$:

$12.0 - 7.50I - 10.0 = 0 \to I = 0.267\ \text{A}$.

When the switch is opened, the initial current in the inductor remains at 0.267 A.

$IR = \Delta V$: $\quad (0.267\ \text{A})R \le 80.0\ \text{V}$

$$\boxed{R \le 300\ \Omega}$$



(a) (b)

**FIG. P32.73**

**P32.74** (a) $$L_1 = \frac{\mu_0 N_1^2 A}{\ell_1} = \frac{\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)(1\,000)^2\left(1.00 \times 10^{-4}\ \text{m}^2\right)}{0.500\ \text{m}} = 2.51 \times 10^{-4}\ \text{H} = \boxed{251\ \mu\text{H}}$$

(b) $$M = \frac{N_2 \Phi_2}{I_1} = \frac{N_2 \Phi_1}{I_1} = \frac{N_2 BA}{I_1} = \frac{N_2\left[\mu_0 (N_1/\ell_1) I_1\right] A}{I_1} = \frac{\mu_0 N_1 N_2 A}{\ell_1}$$

$$M = \frac{\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)(1\,000)(100)\left(1.00 \times 10^{-4}\ \text{m}^2\right)}{0.500\ \text{m}} = 2.51 \times 10^{-5}\ \text{H} = \boxed{25.1\ \mu\text{H}}$$

(c) $$\varepsilon_1 = -M\frac{dI_2}{dt},\ \text{or}\ I_1 R_1 = -M\frac{dI_2}{dt}\ \text{and}\ I_1 = \frac{dQ_1}{dt} = -\frac{M}{R_1}\frac{dI_2}{dt}$$

$$Q_1 = -\frac{M}{R_1}\int_0^{t_f} dI_2 = -\frac{M}{R_1}\left(I_{2f} - I_{2i}\right) = -\frac{M}{R_1}\left(0 - I_{2i}\right) = \frac{M I_{2i}}{R_1}$$

$$Q_1 = \frac{\left(2.51 \times 10^{-5}\ \text{H}\right)(1.00\ \text{A})}{1\,000\Omega} = 2.51 \times 10^{-8}\ \text{C} = \boxed{25.1\ \text{nC}}$$

**P32.75** (a) It has a magnetic field, and it stores energy, so $L = \frac{2U}{I^2}$ is non-zero.

(b) Every field line goes through the rectangle between the conductors.

(c) $\Phi = LI$ so $L = \frac{\Phi}{I} = \frac{1}{I}\int_{y=a}^{w-a} B\,dA$

$$L = \frac{1}{I}\int_{a}^{w-a} x\,dy\left(\frac{\mu_0 I}{2\pi y} + \frac{\mu_0 I}{2\pi(w-y)}\right) = \frac{2}{I}\int \frac{\mu_0 I x}{2\pi y}dy = \frac{2\mu_0 x}{2\pi}\ln y\Big|_a^{w-a}.$$

Thus $L = \frac{\mu_0 x}{\pi}\ln\left(\frac{w-a}{a}\right)$.

**P32.76** For an *RL* circuit,

$I(t) = I_{\max}e^{-(R/L)t}$: $\frac{I(t)}{I_{\max}} = 1 - 10^{-9} = e^{-(R/L)t} \cong 1 - \frac{R}{L}t$

$\frac{R}{L}t = 10^{-9}$ so $R_{\max} = \frac{(3.14\times10^{-8})(10^{-9})}{(2.50\text{ yr})(3.16\times10^{7}\text{ s/yr})} = \boxed{3.97\times10^{-25}\ \Omega}$.

(If the ring were of purest copper, of diameter 1 cm, and cross-sectional area $1\text{ mm}^2$, its resistance would be at least $10^{-6}\ \Omega$).

**P32.77** (a) $U_B = \frac{1}{2}LI^2 = \frac{1}{2}(50.0\text{ H})(50.0\times10^3\text{ A})^2 = \boxed{6.25\times10^{10}\text{ J}}$

(b) Two adjacent turns are parallel wires carrying current in the same direction. Since the loops have such large radius, a one-meter section can be regarded as straight.

Then one wire creates a field of $B = \frac{\mu_0 I}{2\pi r}$.

This causes a force on the next wire of $F = I\ell B\sin\theta$

giving $F = I\ell\frac{\mu_0 I}{2\pi r}\sin 90° = \frac{\mu_0 \ell I^2}{2\pi r}$.

Evaluating the force, $F = (4\pi\times10^{-7}\text{ N/A}^2)\frac{(1.00\text{ m})(50.0\times10^3\text{ A})^2}{2\pi(0.250\text{ m})} = \boxed{2\,000\text{ N}}$.

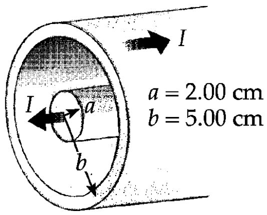


**FIG. P32.73**

**P32.78** $\mathcal{P} = I\Delta V \qquad I = \frac{\mathcal{P}}{\Delta V} = \frac{1.00\times 10^9\ \text{W}}{200\times 10^3\ \text{V}} = 5.00\times 10^3\ \text{A}$

From Ampere's law, $B(2\pi r) = \mu_0 I_{\text{enclosed}}$ or $B = \frac{\mu_0 I_{\text{enclosed}}}{2\pi r}$.

(a) At $r = a = 0.020\,0\ \text{m}$, $I_{\text{enclosed}} = 5.00\times 10^3\ \text{A}$

and $$B = \frac{\left(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(5.00\times 10^3\ \text{A}\right)}{2\pi(0.020\,0\ \text{m})} = 0.050\,0\ \text{T} = \boxed{50.0\ \text{mT}}.$$

(b) At $r = b = 0.050\,0\ \text{m}$, $I_{\text{enclosed}} = I = 5.00\times 10^3\ \text{A}$

and $$B = \frac{\left(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(5.00\times 10^3\ \text{A}\right)}{2\pi(0.050\,0\ \text{m})} = 0.020\,0\ \text{T} = \boxed{20.0\ \text{mT}}.$$

(c) $$U = \int u\,dV = \int_a^b \frac{[B(r)]^2(2\pi r\ell\,dr)}{2\mu_0} = \frac{\mu_0 I^2 \ell}{4\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 I^2 \ell}{4\pi}\ln\left(\frac{b}{a}\right)$$

$$U = \frac{\left(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(5.00\times 10^3\ \text{A}\right)^2\left(1\,000\times 10^3\ \text{m}\right)}{4\pi}\ln\left(\frac{5.00\ \text{cm}}{2.00\ \text{cm}}\right) = 2.29\times 10^6\ \text{J} = \boxed{2.29\ \text{MJ}}$$

(d) The magnetic field created by the inner conductor exerts a force of repulsion on the current in the outer sheath. The strength of this field, from part (b), is 20.0 mT. Consider a small rectangular section of the outer cylinder of length $\ell$ and width $w$.

It carries a current of $$\left(5.00\times 10^3\ \text{A}\right)\left(\frac{w}{2\pi(0.050\,0\ \text{m})}\right)$$

and experiences an outward force

$$F = I\ell B\sin\theta = \frac{\left(5.00\times 10^3\ \text{A}\right)w}{2\pi(0.050\,0\ \text{m})}\ell\left(20.0\times 10^{-3}\ \text{T}\right)\sin 90.0^\circ.$$

The pressure on it is $$P = \frac{F}{A} = \frac{F}{w\ell} = \frac{\left(5.00\times 10^3\ \text{A}\right)\left(20.0\times 10^{-3}\ \text{T}\right)}{2\pi(0.050\,0\ \text{m})} = \boxed{318\ \text{Pa}}.$$

**P32.79** (a) $$B=\frac{\mu_0 NI}{\ell}=\frac{\left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)(1\,400)(2.00\ \mathrm{A})}{1.20\ \mathrm{m}}=\boxed{2.93\times10^{-3}\ \mathrm{T\ (upward)}}$$

(b) $$u=\frac{B^2}{2\mu_0}=\frac{\left(2.93\times10^{-3}\ \mathrm{T}\right)^2}{2\left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)}=\left(3.42\ \mathrm{J/m^3}\right)\left(\frac{1\ \mathrm{N\cdot m}}{1\ \mathrm{J}}\right)=3.42\ \mathrm{N/m^2}=\boxed{3.42\ \mathrm{Pa}}$$

(c) To produce a downward magnetic field, the surface of the superconductor must carry a $\boxed{\text{clockwise}}$ current.

(d) The vertical component of the field of the solenoid exerts an inward force on the superconductor. The total horizontal force is zero. Over the top end of the solenoid, its field diverges and has a radially outward horizontal component. This component exerts upward force on the clockwise superconductor current. The total force on the core is $\boxed{\text{upward}}$. You can think of it as a force of repulsion between the solenoid with its north end pointing up, and the core, with its north end pointing down.

(e) $$F=PA=(3.42\ \mathrm{Pa})\left[\pi\left(1.10\times10^{-2}\ \mathrm{m}\right)^2\right]=\boxed{1.30\times10^{-3}\ \mathrm{N}}$$

Note that we have not proven that energy density is pressure. In fact, it is not in some cases; Equation 21.2 shows that the pressure is two-thirds of the translational energy density in an ideal gas.

## ANSWERS TO EVEN PROBLEMS

| | |
|---|---|
| **P32.2** | 1.36 $\mu$H |
| **P32.4** | 240 nWb |
| **P32.6** | 19.2 $\mu$Wb |
| **P32.8** | (a) 360 mV; (b) 180 mV; (c) $t=3.00$ s |
| **P32.10** | (a) 15.8 $\mu$H; (b) 12.6 mH |
| **P32.12** | see the solution |
| **P32.14** | 1.92 Ω |
| **P32.16** | see the solution |
| **P32.18** | 92.8 V |
| **P32.20** | 30.0 mH |
| **P32.22** | 7.67 mH |
| **P32.24** | (a) 1.00 kΩ; (b) 3.00 ms |
| **P32.26** | (a) 1.00 A; (b) $\Delta V_{12}=12.0$ V, $\Delta V_{1\,200}=1.20$ kV, $\Delta V_L=1.21$ kV; (c) 7.62 ms |
| **P32.28** | (a), (b), and (c) see the solution;<br>(d) yes; see the solution |
| **P32.30** | (a) 8.06 $\mathrm{MJ/m^3}$; (b) 6.32 kJ |
| **P32.32** | (a) 27.8 J; (b) 18.5 ms |
| **P32.34** | see the solution |
| **P32.36** | (a) 20.0 W; (b) 20.0 W; (c) 0;(d) 20.0 J |
| **P32.38** | $\frac{2\pi B_0^2R^3}{\mu_0}=2.70\times10^{18}$ J |
| **P32.40** | 1.00 V |
| **P32.42** | 781 pH |
| **P32.44** | $M=\frac{N_1N_2\pi\mu_0R_1^2R_2^2}{2\left(x^2+R_1^2\right)^{3/2}}$ |

**P32.46** 400 mA

**P32.48** 281 mH

**P32.50** 220 mH

**P32.52** (a) 503 Hz; (b) 12.0 $\mu$C; (c) 37.9 mA; (d) 72.0 $\mu$J

**P32.54** (a) 2.51 kHz; (b) 69.9 $\Omega$

**P32.56** see the solution

**P32.58** (a) $0.693\left(\frac{2L}{R}\right)$; (b) $0.347\left(\frac{2L}{R}\right)$

**P32.60** $\frac{9t^2}{\pi^2 C}$

**P32.62** (a) $\varepsilon_L = -LK$; (b) $\Delta V_c = \frac{-Kt^2}{2C}$; (c) $t = 2\sqrt{LC}$

**P32.64** (a) see the solution; (b) 91.2 $\mu$H; (c) 90.9 $\mu$H, 0.3% smaller

**P32.66** (a) 127; (b) 0.522 $\Omega$; (c) 76.8 mH

**P32.68** (a) 20.0 ms; (b) 37.9 V; (c) 3.04 mV; (d) 104 mA

**P32.70** 95.6 mH

**P32.72** (a) $I_L = 0$, $I_C = \frac{\varepsilon_0}{R}$, $I_R = \frac{\varepsilon_0}{R}$, $\Delta V_L = \varepsilon_0$, $\Delta V_C = 0$, $\Delta V_R = \varepsilon_0$; (b) $I_L = 0$, $I_C = 0$, $I_R = 0$, $\Delta V_L = 0$, $\Delta V_C = \varepsilon_0$, $\Delta V_R = 0$

**P32.74** (a) 251 $\mu$H; (b) 25.1 $\mu$H; (c) 25.1 nC

**P32.76** $3.97 \times 10^{-25}$ $\Omega$

**P32.78** (a) 50.0 mT; (b) 20.0 mT; (c) 2.29 MJ; (d) 318 Pa

# 33

# Alternating Current Circuits

**CHAPTER OUTLINE**

33.1 AC Sources
33.2 Resistors in an AC Circuit
33.3 Inductors in an AC Circuit
33.4 Capacitors in an AC Circuit
33.5 The RLC Series Circuit
33.6 Power in an AC Circuit
33.7 Resonance in a Series RLC Circuit
33.8 The Transformer and Power Transmission
33.9 Rectifiers and Filters

## ANSWERS TO QUESTIONS

**Q33.1** If the current is positive half the time and negative half the time, the average current can be zero. The rms current is not zero. By squaring all of the values of the current, they all become positive. The average (mean) of these positive values is also positive, as is the square root of the average.

**Q33.2** $\Delta V_{avg} = \frac{\Delta V_{\max}}{2}$, $\Delta V_{rms} = \frac{\Delta V_{\max}}{\sqrt{2}}$

**Q33.3** AC ammeters and voltmeters read rms values. With an oscilloscope you can read a maximum voltage, or test whether the average is zero.

**Q33.4** Suppose the voltage across an inductor varies sinusoidally. Then the current in the inductor will have its instantaneous peak positive value $\frac{1}{4}$ cycle *after* the voltage peaks. The voltage is zero and going positive $\frac{1}{4}$ cycle (90°) before the current is zero and going positive.

**Q33.5** If it is run directly from the electric line, a fluorescent light tube can dim considerably twice in every cycle of the AC current that drives it. Looking at one sinusoidal cycle, the voltage passes through zero twice. We don't notice the flickering due to a phenomenon called retinal imaging. We do not notice that the lights turn on and off since our retinas continue to send information to our brains after the light has turned off. For example, most TV screens refresh at between 60 to 75 times per second, yet we do not see the evening news flickering. Home video cameras record information at frequencies as low as 30 frames per second, yet we still see them as continuous action. A vivid display of retinal imaging is that persistent purple spot you see after someone has taken a picture of you with a flash camera.

**Q33.6** The capacitive reactance is proportional to the inverse of the frequency. At higher and higher frequencies, the capacitive reactance approaches zero, making a capacitor behave like a wire. As the frequency goes to zero, the capacitive reactance approaches infinity—the resistance of an open circuit.

**Q33.7** The second letter in each word stands for the circuit element. For an inductor $L$, the emf $\varepsilon$ leads the current $I$—thus ELI. For a capacitor $C$, the current leads the voltage across the device. In a circuit in which the capacitive reactance is larger than the inductive reactance, the current leads the source emf—thus ICE.

**Q33.8** The voltages are not added in a scalar form, but in a vector form, as shown in the phasor diagrams throughout the chapter. Kirchhoff's loop rule is true at any instant, but the voltages across different circuit elements are not simultaneously at their maximum values. Do not forget that an inductor can induce an emf in itself and that the voltage across it is 90° *ahead* of the current in the circuit in phase.

**Q33.9** In an RLC series circuit, the phase angle depends on the source frequency. At very low frequency the capacitor dominates the impedance and the phase angle is near –90°. The phase angle is zero at the resonance frequency, where the inductive and capacitive reactances are equal. At very high frequencies $\phi$ approaches +90°.

**Q33.10** $-90° \le \phi \le 90°$. The extremes are reached when there is no significant resistance in the circuit.

**Q33.11** The resistance remains unchanged, the inductive resistance doubles, and the capacitive reactance is reduced by one half.

**Q33.12** The power factor, as seen in equation 33.29, is the cosine of the phase angle between the current and applied voltage. Maximum power will be delivered if $\Delta V$ and $I$ are in phase. If $\Delta V$ and $I$ are 90° out of phase, the source voltage drives a net current of zero in each cycle and the average power is zero.

**Q33.13** The person is doing work at a rate of $\mathcal{P} = Fv\cos\theta$. One can consider the emf as the "force" that moves the charges through the circuit, and the current as the "speed" of the moving charges. The $\cos\theta$ factor measures the effectiveness of the cause in producing the effect. Theta is an angle in real space for the vacuum cleaner and phi is the analogous angle of phase difference between the emf and the current in the circuit.

**Q33.14** As mentioned in Question 33.5, lights that are powered by alternating current flicker or get slightly brighter and dimmer at twice the frequency of the AC power source. Even if you tried using two banks of lights, one driven by AC 180° of phase from the other, you would not have a stable light source, but one that exhibits a "ripple" in intensity.

**Q33.15** In 1881, an assassin shot President James Garfield. The bullet was lost in his body. Alexander Graham Bell invented the metal detector in an effort to save the President's life. The coil is preserved in the Smithsonian Institution. The detector was thrown off by metal springs in Garfield's mattress, a new invention itself. Surgeons went hunting for the bullet in the wrong place and Garfield died.

**Q33.16** As seen in Example 33.8, it is far more economical to transmit at high voltage than at low voltage, as the $I^2R$ loss on the transmission line is significantly lower. Transmitting power at high voltage permits the use of step-down transformers to make "low" voltages and high currents available to the end user.

**Q33.17** Insulation and safety limit the voltage of a transmission line. For an underground cable, the thickness and dielectric strength of the insulation between the conductors determines the maximum voltage that can be applied, just as with a capacitor. For an overhead line on towers, the designer must consider electrical breakdown of the surrounding air, possible accidents, sparking across the insulating supports, ozone production, and inducing voltages in cars, fences, and the roof gutters of nearby houses. Nuisance effects include noise, electrical noise, and a prankster lighting a hand-held fluorescent tube under the line.

**Q33.18** No. A voltage is only induced in the secondary coil if the flux through the core changes in time.

**Q33.19** This person needs to consider the difference between the power delivered by a power plant and $I^2R$ losses in transmission lines. At lower voltages, transmission lines must carry higher currents to transmit the same power, as seen in Example 33.8. The high transmitted current at low voltage actually results in more internal energy production than a lower current at high voltage. In his formula $\frac{(\Delta V)^2}{R}$, the $\Delta V$ does not represent the line voltage but the potential difference between the ends of one conductor. This is very small when the current is small.

**Q33.20** The $Q$ factor determines the selectivity of the radio receiver. For example, a receiver with a very low $Q$ factor will respond to a wide range of frequencies and might pick up several adjacent radio stations at the same time. To discriminate between 102.5 MHz and 102.7 MHz requires a high-$Q$ circuit. Typically, lowering the resistance in the circuit is the way to get a higher quality resonance.

**Q33.21** Both coils are wrapped around the same core so that nearly all of the magnetic flux created by the primary passes through the secondary coil, and thus induces current in the secondary when the current in the primary changes.

**Q33.22** The frequency of a DC signal is zero, making the capacitive reactance at DC infinite. The capacitor then acts as an open switch. An AC signal has a non-zero frequency, and thus the capacitive reactance is finite, allowing a signal to pass from Circuit A to Circuit B.

## SOLUTIONS TO PROBLEMS

### Section 33.1 AC Sources

### Section 33.2 Resistors in an AC Circuit

**P33.1** $\Delta v(t) = \Delta V_{\max}\sin(\omega t) = \sqrt{2}\Delta V_{\text{rms}}\sin(\omega t) = 200\sqrt{2}\sin[2\pi(100t)] = \boxed{(283\text{ V})\sin(628t)}$

**P33.2** $\Delta V_{\text{rms}} = \frac{170\text{ V}}{\sqrt{2}} = 120\text{ V}$

(a) $\mathscr{P} = \frac{(\Delta V_{\text{rms}})^2}{R} \to R = \frac{(120\text{ V})^2}{75.0\text{ W}} = \boxed{193\ \Omega}$

(b) $R = \frac{(120\text{ V})^2}{100\text{ W}} = \boxed{144\ \Omega}$

**P33.3** Each meter reads the rms value.

$$\Delta V_{\text{rms}} = \frac{100\text{ V}}{\sqrt{2}} = \boxed{70.7\text{ V}}$$

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{R} = \frac{70.7\text{ V}}{24.0\ \Omega} = \boxed{2.95\text{ A}}$$

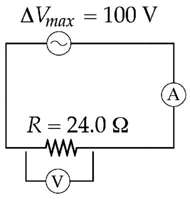


**FIG. P33.3**

**P33.4** (a) $\Delta v_R = \Delta V_{\max} \sin \omega t$

$\Delta v_R = 0.250(\Delta V_{\max})$, so $\sin \omega t = 0.250$, or $\omega t = \sin^{-1}(0.250)$.

The smallest angle for which this is true is $\omega t = 0.253$ rad Thus, if $t = 0.010\,0$ s,

$$\omega = \frac{0.253 \text{ rad}}{0.010\,0 \text{ s}} = \boxed{25.3 \text{ rad/s}}.$$

(b) The second time when $\Delta v_R = 0.250(\Delta V_{\max})$, $\omega t = \sin^{-1}(0.250)$ again. For this occurrence, $\omega t = \pi - 0.253 \text{ rad} = 2.89 \text{ rad}$ (to understand why this is true, recall the identity $\sin(\pi - \theta) = \sin \theta$ from trigonometry). Thus,

$$t = \frac{2.89 \text{ rad}}{25.3 \text{ rad/s}} = \boxed{0.114 \text{ s}}.$$

**P33.5** $i_R = I_{\max} \sin \omega t$ becomes $0.600 = \sin(\omega\, 0.007\,00)$.

Thus, $(0.007\,00)\omega = \sin^{-1}(0.600) = 0.644$

and $\omega = 91.9 \text{ rad/s} = 2\pi f$ so $\boxed{f = 14.6 \text{ Hz}}$.

**P33.6** $\mathscr{P} = I_{\text{rms}}(\Delta V_{\text{rms}})$ and $\Delta V_{\text{rms}} = 120$ V for each bulb (parallel circuit), so:

$$I_1 = I_2 = \frac{\mathscr{P}_1}{\Delta V_{\text{rms}}} = \frac{150 \text{ W}}{120 \text{ V}} = \boxed{1.25 \text{ A}}, \text{ and } R_1 = \frac{\Delta V_{\text{rms}}}{I_1} = \frac{120 \text{ V}}{1.25 \text{ A}} = \boxed{96.0\ \Omega} = R_2$$

$$I_3 = \frac{\mathscr{P}_3}{\Delta V_{\text{rms}}} = \frac{100 \text{ W}}{120 \text{ V}} = \boxed{0.833 \text{ A}}, \text{ and } R_3 = \frac{\Delta V_{\text{rms}}}{I_3} = \frac{120 \text{ V}}{0.833 \text{ A}} = \boxed{144\ \Omega}.$$

**P33.7** $\Delta V_{\max} = 15.0$ V and $R_{\text{total}} = 8.20\ \Omega + 10.4\ \Omega = 18.6\ \Omega$

$$I_{\max} = \frac{\Delta V_{\max}}{R_{\text{total}}} = \frac{15.0 \text{ V}}{18.6\ \Omega} = 0.806 \text{ A} = \sqrt{2} I_{\text{rms}}$$

$$\mathscr{P}_{\text{speaker}} = I_{\text{rms}}^2 R_{\text{speaker}} = \left(\frac{0.806 \text{ A}}{\sqrt{2}}\right)^2 (10.4\ \Omega) = \boxed{3.38 \text{ W}}$$

## Section 33.3 **Inductors in an AC Circuit**

**P33.8** For $I_{\max} = 80.0$ mA, $I_{\text{rms}} = \frac{80.0 \text{ mA}}{\sqrt{2}} = 56.6$ mA

$$(X_L)_{\min} = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \frac{50.0 \text{ V}}{0.056\,6 \text{ A}} = 884\ \Omega$$

$$X_L = 2\pi f L \rightarrow L = \frac{X_L}{2\pi f} \geq \frac{884\ \Omega}{2\pi(20.0)} \geq \boxed{7.03 \text{ H}}$$

**P33.9** (a)
$$X_L=\frac{\Delta V_{\max}}{I_{\max}}=\frac{100}{7.50}=13.3\ \Omega$$
$$L=\frac{X_L}{\omega}=\frac{13.3}{2\pi(50.0)}=0.042\,4\ \text{H}=\boxed{42.4\ \text{mH}}$$

(b)
$$X_L=\frac{\Delta V_{\max}}{I_{\max}}=\frac{100}{2.50}=40.0\ \Omega$$
$$\omega=\frac{X_L}{L}=\frac{40.0}{42.4\times10^{-3}}=\boxed{942\ \text{rad/s}}$$

**P33.10** At 50.0 Hz, $X_L=2\pi(50.0\ \text{Hz})L=2\pi(50.0\ \text{Hz})\left(\frac{X_L|_{60.0\ \text{Hz}}}{2\pi(60.0\ \text{Hz})}\right)=\frac{50.0}{60.0}(54.0\ \Omega)=45.0\ \Omega$

$$I_{\max}=\frac{\Delta V_{\max}}{X_L}=\frac{\sqrt{2}(\Delta V_{\text{rms}})}{X_L}=\frac{\sqrt{2}(100\ \text{V})}{45.0\ \Omega}=\boxed{3.14\ \text{A}}.$$

**P33.11**
$$i_L(t)=\frac{\Delta V_{\max}}{\omega L}\sin\left(\omega t-\frac{\pi}{2}\right)=\frac{(80.0\ \text{V})\sin\left[(65.0\pi)(0.015\,5)-\pi/2\right]}{(65.0\pi\ \text{rad/s})\left(70.0\times10^{-3}\ \text{H}\right)}$$
$$i_L(t)=(5.60\ \text{A})\sin(1.59\ \text{rad})=\boxed{5.60\ \text{A}}$$

**P33.12**
$$\omega=2\pi f=2\pi(60.0/\text{s})=377\ \text{rad/s}$$
$$X_L=\omega L=(377/\text{s})(0.020\,0\ \text{V}\cdot\text{s/A})=7.54\ \Omega$$
$$I_{\text{rms}}=\frac{\Delta V_{\text{rms}}}{X_L}=\frac{120\ \text{V}}{7.54\ \Omega}=15.9\ \text{A}$$
$$I_{\max}=\sqrt{2}I_{\text{rms}}=\sqrt{2}(15.9\ \text{A})=22.5\ \text{A}$$
$$i(t)=I_{\max}\sin\omega t=(22.5\ \text{A})\sin\left(\frac{2\pi(60.0)}{\text{s}}\cdot\frac{1\ \text{s}}{180}\right)=(22.5\ \text{A})\sin120°=19.5\ \text{A}$$
$$U=\frac{1}{2}Li^2=\frac{1}{2}(0.020\,0\ \text{V}\cdot\text{s/A})(19.5\ \text{A})^2=\boxed{3.80\ \text{J}}$$

**P33.13** $L=\frac{N\Phi_B}{I}$ where $\Phi_B$ is the flux through each turn. $N\Phi_{B,\max}=LI_{\max}=\frac{X_L}{\omega}\frac{\left(\Delta V_{L,\max}\right)}{X_L}$

$$N\Phi_{B,\max}=\frac{\sqrt{2}\left(\Delta V_{L,\text{rms}}\right)}{2\pi f}=\frac{120\ \text{V}\cdot\text{s}}{\sqrt{2}\pi(60.0)}\left(\frac{\text{T}\cdot\text{C}\cdot\text{m}}{\text{N}\cdot\text{s}}\right)\left(\frac{\text{N}\cdot\text{m}}{\text{J}}\right)\left(\frac{\text{J}}{\text{V}\cdot\text{C}}\right)=\boxed{0.450\ \text{T}\cdot\text{m}^2}.$$

## Section 33.4 **Capacitors in an AC Circuit**

**P33.14** (a) $X_C = \frac{1}{2\pi f C}: \ \frac{1}{2\pi f\left(22.0\times10^{-6}\right)} < 175\ \Omega$

$$\frac{1}{2\pi\left(22.0\times10^{-6}\right)(175)} < f \qquad \boxed{f > 41.3\text{ Hz}}$$

(b) $X_C \propto \frac{1}{C}$, so $X(44) = \frac{1}{2}X(22)$: $\boxed{X_C < 87.5\ \Omega}$

**P33.15** $I_{\max} = \sqrt{2}I_{\text{rms}} = \frac{\sqrt{2}(\Delta V_{\text{rms}})}{X_C} = \sqrt{2}(\Delta V_{\text{rms}})2\pi f C$

(a) $I_{\max} = \sqrt{2}(120\text{ V})2\pi(60.0/\text{s})\left(2.20\times10^{-6}\ \text{C/V}\right) = \boxed{141\text{ mA}}$

(b) $I_{\max} = \sqrt{2}(240\text{ V})2\pi(50.0/\text{s})\left(2.20\times10^{-6}\ \text{F}\right) = \boxed{235\text{ mA}}$

**P33.16** $Q_{\max} = C(\Delta V_{\max}) = C\left[\sqrt{2}(\Delta V_{\text{rms}})\right] = \boxed{\sqrt{2}C(\Delta V_{\text{rms}})}$

**P33.17** $I_{\max} = (\Delta V_{\max})\omega C = (48.0\text{ V})(2\pi)\left(90.0\text{ s}^{-1}\right)\left(3.70\times10^{-6}\ \text{F}\right) = \boxed{100\text{ mA}}$

**P33.18** $X_C = \frac{1}{\omega C} = \frac{1}{2\pi(60.0/\text{s})\left(1.00\times10^{-3}\ \text{C/V}\right)} = 2.65\ \Omega$

$v_C(t) = \Delta V_{\max}\sin\omega t$, to be zero at $t = 0$

$$i_C = \frac{\Delta V_{\max}}{X_C}\sin(\omega t + \phi) = \frac{\sqrt{2}(120\text{ V})}{2.65\ \Omega}\sin\left[2\pi\frac{60\text{ s}^{-1}}{180\text{ s}^{-1}} + 90.0°\right] = (64.0\text{ A})\sin(120°+90.0°) = \boxed{-32.0\text{ A}}$$

## Section 33.5 **The RLC Series Circuit**

**P33.19** (a) $X_L = \omega L = 2\pi(50.0)\left(400\times10^{-3}\right) = 126\ \Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi(50.0)\left(4.43\times10^{-6}\right)} = 719\ \Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{500^2 + (126-719)^2} = 776\ \Omega$$

$$\Delta V_{\max} = I_{\max}Z = \left(250\times10^{-3}\right)(776) = \boxed{194\text{ V}}$$

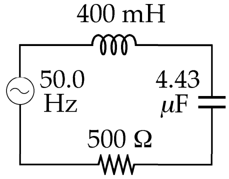


**FIG. P33.19**

(b) $\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{126-719}{500}\right) = \boxed{-49.9°}$. Thus, the $\boxed{\text{Current leads the voltage.}}$

**P33.20** $\omega L = \frac{1}{\omega C} \to \omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\left(57.0\times10^{-6}\right)\left(57.0\times10^{-6}\right)}} = 1.75\times10^4\ \text{rad/s}$

$$f = \frac{\omega}{2\pi} = \boxed{2.79\text{ kHz}}$$

**P33.21** (a) $X_L = \omega L = 2\pi\left(50.0\ \text{s}^{-1}\right)\left(250\times10^{-3}\ \text{H}\right) = \boxed{78.5\ \Omega}$

(b) $X_C = \dfrac{1}{\omega C} = \left[2\pi\left(50.0\ \text{s}^{-1}\right)\left(2.00\times10^{-6}\ \text{F}\right)\right]^{-1} = \boxed{1.59\ \text{k}\Omega}$

(c) $Z = \sqrt{R^2 + (X_L - X_C)^2} = \boxed{1.52\ \text{k}\Omega}$

(d) $I_{\max} = \dfrac{\Delta V_{\max}}{Z} = \dfrac{210\ \text{V}}{1.52\times10^3\ \Omega} = \boxed{138\ \text{mA}}$

(e) $\phi = \tan^{-1}\left[\dfrac{X_L - X_C}{R}\right] = \tan^{-1}(-10.1) = \boxed{-84.3°}$

**P33.22** (a) $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{68.0^2 + (16.0 - 101)^2} = \boxed{109\ \Omega}$

$X_L = \omega L = (100)(0.160) = 16.0\ \Omega$

$X_C = \dfrac{1}{\omega C} = \dfrac{1}{(100)\left(99.0\times10^{-6}\right)} = 101\ \Omega$

(b) $I_{\max} = \dfrac{\Delta V_{\max}}{Z} = \dfrac{40.0\ \text{V}}{109\ \Omega} = \boxed{0.367\ \text{A}}$

(c) $\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{16.0 - 101}{68.0} = -1.25$ :

$\phi = -0.896\ \text{rad} = -51.3°$

$\boxed{I_{\max} = 0.367\ \text{A}}$ $\boxed{\omega = 100\ \text{rad/s}}$ $\boxed{\phi = -0.896\ \text{rad} = -51.3°}$

**P33.23** $X_L = 2\pi f L = 2\pi(60.0)(0.460) = 173\ \Omega$

$X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{2\pi(60.0)\left(21.0\times10^{-6}\right)} = 126\ \Omega$

(a) $\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{173\ \Omega - 126\ \Omega}{150\ \Omega} = 0.314$

$\phi = 0.304\ \text{rad} = \boxed{17.4°}$

(b) Since $X_L > X_C$, $\phi$ is positive; so $\boxed{\text{voltage leads the current}}$.

***P33.24** For the source-capacitor circuit, the rms source voltage is $\Delta V_s = (25.1\ \text{mA})X_C$. For the circuit with resistor, $\Delta V_s = (15.7\ \text{mA})\sqrt{R^2 + X_C^2} = (25.1\ \text{mA})X_C$. This gives $R = 1.247X_C$. For the circuit with ideal inductor, $\Delta V_s = (68.2\ \text{mA})\left|X_L - X_C\right| = (25.1\ \text{mA})X_C$. So $\left|X_L - X_C\right| = 0.368\,0X_C$. Now for the full circuit

$$\Delta V_s = I\sqrt{R^2 + (X_L - X_C)^2}$$

$$(25.1\ \text{mA})X_C = I\sqrt{(1.247X_C)^2 + (0.368X_C)^2}$$

$$\boxed{I = 19.3\ \text{mA}}$$

**P33.25**

$$X_C=\frac{1}{2\pi fC}=\frac{1}{2\pi(60.0\text{ Hz})\left(20.0\times10^{-12}\text{ F}\right)}=1.33\times10^{8}\ \Omega$$

$$Z=\sqrt{\left(50.0\times10^{3}\ \Omega\right)^{2}+\left(1.33\times10^{8}\ \Omega\right)^{2}}\approx1.33\times10^{8}\ \Omega$$

$$I_{\text{rms}}=\frac{\Delta V_{\text{rms}}}{Z}=\frac{5\,000\text{ V}}{1.33\times10^{8}\ \Omega}=3.77\times10^{-5}\text{ A}$$

$$(\Delta V_{\text{rms}})_{\text{body}}=I_{\text{rms}}R_{\text{body}}=\left(3.77\times10^{-5}\text{ A}\right)\left(50.0\times10^{3}\ \Omega\right)=\boxed{1.88\text{ V}}$$

**P33.26**

$$X_C=\frac{1}{\omega C}=\frac{1}{2\pi(50.0)\left(65.0\times10^{-6}\right)}=49.0\ \Omega$$

$$X_L=\omega L=2\pi(50.0)\left(185\times10^{-3}\right)=58.1\ \Omega$$

$$Z=\sqrt{R^2+(X_L-X_C)^2}=\sqrt{(40.0)^2+(58.1-49.0)^2}=41.0\ \Omega$$

$$I_{\max}=\frac{\Delta V_{\max}}{Z}=\frac{150}{41.0}=3.66\text{ A}$$

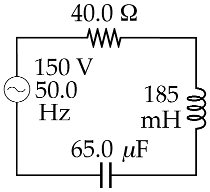


**FIG. P33.26**

(a) $\Delta V_R=I_{\max}R=(3.66)(40)=\boxed{146\text{ V}}$

(b) $\Delta V_L=I_{\max}X_L=(3.66)(58.1)=212.5=\boxed{212\text{ V}}$

(c) $\Delta V_C=I_{\max}X_C=(3.66)(49.0)=179.1\text{ V}=\boxed{179\text{ V}}$

(d) $\Delta V_L-\Delta V_C=212.5-179.1=\boxed{33.4\text{ V}}$

**P33.27**

$$R=300\ \Omega$$

$$X_L=\omega L=2\pi\left(\frac{500}{\pi}\text{ s}^{-1}\right)(0.200\text{ H})=200\ \Omega$$

$$X_C=\frac{1}{\omega C}=\left[2\pi\left(\frac{500}{\pi}\text{ s}^{-1}\right)\left(11.0\times10^{-6}\text{ F}\right)\right]^{-1}=90.9\ \Omega$$

$$Z=\sqrt{R^2+(X_L-X_C)^2}=319\ \Omega\text{ and}$$

$$\phi=\tan^{-1}\left(\frac{X_L-X_C}{R}\right)=20.0°$$

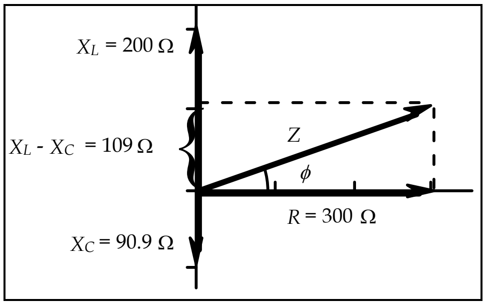


**FIG. P33.27**

***P33.28** Let $X_c$ represent the initial capacitive reactance. Moving the plates to half their original separation doubles the capacitance and cuts $X_C=\dfrac{1}{\omega C}$ in half. For the current to double, the total impedance must be cut in half: $Z_i=2Z_f$, $\sqrt{R^2+(X_L-X_C)^2}=2\sqrt{R^2+\left(X_L-\dfrac{X_C}{2}\right)^2}$,

$$R^2+(R-X_C)^2=4\left(R^2+\left(R-\frac{X_C}{2}\right)^2\right)$$

$$2R^2-2RX_C+X_C^2=8R^2-4RX_C+X_C^2$$

$$\boxed{X_C=3R}$$

**P33.29** (a)
$$X_L = 2\pi(100\text{ Hz})(20.5\text{ H}) = 1.29\times10^4\ \Omega$$
$$Z = \frac{\Delta V_{\text{rms}}}{I_{\text{rms}}} = \frac{200\text{ V}}{4.00\text{ A}} = 50.0\ \Omega$$
$$(X_L - X_C)^2 = Z^2 - R^2 = (50.0\ \Omega)^2 - (35.0\ \Omega)^2$$
$$X_L - X_C = 1.29\times10^4\ \Omega - \frac{1}{2\pi(100\text{ Hz})C} = \pm35.7\ \Omega \quad \boxed{C = 123\text{ nF or }124\text{ nF}}$$

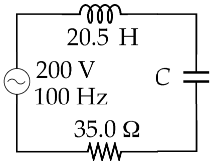


FIG. P33.29

(b)
$$\Delta V_{L,\text{ rms}} = I_{\text{rms}}X_L = (4.00\text{ A})(1.29\times10^4\ \Omega) = \boxed{51.5\text{ kV}}$$

Notice that this is a very large voltage!

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## Section 33.6 Power in an AC Circuit

**P33.30**
$$X_L = \omega L = [(1\,000/\text{s})(0.050\,0\text{ H})] = 50.0\ \Omega$$
$$X_C = \frac{1}{\omega C} = [(1\,000/\text{s})(50.0\times10^{-6}\text{ F})]^{-1} = 20.0\ \Omega$$
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$
$$Z = \sqrt{(40.0)^2 + (50.0 - 20.0)^2} = 50.0\ \Omega$$

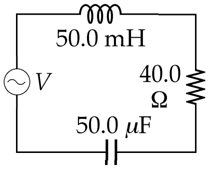

FIG. P33.30

(a)
$$I_{\text{rms}} = \frac{(\Delta V_{\text{rms}})}{Z} = \frac{100\text{ V}}{50.0\ \Omega}$$
$$I_{\text{rms}} = \boxed{2.00\text{ A}}$$
$$\phi = \arctan\left(\frac{X_L - X_C}{R}\right)$$
$$\phi = \arctan\frac{30.0\ \Omega}{40.0\ \Omega} = 36.9°$$

(b)
$$\mathcal{P} = (\Delta V_{\text{rms}})I_{\text{rms}}\cos\phi = 100\text{ V}(2.00\text{ A})\cos36.9° = \boxed{160\text{ W}}$$

(c)
$$\mathcal{P}_R = I_{\text{rms}}^2R = (2.00\text{ A})^2\,40.0\ \Omega = \boxed{160\text{ W}}$$

**P33.31**
$$\omega = 1\,000\text{ rad/s}, \quad R = 400\ \Omega, \quad C = 5.00\times10^{-6}\text{ F}, \quad L = 0.500\text{ H}$$
$$\Delta V_{\max} = 100\text{ V}, \quad \omega L = 500\ \Omega, \quad \left(\frac{1}{\omega C}\right) = 200\ \Omega$$
$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{400^2 + 300^2} = 500\ \Omega$$
$$I_{\max} = \frac{\Delta V_{\max}}{Z} = \frac{100}{500} = 0.200\text{ A}$$

The average power dissipated in the circuit is $\mathcal{P} = I_{\text{rms}}^2R = \left(\frac{I_{\max}^2}{2}\right)R$.

$$\mathcal{P} = \frac{(0.200\text{ A})^2}{2}(400\ \Omega) = \boxed{8.00\text{ W}}$$

**P33.32** $Z = \sqrt{R^2 + (X_L - X_C)^2}$ or $(X_L - X_C) = \sqrt{Z^2 - R^2}$

$$(X_L - X_C) = \sqrt{(75.0\ \Omega)^2 - (45.0\ \Omega)^2} = 60.0\ \Omega$$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{60.0\ \Omega}{45.0\ \Omega}\right) = 53.1°$$

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{Z} = \frac{210\ \text{V}}{75.0\ \Omega} = 2.80\ \text{A}$$

$$\mathcal{P} = (\Delta V_{\text{rms}})I_{\text{rms}}\cos\phi = (210\ \text{V})(2.80\ \text{A})\cos(53.1°) = \boxed{353\ \text{W}}$$

**P33.33** (a) $\mathcal{P} = I_{\text{rms}}(\Delta V_{\text{rms}})\cos\phi = (9.00)180\cos(-37.0°) = 1.29\times 10^3\ \text{W}$

$\mathcal{P} = I_{\text{rms}}^2 R$ so $1.29\times 10^3 = (9.00)^2 R$ and $R = \boxed{16.0\ \Omega}$.

(b) $\tan\phi = \dfrac{X_L - X_C}{R}$ becomes $\tan(-37.0°) = \dfrac{X_L - X_C}{16}$: so $X_L - X_C = \boxed{-12.0\ \Omega}$.

**P33.34** $X_L = \omega L = 2\pi(60.0/\text{s})(0.025\,0\ \text{H}) = 9.42\ \Omega$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(20.0)^2 + (9.42)^2}\ \Omega = 22.1\ \Omega$$

(a) $I_{\text{rms}} = \dfrac{\Delta V_{\text{rms}}}{Z} = \dfrac{120\ \text{V}}{22.1\ \Omega} = \boxed{5.43\ \text{A}}$

(b) $\phi = \tan^{-1}\left(\dfrac{9.42}{20.0}\right) = 25.2°$ so power factor $= \cos\phi = \boxed{0.905}$.

(c) We require $\phi = 0$. Thus, $X_L = X_C$: $9.42\ \Omega = \dfrac{1}{2\pi\left(60.0\ \text{s}^{-1}\right)C}$

and $C = \boxed{281\ \mu\text{F}}$.

(d) $\mathcal{P}_b = \mathcal{P}_d$ or $(\Delta V_{\text{rms}})_b (I_{\text{rms}})_b \cos\phi_b = \dfrac{(\Delta V_{\text{rms}})_d^2}{R}$

$$(\Delta V_{\text{rms}})_d = \sqrt{R(\Delta V_{\text{rms}})_b (I_{\text{rms}})_b \cos\phi_b} = \sqrt{(20.0\ \Omega)(120\ \text{V})(5.43\ \text{A})(0.905)} = \boxed{109\ \text{V}}$$

**P33.35** Consider a two-wire transmission line:

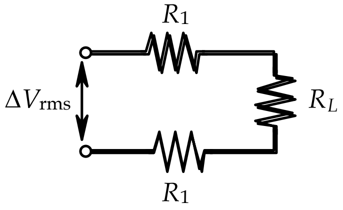

**FIG. P33.35**

$I_{\text{rms}} = \dfrac{\mathcal{P}}{\Delta V_{\text{rms}}}$ and power loss $= I_{\text{rms}}^2 R_{\text{line}} = \dfrac{\mathcal{P}}{100}$.

Thus, $\left(\dfrac{\mathcal{P}}{\Delta V_{\text{rms}}}\right)^2 (2R_1) = \dfrac{\mathcal{P}}{100}$ or $R_1 = \dfrac{(\Delta V_{\text{rms}})^2}{200\mathcal{P}}$

$R_1 = \dfrac{\rho d}{A} = \dfrac{(\Delta V_{\text{rms}})^2}{200\mathcal{P}}$ or $A = \dfrac{\pi(2r)^2}{4} = \dfrac{200\rho\mathcal{P}d}{(\Delta V_{\text{rms}})^2}$

and the diameter is $2r = \boxed{\sqrt{\dfrac{800\rho\mathcal{P}d}{\pi(\Delta V)^2}}}$.

**P33.36** One-half the time, the left side of the generator is positive, the top diode conducts, and the bottom diode switches off. The power supply sees resistance

$$\left[\frac{1}{2R}+\frac{1}{2R}\right]^{-1}=R \text{ and the power is } \frac{(\Delta V_{\text{rms}})^2}{R}.$$

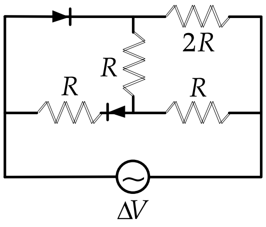

**FIG. P33.36**

The other half of the time the right side of the generator is positive, the upper diode is an open circuit, and the lower diode has zero resistance. The equivalent resistance is then

$$R_{\text{eq}}=R+\left[\frac{1}{3R}+\frac{1}{R}\right]^{-1}=\frac{7R}{4} \qquad \text{and} \qquad \mathcal{P}=\frac{(\Delta V_{\text{rms}})^2}{R_{\text{eq}}}=\frac{4(\Delta V_{\text{rms}})^2}{7R}.$$

The overall time average power is: $\dfrac{\left[(\Delta V_{\text{rms}})^2/R\right]+\left[4(\Delta V_{\text{rms}})^2/7R\right]}{2}=\boxed{\dfrac{11(\Delta V_{\text{rms}})^2}{14R}}$.

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Section 33.7 **Resonance in a Series RLC Circuit**

**P33.37** $\omega_0=2\pi\left(99.7\times10^6\right)=6.26\times10^8 \text{ rad/s}=\dfrac{1}{\sqrt{LC}}$

$$C=\frac{1}{\omega_0^2L}=\frac{1}{\left(6.26\times10^8\right)^2\left(1.40\times10^{-6}\right)}=\boxed{1.82 \text{ pF}}$$

**P33.38** At resonance, $\dfrac{1}{2\pi fC}=2\pi fL$ and $\dfrac{1}{(2\pi f)^2L}=C$.

The range of values for $C$ is $\boxed{46.5 \text{ pF to } 419 \text{ pF}}$.

***P33.39** (a) $f=\dfrac{1}{2\pi\sqrt{LC}}$

$$C=\frac{1}{4\pi^2f^2L}=\frac{1}{4\pi^2\left(10^{10}/\text{s}\right)^2 400\times10^{-12}\text{ Vs}}\frac{\text{A}}{}\left(\frac{\text{C}}{\text{As}}\right)=\boxed{6.33\times10^{-13}\text{ F}}$$

(b) $C=\dfrac{\kappa\in_0 A}{d}=\dfrac{\kappa\in_0 \ell^2}{d}$

$$\ell=\left(\frac{Cd}{\kappa\in_0}\right)^{1/2}=\left(\frac{6.33\times10^{-13}\text{ F}\times10^{-3}\text{ mm}}{1\times8.85\times10^{-12}\text{ F}}\right)^{1/2}=\boxed{8.46\times10^{-3}\text{ m}}$$

(c) $X_L=2\pi fL=2\pi\times10^{10}/\text{s}\times400\times10^{-12}\text{ Vs/A}=\boxed{25.1\ \Omega}$

**P33.40** $L = 20.0\text{ mH}$, $C = 1.00\times 10^{-7}$, $R = 20.0\ \Omega$, $\Delta V_{\text{max}} = 100\text{ V}$

(a) The resonant frequency for a series –*RLC* circuit is $f = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC}} = \boxed{3.56\text{ kHz}}$.

(b) At resonance, $I_{\text{max}} = \dfrac{\Delta V_{\text{max}}}{R} = \boxed{5.00\text{ A}}$.

(c) From Equation 33.38, $Q = \dfrac{\omega_0 L}{R} = \boxed{22.4}$.

(d) $\Delta V_{L,\text{ max}} = X_L I_{\text{max}} = \omega_0 L I_{\text{max}} = \boxed{2.24\text{ kV}}$

**P33.41** The resonance frequency is $\omega_0 = \dfrac{1}{\sqrt{LC}}$. Thus, if $\omega = 2\omega_0$,

$$X_L = \omega L = \left(\frac{2}{\sqrt{LC}}\right)L = 2\sqrt{\frac{L}{C}} \quad \text{and} \quad X_C = \frac{1}{\omega C} = \frac{\sqrt{LC}}{2C} = \frac{1}{2}\sqrt{\frac{L}{C}}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + 2.25\left(\frac{L}{C}\right)} \quad \text{so} \quad I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{Z} = \frac{\Delta V_{\text{rms}}}{\sqrt{R^2 + 2.25(L/C)}}$$

and the energy delivered in one period is $E = \mathcal{P}\Delta t$:

$$E = \frac{(\Delta V_{\text{rms}})^2 R}{R^2 + 2.25(L/C)}\left(\frac{2\pi}{\omega}\right) = \frac{(\Delta V_{\text{rms}})^2 RC}{R^2C + 2.25L}\left(\pi\sqrt{LC}\right) = \frac{4\pi(\Delta V_{\text{rms}})^2 RC\sqrt{LC}}{4R^2C + 9.00L}.$$

With the values specified for this circuit, this gives:

$$E = \frac{4\pi(50.0\text{ V})^2(10.0\ \Omega)\left(100\times 10^{-6}\text{ F}\right)^{3/2}\left(10.0\times 10^{-3}\text{ H}\right)^{1/2}}{4(10.0\ \Omega)^2\left(100\times 10^{-6}\text{ F}\right) + 9.00\left(10.0\times 10^{-3}\text{ H}\right)} = \boxed{242\text{ mJ}}.$$

**P33.42** The resonance frequency is $\omega_0 = \dfrac{1}{\sqrt{LC}}$. Thus, if $\omega = 2\omega_0$,

$$X_L = \omega L = \left(\frac{2}{\sqrt{LC}}\right)L = 2\sqrt{\frac{L}{C}} \quad \text{and} \quad X_C = \frac{1}{\omega C} = \frac{\sqrt{LC}}{2C} = \frac{1}{2}\sqrt{\frac{L}{C}}.$$

Then $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + 2.25\left(\dfrac{L}{C}\right)}$ so $I_{\text{rms}} = \dfrac{\Delta V_{\text{rms}}}{Z} = \dfrac{\Delta V_{\text{rms}}}{\sqrt{R^2 + 2.25(L/C)}}$

and the energy delivered in one period is

$$E = \mathcal{P}\Delta t = \frac{(\Delta V_{\text{rms}})^2 R}{R^2 + 2.25(L/C)}\left(\frac{2\pi}{\omega}\right) = \frac{(\Delta V_{\text{rms}})^2 RC}{R^2C + 2.25L}\left(\pi\sqrt{LC}\right) = \boxed{\frac{4\pi(\Delta V_{\text{rms}})^2 RC\sqrt{LC}}{4R^2C + 9.00L}}.$$

**P33.43** For the circuit of Problem 22, $\omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{\left(160 \times 10^{-3}\text{ H}\right)\left(99.0 \times 10^{-6}\text{ F}\right)}} = 251\text{ rad/s}$

$$Q = \frac{\omega_0 L}{R} = \frac{(251\text{ rad/s})\left(160 \times 10^{-3}\text{ H}\right)}{68.0\ \Omega} = \boxed{0.591}.$$

For the circuit of Problem 23, $Q = \dfrac{\omega_0 L}{R} = \dfrac{L}{R\sqrt{LC}} = \dfrac{1}{R}\sqrt{\dfrac{L}{C}} = \dfrac{1}{150\ \Omega}\sqrt{\dfrac{460 \times 10^{-3}\text{ H}}{21.0 \times 10^{-6}\text{ F}}} = \boxed{0.987}$.

The $\boxed{\text{circuit of Problem 23}}$ has a sharper resonance.

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## Section 33.8 **The Transformer and Power Transmission**

**P33.44** (a) $\Delta V_{2,\text{ rms}} = \dfrac{1}{13}(120\text{ V}) = \boxed{9.23\text{ V}}$

(b) $\Delta V_{1,\text{ rms}} I_{1,\text{ rms}} = \Delta V_{2,\text{ rms}} I_{2,\text{ rms}}$

$(120\text{ V})(0.350\text{ A}) = (9.23\text{ V}) I_{2,\text{ rms}}$

$I_{2,\text{ rms}} = \dfrac{42.0\text{ W}}{9.23\text{ V}} = \boxed{4.55\text{ A}}$ for a transformer with no energy loss.

(c) $\mathscr{P} = \boxed{42.0\text{ W}}$ from part (b).

**P33.45** $\left(\Delta V_{\text{out}}\right)_{\max} = \dfrac{N_2}{N_1}\left(\Delta V_{\text{in}}\right)_{\max} = \left(\dfrac{2\,000}{350}\right)(170\text{ V}) = 971\text{ V}$

$\left(\Delta V_{\text{out}}\right)_{\text{rms}} = \dfrac{(971\text{ V})}{\sqrt{2}} = \boxed{687\text{ V}}$

**P33.46** (a) $\left(\Delta V_{2,\text{ rms}}\right) = \dfrac{N_2}{N_1}\left(\Delta V_{1,\text{ rms}}\right)$ $\qquad N_2 = \dfrac{(2\,200)(80)}{110} = \boxed{1\,600\text{ windings}}$

(b) $I_{1,\text{ rms}}\left(\Delta V_{1,\text{ rms}}\right) = I_{2,\text{ rms}}\left(\Delta V_{2,\text{ rms}}\right)$ $\qquad I_{1,\text{ rms}} = \dfrac{(1.50)(2\,200)}{110} = \boxed{30.0\text{ A}}$

(c) $0.950 I_{1,\text{ rms}}\left(\Delta V_{1,\text{ rms}}\right) = I_{2,\text{ rms}}\left(\Delta V_{2,\text{ rms}}\right)$ $\qquad I_{1,\text{ rms}} = \dfrac{(1.20)(2\,200)}{110(0.950)} = \boxed{25.3\text{ A}}$

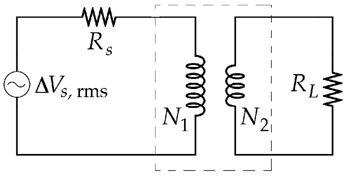


**FIG. P33.47**

**P33.47** The rms voltage across the transformer primary is

$$\frac{N_1}{N_2}\left(\Delta V_{2,\text{ rms}}\right)$$

so the source voltage is $\Delta V_{s,\text{ rms}} = I_{1,\text{ rms}}R_s + \frac{N_1}{N_2}\left(\Delta V_{2,\text{ rms}}\right)$.

The secondary current is $\frac{\left(\Delta V_{2,\text{ rms}}\right)}{R_L}$, so the primary current is

$\frac{N_2}{N_1}\frac{\left(\Delta V_{2,\text{ rms}}\right)}{R_L} = I_{1,\text{ rms}}$.

Then $\Delta V_{s,\text{ rms}} = \frac{N_2\left(\Delta V_{2,\text{ rms}}\right)R_s}{N_1 R_L} + \frac{N_1\left(\Delta V_{2,\text{ rms}}\right)}{N_2}$

and $R_s = \frac{N_1 R_L}{N_2\left(\Delta V_{2,\text{ rms}}\right)}\left(\Delta V_{s,\text{ rms}} - \frac{N_1\left(\Delta V_{2,\text{ rms}}\right)}{N_2}\right) = \frac{5(50.0\ \Omega)}{2(25.0\ \text{V})}\left(80.0\ \text{V} - \frac{5(25.0\ \text{V})}{2}\right) = \boxed{87.5\ \Omega}$.

**P33.48** (a) $\Delta V_{2,\text{ rms}} = \frac{N_2}{N_1}\left(\Delta V_{1,\text{ rms}}\right)$ $\qquad \frac{N_2}{N_1} = \frac{\Delta V_{2,\text{ rms}}}{\Delta V_{1,\text{ rms}}} = \frac{10.0\times 10^3\ \text{V}}{120\ \text{V}} = \boxed{83.3}$

(b) $I_{2,\text{ rms}}\left(\Delta V_{2,\text{ rms}}\right) = 0.900 I_{1,\text{ rms}}\left(\Delta V_{1,\text{ rms}}\right)$

$I_{2,\text{ rms}}\left(10.0\times 10^3\ \text{V}\right) = 0.900\left(\frac{120\ \text{V}}{24.0\ \Omega}\right)(120\ \text{V})$ $\qquad I_{2,\text{ rms}} = \boxed{54.0\ \text{mA}}$

(c) $Z_2 = \frac{\Delta V_{2,\text{ rms}}}{I_{2,\text{ rms}}} = \frac{10.0\times 10^3\ \text{V}}{0.054\ \text{A}} = \boxed{185\ \text{k}\Omega}$

**P33.49** (a) $R = \left(4.50\times 10^{-4}\ \Omega/\text{M}\right)\left(6.44\times 10^5\ \text{m}\right) = 290\ \Omega$ and $I_{\text{rms}} = \frac{\mathscr{P}}{\Delta V_{\text{rms}}} = \frac{5.00\times 10^6\ \text{W}}{5.00\times 10^5\ \text{V}} = 10.0\ \text{A}$

$\mathscr{P}_{\text{loss}} = I_{\text{rms}}^2 R = (10.0\ \text{A})^2(290\ \Omega) = \boxed{29.0\ \text{kW}}$

(b) $\frac{\mathscr{P}_{\text{loss}}}{\mathscr{P}} = \frac{2.90\times 10^4}{5.00\times 10^6} = \boxed{5.80\times 10^{-3}}$

(c) It is impossible to transmit so much power at such low voltage. Maximum power transfer occurs when load resistance equals the line resistance of $290\ \Omega$, and is

$\frac{\left(4.50\times 10^3\ \text{V}\right)^2}{2\cdot 2(290\ \Omega)} = 17.5\ \text{kW}$ far below the required 5 000 kW.

## Section 33.9 Rectifiers and Filters

***P33.50** (a) Input power $= 8\text{ W}$

Useful output power $= I\Delta V = 0.3\text{ A}(9\text{ V}) = 2.7\text{ W}$

$$\text{efficiency} = \frac{\text{useful output}}{\text{total input}} = \frac{2.7\text{ W}}{8\text{ W}} = \boxed{0.34} = 34\%$$

(b) Total input power = Total output power

$8\text{ W} = 2.7\text{ W} + \text{wasted power}$

$\text{wasted power} = \boxed{5.3\text{ W}}$

(c) $$E = \mathscr{P}\Delta t = 8\text{ W}(6)(31\text{ d})\left(\frac{86\,400\text{ s}}{1\text{ d}}\right)\left(\frac{1\text{ J}}{1\text{ Ws}}\right) = 1.29\times10^{8}\text{ J}\left(\frac{\$0.135}{3.6\times10^{6}\text{ J}}\right) = \boxed{\$4.8}$$

***P33.51** (a) The input voltage is $\Delta V_{\text{in}} = IZ = I\sqrt{R^2 + X_C^2} = I\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$. The output voltage is $\Delta V_{\text{out}} = IR$. The gain ratio is $\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \frac{IR}{I\sqrt{R^2 + (1/\omega C)^2}} = \frac{R}{\sqrt{R^2 + (1/\omega C)^2}}$.

(b) As $\omega \to 0$, $\frac{1}{\omega C} \to \infty$ and $\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} \to \boxed{0}$

As $\omega \to \infty$, $\frac{1}{\omega C} \to 0$ and $\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} \to \frac{R}{R} = \boxed{1}$

(c) $$\frac{1}{2} = \frac{R}{\sqrt{R^2 + (1/\omega C)^2}}$$

$$R^2 + \frac{1}{\omega^2 C^2} = 4R^2 \qquad \omega^2 C^2 = \frac{1}{3R^2} \qquad \omega = 2\pi f = \frac{1}{\sqrt{3}RC} \qquad \boxed{f = \frac{1}{2\pi\sqrt{3}RC}}$$

**P33.52** (a) The input voltage is $\Delta V_{\text{in}} = IZ = I\sqrt{R^2 + X_C^2} = I\sqrt{R^2 + (1/\omega C)^2}$. The output voltage is $\Delta V_{\text{out}} = IX_C = \frac{I}{\omega C}$. The gain ratio is $\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \frac{I/\omega C}{I\sqrt{R^2 + (1/\omega C)^2}} = \frac{1/\omega C}{\sqrt{R^2 + (1/\omega C)^2}}$.

(b) As $\omega \to 0$, $\frac{1}{\omega C} \to \infty$ and $R$ becomes negligible in comparison. Then $\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} \to \frac{1/\omega C}{1/\omega C} = \boxed{1}$. As $\omega \to \infty$, $\frac{1}{\omega C} \to 0$ and $\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} \to \boxed{0}$.

(c) $$\frac{1}{2} = \frac{1/\omega C}{\sqrt{R^2 + (1/\omega C)^2}} \qquad R^2 + \left(\frac{1}{\omega C}\right)^2 = \frac{4}{\omega^2 C^2} \qquad R^2\omega^2 C^2 = 3 \qquad \omega = 2\pi f = \frac{\sqrt{3}}{RC}$$

$$\boxed{f = \frac{\sqrt{3}}{2\pi RC}}$$

**P33.53** For this RC high-pass filter, $\dfrac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \dfrac{R}{\sqrt{R^2 + X_C^2}}$.

(a) When $\dfrac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = 0.500$,

then $\dfrac{0.500\ \Omega}{\sqrt{(0.500\ \Omega)^2 + X_C^2}} = 0.500$ or $X_C = 0.866\ \Omega$.

If this occurs at $f = 300$ Hz, the capacitance is

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi(300\ \text{Hz})(0.866\ \Omega)} = 6.13 \times 10^{-4}\ \text{F} = \boxed{613\ \mu\text{F}}.$$

(b) With this capacitance and a frequency of 600 Hz,

$$X_C = \frac{1}{2\pi(600\ \text{Hz})\left(6.13 \times 10^{-4}\ \text{F}\right)} = 0.433\ \Omega$$

$$\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \frac{R}{\sqrt{R^2 + X_C^2}} = \frac{0.500\ \Omega}{\sqrt{(0.500\ \Omega)^2 + (0.433\ \Omega)^2}} = \boxed{0.756}.$$

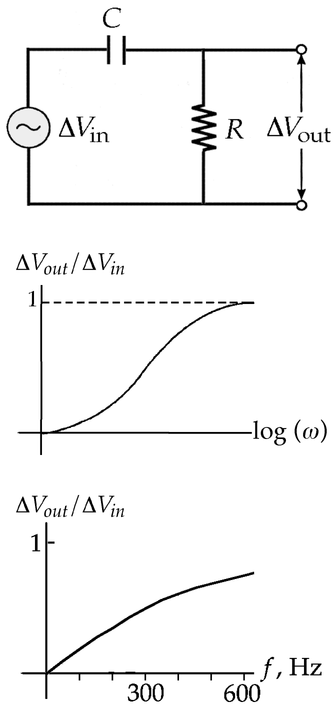


**FIG. P33.53**

**P33.54** For the filter circuit, $\dfrac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \dfrac{X_C}{\sqrt{R^2 + X_C^2}}$.

(a) At $f = 600$ Hz,
$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(600\ \text{Hz})\left(8.00 \times 10^{-9}\ \text{F}\right)} = 3.32 \times 10^4\ \Omega$$

and
$$\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \frac{3.32 \times 10^4\ \Omega}{\sqrt{(90.0\ \Omega)^2 + \left(3.32 \times 10^4\ \Omega\right)^2}} \approx \boxed{1.00}.$$

(b) At $f = 600$ kHz,
$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi\left(600 \times 10^3\ \text{Hz}\right)\left(8.00 \times 10^{-9}\ \text{F}\right)} = 33.2\ \Omega$$

and
$$\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \frac{33.2\ \Omega}{\sqrt{(90.0\ \Omega)^2 + (33.2\ \Omega)^2}} = \boxed{0.346}.$$

**P33.55** $$\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}}=\frac{R}{\sqrt{R^2+(X_L-X_C)^2}}$$

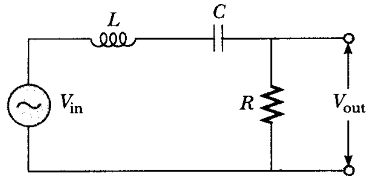


**FIG. P33.55(a)**

(a) At 200 Hz: $\dfrac{1}{4}=\dfrac{(8.00\ \Omega)^2}{(8.00\ \Omega)^2+\left[400\pi L-1/400\pi C\right]^2}$.

At 4 000 Hz: $(8.00\ \Omega)^2+\left[8\,000\pi L-\dfrac{1}{8\,000\pi C}\right]^2=4(8.00\ \Omega)^2$.

At the low frequency, $X_L-X_C<0$. This reduces to $400\pi L-\dfrac{1}{400\pi C}=-13.9\ \Omega$. [1]

For the high frequency half-voltage point, $8\,000\pi L-\dfrac{1}{8\,000\pi C}=+13.9\ \Omega$. [2]

Solving Equations (1) and (2) simultaneously gives $C=\boxed{54.6\ \mu\text{F}}$ and $L=\boxed{580\ \mu\text{H}}$.

(b) When $X_L=X_C$, $\dfrac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}}=\left(\dfrac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}}\right)_{\max}=\boxed{1.00}$.

(c) $X_L=X_C$ requires $f_0=\dfrac{1}{2\pi\sqrt{LC}}=\dfrac{1}{2\pi\sqrt{\left(5.80\times10^{-4}\ \text{H}\right)\left(5.46\times10^{-5}\ \text{F}\right)}}=\boxed{894\ \text{Hz}}$.

(d) At 200 Hz, $\dfrac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}}=\dfrac{R}{Z}=\dfrac{1}{2}$ and $X_C>X_L$,

so the phasor diagram is as shown:

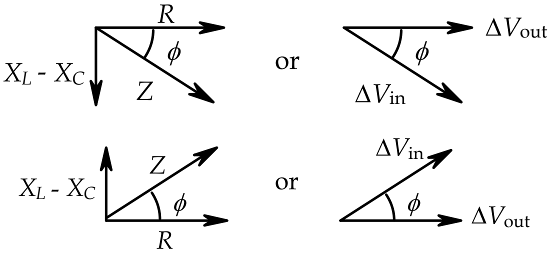


**FIG. P33.55(d)**

$\phi=-\cos^{-1}\left(\dfrac{R}{Z}\right)=-\cos^{-1}\left(\dfrac{1}{2}\right)$ so

$\boxed{\Delta V_{\text{out}}\text{ leads }\Delta V_{\text{in}}\text{ by }60.0°}$.

At $f_0$, $X_L=X_C$ so

$\boxed{\Delta V_{\text{out}}\text{ and }\Delta V_{\text{in}}\text{ have a phase difference of }0°}$.

At 4 000 Hz, $\dfrac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}}=\dfrac{R}{Z}=\dfrac{1}{2}$ and $X_L-X_C>0$.

Thus, $\phi=\cos^{-1}\left(\dfrac{1}{2}\right)=60.0°$

or $\boxed{\Delta V_{\text{out}}\text{ lags }\Delta V_{\text{in}}\text{ by }60.0°}$.

(e) At 200 Hz and at 4 kHz,

$$\mathcal{P}=\frac{\left(\Delta V_{\text{out, rms}}\right)^2}{R}=\frac{\left((1/2)\Delta V_{\text{in, rms}}\right)^2}{R}=\frac{(1/2)\left[(1/2)\Delta V_{\text{in, max}}\right]^2}{R}=\frac{(10.0\ \text{V})^2}{8(8.00\ \Omega)}=\boxed{1.56\ \text{W}}.$$

At $f_0$, $\mathcal{P}=\dfrac{\left(\Delta V_{\text{out, rms}}\right)^2}{R}=\dfrac{\left(\Delta V_{\text{in, rms}}\right)^2}{R}=\dfrac{(1/2)\left[\Delta V_{\text{in, max}}\right]^2}{R}=\dfrac{(10.0\ \text{V})^2}{2(8.00\ \Omega)}=\boxed{6.25\ \text{W}}$.

(f) We take: $Q=\dfrac{\omega_0 L}{R}=\dfrac{2\pi f_0 L}{R}=\dfrac{2\pi(894\ \text{Hz})\left(5.80\times10^{-4}\ \text{H}\right)}{8.00\ \Omega}=\boxed{0.408}$.

## Additional Problems

**P33.56** The equation for $\Delta v(t)$ during the first period (using $y=mx+b$) is:

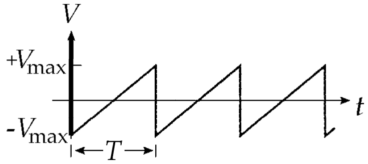


FIG. P33.56

$$\Delta v(t)=\frac{2(\Delta V_{\max})t}{T}-\Delta V_{\max}$$

$$\left[(\Delta v)^2\right]_{\text{ave}}=\frac{1}{T}\int_0^T\left[\Delta v(t)\right]^2dt=\frac{(\Delta V_{\max})^2}{T}\int_0^T\left[\frac{2}{T}t-1\right]^2dt$$

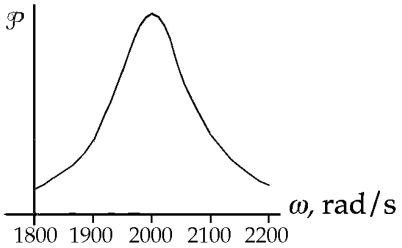
$$\left[(\Delta v)^2\right]_{\text{ave}}=\frac{(\Delta V_{\max})^2}{T}\left(\frac{T}{2}\right)\frac{\left[2t/T-1\right]^3}{3}\Bigg|_{t=0}^{t=T}=\frac{(\Delta V_{\max})^2}{6}\left[(+1)^3-(-1)^3\right]=\frac{(\Delta V_{\max})^2}{3}$$

$$\Delta V_{\text{rms}}=\sqrt{\left[(\Delta v)^2\right]_{\text{ave}}}=\sqrt{\frac{(\Delta V_{\max})^2}{3}}=\boxed{\frac{\Delta V_{\max}}{\sqrt{3}}}$$

**P33.57** $\omega_0=\dfrac{1}{\sqrt{LC}}=\dfrac{1}{\sqrt{(0.050\,0\text{ H})\left(5.00\times10^{-6}\text{ F}\right)}}=2\,000\text{ s}^{-1}$

so the operating angular frequency of the circuit is

$\omega=\dfrac{\omega_0}{2}=1\,000\text{ s}^{-1}$.



$(Q\approx12.5)$

FIG. P33.57

Using Equation 33.37, $\mathscr{P}=\dfrac{(\Delta V_{\text{rms}})^2R\omega^2}{R^2\omega^2+L^2\left(\omega^2-\omega_0^2\right)^2}$

$$P=\frac{(400)^2(8.00)(1\,000)^2}{(8.00)^2(1\,000)^2+(0.050\,0)^2\left[(1.00-4.00)\times10^6\right]^2}=\boxed{56.7\text{ W}}.$$

***P33.58** The angular frequency is $\omega=2\pi60/\text{s}=377/\text{s}$. When $S$ is open, $R$, $L$, and $C$ are in series with the source:

$$R^2+(X_L-X_C)^2=\left(\frac{\Delta V_s}{I}\right)^2=\left(\frac{20\text{ V}}{0.183\text{ A}}\right)^2=1.194\times10^4\ \Omega^2. \qquad (1)$$

When $S$ is in position 1, a parallel combination of two $R$'s presents equivalent resistance $\dfrac{R}{2}$, in series with $L$ and $C$:

$$\left(\frac{R}{2}\right)^2+(X_L-X_C)^2=\left(\frac{20\text{ V}}{0.298\text{ A}}\right)^2=4.504\times10^3\ \Omega^2. \qquad (2)$$

When $S$ is in position 2, the current by passes the inductor. $R$ and $C$ are in series with the source:

$$R^2+X_C^2=\left(\frac{20\text{ V}}{0.137\text{ A}}\right)^2=2.131\times10^4\ \Omega^2. \qquad (3)$$

Take equation (1) minus equation (2):

$\dfrac{3}{4}R^2=7.440\times10^3\ \Omega^2\quad\boxed{R=99.6\ \Omega}$

*continued on next page*

(only the positive root is physical.) Now equation (3) gives

$$X_C = \left[2.131\times 10^4 - (99.6)^2\right]^{1/2}\ \Omega = 106.7\ \Omega = \frac{1}{\omega C} \text{ (only the positive root is physical.)}$$

$$C = (\omega X_C)^{-1} = \left[(377/\text{s})106.7\ \Omega\right]^{-1} = \boxed{2.49\times 10^{-5}\ \text{F} = C}.$$

Now equation (1) gives

$$X_L - X_C = \pm\left[1.194\times 10^4 - (99.6)^2\right]^{1/2}\ \Omega = \pm 44.99\ \Omega$$

$$X_L = 106.7\ \Omega + 44.99\ \Omega = 61.74\ \Omega \text{ or } 151.7\ \Omega = \omega L$$

$$L = \frac{X_L}{\omega} = \boxed{0.164\ \text{H or } 0.402\ \text{H} = L}$$

**P33.59** The resistance of the circuit is $R = \frac{\Delta V}{I} = \frac{12.0\ \text{V}}{0.630\ \text{A}} = 19.0\ \Omega$.

The impedance of the circuit is $Z = \frac{\Delta V_{\text{rms}}}{I_{\text{rms}}} = \frac{24.0\ \text{V}}{0.570\ \text{A}} = 42.1\ \Omega$.

$$Z^2 = R^2 + \omega^2 L^2$$

$$L = \frac{1}{\omega}\sqrt{Z^2 - R^2} = \frac{1}{377}\sqrt{(42.1)^2 - (19.0)^2} = \boxed{99.6\ \text{mH}}$$

***P33.60** The lowest-frequency standing-wave state is NAN. The distance between the clamps we represent as $L = d_{\text{NN}} = \frac{\lambda}{2}$. The speed of transverse waves on the string is $v = f\lambda = \sqrt{\frac{T}{\mu}} = f2L$. The magnetic force on the wire oscillates at 60 Hz, so the wire will oscillate in resonance at 60 Hz.

$$\frac{T}{0.019\ \text{kg/m}} = (60/\text{s})^2 4L^2 \qquad \boxed{T = \left(274\ \text{kg/ms}^2\right)L^2}$$

Any values of $T$ and $L$ related according to this expression will work, including $\boxed{\text{if } L = 0.200\ \text{m} \quad T = 10.9\ \text{N}}$. We did not need to use the value of the current and magnetic field. If we assume the subsection of wire in the field is 2 cm wide, we can find the rms value of the magnetic force:

$$F_B = I\ell B\sin\theta = (9\ \text{A})(0.02\ \text{m})(0.015\,3T)\sin 90° = 2.75\ \text{mN}.$$

So a small force can produce an oscillation of noticeable amplitude if internal friction is small.

**P33.61** (a) When $\omega L$ is very large, the bottom branch carries negligible current. Also, $\frac{1}{\omega C}$ will be negligible compared to $200\ \Omega$ and $\frac{45.0\ \text{V}}{200\ \Omega} = \boxed{225\ \text{mA}}$ flows in the power supply and the top branch.

(b) Now $\frac{1}{\omega C} \to \infty$ and $\omega L \to 0$ so the generator and bottom branch carry $\boxed{450\ \text{mA}}$.

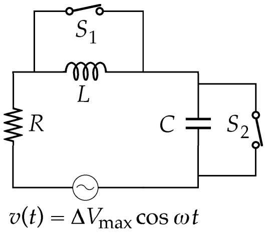


**FIG. P33.62**

**P33.62** (a) With both switches closed, the current goes only through generator and resistor.

$$\boxed{i(t) = \frac{\Delta V_{\max}}{R} \cos \omega t}$$

(b) $$\boxed{\mathscr{P} = \frac{1}{2}\frac{(\Delta V_{\max})^2}{R}}$$

(c) $$\boxed{i(t) = \frac{\Delta V_{\max}}{\sqrt{R^2 + \omega^2 L^2}} \cos\left[\omega t + \arctan\left(\frac{\omega L}{R}\right)\right]}$$

(d) For $$0 = \phi = \arctan\left(\frac{\omega_0 L - (1/\omega_0 C)}{R}\right).$$

We require $\omega_0 L = \dfrac{1}{\omega_0 C}$, so $\boxed{C = \dfrac{1}{\omega_0^2 L}}$.

(e) At this resonance frequency, $Z = \boxed{R}$.

(f) $$U = \frac{1}{2} C (\Delta V_C)^2 = \frac{1}{2} C I^2 X_C^2$$

$$U_{\max} = \frac{1}{2} C I_{\max}^2 X_C^2 = \frac{1}{2} C \frac{(\Delta V_{\max})^2}{R^2} \frac{1}{\omega_0^2 C^2} = \boxed{\frac{(\Delta V_{\max})^2 L}{2R^2}}$$

(g) $$U_{\max} = \frac{1}{2} L I_{\max}^2 = \boxed{\frac{1}{2} L \frac{(\Delta V_{\max})^2}{R^2}}$$

(h) Now $\omega = 2\omega_0 = \dfrac{2}{\sqrt{LC}}$.

So $$\phi = \arctan\left(\frac{\omega L - (1/\omega C)}{R}\right) = \arctan\left(\frac{2\sqrt{L/C} - (1/2)\sqrt{L/C}}{R}\right) = \boxed{\arctan\left(\frac{3}{2R}\sqrt{\frac{L}{C}}\right)}.$$

(i) Now $\omega L = \dfrac{1}{2}\dfrac{1}{\omega C}$ $\quad \omega = \boxed{\dfrac{1}{\sqrt{2LC}}} = \dfrac{\omega_0}{\sqrt{2}}$.

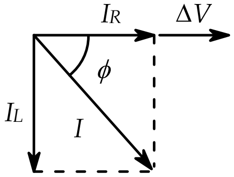


**FIG. P33.63**

**P33.63** (a) $$I_{R,\,\text{rms}} = \frac{\Delta V_{\text{rms}}}{R} = \frac{100\text{ V}}{80.0\ \Omega} = \boxed{1.25\text{ A}}$$

(b) The total current will $\boxed{\text{lag}}$ the applied voltage as seen in the phasor diagram at the right.

$$I_{L,\,\text{rms}} = \frac{\Delta V_{\text{rms}}}{X_L} = \frac{100\text{ V}}{2\pi(60.0\text{ s}^{-1})(0.200\text{ H})} = 1.33\text{ A}$$

Thus, the phase angle is: $\phi = \tan^{-1}\left(\dfrac{I_{L,\,\text{rms}}}{I_{R,\,\text{rms}}}\right) = \tan^{-1}\left(\dfrac{1.33\text{ A}}{1.25\text{ A}}\right) = \boxed{46.7°}$.

**P33.64** Suppose each of the 20 000 people uses an average power of 500 W. (This means 12 kWh per day, or \$36 per 30 days at 10¢ per kWh). Suppose the transmission line is at 20 kV. Then

$$I_{\text{rms}} = \frac{\mathcal{P}}{\Delta V_{\text{rms}}} = \frac{(20\,000)(500\text{ W})}{20\,000\text{ V}} \boxed{\sim 10^3\text{ A}}.$$

If the transmission line had been at 200 kV, the current would be only $\boxed{\sim 10^2\text{ A}}$.

**P33.65** $R = 200\ \Omega$, $L = 663\text{ mH}$, $C = 26.5\ \mu\text{F}$, $\omega = 377\text{ s}^{-1}$, $\Delta V_{\text{max}} = 50.0\text{ V}$

$$\omega L = 250\ \Omega,\ \left(\frac{1}{\omega C}\right) = 100\ \Omega,\ Z = \sqrt{R^2 + (X_L - X_C)^2} = 250\ \Omega$$

(a) $$I_{\text{max}} = \frac{\Delta V_{\text{max}}}{Z} = \frac{50.0\text{ V}}{250\ \Omega} = \boxed{0.200\text{ A}}$$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \boxed{36.8°}\ \ (\Delta V\ \text{leads}\ I)$$

(b) $$\Delta V_{R,\text{ max}} = I_{\text{max}}R = \boxed{40.0\text{ V}}\ \text{at}\ \boxed{\phi = 0°}$$

(c) $$\Delta V_{C,\text{ max}} = \frac{I_{\text{max}}}{\omega C} = \boxed{20.0\text{ V}}\ \text{at}\ \boxed{\phi = -90.0°}\ \ (I\ \text{leads}\ \Delta V)$$

(d) $$\Delta V_{L,\text{ max}} = I_{\text{max}}\omega L = \boxed{50.0\text{ V}}\ \text{at}\ \boxed{\phi = +90.0°}\ \ (\Delta V\ \text{leads}\ I)$$

**P33.66** $L = 2.00\text{ H}$, $C = 10.0 \times 10^{-6}\text{ F}$, $R = 10.0\ \Omega$, $\Delta v(t) = (100 \sin \omega t)$

(a) The resonant frequency $\omega_0$ produces the maximum current and thus the maximum power delivery to the resistor.

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(2.00)(10.0 \times 10^{-6})}} = \boxed{224\text{ rad/s}}$$

(b) $$\mathcal{P} = \frac{(\Delta V_{\text{max}})^2}{2R} = \frac{(100)^2}{2(10.0)} = \boxed{500\text{ W}}$$

(c) $$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{Z} = \frac{\Delta V_{\text{rms}}}{\sqrt{R^2 + (\omega L - (1/\omega C))^2}} \quad \text{and} \quad (I_{\text{rms}})_{\text{max}} = \frac{\Delta V_{\text{rms}}}{R}$$

$$I_{\text{rms}}^2 R = \frac{1}{2}\left(I_{\text{rms}}^2\right)_{\text{max}} R \quad \text{or} \quad \frac{(\Delta V_{\text{rms}})^2}{Z^2}R = \frac{1}{2}\frac{(\Delta V_{\text{rms}})^2}{R^2}R.$$

This occurs where $Z^2 = 2R^2$: $$R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = 2R^2$$

$$\omega^4 L^2C^2 - 2L\omega^2 C - R^2\omega^2 C^2 + 1 = 0 \quad \text{or} \quad L^2C^2\omega^4 - \left(2LC + R^2C^2\right)\omega^2 + 1 = 0$$

$$\left[(2.00)^2\left(10.0 \times 10^{-6}\right)^2\right]\omega^4 - \left[2(2.00)\left(10.0 \times 10^{-6}\right) + (10.0)^2\left(10.0 \times 10^{-6}\right)^2\right]\omega^2 + 1 = 0.$$

Solving this quadratic equation, we find that $\omega^2 = 51\,130$, or $48\,894$

$$\omega_1 = \sqrt{48\,894} = \boxed{221\text{ rad/s}} \quad \text{and} \quad \omega_2 = \sqrt{51\,130} = \boxed{226\text{ rad/s}}.$$

**P33.67** (a) From Equation 33.41, $\dfrac{N_1}{N_2} = \dfrac{\Delta V_1}{\Delta V_2}$.

Let input impedance $Z_1 = \dfrac{\Delta V_1}{I_1}$ and the output impedance $Z_2 = \dfrac{\Delta V_2}{I_2}$

so that $\dfrac{N_1}{N_2} = \dfrac{Z_1 I_1}{Z_2 I_2}$. But from Eq. 33.42, $\dfrac{I_1}{I_2} = \dfrac{\Delta V_2}{\Delta V_1} = \dfrac{N_2}{N_1}$.

So, combining with the previous result we have $\boxed{\dfrac{N_1}{N_2} = \sqrt{\dfrac{Z_1}{Z_2}}}$.

(b) $\dfrac{N_1}{N_2} = \sqrt{\dfrac{Z_1}{Z_2}} = \sqrt{\dfrac{8\,000}{8.00}} = \boxed{31.6}$

**P33.68** $P = I_{\text{rms}}^2 R = \left(\dfrac{\Delta V_{\text{rms}}}{Z}\right)^2 R$, so $250\text{ W} = \dfrac{(120\text{ V})^2}{Z^2}(40.0\ \Omega)$: $Z = \sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}$

$$250 = \frac{(120)^2(40.0)}{(40.0)^2 + \left[2\pi f(0.185) - \left[1/2\pi f\left(65.0\times 10^{-6}\right)\right]\right]^2} \quad \text{and} \quad 250 = \frac{576\,000 f^2}{1\,600 f^2 + \left(1.162\,4 f^2 - 2\,448.5\right)^2}$$

$$1 = \frac{2\,304 f^2}{1\,600 f^2 + 1.351\,1 f^4 - 5\,692.3 f^2 + 5\,995\,300} \quad \text{so} \quad 1.351\,1 f^4 - 6\,396.3 f^2 + 5\,995\,300 = 0$$

$$f^2 = \frac{6\,396.3 \pm \sqrt{(6\,396.3)^2 - 4(1.351\,1)(5\,995\,300)}}{2(1.351\,1)} = 3\,446.5 \text{ or } 1\,287.4$$

$f = \boxed{58.7\text{ Hz or }35.9\text{ Hz}}$

**P33.69** $I_R = \dfrac{\Delta V_{\text{rms}}}{R}$; $I_L = \dfrac{\Delta V_{\text{rms}}}{\omega L}$; $I_C = \dfrac{\Delta V_{\text{rms}}}{(\omega C)^{-1}}$

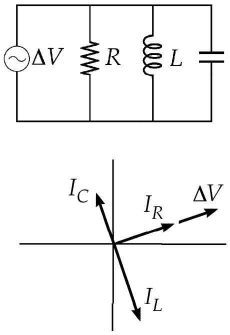


**FIG. P33.69**

(a) $I_{\text{rms}} = \sqrt{I_R^2 + (I_C - I_L)^2} = \boxed{\Delta V_{\text{rms}} \sqrt{\left(\dfrac{1}{R^2}\right) + \left(\omega C - \dfrac{1}{\omega L}\right)^2}}$

(b) $\tan\phi = \dfrac{I_C - I_L}{I_R} = \Delta V_{\text{rms}}\left[\dfrac{1}{X_C} - \dfrac{1}{X_L}\right]\left(\dfrac{1}{\Delta V_{\text{rms}}/R}\right)$

$\boxed{\tan\phi = R\left[\dfrac{1}{X_C} - \dfrac{1}{X_L}\right]}$

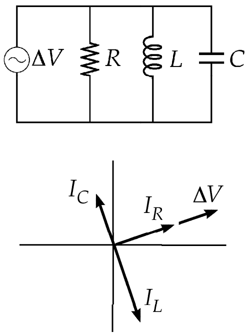


**FIG. P33.70**

**P33.70** (a) $$I_{\text{rms}}=\Delta V_{\text{rms}}\sqrt{\frac{1}{R^2}+\left(\omega C-\frac{1}{\omega L}\right)^2}$$

$$\Delta V_{\text{rms}}\to\left(\Delta V_{\text{rms}}\right)_{\max}\text{ when }\omega C=\frac{1}{\omega L}$$

$$f=\frac{1}{2\pi\sqrt{LC}}=\frac{1}{2\pi\sqrt{200\times10^{-3}\text{ H}\left(0.150\times10^{-6}\text{ F}\right)}}=\boxed{919\text{ Hz}}$$

(b) $$I_R=\frac{\Delta V_{\text{rms}}}{R}=\frac{120\text{ V}}{80.0\ \Omega}=\boxed{1.50\text{ A}}$$

$$I_L=\frac{\Delta V_{\text{rms}}}{\omega L}=\frac{120\text{ V}}{\left(374\text{ s}^{-1}\right)(0.200\text{ H})}=\boxed{1.60\text{ A}}$$

$$I_C=\Delta V_{\text{rms}}(\omega C)=(120\text{ V})\left(374\text{ s}^{-1}\right)\left(0.150\times10^{-6}\text{ F}\right)=\boxed{6.73\text{ mA}}$$

(c) $$I_{\text{rms}}=\sqrt{I_R^2+\left(I_C-I_L\right)^2}=\sqrt{(1.50)^2+(0.006\,73-1.60)^2}=\boxed{2.19\text{ A}}$$

(d) $$\phi=\tan^{-1}\left[\frac{I_C-I_L}{I_R}\right]=\tan^{-1}\left[\frac{0.006\,73-1.60}{1.50}\right]=\boxed{-46.7^\circ}$$

The $\boxed{\text{current is lagging the voltage}}$.

**P33.71** (a) $$X_L=X_C=1\,884\ \Omega\text{ when }f=2\,000\text{ Hz}$$

$$L=\frac{X_L}{2\pi f}=\frac{1\,884\ \Omega}{4\,000\pi\text{ rad/s}}=0.150\text{ H and}$$

$$C=\frac{1}{(2\pi f)X_C}=\frac{1}{(4\,000\pi\text{ rad/s})(1\,884\ \Omega)}=42.2\text{ nF}$$

$$X_L=2\pi f(0.150\text{ H})\qquad X_C=\frac{1}{(2\pi f)\left(4.22\times10^{-8}\text{ F}\right)}$$

$$Z=\sqrt{(40.0\ \Omega)^2+\left(X_L-X_C\right)^2}$$

| $f$ (Hz) | $X_L$ (Ω) | $X_C$ (Ω) | $Z$ (Ω) |
|---|---|---|---|
| 300 | 283 | 12 600 | 1 2300 |
| 600 | 565 | 6 280 | 5 720 |
| 800 | 754 | 4 710 | 3 960 |
| 1 000 | 942 | 3 770 | 2 830 |
| 1 500 | 1 410 | 2 510 | 1 100 |
| 2 000 | 1 880 | 1 880 | 40 |
| 3 000 | 2 830 | 1 260 | 1 570 |
| 4 000 | 3 770 | 942 | 2 830 |
| 6 000 | 5 650 | 628 | 5 020 |
| 10 000 | 9 420 | 377 | 9 040 |

(b)

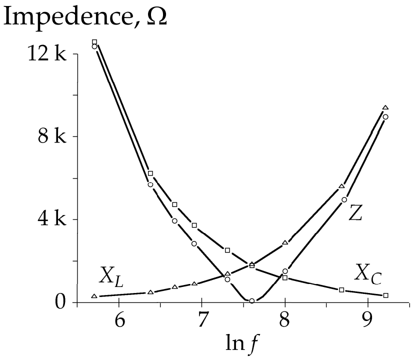


**FIG. P33.71(b)**

**P33.72** $\omega_0 = \frac{1}{\sqrt{LC}} = 1.00 \times 10^6 \text{ rad/s}$

For each angular frequency, we find

$$Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$$

then $I = \frac{1.00 \text{ V}}{Z}$

and $\mathscr{P} = I^2(1.00\ \Omega)$.

The full width at half maximum is:

$$\Delta f = \frac{\Delta\omega}{2\pi} = \frac{(1.000\,5 - 0.999\,5)\omega_0}{2\pi}$$

$$\Delta f = \frac{1.00 \times 10^3 \text{ s}^{-1}}{2\pi} = 159 \text{ Hz}$$

while

$$\frac{R}{2\pi L} = \frac{1.00\ \Omega}{2\pi(1.00 \times 10^{-3} \text{ H})} = 159 \text{ Hz}.$$

| $\frac{\omega}{\omega_0}$ | $\omega L$ (Ω) | $\frac{1}{\omega C}$ (Ω) | $Z$ (Ω) | $P = I^2R$ (W) |
|---|---|---|---|---|
| 0.9990 | 999.0 | 1001.0 | 2.24 | 0.19984 |
| 0.9991 | 999.1 | 1000.9 | 2.06 | 0.23569 |
| 0.9993 | 999.3 | 1000.7 | 1.72 | 0.33768 |
| 0.9995 | 999.5 | 1000.5 | 1.41 | 0.49987 |
| 0.9997 | 999.7 | 1000.3 | 1.17 | 0.73524 |
| 0.9999 | 999.9 | 1000.1 | 1.02 | 0.96153 |
| 1.0000 | 1000 | 1000.0 | 1.00 | 1.00000 |
| 1.0001 | 1000.1 | 999.9 | 1.02 | 0.96154 |
| 1.0003 | 1000.3 | 999.7 | 1.17 | 0.73535 |
| 1.0005 | 1000.5 | 999.5 | 1.41 | 0.50012 |
| 1.0007 | 1000.7 | 999.3 | 1.72 | 0.33799 |
| 1.0009 | 1000.9 | 999.1 | 2.06 | 0.23601 |
| 1.0010 | 1001 | 999.0 | 2.24 | 0.20016 |

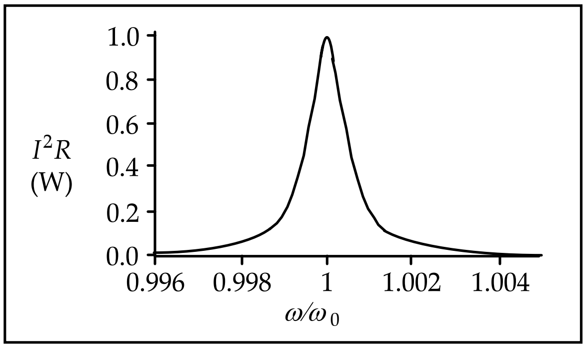


**FIG. P33.72**

**P33.73** $\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \frac{R}{\sqrt{R^2 + (1/\omega C)^2}} = \frac{R}{\sqrt{R^2 + (1/2\pi f C)^2}}$

(a) $\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \frac{1}{2}$ when $\frac{1}{\omega C} = R\sqrt{3}$.

Hence, $f = \frac{\omega}{2\pi} = \frac{1}{2\pi RC\sqrt{3}} = \boxed{1.84 \text{ kHz}}$.

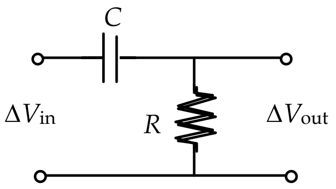


**FIG. P33.73**

*continued on next page*

(b)

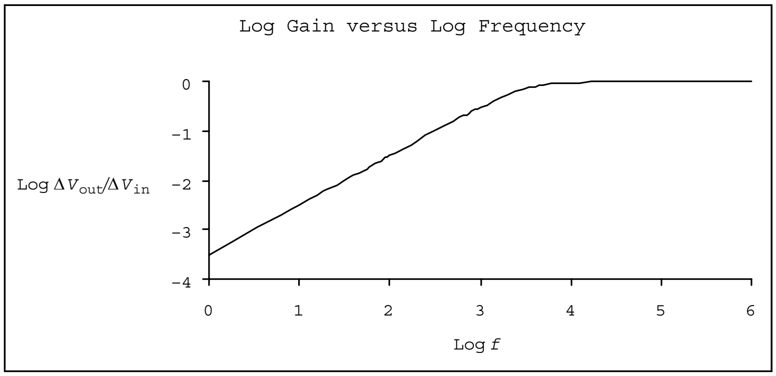


**FIG. P33.73(b)**

## ANSWERS TO EVEN PROBLEMS

| | |
|---|---|
| **P33.2** | (a) $193\ \Omega$; (b) $144\ \Omega$ |
| **P33.4** | (a) 25.3 rad/s; (b) 0.114 s |
| **P33.6** | $1.25\ \text{A}$ and $96.0\ \Omega$ for bulbs 1 and 2;<br>$0.833\ \text{A}$ and $144\ \Omega$ for bulb 3 |
| **P33.8** | $7.03\ \text{H}$ or more |
| **P33.10** | 3.14 A |
| **P33.12** | 3.80 J |
| **P33.14** | (a) greater than $41.3\ \text{Hz}$;<br>(b) less than $87.5\ \Omega$ |
| **P33.16** | $\sqrt{2}C(\Delta V_{\text{rms}})$ |
| **P33.18** | –32.0 A |
| **P33.20** | 2.79 kHz |
| **P33.22** | (a) $109\ \Omega$; (b) $0.367\ \text{A}$; (c) $I_{\max} = 0.367\ \text{A}$,<br>$\omega = 100\ \text{rad/s}$, $\phi = -0.896\ \text{rad}$ |
| **P33.24** | 19.3 mA |
| **P33.26** | (a) $146\ \text{V}$; (b) $212\ \text{V}$; (c) $179\ \text{V}$; (d) $33.4\ \text{V}$ |
| **P33.28** | $X_C = 3R$ |
| **P33.30** | (a) $2.00\ \text{A}$; (b) $160\ \text{W}$; (c) see the solution |
| **P33.32** | 353 W |
| **P33.34** | (a) $5.43\ \text{A}$; (b) $0.905$; (c) $281\ \mu\text{F}$; (d) $109\ \text{V}$ |
| **P33.36** | $\dfrac{11(\Delta V_{\text{rms}})^2}{14R}$ |
| **P33.38** | 46.5 pF to 419 pF |
| **P33.40** | (a) $3.56\ \text{kHz}$; (b) $5.00\ \text{A}$; (c) $22.4$;<br>(d) $2.24\ \text{kV}$ |
| **P33.42** | $\dfrac{4\pi(\Delta V_{\text{rms}})^2 RC\sqrt{LC}}{4R^2C + 9L}$ |
| **P33.44** | (a) $9.23\ \text{V}$; (b) $4.55\ \text{A}$; (c) $42.0\ \text{W}$ |
| **P33.46** | (a) $1\,600$ turns; (b) $30.0\ \text{A}$; (c) $25.3\ \text{A}$ |
| **P33.48** | (a) $83.3$; (b) $54.0\ \text{mA}$; (c) $185\ \text{k}\Omega$ |
| **P33.50** | (a) 0.34; (b) 5.3 W; (c) \$4.8 |
| **P33.52** | (a) see the solution; (b) 1; 0; (c) $\dfrac{\sqrt{3}}{2\pi RC}$ |
| **P33.54** | (a) 1.00; (b) 0.346 |
| **P33.56** | see the solution |
| **P33.58** | $R = 99.6\ \Omega$, $C = 24.9\ \mu\text{F}$, $L = 164\ \text{mH}$ or<br>$402\ \text{mH}$ |

**P33.60** $L = 0.200\ \text{m}$ and $T = 10.9\ \text{N}$, or any values related by $T = \left(274\ \text{kg/ms}^2\right)L^2$

**P33.62** (a) $i(t) = \dfrac{\Delta V_{\max}}{R}\cos\omega t$; (b) $\mathscr{P} = \dfrac{\left(\Delta V_{\max}\right)^2}{2R}$;
(c) $i(t) = \dfrac{\Delta V_{\max}}{\sqrt{R^2 + \omega^2 L^2}}\cos\left[\omega t + \tan^{-1}\left(\dfrac{\omega L}{R}\right)\right]$;
(d) $C = \dfrac{1}{\omega_0^2 L}$; (e) $Z = R$; (f) $\dfrac{\left(\Delta V_{\max}\right)^2 L}{2R^2}$;
(g) $\dfrac{\left(\Delta V_{\max}\right)^2 L}{2R^2}$; (h) $\tan^{-1}\left(\dfrac{3}{2R}\sqrt{\dfrac{L}{C}}\right)$;
(i) $\dfrac{1}{\sqrt{2LC}}$

**P33.64** $\sim 10^3\ \text{A}$

**P33.66** (a) $224\ \text{rad/s}$; (b) $500\ \text{W}$;
(c) $221\ \text{rad/s}$ and $226\ \text{rad/s}$

**P33.68** either 58.7 Hz or 35.9 Hz

**P33.70** (a) 919 Hz;
(b) $I_R = 1.50\ \text{A}$, $I_L = 1.60\ \text{A}$, $I_C = 6.73\ \text{mA}$;
(c) 2.19 A; (d) –46.7°; current lagging

**P33.72** see the solution

# 34

# Electromagnetic Waves

## CHAPTER OUTLINE

34.1 Maxwell's Equations and Hertz's Discoveries
34.2 Plane Electromagnetic Waves
34.3 Energy Carried by Electromagnetic Waves
34.4 Momentum and Radiation Pressure
34.5 Production of Electromagnetic Waves by an Antenna
34.6 The Spectrum of Electromagnetic Waves

## ANSWERS TO QUESTIONS

**Q34.1** Radio waves move at the speed of light. They can travel around the curved surface of the Earth, bouncing between the ground and the ionosphere, which has an altitude that is small when compared to the radius of the Earth. The distance across the lower forty-eight states is approximately 5 000 km, requiring a transit time of $\frac{5\times10^6\text{ m}}{3\times10^8\text{ m/s}}\sim10^{-2}\text{ s}$. To go halfway around the Earth takes only 0.07 s. In other words, a speech can be heard on the other side of the world before it is heard at the back of a large room.

**Q34.2** The Sun's angular speed in our sky is our rate of rotation, $\frac{360°}{24\text{ h}}=15°/\text{h}$. In 8.3 minutes it moves west by $\theta=\omega t=(15°/\text{h})\left(\frac{1\text{ h}}{60\text{ min}}\right)(8.3\text{ min})=2.1°$. This is about four times the angular diameter of the Sun.

**Q34.3** Energy moves. No matter moves. You could say that electric and magnetic fields move, but it is nicer to say that the fields at one point stay at that point and oscillate. The fields vary in time, like sports fans in the grandstand when the crowd does the wave. The fields constitute the medium for the wave, and energy moves.

**Q34.4** No. If a single wire carries DC current, it does not emit electromagnetic waves. In this case, there is a constant magnetic field around the wire. Alternately, if the cable is a coaxial cable, it ideally does not emit electromagnetic waves even while carrying AC current.

**Q34.5** Acceleration of electric charge.

**Q34.6** The changing magnetic field of the solenoid induces eddy currents in the conducting core. This is accompanied by $I^2R$ conversion of electrically-transmitted energy into internal energy in the conductor.

**Q34.7** A wire connected to the terminals of a battery does not radiate electromagnetic waves. The battery establishes an electric field, which produces current in the wire. The current in the wire creates a magnetic field. Both fields are constant in time, so no electromagnetic induction or "magneto-electric induction" happens. Neither field creates a new cycle of the other field. No wave propagation occurs.

**Q34.8** No. Static electricity is just that: static. Without acceleration of the charge, there can be no electromagnetic wave.

**Q34.9**

| *Sound* | *Light* |
|---|---|
| The world of sound extends to the top of the atmosphere and stops there; sound requires a material medium. Sound propagates by a chain reaction of density and pressure disturbances recreating each other. Sound in air moves at hundreds of meters per second. Audible sound has frequencies over a range of three decades (ten octaves) from 20 Hz to 20 kHz. Audible sound has wavelengths of ordinary size (1.7 cm to 17 m). Sound waves are longitudinal. | The universe of light fills the whole universe. Light moves through materials, but faster in a vacuum. Light propagates by a chain reaction of electric and magnetic fields recreating each other. Light in air moves at hundreds of millions of meters per second. Visible light has frequencies over a range of less than one octave, from 430 to 750 **Tera**hertz. Visible light has wavelengths of very small size (400 nm to 700 nm). Light waves are transverse. |

Sound and light can both be reflected, refracted, or absorbed to produce internal energy. Both have amplitude and frequency set by the source, speed set by the medium, and wavelength set by both source and medium. Sound and light both exhibit the Doppler effect, standing waves, beats, interference, diffraction, and resonance. Both can be focused to make images. Both are described by wave functions satisfying wave equations. **Both carry energy**. If the source is small, their intensities both follow an inverse-square law. Both are waves.

**Q34.10** The Poynting vector **S** describes the energy flow associated with an electromagnetic wave. The direction of **S** is along the direction of propagation and the magnitude of **S** is the rate at which electromagnetic energy crosses a unit surface area perpendicular to the direction of **S**.

**Q34.11** Photons carry momentum. Recalling what we learned in Chapter 9, the impulse imparted to a particle that bounces elastically is twice that imparted to an object that sticks to a massive wall. Similarly, the impulse, and hence the pressure exerted by a photon reflecting from a surface must be twice that exerted by a photon that is absorbed.

**Q34.12** Different stations have transmitting antennas at different locations. For best reception align your rabbit ears perpendicular to the straight-line path from your TV to the transmitting antenna. The transmitted signals are also polarized. The polarization direction of the wave can be changed by reflection from surfaces—including the atmosphere—and through Kerr rotation—a change in polarization axis when passing through an organic substance. In your home, the plane of polarization is determined by your surroundings, so antennas need to be adjusted to align with the polarization of the wave.

**Q34.13** You become part of the receiving antenna! You are a big sack of salt water. Your contribution usually increases the gain of the antenna by a few tenths of a dB, enough to noticeably improve reception.

**Q34.14** On the TV set, each side of the dipole antenna is a 1/4 of the wavelength of the VHF radio wave. The electric field of the wave moves free charges in the antenna in electrical resonance, giving maximum current in the center of the antenna, where the cable connects it to the receiver.

**Q34.15** The loop antenna is essentially a solenoid. As the UHF radio wave varies the magnetic field inside the loop, an AC emf is induced in the loop as described by Faraday's and Lenz's laws. This signal is then carried down a cable to the UHF receiving circuit in the TV. An excellent reference for antennas and all things radio is the *ARRL Handbook*.

**Q34.16** The voltage induced in the loop antenna is proportional to the rate of change of the magnetic field in the wave. A wave of higher frequency induces a larger emf in direct proportion. The instantaneous voltage between the ends of a dipole antenna is the distance between the ends multiplied by the electric field of the wave. It does not depend on the frequency of the wave.

**Q34.17** The radiation resistance of a broadcast antenna is the equivalent resistance that would take the same power that the antenna radiates, and convert it into internal energy.

**Q34.18** Consider a typical metal rod antenna for a car radio. The rod detects the electric field portion of the carrier wave. Variations in the amplitude of the incoming radio wave cause the electrons in the rod to vibrate with amplitudes emulating those of the carrier wave. Likewise, for frequency modulation, the variations of the frequency of the carrier wave cause constant-amplitude vibrations of the electrons in the rod but at frequencies that imitate those of the carrier.

**Q34.19** The frequency of EM waves in a microwave oven, typically 2.45 GHz, is chosen to be in a band of frequencies absorbed by water molecules. The plastic and the glass contain no water molecules. Plastic and glass have very different absorption frequencies from water, so they may not absorb any significant microwave energy and remain cool to the touch.

**Q34.20** People of all the world's races have skin the same color in the infrared. When you blush or exercise or get excited, you stand out like a beacon in an infrared group picture. The brightest portions of your face show where you radiate the most. Your nostrils and the openings of your ear canals are bright; brighter still are just the pupils of your eyes.

**Q34.21** Light bulbs and the toaster shine brightly in the infrared. Somewhat fainter are the back of the refrigerator and the back of the television set, while the TV screen is dark. The pipes under the sink show the same weak sheen as the walls until you turn on the faucets. Then the pipe on the right turns very black while that on the left develops a rich glow that quickly runs up along its length. The food on your plate shines; so does human skin, the same color for all races. Clothing is dark as a rule, but your bottom glows like a monkey's rump when you get up from a chair, and you leave behind a patch of the same blush on the chair seat. Your face shows you are lit from within, like a jack-o-lantern: your nostrils and the openings of your ear canals are bright; brighter still are just the pupils of your eyes.

**Q34.22** Welding produces ultraviolet light, along with high intensity visible and infrared.

**Q34.23** 12.2 cm waves have a frequency of 2.46 GHz. If the $Q$ value of the phone is low (namely if it is cheap), and your microwave oven is not well shielded (namely, if it is also cheap), the phone can likely pick up interference from the oven. If the phone is well constructed and has a high $Q$ value, then there should be no interference at all.

## SOLUTIONS TO PROBLEMS

### Section 34.1 Maxwell's Equations and Hertz's Discoveries

***P34.1** (a) The rod creates the same electric field that it would if stationary. We apply Gauss's law to a cylinder of radius $r = 20\text{ cm}$ and length $\ell$:

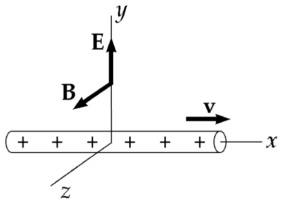


**FIG. P34.1**

$$\oint \mathbf{E}\cdot d\mathbf{A} = \frac{q_{\text{inside}}}{\epsilon_0}$$

$$E(2\pi r l)\cos 0° = \frac{\lambda l}{\epsilon_0}$$

$$\mathbf{E} = \frac{\lambda}{2\pi\epsilon_0 r}\text{ radially outward} = \frac{\left(35\times10^{-9}\text{ C/m}\right)\text{N}\cdot\text{m}^2}{2\pi\left(8.85\times10^{-12}\text{ C}^2\right)(0.2\text{ m})}\hat{\mathbf{j}} = \boxed{3.15\times10^{3}\,\hat{\mathbf{j}}\text{ N/C}}.$$

(b) The charge in motion constitutes a current of $\left(35\times10^{-9}\text{ C/m}\right)\left(15\times10^{6}\text{ m/s}\right) = 0.525\text{ A}$. This current creates a magnetic field.

$$\mathbf{B} = \frac{\mu_0 I}{2\pi r} = \frac{\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)(0.525\text{ A})}{2\pi(0.2\text{ m})}\hat{\mathbf{k}} = \boxed{5.25\times10^{-7}\,\hat{\mathbf{k}}\text{ T}}$$

(c) The Lorentz force on the electron is $F = q\mathbf{E} + q\mathbf{v}\times\mathbf{B}$

$$F = \left(-1.6\times10^{-19}\text{ C}\right)\left(3.15\times10^{3}\,\hat{\mathbf{j}}\text{ N/C}\right) + \left(-1.6\times10^{-19}\text{ C}\right)\left(240\times10^{6}\,\hat{\mathbf{i}}\text{ m/s}\right)\times\left(5.25\times10^{-7}\,\hat{\mathbf{k}}\,\frac{\text{N}\cdot\text{s}}{\text{C}\cdot\text{m}}\right)$$

$$F = 5.04\times10^{-16}\left(-\hat{\mathbf{j}}\right)\text{N} + 2.02\times10^{-17}\left(+\hat{\mathbf{j}}\right)\text{N} = \boxed{4.83\times10^{-16}\left(-\hat{\mathbf{j}}\right)\text{N}}$$

### Section 34.2 Plane Electromagnetic Waves

**P34.2** (a) Since the light from this star travels at $3.00\times10^{8}\text{ m/s}$

the last bit of light will hit the Earth in $\dfrac{6.44\times10^{18}\text{ m}}{3.00\times10^{8}\text{ m/s}} = 2.15\times10^{10}\text{ s} = 680\text{ years}$.

Therefore, it will disappear from the sky in the year $2\,004 + 680 = \boxed{2.68\times10^{3}\text{ C.E.}}$.
The star is 680 light-years away.

(b) $$\Delta t = \frac{\Delta x}{v} = \frac{1.496\times10^{11}\text{ m}}{3\times10^{8}\text{ m/s}} = \boxed{499\text{ s}} = 8.31\text{ min}$$

(c) $$\Delta t = \frac{\Delta x}{v} = \frac{2\left(3.84\times10^{8}\text{ m}\right)}{3\times10^{8}\text{ m/s}} = \boxed{2.56\text{ s}}$$

(d) $$\Delta t = \frac{\Delta x}{v} = \frac{2\pi\left(6.37\times10^{6}\text{ m}\right)}{3\times10^{8}\text{ m/s}} = \boxed{0.133\text{ s}}$$

(e) $$\Delta t = \frac{\Delta x}{v} = \frac{10\times10^{3}\text{ m}}{3\times10^{8}\text{ m/s}} = \boxed{3.33\times10^{-5}\text{ s}}$$

**P34.3** $v=\dfrac{1}{\sqrt{\kappa\mu_0\in_0}}=\dfrac{1}{\sqrt{1.78}}c=0.750c=\boxed{2.25\times10^{8}\ \text{m/s}}$

**P34.4** $\dfrac{E}{B}=c$

or $\dfrac{220}{B}=3.00\times10^{8}$

so $B=7.33\times10^{-7}\ \text{T}=\boxed{733\ \text{nT}}$.

**P34.5** (a) $f\lambda=c$

or $f(50.0\ \text{m})=3.00\times10^{8}\ \text{m/s}$

so $\boxed{f=6.00\times10^{6}\ \text{Hz}=6.00\ \text{MHz}}$.

(b) $\dfrac{E}{B}=c$

or $\dfrac{22.0}{B_{\text{max}}}=3.00\times10^{8}$

so $\mathbf{B}_{\text{max}}=\boxed{-73.3\hat{\mathbf{k}}\ \text{nT}}$.

(c) $k=\dfrac{2\pi}{\lambda}=\dfrac{2\pi}{50.0}=0.126\ \text{m}^{-1}$

and $\omega=2\pi f=2\pi\left(6.00\times10^{6}\ \text{s}^{-1}\right)=3.77\times10^{7}\ \text{rad/s}$

$\mathbf{B}=\mathbf{B}_{\text{max}}\cos(kx-\omega t)=\boxed{-73.3\cos\left(0.126x-3.77\times10^{7}t\right)\hat{\mathbf{k}}\ \text{nT}}$.

**P34.6** $\omega=2\pi f=6.00\pi\times10^{9}\ \text{s}^{-1}=1.88\times10^{10}\ \text{s}^{-1}$

$k=\dfrac{2\pi}{\lambda}=\dfrac{\omega}{c}=\dfrac{6.00\pi\times10^{9}\ \text{s}^{-1}}{3.00\times10^{8}\ \text{m/s}}=20.0\pi=62.8\ \text{m}^{-1}$

$B_{\text{max}}=\dfrac{E}{c}=\dfrac{300\ \text{V/m}}{3.00\times10^{8}\ \text{m/s}}=1.00\ \mu\text{T}$

$\boxed{E=(300\ \text{V/m})\cos\left(62.8x-1.88\times10^{10}t\right)}$

$\boxed{B=(1.00\ \mu\text{T})\cos\left(62.8x-1.88\times10^{10}t\right)}$

**P34.7** (a) $B=\dfrac{E}{c}=\dfrac{100\ \text{V/m}}{3.00\times10^{8}\ \text{m/s}}=3.33\times10^{-7}\ \text{T}=\boxed{0.333\ \mu\text{T}}$

(b) $\lambda=\dfrac{2\pi}{k}=\dfrac{2\pi}{1.00\times10^{7}\ \text{m}^{-1}}=\boxed{0.628\ \mu\text{m}}$

(c) $f=\dfrac{c}{\lambda}=\dfrac{3.00\times10^{8}\ \text{m/s}}{6.28\times10^{-7}\ \text{m}}=\boxed{4.77\times10^{14}\ \text{Hz}}$

**P34.8** $E = E_{\max}\cos(kx - \omega t)$

$$\frac{\partial E}{\partial x} = -E_{\max}\sin(kx - \omega t)(k)$$

$$\frac{\partial E}{\partial t} = -E_{\max}\sin(kx - \omega t)(-\omega)$$

$$\frac{\partial^2 E}{\partial x^2} = -E_{\max}\cos(kx - \omega t)\left(k^2\right)$$

$$\frac{\partial^2 E}{\partial t^2} = -E_{\max}\cos(kx - \omega t)(-\omega)^2$$

We must show: $\dfrac{\partial E}{\partial x^2} = \mu_0 \in_0 \dfrac{\partial^2 E}{\partial t^2}$.

That is, $-\left(k^2\right)E_{\max}\cos(kx - \omega t) = -\mu_0 \in_0 (-\omega)^2 E_{\max}\cos(kx - \omega t)$.

But this is true, because $\dfrac{k^2}{\omega^2} = \left(\dfrac{1}{f\lambda}\right)^2 = \dfrac{1}{c^2} = \mu_0 \in_0$.

The proof for the wave of magnetic field follows precisely the same steps.

**P34.9** In the fundamental mode, there is a single loop in the standing wave between the plates. Therefore, the distance between the plates is equal to half a wavelength.

$$\lambda = 2L = 2(2.00\text{ m}) = 4.00\text{ m}$$

Thus, $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^8\text{ m/s}}{4.00\text{ m}} = 7.50\times10^7\text{ Hz} = \boxed{75.0\text{ MHz}}$.

**P34.10** $d_{\text{A to A}} = 6\text{ cm} \pm 5\% = \dfrac{\lambda}{2}$

$\lambda = 12\text{ cm} \pm 5\%$

$v = \lambda f = (0.12\text{ m} \pm 5\%)\left(2.45\times10^9\text{ s}^{-1}\right) = \boxed{2.9\times10^8\text{ m/s} \pm 5\%}$

## Section 34.3 Energy Carried by Electromagnetic Waves

**P34.11** $S = I = \dfrac{U}{At} = \dfrac{Uc}{V} = uc$ $\qquad \dfrac{\text{Energy}}{\text{Unit Volume}} = u = \dfrac{I}{c} = \dfrac{1\,000\text{ W/m}^2}{3.00\times10^8\text{ m/s}} = \boxed{3.33\ \mu\text{J/m}^3}$

**P34.12** $S_{\text{av}} = \dfrac{\overline{\mathcal{P}}}{4\pi r^2} = \dfrac{4.00\times10^3\text{ W}}{4\pi(4.00\times1\,609\text{ m})^2} = 7.68\ \mu\text{W/m}^2$

$E_{\max} = \sqrt{2\mu_0 c S_{\text{av}}} = 0.076\,1\text{ V/m}$

$\Delta V_{\max} = E_{\max}L = (76.1\text{ mV/m})(0.650\text{ m}) = \boxed{49.5\text{ mV (amplitude)}}$ or 35.0 mV (rms)

**P34.13** $r = (5.00\ \text{mi})(1\,609\ \text{m/mi}) = 8.04\times 10^3\ \text{m}$

$$S = \frac{\overline{\mathscr{P}}}{4\pi r^2} = \frac{250\times 10^3\ \text{W}}{4\pi\left(8.04\times 10^3\ \text{W}\right)^2} = \boxed{307\ \mu\text{W/m}^2}$$

**P34.14** $I = \dfrac{100\ \text{W}}{4\pi(1.00\ \text{m})^2} = 7.96\ \text{W/m}^2$

$$u = \frac{I}{c} = 2.65\times 10^{-8}\ \text{J/m}^3 = 26.5\ \text{nJ/m}^3$$

(a) $u_E = \dfrac{1}{2}u = \boxed{13.3\ \text{nJ/m}^3}$

(b) $u_B = \dfrac{1}{2}u = \boxed{13.3\ \text{nJ/m}^3}$

(c) $I = \boxed{7.96\ \text{W/m}^2}$

**P34.15** Power output = (power input)(efficiency).

Thus, $\text{Power input} = \dfrac{\text{Power out}}{\text{eff}} = \dfrac{1.00\times 10^6\ \text{W}}{0.300} = 3.33\times 10^6\ \text{W}$

and $A = \dfrac{\mathscr{P}}{I} = \dfrac{3.33\times 10^6\ \text{W}}{1.00\times 10^3\ \text{W/m}^2} = \boxed{3.33\times 10^3\ \text{m}^2}$.

**P34.16** $I = \dfrac{B_{\max}^2 c}{2\mu_0} = \dfrac{\mathscr{P}}{4\pi r^2}$

$$B_{\max} = \sqrt{\left(\frac{\mathscr{P}}{4\pi r^2}\right)\left(\frac{2\mu_0}{c}\right)} = \sqrt{\frac{\left(10.0\times 10^3\right)(2)\left(4\pi\times 10^{-7}\right)}{4\pi\left(5.00\times 10^3\right)^2\left(3.00\times 10^8\right)}} = \boxed{5.16\times 10^{-10}\ \text{T}}$$

Since the magnetic field of the Earth is approximately $5\times 10^{-5}\ \text{T}$, the Earth's field is some 100 000 times stronger.

**P34.17** (a) $\mathscr{P} = I^2 R = 150\ \text{W}$

$$A = 2\pi r L = 2\pi\left(0.900\times 10^{-3}\ \text{m}\right)(0.080\,0\ \text{m}) = 4.52\times 10^{-4}\ \text{m}^2$$

$$S = \frac{\mathscr{P}}{A} = \boxed{332\ \text{kW/m}^2}\ \text{(points radially inward)}$$

(b) $B = \dfrac{\mu_0 I}{2\pi r} = \dfrac{\mu_0(1.00)}{2\pi\left(0.900\times 10^{-3}\right)} = \boxed{222\ \mu\text{T}}$

$$E = \frac{\Delta V}{\Delta x} = \frac{IR}{L} = \frac{150\ \text{V}}{0.080\,0\ \text{m}} = \boxed{1.88\ \text{kV/m}}$$

Note: $S = \dfrac{EB}{\mu_0} = 332\ \text{kW/m}^2$

***P34.18** (a)

$$I = \frac{E_{\max}^2}{2\mu_0 c} = \frac{\left(3\times10^6\ \text{V/m}\right)^2}{2\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(3\times10^8\ \text{m/s}\right)}\left(\frac{\text{J}}{\text{V}\cdot\text{C}}\right)^2\left(\frac{\text{C}}{\text{A}\cdot\text{s}}\right)\left(\frac{\text{T}\cdot\text{C}\cdot\text{m}}{\text{N}\cdot\text{s}}\right)\left(\frac{\text{N}\cdot\text{m}}{\text{J}}\right)$$

$$I = \boxed{1.19\times10^{10}\ \text{W/m}^2}$$

(b)

$$\mathscr{P} = IA = \left(1.19\times10^{10}\ \text{W/m}^2\right)\pi\left(\frac{5\times10^{-3}\ \text{m}}{2}\right)^2 = \boxed{2.34\times10^5\ \text{W}}$$

**P34.19** (a)

$$\mathbf{E}\cdot\mathbf{B} = \left(80.0\hat{\mathbf{i}} + 32.0\hat{\mathbf{j}} - 64.0\hat{\mathbf{k}}\right)(\text{N/C})\cdot\left(0.200\hat{\mathbf{i}} + 0.080\,0\hat{\mathbf{j}} + 0.290\hat{\mathbf{k}}\right)\mu\text{T}$$

$$\mathbf{E}\cdot\mathbf{B} = (16.0 + 2.56 - 18.56)\ \text{N}^2\cdot\text{s/C}^2\cdot\text{m} = \boxed{0}$$

(b)

$$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E}\times\mathbf{B} = \frac{\left[\left(80.0\hat{\mathbf{i}} + 32.0\hat{\mathbf{j}} - 64.0\hat{\mathbf{k}}\right)\ \text{N/C}\right]\times\left[\left(0.200\hat{\mathbf{i}} + 0.080\,0\hat{\mathbf{j}} + 0.290\hat{\mathbf{k}}\right)\mu\text{T}\right]}{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}}$$

$$\mathbf{S} = \frac{\left(6.40\hat{\mathbf{k}} - 23.2\hat{\mathbf{j}} - 6.40\hat{\mathbf{k}} + 9.28\hat{\mathbf{i}} - 12.8\hat{\mathbf{j}} + 5.12\hat{\mathbf{i}}\right)\times10^{-6}\ \text{W/m}^2}{4\pi\times10^{-7}}$$

$$\mathbf{S} = \boxed{\left(11.5\hat{\mathbf{i}} - 28.6\hat{\mathbf{j}}\right)\ \text{W/m}^2} = 30.9\ \text{W/m}^2\ \text{at } -68.2°\ \text{from the } +x \text{ axis.}$$

***P34.20** The energy put into the water in each container by electromagnetic radiation can be written as $e\mathscr{P}\Delta t = eIA\Delta t$ where $e$ is the percentage absorption efficiency. This energy has the same effect as heat in raising the temperature of the water:

$eIA\Delta t = mc\Delta T = \rho Vc\Delta T$

$$\Delta T = \frac{eI\ell^2\Delta t}{\rho\ell^3 c} = \frac{eI\Delta t}{\rho\ell c}$$

where $\ell$ is the edge dimension of the container and $c$ the specific heat of water. For the small container,

$$\Delta T = \frac{0.7\left(25\times10^3\ \text{W/m}^2\right)480\ \text{s}}{\left(10^3\ \text{kg/m}^3\right)(0.06\ \text{m})4\,186\ \text{J/kg}\cdot°\text{C}} = \boxed{33.4°\text{C}}.$$

For the larger,

$$\Delta T = \frac{0.91\left(25\ \text{J/s}\cdot\text{m}^2\right)480\ \text{s}}{\left(0.12/\text{m}^2\right)4\,186\ \text{J/°C}} = \boxed{21.7°\text{C}}.$$

**P34.21** We call the current $I_{\text{rms}}$ and the intensity $I$. The power radiated at this frequency is

$$\mathscr{P} = (0.010\,0)(\Delta V_{\text{rms}})I_{\text{rms}} = \frac{0.010\,0(\Delta V_{\text{rms}})^2}{R} = 1.31\ \text{W}.$$

If it is isotropic, the intensity one meter away is

$$I = \frac{\mathscr{P}}{A} = \frac{1.31\ \text{W}}{4\pi(1.00\ \text{m})^2} = 0.104\ \text{W/m}^2 = S_{\text{av}} = \frac{c}{2\mu_0}B_{\max}^2$$

$$B_{\max} = \sqrt{\frac{2\mu_0 I}{c}} = \sqrt{\frac{2\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(0.104\ \text{W/m}^2\right)}{3.00\times10^8\ \text{m/s}}} = \boxed{29.5\ \text{nT}}$$

**P34.22** (a) $$\text{efficiency} = \frac{\text{useful power output}}{\text{total power input}} \times 100\% = \left(\frac{700\text{ W}}{1\,400\text{ W}}\right) \times 100\% = \boxed{50.0\%}$$

(b) $$S_{\text{av}} = \frac{\mathscr{P}}{A} = \frac{700\text{ W}}{(0.068\,3\text{ m})(0.038\,1\text{ m})} = 2.69 \times 10^5\ \text{W/m}^2$$

$$\mathbf{S}_{\text{av}} = \boxed{269\ \text{kW/m}^2\ \text{toward the oven chamber}}$$

(c) $$S_{\text{av}} = \frac{E_{\max}^2}{2\mu_0 c}$$

$$E_{\max} = \sqrt{2\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(3.00 \times 10^8\ \text{m/s}\right)\left(2.69 \times 10^5\ \text{W/m}^2\right)} = 1.42 \times 10^4\ \text{V/m}$$
$$= \boxed{14.2\ \text{kV/m}}$$

**P34.23** (a) $$B_{\max} = \frac{E_{\max}}{c}: \qquad B_{\max} = \frac{7.00 \times 10^5\ \text{N/C}}{3.00 \times 10^8\ \text{m/s}} = \boxed{2.33\ \text{mT}}$$

(b) $$I = \frac{E_{\max}^2}{2\mu_0 c}: \qquad I = \frac{\left(7.00 \times 10^5\right)^2}{2\left(4\pi \times 10^{-7}\right)\left(3.00 \times 10^8\right)} = \boxed{650\ \text{MW/m}^2}$$

(c) $$I = \frac{\mathscr{P}}{A}: \qquad \mathscr{P} = IA = \left(6.50 \times 10^8\ \text{W/m}^2\right)\left[\frac{\pi}{4}\left(1.00 \times 10^{-3}\ \text{m}\right)^2\right] = \boxed{510\ \text{W}}$$

**P34.24** (a) $$I = \frac{\left(10.0 \times 10^{-3}\right)\text{W}}{\pi\left(0.800 \times 10^{-3}\ \text{m}\right)^2} = \boxed{4.97\ \text{kW/m}^2}$$

(b) $$u_{\text{av}} = \frac{I}{c} = \frac{4.97 \times 10^3\ \text{J/m}^2\cdot\text{s}}{3.00 \times 10^8\ \text{m/s}} = \boxed{16.6\ \mu\text{J/m}^3}$$

**P34.25** (a) $$E = cB = \left(3.00 \times 10^8\ \text{m/s}\right)\left(1.80 \times 10^{-6}\ \text{T}\right) = \boxed{540\ \text{V/m}}$$

(b) $$u_{\text{av}} = \frac{B^2}{\mu_0} = \frac{\left(1.80 \times 10^{-6}\right)^2}{4\pi \times 10^{-7}} = \boxed{2.58\ \mu\text{J/m}^3}$$

(c) $$S_{\text{av}} = cu_{\text{av}} = \left(3.00 \times 10^8\right)\left(2.58 \times 10^{-6}\right) = \boxed{773\ \text{W/m}^2}$$

(d) This is $\boxed{\text{77.3\% of the intensity in Example 34.5}}$. It may be cloudy, or the Sun may be setting.

## Section 34.4 **Momentum and Radiation Pressure**

**P34.26** The pressure $P$ upon the mirror is $P=\frac{2S_{\text{av}}}{c}$

where $A$ is the cross-sectional area of the beam and $S_{\text{av}}=\frac{\mathcal{P}}{A}$.

The force on the mirror is then $F=PA=\frac{2}{c}\left(\frac{\mathcal{P}}{A}\right)A=\frac{2\mathcal{P}}{c}$.

Therefore, $F=\frac{2\left(100\times10^{-3}\right)}{\left(3\times10^{8}\right)}=\boxed{6.67\times10^{-10}\text{ N}}$.

**P34.27** For complete absorption, $P=\frac{S}{c}=\frac{25.0}{3.00\times10^{8}}=\boxed{83.3\text{ nPa}}$.

**P34.28** (a) The radiation pressure is $\frac{2\left(1\,340\text{ W/m}^2\right)}{3.00\times10^{8}\text{ m/s}^2}=8.93\times10^{-6}\text{ N/m}^2$.

Multiplying by the total area, $A=6.00\times10^{5}\text{ m}^2$ gives: $F=\boxed{5.36\text{ N}}$.

(b) The acceleration is: $a=\frac{F}{m}=\frac{5.36\text{ N}}{6\,000\text{ kg}}=\boxed{8.93\times10^{-4}\text{ m/s}^2}$.

(c) It will arrive at time $t$ where $d=\frac{1}{2}at^2$

or $t=\sqrt{\frac{2d}{a}}=\sqrt{\frac{2\left(3.84\times10^{8}\text{ m}\right)}{\left(8.93\times10^{-4}\text{ m/s}^2\right)}}=9.27\times10^{5}\text{ s}=\boxed{10.7\text{ days}}$.

**P34.29** $I=\frac{\mathcal{P}}{\pi r^2}=\frac{E_{\max}^2}{2\mu_0 c}$

(a) $E_{\max}=\sqrt{\frac{\mathcal{P}(2\mu_0 c)}{\pi r^2}}=\boxed{1.90\text{ kN/C}}$

(b) $\frac{15\times10^{-3}\text{ J/s}}{3.00\times10^{8}\text{ m/s}}(1.00\text{ m})=\boxed{50.0\text{ pJ}}$

(c) $p=\frac{U}{c}=\frac{5\times10^{-11}}{3.00\times10^{8}}=\boxed{1.67\times10^{-19}\text{ kg}\cdot\text{m/s}}$

**P34.30** (a) If $\mathcal{P}_S$ is the total power radiated by the Sun, and $r_E$ and $r_M$ are the radii of the orbits of the planets Earth and Mars, then the intensities of the solar radiation at these planets are:

$$I_E = \frac{\mathcal{P}_S}{4\pi r_E^2}$$

and
$$I_M = \frac{\mathcal{P}_S}{4\pi r_M^2}.$$

Thus,
$$I_M = I_E\left(\frac{r_E}{r_M}\right)^2 = \left(1\,340\ \text{W/m}^2\right)\left(\frac{1.496\times10^{11}\ \text{m}}{2.28\times10^{11}\ \text{m}}\right)^2 = \boxed{577\ \text{W/m}^2}.$$

(b) Mars intercepts the power falling on its circular face:

$$\mathcal{P}_M = I_M\left(\pi R_M^2\right) = \left(577\ \text{W/m}^2\right)\left[\pi\left(3.37\times10^6\ \text{m}\right)^2\right] = \boxed{2.06\times10^{16}\ \text{W}}.$$

(c) If Mars behaves as a perfect absorber, it feels pressure $P = \frac{S_M}{c} = \frac{I_M}{c}$

and force
$$F = PA = \frac{I_M}{c}\left(\pi R_M^2\right) = \frac{\mathcal{P}_M}{c} = \frac{2.06\times10^{16}\ \text{W}}{3.00\times10^8\ \text{m/s}} = \boxed{6.87\times10^7\ \text{N}}.$$

(d) The attractive gravitational force exerted on Mars by the Sun is

$$F_g = \frac{GM_SM_M}{r_M^2} = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.991\times10^{30}\ \text{kg}\right)\left(6.42\times10^{23}\ \text{kg}\right)}{\left(2.28\times10^{11}\ \text{m}\right)^2} = 1.64\times10^{21}\ \text{N}$$

which is $\boxed{\sim10^{13}\ \text{times stronger}}$ than the repulsive force of part (c).

**P34.31** (a) The total energy absorbed by the surface is

$$U = \left(\frac{1}{2}I\right)At = \left[\frac{1}{2}\left(750\ \text{W/m}^2\right)\right]\left(0.500\times1.00\ \text{m}^2\right)(60.0\ \text{s}) = \boxed{11.3\ \text{kJ}}.$$

(b) The total energy incident on the surface in this time is $2U = 22.5\ \text{kJ}$, with $U = 11.3\ \text{kJ}$ being absorbed and $U = 11.3\ \text{kJ}$ being reflected. The total momentum transferred to the surface is

$$p = (\text{momentum from absorption}) + (\text{momentum from reflection})$$

$$p = \left(\frac{U}{c}\right) + \left(\frac{2U}{c}\right) = \frac{3U}{c} = \frac{3\left(11.3\times10^3\ \text{J}\right)}{3.00\times10^8\ \text{m/s}} = \boxed{1.13\times10^{-4}\ \text{kg}\cdot\text{m/s}}$$

***P34.32** The radiation pressure on the disk is $P=\frac{S}{c}=\frac{I}{c}=\frac{F}{A}=\frac{F}{\pi r^2}$.

Then $F=\frac{\pi r^2 I}{c}$.

Take torques about the hinge: $\sum \tau = 0$

$$H_x(0)+H_y(0)-mgr\sin\theta+\frac{\pi r^2 Ir}{c}=0$$

$$\theta=\sin^{-1}\frac{\pi r^2 I}{mgc}=\sin^{-1}\frac{\pi(0.4\text{ m})^2 10^7\text{ W s}^2\text{ s}}{(0.024\text{ kg})\text{ m}^2(9.8\text{ m})\left(3\times 10^8\text{ m}\right)}\left(\frac{1\text{ kg m}^2}{1\text{ W s}^3}\right)$$

$$=\sin^{-1}0.0712=\boxed{4.09°}$$

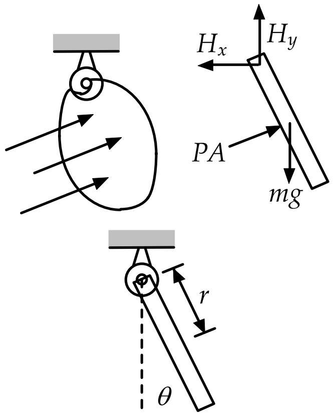


**FIG. P34.32**

## Section 34.5 **Production of Electromagnetic Waves by an Antenna**

**P34.33** $\lambda=\frac{c}{f}=536\text{ m}$ so $h=\frac{\lambda}{4}=\boxed{134\text{ m}}$

$\lambda=\frac{c}{f}=188\text{ m}$ so $h=\frac{\lambda}{4}=\boxed{46.9\text{ m}}$

**P34.34** $\mathcal{P}=\frac{(\Delta V)^2}{R}$ or $\mathcal{P}\propto(\Delta V)^2$

$\Delta V=(-)E_y\cdot\Delta y=E_y\cdot\ell\cos\theta$

$\Delta V\propto\cos\theta$ so $\mathcal{P}\propto\cos^2\theta$

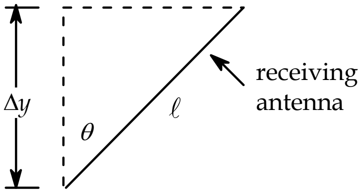


**FIG. P34.34**

(a) $\theta=15.0°$: $\mathcal{P}=\mathcal{P}_{\text{max}}\cos^2(15.0°)=0.933\mathcal{P}_{\text{max}}=\boxed{93.3\%}$

(b) $\theta=45.0°$: $\mathcal{P}=\mathcal{P}_{\text{max}}\cos^2(45.0°)=0.500\mathcal{P}_{\text{max}}=\boxed{50.0\%}$

(c) $\theta=90.0°$: $\mathcal{P}=\mathcal{P}_{\text{max}}\cos^2(90.0°)=\boxed{0}$

**P34.35** (a) Constructive interference occurs when $d\cos\theta = n\lambda$ for some integer $n$.

$$\cos\theta = n\frac{\lambda}{d} = n\left(\frac{\lambda}{\lambda/2}\right) = 2n$$

$$n = 0,\ \pm 1,\ \pm 2,\ \ldots$$

$$\therefore \boxed{\text{strong signal @ } \theta = \cos^{-1} 0 = 90°,\ 270°}$$

(b) Destructive interference occurs when

$$d\cos\theta = \left(\frac{2n+1}{2}\right)\lambda: \qquad \cos\theta = 2n+1$$

$$\therefore \boxed{\text{weak signal @ } \theta = \cos^{-1}(\pm 1) = 0°,\ 180°}$$

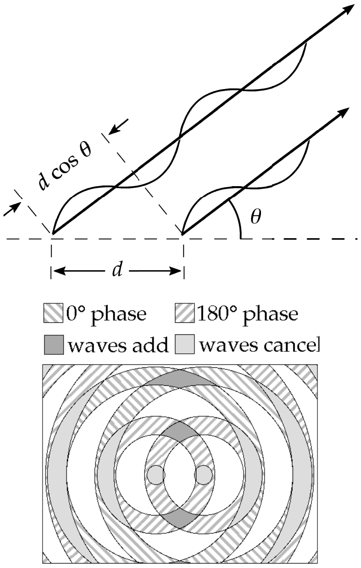


**FIG. P34.35**

**P34.36** For the proton, $\sum F = ma$ yields $qvB\sin 90.0° = \dfrac{mv^2}{R}$.

The period of the proton's circular motion is therefore: $T = \dfrac{2\pi R}{v} = \dfrac{2\pi m}{qB}$.

The frequency of the proton's motion is $f = \dfrac{1}{T}$.

The charge will radiate electromagnetic waves at this frequency, with $\lambda = \dfrac{c}{f} = cT = \boxed{\dfrac{2\pi mc}{qB}}$.

***P34.37** (a) The magnetic field $\mathbf{B} = \frac{1}{2}\mu_0 J_{\text{max}}\cos(kx - \omega t)\hat{\mathbf{k}}$ applies for $x > 0$, since it describes a wave moving in the $\hat{\mathbf{i}}$ direction. The electric field direction must satisfy $\mathbf{S} = \frac{1}{\mu_0}\mathbf{E}\times\mathbf{B}$ as $\hat{\mathbf{i}} = \hat{\mathbf{j}}\times\hat{\mathbf{k}}$ so the direction of the electric field is $\hat{\mathbf{j}}$ when the cosine is positive. For its magnitude we have $E = cB$, so altogether we have $\boxed{\mathbf{E} = \frac{1}{2}\mu_0 c J_{\text{max}}\cos(kx - \omega t)\hat{\mathbf{j}}}$.

(b)

$$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E}\times\mathbf{B} = \frac{1}{\mu_0}\frac{1}{4}\mu_0^2 c J_{\text{max}}^2\cos^2(kx - \omega t)\hat{\mathbf{i}}$$

$$\boxed{\mathbf{S} = \frac{1}{4}\mu_0 c J_{\text{max}}^2\cos^2(kx - \omega t)\hat{\mathbf{i}}}$$

(c) The intensity is the magnitude of the Poynting vector averaged over one or more cycles. The average of the cosine-squared function is $\frac{1}{2}$, so $\boxed{I = \frac{1}{8}\mu_0 c J_{\text{max}}^2}$.

(d)

$$J_{\text{max}} = \sqrt{\frac{8I}{\mu_0 c}} = \sqrt{\frac{8\left(570\ \text{W/m}^2\right)}{4\pi\times 10^{-7}\,(\text{Tm/A})3\times 10^8\ \text{m/s}}} = \boxed{3.48\ \text{A/m}}$$

## Section 34.6 **The Spectrum of Electromagnetic Waves**

**P34.38** From the electromagnetic spectrum chart and accompanying text discussion, the following identifications are made:

| Frequency, $f$ | Wavelength, $\lambda = \frac{c}{f}$ | Classification |
|---|---|---|
| $2\text{ Hz} = 2\times10^{0}\text{ Hz}$ | 150 Mm | Radio |
| $2\text{ kHz} = 2\times10^{3}\text{ Hz}$ | 150 km | Radio |
| $2\text{ MHz} = 2\times10^{6}\text{ Hz}$ | 150 m | Radio |
| $2\text{ GHz} = 2\times10^{9}\text{ Hz}$ | 15 cm | Microwave |
| $2\text{ THz} = 2\times10^{12}\text{ Hz}$ | 150 $\mu$m | Infrared |
| $2\text{ PHz} = 2\times10^{15}\text{ Hz}$ | 150 nm | Ultraviolet |
| $2\text{ EHz} = 2\times10^{18}\text{ Hz}$ | 150 pm | X-ray |
| $2\text{ ZHz} = 2\times10^{21}\text{ Hz}$ | 150 fm | Gamma ray |
| $2\text{ YHz} = 2\times10^{24}\text{ Hz}$ | 150 am | Gamma ray |

| Wavelength, $\lambda$ | Frequency, $f = \frac{c}{\lambda}$ | Classification |
|---|---|---|
| $2\text{ km} = 2\times10^{3}\text{ m}$ | $1.5\times10^{5}\text{ Hz}$ | Radio |
| $2\text{ m} = 2\times10^{0}\text{ m}$ | $1.5\times10^{8}\text{ Hz}$ | Radio |
| $2\text{ mm} = 2\times10^{-3}\text{ m}$ | $1.5\times10^{11}\text{ Hz}$ | Microwave |
| $2\ \mu\text{m} = 2\times10^{-6}\text{ m}$ | $1.5\times10^{14}\text{ Hz}$ | Infrared |
| $2\text{ nm} = 2\times10^{-9}\text{ m}$ | $1.5\times10^{17}\text{ Hz}$ | Ultraviolet/X-ray |
| $2\text{ pm} = 2\times10^{-12}\text{ m}$ | $1.5\times10^{20}\text{ Hz}$ | X-ray/Gamma ray |
| $2\text{ fm} = 2\times10^{-15}\text{ m}$ | $1.5\times10^{23}\text{ Hz}$ | Gamma ray |
| $2\text{ am} = 2\times10^{-18}\text{ m}$ | $1.5\times10^{26}\text{ Hz}$ | Gamma ray |

**P34.39** $$f = \frac{c}{\lambda} = \frac{3.00\times10^{8}\text{ m/s}}{5.50\times10^{-7}\text{ m}} = \boxed{5.45\times10^{14}\text{ Hz}}$$

**P34.40** (a) $$f = \frac{c}{\lambda} = \frac{3\times10^{8}\text{ m/s}}{1.7\text{ m}}\ \boxed{\sim10^{8}\text{ Hz}}\ \boxed{\text{radio wave}}$$

(b) 1 000 pages, 500 sheets, is about 3 cm thick so one sheet is about $6\times10^{-5}$ m thick.

$$f = \frac{3.00\times10^{8}\text{ m/s}}{6\times10^{-5}\text{ m}}\ \boxed{\sim10^{13}\text{ Hz}}\ \boxed{\text{infrared}}$$

**P34.41** (a) $f\lambda = c$ gives $(5.00\times10^{19}\text{ Hz})\lambda = 3.00\times10^{8}\text{ m/s}$: $\boxed{\lambda = 6.00\times10^{-12}\text{ m} = 6.00\text{ pm}}$

(b) $f\lambda = c$ gives $(4.00\times10^{9}\text{ Hz})\lambda = 3.00\times10^{8}\text{ m/s}$: $\boxed{\lambda = 0.075\text{ m} = 7.50\text{ cm}}$

**P34.42** (a) $\lambda = \frac{c}{f} = \frac{3.00\times 10^8 \text{ m/s}}{1\,150\times 10^3 \text{ s}^{-1}} = 261 \text{ m}$ so $\frac{180 \text{ m}}{261 \text{ m}} = \boxed{0.690 \text{ wavelengths}}$

(b) $\lambda = \frac{c}{f} = \frac{3.00\times 10^8 \text{ m/s}}{98.1\times 10^6 \text{ s}^{-1}} = 3.06 \text{ m}$ so $\frac{180 \text{ m}}{3.06 \text{ m}} = \boxed{58.9 \text{ wavelengths}}$

**P34.43** Time to reach object $= \frac{1}{2}(\text{total time of flight}) = \frac{1}{2}\left(4.00\times 10^{-4} \text{ s}\right) = 2.00\times 10^{-4} \text{ s}$.

Thus, $d = vt = \left(3.00\times 10^8 \text{ m/s}\right)\left(2.00\times 10^{-4} \text{ s}\right) = 6.00\times 10^4 \text{ m} = \boxed{60.0 \text{ km}}$.

**P34.44** The time for the radio signal to travel 100 km is: $\Delta t_r = \frac{100\times 10^3 \text{ m}}{3.00\times 10^8 \text{ m/s}} = 3.33\times 10^{-4} \text{ s}$.

The sound wave travels 3.00 m across the room in: $\Delta t_s = \frac{3.00 \text{ m}}{343 \text{ m/s}} = 8.75\times 10^{-3} \text{ s}$.

Therefore, $\boxed{\text{listeners 100 km away}}$ will receive the news before the people in the news room by a total time difference of

$\Delta t = 8.75\times 10^{-3} \text{ s} - 3.33\times 10^{-4} \text{ s} = 8.41\times 10^{-3} \text{ s}$.

**P34.45** The wavelength of an ELF wave of frequency 75.0 Hz is $\lambda = \frac{c}{f} = \frac{3.00\times 10^8 \text{ m/s}}{75.0 \text{ Hz}} = 4.00\times 10^6 \text{ m}$.

The length of a quarter-wavelength antenna would be $L = 1.00\times 10^6 \text{ m} = \boxed{1.00\times 10^3 \text{ km}}$

or $L = (1\,000 \text{ km})\left(\frac{0.621 \text{ mi}}{1.00 \text{ km}}\right) = \boxed{621 \text{ mi}}$.

Thus, while the project may be theoretically possible, it is not very practical.

**P34.46** (a) For the AM band, $\lambda_{\max} = \frac{c}{f_{\min}} = \frac{3.00\times 10^8 \text{ m/s}}{540\times 10^3 \text{ Hz}} = \boxed{556 \text{ m}}$

$\lambda_{\min} = \frac{c}{f_{\max}} = \frac{3.00\times 10^8 \text{ m/s}}{1\,600\times 10^3 \text{ Hz}} = \boxed{187 \text{ m}}$.

(b) For the FM band, $\lambda_{\max} = \frac{c}{f_{\min}} = \frac{3.00\times 10^8 \text{ m/s}}{88.0\times 10^6 \text{ Hz}} = \boxed{3.41 \text{ m}}$

$\lambda_{\min} = \frac{c}{f_{\max}} = \frac{3.00\times 10^8 \text{ m/s}}{108\times 10^6 \text{ Hz}} = \boxed{2.78 \text{ m}}$.

## Additional Problems

**P34.47** (a) $\mathscr{P} = SA$: $\qquad \mathscr{P} = \left(1\,340\ \text{W/m}^2\right)\left[4\pi\left(1.496 \times 10^{11}\ \text{m}\right)^2\right] = \boxed{3.77 \times 10^{26}\ \text{W}}$

(b) $S = \dfrac{cB_{\max}^2}{2\mu_0}$ so $B_{\max} = \sqrt{\dfrac{2\mu_0 S}{c}} = \sqrt{\dfrac{2\left(4\pi \times 10^{-7}\ \text{N/A}^2\right)\left(1\,340\ \text{W/m}^2\right)}{3.00 \times 10^8\ \text{m/s}}} = \boxed{3.35\ \mu\text{T}}$

$S = \dfrac{E_{\max}^2}{2\mu_0 c}$ so $E_{\max} = \sqrt{2\mu_0 cS} = \sqrt{2\left(4\pi \times 10^{-7}\right)\left(3.00 \times 10^8\right)(1\,340)} = \boxed{1.01\ \text{kV/m}}$

**P34.48** Suppose you cover a 1.7 m-by-0.3 m section of beach blanket. Suppose the elevation angle of the Sun is 60°. Then the target area you fill in the Sun's field of view is

$$(1.7\ \text{m})(0.3\ \text{m})\cos 30° = 0.4\ \text{m}^2\,.$$

Now $I = \dfrac{\mathscr{P}}{A} = \dfrac{U}{At}$ $\qquad U = IAt = \left(1\,340\ \text{W/m}^2\right)\left[(0.6)(0.5)\left(0.4\ \text{m}^2\right)\right](3\,600\ \text{s})\boxed{\sim 10^6\ \text{J}}$.

**P34.49** (a) $\varepsilon = -\dfrac{d\Phi_B}{dt} = -\dfrac{d}{dt}(BA\cos\theta)$ $\qquad \varepsilon = -A\dfrac{d}{dt}\left(B_{\max}\cos\omega t\cos\theta\right) = AB_{\max}\omega\left(\sin\omega t\cos\theta\right)$

$\varepsilon(t) = 2\pi f B_{\max} A\sin 2\pi f t\cos\theta$ $\qquad \varepsilon(t) = 2\pi^2 r^2 f B_{\max}\cos\theta\sin 2\pi f t$

Thus, $\qquad \boxed{\varepsilon_{\max} = 2\pi^2 r^2 f\, B_{\max}\cos\theta}$

where $\theta$ is the angle between the magnetic field and the normal to the loop.

(b) If **E** is vertical, **B** is horizontal, so $\boxed{\text{the plane of the loop should be vertical}}$

and $\boxed{\text{the plane should contain the line of sight of the transmitter}}$.

**P34.50** (a) $F_{\text{grav}} = \dfrac{GM_S m}{R^2} = \left(\dfrac{GM_S}{R^2}\right)\left[\rho\left(\dfrac{4}{3}\pi r^3\right)\right]$

where $M_S =$ mass of Sun, $r =$ radius of particle and $R =$ distance from Sun to particle.

Since $\qquad F_{rad} = \dfrac{S\pi r^2}{c}$,

$$\frac{F_{rad}}{F_{\text{grav}}} = \left(\frac{1}{r}\right)\left(\frac{3SR^2}{4cGM_S\rho}\right) \propto \frac{1}{r}\,.$$

(b) From the result found in part (a), when $F_{\text{grav}} = F_{rad}$,

we have $r = \dfrac{3SR^2}{4cGM_S\rho}$

$$r = \frac{3\left(214\ \text{W/m}^2\right)\left(3.75 \times 10^{11}\ \text{m}\right)^2}{4\left(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.991 \times 10^{30}\ \text{kg}\right)\left(1\,500\ \text{kg/m}^3\right)\left(3.00 \times 10^8\ \text{m/s}\right)}$$

$$= \boxed{3.78 \times 10^{-7}\ \text{m}}$$

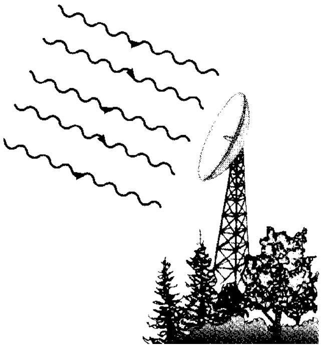


FIG. P34.51

**P34.51** (a) $B_{\max} = \frac{E_{\max}}{c} = \boxed{6.67 \times 10^{-16}\ \text{T}}$

(b) $S_{\text{av}} = \frac{E_{\max}^2}{2\mu_0 c} = \boxed{5.31 \times 10^{-17}\ \text{W/m}^2}$

(c) $\mathscr{P} = S_{\text{av}} A = \boxed{1.67 \times 10^{-14}\ \text{W}}$

(d) $F = PA = \left(\frac{S_{\text{av}}}{c}\right)A = \boxed{5.56 \times 10^{-23}\ \text{N}}$ (≅ the weight of 3 000 H atoms!)

**P34.52** (a) The power incident on the mirror is: $\mathscr{P}_I = IA = \left(1\,340\ \text{W/m}^2\right)\left[\pi(100\ \text{m})^2\right] = 4.21 \times 10^7\ \text{W}$.

The power reflected through the atmosphere is $\mathscr{P}_R = 0.746\left(4.21 \times 10^7\ \text{W}\right) = \boxed{3.14 \times 10^7\ \text{W}}$.

(b) $S = \frac{\mathscr{P}_R}{A} = \frac{3.14 \times 10^7\ \text{W}}{\pi\left(4.00 \times 10^3\ \text{m}\right)^2} = \boxed{0.625\ \text{W/m}^2}$

(c) Noon sunshine in Saint Petersburg produces this power-per-area on a horizontal surface:

$\frac{\mathscr{P}_N}{A} = 0.746\left(1\,340\ \text{W/m}^2\right)\sin 7.00° = 122\ \text{W/m}^2$.

The radiation intensity received from the mirror is

$\left(\frac{0.625\ \text{W/m}^2}{122\ \text{W/m}^2}\right)100\% = \boxed{0.513\%}$ of that from the noon Sun in January.

**P34.53** $u = \frac{1}{2}\epsilon_0 E_{\max}^2$ $\qquad E_{\max} = \sqrt{\frac{2u}{\epsilon_0}} = \boxed{95.1\ \text{mV/m}}$

**P34.54** The area over which we model the antenna as radiating is the lateral surface of a cylinder,

$$A = 2\pi r\ell = 2\pi\left(4.00 \times 10^{-2}\ \text{m}\right)(0.100\ \text{m}) = 2.51 \times 10^{-2}\ \text{m}^2.$$

(a) The intensity is then: $S = \frac{\mathscr{P}}{A} = \frac{0.600\ \text{W}}{2.51 \times 10^{-2}\ \text{m}^2} = \boxed{23.9\ \text{W/m}^2}$.

(b) The standard is:

$0.570\ \text{mW/cm}^2 = 0.570\left(\text{mW/cm}^2\right)\left(\frac{1.00 \times 10^{-3}\ \text{W}}{1.00\ \text{mW}}\right)\left(\frac{1.00 \times 10^4\ \text{cm}^2}{1.00\ \text{m}^2}\right) = 5.70\ \text{W/m}^2$.

While it is on, the telephone is over the standard by $\frac{23.9\ \text{W/m}^2}{5.70\ \text{W/m}^2} = \boxed{4.19\ \text{times}}$.

**P34.55** (a) $$B_{\max}=\frac{E_{\max}}{c}=\frac{175\ \text{V/m}}{3.00\times10^{8}\ \text{m/s}}=\boxed{5.83\times10^{-7}\ \text{T}}$$

$$k=\frac{2\pi}{\lambda}=\frac{2\pi}{0.015\,0\ \text{m}}=\boxed{419\ \text{rad/m}}\qquad \omega=kc=\boxed{1.26\times10^{11}\ \text{rad/s}}$$

Since **S** is along $x$, and **E** is along $y$, **B** must be in $\boxed{\text{the } z \text{ direction}}$. (That is $\mathbf{S}\propto\mathbf{E}\times\mathbf{B}$.)

(b) $$S_{av}=\frac{E_{\max}B_{\max}}{2\mu_0}=40.6\ \text{W/m}^2\qquad \mathbf{S}_{av}=\boxed{\left(40.6\ \text{W/m}^2\right)\hat{\mathbf{i}}}$$

(c) $$P_r=\frac{2S}{c}=\boxed{2.71\times10^{-7}\ \text{N/m}^2}$$

(d) $$a=\frac{\sum F}{m}=\frac{PA}{m}=\frac{\left(2.71\times10^{-7}\ \text{N/m}^2\right)\left(0.750\ \text{m}^2\right)}{0.500\ \text{kg}}=4.06\times10^{-7}\ \text{m/s}^2\qquad \mathbf{a}=\boxed{\left(406\ \text{nm/s}^2\right)\hat{\mathbf{i}}}$$

**P34.56** Of the intensity $S=1\,340\ \text{W/m}^2$

the 38.0% that is reflected exerts a pressure $P_1=\frac{2S_r}{c}=\frac{2(0.380)S}{c}$.

The absorbed light exerts pressure $P_2=\frac{S_a}{c}=\frac{0.620S}{c}$.

Altogether the pressure at the subsolar point on Earth is

(a) $$P_{\text{total}}=P_1+P_2=\frac{1.38S}{c}=\frac{1.38\left(1\,340\ \text{W/m}^2\right)}{3.00\times10^{8}\ \text{m/s}}=\boxed{6.16\times10^{-6}\ \text{Pa}}$$

(b) $$\frac{P_a}{P_{\text{total}}}=\frac{1.01\times10^{5}\ \text{N/m}^2}{6.16\times10^{-6}\ \text{N/m}^2}=\boxed{1.64\times10^{10}\ \text{times smaller than atmospheric pressure}}$$

**P34.57** (a) $$P=\frac{F}{A}=\frac{I}{c}\qquad F=\frac{IA}{c}=\frac{\mathcal{P}}{c}=\frac{100\ \text{J/s}}{3.00\times10^{8}\ \text{m/s}}=3.33\times10^{-7}\ \text{N}=(110\ \text{kg})a$$

$$a=3.03\times10^{-9}\ \text{m/s}^2\ \text{and}\qquad x=\frac{1}{2}at^2$$

$$t=\sqrt{\frac{2x}{a}}=8.12\times10^{4}\ \text{s}=\boxed{22.6\ \text{h}}$$

(b) $$0=(107\ \text{kg})v-(3.00\ \text{kg})(12.0\ \text{m/s}-v)=(107\ \text{kg})v-36.0\ \text{kg}\cdot\text{m/s}+(3.00\ \text{kg})v$$

$$v=\frac{36.0}{110}=0.327\ \text{m/s}\qquad t=\boxed{30.6\ \text{s}}$$

**P34.58** The mirror intercepts power $\mathcal{P} = I_1A_1 = \left(1.00\times10^3 \text{ W/m}^2\right)\left[\pi(0.500 \text{ m})^2\right] = 785 \text{ W}$.

In the image,

(a) $I_2 = \dfrac{\mathcal{P}}{A_2}$: $\quad I_2 = \dfrac{785 \text{ W}}{\pi(0.020\,0 \text{ m})^2} = \boxed{625 \text{ kW/m}^2}$

(b) $I_2 = \dfrac{E_{\max}^2}{2\mu_0 c}$ so $\quad E_{\max} = \sqrt{2\mu_0 c I_2} = \sqrt{2\left(4\pi\times10^{-7}\right)\left(3.00\times10^8\right)\left(6.25\times10^5\right)} = \boxed{21.7 \text{ kN/C}}$

$$B_{\max} = \frac{E_{\max}}{c} = \boxed{72.4\ \mu\text{T}}$$

(c) $0.400\mathcal{P}\Delta t = mc\Delta T$

$$0.400(785 \text{ W})\Delta t = (1.00 \text{ kg})(4\,186 \text{ J/kg}\cdot°\text{C})(100°\text{C} - 20.0°\text{C})$$

$$\Delta t = \frac{3.35\times10^5 \text{ J}}{314 \text{ W}} = 1.07\times10^3 \text{ s} = \boxed{17.8 \text{ min}}$$

**P34.59** Think of light going up and being absorbed by the bead which presents a face area $\pi r_b^2$.

The light pressure is $P = \dfrac{S}{c} = \dfrac{I}{c}$.

(a) $F_\ell = \dfrac{I\pi r_b^2}{c} = mg = \rho\dfrac{4}{3}\pi r_b^3 g$ and $\quad I = \dfrac{4\rho g c}{3}\left(\dfrac{3m}{4\pi\rho}\right)^{1/3} = \boxed{8.32\times10^7 \text{ W/m}^2}$

(b) $\mathcal{P} = IA = \left(8.32\times10^7 \text{ W/m}^2\right)\pi\left(2.00\times10^{-3} \text{ m}\right)^2 = \boxed{1.05 \text{ kW}}$

**P34.60** Think of light going up and being absorbed by the bead, which presents face area $\pi r_b^2$.

If we take the bead to be perfectly absorbing, the light pressure is $P = \dfrac{S_{\text{av}}}{c} = \dfrac{I}{c} = \dfrac{F_\ell}{A}$.

(a) $F_\ell = F_g$

so $\quad I = \dfrac{F_\ell c}{A} = \dfrac{F_g c}{A} = \dfrac{mgc}{\pi r_b^2}$.

From the definition of density, $\quad \rho = \dfrac{m}{V} = \dfrac{m}{(4/3)\pi r_b^3}$

so $\quad \dfrac{1}{r_b} = \left(\dfrac{(4/3)\pi\rho}{m}\right)^{1/3}$.

Substituting for $r_b$, $\quad I = \dfrac{mgc}{\pi}\left(\dfrac{4\pi\rho}{3m}\right)^{2/3} = gc\left(\dfrac{4\rho}{3}\right)^{2/3}\left(\dfrac{m}{\pi}\right)^{1/3} = \boxed{\dfrac{4\rho gc}{3}\left(\dfrac{3m}{4\pi\rho}\right)^{1/3}}$.

(b) $\mathcal{P} = IA$ $\quad \mathcal{P} = \boxed{\dfrac{4\pi r^2\rho gc}{3}\left(\dfrac{3m}{4\pi\rho}\right)^{1/3}}$

**P34.61** (a) $\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{20.0 \times 10^9 \text{ s}^{-1}} = \boxed{1.50 \text{ cm}}$

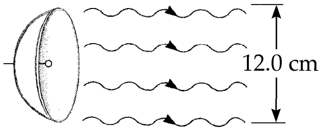


**FIG. P34.61**

(b) $U = \mathscr{P}(\Delta t) = \left(25.0 \times 10^3 \text{ J/s}\right)\left(1.00 \times 10^{-9} \text{ s}\right) = 25.0 \times 10^{-6} \text{ J}$

$= \boxed{25.0\ \mu\text{J}}$

(c) $u_{\text{av}} = \frac{U}{V} = \frac{U}{\left(\pi r^2\right)\ell} = \frac{U}{\left(\pi r^2\right)c(\Delta t)} = \frac{25.0 \times 10^{-6} \text{ J}}{\pi(0.060\,0 \text{ m})^2\left(3.00 \times 10^8 \text{ m/s}\right)\left(1.00 \times 10^{-9} \text{ s}\right)}$

$u_{\text{av}} = 7.37 \times 10^{-3} \text{ J/m}^3 = \boxed{7.37 \text{ mJ/m}^3}$

(d) $E_{\max} = \sqrt{\frac{2u_{\text{av}}}{\epsilon_0}} = \sqrt{\frac{2\left(7.37 \times 10^{-3} \text{ J/m}^3\right)}{8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2}} = 4.08 \times 10^4 \text{ V/m} = \boxed{40.8 \text{ kV/m}}$

$B_{\max} = \frac{E_{\max}}{c} = \frac{4.08 \times 10^4 \text{ V/m}}{3.00 \times 10^8 \text{ m/s}} = 1.36 \times 10^{-4} \text{ T} = \boxed{136\ \mu\text{T}}$

(e) $F = PA = \left(\frac{S}{c}\right)A = u_{\text{av}}A = \left(7.37 \times 10^{-3} \text{ J/m}^3\right)\pi(0.060\,0 \text{ m})^2 = 8.33 \times 10^{-5} \text{ N} = \boxed{83.3\ \mu\text{N}}$

**P34.62** (a) On the right side of the equation, $\frac{\text{C}^2\left(\text{m/s}^2\right)^2}{\left(\text{C}^2/\text{N}\cdot\text{m}^2\right)(\text{m/s})^3} = \frac{\text{N}\cdot\text{m}^2\cdot\text{C}^2\cdot\text{m}^2\cdot\text{s}^3}{\text{C}^2\cdot\text{s}^4\cdot\text{m}^3} = \frac{\text{N}\cdot\text{m}}{\text{s}} = \frac{\text{J}}{\text{s}} = \text{W}.$

(b) $F = ma = qE$ or $a = \frac{qE}{m} = \frac{\left(1.60 \times 10^{-19} \text{ C}\right)(100 \text{ N/C})}{9.11 \times 10^{-31} \text{ kg}} = \boxed{1.76 \times 10^{13} \text{ m/s}^2}.$

The radiated power is then: $\mathscr{P} = \frac{q^2a^2}{6\pi \epsilon_0 c^3} = \frac{\left(1.60 \times 10^{-19}\right)^2\left(1.76 \times 10^{13}\right)^2}{6\pi\left(8.85 \times 10^{-12}\right)\left(3.00 \times 10^8\right)^3} = \boxed{1.75 \times 10^{-27} \text{ W}}.$

(c) $F = ma_c = m\left(\frac{v^2}{r}\right) = qvB$ so $v = \frac{qBr}{m}.$

The proton accelerates at $a = \frac{v^2}{r} = \frac{q^2B^2r}{m^2} = \frac{\left(1.60 \times 10^{-19}\right)^2(0.350)^2(0.500)}{\left(1.67 \times 10^{-27}\right)^2} = 5.62 \times 10^{14} \text{ m/s}^2.$

The proton then radiates $P = \frac{q^2a^2}{6\pi \epsilon_0 c^3} = \frac{\left(1.60 \times 10^{-19}\right)^2\left(5.62 \times 10^{14}\right)^2}{6\pi\left(8.85 \times 10^{-12}\right)\left(3.00 \times 10^8\right)^3} = \boxed{1.80 \times 10^{-24} \text{ W}}.$

**P34.63** $P = \frac{S}{c} = \frac{\text{Power}}{Ac} = \frac{\mathscr{P}}{2\pi r\ell c} = \frac{60.0 \text{ W}}{2\pi(0.050\,0 \text{ m})(1.00 \text{ m})\left(3.00 \times 10^8 \text{ m/s}\right)} = \boxed{6.37 \times 10^{-7} \text{ Pa}}$

**P34.64** $F=PA=\frac{SA}{c}=\frac{(\mathcal{P}/A)A}{c}=\frac{\mathcal{P}}{c}$, $\tau=F\left(\frac{\ell}{2}\right)=\frac{\mathcal{P}\ell}{2c}$, and $\tau=\kappa\theta$.

Therefore, $\theta=\frac{\mathcal{P}\ell}{2c\kappa}=\frac{\left(3.00\times10^{-3}\right)(0.060\,0)}{2\left(3.00\times10^{8}\right)\left(1.00\times10^{-11}\right)}=\boxed{3.00\times10^{-2}\text{ deg}}$.

**P34.65** The light intensity is $I=S_{\text{av}}=\frac{E^2}{2\mu_0c}$.

The light pressure is $P=\frac{S}{c}=\frac{E^2}{2\mu_0c^2}=\frac{1}{2}\in_0E^2$.

For the asteroid, $PA=ma$ and $a=\boxed{\frac{\in_0E^2A}{2m}}$.

**P34.66** $f=90.0\text{ MHz}$, $E_{\max}=2.00\times10^{-3}\text{ V/m}=200\text{ mV/m}$

(a) $\lambda=\frac{c}{f}=\boxed{3.33\text{ m}}$

$T=\frac{1}{f}=1.11\times10^{-8}\text{ s}=\boxed{11.1\text{ ns}}$

$B_{\max}=\frac{E_{\max}}{c}=6.67\times10^{-12}\text{ T}=\boxed{6.67\text{ pT}}$

(b) $\boxed{\mathbf{E}=(2.00\text{ mV/m})\cos2\pi\left(\frac{x}{3.33\text{ m}}-\frac{t}{11.1\text{ ns}}\right)\hat{\mathbf{j}}}$ $\boxed{\mathbf{B}=(6.67\text{ pT})\hat{\mathbf{k}}\cos2\pi\left(\frac{x}{3.33\text{ m}}-\frac{t}{11.1\text{ ns}}\right)}$

(c) $I=\frac{E_{\max}^2}{2\mu_0c}=\frac{\left(2.00\times10^{-3}\right)^2}{2\left(4\pi\times10^{-7}\right)\left(3.00\times10^{8}\right)}=\boxed{5.31\times10^{-9}\text{ W/m}^2}$

(d) $I=cu_{\text{av}}$ so $u_{\text{av}}=\boxed{1.77\times10^{-17}\text{ J/m}^3}$

(e) $P=\frac{2I}{c}=\frac{(2)\left(5.31\times10^{-9}\right)}{3.00\times10^{8}}=\boxed{3.54\times10^{-17}\text{ Pa}}$

***P34.67** (a) $m = \rho V = \rho \frac{1}{2}\frac{4}{3}\pi r^3$

$$r = \left(\frac{6m}{\rho 4\pi}\right)^{1/3} = \left(\frac{6(8.7\text{ kg})}{(990\text{ kg/m}^3)4\pi}\right)^{1/3} = \boxed{0.161\text{ m}}$$

(b) $A = \frac{1}{2}4\pi r^2 = 2\pi(0.161\text{ m})^2 = \boxed{0.163\text{ m}^2}$

(c) $I = e\sigma T^4 = 0.970(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)(304\text{ K})^4 = \boxed{470\text{ W/m}^2}$

(d) $\mathscr{P} = IA = (470\text{ W/m}^2)0.163\text{ m}^2 = \boxed{76.8\text{ W}}$

(e) $I = \frac{E_{\text{max}}^2}{2\mu_0 c}$

$$E_{\text{max}} = (2\mu_0 cI)^{1/2} = \left[(8\pi\times10^{-7}\text{ Tm/A})(3\times10^8\text{ m/s})(470\text{ W/m}^2)\right]^{1/2} = \boxed{595\text{ N/C}}$$

(f) $E_{\text{max}} = cB_{\text{max}}$

$$B_{\text{max}} = \frac{595\text{ N/C}}{3\times10^8\text{ m/s}} = \boxed{1.98\ \mu\text{T}}$$

(g) The sleeping cats are uncharged and nonmagnetic. They carry no macroscopic current. They are a source of infrared radiation. They glow not by visible-light emission but by infrared emission.

(h) Each kitten has radius $r_k = \left(\frac{6(0.8)}{990\times4\pi}\right)^{1/3} = 0.072\,8\text{ m}$ and radiating area $2\pi(0.072\,8\text{ m})^2 = 0.033\,3\text{ m}^2$. Eliza has area $2\pi\left(\frac{6(5.5)}{990\times4\pi}\right)^{2/3} = 0.120\text{ m}^2$. The total glowing area is $0.120\text{ m}^2 + 4(0.033\,3\text{ m}^2) = 0.254\text{ m}^2$ and has power output $\mathscr{P} = IA = (470\text{ W/m}^2)0.254\text{ m}^2 = \boxed{119\text{ W}}$.

**P34.68** (a) At steady state, $\mathscr{P}_{\text{in}} = \mathscr{P}_{\text{out}}$ and the power radiated out is $\mathscr{P}_{\text{out}} = e\sigma AT^4$.

Thus, $0.900(1\,000\text{ W/m}^2)A = 0.700(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)AT^4$

or $$T = \left[\frac{900\text{ W/m}^2}{0.700(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)}\right]^{1/4} = \boxed{388\text{ K}} = 115^\circ\text{C}.$$

(b) The box of horizontal area $A$, presents projected area $A\sin 50.0^\circ$ perpendicular to the sunlight. Then by the same reasoning,

$$0.900(1\,000\text{ W/m}^2)A\sin 50.0^\circ = 0.700(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)AT^4$$

or $$T = \left[\frac{(900\text{ W/m}^2)\sin 50.0^\circ}{0.700(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)}\right]^{1/4} = \boxed{363\text{ K}} = 90.0^\circ\text{C}.$$

**P34.69** We take $R$ to be the planet's distance from its star. The planet, of radius $r$, presents a $\boxed{\text{projected area } \pi r^2}$ perpendicular to the starlight. $\boxed{\text{It radiates over area } 4\pi r^2.}$

At steady-state, $\mathscr{P}_{\text{in}} = \mathscr{P}_{\text{out}}$: $\qquad eI_{\text{in}}\left(\pi r^2\right) = e\sigma\left(4\pi r^2\right)T^4$

$$e\left(\frac{6.00\times 10^{23}\text{ W}}{4\pi R^2}\right)\left(\pi r^2\right) = e\sigma\left(4\pi r^2\right)T^4 \text{ so that } 6.00\times 10^{23}\text{ W} = 16\pi\sigma R^2 T^4$$

$$R = \sqrt{\frac{6.00\times 10^{23}\text{ W}}{16\pi\sigma T^4}} = \sqrt{\frac{6.00\times 10^{23}\text{ W}}{16\pi\left(5.67\times 10^{-8}\text{ W/m}^2\cdot\text{K}^4\right)(310\text{ K})^4}} = \boxed{4.77\times 10^9\text{ m} = 4.77\text{ Gm}}.$$

## ANSWERS TO EVEN PROBLEMS

**P34.2** (a) $2.68\times 10^3$ AD; (b) 8.31 min; (c) 2.56 s; (d) 0.133 s; (e) 33.3 $\mu$s

**P34.4** 733 nT

**P34.6** $E = (300\text{ V/m})\cos\left(62.8x - 1.88\times 10^{10}t\right)$; $B = (1.00\ \mu\text{T})\cos\left(62.8x - 1.88\times 10^{10}t\right)$

**P34.8** see the solution

**P34.10** $2.9\times 10^8$ m/s ± 5%

**P34.12** 49.5 mV

**P34.14** (a) 13.3 nJ/m$^3$; (b) 13.3 nJ/m$^3$; (c) 7.96 W/m$^2$

**P34.16** 516 pT, $\sim 10^5$ times stronger than the Earth's field

**P34.18** (a) 11.9 GW/m$^2$; (b) 234 kW

**P34.20** 33.4°C for the smaller container and 21.7°C for the larger

**P34.22** (a) 50.0%; (b) 269 kW/m$^2$ toward the oven chamber; (c) 14.2 kV/m

**P34.24** (a) 4.97 kW/m$^2$; (b) 16.6 $\mu$J/m$^3$

**P34.26** 667 pN

**P34.28** (a) 5.36 N; (b) 893 $\mu$m/s$^2$; (c) 10.7 days

**P34.30** (a) 577 W/m$^2$; (b) $2.06\times 10^{16}$ W; (c) 68.7 MN; (d) The gravitational force is $\sim 10^{13}$ times stronger and in the opposite direction.

**P34.32** 4.09°

**P34.34** (a) 93.3%; (b) 50.0%; (c) 0

**P34.36** $\dfrac{2\pi m_p c}{eB}$

**P34.38** see the solution

**P34.40** (a) $\sim 10^8$ Hz radio wave; (b) $\sim 10^{13}$ Hz infrared light

**P34.42** (a) 0.690 wavelengths; (b) 58.9 wavelengths

**P34.44** The radio audience gets the news 8.41 ms sooner.

**P34.46** (a) 187 m to 556 m; (b) 2.78 m to 3.41 m

**P34.48** $\sim 10^6$ J

**P34.50** (a) see the solution; (b) 378 nm

**P34.52** (a) 31.4 MW; (b) 0.625 W/m$^2$; (c) 0.513%

**P34.54** (a) 23.9 W/m$^2$; (b) 4.19 times the standard

**P34.56** (a) 6.16 $\mu$Pa; (b) $1.64\times 10^{10}$ times less than atmospheric pressure

**P34.58** (a) $625\ \text{kW/m}^2$;
(b) $21.7\ \text{kN/C}$ and $72.4\ \mu\text{T}$; (c) 17.8 min

**P34.60** (a) $\left(\dfrac{16m\rho^2}{9\pi}\right)^{1/3} gc$; (b) $\left(\dfrac{16\pi^2 m\rho^2}{9}\right)^{1/3} r^2 gc$

**P34.62** (a) see the solution;
(b) $17.6\ \text{Tm/s}^2$, $1.75\times10^{-27}\ \text{W}$;
(c) $1.80\times10^{-24}\ \text{W}$

**P34.64** $3.00\times10^{-2}\ \text{deg}$

**P34.66** (a) $3.33\ \text{m}$, $11.1\ \text{ns}$, $6.67\ \text{pT}$;
(b) $\mathbf{E}=(2.00\ \text{mV/m})\cos 2\pi\left(\dfrac{x}{3.33\ \text{m}}-\dfrac{t}{11.1\ \text{ns}}\right)\hat{\mathbf{j}}$;
$\mathbf{B}=(6.67\ \text{pT})\hat{\mathbf{k}}\cos 2\pi\left(\dfrac{x}{3.33\ \text{m}}-\dfrac{t}{11.1\ \text{ns}}\right)$;
(c) $5.31\ \text{nW/m}^2$; (d) $1.77\times10^{-17}\ \text{J/m}^3$;
(e) $3.54\times10^{-17}\ \text{Pa}$

**P34.68** (a) 388 K; (b) 363 K

# 35

# The Nature of Light and the Laws of Geometric Optics

### CHAPTER OUTLINE

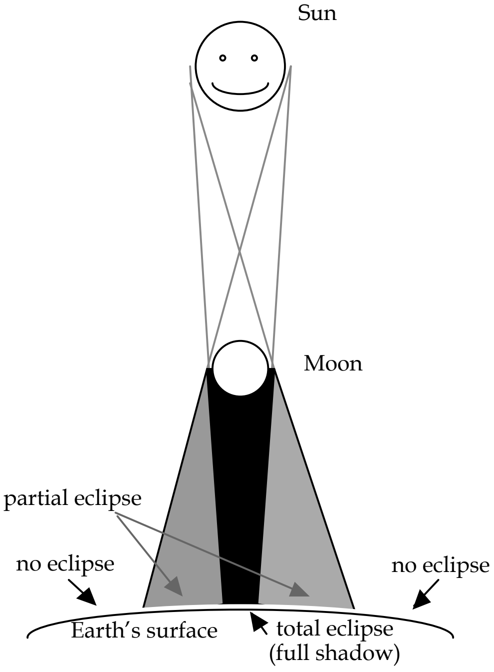
35.1 The Nature of Light
35.2 Measurements of the Speed of Light
35.3 The Ray Approximation in Geometric Optics
35.4 Reflection
35.5 Refraction
35.6 Huygen's Principle
35.7 Dispersion and Prisms
35.8 Total Internal Reflection
35.9 Fermat's Principle

## ANSWERS TO QUESTIONS

**Q35.1** The ray approximation, predicting sharp shadows, is valid for $\lambda << d$. For $\lambda \sim d$ diffraction effects become important, and the light waves will spread out noticeably beyond the slit.

**Q35.2** Light travels through a vacuum at a speed of 300 000 km per second. Thus, an image we see from a distant star or galaxy must have been generated some time ago. For example, the star Altair is 16 light-years away; if we look at an image of Altair today, we know only what was happening 16 years ago. This may not initially seem significant, but astronomers who look at other galaxies can gain an idea of what galaxies looked like when they were significantly younger. Thus, it actually makes sense to speak of "looking backward in time."

**Q35.3**



**Note: Figure not at all to scale**

**FIG. Q35.3**

**Q35.4** With a vertical shop window, streetlights and his own reflection can impede the window shopper's clear view of the display. The tilted shop window can put these reflections out of the way. Windows of airport control towers are also tilted like this, as are automobile windshields.

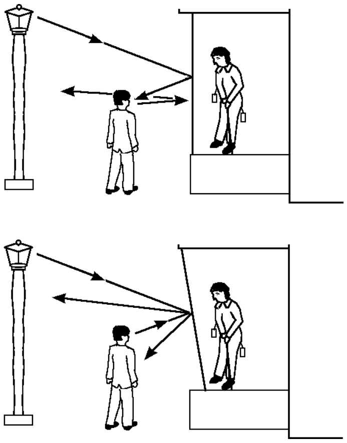


**FIG. Q35.4**

**Q35.5** We assume that you and the child are always standing close together. For a flat wall to make an echo of a sound that you make, you must be standing along a normal to the wall. You must be on the order of 100 m away, to make the transit time sufficiently long that you can hear the echo separately from the original sound. Your sound must be loud enough so that you can hear it even at this considerable range. In the picture, the dashed rectangle represents an area in which you can be standing. The arrows represent rays of sound.

Now suppose two vertical perpendicular walls form an inside corner that you can see. Some of the sound you radiate horizontally will be headed generally toward the corner. It will reflect from both walls with high efficiency to reverse in direction and come back to you. You can stand anywhere reasonably far away to hear a retroreflected echo of sound you produce.

If the two walls are not perpendicular, the inside corner will not produce retroreflection. You will generally hear no echo of your shout or clap.

If two perpendicular walls have a reasonably narrow gap between them at the corner, you can still hear a clear echo. It is not the corner line itself that retroreflects the sound, but the perpendicular walls on both sides of the corner. Diagram (b) applies also in this case.

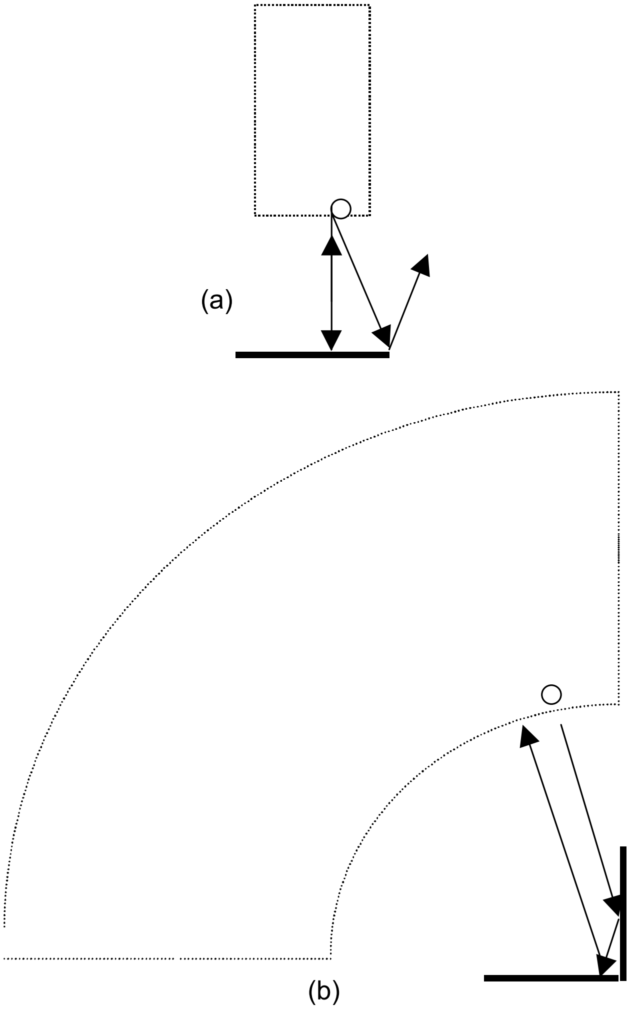


**FIG. Q35.4**

**Q35.6** The stealth fighter is designed so that adjacent panels are not joined at right angles, to prevent any retroreflection of radar signals. This means that radar signals directed at the fighter will not be channeled back toward the detector by reflection. Just as with sound, radar signals can be treated as *diverging* rays, so that any ray that is by chance reflected back to the detector will be too weak in intensity to distinguish from background noise. This author is still waiting for the automotive industry to utilize this technology.

**Q35.7** An echo is an example of the reflection of sound. Hearing the noise of a distant highway on a cold morning, when you cannot hear it after the ground warms up, is an example of acoustical refraction. You can use a rubber inner tube inflated with helium as an acoustical lens to concentrate sound in the way a lens can focus light. At your next party, see if you can experimentally find the approximate focal point!

**Q35.8** No. If the incidence angle is zero, then the ray does not change direction. Also, if the ray travels from a medium of relatively high index of refraction to one of lower index of refraction, it will bend away from the normal.

**Q35.9** Suppose the light moves into a medium of higher refractive index. Then its wavelength decreases. The frequency remains constant. The speed diminishes by a factor equal to the index of refraction.

**Q35.10** If a laser beam enters a sugar solution with a concentration gradient (density and index of refraction increasing with depth) then the laser beam will be progressively bent downward (toward the normal) as it passes into regions of greater index of refraction.

**Q35.11** As measured from the diagram, the incidence angle is 60°, and the refraction angle is 35°. Using equation 35.3, $\frac{\sin\theta_2}{\sin\theta_1}=\frac{v_2}{v_1}$, then $\frac{\sin 35°}{\sin 60°}=\frac{v_2}{c}$ and the speed of light in Lucite is $2.0\times10^8\ \text{m/s}$.

The frequency of the light does not change upon refraction. Knowing the wavelength in a vacuum, we can use the speed of light in a vacuum to determine the frequency: $c=f\lambda$, thus $3.00\times10^8=f\left(632.8\times10^{-9}\right)$, so the frequency is 474.1 THz. To find the wavelength of light in Lucite, we use the same wave speed relation, $v=f\lambda$, so $2.0\times10^8=\left(4.741\times10^{14}\right)\lambda$, so $\lambda_{\text{Lucite}}=420\ \text{nm}$.

**Q35.12** Blue light would be refracted at a smaller angle from the normal, since the index of refraction for blue light—a smaller wavelength than red light—is larger.

**Q35.13** The index of refraction of water is 1.33, quite different from 1.00 for air. Babies learn that the refraction of light going through the water indicates the water is there. On the other hand, the index of refraction of liquid helium is close to that of air, so it gives little visible evidence of its presence.

**Q35.14** The outgoing beam would be a rainbow, with the different colors of light traveling parallel to each other. The white light would undergo dispersion upon refraction into the slab, with blue light bending towards the normal more than the red light. Upon refraction out of the block, all rays of light would exit the slab at the same angle at which they entered the slab, but offset from each other.

**Q35.15** Diamond has higher index of refraction than glass and consequently a smaller critical angle for total internal reflection. A brilliant-cut diamond is shaped to admit light from above, reflect it totally at the converging facets on the underside of the jewel, and let the light escape only at the top. Glass will have less light internally reflected.

**Q35.16** Light coming up from underwater is bent away from the normal. Therefore the part of the oar that is submerged appears bent upward.

**Q35.17** Highly silvered mirrors reflect about 98% of the incident light. With a 2-mirror periscope, that results in approximately a 4% decrease in intensity of light as the light passes through the periscope. This may not seem like much, but in low-light conditions, that lost light may mean the difference between being able to distinguish an enemy armada or an iceberg from the sky beyond. Using prisms results in total internal reflection, meaning that 100% of the incident light is reflected through the periscope. That is the "total" in total internal reflection.

**Q35.18** Sound travels faster in the warmer air, and thus the sound traveling through the warm air aloft will refract much like the light refracting through the nonuniform sugar-water solution in Question 35.10. Sound that would normally travel up over the tree-tops can be refracted back towards the ground.

**Q35.19** The light with the greater change in speed will have the larger deviation. If the glass has a higher index than the surrounding medium, $X$ travels slower in the glass.

**Q35.20** Immediately around the dark shadow of my head, I see a halo brighter than the rest of the dewy grass. It is called the *heiligenschein*. Cellini believed that it was a miraculous sign of divine favor pertaining to him alone. Apparently none of the people to whom he showed it told him that they could see halos around their own shadows but not around Cellini's. Thoreau knew that each person had his own halo. He did not draw any ray diagrams but assumed that it was entirely natural. Between Cellini's time and Thoreau's, the Enlightenment and Newton's explanation of the rainbow had happened. Today the effect is easy to see, whenever your shadow falls on a retroreflecting traffic sign, license plate, or road stripe. When a bicyclist's shadow falls on a paint stripe marking the edge of the road, her halo races along with her. It is a shame that few people are sufficiently curious observers of the natural world to have noticed the phenomenon.

**Q35.21** Suppose the Sun is low in the sky and an observer faces away from the Sun toward a large uniform rain shower. A ray of light passing overhead strikes a drop of water. The light is refracted first at the front surface of the drop, with the violet light deviating the most and the red light the least. At the back of the drop the light is reflected and it returns to the front surface where it again undergoes refraction with additional dispersion as it moves from water into air. The rays leave the drop so that the angle between the incident white light and the most intense returning violet light is 40°, and the angle between the white light and the most intense returning red light is 42°. The observer can see a ring of raindrops shining violet, a ring with angular radius 40° around her shadow. From the locus of directions at 42° away from the antisolar direction, the observer receives red light. The other spectral colors make up the rainbow in between. An observer of a rainbow sees violet light at 40° angular separation from the direction opposite the Sun, then the other spectral colors, and then red light on the outside the rainbow, with angular radius 42°.

**Q35.22** At the altitude of the plane the surface of the Earth need not block off the lower half of the rainbow. Thus, the full circle can be seen. You can see such a rainbow by climbing on a stepladder above a garden sprinkler in the middle of a sunny day. Set the sprinkler for fine mist. Do not let the slippery children fall from the ladder.

**Q35.23** Total internal reflection occurs only when light moving originally in a medium of high index of refraction falls on an interface with a medium of lower index of refraction. Thus, light moving from air ($n = 1$) to water ($n = 1.33$) cannot undergo total internal reflection.

**Q35.24** A mirage occurs when light changes direction as it moves between batches of air having different indices of refraction because they have different densities at different temperatures. When the sun makes a blacktop road hot, an apparent wet spot is bright due to refraction of light from the bright sky. The light, originally headed a little below the horizontal, always bends up as it first enters and then leaves sequentially hotter, lower-density, lower-index layers of air closer to the road surface.

## SOLUTIONS TO PROBLEMS

### Section 35.1 **The Nature of Light**

### Section 35.2 **Measurements of the Speed of Light**

**P35.1** The Moon's radius is $1.74 \times 10^6$ m and the Earth's radius is $6.37 \times 10^6$ m. The total distance traveled by the light is:

$$d = 2\left(3.84 \times 10^8 \text{ m} - 1.74 \times 10^6 \text{ m} - 6.37 \times 10^6 \text{ m}\right) = 7.52 \times 10^8 \text{ m}.$$

This takes 2.51 s, so $v = \dfrac{7.52 \times 10^8 \text{ m}}{2.51 \text{ s}} = 2.995 \times 10^8 \text{ m/s} = \boxed{299.5 \text{ Mm/s}}$.

**P35.2** $\Delta x = ct$; $\qquad c = \dfrac{\Delta x}{t} = \dfrac{2\left(1.50 \times 10^8 \text{ km}\right)(1\,000 \text{ m/km})}{(22.0 \text{ min})(60.0 \text{ s/min})} = 2.27 \times 10^8 \text{ m/s} = \boxed{227 \text{ Mm/s}}$

**P35.3** The experiment is most convincing if the wheel turns fast enough to pass outgoing light through one notch and returning light through the next: $t = \dfrac{2\ell}{c}$

$$\theta = \omega t = \omega\left(\frac{2\ell}{c}\right) \qquad \text{so} \qquad \omega = \frac{c\theta}{2\ell} = \frac{\left(2.998 \times 10^8\right)\left[2\pi/(720)\right]}{2\left(11.45 \times 10^3\right)} = \boxed{114 \text{ rad/s}}.$$

The returning light would be blocked by a tooth at one-half the angular speed, giving another data point.

**P35.4** (a) For the light beam to make it through both slots, the time for the light to travel the distance $d$ must equal the time for the disk to rotate through the angle $\theta$, if $c$ is the speed of light,

$$\frac{d}{c} = \frac{\theta}{\omega}, \text{ so } \boxed{c = \frac{d\omega}{\theta}}.$$

(b) We are given that

$$d = 2.50 \text{ m}, \; \theta = \frac{1.00°}{60.0}\left(\frac{\pi \text{ rad}}{180°}\right) = 2.91 \times 10^{-4} \text{ rad}, \; \omega = 5\,555 \text{ rev/s}\left(\frac{2\pi \text{ rad}}{1.00 \text{ rev}}\right) = 3.49 \times 10^4 \text{ rad/s}$$

$$c = \frac{d\omega}{\theta} = \frac{(2.50 \text{ m})\left(3.49 \times 10^4 \text{ rad/s}\right)}{2.91 \times 10^{-4} \text{ rad}} = 3.00 \times 10^8 \text{ m/s} = \boxed{300 \text{ Mm/s}}$$

### Section 35.3 The Ray Approximation in Geometric Optics

### Section 35.4 Reflection

### Section 35.5 Refraction

***P35.5** (a) Let $AB$ be the originally horizontal ceiling, $BC$ its originally vertical normal, $AD$ the new ceiling and $DE$ its normal. Then angle $BAD = \phi$. By definition $DE$ is perpendicular to $AD$ and $BC$ is perpendicular to $AB$. Then the angle between $DE$ extended and $BC$ is $\phi$ because angles are equal when their sides are perpendicular, right side to right side and left side to left side.

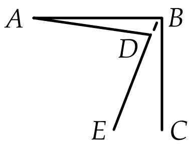


**FIG. P35.5(a)**

(b) Now $CBE = \phi$ is the angle of incidence of the vertical light beam. Its angle of reflection is also $\phi$. The angle between the vertical incident beam and the reflected beam is $2\phi$.

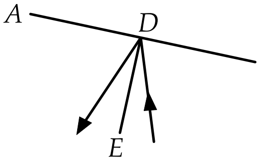


**FIG. P35.5(b)**

(c) $\tan 2\phi = \dfrac{1.40 \text{ cm}}{720 \text{ cm}} = 0.001\,94$ $\boxed{\phi = 0.055\,7°}$

**P35.6** (a) From geometry, $1.25 \text{ m} = d \sin 40.0°$

so $d = \boxed{1.94 \text{ m}}$.

(b) $\boxed{50.0° \text{ above the horizontal}}$

or parallel to the incident ray.

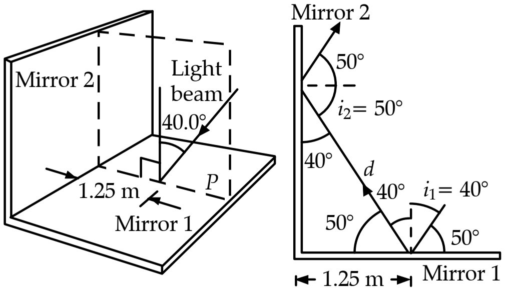


**FIG. P35.6**

***P35.7** (a) **Method One:**
The incident ray makes angle $\alpha = 90° - \theta_1$ with the first mirror. In the picture, the law of reflection implies that

$$\theta_1 = \theta_1' .$$

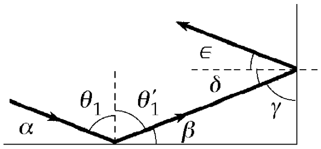


**FIG. P35.7**

Then $\beta = 90° - \theta_1' = 90 - \theta_1 = \alpha$.

In the triangle made by the mirrors and the ray passing between them,

$$\beta + 90° + \gamma = 180°$$

$$\gamma = 90° - \beta$$

Further, $\delta = 90° - \gamma = \beta = \alpha$

and $\in = \delta = \alpha$.

Thus the final ray makes the same angle with the first mirror as did the incident ray. Its direction is opposite to the incident ray.

*continued on next page*

**Method Two:**

The vector velocity of the incident light has a component $v_y$ perpendicular to the first mirror and a component $v_x$ perpendicular to the second. The $v_y$ component is reversed upon the first reflection, which leaves $v_x$ unchanged. The second reflection reverses $v_x$ and leaves $v_y$ unchanged. The doubly reflected ray then has velocity opposite to the incident ray.

(b) The incident ray has velocity $v_x\hat{\mathbf{i}}+v_y\hat{\mathbf{j}}+v_z\hat{\mathbf{k}}$. Each reflection reverses one component and leaves the other two unchanged. After all the reflections, the light has velocity $-v_x\hat{\mathbf{i}}-v_y\hat{\mathbf{j}}-v_z\hat{\mathbf{k}}$, opposite to the incident ray.

**P35.8** The incident light reaches the left-hand mirror at distance

$$(1.00\text{ m})\tan 5.00° = 0.087\,5\text{ m}$$

above its bottom edge. The reflected light first reaches the right-hand mirror at height

$$2(0.087\,5\text{ m}) = 0.175\text{ m}.$$

It bounces between the mirrors with this distance between points of contact with either.

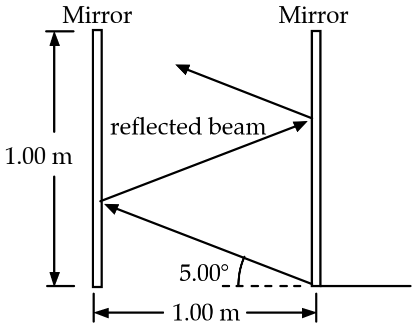


FIG. P35.8

Since $\dfrac{1.00\text{ m}}{0.175\text{ m}} = 5.72$

the light reflects $\boxed{\text{five times from the right-hand mirror and six times from the left}}$.

***P35.9** Let $d$ represent the perpendicular distance from the person to the mirror. The distance between lamp and peson measured parallel to the mirror can be written in two ways: $2d\tan\theta + d\tan\theta = d\tan\phi$. The condition on the distance traveled by the light is $\dfrac{2d}{\cos\phi} = \dfrac{2d}{\cos\theta} + \dfrac{d}{\cos\theta}$. We have the two equations $3\tan\theta = \tan\phi$ and $2\cos\theta = 3\cos\phi$. To eliminate $\phi$ we write

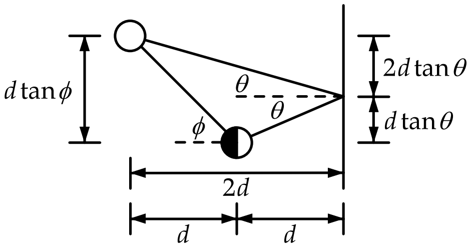


FIG. P35.9

$$\frac{9\sin^2\theta}{\cos^2\theta} = \frac{\sin^2\phi}{\cos^2\phi} \qquad 4\cos^2\theta = 9\cos^2\phi$$

$$9\cos^2\phi\sin^2\theta = \cos^2\theta\left(1-\cos^2\phi\right)$$

$$4\cos^2\theta\sin^2\theta = \cos^2\theta\left(1-\frac{4}{9}\cos^2\theta\right)$$

$$4\sin^2\theta = 1-\frac{4}{9}\left(1-\sin^2\theta\right) \qquad 36\sin^2\theta = 9-4+4\sin^2\theta$$

$$\sin^2\theta = \frac{5}{32} \qquad \theta = \boxed{23.3°}$$

**P35.10** Using Snell's law, $\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1$

$$\theta_2 = \boxed{25.5°}$$

$$\lambda_2 = \frac{\lambda_1}{n_1} = \boxed{442 \text{ nm}}.$$

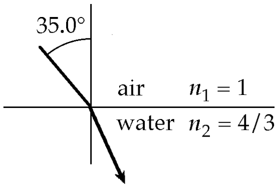


**FIG. P35.10**

***P35.11** The law of refraction $n_1 \sin\theta_1 = n_2 \sin\theta_2$ can be put into the more general form

$$\frac{c}{v_1}\sin\theta_1 = \frac{c}{v_2}\sin\theta_2$$

$$\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}$$

In this form it applies to all kinds of waves that move through space.

$$\frac{\sin 3.5°}{343 \text{ m/s}} = \frac{\sin\theta_2}{1\,493 \text{ m/s}}$$

$$\sin\theta_2 = 0.266$$

$$\theta_2 = \boxed{15.4°}$$

The wave keeps constant frequency in

$$f = \frac{v_1}{\lambda_1} = \frac{v_2}{\lambda_2}$$

$$\lambda_2 = \frac{v_2\lambda_1}{v_1} = \frac{1\,493 \text{ m/s}(0.589 \text{ m})}{343 \text{ m/s}} = \boxed{2.56 \text{ m}}$$

**P35.12** (a) $f = \frac{c}{\lambda} = \frac{3.00\times10^8 \text{ m/s}}{6.328\times10^{-7} \text{ m}} = \boxed{4.74\times10^{14} \text{ Hz}}$

(b) $\lambda_{\text{glass}} = \frac{\lambda_{\text{air}}}{n} = \frac{632.8 \text{ nm}}{1.50} = \boxed{422 \text{ nm}}$

(c) $v_{\text{glass}} = \frac{c_{\text{air}}}{n} = \frac{3.00\times10^8 \text{ m/s}}{1.50} = 2.00\times10^8 \text{ m/s} = \boxed{200 \text{ Mm/s}}$

**P35.13** $n_1 \sin\theta_1 = n_2 \sin\theta_2$

$$\sin\theta_1 = 1.333\sin 45°$$

$$\sin\theta_1 = (1.33)(0.707) = 0.943$$

$$\theta_1 = 70.5° \rightarrow \boxed{19.5° \text{ above the horizon}}$$

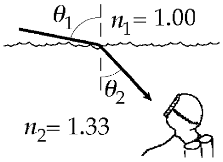


**FIG. P35.13**

***P35.14** We find the angle of incidence:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$1.333\sin\theta_1 = 1.52\sin 19.6°$$

$$\theta_1 = 22.5°$$

The angle of reflection of the beam in water is then also $\boxed{22.5°}$.

**P35.15** (a) $n_1 \sin\theta_1 = n_2 \sin\theta_2$

$1.00 \sin 30.0° = n \sin 19.24°$

$n = \boxed{1.52}$

(c) $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^{8}\ \text{m/s}}{6.328\times10^{-7}\ \text{m}} = \boxed{4.74\times10^{14}\ \text{Hz}}$ in air and in syrup.

(d) $v = \dfrac{c}{n} = \dfrac{3.00\times10^{8}\ \text{m/s}}{1.52} = 1.98\times10^{8}\ \text{m/s} = \boxed{198\ \text{Mm/s}}$

(b) $\lambda = \dfrac{v}{f} = \dfrac{1.98\times10^{8}\ \text{m/s}}{4.74\times10^{14}/\text{s}} = \boxed{417\ \text{nm}}$

**P35.16** (a) Flint Glass: $v = \dfrac{c}{n} = \dfrac{3.00\times10^{8}\ \text{m/s}}{1.66} = 1.81\times10^{8}\ \text{m/s} = \boxed{181\ \text{Mm/s}}$

(b) Water: $v = \dfrac{c}{n} = \dfrac{3.00\times10^{8}\ \text{m/s}}{1.333} = 2.25\times10^{8}\ \text{m/s} = \boxed{225\ \text{Mm/s}}$

(c) Cubic Zirconia: $v = \dfrac{c}{n} = \dfrac{3.00\times10^{8}\ \text{m/s}}{2.20} = 1.36\times10^{8}\ \text{m/s} = \boxed{136\ \text{Mm/s}}$

**P35.17** $n_1 \sin\theta_1 = n_2 \sin\theta_2$: $1.333 \sin 37.0° = n_2 \sin 25.0°$

$n_2 = 1.90 = \dfrac{c}{v}$: $v = \dfrac{c}{1.90} = 1.58\times10^{8}\ \text{m/s} = \boxed{158\ \text{Mm/s}}$

**P35.18** $\sin\theta_1 = n_w \sin\theta_2$

$$\sin\theta_2 = \frac{1}{1.333}\sin\theta_1 = \frac{1}{1.333}\sin(90.0°-28.0°) = 0.662$$

$$\theta_2 = \sin^{-1}(0.662) = 41.5°$$

$$h = \frac{d}{\tan\theta_2} = \frac{3.00\ \text{m}}{\tan 41.5°} = \boxed{3.39\ \text{m}}$$

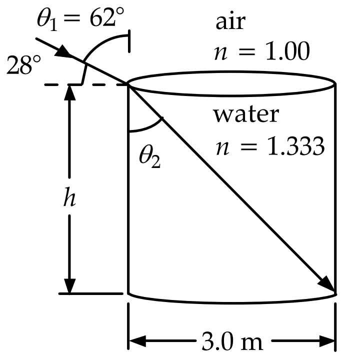


**FIG. P35.18**

**P35.19** $n_1 \sin\theta_1 = n_2 \sin\theta_2$: $\qquad \theta_2 = \sin^{-1}\left(\dfrac{n_1 \sin\theta_1}{n_2}\right)$

$$\theta_2 = \sin^{-1}\left\{\frac{1.00\sin 30°}{1.50}\right\} = \boxed{19.5°}$$

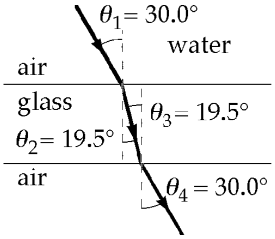


**FIG. P35.19**

$\theta_2$ and $\theta_3$ are alternate interior angles formed by the ray cutting parallel normals.

So,
$$\theta_3 = \theta_2 = \boxed{19.5°}$$
$$1.50\sin\theta_3 = 1.00\sin\theta_4$$
$$\theta_4 = \boxed{30.0°}$$

***P35.20** For $\qquad \alpha + \beta = 90°$

with $\qquad \theta_1' + \alpha + \beta + \theta_2 = 180°$

we have $\qquad \theta_1' + \theta_2 = 90°$ .

Also, $\qquad \theta_1' = \theta_1$

and $\qquad 1\sin\theta_1 = n\sin\theta_2$ .

Then, $\qquad \sin\theta_1 = n\sin(90 - \theta_1) = n\cos\theta_1$

$$\frac{\sin\theta_1}{\cos\theta_1} = n = \tan\theta_1 \qquad \boxed{\theta_1 = \tan^{-1} n} .$$

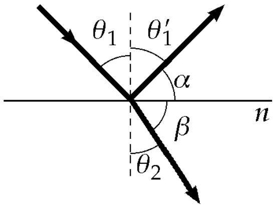


**FIG. P35.20**

**P35.21** At entry, $\qquad n_1 \sin\theta_1 = n_2 \sin\theta_2$

or $\qquad 1.00\sin 30.0° = 1.50\sin\theta_2$

$$\theta_2 = 19.5° .$$

The distance $h$ the light travels in the medium is given by

$$\cos\theta_2 = \frac{2.00 \text{ cm}}{h}$$

or
$$h = \frac{2.00 \text{ cm}}{\cos 19.5°} = 2.12 \text{ cm} .$$

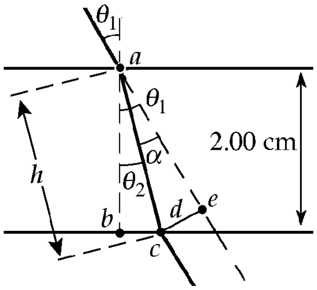


**FIG. P35.21**

The angle of deviation upon entry is $\qquad \alpha = \theta_1 - \theta_2 = 30.0° – 19.5° = 10.5°$ .

The offset distance comes from $\sin\alpha = \dfrac{d}{h}$: $\qquad d = (2.21 \text{ cm})\sin 10.5° = \boxed{0.388 \text{ cm}}$ .

**P35.22** The distance, $h$, traveled by the light is $h = \dfrac{2.00 \text{ cm}}{\cos 19.5°} = 2.12 \text{ cm}$

The speed of light in the material is $\qquad v = \dfrac{c}{h} = \dfrac{3.00 \times 10^8 \text{ m/s}}{1.50} = 2.00 \times 10^8 \text{ m/s}$

Therefore, $\qquad t = \dfrac{h}{v} = \dfrac{2.12 \times 10^{-2} \text{ m}}{2.00 \times 10^8 \text{ m/s}} = 1.06 \times 10^{-10} \text{ s} = \boxed{106 \text{ ps}}$ .

**P35.23** Applying Snell's law at the air-oil interface,

$$n_{\text{air}} \sin\theta = n_{\text{oil}} \sin 20.0°$$

yields $\boxed{\theta = 30.4°}$.

Applying Snell's law at the oil-water interface

$$n_w \sin\theta' = n_{\text{oil}} \sin 20.0°$$

yields $\boxed{\theta' = 22.3°}$.

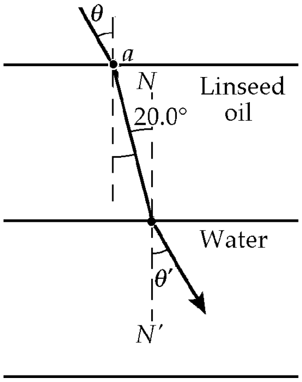


FIG. P35.23

***P35.24** For sheets 1 and 2 as described,

$n_1 \sin 26.5° = n_2 \sin 31.7°$
$0.849 n_1 = n_2$

For the trial with sheets 3 and 2,

$n_3 \sin 26.5° = n_2 \sin 36.7°$
$0.747 n_3 = n_2$

Now

$0.747 n_3 = 0.849 n_1$
$n_3 = 1.14 n_1$

For the third trial,

$n_1 \sin 26.5° = n_3 \sin\theta_3 = 1.14 n_1 \sin\theta_3$
$\theta_3 = \boxed{23.1°}$

**P35.25** Consider glass with an index of refraction of 1.5, which is 3 mm thick. The speed of light in the glass is

$$\frac{3\times10^8 \text{ m/s}}{1.5} = 2\times10^8 \text{ m/s}.$$

The extra travel time is $\dfrac{3\times10^{-3}\text{ m}}{2\times10^8\text{ m/s}} - \dfrac{3\times10^{-3}\text{ m}}{3\times10^8\text{ m/s}} \boxed{\sim 10^{-11}\text{ s}}$.

For light of wavelength 600 nm in vacuum and wavelength $\dfrac{600\text{ nm}}{1.5} = 400\text{ nm}$ in glass,

the extra optical path, in wavelengths, is $\dfrac{3\times10^{-3}\text{ m}}{4\times10^{-7}\text{ m}} - \dfrac{3\times10^{-3}\text{ m}}{6\times10^{-7}\text{ m}} \boxed{\sim 10^3\text{ wavelengths}}$.

**P35.26** Refraction proceeds according to $(1.00)\sin\theta_1 = (1.66)\sin\theta_2$. (1)

(a) For the normal component of velocity to be constant, $v_1\cos\theta_1 = v_2\cos\theta_2$

or $(c)\cos\theta_1 = \left(\dfrac{c}{1.66}\right)\cos\theta_2$. (2)

We multiply Equations (1) and (2), obtaining: $\sin\theta_1\cos\theta_1 = \sin\theta_2\cos\theta_2$

or $\sin 2\theta_1 = \sin 2\theta_2$.

The solution $\theta_1 = \theta_2 = 0$ does not satisfy Equation (2) and must be rejected. The physical solution is $2\theta_1 = 180° - 2\theta_2$ or $\theta_2 = 90.0° - \theta_1$. Then Equation (1) becomes:

$\sin\theta_1 = 1.66\cos\theta_1$, or $\tan\theta_1 = 1.66$

which yields $\theta_1 = \boxed{58.9°}$.

(b) Light entering the glass slows down and makes a smaller angle with the normal. Both effects reduce the velocity component parallel to the surface of the glass, so that component cannot remain constant, or will remain constant only in the trivial case $\boxed{\theta_1 = \theta_2 = 0}$.

**P35.27** See the sketch showing the path of the light ray. $\alpha$ and $\gamma$ are angles of incidence at mirrors 1 and 2.

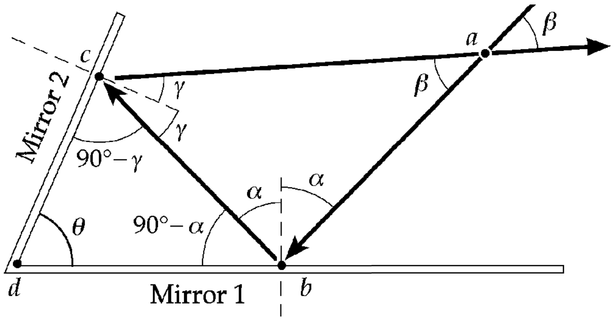


FIG. P35.27

For triangle *abca*,

$$2\alpha + 2\gamma + \beta = 180°$$

or $\beta = 180° - 2(\alpha + \gamma)$. (1)

Now for triangle *bcdb*,

$$(90.0° - \alpha) + (90.0° - \gamma) + \theta = 180°$$

or $\theta = \alpha + \gamma$. (2)

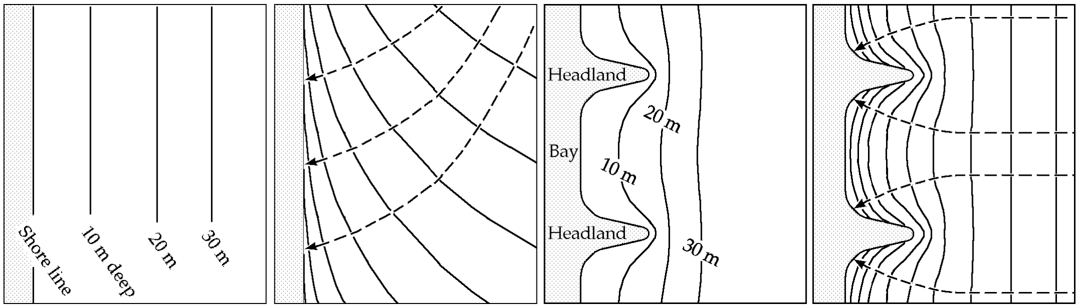
Substituting Equation (2) into Equation (1) gives $\boxed{\beta = 180° - 2\theta}$.

Note: From Equation (2), $\gamma = \theta - \alpha$. Thus, the ray will follow a path like that shown only if $\alpha < 0$. For $\alpha > 0$, $\gamma$ is negative and multiple reflections from each mirror will occur before the incident and reflected rays intersect.

## Section 35.6 Huygen's Principle

***P35.28** (a) For the diagrams of contour lines and wave fronts and rays, see Figures (a) and (b) below. As the waves move to shallower water, the wave fronts bend to become more nearly parallel to the contour lines.

(b) For the diagrams of contour lines and wave fronts and rays, see Figures (c) and (d) below. We suppose that the headlands are steep underwater, as they are above water. The rays are everywhere perpendicular to the wave fronts of the incoming refracting waves. As shown, the rays bend toward the headlands and deliver more energy per length at the headlands.



(a) Contour lines (b) Wave fronts and rays (c) Contour lines (d) Wave fronts and rays

**FIG. P35.28**

## Section 35.7 Dispersion and Prisms

**P35.29** From Fig 35.21 $n_v = 1.470$ at 400 nm and $n_r = 1.458$ at 700 nm.

Then $1.00\sin\theta = 1.470\sin\theta_v$ and $1.00\sin\theta = 1.458\sin\theta_r$

$$\delta_r - \delta_v = \theta_r - \theta_v = \sin^{-1}\left(\frac{\sin\theta}{1.458}\right) - \sin^{-1}\left(\frac{\sin\theta}{1.470}\right)$$

$$\Delta\delta = \sin^{-1}\left(\frac{\sin 30.0°}{1.458}\right) - \sin^{-1}\left(\frac{\sin 30.0°}{1.470}\right) = \boxed{0.171°}$$

**P35.30** $n(700\text{ nm}) = 1.458$

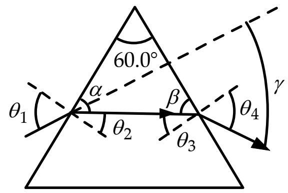


**FIG. P35.30**

(a) $(1.00)\sin 75.0° = 1.458\sin\theta_2$ ; $\theta_2 = \boxed{41.5°}$

(b) Let $\theta_3 + \beta = 90.0°$ , $\theta_2 + \alpha = 90.0°$ then $\alpha + \beta + 60.0° = 180°$.

So $60.0° - \theta_2 - \theta_3 = 0 \Rightarrow 60.0° - 41.5° = \theta_3 = \boxed{18.5°}$.

(c) $1.458\sin 18.5° = 1.00\sin\theta_4$ $\theta_4 = \boxed{27.6°}$

(d) $\gamma = (\theta_1 - \theta_2) + [\beta - (90.0° - \theta_4)]$

$\gamma = 75.0° - 41.5° + (90.0° - 18.5°) - (90.0° - 27.6°) = \boxed{42.6°}$

**P35.31** Taking $\Phi$ to be the apex angle and $\delta_{\text{min}}$ to be the angle of minimum deviation, from Equation 35.9, the index of refraction of the prism material is

$$n = \frac{\sin\left[(\Phi + \delta_{\text{min}})/2\right]}{\sin(\Phi/2)}$$

Solving for $\delta_{\text{min}}$, $\delta_{\text{min}} = 2\sin^{-1}\left(n\sin\frac{\Phi}{2}\right) - \Phi = 2\sin^{-1}\left[(2.20)\sin(25.0°)\right] - 50.0° = \boxed{86.8°}$ .

**P35.32** Note for use in every part: $\Phi + (90.0° - \theta_2) + (90.0° - \theta_3) = 180°$

so $\theta_3 = \Phi - \theta_2$ .

At the first surface the deviation is $\alpha = \theta_1 - \theta_2$ .

At exit, the deviation is $\beta = \theta_4 - \theta_3$ .

The total deviation is therefore $\delta = \alpha + \beta = \theta_1 + \theta_4 - \theta_2 - \theta_3 = \theta_1 + \theta_4 - \Phi$ .

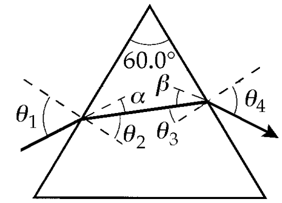


**FIG. P35.32**

(a) At entry: $n_1\sin\theta_1 = n_2\sin\theta_2$ or $\theta_2 = \sin^{-1}\left(\frac{\sin 48.6°}{1.50}\right) = 30.0°$ .

Thus, $\theta_3 = 60.0° - 30.0° = 30.0°$ .

At exit: $1.50\sin 30.0° = 1.00\sin\theta_4$ or $\theta_4 = \sin^{-1}\left[1.50\sin(30.0°)\right] = 48.6°$

so the path through the prism is symmetric when $\theta_1 = 48.6°$ .

(b) $\delta = 48.6° + 48.6° - 60.0° = \boxed{37.2°}$

(c) At entry: $\sin\theta_2 = \frac{\sin 45.6°}{1.50} \Rightarrow \theta_2 = 28.4°$ $\theta_3 = 60.0° - 28.4° = 31.6°$ .

At exit: $\sin\theta_4 = 1.50\sin(31.6°) \Rightarrow \theta_4 = 51.7°$ $\delta = 45.6° + 51.7° - 60.0° = \boxed{37.3°}$ .

(d) At entry: $\sin\theta_2 = \frac{\sin 51.6°}{1.50} \Rightarrow \theta_2 = 31.5°$ $\theta_3 = 60.0° - 31.5° = 28.5°$ .

At exit: $\sin\theta_4 = 1.50\sin(28.5°) \Rightarrow \theta_4 = 45.7°$ $\delta = 51.6° + 45.7° - 60.0° = \boxed{37.3°}$ .

**P35.33** At the first refraction, $1.00\sin\theta_1 = n\sin\theta_2$ .

The critical angle at the second surface is given by $n\sin\theta_3 = 1.00$ :

or $\theta_3 = \sin^{-1}\left(\frac{1.00}{1.50}\right) = 41.8°$ .

But, $\theta_2 = 60.0° - \theta_3$ .

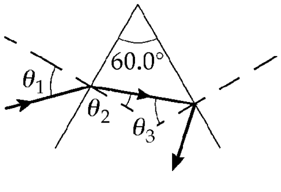


**FIG. P35.33**

Thus, to avoid total internal reflection at the second surface (i.e., have $\theta_3 < 41.8°$)

it is necessary that $\theta_2 > 18.2°$ .

Since $\sin\theta_1 = n\sin\theta_2$, this becomes $\sin\theta_1 > 1.50\sin 18.2° = 0.468$

or $\theta_1 > \boxed{27.9°}$ .

**P35.34** At the first refraction, $1.00\sin\theta_1 = n\sin\theta_2$.

The critical angle at the second surface is given by

$n\sin\theta_3 = 1.00$, or $\theta_3 = \sin^{-1}\left(\dfrac{1.00}{n}\right)$.

But $(90.0°-\theta_2)+(90.0°-\theta_3)+\Phi = 180°$

which gives $\theta_2 = \Phi - \theta_3$.

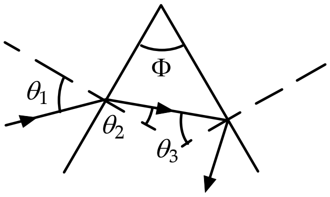


**FIG. P35.34**

Thus, to have $\theta_3 < \sin^{-1}\left(\dfrac{1.00}{n}\right)$ and avoid total internal reflection at the second surface,

it is necessary that $\theta_2 > \Phi - \sin^{-1}\left(\dfrac{1.00}{n}\right)$.

Since $\sin\theta_1 = n\sin\theta_2$, this requirement becomes $\sin\theta_1 > n\sin\left[\Phi - \sin^{-1}\left(\dfrac{1.00}{n}\right)\right]$

or $\theta_1 > \boxed{\sin^{-1}\left(n\sin\left[\Phi - \sin^{-1}\left(\dfrac{1.00}{n}\right)\right]\right)}$.

Through the application of trigonometric identities, $\theta_1 > \boxed{\sin^{-1}\left(\sqrt{n^2-1}\,\sin\Phi - \cos\Phi\right)}$.

**P35.35** For the incoming ray, $\sin\theta_2 = \dfrac{\sin\theta_1}{n}$.

Using the figure to the right, $(\theta_2)_{\text{violet}} = \sin^{-1}\left(\dfrac{\sin 50.0°}{1.66}\right) = 27.48°$

$(\theta_2)_{\text{red}} = \sin^{-1}\left(\dfrac{\sin 50.0°}{1.62}\right) = 28.22°$.

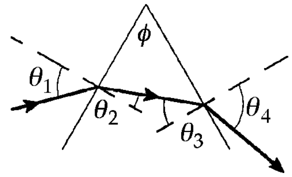


**FIG. P35.35**

For the outgoing ray, $\theta_3 = 60.0°-\theta_2$

and $\sin\theta_4 = n\sin\theta_3$: $(\theta_4)_{\text{violet}} = \sin^{-1}[1.66\sin 32.52°] = 63.17°$

$(\theta_4)_{\text{red}} = \sin^{-1}[1.62\sin 31.78°] = 58.56°$.

The angular dispersion is the difference $\Delta\theta_4 = (\theta_4)_{\text{violet}} - (\theta_4)_{\text{red}} = 63.17°-58.56° = \boxed{4.61°}$.

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## Section 35.8 **Total Internal Reflection**

**P35.36** $n\sin\theta = 1$. From Table 35.1,

(a) $\theta = \sin^{-1}\left(\dfrac{1}{2.419}\right) = \boxed{24.4°}$

(b) $\theta = \sin^{-1}\left(\dfrac{1}{1.66}\right) = \boxed{37.0°}$

(c) $\theta = \sin^{-1}\left(\dfrac{1}{1.309}\right) = \boxed{49.8°}$

**P35.37** $\sin\theta_c = \frac{n_2}{n_1}$: $\quad \theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

(a) Diamond: $\theta_c = \sin^{-1}\left(\frac{1.333}{2.419}\right) = \boxed{33.4°}$

(b) Flint glass: $\theta_c = \sin^{-1}\left(\frac{1.333}{1.66}\right) = \boxed{53.4°}$

(c) Ice: Since $n_2 > n_1$, $\boxed{\text{there is no critical angle}}$.

**P35.38** $\sin\theta_c = \frac{n_{\text{air}}}{n_{\text{pipe}}} = \frac{1.00}{1.36} = 0.735 \qquad \theta_c = 47.3°$

Geometry shows that the angle of refraction at the end is $\phi = 90.0° - \theta_c = 90.0° - 47.3° = 42.7°$.

Then, Snell's law at the end, $\quad 1.00\sin\theta = 1.36\sin 42.7°$

gives $\quad \boxed{\theta = 67.2°}$.

The 2-$\mu$m diameter is unnecessary information.

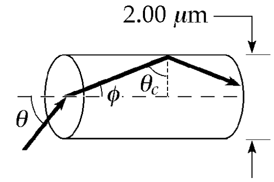


FIG. P35.38

**P35.39** $\sin\theta_c = \frac{n_2}{n_1}$

$n_2 = n_1 \sin 88.8° = (1.000\,3)(0.999\,8) = \boxed{1.000\,08}$

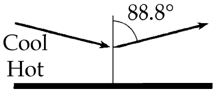


FIG. P35.39

***P35.40** (a) A ray along the inner edge will escape if any ray escapes. Its angle of incidence is described by $\sin\theta = \frac{R-d}{R}$ and by $n\sin\theta > 1\sin 90°$. Then

$$\frac{n(R-d)}{R} > 1 \qquad nR - nd > R \qquad nR - R > nd \qquad R > \boxed{\frac{nd}{n-1}}.$$

(b) As $d \to 0$, $R_{\text{min}} \to 0$. This is reasonable.
As $n$ increases, $R_{\text{min}}$ decreases. This is reasonable.
As $n$ decreases toward 1, $R_{\text{min}}$ increases. This is reasonable.

(c) $R_{\text{min}} = \frac{1.40\left(100\times10^{-6}\text{ m}\right)}{0.40} = \boxed{350\times10^{-6}\text{ m}}$

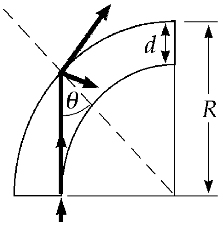


FIG. P35.40

**P35.41** From Snell's law, $n_1\sin\theta_1 = n_2\sin\theta_2$.

At the extreme angle of viewing, $\theta_2 = 90.0°$

$(1.59)(\sin\theta_1) = (1.00)\sin 90.0°$.

So $\quad \theta_1 = 39.0°$.

Therefore, the depth of the air bubble is

$$\frac{r_d}{\tan\theta_1} < d < \frac{r_p}{\tan\theta_1}$$

or $\quad \boxed{1.08\text{ cm} < d < 1.17\text{ cm}}$.

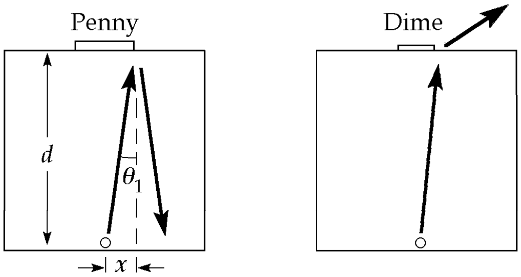


FIG. P35.41

**P35.42** (a) $$\frac{\sin\theta_2}{\sin\theta_1}=\frac{v_2}{v_1}$$

and $\theta_2 = 90.0°$ at the critical angle

$$\frac{\sin 90.0°}{\sin\theta_c}=\frac{1\,850\text{ m/s}}{343\text{ m/s}}$$

so $\theta_c=\sin^{-1}(0.185)=\boxed{10.7°}$.

(b) Sound can be totally reflected if it is traveling in the medium where it travels slower: $\boxed{\text{air}}$.

(c) $\boxed{\text{Sound in air falling on the wall from most directions is 100\% reflected}}$, so the wall is a good mirror.

**P35.43** For plastic with index of refraction $\boxed{n \geq 1.42}$ surrounded by air, the critical angle for total internal reflection is given by

$$\theta_c=\sin^{-1}\left(\frac{1}{n}\right)\leq \sin^{-1}\left(\frac{1}{1.42}\right)=44.8°.$$

In the gasoline gauge, skylight from above travels down the plastic. The rays close to the vertical are totally reflected from the sides of the slab and from both facets at the lower end of the plastic, where it is not immersed in gasoline. This light returns up inside the plastic and makes it look bright. Where the plastic is immersed in gasoline, with index of refraction about 1.50, total internal reflection should not happen. The light passes out of the lower end of the plastic with little reflected, making this part of the gauge look dark. To frustrate total internal reflection in the gasoline, the index of refraction of the plastic should be $\boxed{n<2.12}$.

since $$\theta_c=\sin^{-1}\left(\frac{1.50}{2.12}\right)=45.0°.$$

## Section 35.9 **Fermat's Principle**

**P35.44** Assume the lifeguard's initial path makes angle $\theta_1$ with the north-south normal to the shoreline, and angle $\theta_2$ with this normal in the water. By Fermat's principle, his path should follow the law of refraction:

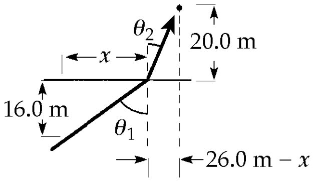


**FIG. P35.44**

$$\frac{\sin\theta_1}{\sin\theta_2}=\frac{v_1}{v_2}=\frac{7.00\text{ m/s}}{1.40\text{ m/s}}=5.00 \text{ or } \theta_2=\sin^{-1}\left(\frac{\sin\theta_1}{5}\right).$$

The lifeguard on land travels eastward a distance $x=(16.0\text{ m})\tan\theta_1$. Then in the water, he travels $26.0\text{ m}-x=(20.0\text{ m})\tan\theta_2$ further east. Thus, $26.0\text{ m}=(16.0\text{ m})\tan\theta_1+(20.0\text{ m})\tan\theta_2$

or $26.0\text{ m}=(16.0\text{ m})\tan\theta_1+(20.0\text{ m})\tan\left[\sin^{-1}\left(\frac{\sin\theta_1}{5}\right)\right]$.

We home in on the solution as follows:

| $\theta_1$ (deg) | 50.0 | 60.0 | 54.0 | 54.8 | 54.81 |
|---|---|---|---|---|---|
| right-hand side | 22.2 m | 31.2 m | 25.3 m | 25.99 m | 26.003 m |

The lifeguard should start running at $\boxed{54.8° \text{ east of north}}$.

**Additional Problems**

***P35.45** Scattered light leaves the center of the photograph (a) in all horizontal directions between $\theta_1 = 0°$ and 90° from the normal. When it immediately enters the water (b), it is gathered into a fan between 0° and $\theta_{2\ \text{max}}$ given by

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$1.00 \sin 90 = 1.333 \sin\theta_{2\ \text{max}}$$
$$\theta_{2\ \text{max}} = 48.6°$$

The light leaves the cylinder without deviation, so the viewer only receives light from the center of the photograph when he has turned by an angle less than 48.6°. When the paperweight is turned farther, light at the back surface undergoes total internal reflection (c). The viewer sees things outside the globe on the far side.

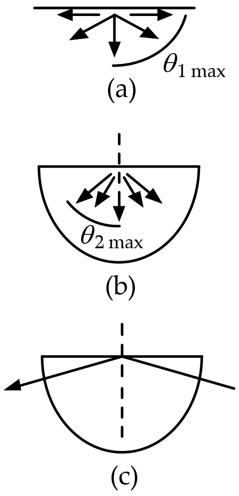


**FIG. P35.45**

**P35.46** Let $n(x)$ be the index of refraction at distance $x$ below the top of the atmosphere and $n(x=h)=n$ be its value at the planet surface.

Then, $$n(x) = 1.000 + \left(\frac{n-1.000}{h}\right)x.$$

(a) The total time interval required to traverse the atmosphere is

$$\Delta t = \int_0^h \frac{dx}{v} = \int_0^h \frac{n(x)}{c}dx: \qquad \Delta t = \frac{1}{c}\int_0^h \left[1.000 + \left(\frac{n-1.000}{h}\right)x\right]dx$$

$$\Delta t = \frac{h}{c} + \frac{(n-1.000)}{ch}\left(\frac{h^2}{2}\right) = \boxed{\frac{h}{c}\left(\frac{n+1.000}{2}\right)}.$$

(b) The travel time in the absence of an atmosphere would be $\frac{h}{c}$.

Thus, the time in the presence of an atmosphere is $\boxed{\left(\frac{n+1.000}{2}\right)\text{ times larger}}$.

**P35.47** Let the air and glass be medium 1 and 2, respectively. By Snell's law, $n_2 \sin\theta_2 = n_1 \sin\theta_1$

or $1.56 \sin\theta_2 = \sin\theta_1$.

But the conditions of the problem are such that $\theta_1 = 2\theta_2$. $1.56 \sin\theta_2 = \sin 2\theta_2$.

We now use the double-angle trig identity suggested. $1.56 \sin\theta_2 = 2\sin\theta_2 \cos\theta_2$

or $\cos\theta_2 = \frac{1.56}{2} = 0.780$.

Thus, $\theta_2 = 38.7°$ and $\theta_1 = 2\theta_2 = \boxed{77.5°}$.

**P35.48** (a) $\theta_1' = \theta_1 = \boxed{30.0°}$ $n_1 \sin\theta_1 = n_2 \sin\theta_2$

$$1.00 \sin 30.0° = 1.55 \sin\theta_2$$

$$\theta_2 = \boxed{18.8°}$$

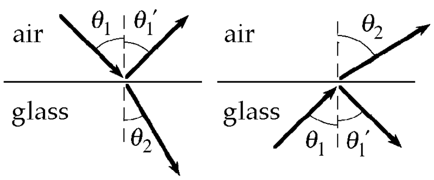


**FIG. P35.48**

(b) $\theta_1' = \theta_1 = \boxed{30.0°}$ $\theta_2 = \sin^{-1}\left(\dfrac{n_1 \sin\theta_1}{n_2}\right)$

$$= \sin^{-1}\left(\frac{1.55 \sin 30.0°}{1}\right) = \boxed{50.8°}$$

(c), (d) The other entries are computed similarly, and are shown in the table below.

| (c) air into glass, angles in degrees | | | (d) glass into air, angles in degrees | | |
|---|---|---|---|---|---|
| incidence | reflection | refraction | incidence | reflection | refraction |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 10.0 | 10.0 | 6.43 | 10.0 | 10.0 | 15.6 |
| 20.0 | 20.0 | 12.7 | 20.0 | 20.0 | 32.0 |
| 30.0 | 30.0 | 18.8 | 30.0 | 30.0 | 50.8 |
| 40.0 | 40.0 | 24.5 | 40.0 | 40.0 | 85.1 |
| 50.0 | 50.0 | 29.6 | 50.0 | 50.0 | none* |
| 60.0 | 60.0 | 34.0 | 60.0 | 60.0 | none* |
| 70.0 | 70.0 | 37.3 | 70.0 | 70.0 | none* |
| 80.0 | 80.0 | 39.4 | 80.0 | 80.0 | none* |
| 90.0 | 90.0 | 40.2 | 90.0 | 90.0 | none* |

*total internal reflection

**P35.49** For water, $\sin\theta_c = \dfrac{1}{4/3} = \dfrac{3}{4}$.

Thus $\theta_c = \sin^{-1}(0.750) = 48.6°$

and $d = 2\left[(1.00\text{ m})\tan\theta_c\right]$

$$d = (2.00\text{ m})\tan 48.6° = \boxed{2.27\text{ m}}.$$

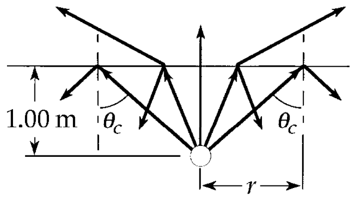


**FIG. P35.49**

**P35.50** Call $\theta_1$ the angle of incidence and of reflection on the left face and $\theta_2$ those angles on the right face. Let $\alpha$ represent the complement of $\theta_1$ and $\beta$ be the complement of $\theta_2$. Now $\alpha = \gamma$ and $\beta = \delta$ because they are pairs of alternate interior angles. We have

$$A = \gamma + \delta = \alpha + \beta$$

and $B = \alpha + A + \beta = \alpha + \beta + A = \boxed{2A}$.

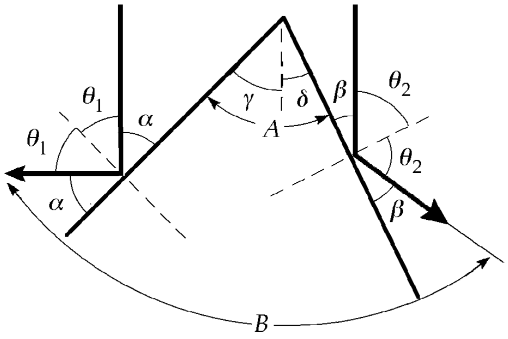


**FIG. P35.50**

**P35.51** (a) We see the Sun moving from east to west across the sky. Its angular speed is

$$\omega=\frac{\Delta\theta}{\Delta t}=\frac{2\pi\text{ rad}}{86\,400\text{ s}}=7.27\times10^{-5}\text{ rad/s}.$$

The direction of sunlight crossing the cell from the window changes at this rate, moving on the opposite wall at speed

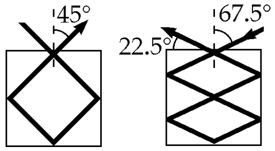
$$v=r\omega=(2.37\text{ m})\left(7.27\times10^{-5}\text{ rad/s}\right)=1.72\times10^{-4}\text{ m/s}=\boxed{0.172\text{ mm/s}}.$$

(b) The mirror folds into the cell the motion that would occur in a room twice as wide:

$$v=r\omega=2(0.174\text{ mm/s})=\boxed{0.345\text{ mm/s}}.$$

(c), (d) As the Sun moves southward and upward at 50.0°, we may regard the corner of the window as fixed, and both patches of light move $\boxed{\text{northward and downward at }50.0°}$.

***P35.52** (a) $\boxed{45.0°}$ as shown in the first figure to the right.



(b) $\boxed{\text{Yes}}$

If grazing angle is halved, the number of reflections from the side faces is doubled.

**FIG. P35.52**

**P35.53** Horizontal light rays from the setting Sun pass above the hiker. The light rays are twice refracted and once reflected, as in Figure (b). The most intense light reaching the hiker, that which represents the visible rainbow, is located between angles of 40° and 42° from the hiker's shadow.

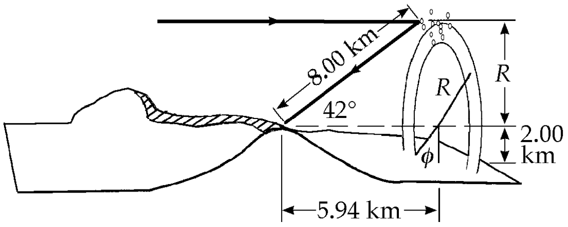


Figure (a)

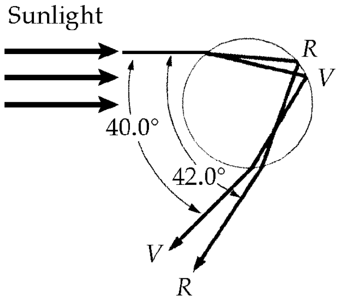


Figure (b)

**FIG. P35.53**

The hiker sees a greater percentage of the violet inner edge, so we consider the red outer edge. The radius $R$ of the circle of droplets is

$$R=(8.00\text{ km})\sin 42.0°=5.35\text{ km}.$$

Then the angle $\phi$, between the vertical and the radius where the bow touches the ground, is given by

$$\cos\phi=\frac{2.00\text{ km}}{R}=\frac{2.00\text{ km}}{5.35\text{ km}}=0.374$$

or $\phi=68.1°$.

The angle filled by the visible bow is $360°-(2\times68.1°)=224°$

so the visible bow is $\frac{224°}{360°}=\boxed{62.2\%\text{ of a circle}}$.

**P35.54** Light passing the top of the pole makes an angle of incidence $\phi_1 = 90.0°-\theta$. It falls on the water surface at distance from the pole

$$s_1 = \frac{L-d}{\tan\theta}$$

and has an angle of refraction $\phi_2$ from $1.00\sin\phi_1 = n\sin\phi_2$.

Then $s_2 = d\tan\phi_2$

and the whole shadow length is

$$s_1+s_2 = \frac{L-d}{\tan\theta} + d\tan\left(\sin^{-1}\left(\frac{\sin\phi_1}{n}\right)\right)$$

$$s_1+s_2 = \frac{L-d}{\tan\theta} + d\tan\left(\sin^{-1}\left(\frac{\cos\theta}{n}\right)\right)$$

$$= \frac{2.00\text{ m}}{\tan 40.0°} + (2.00\text{ m})\tan\left(\sin^{-1}\left(\frac{\cos 40.0°}{1.33}\right)\right) = \boxed{3.79\text{ m}}$$

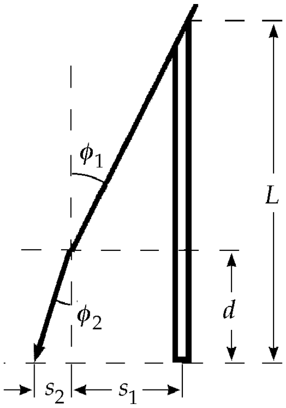


**FIG. P35.54**

**P35.55** As the beam enters the slab,

$$1.00\sin 50.0° = 1.48\sin\theta_2$$

giving $\theta_2 = 31.2°$.

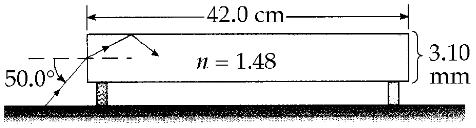


**FIG. P35.55**

The beam then strikes the top of the slab at $x_1 = \dfrac{1.55\text{ mm}}{\tan 31.2°}$ from the left end. Thereafter, the beam strikes a face each time it has traveled a distance of $2x_1$ along the length of the slab. Since the slab is 420 mm long, the beam has an additional $420\text{ mm} - x_1$ to travel after the first reflection. The number of additional reflections is

$$\frac{420\text{ mm}-x_1}{2x_1} = \frac{420\text{ mm} - 1.55\text{ mm}/\tan 31.2°}{3.10\text{ mm}/\tan 31.2°} = 81.5 \qquad \text{or 81 reflections}$$

since the answer must be an integer. The total number of reflections made in the slab is then $\boxed{82}$.

**P35.56** (a) $$\frac{S_1'}{S_1} = \left[\frac{n_2-n_1}{n_2+n_1}\right]^2 = \left[\frac{1.52-1.00}{1.52+1.00}\right]^2 = \boxed{0.042\,6}$$

(b) If medium 1 is glass and medium 2 is air, $\dfrac{S_1'}{S_1} = \left[\dfrac{n_2-n_1}{n_2+n_1}\right]^2 = \left[\dfrac{1.00-1.52}{1.00+1.52}\right]^2 = 0.042\,6$.

There is $\boxed{\text{no difference}}$.

**P35.57** (a) With $n_1 = 1$

and $n_2 = n$

the reflected fractional intensity is $\dfrac{S_1'}{S_1} = \left(\dfrac{n-1}{n+1}\right)^2$.

The remaining intensity must be transmitted:

$$\frac{S_2}{S_1} = 1-\left(\frac{n-1}{n+1}\right)^2 = \frac{(n+1)^2-(n-1)^2}{(n+1)^2} = \frac{n^2+2n+1-n^2+2n-1}{(n+1)^2} = \boxed{\frac{4n}{(n+1)^2}}.$$

*continued on next page*

(b) At entry, $\frac{S_2}{S_1} = 1 - \left(\frac{n-1}{n+1}\right)^2 = \frac{4(2.419)}{(2.419+1)^2} = 0.828$.

At exit, $\frac{S_3}{S_2} = 0.828$.

Overall, $\frac{S_3}{S_1} = \left(\frac{S_3}{S_2}\right)\left(\frac{S_2}{S_1}\right) = (0.828)^2 = 0.685$

or $\boxed{68.5\%}$.

**P35.58** Define $T = \frac{4n}{(n+1)^2}$ as the transmission coefficient for one encounter with an interface. For diamond and air, it is 0.828, as in Problem 57.

As shown in the figure, the total amount transmitted is

$$T^2 + T^2(1-T)^2 + T^2(1-T)^4 + T^2(1-T)^6$$
$$+\ldots + T^2(1-T)^{2n} + \ldots$$

We have $1 - T = 1 - 0.828 = 0.172$ so the total transmission is

$$(0.828)^2\left[1 + (0.172)^2 + (0.172)^4 + (0.172)^6 + \ldots\right].$$

To sum this series, define $F = 1 + (0.172)^2 + (0.172)^4 + (0.172)^6 + \ldots$.

Note that $(0.172)^2 F = (0.172)^2 + (0.172)^4 + (0.172)^6 + \ldots$, and

$1 + (0.172)^2 F = 1 + (0.172)^2 + (0.172)^4 + (0.172)^6 + \ldots = F$.

Then, $1 = F - (0.172)^2 F$ or $F = \frac{1}{1-(0.172)^2}$.

The overall transmission is then $\frac{(0.828)^2}{1-(0.172)^2} = 0.706$ or $\boxed{70.6\%}$.

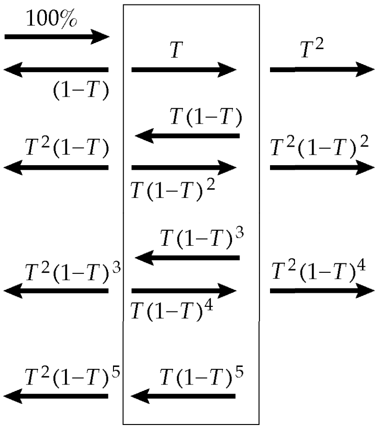


FIG. P35.58

**P35.59** Define $n_1$ to be the index of refraction of the surrounding medium and $n_2$ to be that for the prism material. We can use the critical angle of 42.0° to find the ratio $\frac{n_2}{n_1}$:

$$n_2 \sin 42.0° = n_1 \sin 90.0°.$$

So, $\frac{n_2}{n_1} = \frac{1}{\sin 42.0°} = 1.49$.

Call the angle of refraction $\theta_2$ at the surface 1. The ray inside the prism forms a triangle with surfaces 1 and 2, so the sum of the interior angles of this triangle must be 180°.

Thus, $(90.0° - \theta_2) + 60.0° + (90.0° - 42.0°) = 180°$.

Therefore, $\theta_2 = 18.0°$.

Applying Snell's law at surface 1, $n_1 \sin\theta_1 = n_2 \sin 18.0°$

$$\sin\theta_1 = \left(\frac{n_2}{n_1}\right)\sin\theta_2 = 1.49 \sin 18.0° \qquad \boxed{\theta_1 = 27.5°}.$$

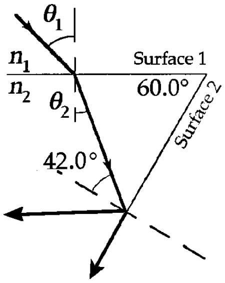


FIG. P35.59

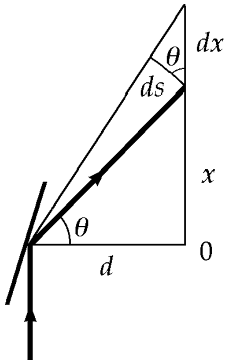


FIG. P35.60

***P35.60** (a) As the mirror turns through angle $\theta$, the angle of incidence increases by $\theta$ and so does the angle of reflection. The incident ray is stationary, so the reflected ray turns through angle $2\theta$. The angular speed of the reflected ray is $2\omega_m$. The speed of the dot of light on the circular wall is $\boxed{2\omega_m R}$.

(b) The two angles marked $\theta$ in the figure to the right are equal because their sides are perpendicular, right side to right side and left side to left side.

We have $\cos\theta = \dfrac{d}{\sqrt{x^2+d^2}} = \dfrac{ds}{dx}$

and $\dfrac{ds}{dt} = 2\omega_m\sqrt{x^2+d^2}$.

So $\dfrac{dx}{dt} = \dfrac{ds}{dt}\dfrac{\sqrt{x^2+d^2}}{d} = \boxed{2\omega_m \dfrac{x^2+d^2}{d}}$.

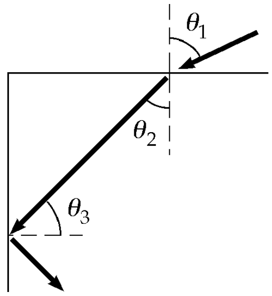


FIG. P35.61

**P35.61** (a) For polystyrene *surrounded by air*, internal reflection requires

$$\theta_3 = \sin^{-1}\left(\frac{1.00}{1.49}\right) = 42.2°.$$

Then from geometry, $\theta_2 = 90.0° - \theta_3 = 47.8°$.

From Snell's law, $\sin\theta_1 = 1.49\sin 47.8° = 1.10$.

This has no solution.

Therefore, total internal reflection $\boxed{\text{always happens}}$.

(b) For polystyrene *surrounded by water*, $\theta_3 = \sin^{-1}\left(\dfrac{1.33}{1.49}\right) = 63.2°$

and $\theta_2 = 26.8°$.

From Snell's law, $\theta_1 = \boxed{30.3°}$.

(c) $\boxed{\text{No internal refraction is possible}}$

since the beam is initially traveling in a medium of lower index of refraction.

***P35.62** The picture illustrates optical sunrise. At the center of the earth,

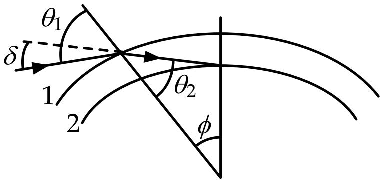


**FIG. P35.62**

$$\cos\phi = \frac{6.37\times10^6\ \text{m}}{6.37\times10^6\ \text{m}+8\,614}$$

$$\phi = 2.98°$$

$$\theta_2 = 90 - 2.98° = 87.0°$$

At the top of the atmosphere

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$1 \sin\theta_1 = 1.000\,293 \sin 87.0°$$

$$\theta_1 = 87.4°$$

Deviation upon entry is

$$\delta = |\theta_1 - \theta_2|$$

$$\delta = 87.364° – 87.022° = 0.342°$$

Sunrise of the optical day is before geometric sunrise by $0.342°\left(\frac{86\,400\ \text{s}}{360°}\right) = 82.2\ \text{s}$. Optical sunset occurs later too, so the optical day is longer by $\boxed{164\ \text{s}}$.

**P35.63** $\tan\theta_1 = \frac{4.00\ \text{cm}}{h}$

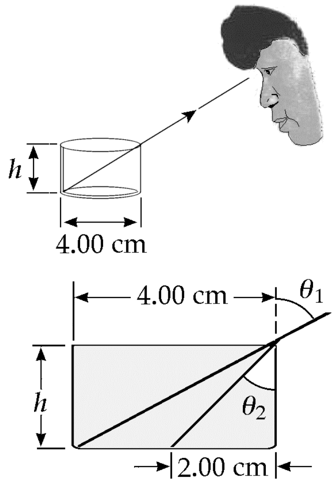


**FIG. P35.63**

and $\tan\theta_2 = \frac{2.00\ \text{cm}}{h}$

$$\tan^2\theta_1 = (2.00\tan\theta_2)^2 = 4.00\tan^2\theta_2$$

$$\frac{\sin^2\theta_1}{1-\sin^2\theta_1} = 4.00\left(\frac{\sin^2\theta_2}{1-\sin^2\theta_2}\right). \qquad (1)$$

Snell's law in this case is: $n_1\sin\theta_1 = n_2\sin\theta_2$

$$\sin\theta_1 = 1.333\sin\theta_2.$$

Squaring both sides, $\sin^2\theta_1 = 1.777\sin^2\theta_2. \qquad (2)$

Substituting (2) into (1), $\frac{1.777\sin^2\theta_2}{1-1.777\sin^2\theta_2} = 4.00\left(\frac{\sin^2\theta_2}{1-\sin^2\theta_2}\right).$

Defining $x = \sin^2\theta$, $\frac{0.444}{1-1.777x} = \frac{1}{1-x}.$

Solving for $x$, $0.444 - 0.444x = 1 - 1.777x$ and $x = 0.417.$

From $x$ we can solve for $\theta_2$: $\theta_2 = \sin^{-1}\sqrt{0.417} = 40.2°.$

Thus, the height is $h = \frac{2.00\ \text{cm}}{\tan\theta_2} = \frac{2.00\ \text{cm}}{\tan 40.2°} = \boxed{2.36\ \text{cm}}.$

**P35.64** $\delta = \theta_1 - \theta_2 = 10.0°$ and $n_1 \sin\theta_1 = n_2 \sin\theta_2$

with $n_1 = 1$, $n_2 = \frac{4}{3}$.

Thus, $\theta_1 = \sin^{-1}(n_2 \sin\theta_2) = \sin^{-1}[n_2 \sin(\theta_1 - 10.0°)]$.

(You can use a calculator to home in on an approximate solution to this equation, testing different values of $\theta_1$ until you find that $\theta_1 = \boxed{36.5°}$. Alternatively, you can solve for $\theta_1$ exactly, as shown below.)

We are given that $\sin\theta_1 = \frac{4}{3}\sin(\theta_1 - 10.0°)$.

This is the sine of a difference, so $\frac{3}{4}\sin\theta_1 = \sin\theta_1 \cos 10.0° - \cos\theta_1 \sin 10.0°$.

Rearranging, $\sin 10.0° \cos\theta_1 = \left(\cos 10.0° - \frac{3}{4}\right)\sin\theta_1$

$\frac{\sin 10.0°}{\cos 10.0° - 0.750} = \tan\theta_1$ and $\theta_1 = \tan^{-1}(0.740) = \boxed{36.5°}$.

**P35.65** To derive the law of *reflection*, locate point *O* so that the time of travel from point *A* to point *B* will be minimum.

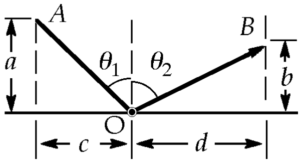


FIG. P35.65

The *total* light path is $L = a\sec\theta_1 + b\sec\theta_2$.

The time of travel is $t = \left(\frac{1}{v}\right)(a\sec\theta_1 + b\sec\theta_2)$.

If point *O* is displaced by *dx*, then

$$dt = \left(\frac{1}{v}\right)(a\sec\theta_1 \tan\theta_1 d\theta_1 + b\sec\theta_2 \tan\theta_2 d\theta_2) = 0 \qquad (1)$$

(since for minimum time $dt = 0$).

Also, $c + d = a\tan\theta_1 + b\tan\theta_2 = \text{constant}$

so, $$a\sec^2\theta_1 d\theta_1 + b\sec^2\theta_2 d\theta_2 = 0. \qquad (2)$$

Divide equations (1) and (2) to find $\boxed{\theta_1 = \theta_2}$.

**P35.66** Observe in the sketch that the angle of incidence at point *P* is $\gamma$, and using triangle *OPQ*:

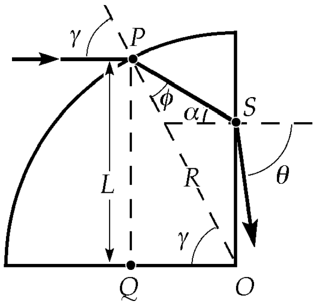


**FIG. P35.66**

$$\sin\gamma = \frac{L}{R}.$$

Also, $\cos\gamma = \sqrt{1-\sin^2\gamma} = \dfrac{\sqrt{R^2-L^2}}{R}.$

Applying Snell's law at point *P*, $1.00\sin\gamma = n\sin\phi$.

Thus, $\sin\phi = \dfrac{\sin\gamma}{n} = \dfrac{L}{nR}$

and $\cos\phi = \sqrt{1-\sin^2\phi} = \dfrac{\sqrt{n^2R^2-L^2}}{nR}.$

From triangle *OPS*, $\phi + (\alpha + 90.0°) + (90.0° - \gamma) = 180°$ or the angle of incidence at point *S* is $\alpha = \gamma - \phi$. Then, applying Snell's law at point *S*

gives $1.00\sin\theta = n\sin\alpha = n\sin(\gamma - \phi)$

or $\sin\theta = n[\sin\gamma\cos\phi - \cos\gamma\sin\phi] = n\left[\left(\dfrac{L}{R}\right)\dfrac{\sqrt{n^2R^2-L^2}}{nR} - \dfrac{\sqrt{R^2-L^2}}{R}\left(\dfrac{L}{nR}\right)\right]$

$$\sin\theta = \frac{L}{R^2}\left(\sqrt{n^2R^2-L^2} - \sqrt{R^2-L^2}\right)$$

and $\theta = \boxed{\sin^{-1}\left[\dfrac{L}{R^2}\left(\sqrt{n^2R^2-L^2} - \sqrt{R^2-L^2}\right)\right]}.$

**P35.67** As shown in the sketch, the angle of incidence at point *A* is:

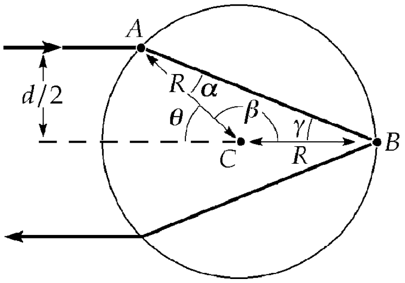


**FIG. P35.67**

$$\theta = \sin^{-1}\left(\frac{d/2}{R}\right) = \sin^{-1}\left(\frac{1.00\text{ m}}{2.00\text{ m}}\right) = 30.0°.$$

If the emerging ray is to be parallel to the incident ray, the path must be symmetric about the centerline *CB* of the cylinder. In the isosceles triangle *ABC*,

$$\gamma = \alpha \quad \text{and} \quad \beta = 180° - \theta.$$

Therefore, $\alpha + \beta + \gamma = 180°$

becomes $2\alpha + 180° - \theta = 180°$

or $\alpha = \dfrac{\theta}{2} = 15.0°.$

Then, applying Snell's law at point *A*,

$$n\sin\alpha = 1.00\sin\theta$$

or $n = \dfrac{\sin\theta}{\sin\alpha} = \dfrac{\sin 30.0°}{\sin 15.0°} = \boxed{1.93}.$

***P35.68** (a) The apparent radius of the glowing sphere is $R_3$ as shown. For it

$$\sin\theta_1 = \frac{R_1}{R_2}$$

$$\sin\theta_2 = \frac{R_3}{R_2}$$

$$n\sin\theta_1 = 1\sin\theta_2$$

$$n\frac{R_1}{R_2} = \frac{R_3}{R_2} \qquad \boxed{R_3 = nR_1}$$

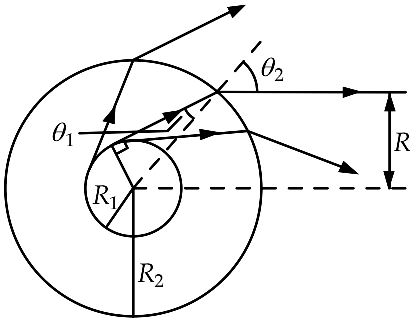


FIG. P35.68(a)

(b) If $nR_1 > R_2$, then $\sin\theta_2$ cannot be equal to $\frac{nR_1}{R_2}$. The ray considered in part (a) undergoes total internal reflection. In this case a ray escaping the atmosphere as shown here is responsible for the apparent radius of the glowing sphere and $\boxed{R_3 = R_2}$.

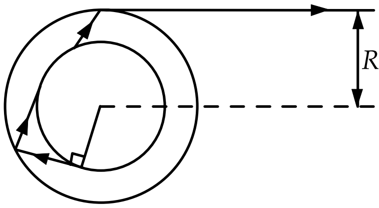


FIG. P35.68(b)

**P35.69** (a) At the boundary of the air and glass, the critical angle is given by $\sin\theta_c = \frac{1}{n}$.

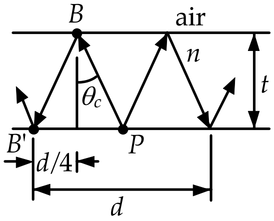


FIG. P35.69

Consider the critical ray $PBB'$: $\tan\theta_c = \frac{d/4}{t}$ or $\frac{\sin\theta_c}{\cos\theta_c} = \frac{d}{4t}$.

Squaring the last equation gives: $\frac{\sin^2\theta_c}{\cos^2\theta_c} = \frac{\sin^2\theta_c}{1-\sin^2\theta_c} = \left(\frac{d}{4t}\right)^2$.

Since $\sin\theta_c = \frac{1}{n}$, this becomes $\frac{1}{n^2-1} = \left(\frac{d}{4t}\right)^2$ or $\boxed{n = \sqrt{1+\left(\frac{4t}{d}\right)^2}}$.

(b) Solving for $d$, $d = \frac{4t}{\sqrt{n^2-1}}$.

Thus, if $n = 1.52$ and $t = 0.600\text{ cm}$, $d = \frac{4(0.600\text{ cm})}{\sqrt{(1.52)^2-1}} = \boxed{2.10\text{ cm}}$.

(c) Since violet light has a larger index of refraction, it will lead to a smaller critical angle and the inner edge of the white halo will be tinged with $\boxed{\text{violet}}$ light.

**P35.70** From the sketch, observe that the angle of incidence at point $A$ is the same as the prism angle $\theta$ at point $O$. Given that $\theta = 60.0°$, application of Snell's law at point $A$ gives

$$1.50\sin\beta = 1.00\sin 60.0° \text{ or } \beta = 35.3° .$$

From triangle $AOB$, we calculate the angle of incidence (and reflection) at point $B$.

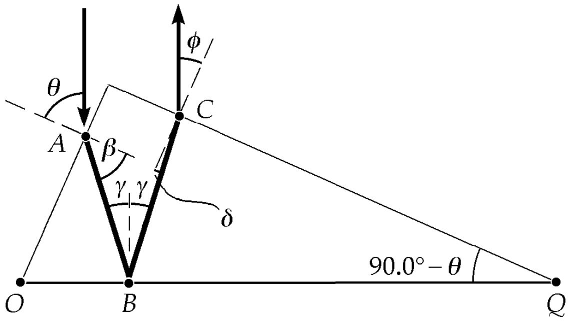


FIG. P35.70

$\theta = (90.0° - \beta) + (90.0° - \gamma) = 180°$ so $\gamma = \theta - \beta = 60.0° - 35.3° = 24.7°$ .

Now, using triangle $BCQ$: $(90.0° - \gamma) + (90.0° - \delta) + (90.0° - \theta) = 180°$ .

Thus the angle of incidence at point $C$ is $\delta = (90.0° - \theta) - \gamma = 30.0° - 24.7° = 5.30°$ .

Finally, Snell's law applied at point $C$ gives $1.00\sin\phi = 1.50\sin 5.30°$

or $\phi = \sin^{-1}(1.50\sin 5.30°) = \boxed{7.96°}$ .

**P35.71** (a) Given that $\theta_1 = 45.0°$ and $\theta_2 = 76.0°$.

Snell's law at the first surface gives

$$n\sin\alpha = 1.00\sin 45.0° \quad (1)$$

Observe that the angle of incidence at the second surface is

$$\beta = 90.0° - \alpha .$$

Thus, Snell's law at the second surface yields

$$n\sin\beta = n\sin(90.0° - \alpha) = 1.00\sin 76.0°$$

or $n\cos\alpha = \sin 76.0°$ . (2)

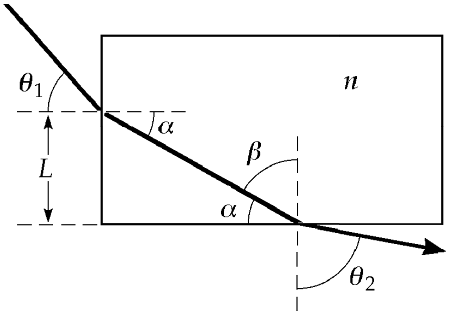


FIG. P35.71

Dividing Equation (1) by Equation (2), $\tan\alpha = \dfrac{\sin 45.0°}{\sin 76.0°} = 0.729$

or $\alpha = 36.1°$ .

Then, from Equation (1), $n = \dfrac{\sin 45.0°}{\sin\alpha} = \dfrac{\sin 45.0°}{\sin 36.1°} = \boxed{1.20}$ .

(b) From the sketch, observe that the distance the light travels in the plastic is $d = \dfrac{L}{\sin\alpha}$. Also, the speed of light in the plastic is $v = \dfrac{c}{n}$, so the time required to travel through the plastic is

$$\Delta t = \frac{d}{v} = \frac{nL}{c\sin\alpha} = \frac{1.20(0.500\text{ m})}{\left(3.00\times10^{8}\text{ m/s}\right)\sin 36.1°} = 3.40\times10^{-9}\text{ s} = \boxed{3.40\text{ ns}} .$$

**P35.72**

| $\sin\theta_1$ | $\sin\theta_2$ | $\dfrac{\sin\theta_1}{\sin\theta_2}$ |
|---|---|---|
| 0.174 | 0.131 | 1.330 4 |
| 0.342 | 0.261 | 1.312 9 |
| 0.500 | 0.379 | 1.317 7 |
| 0.643 | 0.480 | 1.338 5 |
| 0.766 | 0.576 | 1.328 9 |
| 0.866 | 0.647 | 1.339 0 |
| 0.940 | 0.711 | 1.322 0 |
| 0.985 | 0.740 | 1.331 5 |

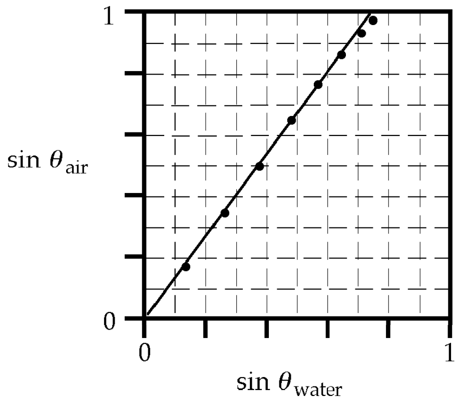


**FIG. P35.72**

The straightness of the graph line demonstrates Snell's proportionality.

The slope of the line is $\bar{n} = 1.327\,6 \pm 0.01$

and $n = \boxed{1.328 \pm 0.8\%}$.

## ANSWERS TO EVEN PROBLEMS

| | |
|---|---|
| **P35.2** | 227 Mm/s |
| **P35.4** | (a) see the solution; (b) 300 Mm/s |
| **P35.6** | (a) 1.94 m; (b) 50.0° above the horizontal: antiparallel to the incident ray |
| **P35.8** | five times by the right-hand mirror and six times by the left-hand mirror |
| **P35.10** | 25.5°; 442 nm |
| **P35.12** | (a) 474 THz; (b) 422 nm; (c) 200 Mm/s |
| **P35.14** | 22.5° |
| **P35.16** | (a) 181 Mm/s; (b) 225 Mm/s; (c) 136 Mm/s |
| **P35.18** | 3.39 m |
| **P35.20** | $\theta_1 = \tan^{-1} n$ |
| **P35.22** | 106 ps |
| **P35.24** | 23.1° |
| **P35.26** | (a) 58.9°; (b) Only if $\theta_1 = \theta_2 = 0$ |
| **P35.28** | see the solution |
| **P35.30** | (a) 41.5°; (b) 18.5°; (c) 27.6°; (d) 42.6° |
| **P35.32** | (a) see the solution; (b) 37.2°; (c) 37.3°; (d) 37.3° |
| **P35.34** | $\sin^{-1}\left(\sqrt{n^2-1}\sin\Phi - \cos\Phi\right)$ |
| **P35.36** | (a) 24.4°; (b) 37.0°; (c) 49.8° |
| **P35.38** | 67.2 |
| **P35.40** | (a) $\dfrac{nd}{n-1}$; (b) yes; (c) 350 $\mu$m |
| **P35.42** | (a) 10.7°; (b) air; (c) Sound falling on the wall from most directions is 100% reflected. |
| **P35.44** | 54.8° east of north |
| **P35.46** | (a) $\dfrac{h}{c}\left(\dfrac{n+1}{2}\right)$; (b) larger by $\dfrac{n+1}{2}$ times |
| **P35.48** | see the solution |
| **P35.50** | see the solution |
| **P35.52** | (a) 45.0°; (b) yes; see the solution |
| **P35.54** | 3.79 m |

**P35.56** (a) 0.042 6; (b) no difference

**P35.58** 0.706

**P35.60** (a) $2\omega_m R$; (b) $2\omega_m \dfrac{x^2+d^2}{d}$

**P35.62** 164 s

**P35.64** 36.5°

**P35.66** $\theta = \sin^{-1}\left[\dfrac{L}{R^2}\left(\sqrt{n^2R^2-L^2}-\sqrt{R^2-L^2}\right)\right]$

**P35.68** (a) $nR_1$; (b) $R_2$

**P35.70** 7.96°

**P35.72** see the solution; $n = 1.328 \pm 0.8\%$

# 36

# Image Formation

## CHAPTER OUTLINE

36.1 Images Formed by Flat Mirrors
36.2 Images Formed by Spherical Mirrors
36.3 Images Formed by Refraction
36.4 Thin Lenses
36.5 Lens Aberrations
36.6 The Camera
36.7 The Eye
36.8 The Simple Magnifier
36.9 The Compound Microscope
36.10 The Telescope

## ANSWERS TO QUESTIONS

**Q36.1** The mirror shown in the textbook picture produces an inverted image. It actually reverses top and bottom. It is not true in the same sense that "Most mirrors reverse left and right." Mirrors don't actually flip images side to side—we just assign the labels "left" and "right" to images as if they were real people mimicking us. If you stand face to face with a real person and raise your left hand, then he or she would have to raise his or her *right* hand to "mirror" your movement. Try this while facing a mirror. For sake of argument, let's assume you are facing north and wear a watch on your left hand, which is on the western side. If you raise your left hand, you might say that your image raises its right hand, based on the labels we assign to other people. But your image raises its western-side hand, which is the hand with the watch.

**Q36.2** With a concave spherical mirror, for objects beyond the focal length the image will be real and inverted. For objects inside the focal length, the image will be virtual, upright, and magnified. Try a shaving or makeup mirror as an example.

**Q36.3** With a convex spherical mirror, all images of real objects are upright, virtual and smaller than the object. As seen in Question 36.2, you only get a change of orientation when you pass the focal point—but the focal point of a convex mirror is on the non-reflecting side!

**Q36.4** The mirror equation and the magnification equation apply to plane mirrors. A curved mirror is made flat by increasing its radius of curvature without bound, so that its focal length goes to infinity. From $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}=0$ we have $\frac{1}{p}=-\frac{1}{q}$; therefore, $p=-q$. The virtual image is as far behind the mirror as the object is in front. The magnification is $M=-\frac{q}{p}=\frac{p}{p}=1$. The image is right side up and actual size.

**Q36.5** Stones at the bottom of a clear stream always appears closer to the surface because light is refracted away from the normal at the surface. Example 36.8 in the textbook shows that its apparent depth is three quarters of its actual depth.

**Q36.6** For definiteness, we consider real objects ($p > 0$).

(a) For $M = -\frac{q}{p}$ to be negative, $q$ must be positive. This will happen in $\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$ if $p > f$, if the object is farther than the focal point.

(b) For $M = -\frac{q}{p}$ to be positive, $q$ must be negative.
From $\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$ we need $p < f$.

(c) For a real image, $q$ must be positive.
As in part (a), it is sufficient for $p$ to be larger than $f$.

(d) For $q < 0$ we need $p < f$.

(e) For $|M| > 1$, we consider separately $M < -1$ and $M > 1$.

If $M = -\frac{q}{p} < -1$, we need $\frac{q}{p} > 1$ or $q > p$

or $\frac{1}{q} < \frac{1}{p}$

From $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$, $\frac{1}{p} + \frac{1}{p} > \frac{1}{f}$ or $\frac{2}{p} > \frac{1}{f}$

or $\frac{p}{2} < f$ or $p < 2f$.

Now if $-\frac{q}{p} > 1$ or $-q > p$ or $q < -p$

we may require $q < 0$, since then $\frac{1}{p} = \frac{1}{f} - \frac{1}{q}$ with $\frac{1}{f} > 0$

gives $\frac{1}{p} > -\frac{1}{q}$ as required or $-p > q$.

For $q < 0$ in $\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$ we need $p < f$.

Thus the overall condition for an enlarged image is simply $p < 2f$.

(f) For $|M| < 1$, we have the reverse of part (e), requiring $p > 2f$.

**Q36.7** Using the same analysis as in Question 36.6 except $f < 0$.

(a) Never.

(b) Always.

(c) Never, for light rays passing through the lens will always diverge.

(d) Always.

(e) Never.

(f) Always.

**Q36.8** We assume the lens has a refractive index higher than its surroundings. For the biconvex lens in Figure 36.27(a), $R_1 > 0$ and $R_2 < 0$. Then all terms in $(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ are positive and $f > 0$. For the other two lenses in part (a) of the figure, $R_1$ and $R_2$ are both positive but $R_1$ is less than $R_2$. Then $\frac{1}{R_1} > \frac{1}{R_2}$ and the focal length is again positive.

For the biconcave lens and the plano-concave lens in Figure 36.27(b), $R_1 < 0$ and $R_2 > 0$. Then both terms are negative in $\frac{1}{R_1}-\frac{1}{R_2}$ and the focal length is negative. For the middle lens in part (b) of the figure, $R_1$ and $R_2$ are both positive but $R_1$ is greater than $R_2$. Then $\frac{1}{R_1} < \frac{1}{R_2}$ and the focal length is again negative.

**Q36.9** Both words are inverted. However OXIDE has up-down symmetry whereas LEAD does not.

**Q36.10** An infinite number. In general, an infinite number of rays leave each point of any object and travel in all directions. Note that the three principal rays that we use for imaging are just a subset of the infinite number of rays. All three principal rays can be drawn in a ray diagram, provided that we extend the plane of the lens as shown in Figure Q36.10.

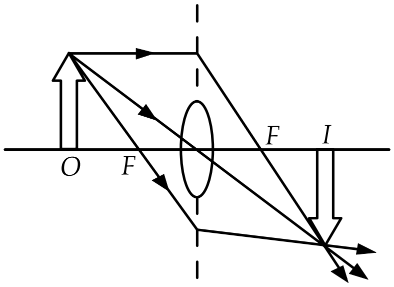


**FIG. Q36.10**

**Q36.11** In this case, the index of refraction of the lens material is less than that of the surrounding medium. Under these conditions, a biconvex lens will be diverging.

**Q36.12** Chromatic aberration arises because a material medium's refractive index can be frequency dependent. A mirror changes the direction of light by reflection, not refraction. Light of all wavelengths follows the same path according to the law of reflection, so no chromatic aberration happens.

**Q36.13** This is a convex mirror. The mirror gives the driver a wide field of view and an upright image with the possible disadvantage of having objects appear diminished. Your brain can then interpret them as farther away than the objects really are.

**Q36.14** As pointed out in Question 36.11, if the converging lens is immersed in a liquid with an index of refraction significantly greater than that of the lens itself, it will make light from a distant source diverge. This is not the case with a converging (concave) mirror, as the law of reflection has nothing to do with the indices of refraction.

**Q36.15** As in the diagram, let the center of curvature *C* of the fishbowl and the bottom of the fish define the optical axis, intersecting the fishbowl at vertex *V*. A ray from the top of the fish that reaches the bowl surface along a radial line through *C* has angle of incidence zero and angle of refraction zero. This ray exits from the bowl unchanged in direction. A ray from the top of the fish to *V* is refracted to bend away from the normal. Its extension back inside the fishbowl determines the location of the image and the characteristics of the image. The image is upright, virtual, and enlarged.

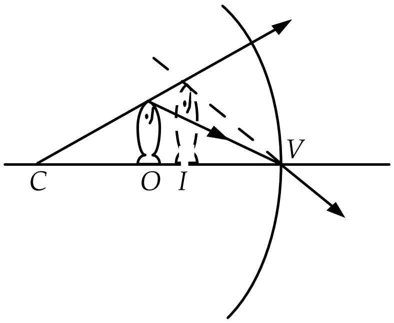


**FIG. Q36.15**

**Q36.16** Because when you look at the ƎƆИA⅃UᗺMA in your rear view mirror, the apparent left-right inversion clearly displays the name of the AMBULANCE behind you. Do not jam on your brakes when a MIAMI city bus is right behind you.

**Q36.17** The entire image is visible, but only at half the intensity. Each point on the object is a source of rays that travel in all directions. Thus, light from all parts of the object goes through all unblocked parts of the lens and forms an image. If you block part of the lens, you are blocking some of the rays, but the remaining ones still come from all parts of the object.

**Q36.18** With the meniscus design, when you direct your gaze near the outer circumference of the lens you receive a ray that has passed through glass with more nearly parallel surfaces of entry and exit. Thus, the lens minimally distorts the direction to the object you are looking at. If you wear glasses, turn them around and look through them the wrong way to maximize this distortion.

**Q36.19** The eyeglasses on the left are diverging lenses that correct for nearsightedness. If you look carefully at the edge of the person's face through the lens, you will see that everything viewed through these glasses is reduced in size. The eyeglasses on the right are converging lenses, which correct for farsightedness. These lenses make everything that is viewed through them look larger.

**Q36.20** The eyeglass wearer's eye is at an object distance from the lens that is quite small—the eye is on the order of $10^{-2}$ meter from the lens. The focal length of an eyeglass lens is several decimeters, positive or negative. Therefore the image distance will be similar in magnitude to the object distance. The onlooker sees a sharp image of the eye behind the lens. Look closely at the left side of Figure Q36.19 and notice that the wearer's eyes seem not only to be smaller, but also positioned a bit behind the plane of his face—namely where they would be if he was not wearing glasses. Similarly, in the right half of Figure Q36.19, his eyes seem to be in front of the plane of his face and magnified. We as observers take this light information coming from the object through the lens and perceive or photograph the image as if it were an object.

**Q36.21** In the diagram, only two of the three principal rays have been used to locate images to reduce the amount of visual clutter. The upright shaded arrows are the objects, and the correspondingly numbered inverted arrows are the images. As you can see, object 2 is closer to the focal point than object 1, and image 2 is farther to the left than image 1.

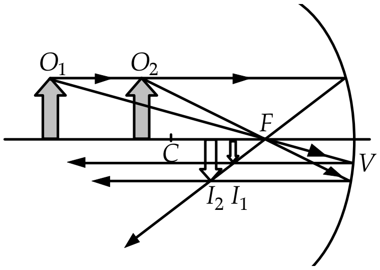


**FIG. Q36.21**

**Q36.22** Absolutely. Only absorbed light, not transmitted light, contributes internal energy to a transparent object. A clear lens can stay ice-cold and solid as megajoules of light energy pass through it.

**Q36.23** One can change the *f* number either by changing the focal length (if using a "zoom" lens) or by changing the aperture of the camera lens. As the *f* number increases, the exposure time required increases also, as both increasing the focal length or decreasing the aperture decreases the light intensity reaching the film.

**Q36.24** Make the mirror an efficient reflector (shiny). Make it reflect to the image even rays far from the axis, by giving it a parabolic shape. Most important, make it large in diameter to intercept a lot of solar power. And you get higher temperature if the image is smaller, as you get with shorter focal length; and if the furnace enclosure is an efficient absorber (black).

**Q36.25** For the explanation, we ignore the lens and consider two objects. Hold your two thumbs parallel and extended upward in front of you, at different distances from your nose. Alternately close your left eye and your right eye. You see both thumbs jump back and forth against the background of more distant objects. Parallax by definition is this apparent motion of a stationary object (one thumb) caused by motion of the observer (jumping from right eye to left eye). Your nearer thumb jumps by a larger angle against the background than your farther thumb does. They will jump by the same amount only if they are equally distant from your face. The method of parallax for adjusting one object so that it is the same distance away from you as another object will work even if one 'object' is an image.

**Q36.26** The artist's statements are accurate, perceptive, and eloquent. The image you see is "almost one's whole surroundings," including things behind you and things farther in front of you than the globe is, but nothing eclipsed by the opaque globe or by your head. For example, we cannot see Escher's index and middle fingers or their reflections in the globe.

The point halfway between your eyes is indeed the focus in a figurative sense, but it is not an optical focus. The principal axis will always lie in a line that runs through the center of the sphere and the bridge of your nose. Outside the globe, you are at the center of your observable universe. If you wink at the ball, the center of the looking-glass world hops over to the location of the image of your open eye.

**Q36.27** The three mirrors, two of which are shown as M and N in the figure to the right, reflect any incident ray back parallel to its original direction. When you look into the corner you see image $I_3$ of yourself.

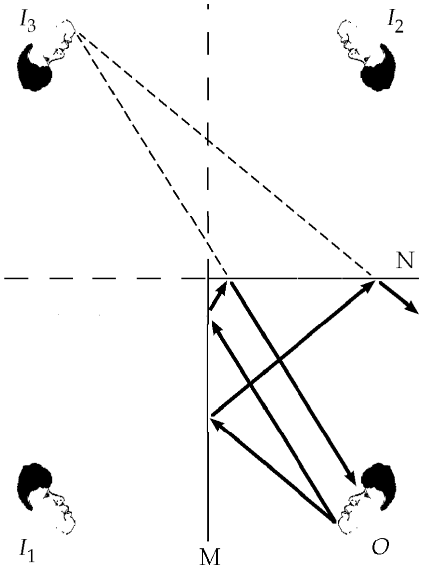


FIG. Q36.21

**Q36.28** You have likely seen a Fresnel mirror for sound. The diagram represents first a side view of a band shell. It is a concave mirror for sound, designed to channel sound into a beam toward the audience in front of the band shell. Sections of its surface can be kept at the right orientations as they are pushed around inside a rectangular box to form an auditorium with good diffusion of sound from stage to audience, with a floor plan suggested by the second part of the diagram.

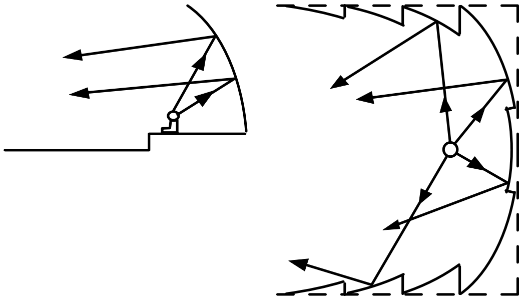


**FIG. Q36.28**

## SOLUTIONS TO PROBLEMS

### Section 36.1 **Images Formed by Flat Mirrors**

**P36.1** I stand 40 cm from my bathroom mirror. I scatter light, which travels to the mirror and back to me in time

$$\frac{0.8\text{ m}}{3\times10^{8}\text{ m/s}}\boxed{\sim10^{-9}\text{ s}}$$

showing me a view of myself as I was at that look-back time. I'm no Dorian Gray!

**P36.2** The virtual image is as far behind the mirror as the choir is in front of the mirror. Thus, the image is 5.30 m behind the mirror. The image of the choir is $0.800\text{ m}+5.30\text{ m}=6.10\text{ m}$ from the organist. Using similar triangles:

$$\frac{h'}{0.600\text{ m}}=\frac{6.10\text{ m}}{0.800\text{ m}}$$

or
$$h'=(0.600\text{ m})\left(\frac{6.10\text{ m}}{0.800\text{ m}}\right)=\boxed{4.58\text{ m}}.$$

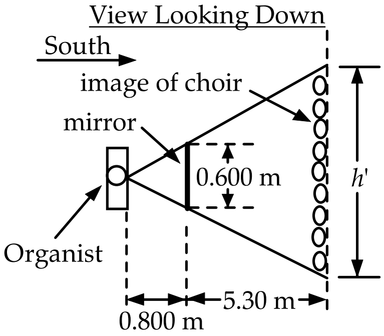


**FIG. P36.2**

**P36.3** The flatness of the mirror is described

by $R = \infty,\ f = \infty$

and $\frac{1}{f} = 0$.

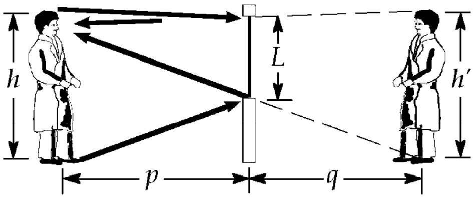


**FIG. P36.3**

By our general mirror equation,

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} = 0$$

or $q = -p$.

Thus, the image is as far behind the mirror as the person is in front. The magnification is then

$$M = \frac{-q}{p} = 1 = \frac{h'}{h}$$

so $h' = h = 70.0$ inches.

The required height of the mirror is defined by the triangle from the person's eyes to the top and bottom of his image, as shown. From the geometry of the triangle, we see that the mirror height must be:

$$h'\left(\frac{p}{p-q}\right) = h'\left(\frac{p}{2p}\right) = \frac{h'}{2}.$$

Thus, the mirror must be $\boxed{\text{at least 35.0 inches high}}$.

**P36.4** A graphical construction produces 5 images, with images $I_1$ and $I_2$ directly into the mirrors from the object $O$,

and $(O, I_3, I_4)$

and $(I_2, I_1, I_5)$

forming the vertices of equilateral triangles.

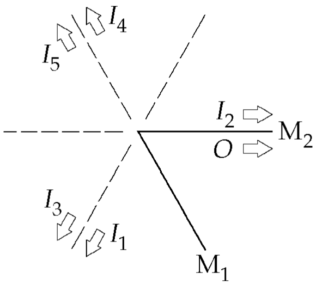


**FIG. P36.4**

**P36.5** (1) The first image in the left mirror is 5.00 ft behind the mirror, or $\boxed{10.0\text{ ft}}$ from the position of the person.

(2) The first image in the right mirror is located 10.0 ft behind the right mirror, but this location is 25.0 ft from the left mirror. Thus, the second image in the left mirror is 25.0 ft behind the mirror, or $\boxed{30.0\text{ ft}}$ from the person.

(3) The first image in the left mirror forms an image in the right mirror. This first image is 20.0 ft from the right mirror, and, thus, an image 20.0 ft behind the right mirror is formed. This image in the right mirror also forms an image in the left mirror. The distance from this image in the right mirror to the left mirror is 35.0 ft. The third image in the left mirror is, thus, 35.0 ft behind the mirror, or $\boxed{40.0\text{ ft}}$ from the person.

***P36.6** (a) The flat mirrors have

$$R \to \infty$$

and $f \to \infty$.

The upper mirror $M_1$ produces a virtual, actual sized image $I_1$ according to

$$\frac{1}{p_1}+\frac{1}{q_1}=\frac{1}{f}=\frac{1}{\infty}=0$$
$$q_1=-p_1$$

with $M_1=-\frac{q_1}{p_1}=+1$.

As shown, this image is above the upper mirror. It is the object for mirror $M_2$, at object distance

$$p_2=p_1+h.$$

The lower mirror produces a virtual, actual-size, right-side-up image according to

$$\frac{1}{p_2}+\frac{1}{q_2}=0$$
$$q_2=-p_2=-(p_1+h)$$

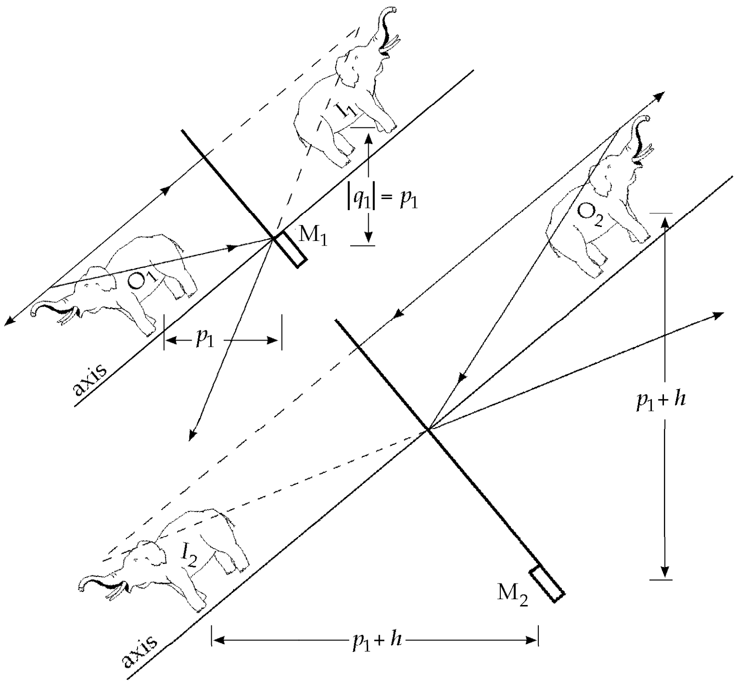


FIG. P36.6

with $M_2=-\frac{q_2}{p_2}=+1$ and $M_{\text{overall}}=M_1M_2=1$.

Thus the final image is at distance $\boxed{p_1+h}$ behind the lower mirror.

(b) It is $\boxed{\text{virtual}}$.

(c) $\boxed{\text{Upright}}$

(d) With magnification $\boxed{+1}$.

(e) It $\boxed{\text{does not appear to be reversed}}$ left and right. In a top view of the periscope, parallel rays from the right and left sides of the object stay parallel and on the right and left.

## Section 36.2 **Images Formed by Spherical Mirrors**

**P36.7** For a concave mirror, both *R* and *f* are positive.

We also know that $$f = \frac{R}{2} = 10.0\text{ cm}.$$

(a) $$\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{10.0\text{ cm}} - \frac{1}{40.0\text{ cm}} = \frac{3}{40.0\text{ cm}}$$

and $$\boxed{q = 13.3\text{ cm}}$$

$$M = \frac{q}{p} = -\frac{13.3\text{ cm}}{40.0\text{ cm}} = \boxed{-0.333}.$$

The image is 13.3 cm in front of the mirror, $\boxed{\text{real, and inverted}}$.

(b) $$\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{10.0\text{ cm}} - \frac{1}{20.0\text{ cm}} = \frac{1}{20.0\text{ cm}}$$

and $$\boxed{q = 20.0\text{ cm}}$$

$$M = \frac{q}{p} = -\frac{20.0\text{ cm}}{20.0\text{ cm}} = \boxed{-1.00}.$$

The image is 20.0 cm in front of the mirror, $\boxed{\text{real, and inverted}}$.

(c) $$\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{10.0\text{ cm}} - \frac{1}{10.0\text{ cm}} = 0$$

Thus, $q$ = infinity.

$\boxed{\text{No image is formed}}$. The rays are reflected parallel to each other.

**P36.8** $$\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = -\frac{1}{0.275\text{ m}} - \frac{1}{10.0\text{ m}} \quad \text{gives} \quad \boxed{q = -0.267\text{ m}}.$$

Thus, the image is $\boxed{\text{virtual}}$.

$$M = \frac{-q}{p} = -\frac{-0.267}{10.0\text{ m}} = \boxed{0.026\,7}$$

Thus, the image is $\boxed{\text{upright}}$ $(+M)$ and $\boxed{\text{diminished}}$ $(|M| < 1)$.

**P36.9** (a) $\frac{1}{p}+\frac{1}{q}=\frac{2}{R}$ gives $\frac{1}{30.0\text{ cm}}+\frac{1}{q}=\frac{2}{(-40.0\text{ cm})}$

$\frac{1}{q}=-\frac{2}{40.0\text{ cm}}-\frac{1}{30.0\text{ cm}}=-0.083\,3\text{ cm}^{-1}$ so $q=\boxed{-12.0\text{ cm}}$

$$M=\frac{-q}{p}=-\frac{(-12.0\text{ cm})}{30.0\text{ cm}}=\boxed{0.400}.$$

(b) $\frac{1}{p}+\frac{1}{q}=\frac{2}{R}$ gives $\frac{1}{60.0\text{ cm}}+\frac{1}{q}=\frac{2}{(-40.0\text{ cm})}$

$\frac{1}{q}=-\frac{2}{40.0\text{ cm}}-\frac{1}{60.0\text{ cm}}=-0.066\,6\text{ cm}^{-1}$ so $q=\boxed{-15.0\text{ cm}}$

$$M=\frac{-q}{p}=-\frac{(-15.0\text{ cm})}{60.0\text{ cm}}=\boxed{0.250}.$$

(c) Since $M>0$, the images are $\boxed{\text{upright}}$.

**P36.10** With radius 2.50 m, the cylindrical wall is a highly efficient mirror for sound, with focal length

$$f=\frac{R}{2}=1.25\text{ m}.$$

In a vertical plane the sound disperses as usual, but that radiated in a horizontal plane is concentrated in a sound image at distance $q$ from the back of the niche, where

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f} \quad \text{so} \quad \frac{1}{2.00\text{ m}}+\frac{1}{q}=\frac{1}{1.25\text{ m}}$$

$$q=\boxed{3.33\text{ m}}.$$

**P36.11** (a) $\frac{1}{p}+\frac{1}{q}=\frac{2}{R}$ becomes $\frac{1}{q}=\frac{2}{60.0\text{ cm}}-\frac{1}{90.0\text{ cm}}$

$q=\boxed{45.0\text{ cm}}$ and $M=\frac{-q}{p}=-\frac{45.0\text{ cm}}{90.0\text{ cm}}=\boxed{-0.500}$.

(b) $\frac{1}{p}+\frac{1}{q}=\frac{2}{R}$ becomes $\frac{1}{q}=\frac{2}{60.0\text{ cm}}-\frac{1}{20.0\text{ cm}}$

$q=\boxed{-60.0\text{ cm}}$ and $M=\frac{-q}{p}=-\frac{(-60.0\text{ cm})}{(20.0\text{ cm})}=\boxed{3.00}$.

(c) The image (a) is real, inverted and diminished. That of (b) is virtual, upright, and enlarged. The ray diagrams are similar to Figure 36.15(a) and 36.15(b) in the text, respectively.

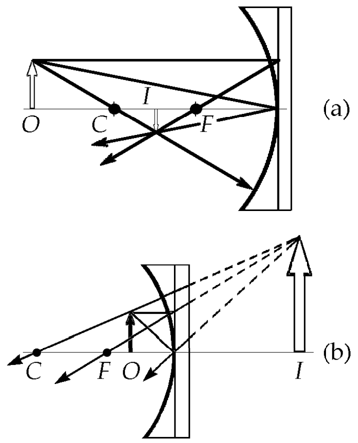


FIG. P36.11

**P36.12** For a concave mirror, $R$ and $f$ are positive. Also, for an erect image, $M$ is positive. Therefore,

$$M=-\frac{q}{p}=4 \text{ and } q=-4p.$$

$$\frac{1}{f}=\frac{1}{p}+\frac{1}{q} \text{ becomes } \frac{1}{40.0\text{ cm}}=\frac{1}{p}-\frac{1}{4p}=\frac{3}{4p}\text{; from which, } p=\boxed{30.0\text{ cm}}.$$

***P36.13** The ball is a convex mirror with $R=-4.25$ cm and $f=\frac{R}{2}=-2.125$ cm. We have

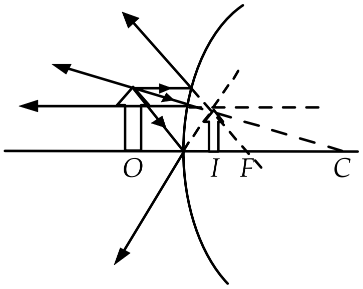


FIG. P36.13

$$M=\frac{3}{4}=-\frac{q}{p}$$

$$q=-\frac{3}{4}p$$

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

$$\frac{1}{p}+\frac{1}{-(3/4)p}=\frac{1}{-2.125\text{ cm}}$$

$$\frac{3}{3p}-\frac{4}{3p}=\frac{1}{-2.125\text{ cm}}$$

$$3p=2.125\text{ cm}$$

$\boxed{p=0.708\text{ cm}}$ in front of the sphere.

The image is upright, virtual, and diminished.

***P36.14** (a) $M=-4=-\frac{q}{p}$ $\quad q=4p$

$q-p=0.60\text{ m}=4p-p$ $\quad p=0.2\text{ m}$ $\quad q=0.8\text{ m}$

$\frac{1}{f}=\frac{1}{p}+\frac{1}{q}=\frac{1}{0.2\text{ m}}+\frac{1}{0.8\text{ m}}$ $\quad f=\boxed{160\text{ mm}}$

(b) $M=+\frac{1}{2}=-\frac{q}{p}$ $\quad p=-2q$

$|q|+p=0.20\text{ m}=-q+p=-q-2q$

$q=-66.7\text{ mm}$ $\quad p=133\text{ mm}$

$\frac{1}{p}+\frac{1}{q}=\frac{2}{R}=\frac{1}{0.133\text{ m}}+\frac{1}{-0.066\,7\text{ m}}$ $\quad R=\boxed{-267\text{ mm}}$

***P36.15** $M = -\dfrac{q}{p}$

$q = -Mp = -0.013(30\text{ cm}) = -0.39\text{ cm}$

$\dfrac{1}{p} + \dfrac{1}{q} = \dfrac{1}{f} = \dfrac{2}{R}$

$\dfrac{1}{30\text{ cm}} + \dfrac{1}{-0.39\text{ cm}} = \dfrac{2}{R}$

$R = \dfrac{2}{-2.53\text{ m}^{-1}} = -0.790\text{ cm}$

The cornea is convex, with radius of curvature $\boxed{0.790\text{ cm}}$.

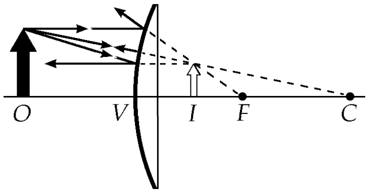


**FIG. P36.15**

***P36.16** With

$M = \dfrac{h'}{h} = \dfrac{+4.00\text{ cm}}{10.0\text{ cm}} = +0.400 = -\dfrac{q}{p}$

$q = -0.400p$

the image must be virtual.

(a) It is a $\boxed{\text{convex}}$ mirror that produces a diminished upright virtual image.

(b) We must have

$p + |q| = 42.0\text{ cm} = p - q$

$p = 42.0\text{ cm} + q$

$p = 42.0\text{ cm} - 0.400p$

$p = \dfrac{42.0\text{ cm}}{1.40} = 30.0\text{ cm}$

The mirror is $\boxed{\text{at the 30.0 cm mark}}$.

(c) $\dfrac{1}{p} + \dfrac{1}{q} = \dfrac{1}{f} = \dfrac{1}{30\text{ cm}} + \dfrac{1}{-0.4(30\text{ cm})} = \dfrac{1}{f} = -0.050\,0/\text{cm}$ $\boxed{f = -20.0\text{ cm}}$

The ray diagram looks like Figure 36.15(c) in the text.

**P36.17** (a) $q = (p + 5.00\text{ m})$ and, since the image must be real,

$M = -\dfrac{q}{p} = -5$ or $q = 5p$.

Therefore, $p + 5.00\text{ m} = 5p$

or $p = 1.25\text{ m}$ and $q = 6.25\text{ m}$.

From $\dfrac{1}{p} + \dfrac{1}{q} = \dfrac{2}{R}$, $R = \dfrac{2pq}{p+q} = \dfrac{2(1.25)(6.25)}{1.25 + 6.25}$

$= \boxed{2.08\text{ m(concave)}}$

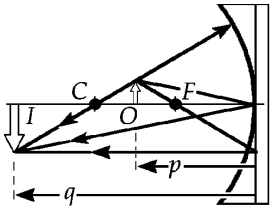


**FIG. P36.17**

(b) From part (a), $p = 1.25\text{ m}$; the mirror should be $\boxed{1.25\text{ m}}$ in front of the object.

**P36.18** Assume that the object distance is the same in both cases (i.e., her face is the same distance from the hubcap regardless of which way it is turned). Also realize that the near image ( $q = -10.0$ cm ) occurs when using the convex side of the hubcap. Applying the mirror equation to both cases gives:

(concave side: $R = |R|$, $q = -30.0$ cm )

$$\frac{1}{p} - \frac{1}{30.0} = \frac{2}{|R|}$$

or

$$\frac{2}{|R|} = \frac{30.0\text{ cm} - p}{(30.0\text{ cm})p} \tag{1}$$

(convex side: $R = -|R|$, $q = -10.0$ cm )

$$\frac{1}{p} - \frac{1}{10.0} = -\frac{2}{|R|}$$

or

$$\frac{2}{|R|} = \frac{p - 10.0\text{ cm}}{(10.0\text{ cm})p}. \tag{2}$$

(a) Equating Equations (1) and (2) gives:

$$\frac{30.0\text{ cm} - p}{3.00} = p - 10.0\text{ cm}$$

or $p = 15.0$ cm .

Thus, her face is $\boxed{15.0\text{ cm}}$ from the hubcap.

(b) Using the above result ( $p = 15.0$ cm ) in Equation (1) gives:

$$\frac{2}{|R|} = \frac{30.0\text{ cm} - 15.0\text{ cm}}{(30.0\text{ cm})(15.0\text{ cm})}$$

or

$$\frac{2}{|R|} = \frac{1}{30.0\text{ cm}}$$

and $|R| = 60.0$ cm .

The radius of the hubcap is $\boxed{60.0\text{ cm}}$ .

***P36.19** (a) The flat mirror produces an image according to

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} = \frac{2}{R} \qquad \frac{1}{24\text{ cm}} + \frac{1}{q} = \frac{1}{\infty} = 0 \qquad q = -24.0\text{ m}.$$

The image is 24.0 m behind the mirror, distant from your eyes by

$$1.55\text{ m} + 24.0\text{ m} = \boxed{25.6\text{ m}}.$$

(b) The image is the same size as the object, so $\theta = \frac{h}{d} = \frac{1.50\text{ m}}{25.6\text{ m}} = \boxed{0.058\,7\text{ rad}}$.

(c)

$$\frac{1}{p} + \frac{1}{q} = \frac{2}{R} \qquad \frac{1}{24\text{ m}} + \frac{1}{q} = \frac{2}{(-2\text{ m})} \qquad q = \frac{1}{-(1/1\text{ m}) - (1/24\text{ m})} = -0.960\text{ m}$$

This image is distant from your eyes by $1.55\text{ m} + 0.960\text{ m} = \boxed{2.51\text{ m}}$.

*continued on next page*

(d) The image size is given by $M = \frac{h'}{h} = -\frac{q}{p}$ $\qquad h' = -h\frac{q}{p} = -1.50\text{ m}\left(\frac{-0.960\text{ m}}{24\text{ m}}\right) = 0.060\,0\text{ m}$.

So its angular size at your eye is $\qquad \theta' = \frac{h'}{d} = \frac{0.06\text{ m}}{2.51\text{ m}} = \boxed{0.023\,9\text{ rad}}$.

(e) Your brain assumes that the car is 1.50 m high and calculate its distance as

$$d' = \frac{h}{\theta'} = \frac{1.50\text{ m}}{0.023\,9} = \boxed{62.8\text{ m}}.$$

**P36.20** (a) The image starts from a point whose height above the mirror vertex is given by

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} = \frac{2}{R} \qquad \frac{1}{3.00\text{ m}} + \frac{1}{q} = \frac{1}{0.500\text{ m}}. \qquad \text{Therefore,} \qquad q = 0.600\text{ m}.$$

As the ball falls, $p$ decreases and $q$ increases. Ball and image pass when $q_1 = p_1$. When this is true,

$$\frac{1}{p_1} + \frac{1}{p_1} = \frac{1}{0.500\text{ m}} = \frac{2}{p_1} \qquad \text{or} \qquad p_1 = 1.00\text{ m}.$$

As the ball passes the focal point, the image switches from infinitely far above the mirror to infinitely far below the mirror. As the ball approaches the mirror from above, the virtual image approaches the mirror from below, reaching it together when $p_2 = q_2 = 0$.

(b) The falling ball passes its real image when it has fallen

$$3.00\text{ m} - 1.00\text{ m} = 2.00\text{ m} = \frac{1}{2}gt^2\text{, or when } t = \sqrt{\frac{2(2.00\text{ m})}{9.80\text{ m/s}^2}} = \boxed{0.639\text{ s}}.$$

The ball reaches its virtual image when it has traversed

$$3.00\text{ m} - 0 = 3.00\text{ m} = \frac{1}{2}gt^2\text{, or at } t = \sqrt{\frac{2(3.00\text{ m})}{9.80\text{ m/s}^2}} = \boxed{0.782\text{ s}}.$$

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## Section 36.3 **Images Formed by Refraction**

**P36.21** $\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R} = 0$ and $R \to \infty$

$$q = -\frac{n_2}{n_1}p = -\frac{1}{1.309}(50.0\text{ cm}) = -38.2\text{ cm}$$

Thus, the virtual image of the dust speck is $\boxed{\text{38.2 cm below the top surface}}$ of the ice.

**P36.22** When $R \to \infty$, the equation describing image formation at a single refracting surface becomes $q = -p\left(\frac{n_2}{n_1}\right)$. We use this to locate the final images of the two surfaces of the glass plate. First, find the image the glass forms of the *bottom* of the plate.

$$q_{B1} = -\left(\frac{1.33}{1.66}\right)(8.00\text{ cm}) = -6.41\text{ cm}$$

This virtual image is 6.41 cm below the top surface of the glass of 18.41 cm below the water surface. Next, use this image as an object and locate the image the water forms of the bottom of the plate.

$q_{B2} = -\left(\frac{1.00}{1.33}\right)(18.41\text{ cm}) = -13.84\text{ cm}$ or 13.84 cm below the water surface.

Now find image the water forms of the *top* surface of the glass.

$q_3 = -\left(\frac{1}{1.33}\right)(12.0\text{ cm}) = -9.02\text{ cm}$ or 9.02 cm below the water surface.

Therefore, the apparent thickness of the glass is $\Delta t = 13.84\text{ cm} - 9.02\text{ cm} = \boxed{4.82\text{ cm}}$.

**P36.23** From Equation 36.8 $\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$.

Solve for $q$ to find $q = \frac{n_2 R p}{p(n_2 - n_1) - n_1 R}$.

In this case, $n_1 = 1.50$, $n_2 = 1.00$, $R = -15.0\text{ cm}$

and $p = 10.0\text{ cm}$.

So $q = \frac{(1.00)(-15.0\text{ cm})(10.0\text{ cm})}{(10.0\text{ cm})(1.00 - 1.50) - (1.50)(-15.0\text{ cm})} = -8.57\text{ cm}$.

Therefore, the $\boxed{\text{apparent depth is 8.57 cm}}$.

**P36.24** $\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$ so $\frac{1.00}{\infty} + \frac{1.40}{21.0\text{ mm}} = \frac{1.40 - 1.00}{6.00\text{ mm}}$

and $0.066\,7 = 0.066\,7$.

They agree. $\boxed{\text{The image is inverted, real and diminished.}}$

**P36.25** $\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$ becomes $\frac{1.00}{p} + \frac{1.50}{q} = \frac{1.50 - 1.00}{6.00\text{ cm}} = \frac{1}{12.0\text{ cm}}$

(a) $\frac{1.00}{20.0\text{ cm}} + \frac{1.50}{q} = \frac{1}{12.0\text{ cm}}$ or $q = \frac{1.50}{\left[(1.00/12.0\text{ cm}) - (1.00/20.0\text{ cm})\right]} = \boxed{45.0\text{ cm}}$

(b) $\frac{1.00}{10.0\text{ cm}} + \frac{1.50}{q} = \frac{1}{12.0\text{ cm}}$ or $q = \frac{1.50}{\left[(1.00/12.0\text{ cm}) - (1.00/10.0\text{ cm})\right]} = \boxed{-90.0\text{ cm}}$

(c) $\frac{1.00}{3.0\text{ cm}} + \frac{1.50}{q} = \frac{1}{12.0\text{ cm}}$ or $q = \frac{1.50}{\left[(1.00/12.0\text{ cm}) - (1.00/3.0\text{ cm})\right]} = \boxed{-6.00\text{ cm}}$

**P36.26** $p = \infty$ and $q = +2R$

$$\frac{1.00}{p} + \frac{n_2}{q} = \frac{n_2 - 1.00}{R}$$

$$0 + \frac{n_2}{2R} = \frac{n_2 - 1.00}{R} \qquad \text{so} \qquad \boxed{n_2 = 2.00}$$

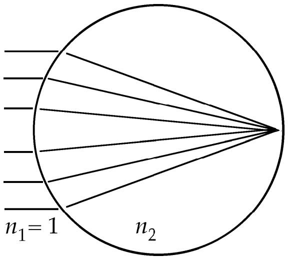


**FIG. P36.26**

**P36.27** For a plane surface, $\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$ becomes $q = -\frac{n_2 p}{n_1}$.

Thus, the magnitudes of the rate of change in the image and object positions are related by

$$\left|\frac{dq}{dt}\right| = \frac{n_2}{n_1}\left|\frac{dp}{dt}\right|.$$

If the fish swims toward the wall with a speed of $2.00 \text{ cm/s}$, the speed of the image is given by

$$v_{\text{image}} = \left|\frac{dq}{dt}\right| = \frac{1.00}{1.33}(2.00 \text{ cm/s}) = \boxed{1.50 \text{ cm/s}}.$$

## Section 36.4 Thin Lenses

**P36.28** Let $R_1 =$ outer radius and $R_2 =$ inner radius

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = (1.50-1)\left[\frac{1}{2.00 \text{ m}} - \frac{1}{2.50 \text{ cm}}\right] = 0.050\,0 \text{ cm}^{-1}$$

so $f = \boxed{20.0 \text{ cm}}$.

**P36.29** (a)

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = (0.440)\left[\frac{1}{12.0 \text{ cm}} - \frac{1}{(-18.0 \text{ cm})}\right]$$

$$f = \boxed{16.4 \text{ cm}}$$

(b)

$$\frac{1}{f} = (0.440)\left[\frac{1}{18.0 \text{ cm}} - \frac{1}{(-12.0 \text{ cm})}\right]$$

$$f = \boxed{16.4 \text{ cm}}$$

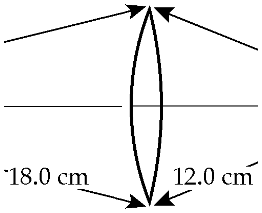


**FIG. P36.29**

**P36.30** For a converging lens, *f* is positive. We use $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$.

(a) $\frac{1}{q}=\frac{1}{f}-\frac{1}{p}=\frac{1}{20.0\text{ cm}}-\frac{1}{40.0\text{ cm}}=\frac{1}{40.0\text{ cm}}$ $\boxed{q=40.0\text{ cm}}$

$M=-\frac{q}{p}=-\frac{40.0}{40.0}=\boxed{-1.00}$

The image is $\boxed{\text{real, inverted}}$, and located 40.0 cm past the lens.

(b) $\frac{1}{q}=\frac{1}{f}-\frac{1}{p}=\frac{1}{20.0\text{ cm}}-\frac{1}{20.0\text{ cm}}=0$ $\boxed{q=\text{infinity}}$

$\boxed{\text{No image}}$ is formed. The rays emerging from the lens are parallel to each other.

(c) $\frac{1}{q}=\frac{1}{f}-\frac{1}{p}=\frac{1}{20.0\text{ cm}}-\frac{1}{10.0\text{ cm}}=-\frac{1}{20.0\text{ cm}}$ $\boxed{q=-20.0\text{ cm}}$

$M=-\frac{q}{p}=-\frac{(-20.0)}{10.0}=\boxed{2.00}$

The image is $\boxed{\text{upright, virtual}}$ and 20.0 cm in front of the lens.

**P36.31** (a) $\frac{1}{q}=\frac{1}{f}-\frac{1}{p}=\frac{1}{25.0\text{ cm}}-\frac{1}{26.0\text{ cm}}$ $\boxed{q=650\text{ cm}}$

The image is $\boxed{\text{real, inverted, and enlarged}}$.

(b) $\frac{1}{q}=\frac{1}{f}-\frac{1}{p}=\frac{1}{25.0\text{ cm}}-\frac{1}{24.0\text{ cm}}$ $\boxed{q=-600\text{ cm}}$

The image is $\boxed{\text{virtual, upright, and enlarged}}$.

**P36.32** (a) $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$: $\frac{1}{32.0\text{ cm}}+\frac{1}{8.00\text{ cm}}=\frac{1}{f}$

so $\boxed{f=6.40\text{ cm}}$

(b) $M=-\frac{q}{p}=-\frac{8.00\text{ cm}}{32.0\text{ cm}}=\boxed{-0.250}$

(c) Since $f>0$, the lens is $\boxed{\text{converging}}$.

**P36.33** We are looking at an enlarged, upright, virtual image:

$$M=\frac{h'}{h}=2=-\frac{q}{p}\qquad\text{so}\qquad p=-\frac{q}{2}=-\frac{(-2.84\text{ cm})}{2}=+1.42\text{ cm}$$

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}\qquad\text{gives}\qquad \frac{1}{1.42\text{ cm}}+\frac{1}{(-2.84\text{ cm})}=\frac{1}{f}$$

$$\boxed{f=2.84\text{ cm}}\,.$$

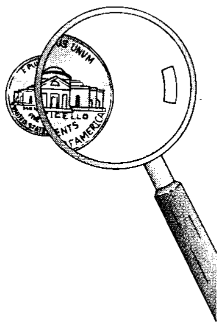


FIG. P36.33

**P36.34** (a) $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$: $\frac{1}{p}+\frac{1}{-30.0\text{ cm}}=\frac{1}{12.5\text{ cm}}$

$$p=8.82\text{ cm}\qquad M=-\frac{q}{p}=-\frac{(-30.0)}{8.82}=\boxed{3.40,\text{ upright}}$$

(b) See the figure to the right.

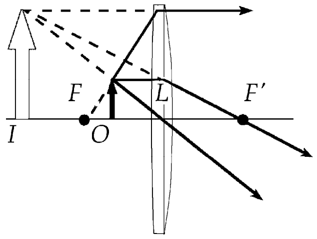


FIG. P36.34(b)

**P36.35** $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$: $p^{-1}+q^{-1}=\text{constant}$

We may differentiate through with respect to $p$: $-1p^{-2}-1q^{-2}\frac{dq}{dp}=0$

$$\frac{dq}{dp}=-\frac{q^2}{p^2}=-M^2\,.$$

**P36.36** The image is inverted: $M=\frac{h'}{h}=\frac{-1.80\text{ m}}{0.0240\text{ m}}=-75.0=\frac{-q}{p}$ $q=75.0p$.

(b) $q+p=3.00\text{ m}=75.0p+p$ $p=\boxed{39.5\text{ mm}}$

(a) $q=2.96\text{ m}$ $\frac{1}{f}=\frac{1}{p}+\frac{1}{q}=\frac{1}{0.0395\text{ m}}+\frac{1}{2.96\text{ m}}$ $f=\boxed{39.0\text{ mm}}$

**P36.37** (a) $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$ $\frac{1}{20.0\text{ cm}}+\frac{1}{q}=\frac{1}{(-32.0\text{ cm})}$

so $q=-\left(\frac{1}{20.0}+\frac{1}{32.0}\right)^{-1}=\boxed{-12.3\text{ cm}}$

The image is 12.3 cm to the left of the lens.

(b) $M=-\frac{q}{p}=-\frac{(-12.3\text{ cm})}{20.0\text{ cm}}=\boxed{0.615}$

(c) See the ray diagram to the right.

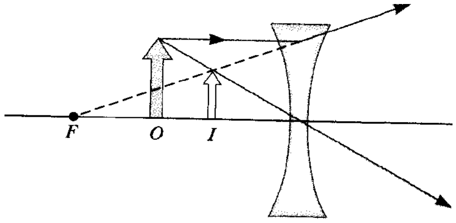


FIG. P36.37

***P36.38** In

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

$p^{-1}+q^{-1}=\text{constant}$,

we differentiate with respect to time

$$-1\left(p^{-2}\right)\frac{dp}{dt}-1\left(q^{-2}\right)\frac{dq}{dt}=0$$

$$\frac{dq}{dt}=\frac{-q^2}{p^2}\frac{dp}{dt}.$$

We must find the momentary image location $q$:

$$\frac{1}{20\text{ m}}+\frac{1}{q}=\frac{1}{0.3\text{ m}}$$

$q=0.305\text{ m}$.

Now $\frac{dq}{dt}=-\frac{(0.305\text{ m})^2}{(20\text{ m})^2}5\text{ m/s}=-0.001\,16\text{ m/s}=\boxed{1.16\text{ mm/s toward the lens}}$.

***P36.39** (a) $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}\qquad\frac{1}{480\text{ cm}}+\frac{1}{q}=\frac{1}{7.00\text{ cm}}\qquad q=\boxed{7.10\text{ cm}}$

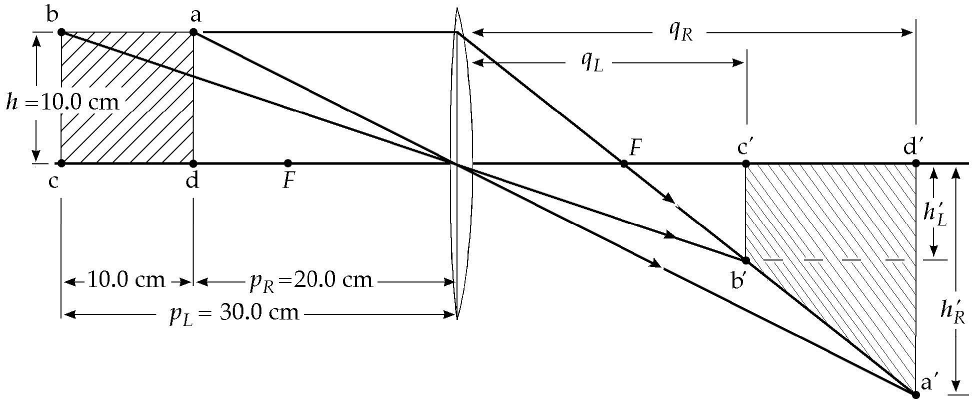
(b) $M=\frac{h'}{h}=-\frac{q}{p}\quad h'=\frac{-hq}{p}=\frac{-(5.00\text{ mm})(7.10\text{ cm})}{480\text{ cm}}=-0.074\,0\text{ mm}$

diameter of illuminated spot $=\boxed{74.0\ \mu\text{m}}$

(c) $I=\frac{\mathcal{P}}{A}=\frac{\mathcal{P}4}{\pi d^2}=\frac{0.100\text{ W}(4)}{\pi\left(74.0\times10^{-6}\text{ m}\right)^2}=\boxed{2.33\times10^{7}\text{ W/m}^2}$

**P36.40** (a) $\frac{1}{f}=(n-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]=(1.50-1)\left[\frac{1}{15.0\text{ cm}}-\frac{1}{(-12.0\text{ cm})}\right]$ or $\boxed{f=13.3\text{ cm}}$

(b)



The square is imaged as a trapezoid.

FIG. P36.40(b)

*continued on next page*

(c) To find the area, first find $q_R$ and $q_L$, along with the heights $h'_R$ and $h'_L$, using the thin lens equation.

$$\frac{1}{p_R}+\frac{1}{q_R}=\frac{1}{f} \quad \text{becomes} \quad \frac{1}{20.0\text{ cm}}+\frac{1}{q_R}=\frac{1}{13.3\text{ cm}} \quad \text{or} \quad q_R = 40.0\text{ cm}$$

$$h'_R = hM_R = h\left(\frac{-q_R}{p_R}\right)=(10.0\text{ cm})(-2.00)=-20.0\text{ cm}$$

$$\frac{1}{30.0\text{ cm}}+\frac{1}{q_L}=\frac{1}{13.3\text{ cm}} \quad \text{or} \quad q_L = 24.0\text{ cm}$$

$$h'_L = hM_L=(10.0\text{ cm})(-0.800)=-8.00\text{ cm}$$

Thus, the area of the image is: $\text{Area } = |q_R-q_L||h'_L|+\frac{1}{2}|q_R-q_L||h'_R-h'_L|=\boxed{224\text{ cm}^2}$.

**P36.41** (a) The image distance is: $q=d-p$.

Thus, $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$ becomes $\frac{1}{p}+\frac{1}{d-p}=\frac{1}{f}$.

This reduces to a quadratic equation: $p^2+(-d)p+fd=0$

which yields: $\boxed{p=\frac{d\pm\sqrt{d^2-4fd}}{2}=\frac{d}{2}\pm\sqrt{\frac{d^2}{4}-fd}}$.

Since $f<\frac{d}{4}$, both solutions are meaningful and the two solutions are not equal to each other. Thus, there are two distinct lens positions that form an image on the screen.

(b) The smaller solution for $p$ gives a larger value for $q$, with a $\boxed{\text{real, enlarged, inverted image}}$.

The larger solution for $p$ describes a $\boxed{\text{real, diminished, inverted image}}$.

**P36.42** To properly focus the image of a distant object, the lens must be at a distance equal to the focal length from the film ($q_1 = 65.0$ mm). For the closer object:

$$\frac{1}{p_2}+\frac{1}{q_2}=\frac{1}{f}$$

becomes $$\frac{1}{2\,000\text{ mm}}+\frac{1}{q_2}=\frac{1}{65.0\text{ mm}}$$

and $$q_2=(65.0\text{ mm})\left(\frac{2\,000}{2\,000-65.0}\right).$$

The lens must be moved $\boxed{\text{away from the film}}$ by a distance

$$D=q_2-q_1=(65.0\text{ mm})\left(\frac{2\,000}{2\,000-65.0}\right)-65.0\text{ mm}=\boxed{2.18\text{ mm}}.$$

***P36.43** In the first arrangement the lens is used as a magnifying glass, producing an upright, virtual enlarged image:

$$M = \frac{h'}{h} = \frac{120\text{ cm}}{3.6\text{ cm}} = 33.3 = -\frac{q}{p}$$

$$q = -33.3p = -33.3(20\text{ cm}) = -667\text{ cm}$$

For the lens,

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} = \frac{1}{20\text{ cm}} + \frac{1}{-667\text{ cm}} = \frac{1}{f}$$

$$f = 20.62\text{ cm}$$

In the second arrangement the lens us used as a projection lens to produce a real inverted enlarged image:

$$-\frac{120\text{ cm}}{3.6\text{ cm}} = -33.3 = -\frac{q_2}{p_2} \qquad q_2 = 33.3p_2$$

$$\frac{1}{p_2} + \frac{1}{33.3p_2} = \frac{1}{20.62\text{ cm}} \qquad \frac{34.3}{33.3p_2} = \frac{1}{20.62\text{ cm}} \qquad p_2 = 21.24\text{ cm}$$

The lens was moved $21.24\text{ cm} - 20.0\text{ cm} = \boxed{1.24\text{ cm}}$.

## Section 36.5 **Lens Aberrations**

**P36.44** (a) The focal length of the lens is given by

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.53 - 1.00)\left(\frac{1}{-32.5\text{ cm}} - \frac{1}{42.5\text{ cm}}\right)$$

$$f = -34.7\text{ cm}$$

Note that $R_1$ is negative because the center of curvature of the first surface is on the virtual image side.

When $p = \infty$

the thin lens equation gives $q = f$.

Thus, the violet image of a very distant object is formed

at $\boxed{q = -34.7\text{ cm}}$.

The image is $\boxed{\text{virtual, upright and diminshed}}$.

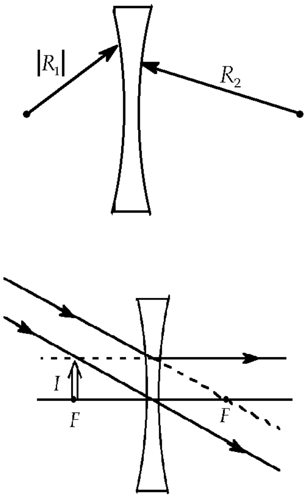


**FIG. P36.44**

(b) The same ray diagram and image characteristics apply for red light.

Again, $q = f$

and now $\frac{1}{f} = (1.51 - 1.00)\left(\frac{1}{-32.5\text{ cm}} - \frac{1}{42.5\text{ cm}}\right)$

giving $f = \boxed{-36.1\text{ cm}}$.

**P36.45** Ray $h_1$ is undeviated at the plane surface and strikes the second surface at angle of incidence given by

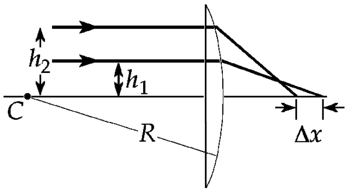


**FIG. P36.45**

$$\theta_1 = \sin^{-1}\left(\frac{h_1}{R}\right) = \sin^{-1}\left(\frac{0.500\text{ cm}}{20.0\text{ cm}}\right) = 1.43°.$$

Then, $$1.00\sin\theta_2 = 1.60\sin\theta_1 = (1.60)\left(\frac{0.500}{20.0\text{ cm}}\right)$$

so $$\theta_2 = 2.29°.$$

The angle this emerging ray makes with the horizontal is $\theta_2 - \theta_1 = 0.860°$.

It crosses the axis at a point farther out by $f_1$

where $$f_1 = \frac{h_1}{\tan(\theta_2 - \theta_1)} = \frac{0.500\text{ cm}}{\tan(0.860°)} = 33.3\text{ cm}.$$

The point of exit for this ray is distant axially from the lens vertex by

$$20.0\text{ cm} - \sqrt{(20.0\text{ cm})^2 - (0.500\text{ cm})^2} = 0.006\,25\text{ cm}$$

so ray $h_1$ crosses the axis at this distance from the vertex:

$$x_1 = 33.3\text{ cm} - 0.006\,25\text{ cm} = 33.3\text{ cm}.$$

Now we repeat this calculation for ray $h_2$:

$$\theta = \sin^{-1}\left(\frac{12.0\text{ cm}}{20.0\text{ cm}}\right) = 36.9°$$

$$1.00\sin\theta_2 = 1.60\sin\theta_1 = (1.60)\left(\frac{12.00}{20.0}\right) \qquad \theta_2 = 73.7°$$

$$f_2 = \frac{h_2}{\tan(\theta_1 - \theta_2)} = \frac{12.0\text{ cm}}{\tan 36.8°} = 16.0\text{ cm}$$

$$x_2 = (16.0\text{ cm})\left(20.0\text{ cm} - \sqrt{(20.0\text{ cm})^2 - (12.0\text{ cm})^2}\right) = 12.0\text{ cm}.$$

Now $$\Delta x = 33.3\text{ cm} - 12.0\text{ cm} = \boxed{21.3\text{ cm}}.$$

## Section 36.6 **The Camera**

***P36.46** The same light intensity is received from the subject, and the same light energy on the film is required:

$$IA_1\Delta t_1 = IA_2\Delta t_2$$

$$\frac{\pi d_1^2}{4}\Delta t_1 = \frac{\pi d_2^2}{4}\Delta t_2$$

$$\left(\frac{f}{4}\right)^2\left(\frac{1}{16}\text{ s}\right) = d_2^2\left(\frac{1}{128}\text{ s}\right)$$

$$d_2 = \sqrt{\frac{128}{16}}\frac{f}{4} = \boxed{\frac{f}{1.41}}$$

## Section 36.7 **The Eye**

**P36.47** $P=\frac{1}{f}=\frac{1}{p}+\frac{1}{q}=\frac{1}{\infty}-\frac{1}{0.250\text{ m}}=-4.00\text{ diopters}=\boxed{-4.00\text{ diopters, a diverging lens}}$

**P36.48** For starlight going through Nick's glasses, $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$

$$\frac{1}{\infty}+\frac{1}{(-0.800\text{ m})}=\frac{1}{f}=-1.25\text{ diopters}.$$

For a nearby object, $\frac{1}{p}+\frac{1}{(-0.180\text{ m})}=-1.25\text{ m}^{-1}$, so $p=\boxed{23.2\text{ cm}}$.

**P36.49** Consider an object at infinity, imaged at the person's far point:

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}\qquad \frac{1}{\infty}+\frac{1}{q}=-4.00\text{ m}^{-1}\qquad q=-25.0\text{ cm}.$$

The person's far point is $25.0\text{ cm}+2.00\text{ cm}=27.0\text{ cm}$ from his eyes. For the contact lenses we want

$$\frac{1}{\infty}+\frac{1}{(-0.270\text{ m})}=\frac{1}{f}=\boxed{-3.70\text{ diopters}}.$$

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## Section 36.8 **The Simple Magnifier**

## Section 36.9 **The Compound Microscope**

## Section 36.10 **The Telescope**

**P36.50** (a) From the thin lens equation: $\frac{1}{p}+\frac{1}{(-25.0\text{ cm})}=\frac{1}{5.00\text{ cm}}$ or $p=\boxed{4.17\text{ cm}}$.

(b) $M=-\frac{q}{p}=1+\frac{25.0\text{ cm}}{f}=1+\frac{25.0\text{ cm}}{5.00\text{ cm}}=\boxed{6.00}$

**P36.51** Using Equation 36.24, $M\approx-\left(\frac{L}{f_o}\right)\left(\frac{25.0\text{ cm}}{f_e}\right)=-\left(\frac{23.0\text{ cm}}{0.400\text{ cm}}\right)\left(\frac{25.0\text{ cm}}{2.50\text{ cm}}\right)=\boxed{-575}$.

**P36.52** $M=M_om_e=M_o\left(\frac{25.0\text{ cm}}{f_e}\right)\Rightarrow f_e=\left(\frac{M_o}{M}\right)(25.0\text{ cm})=\left(\frac{-12.0}{-140}\right)(25.0\text{ cm})=\boxed{2.14\text{ cm}}$

**P36.53** $f_o=20.0\text{ m}\qquad f_e=0.025\,0\text{ m}$

(a) The angular magnification produced by this telescope is: $m=-\frac{f_o}{f_e}=\boxed{-800}$.

(b) Since $m<0$, the image is $\boxed{\text{inverted}}$.

**P36.54** (a) The lensmaker's equation $$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

gives $$q=\frac{1}{1/f-1/p}=\frac{1}{(p-f)/fp}=\frac{fp}{p-f}.$$

Then, $$M=\frac{h'}{h}=-\frac{q}{p}=-\frac{f}{p-f}$$

gives $$\boxed{h'=\frac{hf}{f-p}}.$$

(b) For $p>>f$, $f-p\approx -p$. Then, $$h'=\boxed{-\frac{hf}{p}}.$$

(c) Suppose the telescope observes the space station at the zenith:

$$h'=-\frac{hf}{p}=-\frac{(108.6\text{ m})(4.00\text{ m})}{407\times 10^{3}\text{ m}}=\boxed{-1.07\text{ mm}}.$$

**P36.55** (b) Call the focal length of the objective $f_o$ and that of the eyepiece $-|f_e|$. The distance between the lenses is $f_o-|f_e|$. The objective forms a real diminished inverted image of a very distant object at $q_1=f_o$. This image is a virtual object for the eyepiece at $p_2=-|f_e|$.

For it $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$ becomes $\frac{1}{-|f_e|}+\frac{1}{q}=\frac{1}{-|f_e|}$, $\frac{1}{q_2}=0$

and $\boxed{q_2=\infty}$.

(a) The user views the image as $\boxed{\text{virtual}}$. Letting $h'$ represent the height of the first image, $\theta_o=\frac{h'}{f_o}$ and $\theta=\frac{h'}{|f_e|}$. The angular magnification is

$$m=\frac{\theta}{\theta_o}=\frac{h'/|f_e|}{h'/f_o}=\frac{f_0}{|f_e|}.$$

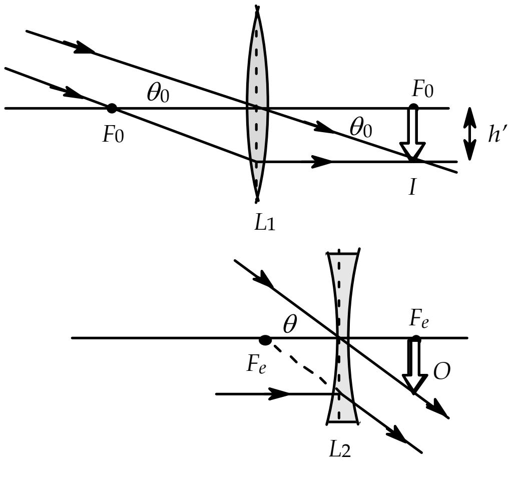

**FIG. P36.55**

(c) Here, $f_o-|f_e|=10.0$ cm and $\frac{f_o}{|f_e|}=3.00$.

Thus, $|f_e|=\frac{f_o}{3.00}$ and $\frac{2}{3}f_o=10.0$ cm.

$$f_o=\boxed{15.0\text{ cm}}$$

$$|f_e|=5.00\text{ cm} \quad\text{and}\quad f_e=\boxed{-5.00\text{ cm}}$$

**P36.56** Let $I_0$ represent the intensity of the light from the nebula and $\theta_0$ its angular diameter. With the first telescope, the image diameter $h'$ on the film is given by $\theta_o = -\dfrac{h'}{f_o}$ as $h' = -\theta_o(2\,000\text{ mm})$.

The light power captured by the telescope aperture is $\mathcal{P}_1 = I_0 A_1 = I_0\left[\dfrac{\pi(200\text{ mm})^2}{4}\right]$, and the light energy focused on the film during the exposure is $E_1 = \mathcal{P}_1 \Delta t_1 = I_0\left[\dfrac{\pi(200\text{ mm})^2}{4}\right](1.50\text{ min})$.

Likewise, the light power captured by the aperture of the second telescope is $\mathcal{P}_2 = I_0 A_2 = I_0\left[\dfrac{\pi(60.0\text{ mm})^2}{4}\right]$ and the light energy is $E_2 = I_0\left[\dfrac{\pi(60.0\text{ mm})^2}{4}\right]\Delta t_2$. Therefore, to have the same light energy per unit area, it is necessary that

$$\frac{I_0\left[\pi(60.0\text{ mm})^2/4\right]\Delta t_2}{\pi\left[\theta_o(900\text{ mm})^2/4\right]} = \frac{I_0\left[\pi(200\text{ mm})^2/4\right](1.50\text{ min})}{\pi\left[\theta_o(2\,000\text{ mm})^2/4\right]}.$$

The required exposure time with the second telescope is

$$\Delta t_2 = \frac{(200\text{ mm})^2(900\text{ mm})^2}{(60.0\text{ mm})^2(2\,000\text{ mm})^2}(1.50\text{ min}) = \boxed{3.38\text{ min}}.$$

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### Additional Problems

**P36.57** Only a diverging lens gives an upright diminished image. The image is virtual and

$d = p - |q| = p + q$: $\quad M = -\dfrac{q}{p}$ so $q = -Mp$ and $d = p - Mp$

$p = \dfrac{d}{1-M}$: $\quad \dfrac{1}{p} + \dfrac{1}{q} = \dfrac{1}{f} = \dfrac{1}{p} + \dfrac{1}{-Mp} = \dfrac{-M+1}{-Mp} = \dfrac{(1-M)^2}{-Md}$

$$f = \frac{-Md}{(1-M)^2} = \frac{-(0.500)(20.0\text{ cm})}{(1-0.500)^2} = \boxed{-40.0\text{ cm}}.$$

**P36.58** If $M < 1$, the lens is diverging and the image is virtual. $\quad d = p - |q| = p + q$

$M = -\dfrac{q}{p}$ so $q = -Mp$ and $d = p - Mp$

$p = \dfrac{d}{1-M}$: $\quad \dfrac{1}{p} + \dfrac{1}{q} = \dfrac{1}{f} = \dfrac{1}{p} + \dfrac{1}{(-Mp)} = \dfrac{(-M+1)}{-Mp} = \dfrac{(1-M)^2}{-Md}$ $\quad \boxed{f = \dfrac{-Md}{(1-M)^2}}$.

If $M > 1$, the lens is converging and the image is still virtual.

Now $\quad d = -q - p$.

We obtain in this case $\quad \boxed{f = \dfrac{Md}{(M-1)^2}}$.

**P36.59** (a)

$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\frac{1}{-65.0\text{ cm}}=(1.66-1)\left(\frac{1}{50.0\text{ cm}}-\frac{1}{R_2}\right)$$

$$\frac{1}{R_2}=\frac{1}{50.0\text{ cm}}+\frac{1}{42.9\text{ cm}}\qquad\text{so}\qquad R_2=\boxed{23.1\text{ cm}}$$

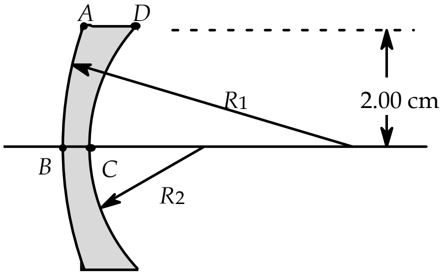

FIG. P36.59

(b) The distance along the axis from *B* to *A* is

$$R_1-\sqrt{R_1^2-(2.00\text{ cm})^2}=50.0\text{ cm}-\sqrt{(50.0\text{ cm})^2-(2.00\text{ cm})^2}=0.040\,0\text{ cm}.$$

Similarly, the axial distance from *C* to *D* is

$$23.1\text{ cm}-\sqrt{(23.1\text{ cm})^2-(2.00\text{ cm})^2}=0.086\,8\text{ cm}.$$

Then, $AD=0.100\text{ cm}-0.040\,0\text{ cm}+0.086\,8\text{ cm}=\boxed{0.147\text{ cm}}$.

***P36.60** We consider light entering the rod. The surface of entry is convex to the object rays, so $R_1=+4.50$ cm

$$\frac{n_1}{p_1}+\frac{n_2}{q_1}=\frac{n_2-n_1}{R_1}\qquad\frac{1.33}{100\text{ cm}}+\frac{1.50}{q_1}=\frac{1.50-1.33}{4.50\text{ cm}}$$

$$\frac{1.50}{q_1}=0.037\,8/\text{cm}-0.013\,3/\text{cm}=0.024\,5/\text{cm}\qquad q_1=61.3\text{ cm}$$

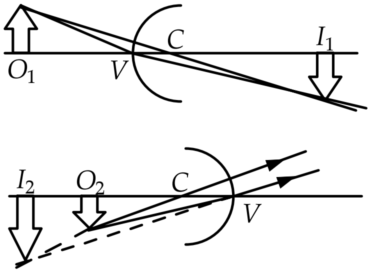

FIG. P36.60

The first image is real, inverted and diminished. To find its magnification we can use two similar triangles in the ray diagram with their vertices meeting at the center of curvature:

$$\frac{h_1}{100\text{ cm}+4.5\text{ cm}}=\frac{|h_1'|}{61.3\text{ cm}-4.5\text{ cm}}\qquad\frac{h_1'}{h_1}=-0.543\,.$$

Now the first image is a real object for the second surface at object distance from its vertex

$$75.0\text{ cm}+4.50\text{ cm}+4.50\text{ cm}-61.3\text{ cm}=22.7\text{ cm}$$

$$\frac{1.50}{22.7\text{ cm}}+\frac{1.33}{q_2}=\frac{1.33-1.50}{-4.50\text{ cm}}$$

$$\frac{1.33}{q_2}=0.037\,8/\text{cm}-0.066\,0/\text{cm}=-0.028\ 2/\text{cm}$$

$$q_2=-47.1\text{ cm}$$

(a) The final image is $\boxed{\text{inside the rod, 47.1 cm from the second surface}}$.

(b) It is $\boxed{\text{virtual, inverted, and enlarged}}$. Again by similar triangles meeting at *C* we have

$$\frac{h_2}{22.7\text{ cm}-4.5\text{ cm}}=\frac{h_2'}{47.1\text{ cm}-4.5\text{ cm}}\qquad\frac{h_2'}{h_2}=2.34\,.$$

Since $h_2=h_1'$, the overall magnification is $M_1M_2=\frac{h_1'}{h_1}\frac{h_2'}{h_2}=\frac{h_2'}{h_1}=(-0.543)(2.34)=-1.27$.

***P36.61** (a) $\frac{1}{q_1}=\frac{1}{f_1}-\frac{1}{p_1}=\frac{1}{5\text{ cm}}-\frac{1}{7.5\text{ cm}} \qquad \therefore q_1=15\text{ cm}$

$M_1=-\frac{q_1}{p_1}=-\frac{15\text{ cm}}{7.5\text{ cm}}=-2$

$M=M_1M_2 \qquad \therefore 1=(-2)M_2$

$\therefore M_2=-\frac{1}{2}=-\frac{q_2}{p_2} \qquad \therefore p_2=2q_2$

$\frac{1}{p_2}+\frac{1}{q_2}=\frac{1}{f_2} \qquad \therefore \frac{1}{2q_2}+\frac{1}{q_2}=\frac{1}{10\text{ cm}} \qquad \therefore q_2=15\text{ cm},\ p_2=30\text{ cm}$

$p_1+q_1+p_2+q_2=7.5\text{ cm}+15\text{ cm}+30\text{ cm}+15\text{ cm}=\boxed{67.5\text{ cm}}$

(b) $\frac{1}{p_1'}+\frac{1}{q_1'}=\frac{1}{f_1}=\frac{1}{5\text{ cm}}$

Solve for $q_1'$ in terms of $p_1'$: $q_1'=\frac{5p_1'}{p_1'-5}$ (1)

$M_1'=-\frac{q_1'}{p_1'}=-\frac{5}{p_1'-5}$, using (1).

$M'=M_1'M_2' \qquad \therefore M_2'=\frac{M'}{M_1'}=-\frac{3}{5}(p_1'-5)=-\frac{q_2'}{p_2'}$

$\therefore q_2'=\frac{3}{5}p_2'(p_1'-5)$ (2)

Substitute (2) into the lens equation $\frac{1}{p_2'}+\frac{1}{q_2'}=\frac{1}{f_2}=\frac{1}{10\text{ cm}}$ and obtain $p_2'$ in terms of $p_1'$:

$p_2'=\frac{10(3p_1'-10)}{3(p_1'-5)}$. (3)

Substituting (3) in (2), obtain $q_2'$ in terms of $p_1'$:

$q_2'=2(3p_1'-10)$. (4)

Now, $p_1'+q_1'+p_2'+q_2'=$ a constant.

Using (1), (3) and (4), and the value obtained in (a):

$p_1'+\frac{5p_1'}{p_1'-5}+\frac{10(3p_1'-10)}{3(p'-5)}+2(3p_1'-10)=67.5$.

This reduces to the quadratic equation

$21p_1'^2-322.5p_1'+1\,212.5=0$,

which has solutions $p_1'=8.784$ cm and 6.573 cm.

Case 1: $p_1'=8.784\text{ cm}$

$\therefore p_1'-p_1=8.784\text{ cm}-7.5\text{ cm}=1.28\text{ cm}$.

From (4): $q_2'=32.7\text{ cm}$

$\therefore q_2'-q_2=32.7\text{ cm}-15\text{ cm}=17.7\text{ cm}$.

Case 2: $p_1'=6.573\text{ cm}$

$\therefore p_1'-p_1=6.573\text{ cm}-7.5\text{ cm}=-0.927\text{ cm}$.

From (4): $q_2'=19.44\text{ cm}$

$\therefore q_2'=q_2=19.44\text{ cm}-15\text{ cm}=4.44\text{ cm}$.

From these results it is concluded that:

| |
|---|
| The lenses can be displaced in two ways. The first lens can be moved 1.28 cm farther from the object and the second lens 17.7 cm toward the object. Alternatively, the first lens can be moved 0.927 cm toward the object and the second lens 4.44 cm toward the object. |

**P36.62** $$\frac{1}{q_1}=\frac{1}{f_1}-\frac{1}{p_1}=\frac{1}{10.0\text{ cm}}-\frac{1}{12.5\text{ cm}}$$

so $q_1=50.0\text{ cm}$ (to left of mirror).

This serves as an object for the lens (a virtual object), so

$$\frac{1}{q_2}=\frac{1}{f_2}-\frac{1}{p_2}=\frac{1}{(-16.7\text{ cm})}-\frac{1}{(-25.0\text{ cm})}\text{ and }q_2=-50.3\text{ cm},$$

meaning 50.3 cm to the right of the lens. Thus, the final image is located $\boxed{25.3\text{ cm to right of mirror}}$.

$$M_1=-\frac{q_1}{p_1}=-\frac{50.0\text{ cm}}{12.5\text{ cm}}=-4.00$$

$$M_2=-\frac{q_2}{p_2}=-\frac{(-50.3\text{ cm})}{(-25.0\text{ cm})}=-2.01$$

$$M=M_1M_2=\boxed{8.05}$$

Thus, the final image is $\boxed{\text{virtual, upright}}$, 8.05 times the size of object, and 25.3 cm to right of the mirror.

**P36.63** We first find the focal length of the mirror.

$$\frac{1}{f}=\frac{1}{p}+\frac{1}{q}=\frac{1}{10.0\text{ cm}}+\frac{1}{8.00\text{ cm}}=\frac{9}{40.0\text{ cm}}\qquad\text{and}\qquad f=4.44\text{ cm}.$$

Hence, if $p=20.0\text{ cm}$, $\frac{1}{q}=\frac{1}{f}-\frac{1}{p}=\frac{1}{4.44\text{ cm}}-\frac{1}{20.0\text{ cm}}=\frac{15.56}{88.8\text{ cm}}$.

Thus, $q=\boxed{5.71\text{ cm}}$, real.

***P36.64** A telescope with an eyepiece decreases the diameter of a beam of parallel rays. When light is sent through the same device in the opposite direction, the beam expands. Send the light first through the diverging lens. It will then be diverging from a virtual image found like this:

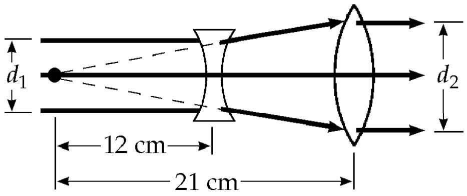

**FIG. P36.64**

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}\qquad\frac{1}{\infty}+\frac{1}{q}=\frac{1}{-12\text{ cm}}$$

$$q=-12\text{ cm}.$$

Use this image as a real object for the converging lens, placing it at the focal point on the object side of the lens, at $p=21\text{ cm}$. Then

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}\qquad\frac{1}{21\text{ cm}}+\frac{1}{q}=\frac{1}{21\text{ cm}}$$

$$q=\infty.$$

The exiting rays will be parallel. The lenses must be $21.0\text{ cm}-12.0\text{ cm}=9.00\text{ cm}$ apart.

By similar triangles, $\frac{d_2}{d_1}=\frac{21\text{ cm}}{12\text{ cm}}=\boxed{1.75\text{ times}}$.

**P36.65** A hemisphere is too thick to be described as a thin lens. The light is undeviated on entry into the flat face. We next consider the light's exit from the second surface, for which $R = -6.00$ cm.

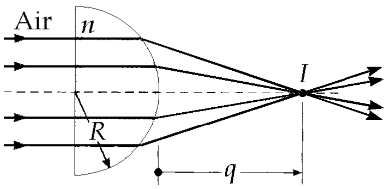

**FIG. P36.65**

The incident rays are parallel, so $p = \infty$.

Then, $$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$$

becomes $$0 + \frac{1}{q} = \frac{1.00 - 1.56}{-6.00 \text{ cm}}$$

and $$\boxed{q = 10.7 \text{ cm}}.$$

**P36.66** (a) $$I = \frac{\mathcal{P}}{4\pi r^2} = \frac{4.50 \text{ W}}{4\pi\left(1.60 \times 10^{-2} \text{ m}\right)^2} = \boxed{1.40 \text{ kW/m}^2}$$

(b) $$I = \frac{\mathcal{P}}{4\pi r^2} = \frac{4.50 \text{ W}}{4\pi(7.20 \text{ m})^2} = \boxed{6.91 \text{ mW/m}^2}$$

(c) $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$: $$\frac{1}{7.20 \text{ m}} + \frac{1}{q} = \frac{1}{0.350 \text{ m}}$$

so $$q = 0.368 \text{ m}$$

and $$M = \frac{h'}{3.20 \text{ cm}} = -\frac{q}{p} = -\frac{0.368 \text{ m}}{7.20 \text{ m}}$$

$$h' = \boxed{0.164 \text{ cm}}$$

(d) The lens intercepts power given by $$\mathcal{P} = IA = \left(6.91 \times 10^{-3} \text{ W/m}^2\right)\left[\frac{\pi}{4}(0.150 \text{ m})^2\right]$$

and puts it all onto the image where $$I = \frac{\mathcal{P}}{A} = \frac{\left(6.91 \times 10^{-3} \text{ W/m}^2\right)\left[\pi(15.0 \text{ cm})^2/4\right]}{\pi(0.164 \text{ cm})^2/4}$$

$$I = \boxed{58.1 \text{ W/m}^2}.$$

**P36.67** From the thin lens equation,
$$q_1 = \frac{f_1 p_1}{p_1 - f_1} = \frac{(-6.00\text{ cm})(12.0\text{ cm})}{12.0\text{ cm} - (-6.00\text{ cm})} = -4.00\text{ cm}.$$

When we require that $q_2 \to \infty$, the thin lens equation becomes $p_2 = f_2$.

In this case, $p_2 = d - (-4.00\text{ cm})$.

Therefore, $d + 4.00\text{ cm} = f_2 = 12.0\text{ cm}$ and $d = \boxed{8.00\text{ cm}}$.

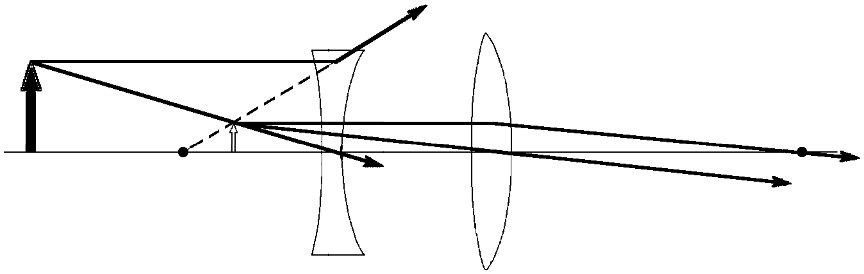

FIG. P36.67

***P36.68** The inverted real image is formed by the lens operating on light directly from the object, on light that has not reflected from the mirror.

For this we have $M = -1.50 = -\dfrac{q}{p}$ $\qquad q = 1.50p$

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \qquad \frac{1}{p} + \frac{1}{1.50p} = \frac{1}{10\text{ cm}} = \frac{2.50}{1.50p} \qquad p = 10\text{ cm}\left(\frac{2.5}{1.5}\right) = 16.7\text{ cm}$$

Then the object is distant from the mirror by $40.0\text{ cm} - 16.7\text{ cm} = 23.3\text{ cm}$.

The second image seen by the person is formed by light that first reflects from the mirror and then goes through the lens. For it to be in the same position as the inverted image, the lens must be receiving light from an image formed by the mirror at the same location as the physical object. The formation of this image is described by

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \qquad \frac{1}{23.3\text{ cm}} + \frac{1}{23.3\text{ cm}} = \frac{1}{f} \qquad f = \boxed{11.7\text{ cm}}.$$

**P36.69** For the mirror, $f = \dfrac{R}{2} = +1.50\text{ m}$. In addition, because the distance to the Sun is so much larger than any other distances, we can take $p = \infty$.

The mirror equation, $\dfrac{1}{p} + \dfrac{1}{q} = \dfrac{1}{f}$, then gives $q = f = \boxed{1.50\text{ m}}$.

Now, in $M = -\dfrac{q}{p} = \dfrac{h'}{h}$

the magnification is nearly zero, but we can be more precise: $\dfrac{h}{p}$ is the angular diameter of the object.

Thus, the image diameter is

$$h' = -\frac{hq}{p} = (-0.533°)\left(\frac{\pi\text{ rad}}{180°}\right)(1.50\text{ m}) = -0.0140\text{ m} = \boxed{-1.40\text{ cm}}.$$

**P36.70** (a) For the light the mirror intercepts,

$$\mathcal{P} = I_0 A = I_0 \pi R_a^2$$

$$350\text{ W} = \left(1\,000\text{ W/m}^2\right)\pi R_a^2$$

and $R_a = \boxed{0.334\text{ m or larger}}$.

(b) In $\dfrac{1}{p}+\dfrac{1}{q}=\dfrac{1}{f}=\dfrac{2}{R}$

we have $p \to \infty$

so $q = \dfrac{R}{2}$

$$M = \frac{h'}{h} = -\frac{q}{p}$$

so $h' = -q\left(\dfrac{h}{p}\right) = -\left(\dfrac{R}{2}\right)\left[0.533°\left(\dfrac{\pi\text{ rad}}{180°}\right)\right] = -\left(\dfrac{R}{2}\right)(9.30\text{ m rad})$

where $\dfrac{h}{p}$ is the angle the Sun subtends. The intensity at the image is

then $I = \dfrac{\mathcal{P}}{\pi h'^2/4} = \dfrac{4I_0\pi R_a^2}{\pi h'^2} = \dfrac{4I_0 R_a^2}{(R/2)^2\left(9.30\times10^{-3}\text{ rad}\right)^2}$

$$120\times10^3\text{ W/m}^2 = \frac{16\left(1\,000\text{ W/m}^2\right)R_a^2}{R^2\left(9.30\times10^{-3}\text{ rad}\right)^2}$$

so $\boxed{\dfrac{R_a}{R} = 0.025\,5\text{ or larger}}$.

**P36.71** In the original situation, $p_1 + q_1 = 1.50\text{ m}$.

In the final situation, $p_2 = p_1 + 0.900\text{ m}$

and $q_2 = q_1 - 0.900\text{ m} = 0.600\text{ m} - p_1$.

Our lens equation is $\dfrac{1}{p_1}+\dfrac{1}{q_1}=\dfrac{1}{f}=\dfrac{1}{p_2}+\dfrac{1}{q_2}$.

Substituting, we have $\dfrac{1}{p_1}+\dfrac{1}{1.50\text{ m}-p_1}=\dfrac{1}{p_1+0.900}+\dfrac{1}{0.600-p_1}$.

Adding the fractions, $\dfrac{1.50\text{ m}-p_1+p_1}{p_1(1.50\text{ m}-p_1)}=\dfrac{0.600-p_1+p_1+0.900}{(p_1+0.900)(0.600-p_1)}$.

Simplified, this becomes $p_1(1.50\text{ m}-p_1)=(p_1+0.900)(0.600-p_1)$.

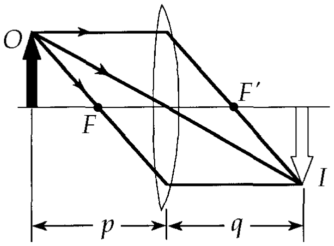

FIG. P36.71

(a) Thus, $p_1 = \dfrac{0.540}{1.80}\text{ m} = \boxed{0.300\text{ m}}$ $\quad p_2 = p_1 + 0.900 = \boxed{1.20\text{ m}}$

(b) $\dfrac{1}{f} = \dfrac{1}{0.300\text{ m}} + \dfrac{1}{1.50\text{ m}-0.300\text{ m}}$ and $f = \boxed{0.240\text{ m}}$

(c) The second image is $\boxed{\text{real, inverted, and diminished}}$

with $M = -\dfrac{q_2}{p_2} = \boxed{-0.250}$.

**P36.72** (a) The lens makers' equation, $\frac{1}{f}=(n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$

becomes: $\frac{1}{5.00\text{ cm}}=(n-1)\left[\frac{1}{9.00\text{ cm}}-\frac{1}{(-11.0\text{ cm})}\right]$ giving $n=\boxed{1.99}$.

(b) As the light passes through the lens for the first time, the thin lens equation

$$\frac{1}{p_1}+\frac{1}{q_1}=\frac{1}{f}$$

becomes: $\frac{1}{8.00\text{ cm}}+\frac{1}{q_1}=\frac{1}{5.00\text{ cm}}$

or $q_1=13.3\text{ cm}$, and $M_1=-\frac{q_1}{p_1}=-\frac{13.3\text{ cm}}{8.00\text{ cm}}=-1.67$.

This image becomes the object for the concave mirror with:

$$p_m=20.0\text{ cm}-q_1=20.0\text{ cm}-13.3\text{ cm}=6.67\text{ cm}$$

and $f=\frac{R}{2}=+4.00\text{ cm}$.

The mirror equation becomes: $\frac{1}{6.67\text{ cm}}+\frac{1}{q_m}=\frac{1}{4.00\text{ cm}}$

giving $q_m=10.0\text{ cm}$

and $M_2=-\frac{q_m}{p_m}=-\frac{10.0\text{ cm}}{6.67\text{ cm}}=-1.50$.

The image formed by the mirror serves as a real object for the lens on the second pass of the light through the lens with:

$$p_3=20.0\text{ cm}-q_m=+10.0\text{ cm}.$$

The thin lens equation yields: $\frac{1}{10.0\text{ cm}}+\frac{1}{q_3}=\frac{1}{5.00\text{ cm}}$

or $q_3=10.0\text{ cm}$

and $M_3=-\frac{q_3}{p_3}=-\frac{10.0\text{ cm}}{10.0\text{ cm}}=-1.00$.

The final image is a real image located $\boxed{\text{10.0 cm to the left of the lens}}$.

The overall magnification is $M_{\text{total}}=M_1M_2M_3=\boxed{-2.50}$.

(c) Since the total magnification is negative, this final image is $\boxed{\text{inverted}}$.

**P36.73** For the objective: $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$ becomes $\frac{1}{3.40\text{ mm}}+\frac{1}{q}=\frac{1}{3.00\text{ mm}}$ so $q=25.5\text{ mm}$.

The objective produces magnification $M_1=-\frac{q}{p}=-\frac{25.5\text{ mm}}{3.40\text{ mm}}=-7.50$.

For the eyepiece as a simple magnifier, $m_e=\frac{25.0\text{ cm}}{f}=\frac{25.0\text{ cm}}{2.50\text{ cm}}=10.0$

and overall $M=M_1m_e=\boxed{-75.0}$.

**P36.74** (a) Start with the second lens: This lens must form a virtual image located 19.0 cm to the left of it (i.e., $q_2 = -19.0\text{ cm}$). The required object distance for this lens is then

$$p_2 = \frac{q_2 f_2}{q_2 - f_2} = \frac{(-19.0\text{ cm})(20.0\text{ cm})}{-19.0\text{ cm} - 20.0\text{ cm}} = \frac{380\text{ cm}}{39.0}.$$

The image formed by the first lens serves as the object for the second lens. Therefore, the image distance for the first lens is

$$q_1 = 50.0\text{ cm} - p_2 = 50.0\text{ cm} - \frac{380\text{ cm}}{39.0} = \frac{1\,570\text{ cm}}{39.0}.$$

The distance the original object must be located to the left of the first lens is then given by

$$\frac{1}{p_1} = \frac{1}{f_1} - \frac{1}{q_1} = \frac{1}{10.0\text{ cm}} - \frac{39.0}{1\,570\text{ cm}} = \frac{157 - 39.0}{1\,570\text{ cm}} = \frac{118}{1\,570\text{ cm}} \quad \text{or} \quad p_1 = \frac{1\,570\text{ cm}}{118} = \boxed{13.3\text{ cm}}.$$

(b) $$M = M_1 M_2 = \left(-\frac{q_1}{p_1}\right)\left(-\frac{q_2}{p_2}\right) = \left[\left(\frac{1\,570\text{ cm}}{39.0}\right)\left(\frac{118}{1\,570\text{ cm}}\right)\right]\left[\frac{(-19.0\text{ cm})(39.0)}{380\text{ cm}}\right] = \boxed{-5.90}$$

(c) Since $M < 0$, the final image is $\boxed{\text{inverted}}$.

**P36.75** (a) $$P = \frac{1}{f} = \frac{1}{p} + \frac{1}{q} = \frac{1}{(0.022\,4\text{ m})} + \frac{1}{\infty} = \boxed{44.6\text{ diopters}}$$

(b) $$P = \frac{1}{f} = \frac{1}{p} + \frac{1}{q} = \frac{1}{(0.330\text{ m})} + \frac{1}{\infty} = \boxed{3.03\text{ diopters}}$$

**P36.76** The object is located at the focal point of the upper mirror. Thus, the upper mirror creates an image at infinity (i.e., parallel rays leave this mirror).

The lower mirror focuses these parallel rays at its focal point, located at the hole in the upper mirror.

Thus, the $\boxed{\text{image is real, inverted, and actual size}}$.

For the upper mirror:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}: \qquad \frac{1}{7.50\text{ cm}} + \frac{1}{q_1} = \frac{1}{7.50\text{ cm}} \qquad q_1 = \infty.$$

For the lower mirror:

$$\frac{1}{\infty} + \frac{1}{q_2} = \frac{1}{7.50\text{ cm}} \qquad q_2 = 7.50\text{ cm}.$$

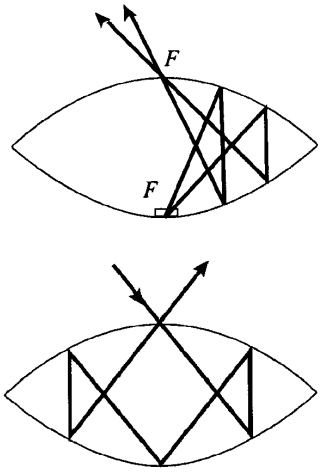

FIG. P36.76

Light directed into the hole in the upper mirror reflects as shown, to behave as if it were reflecting from the hole.

**P36.77** (a) For lens one, as shown in the first figure,

$$\frac{1}{40.0\text{ cm}}+\frac{1}{q_1}=\frac{1}{30.0\text{ cm}}$$

$$q_1=120\text{ cm}$$

$$M_1=-\frac{q_1}{p_1}=-\frac{120\text{ cm}}{40.0\text{ cm}}=-3.00$$

This real image $I_1=O_2$ is a virtual object for the second lens. That is, it is *behind* the lens, as shown in the second figure. The object distance is

$$p_2=110\text{ cm}-120\text{ cm}=-10.0\text{ cm}$$

$$\frac{1}{-10.0\text{ cm}}+\frac{1}{q_2}=\frac{1}{-20.0\text{ cm}}:$$

$$q_2=\boxed{20.0\text{ cm}}$$

$$M_2=-\frac{q_2}{p_2}=-\frac{20.0\text{ cm}}{(-10.0\text{ cm})}=+2.00$$

$$M_{\text{overall}}=M_1M_2=\boxed{-6.00}$$

(b) $M_{\text{overall}}<0$, so final image is $\boxed{\text{inverted}}$.

(c) If lens two is a converging lens (third figure):

$$\frac{1}{-10.0\text{ cm}}+\frac{1}{q_2}=\frac{1}{20.0\text{ cm}}$$

$$q_2=\boxed{6.67\text{ cm}}$$

$$M_2=-\frac{6.67\text{ cm}}{(-10.0\text{ cm})}=+0.667$$

$$M_{\text{overall}}=M_1M_2=\boxed{-2.00}$$

Again, $M_{\text{overall}}<0$ and the final image is $\boxed{\text{inverted}}$.

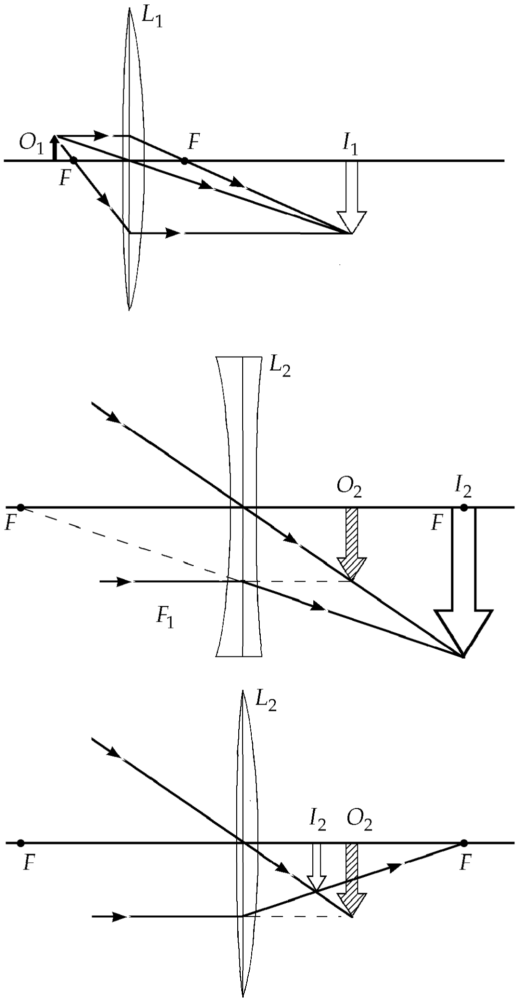

**FIG. P36.77**

***P36.78** The first lens has focal length described by

$$\frac{1}{f_1}=(n_1-1)\left(\frac{1}{R_{11}}-\frac{1}{R_{12}}\right)=(n_1-1)\left(\frac{1}{\infty}-\frac{1}{R}\right)=-\frac{n_1-1}{R}.$$

For the second lens

$$\frac{1}{f_2}=(n_2-1)\left(\frac{1}{R_{21}}-\frac{1}{R_{22}}\right)=(n_2-1)\left(\frac{1}{+R}-\frac{1}{-R}\right)=+\frac{2(n_2-1)}{R}.$$

Let an object be placed at any distance $p_1$ large compared to the thickness of the doublet. The first lens forms an image according to

$$\frac{1}{p_1}+\frac{1}{q_1}=\frac{1}{f_1}$$

$$\frac{1}{q_1}=\frac{-n_1+1}{R}-\frac{1}{p_1}.$$

This virtual $(q_1<0)$ image is a real object for the second lens at distance $p_2=-q_1$. For the second lens

$$\frac{1}{p_2}+\frac{1}{q_2}=\frac{1}{f_2}$$

$$\frac{1}{q_2}=\frac{2n_2-2}{R}-\frac{1}{p_2}=\frac{2n_2-2}{R}+\frac{1}{q_1}=\frac{2n_2-2}{R}+\frac{-n_1+1}{R}-\frac{1}{p_1}=\frac{2n_2-n_1-1}{R}-\frac{1}{p_1}.$$

Then $\frac{1}{p_1}+\frac{1}{q_2}=\frac{2n_2-n_1-1}{R}$ so the doublet behaves like a single lens with $\frac{1}{f}=\frac{2n_2-n_1-1}{R}$.

## ANSWERS TO EVEN PROBLEMS

**P36.2** 4.58 m

**P36.4** see the solution

**P36.6** (a) $p_1+h$; (b) virtual; (c) upright; (d) $+1$; (e) No

**P36.8** at $q=-0.267$ m virtual upright and diminished with $M=0.026\,7$

**P36.10** at 3.33 m from the deepest point of the niche

**P36.12** 30.0 cm

**P36.14** (a) 160 mm; (b) $R=-267$ mm

**P36.16** (a) convex; (b) At the 30.0 cm mark; (c) –20.0 cm

**P36.18** (a) 15.0 cm; (b) 60.0 cm

**P36.20** (a) see the solution;
(b) at 0.639 s and at 0.782 s

**P36.22** 4.82 cm

**P36.24** see the solution;
real, inverted, diminished

**P36.26** 2.00

**P36.28** 20.0 cm

**P36.30** (a) $q=40.0$ cm real, inverted, actual size $M=-1.00$;
(b) $q=\infty$, $M=\infty$, no image is formed;
(c) $q=-20.0$ cm upright, virtual, enlarged $M=+2.00$

**P36.32** (a) 6.40 cm; (b) $-0.250$; (c) converging

**P36.34** (a) 3.40, upright; (b) see the solution

**P36.36** (a) 39.0 mm; (b) 39.5 mm

**P36.38** 1.16 mm/s toward the lens

**P36.40** (a) 13.3 cm;
(b) see the solution; a trapezoid;
(c) 224 $\text{cm}^2$

**P36.42** 2.18 mm away from the film

**P36.44** (a) at $q = -34.7$ cm
virtual, upright and diminshed;
(b) at $q = -36.1$ cm
virtual, upright and diminshed

**P36.46** $\dfrac{f}{1.41}$

**P36.48** 23.2 cm

**P36.50** (a) at 4.17 cm; (b) 6.00

**P36.52** 2.14 cm

**P36.54** (a) see the solution; (b) $h' = -\dfrac{hf}{p}$;
(c) $-1.07$ mm

**P36.56** 3.38 min

**P36.58** if $M < 1$, $f = \dfrac{-Md}{(1-M)^2}$,
if $M > 1$, $f = \dfrac{Md}{(M-1)^2}$

**P36.60** (a) inside the rod, 47.1 cm from the second surface;
(b) virtual, inverted, and enlarged

**P36.62** 25.3 cm to right of mirror,
virtual, upright, enlarged 8.05 times

**P36.64** place the lenses 9.00 cm apart and let light pass through the diverging lens first.
1.75 times

**P36.66** (a) 1.40 $\text{kW/m}^2$; (b) 6.91 $\text{mW/m}^2$;
(c) 0.164 cm; (d) 58.1 $\text{W/m}^2$

**P36.68** 11.7 cm

**P36.70** (a) 0.334 m or larger;
(b) $\dfrac{R_a}{R} = 0.025\,5$ or larger

**P36.72** (a) 1.99;
(b) 10.0 cm to the left of the lens; $-2.50$;
(c) inverted

**P36.74** (a) 13.3 cm; (b) $-5.90$; (c) inverted

**P36.76** see the solution;
real, inverted, and actual size

**P36.78** see the solution

# 37

# Interference of Light Waves

**CHAPTER OUTLINE**

37.1 Conditions for Interference
37.2 Young's Double-Slit Experiment
37.3 Intensity Distribution of the Double-Slit Interference Pattern
37.4 Phasor Addition of Waves
37.5 Change of Phase Due to Reflection
37.6 Interference in Thin Films
37.7 The Michelson Interferometer

## ANSWERS TO QUESTIONS

**Q37.1** (a) Two waves interfere constructively if their path difference is zero, or an integral multiple of the wavelength, according to $\delta = m\lambda$, with $m = 0, 1, 2, 3, \ldots$.

(b) Two waves interfere destructively if their path difference is a half wavelength, or an odd multiple of $\frac{\lambda}{2}$, described by $\delta = \left(m + \frac{1}{2}\right)\lambda$, with $m = 0, 1, 2, 3, \ldots$.

**Q37.2** The light from the flashlights consists of many different wavelengths (that's why it's white) with random time differences between the light waves. There is no *coherence* between the two sources. The light from the two flashlights does not maintain a constant phase relationship over time. These three equivalent statements mean no possibility of an interference pattern.

**Q37.3** Underwater, the wavelength of the light would decrease, $\lambda_{\text{water}} = \frac{\lambda_{\text{air}}}{n_{\text{water}}}$. Since the positions of light and dark bands are proportional to $\lambda$, (according to Equations 37.2 and 37.3), the underwater fringe separations will decrease.

**Q37.4** Every color produces its own pattern, with a spacing between the maxima that is characteristic of the wavelength. With several colors, the patterns are superimposed and it can be difficult to pick out a single maximum. Using monochromatic light can eliminate this problem.

**Q37.5** The threads that are woven together to make the cloth have small meshes between them. These bits of space act as pinholes through which the light diffracts. Since the cloth is a grid of such pinholes, an interference pattern is formed, as when you look through a diffraction grating.

**Q37.6** If the oil film is brightest where it is thinnest, then $n_{\text{air}} < n_{\text{oil}} < n_{\text{water}}$. With this condition, light reflecting from both the top and the bottom surface of the oil film will undergo phase reversal. Then these two beams will be in phase with each other where the film is very thin. This is the condition for constructive interference as the thickness of the oil film decreases toward zero.

**Q37.7** As water evaporates from the 'soap' bubble, the thickness of the bubble wall approaches zero. Since light reflecting from the front of the water surface is phase-shifted 180° and light reflecting from the back of the soap film is phase-shifted 0°, the reflected light meets the conditions for a minimum. Thus the soap film appears black, as in the illustration accompanying textbook Example 37.5, "Interference in a Wedge-Shaped Film."

**Q37.8** If the film is more than a few wavelengths thick, the interference fringes are so close together that you cannot resolve them.

**Q37.9** If $R$ is large, light reflecting from the lower surface of the lens can interfere with light reflecting from the upper surface of the flat. The latter undergoes phase reversal on reflection while the former does not. Where there is negligible distance between the surfaces, at the center of the pattern you will see a dark spot because of the destructive interference associated with the 180° phase shift. Colored rings surround the dark spot. If the lens is a perfect sphere the rings are perfect circles. Distorted rings reveal bumps or hollows on the fine scale of the wavelength of visible light.

**Q37.10** A camera lens will have more than one element, to correct (at least) for chromatic aberration. It will have several surfaces, each of which would reflect some fraction of the incident light. To maximize light throughput the surfaces need antireflective coatings. The coating thickness is chosen to produce destructive interference for reflected light of some wavelength.

**Q37.11** To do Young's double-slit interference experiment with light from an ordinary source, you must first pass the light through a prism or diffraction grating to disperse different colors into different directions. With a single narrow slit you select a single color and make that light diffract to cover both of the slits for the interference experiment. Thus you may have trouble lining things up and you will generally have low light power reaching the screen. The laser light is already monochromatic and coherent across the width of the beam.

**Q37.12** Suppose the coating is intermediate in index of refraction between vacuum and the glass. When the coating is very thin, light reflected from its top and bottom surfaces will interfere constructively, so you see the surface white and brighter. As the thickness reaches one quarter of the wavelength of violet light in the coating, destructive interference for violet will make the surface look red or perhaps orange. Next to interfere destructively are blue, green, yellow, orange, and red, making the surface look red, purple, and then blue. As the coating gets still thicker, we can get constructive interference for violet and then for other colors in spectral order. Still thicker coating will give constructive and destructive interference for several visible wavelengths, so the reflected light will start to look white again.

**Q37.13** Assume the film is higher in refractive index than the medium on both sides of it. The condition for destructive interference of the two transmitted beams is that the waves be out of phase by $\frac{\lambda}{2}$. The ray that reflects through the film undergoes phase reversal both at the bottom and at the top surface. Then this ray should also travel an extra distance of $\frac{\lambda}{2}$. Since this ray passes through two extra thicknesses of film, the thickness should be $\frac{\lambda}{4}$. This is different from the condition for destructive interference of light reflected from the film, but it is the same as the condition for constructive interference of reflected light. The energy of the extra reflected light is energy diverted from light otherwise transmitted.

**Q37.14** The metal body of the airplane is reflecting radio waves broadcast by the television station. The reflected wave that your antenna receives has traveled an extra distance compared to the stronger signal that came straight from the transmitter tower. You receive it with a short time delay. On the television screen you see a faint image offset to the side.

## SOLUTIONS TO PROBLEMS

### Section 37.1 Conditions for Interference

### Section 37.2 Young's Double-Slit Experiment

**P37.1** $$\Delta y_{\text{bright}} = \frac{\lambda L}{d} = \frac{\left(632.8 \times 10^{-9}\right)(5.00)}{2.00 \times 10^{-4}} \text{ m} = \boxed{1.58 \text{ cm}}$$

**P37.2** $$y_{\text{bright}} = \frac{\lambda L}{d} m$$

For $m = 1$, $$\lambda = \frac{yd}{L} = \frac{\left(3.40 \times 10^{-3} \text{ m}\right)\left(5.00 \times 10^{-4} \text{ m}\right)}{3.30 \text{ m}} = \boxed{515 \text{ nm}}$$

**P37.3** Note, with the conditions given, the small angle approximation **does not work well**. That is, $\sin\theta$, $\tan\theta$, and $\theta$ are significantly different. We treat the interference as a Fraunhofer pattern.

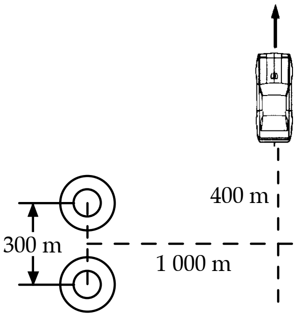

**FIG. P37.3**

(a) At the $m = 2$ maximum, $\tan\theta = \dfrac{400 \text{ m}}{1\,000 \text{ m}} = 0.400$

$$\theta = 21.8°$$

so $$\lambda = \frac{d\sin\theta}{m} = \frac{(300 \text{ m})\sin 21.8°}{2} = \boxed{55.7 \text{ m}}.$$

(b) The next minimum encountered is the $m = 2$ minimum;

and at that point, $$d\sin\theta = \left(m + \frac{1}{2}\right)\lambda$$

which becomes $$d\sin\theta = \frac{5}{2}\lambda$$

or $$\sin\theta = \frac{5}{2}\frac{\lambda}{d} = \frac{5}{2}\left(\frac{55.7 \text{ m}}{300 \text{ m}}\right) = 0.464$$

and $$\theta = 27.7°$$

so $$y = (1\,000 \text{ m})\tan 27.7° = 524 \text{ m}.$$

Therefore, the car must travel an additional $\boxed{124 \text{ m}}$.

If we considered Fresnel interference, we would more precisely find
(a) $\lambda = \dfrac{1}{2}\left(\sqrt{550^2 + 1\,000^2} - \sqrt{250^2 + 1\,000^2}\right) = 55.2 \text{ m}$ and (b) 123 m.

**P37.4** $\lambda = \frac{v}{f} = \frac{354\ \text{m/s}}{2\,000\ \text{s}^{-1}} = 0.177\ \text{m}$

(a) $d\sin\theta = m\lambda$ so $(0.300\ \text{m})\sin\theta = 1(0.177\ \text{m})$ and $\theta = \boxed{36.2°}$

(b) $d\sin\theta = m\lambda$ so $d\sin 36.2° = 1(0.030\,0\ \text{m})$ and $d = \boxed{5.08\ \text{cm}}$

(c) $(1.00\times10^{-6}\ \text{m})\sin 36.2° = (1)\lambda$ so $\lambda = 590\ \text{nm}$

$$f = \frac{c}{\lambda} = \frac{3.00\times10^{8}\ \text{m/s}}{5.90\times10^{-7}\ \text{m}} = \boxed{508\ \text{THz}}$$

**P37.5** In the equation $d\sin\theta = \left(m + \frac{1}{2}\right)\lambda$.

The first minimum is described by $m = 0$

and the tenth by $m = 9$: $\sin\theta = \frac{\lambda}{d}\left(9 + \frac{1}{2}\right)$.

Also, $\tan\theta = \frac{y}{L}$

but for small *θ*, $\sin\theta \approx \tan\theta$.

Thus, $d = \frac{9.5\lambda}{\sin\theta} = \frac{9.5\lambda L}{y}$

$$d = \frac{9.5(5\,890\times10^{-10}\ \text{m})(2.00\ \text{m})}{7.26\times10^{-3}\ \text{m}} = 1.54\times10^{-3}\ \text{m} = \boxed{1.54\ \text{mm}}.$$

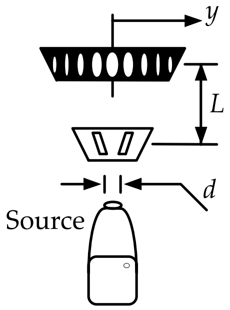

FIG. P37.5

**P37.6** $\lambda = \frac{340\ \text{m/s}}{2\,000\ \text{Hz}} = 0.170\ \text{m}$

Maxima are at $d\sin\theta = m\lambda$:

$m = 0$ gives $\theta = 0°$

$m = 1$ gives $\sin\theta = \frac{\lambda}{d} = \frac{0.170\ \text{m}}{0.350\ \text{m}}$ $\theta = 29.1°$

$m = 2$ gives $\sin\theta = \frac{2\lambda}{d} = 0.971$ $\theta = 76.3°$

$m = 3$ gives $\sin\theta = 1.46$ No solution.

Minima are at $d\sin\theta = \left(m + \frac{1}{2}\right)\lambda$:

$m = 0$ gives $\sin\theta = \frac{\lambda}{2d} = 0.243$ $\theta = 14.1°$

$m = 1$ gives $\sin\theta = \frac{3\lambda}{2d} = 0.729$ $\theta = 46.8°$

$m = 2$ gives $\sin\theta = 1.21$ No solution.

$\boxed{\text{So we have maxima at } 0°,\ 29.1°, \text{ and } 76.3°;\ \text{minima at } 14.1° \text{ and } 46.8°}$.

**P37.7** (a) For the bright fringe,

$$y_{\text{bright}} = \frac{m\lambda L}{d} \text{ where } m = 1$$

$$y = \frac{\left(546.1\times10^{-9}\text{ m}\right)(1.20\text{ m})}{0.250\times10^{-3}\text{ m}} = 2.62\times10^{-3}\text{ m} = \boxed{2.62\text{ mm}}.$$

(b) For the dark bands, $y_{\text{dark}} = \frac{\lambda L}{d}\left(m+\frac{1}{2}\right);\ m = 0, 1, 2, 3, \ldots$

$$y_2 - y_1 = \frac{\lambda L}{d}\left[\left(1+\frac{1}{2}\right)-\left(0+\frac{1}{2}\right)\right] = \frac{\lambda L}{d}(1)$$

$$= \frac{\left(546.1\times10^{-9}\text{ m}\right)(1.20\text{ m})}{0.250\times10^{-3}\text{ m}}$$

$$\Delta y = \boxed{2.62\text{ mm}}.$$

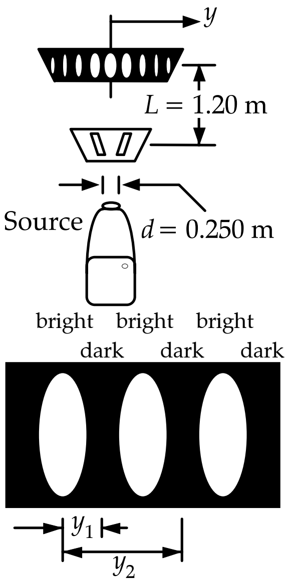

FIG. P37.7

**P37.8** Taking $m = 0$ and $y = 0.200$ mm in Equation 37.6 gives

$$L \approx \frac{2dy}{\lambda} = \frac{2\left(0.400\times10^{-3}\text{ m}\right)\left(0.200\times10^{-3}\text{ m}\right)}{442\times10^{-9}\text{ m}} = 0.362\text{ m}$$

$$L \approx \boxed{36.2\text{ cm}}$$

Geometric optics incorrectly predicts bright regions opposite the slits and darkness in between. But, as this example shows, interference can produce just the opposite.

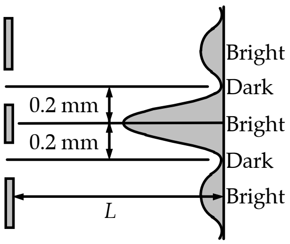

FIG. P37.7

**P37.9** Location of $A =$ central maximum,

Location of $B =$ first minimum.

So, $$\Delta y = \left[y_{\min} - y_{\max}\right] = \frac{\lambda L}{d}\left(0+\frac{1}{2}\right) - 0 = \frac{1}{2}\frac{\lambda L}{d} = 20.0\text{ m}.$$

Thus, $$d = \frac{\lambda L}{2(20.0\text{ m})} = \frac{(3.00\text{ m})(150\text{ m})}{40.0\text{ m}} = \boxed{11.3\text{ m}}.$$

**P37.10** At $30.0°$, $d\sin\theta = m\lambda$

$$\left(3.20\times10^{-4}\text{ m}\right)\sin 30.0° = m\left(500\times10^{-9}\text{ m}\right) \qquad \text{so} \qquad m = 320$$

There are 320 maxima to the right, 320 to the left, and one for $m = 0$ straight ahead.

There are $\boxed{641\text{ maxima}}$.

***P37.11** Observe that the pilot must not only home in on the airport, but must be headed in the right direction when she arrives at the end of the runway.

(a) $\lambda = \frac{c}{f} = \frac{3\times 10^8 \text{ m/s}}{30\times 10^6 \text{ s}^{-1}} = \boxed{10.0 \text{ m}}$

(b) The first side maximum is at an angle given by $d\sin\theta = (1)\lambda$.

$(40 \text{ m})\sin\theta = 10 \text{ m}$ $\qquad\theta = 14.5°$ $\qquad\tan\theta = \frac{y}{L}$

$y = L\tan\theta = (2\,000 \text{ m})\tan 14.5° = \boxed{516 \text{ m}}$

(c) The signal of 10-m wavelength in parts (a) and (b) would show maxima at 0°, 14.5°, 30.0°, 48.6°, and 90°. A signal of wavelength 11.23-m would show maxima at 0°, 16.3°, 34.2°, and 57.3°. The only value in common is 0°. If $\lambda_1$ and $\lambda_2$ were related by a ratio of small integers (a just musical consonance!) in $\frac{\lambda_1}{\lambda_2} = \frac{n_1}{n_2}$, then the equations $d\sin\theta = n_2\lambda_1$ and $d\sin\theta = n_1\lambda_2$ would both be satisfied for the same nonzero angle. The pilot could come flying in with that inappropriate bearing, and run off the runway immediately after touchdown.

***P37.12** In $d\sin\theta = m\lambda$ $\qquad d\frac{y}{L} = m\lambda$ $\qquad y = \frac{m\lambda L}{d}$

$$\frac{dy}{dt} = \frac{m\lambda}{d}\frac{dL}{dt} = \frac{1\left(633\times 10^{-9} \text{ m}\right)}{\left(0.3\times 10^{-3} \text{ m}\right)} 3 \text{ m/s} = \boxed{6.33 \text{ mm/s}}$$

**P37.13** $\phi = \frac{2\pi}{\lambda} d\sin\theta = \frac{2\pi}{\lambda} d\left(\frac{y}{L}\right)$

(a) $\phi = \frac{2\pi}{\left(5.00\times 10^{-7} \text{ m}\right)}\left(1.20\times 10^{-4} \text{ m}\right)\sin(0.500°) = \boxed{13.2 \text{ rad}}$

(b) $\phi = \frac{2\pi}{\left(5.00\times 10^{-7} \text{ m}\right)}\left(1.20\times 10^{-4} \text{ m}\right)\left(\frac{5.00\times 10^{-3} \text{ m}}{1.20 \text{ m}}\right) = \boxed{6.28 \text{ rad}}$

(c) If $\phi = 0.333 \text{ rad} = \frac{2\pi d\sin\theta}{\lambda}$ $\qquad \theta = \sin^{-1}\left(\frac{\lambda\phi}{2\pi d}\right) = \sin^{-1}\left[\frac{\left(5.00\times 10^{-7} \text{ m}\right)(0.333 \text{ rad})}{2\pi\left(1.20\times 10^{-4} \text{ m}\right)}\right]$

$\theta = \boxed{1.27\times 10^{-2} \text{ deg}}$.

(d) If $d\sin\theta = \frac{\lambda}{4}$ $\qquad \theta = \sin^{-1}\left(\frac{\lambda}{4d}\right) = \sin^{-1}\left[\frac{5\times 10^{-7} \text{ m}}{4\left(1.20\times 10^{-4} \text{ m}\right)}\right]$

$\theta = \boxed{5.97\times 10^{-2} \text{ deg}}$.

**P37.14** The path difference between rays 1 and 2 is: $\delta = d\sin\theta_1 - d\sin\theta_2$.

For constructive interference, this path difference must be equal to an integral number of wavelengths: $d\sin\theta_1 - d\sin\theta_2 = m\lambda$, or

$\boxed{d(\sin\theta_1 - \sin\theta_2) = m\lambda}$.

**P37.15** (a) The path difference $\delta = d\sin\theta$ and when $L >> y$

$$\delta = \frac{yd}{L} = \frac{\left(1.80\times10^{-2}\text{ m}\right)\left(1.50\times10^{-4}\text{ m}\right)}{1.40\text{ m}} = 1.93\times10^{-6}\text{ m} = \boxed{1.93\ \mu\text{m}}.$$

(b) $\dfrac{\delta}{\lambda} = \dfrac{1.93\times10^{-6}\text{ m}}{6.43\times10^{-7}\text{ m}} = 3.00$, or $\boxed{\delta = 3.00\lambda}$

(c) Point P will be a $\boxed{\text{maximum}}$ since the path difference is an integer multiple of the wavelength.

## Section 37.3 **Intensity Distribution of the Double-Slit Interference Pattern**

**P37.16** (a) $\dfrac{I}{I_{\max}} = \cos^2\left(\dfrac{\phi}{2}\right)$ (Equation 37.11)

Therefore, $\phi = 2\cos^{-1}\sqrt{\dfrac{I}{I_{\max}}} = 2\cos^{-1}\sqrt{0.640} = \boxed{1.29\text{ rad}}$.

(b) $\delta = \dfrac{\lambda\phi}{2\pi} = \dfrac{(486\text{ nm})(1.29\text{ rad})}{2\pi} = \boxed{99.8\text{ nm}}$

**P37.17** $I_{av} = I_{\max}\cos^2\left(\dfrac{\pi d\sin\theta}{\lambda}\right)$

For small $\theta$, $\sin\theta = \dfrac{y}{L}$

and $I_{av} = 0.750 I_{\max}$

$$y = \frac{\lambda L}{\pi d}\cos^{-1}\sqrt{\frac{I_{av}}{I_{\max}}}$$

$$y = \frac{\left(6.00\times10^{-7}\right)(1.20\text{ m})}{\pi\left(2.50\times10^{-3}\text{ m}\right)}\cos^{-1}\sqrt{\frac{0.750 I_{\max}}{I_{\max}}} = \boxed{48.0\ \mu\text{m}}$$

**P37.18** $I = I_{\max}\cos^2\left(\dfrac{\pi yd}{\lambda L}\right)$

$$\frac{I}{I_{\max}} = \cos^2\left[\frac{\pi\left(6.00\times10^{-3}\text{ m}\right)\left(1.80\times10^{-4}\text{ m}\right)}{\left(656.3\times10^{-9}\text{ m}\right)(0.800\text{ m})}\right] = \boxed{0.968}$$

**P37.19** (a) From Equation 37.8,

$$\phi = \frac{2\pi d}{\lambda}\sin\theta = \frac{2\pi d}{\lambda}\cdot\frac{y}{\sqrt{y^2+D^2}}$$

$$\phi \approx \frac{2\pi yd}{\lambda D} = \frac{2\pi\left(0.850\times10^{-3}\text{ m}\right)\left(2.50\times10^{-3}\text{ m}\right)}{\left(600\times10^{-9}\text{ m}\right)(2.80\text{ m})} = \boxed{7.95\text{ rad}}$$

(b)

$$\frac{I}{I_{\max}} = \frac{\cos^2\left[(\pi d/\lambda)\sin\theta\right]}{\cos^2\left[(\pi d/\lambda)\sin\theta_{\max}\right]} = \frac{\cos^2(\phi/2)}{\cos^2 m\pi}$$

$$\frac{I}{I_{\max}} = \cos^2\frac{\phi}{2} = \cos^2\left(\frac{7.95\text{ rad}}{2}\right) = \boxed{0.453}$$

**P37.20** (a) The resultant amplitude is

$$E_r = E_0\sin\omega t + E_0\sin(\omega t+\phi) + E_0(\omega t+2\phi), \qquad \text{where} \qquad \phi = \frac{2\pi}{\lambda}d\sin\theta.$$

$$E_r = E_0(\sin\omega t + \sin\omega t\cos\phi + \cos\omega t\sin\phi + \sin\omega t\cos 2\phi + \cos\omega t\sin 2\phi)$$

$$E_r = E_0(\sin\omega t)\left(1+\cos\phi+2\cos^2\phi-1\right) + E_0(\cos\omega t)(\sin\phi + 2\sin\phi\cos\phi)$$

$$E_r = E_0(1+2\cos\phi)(\sin\omega t\cos\phi + \cos\omega t\sin\phi) = E_0(1+2\cos\phi)\sin(\omega t+\phi)$$

Then the intensity is $I \propto E_r^2 = E_0^2(1+2\cos\phi)^2\left(\frac{1}{2}\right)$

where the time average of $\sin^2(\omega t+\phi)$ is $\frac{1}{2}$.

From one slit alone we would get intensity $I_{\max} \propto E_0^2\left(\frac{1}{2}\right)$ so

$$\boxed{I = I_{\max}\left[1+2\cos\left(\frac{2\pi d\sin\theta}{\lambda}\right)\right]^2}.$$

(b) Look at the $N = 3$ graph in Figure 37.14. Minimum intensity is zero, attained where $\cos\phi = -\frac{1}{2}$. One relative maximum occurs at $\cos\phi = -1.00$, where $I = I_{\max}$.

The larger local maximum happens where $\cos\phi = +1.00$, giving $I = 9.00I_0$.

The ratio of intensities at primary versus secondary maxima is $\boxed{9.00}$.

## Section 37.4 **Phasor Addition of Waves**

**P37.21** (a) We can use $\sin A + \sin B = 2\sin\left(\frac{A}{2}+\frac{B}{2}\right)\cos\left(\frac{A}{2}-\frac{B}{2}\right)$ to find the sum of the two sine functions to be

$$E_1 + E_2 = (24.0\ \text{kN/C})\sin(15x - 4.5t + 35.0°)\cos 35.0°$$

$$E_1 + E_2 = (19.7\ \text{kN/C})\sin(15x - 4.5t + 35.0°)$$

Thus, the total wave has amplitude $\boxed{19.7\ \text{kN/C}}$ and has a constant phase difference of $\boxed{35.0°}$ from the first wave.

(b) In units of kN/C, the resultant phasor is

$$\mathbf{E}_R = \mathbf{E}_1 + \mathbf{E}_2 = (12.0\hat{\mathbf{i}}) + (12.0\cos 70.0°\,\hat{\mathbf{i}} + 12.0\sin 70.0\hat{\mathbf{j}}) = 16.1\hat{\mathbf{i}} + 11.3\hat{\mathbf{j}}$$

$$\mathbf{E}_R = \sqrt{(16.1)^2 + (11.3)^2}\ \text{at}\ \tan^{-1}\left(\frac{11.3}{16.1}\right) = \boxed{19.7\ \text{kN/C at } 35.0°}$$

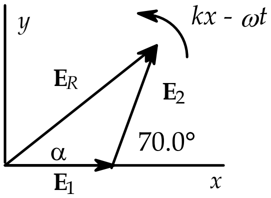

FIG. P37.21(b)

(c)
$$\mathbf{E}_R = 12.0\cos 70.0°\,\hat{\mathbf{i}} + 12.0\sin 70.0°\,\hat{\mathbf{j}}$$
$$+15.5\cos 80.0°\,\hat{\mathbf{i}} - 15.5\sin 80.0°\,\hat{\mathbf{j}}$$
$$+17.0\cos 160°\,\hat{\mathbf{i}} + 17.0\sin 160°\,\hat{\mathbf{j}}$$
$$\mathbf{E}_R = -9.18\hat{\mathbf{i}} + 1.83\hat{\mathbf{j}} = \boxed{9.36\ \text{kN/C at } 169°}$$

The wave function of the total wave is
$E_P = (9.36\ \text{kN/C})\sin(15x - 4.5t + 169°)$.

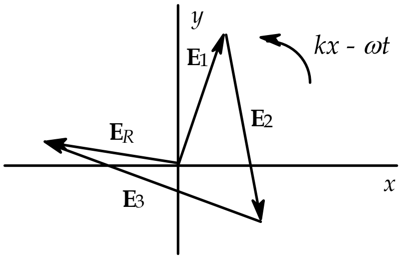

FIG. P37.21(c)

**P37.22** (a)
$$\mathbf{E}_R = E_0\left[\hat{\mathbf{i}} + (\hat{\mathbf{i}}\cos 20.0° + \hat{\mathbf{j}}\sin 20.0°) + (\hat{\mathbf{i}}\cos 40.0° + \hat{\mathbf{j}}\sin 40.0°)\right]$$

$$\mathbf{E}_R = E_0\left[2.71\hat{\mathbf{i}} + 0.985\hat{\mathbf{j}}\right] = 2.88E_0\ \text{at}\ 20.0° = \boxed{2.88E_0\ \text{at } 0.349\ \text{rad}}$$

$$E_P = 2.88E_0\sin(\omega t + 0.349)$$

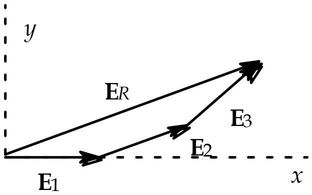

FIG. P37.22(a)

(b)
$$\mathbf{E}_R = E_0\left[\hat{\mathbf{i}} + (\hat{\mathbf{i}}\cos 60.0° + \hat{\mathbf{j}}\sin 60.0°) + (\hat{\mathbf{i}}\cos 120° + \hat{\mathbf{j}}\sin 120°)\right]$$

$$\mathbf{E}_R = E_0\left[1.00\hat{\mathbf{i}} + 1.73\hat{\mathbf{j}}\right] = 2.00E_0\ \text{at}\ 60.0° = \boxed{2.00E_0\ \text{at } \frac{\pi}{3}\ \text{rad}}$$

$$E_P = 2.00E_0\sin\left(\omega t + \frac{\pi}{3}\right)$$

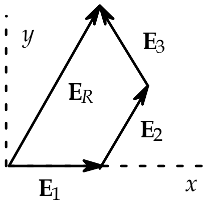

FIG. P37.22(b)

*continued on next page*

(c) $$\mathbf{E}_R = E_0\left[\hat{\mathbf{i}} + \left(\hat{\mathbf{i}}\cos 120° + \hat{\mathbf{j}}\sin 120°\right) + \left(\hat{\mathbf{i}}\cos 240° + \hat{\mathbf{j}}\sin 240°\right)\right]$$

$$\mathbf{E}_R = E_0\left[0\hat{\mathbf{i}} + 0\hat{\mathbf{j}}\right] = \boxed{0}$$

$$E_P = 0$$

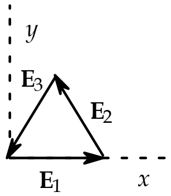

**FIG. P37.22(c)**

(d) $$\mathbf{E}_R = E_0\left[\hat{\mathbf{i}} + \left(\hat{\mathbf{i}}\cos\frac{3\pi}{2} + \hat{\mathbf{j}}\sin\frac{3\pi}{2}\right) + \left(\hat{\mathbf{i}}\cos 3\pi + \hat{\mathbf{j}}\sin 3\pi\right)\right]$$

$$\mathbf{E}_R = E_0\left[0\hat{\mathbf{i}} - 1.00\hat{\mathbf{j}}\right] = E_0 \text{ at } 270° = \boxed{E_0 \text{ at } \frac{3\pi}{2} \text{ rad}}$$

$$E_P = E_0\sin\left(\omega t + \frac{3\pi}{2}\right)$$

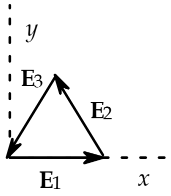

**FIG. P37.22(d)**

**P37.23** $$\mathbf{E}_R = 6.00\hat{\mathbf{i}} + 8.00\hat{\mathbf{j}} = \sqrt{(6.00)^2 + (8.00)^2} \text{ at } \tan^{-1}\left(\frac{8.00}{6.00}\right)$$

$$\mathbf{E}_R = 10.0 \text{ at } 53.1° = 10.0 \text{ at } 0.927 \text{ rad}$$

$$E_P = \boxed{10.0\sin(100\pi t + 0.927)}$$

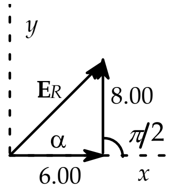

**FIG. P37.23**

**P37.24** If $E_1 = E_{01}\sin\omega t$ and $E_2 = E_{02}\sin(\omega t + \phi)$, then by phasor addition, the amplitude of **E** is

$$E_0 = \sqrt{\left(E_{01} + E_{02}\cos\phi\right)^2 + \left(E_{02}\sin\phi\right)^2} = \boxed{\sqrt{E_{01}^2 + 2E_{01}E_{02}\cos\phi + E_{02}^2}}$$

and the phase angle is found from $\boxed{\sin\theta = \frac{E_{02}\sin\phi}{E_0}}$.

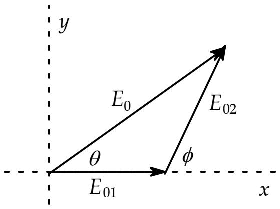

**FIG. P37.24**

**P37.25** $$\mathbf{E}_R = 12.0\hat{\mathbf{i}} + \left(18.0\cos 60.0°\,\hat{\mathbf{i}} + 18.0\sin 60.0°\,\hat{\mathbf{j}}\right)$$

$$\mathbf{E}_R = 21.0\hat{\mathbf{i}} + 15.6\hat{\mathbf{j}} = 26.2 \text{ at } 36.6°$$

$$E_R = \boxed{26.2\sin(\omega t + 36.6°)}$$

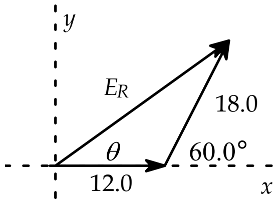

**FIG. P37.25**

**P37.26** Constructive interference occurs where $m = 0, 1, 2, 3, \ldots$, for

$$\left(\frac{2\pi x_1}{\lambda} - 2\pi ft + \frac{\pi}{6}\right) - \left(\frac{2\pi x_2}{\lambda} - 2\pi ft + \frac{\pi}{8}\right) = 2\pi m \qquad \frac{2\pi(x_1 - x_2)}{\lambda} + \left(\frac{\pi}{6} - \frac{\pi}{8}\right) = 2\pi m$$

$$\frac{(x_1 - x_2)}{\lambda} + \frac{1}{12} - \frac{1}{16} = m \qquad \boxed{x_1 - x_2 = \left(m - \frac{1}{48}\right)\lambda \qquad m = 0, 1, 2, 3, \ldots}.$$

**P37.27** See the figure to the right:

$$\boxed{\phi = \frac{\pi}{2}}.$$

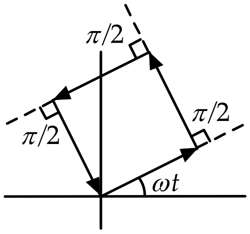

**FIG. P37.27**

**P37.28** $E_R^2 = E_1^2 + E_2^2 - 2E_1E_2\cos\beta$ where $\beta = 180 - \phi$.

Since $I \propto E^2$

$$I_R = \boxed{I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi}.$$

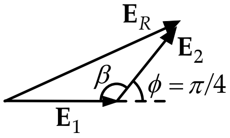

**FIG. P37.28**

**P37.29** Take $\boxed{\phi = \frac{360°}{N}}$ where $N$ defines the number of coherent sources. Then,

$$E_R = \sum_{m=1}^{N} E_0 \sin(\omega t + m\phi) = 0.$$

In essence, the set of $N$ electric field components complete a full circle and return to zero.

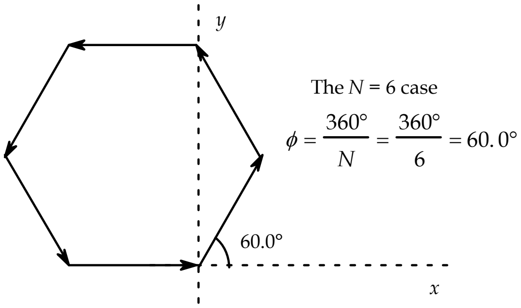

**FIG. P37.29**

## Section 37.5 **Change of Phase Due to Reflection**

## Section 37.6 **Interference in Thin Films**

**P37.30** Light reflecting from the first surface suffers phase reversal. Light reflecting from the second surface does not, but passes twice through the thickness $t$ of the film. So, for constructive interference, we require

$$\frac{\lambda_n}{2} + 2t = \lambda_n$$

where $\lambda_n = \frac{\lambda}{n}$ is the wavelength in the material.

Then $2t = \frac{\lambda_n}{2} = \frac{\lambda}{2n}$

$$\lambda = 4nt = 4(1.33)(115\text{ nm}) = \boxed{612\text{ nm}}.$$

**P37.31** (a) The light reflected from the top of the oil film undergoes phase reversal. Since $1.45 > 1.33$, the light reflected from the bottom undergoes no reversal. For constructive interference of reflected light, we then have

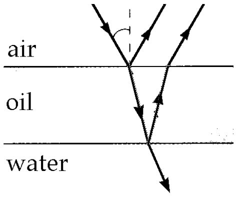

**FIG. P37.31**

$$2nt = \left(m + \frac{1}{2}\right)\lambda$$

or $$\lambda_m = \frac{2nt}{m + (1/2)} = \frac{2(1.45)(280 \text{ nm})}{m + (1/2)}.$$

Substituting for $m$ gives: $m = 0$, $\lambda_0 = 1\,620$ nm (infrared)

$m = 1$, $\lambda_1 = 541$ nm (green)

$m = 2$, $\lambda_2 = 325$ nm (ultraviolet).

Both infrared and ultraviolet light are invisible to the human eye, so the dominant color in reflected light is $\boxed{\text{green}}$.

(b) The dominant wavelengths in the transmitted light are those that produce destructive interference in the reflected light. The condition for destructive interference upon reflection is

$$2nt = m\lambda$$

or $$\lambda_m = \frac{2nt}{m} = \frac{812 \text{ nm}}{m}.$$

Substituting for $m$ gives: $m = 1$, $\lambda_1 = 812$ nm (near infrared)

$m = 2$, $\lambda_2 = 406$ nm (violet)

$m = 3$, $\lambda_3 = 271$ nm (ultraviolet).

Of these, the only wavelength visible to the human eye (and hence the dominate wavelength observed in the transmitted light) is 406 nm. Thus, the dominant color in the transmitted light is $\boxed{\text{violet}}$.

**P37.32** Since $1 < 1.25 < 1.33$, light reflected both from the top and from the bottom surface of the oil suffers phase reversal.

For constructive interference we require $$2t = \frac{m\lambda_{\text{cons}}}{n}$$

and for destructive interference, $$2t = \frac{[m + (1/2)]\lambda_{\text{des}}}{n}.$$

Then $$\frac{\lambda_{\text{cons}}}{\lambda_{\text{dest}}} = 1 + \frac{1}{2m} = \frac{640 \text{ nm}}{512 \text{ nm}} = 1.25 \text{ and } m = 2.$$

Therefore, $$t = \frac{2(640 \text{ nm})}{2(1.25)} = \boxed{512 \text{ nm}}.$$

**P37.33** Treating the anti-reflectance coating like a camera-lens coating,

$$2t=\left(m+\frac{1}{2}\right)\frac{\lambda}{n}.$$

Let $m=0$: $t=\frac{\lambda}{4n}=\frac{3.00\text{ cm}}{4(1.50)}=\boxed{0.500\text{ cm}}$.

This anti-reflectance coating could be easily countered by changing the wavelength of the radar—to 1.50 cm—now creating maximum reflection!

**P37.34** $2nt=\left(m+\frac{1}{2}\right)\lambda$ so $t=\left(m+\frac{1}{2}\right)\frac{\lambda}{2n}$

Minimum $t=\left(\frac{1}{2}\right)\frac{(500\text{ nm})}{2(1.30)}=\boxed{96.2\text{ nm}}$.

**P37.35** Since the light undergoes a 180° phase change at each surface of the film, the condition for *constructive* interference is $2nt=m\lambda$, or $\lambda=\frac{2nt}{m}$. The film thickness is $t=1.00\times10^{-5}\text{ cm}=1.00\times10^{-7}\text{ m}=100\text{ nm}$. Therefore, the wavelengths intensified in the reflected light are

$$\lambda=\frac{2(1.38)(100\text{ nm})}{m}=\frac{276\text{ nm}}{m}\quad\text{where } m=1,\,2,\,3,\,\ldots$$

or $\lambda_1=276\text{ nm}$, $\lambda_2=138\text{ nm}$, . . . . All reflection maxima are in the ultraviolet and beyond.

$\boxed{\text{No visible wavelengths are intensified.}}$

**P37.36** (a) For maximum transmission, we want destructive interference in the light reflected from the front and back surfaces of the film.

If the surrounding glass has refractive index greater than 1.378, light reflected from the front surface suffers no phase reversal and light reflected from the back does undergo phase reversal. This effect by itself would produce destructive interference, so we want the distance down and back to be one whole wavelength in the film: $2t=\frac{\lambda}{n}$.

$$t=\frac{\lambda}{2n}=\frac{656.3\text{ nm}}{2(1.378)}=\boxed{238\text{ nm}}$$

(b) The filter will expand. As *t* increases in $2nt=\lambda$, so does $\boxed{\lambda\text{ increase}}$.

(c) Destructive interference for reflected light happens also for $\lambda$ in $2nt=2\lambda$,

or $\lambda=1.378(238\text{ nm})=\boxed{328\text{ nm}}$ (near ultraviolet).

**P37.37** If the path length difference $\Delta=\lambda$, the transmitted light will be bright. Since $\Delta=2d=\lambda$,

$$d_{\min}=\frac{\lambda}{2}=\frac{580\text{ nm}}{2}=\boxed{290\text{ nm}}.$$

**P37.38** The condition for bright fringes is

$$2t+\frac{\lambda}{2n}=m\frac{\lambda}{n} \qquad m=1,\ 2,\ 3,\ \ldots.$$

From the sketch, observe that

$$t=R(1-\cos\theta)\approx R\left(1-1+\frac{\theta^2}{2}\right)=\frac{R}{2}\left(\frac{r}{R}\right)^2=\frac{r^2}{2R}.$$

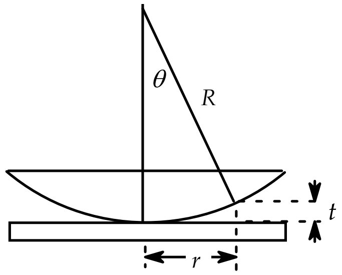

FIG. P37.38

The condition for a bright fringe becomes $\frac{r^2}{R}=\left(m-\frac{1}{2}\right)\frac{\lambda}{n}$.

Thus, for fixed $m$ and $\lambda$, $nr^2=\text{constant}$.

Therefore, $n_{\text{liquid}}r_f^2=n_{\text{air}}r_i^2$ and $n_{\text{liquid}}=(1.00)\frac{(1.50\text{ cm})^2}{(1.31\text{ cm})^2}=\boxed{1.31}$.

**P37.39** For destructive interference in the air,

$$2t=m\lambda.$$

For 30 dark fringes, including the one where the plates meet,

$$t=\frac{29(600\text{ nm})}{2}=8.70\times10^{-6}\text{ m}.$$

Therefore, the *radius* of the wire is

$$r=\frac{t}{2}=\frac{8.70\ \mu\text{m}}{2}=\boxed{4.35\ \mu\text{m}}.$$

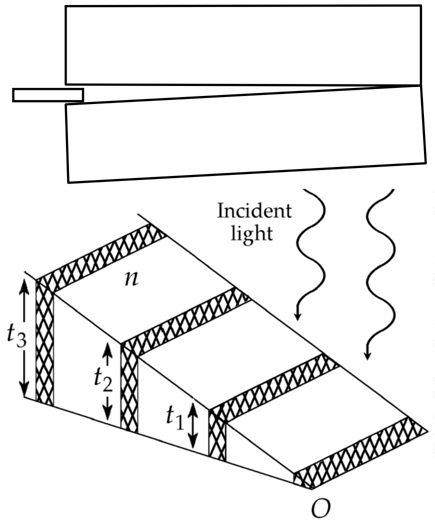

FIG. P37.39

**P37.40** For total darkness, we want destructive interference for reflected light for both 400 nm and 600 nm. With phase reversal at just one reflecting surface, the condition for destructive interference is

$$2n_{\text{air}}t=m\lambda \qquad m=0,\ 1,\ 2,\ \ldots.$$

The least common multiple of these two wavelengths is 1 200 nm, so we get no reflected light at $2(1.00)t=3(400\text{ nm})=2(600\text{ nm})=1\,200\text{ nm}$, so $t=600\text{ nm}$ at this second dark fringe.

By similar triangles, $\frac{600\text{ nm}}{x}=\frac{0.050\,0\text{ mm}}{10.0\text{ cm}}$,

or the distance from the contact point is $x=\left(600\times10^{-9}\text{ m}\right)\left(\frac{0.100\text{ m}}{5.00\times10^{-5}\text{ m}}\right)=\boxed{1.20\text{ mm}}$.

## Section 37.7 **The Michelson Interferometer**

**P37.41** When the mirror on one arm is displaced by $\Delta\ell$, the path difference changes by $2\Delta\ell$. A shift resulting in the reversal between dark and bright fringes requires a path length change of one-half wavelength. Therefore, $2\Delta\ell = \frac{m\lambda}{2}$, where in this case, $m = 250$.

$$\Delta\ell = m\frac{\lambda}{4} = \frac{(250)\left(6.328 \times 10^{-7}\text{ m}\right)}{4} = \boxed{39.6\ \mu\text{m}}$$

**P37.42** Distance $= 2\left(3.82 \times 10^{-4}\text{ m}\right) = 1\,700\lambda$ $\qquad$ $\lambda = 4.49 \times 10^{-7}\text{ m} = \boxed{449\text{ nm}}$

The light is $\boxed{\text{blue}}$.

**P37.43** Counting light going both directions, the number of wavelengths originally in the cylinder is $m_1 = \frac{2L}{\lambda}$. It changes to $m_2 = \frac{2L}{\lambda/n} = \frac{2nL}{\lambda}$ as the cylinder is filled with gas. If $N$ is the number of bright fringes passing, $N = m_2 - m_1 = \frac{2L}{\lambda}(n-1)$, or the index of refraction of the gas is

$$n = \boxed{1 + \frac{N\lambda}{2L}}$$

## Additional Problems

***P37.44** (a) Where fringes of the two colors coincide we have $d\sin\theta = m\lambda = m'\lambda'$, requiring $\frac{\lambda}{\lambda'} = \frac{m'}{m}$.

(b) $\lambda = 430\text{ nm}$, $\lambda' = 510\text{ nm}$

$\therefore \frac{m'}{m} = \frac{430\text{ nm}}{510\text{ nm}} = \frac{43}{51}$, which cannot be reduced any further. Then $m = 51$, $m' = 43$.

$$\theta_m = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left[\frac{(51)\left(430 \times 10^{-9}\text{ m}\right)}{0.025 \times 10^{-3}\text{ m}}\right] = 61.3^\circ$$

$$y_m = L\tan\theta_m = (1.5\text{ m})\tan 61.3^\circ = \boxed{2.74\text{ m}}$$

**P37.45** The wavelength is $\lambda = \frac{c}{f} = \frac{3.00 \times 10^8\text{ m/s}}{60.0 \times 10^6\text{ s}^{-1}} = 5.00\text{ m}$.

Along the line *AB* the two traveling waves going in opposite directions add to give a standing wave. The two transmitters are exactly 2.00 wavelengths apart and the signal from *B*, when it arrives at *A*, will always be in phase with transmitter *B*. Since *B* is 180° out of phase with *A*, the two signals always interfere destructively at the position of *A*.

The first antinode (point of constructive interference) is located at distance

$\frac{\lambda}{4} = \frac{5.00\text{ m}}{4} = \boxed{1.25\text{ m}}$ from the node at *A*.

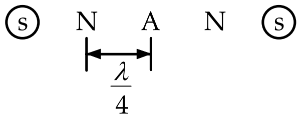

**FIG. P37.46**

***P37.46** Along the line of length $d$ joining the source, two identical waves moving in opposite directions add to give a standing wave. An antinode is halfway between the sources. If $\frac{d}{2} > \frac{\lambda}{2}$, there is space for two more antinodes for a total of three. If $\frac{d}{2} > \lambda$, there will be at least five antinodes, and so on. To repeat, if $\frac{d}{\lambda} > 0$, the number of antinodes is 1 or more. If $\frac{d}{\lambda} > 1$, the number of antinodes is 3 or more. If $\frac{d}{\lambda} > 2$, the number of antinodes is 5 or more. In general,

$$\boxed{\text{The number of antinodes is 1 plus 2 times the greatest integer less than or equal to } \frac{d}{\lambda}.}$$

If $\frac{d}{2} < \frac{\lambda}{4}$, there will be no nodes. If $\frac{d}{2} > \frac{\lambda}{4}$, there will be space for at least two nodes, as shown in the picture. If $\frac{d}{2} > \frac{3\lambda}{4}$, there will be at least four nodes. If $\frac{d}{2} > \frac{5\lambda}{4}$ six or more nodes will fit in, and so on. To repeat, if $2d < \lambda$ the number of nodes is 0. If $2d > \lambda$ the number of nodes is 2 or more. If $2d > 3\lambda$ the number of nodes is 4 or more. If $2d > 5\lambda$ the number of nodes is 6 or more. Again, if $\left(\frac{d}{\lambda} + \frac{1}{2}\right) > 1$, the number of nodes is at least 2. If $\left(\frac{d}{\lambda} + \frac{1}{2}\right) > 2$, the number of nodes is at least 4. If $\left(\frac{d}{\lambda} + \frac{1}{2}\right) > 3$, the number of nodes is at least 6. In general,

$$\boxed{\text{the number of nodes is 2 times the greatest nonzero integer less than } \left(\frac{d}{\lambda} + \frac{1}{2}\right).}$$

Next, we enumerate the zones of constructive interference. They are described by $d\sin\theta = m\lambda$, $m = 0,\ 1,\ 2,\ \ldots$ with $\theta$ counted as positive both left and right of the maximum at $\theta = 0$ in the center. The number of side maxima on each side is the greatest integer satisfying $\sin\theta \le 1$, $d1 \ge m\lambda$, $m \le \frac{d}{\lambda}$. So the total

$$\boxed{\text{number of bright fringes is one plus 2 times the greatest integer less than or equal to } \frac{d}{\lambda}.}$$

It is equal to the number of antinodes on the line joining the sources.

The interference minima are to the left and right at angles described by $d\sin\theta = \left(m + \frac{1}{2}\right)\lambda$, $m = 0,\ 1,\ 2,\ \ldots$. With $\sin\theta < 1$, $d1 > \left(m_{\max} + \frac{1}{2}\right)\lambda$, $m_{\max} < \frac{d}{\lambda} - \frac{1}{2}$ or $m_{\max} + 1 < \frac{d}{\lambda} + \frac{1}{2}$. Let $n = 1,\ 2,\ 3,\ \ldots$. Then the number of side minima is the greatest integer $n$ less than $\frac{d}{\lambda} + \frac{1}{2}$. Counting both left and right, $\boxed{\text{the number of dark fringes is two times the greatest positive integer less than } \left(\frac{d}{\lambda} + \frac{1}{2}\right).}$ It is equal to the number of nodes in the standing wave between the sources.

**P37.47** My middle finger has width $d = 2\text{ cm}$.

(a) Two adjacent directions of constructive interference for 600-nm light are described by

$$d\sin\theta = m\lambda$$
$$\theta_0 = 0$$
$$\left(2\times10^{-2}\text{ m}\right)\sin\theta_1 = 1\left(6\times10^{-7}\text{ m}\right)$$

Thus, $\theta_1 = 2\times10^{-3}$ degree

and $\theta_1 - \theta_0 \boxed{\sim 10^{-3}\text{ degree}}$.

(b) Choose $\theta_1 = 20°$

$$\left(2\times10^{-2}\text{ m}\right)\sin 20° = (1)\lambda$$
$$\lambda = 7\text{ mm}$$

Millimeter waves are $\boxed{\text{microwaves}}$.

$f = \dfrac{c}{\lambda}$: $\quad f = \dfrac{3\times10^{8}\text{ m/s}}{7\times10^{-3}\text{ m}} \boxed{\sim 10^{11}\text{ Hz}}$

**P37.48** If the center point on the screen is to be a dark spot rather than bright, passage through the plastic must delay the light by one-half wavelength. Calling the thickness of the plastic *t*.

$\dfrac{t}{\lambda} + \dfrac{1}{2} = \dfrac{t}{\lambda/n} = \dfrac{nt}{\lambda}$ or $t = \boxed{\dfrac{\lambda}{2(n-1)}}$ where $n$ is the index of refraction for the plastic.

**P37.49** No phase shift upon reflection from the upper surface (glass to air) of the film, but there will be a shift of $\dfrac{\lambda}{2}$ due to the reflection at the lower surface of the film (air to metal). The total phase difference in the two reflected beams is

then $\delta = 2nt + \dfrac{\lambda}{2}$.

For constructive interference, $\delta = m\lambda$

or $2(1.00)t + \dfrac{\lambda}{2} = m\lambda$.

Thus, the film thickness for the $m^{\text{th}}$ order bright fringe is

$$t_m = \left(m - \frac{1}{2}\right)\frac{\lambda}{2} = m\left(\frac{\lambda}{2}\right) - \frac{\lambda}{4}$$

and the thickness for the $m-1$ bright fringe is:

$$t_{m-1} = (m-1)\left(\frac{\lambda}{2}\right) - \frac{\lambda}{4}.$$

Therefore, the change in thickness required to go from one bright fringe to the next is

$$\Delta t = t_m - t_{m-1} = \frac{\lambda}{2}.$$

*continued on next page*

To go through 200 bright fringes, the change in thickness of the air film must be:

$$200\left(\frac{\lambda}{2}\right)=100\lambda .$$

Thus, the increase in the length of the rod is

$$\Delta L=100\lambda=100\left(5.00\times10^{-7}\text{ m}\right)=5.00\times10^{-5}\text{ m} .$$

From $\Delta L=L_i\alpha\Delta t$

we have: $\alpha=\frac{\Delta L}{L_i\Delta T}=\frac{5.00\times10^{-5}\text{ m}}{(0.100\text{ m})(25.0°\text{C})}=\boxed{20.0\times10^{-6}\ °\text{C}^{-1}}$.

**P37.50** Since $1<1.25<1.34$, light reflected from top and bottom surfaces of the oil undergoes phase reversal. The path difference is then 2*t*, which must be equal to

$$m\lambda_n=\frac{m\lambda}{n}$$

for maximum reflection, with $m=1$ for the given first-order condition and $n=1.25$. So

$$t=\frac{m\lambda}{2n}=\frac{1(500\text{ nm})}{2(1.25)}=200\text{ nm} .$$

The volume we assume to be constant: $1.00\text{ m}^3=(200\text{ nm})A$

$$A=\frac{1.00\text{ m}^3}{200\left(10^{-9}\text{ m}\right)}=5.00\times10^{6}\text{ m}^2=\boxed{5.00\text{ km}^2} .$$

**P37.51** One radio wave reaches the receiver *R* directly from the distant source at an angle $\theta$ above the horizontal. The other wave undergoes phase reversal as it reflects from the water at *P*.

Constructive interference first occurs for a path difference of

$$d=\frac{\lambda}{2} \qquad (1)$$

It is equally far from *P* to *R* as from *P* to $R'$, the mirror image of the telescope.

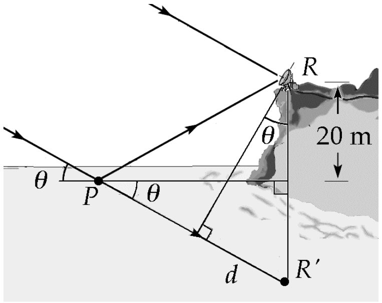

**FIG. P37.51**

The angles $\theta$ in the figure are equal because they each form part of a right triangle with a shared angle at $R'$.

So the path difference is $d=2(20.0\text{ m})\sin\theta=(40.0\text{ m})\sin\theta$.

The wavelength is $\lambda=\frac{c}{f}=\frac{3.00\times10^{8}\text{ m/s}}{60.0\times10^{6}\text{ Hz}}=5.00\text{ m}$.

Substituting for *d* and $\lambda$ in Equation (1), $(40.0\text{ m})\sin\theta=\frac{5.00\text{ m}}{2}$.

Solving for the angle $\theta$, $\sin\theta=\frac{5.00\text{ m}}{80.0\text{ m}}$ and $\boxed{\theta=3.58°}$.

**P37.52** For destructive interference, the path length must differ by $m\lambda$. We may treat this problem as a double slit experiment if we remember the light undergoes a $\frac{\pi}{2}$-phase shift at the mirror. The second slit is the mirror image of the source, 1.00 cm below the mirror plane. Modifying Equation 37.5,

$$y_{\text{dark}} = \frac{m\lambda L}{d} = \frac{1\left(5.00\times 10^{-7}\text{ m}\right)(100\text{ m})}{\left(2.00\times 10^{-2}\text{ m}\right)} = \boxed{2.50\text{ mm}}.$$

**P37.53** $2\sqrt{(15.0\text{ km})^2 + h^2} = 30.175\text{ km}$

$(15.0\text{ km})^2 + h^2 = 227.63$

$h = \boxed{1.62\text{ km}}$

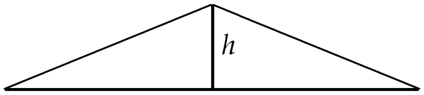

**FIG. P37.53**

**P37.54** For dark fringes, $2nt = m\lambda$

and at the edge of the wedge, $t = \frac{84(500\text{ nm})}{2}$.

When submerged in water, $2nt = m\lambda$

$$m = \frac{2(1.33)(42)(500\text{ nm})}{500\text{ nm}}$$

so $m + 1 = \boxed{113\text{ dark fringes}}$.

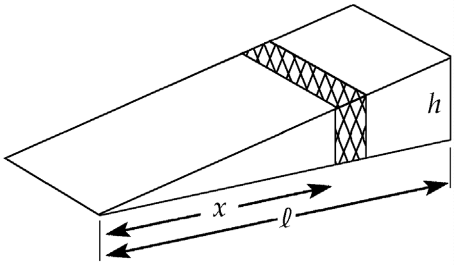

**FIG. P37.54**

**P37.55** From Equation 37.13, $\frac{I}{I_{\max}} = \cos^2\left(\frac{\pi yd}{\lambda L}\right)$.

Let $\lambda_2$ equal the wavelength for which $\frac{I}{I_{\max}} \to \frac{I_2}{I_{\max}} = 0.640$.

Then $\lambda_2 = \frac{\pi yd/L}{\cos^{-1}(I_2/I_{\max})^{1/2}}$.

But $\frac{\pi yd}{L} = \lambda_1 \cos^{-1}\left(\frac{I_1}{I_{\max}}\right)^{1/2} = (600\text{ nm})\cos^{-1}(0.900) = 271\text{ nm}$.

Substituting this value into the expression for $\lambda_2$, $\lambda_2 = \frac{271\text{ nm}}{\cos^{-1}\left(0.640^{1/2}\right)} = \boxed{421\text{ nm}}$.

Note that in this problem, $\cos^{-1}\left(\frac{I}{I_{\max}}\right)^{1/2}$ must be expressed in radians.

**P37.56** At entrance, $1.00\sin 30.0° = 1.38\sin\theta_2$ $\qquad \theta_2 = 21.2°$

Call $t$ the unknown thickness. Then

$$\cos 21.2° = \frac{t}{a} \qquad a = \frac{t}{\cos 21.2°}$$

$$\tan 21.2° = \frac{c}{t} \qquad c = t\tan 21.2°$$

$$\sin\theta_1 = \frac{b}{2c} \qquad b = 2t\tan 21.2°\sin 30.0°$$

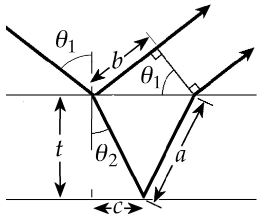

**FIG. P37.56**

The net shift for the second ray, including the phase reversal on reflection of the first, is

$$2an - b - \frac{\lambda}{2}$$

where the factor $n$ accounts for the shorter wavelength in the film. For constructive interference, we require

$$2an - b - \frac{\lambda}{2} = m\lambda\,.$$

The minimum thickness will be given by $\qquad 2an - b - \frac{\lambda}{2} = 0\,.$

$$\frac{\lambda}{2} = 2an - b = 2\frac{nt}{\cos 21.2°} - 2t(\tan 21.2°)\sin 30.0°$$

$$\frac{590\text{ nm}}{2} = \left(\frac{2\times 1.38}{\cos 21.2°} - 2\tan 21.2°\sin 30.0°\right)t = 2.57t \qquad t = \boxed{115\text{ nm}}$$

**P37.57** The shift between the two reflected waves is $\delta = 2na - b - \frac{\lambda}{2}$ where $a$ and $b$ are as shown in the ray diagram, $n$ is the index of refraction, and the term $\frac{\lambda}{2}$ is due to phase reversal at the top surface. For constructive interference, $\delta = m\lambda$ where $m$ has integer values. This condition becomes

$$2na - b = \left(m + \frac{1}{2}\right)\lambda \qquad (1)$$

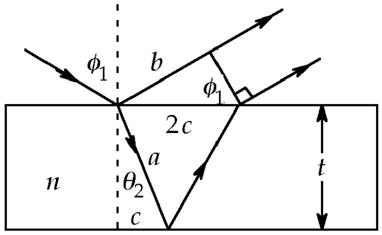

**FIG. P37.57**

From the figure's geometry, $\quad a = \frac{t}{\cos\theta_2}$

$$c = a\sin\theta_2 = \frac{t\sin\theta_2}{\cos\theta_2}$$

$$b = 2c\sin\phi_1 = \frac{2t\sin\theta_2}{\cos\theta_2}\sin\phi_1$$

Also, from Snell's law, $\quad \sin\phi_1 = n\sin\theta_2\,.$

Thus, $\quad b = \frac{2nt\sin^2\theta_2}{\cos\theta_2}\,.$

With these results, the condition for constructive interference given in Equation (1) becomes:

$$2n\left(\frac{t}{\cos\theta_2}\right) - \frac{2nt\sin^2\theta_2}{\cos\theta_2} = \frac{2nt}{\cos\theta_2}\left(1 - \sin^2\theta_2\right) = \left(m + \frac{1}{2}\right)\lambda$$

or

$$\boxed{2nt\cos\theta_2 = \left(m + \frac{1}{2}\right)\lambda}\,.$$

**P37.58** (a) Minimum: $2nt = m\lambda_2$ for $m = 0, 1, 2, \ldots$

Maximum: $2nt = \left(m' + \frac{1}{2}\right)\lambda_1$ for $m' = 0, 1, 2, \ldots$

for $\lambda_1 > \lambda_2$, $\left(m' + \frac{1}{2}\right) < m$

so $m' = m - 1$.

Then $2nt = m\lambda_2 = \left(m - \frac{1}{2}\right)\lambda_1$

$$2m\lambda_2 = 2m\lambda_1 - \lambda_1$$

so $\boxed{m = \frac{\lambda_1}{2(\lambda_1 - \lambda_2)}}$.

(b) $m = \frac{500}{2(500 - 370)} = 1.92 \to 2$ (wavelengths measured to ±5 nm)

Minimum: $2nt = m\lambda_2$

$2(1.40)t = 2(370\text{ nm})$ $t = 264\text{ nm}$

Maximum: $2nt = \left(m - 1 + \frac{1}{2}\right)\lambda = 1.5\lambda$

$2(1.40)t = 1.5(500\text{ nm})$ $t = 268\text{ nm}$

Film thickness $= \boxed{266\text{ nm}}$.

**P37.59** From the sketch, observe that

$$x = \sqrt{h^2 + \left(\frac{d}{2}\right)^2} = \frac{\sqrt{4h^2 + d^2}}{2}.$$

Including the phase reversal due to reflection from the ground, the total shift between the two waves is $\delta = 2x - d - \frac{\lambda}{2}$.

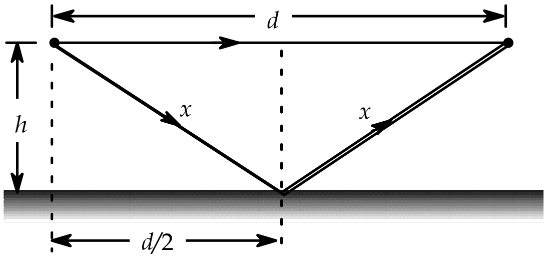

FIG. P37.59

(a) For constructive interference, the total shift must be an integral number of wavelengths, or $\delta = m\lambda$ where $m = 0, 1, 2, 3, \ldots$.

Thus, $2x - d = \left(m + \frac{1}{2}\right)\lambda$ or $\lambda = \frac{4x - 2d}{2m + 1}$.

For the longest wavelength, $m = 0$, giving $\lambda = 4x - 2d = \boxed{2\sqrt{4h^2 + d^2} - 2d}$.

(b) For destructive interference, $\delta = \left(m - \frac{1}{2}\right)\lambda$ where $m = 0, 1, 2, 3, \ldots$.

Thus, $2x - d = m\lambda$ or $\lambda = \frac{2x - d}{m}$.

For the longest wavelength, $m = 1$ giving $\lambda = 2x - d = \boxed{\sqrt{4h^2 + d^2} - d}$.

**P37.60** Bright fringes occur when $2t = \frac{\lambda}{n}\left(m + \frac{1}{2}\right)$

and dark fringes occur when $2t = \left(\frac{\lambda}{n}\right)m$.

The thickness of the film at $x$ is $t = \left(\frac{h}{\ell}\right)x$.

Therefore, $\boxed{x_{\text{bright}} = \frac{\lambda\ell}{2hn}\left(m + \frac{1}{2}\right)}$ and $\boxed{x_{\text{dark}} = \frac{\lambda\ell m}{2hn}}$.

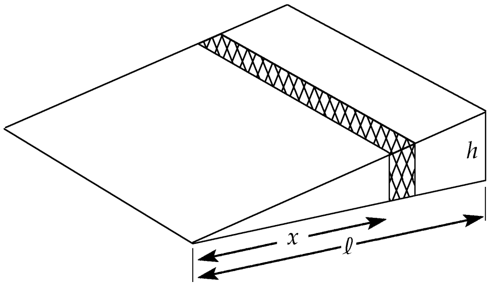

**FIG. P37.60**

**P37.61** Call $t$ the thickness of the film. The central maximum corresponds to zero phase difference. Thus, the added distance $\Delta r$ traveled by the light from the lower slit must introduce a phase difference equal to that introduced by the plastic film. The phase difference $\phi$ is

$$\phi = 2\pi\left(\frac{t}{\lambda_a}\right)(n-1).$$

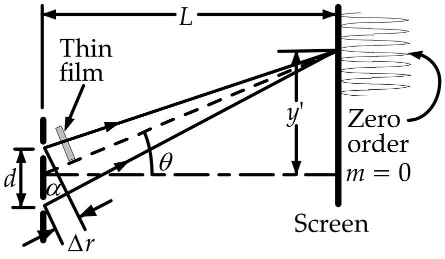

**FIG. P37.61**

The corresponding difference in **path length** $\Delta r$ is

$$\Delta r = \phi\left(\frac{\lambda_a}{2\pi}\right) = 2\pi\left(\frac{t}{\lambda_a}\right)(n-1)\left(\frac{\lambda_a}{2\pi}\right) = t(n-1).$$

Note that the wavelength of the light does not appear in this equation. In the figure, the two rays from the slits are essentially parallel.

Thus the angle $\theta$ may be expressed as $\tan\theta = \frac{\Delta r}{d} = \frac{y'}{L}$.

Eliminating $\Delta r$ by substitution, $\frac{y'}{L} = \frac{t(n-1)}{d}$ gives $\boxed{y' = \frac{t(n-1)L}{d}}$.

**P37.62** The shift between the waves reflecting from the top and bottom surfaces of the film at the point where the film has thickness $t$ is $\delta = 2tn_{\text{film}} + \frac{\lambda}{2}$, with the factor of $\frac{\lambda}{2}$ being due to a phase reversal at *one* of the surfaces.

For the dark rings (destructive interference), the total shift should be $\delta = \left(m + \frac{1}{2}\right)\lambda$ with $m = 0, 1, 2, 3, \ldots$. This requires that

$$t = \frac{m\lambda}{2n_{\text{film}}}.$$

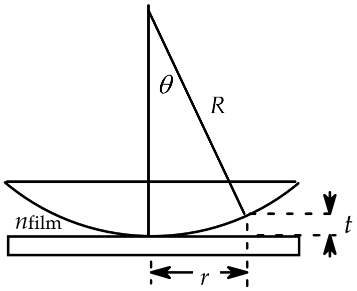

**FIG. P37.62**

To find $t$ in terms of $r$ and $R$, $R^2 = r^2 + (R-t)^2$ so $r^2 = 2Rt + t^2$.

Since $t$ is much smaller than $R$, $t^2 << 2Rt$ and $r^2 \approx 2Rt = 2R\left(\frac{m\lambda}{2n_{\text{film}}}\right)$.

Thus, where $m$ is an integer, $\boxed{r \approx \sqrt{\frac{m\lambda R}{n_{\text{film}}}}}$.

**P37.63** (a) Constructive interference in the reflected light requires $2t=\left(m+\frac{1}{2}\right)\lambda$. The first bright ring has $m=0$ and the 55th has $m=54$, so at the edge of the lens

$$t=\frac{54.5\left(650\times10^{-9}\text{ m}\right)}{2}=17.7\ \mu\text{m}.$$

Now from the geometry in Figure 37.18, the distance from the center of curvature down to the flat side of the lens is

$$\sqrt{R^2-r^2}=R-t \text{ or } R^2-r^2=R^2-2Rt+t^2$$

$$R=\frac{r^2+t^2}{2t}=\frac{\left(5.00\times10^{-2}\text{ m}\right)^2+\left(1.77\times10^{-5}\text{ m}\right)^2}{2\left(1.77\times10^{-5}\text{ m}\right)}=\boxed{70.6\text{ m}}$$

(b) $$\frac{1}{f}=(n-1)\left(\frac{1}{R_2}-\frac{1}{R_2}\right)=0.520\left(\frac{1}{\infty}-\frac{1}{-70.6\text{ m}}\right) \text{ so } f=\boxed{136\text{ m}}$$

***P37.64** Light reflecting from the upper interface of the air layer suffers no phase change, while light reflecting from the lower interface is reversed 180°. Then there is indeed a dark fringe at the outer circumference of the lens, and a dark fringe wherever the air thickness $t$ satisfies $2t=m\lambda$, $m=0, 1, 2, \ldots$.

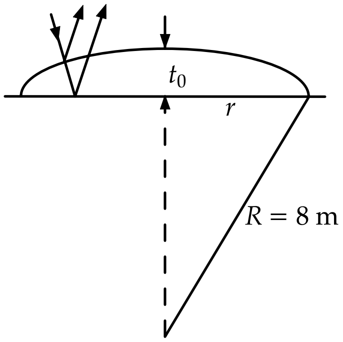

**FIG. P37.64**

(a) At the central dark spot $m=50$ and

$$t_0=\frac{50\lambda}{2}=25\left(589\times10^{-9}\text{ m}\right)=\boxed{1.47\times10^{-5}\text{ m}}.$$

(b) In the right triangle,

$$(8\text{ m})^2=r^2+\left(8\text{ m}-1.47\times10^{-5}\text{ m}\right)^2=r^2+(8\text{ m})^2-2(8\text{ m})\left(1.47\times10^{-5}\text{ m}\right)+2\times10^{-10}\text{ m}^2.$$ The last term is negligible. $r=\sqrt{2(8\text{ m})\left(1.47\times10^{-5}\text{ m}\right)}=\boxed{1.53\times10^{-2}\text{ m}}$

(c) $$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=(1.50-1)\left(\frac{1}{\infty}-\frac{1}{8.00\text{ m}}\right)$$

$$\boxed{f=-16.0\text{ m}}$$

**P37.65** For bright rings the gap $t$ between surfaces is given by $2t=\left(m+\frac{1}{2}\right)\lambda$. The first bright ring has $m=0$ and the hundredth has $m=99$.

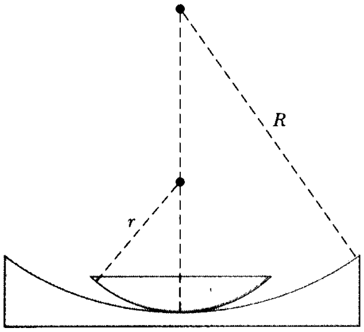

FIG. P37.65

So, $t=\frac{1}{2}(99.5)\left(500\times10^{-9}\text{ m}\right)=24.9\ \mu\text{m}$.

Call $r_b$ the ring radius. From the geometry of the figure at the right,

$$t=r-\sqrt{r^2-r_b^2}-\left(R-\sqrt{R^2-r_b^2}\right)$$

Since $r_b<<r$, we can expand in series:

$$t=r-r\left(1-\frac{1}{2}\frac{r_b^2}{r^2}\right)-R+R\left(1-\frac{1}{2}\frac{r_b^2}{R^2}\right)=\frac{1}{2}\frac{r_b^2}{r}-\frac{1}{2}\frac{r_b^2}{R}$$

$$r_b=\left[\frac{2t}{1/r-1/R}\right]^{1/2}=\left[\frac{2\left(24.9\times10^{-6}\text{ m}\right)}{1/4.00\text{ m}-1/12.0\text{ m}}\right]^{1/2}=\boxed{1.73\text{ cm}}$$

**P37.66**

$$\mathbf{E}_R=\mathbf{E}_1+\mathbf{E}_2+\mathbf{E}_3=\left[\cos\frac{\pi}{6}+3.00\cos\frac{7\pi}{2}+6.00\cos\frac{4\pi}{3}\right]\hat{\mathbf{i}}$$

$$+\left[\sin\frac{\pi}{6}+3.00\sin\frac{7\pi}{2}+6.00\sin\frac{4\pi}{3}\right]\hat{\mathbf{j}}$$

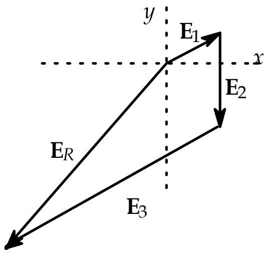

FIG. P37.66

$$\mathbf{E}_R=-2.13\hat{\mathbf{i}}-7.70\hat{\mathbf{j}}$$

$$\mathbf{E}_R=\sqrt{(-2.13)^2+(-7.70)^2}\ \text{ at }\tan^{-1}\left(\frac{-7.70}{-2.13}\right)=7.99\ \text{ at } 4.44\text{ rad}$$

Thus, $E_P=\boxed{7.99\sin(\omega t+4.44\text{ rad})}$.

**P37.67** (a) Bright bands are observed when $2nt=\left(m+\frac{1}{2}\right)\lambda$.

Hence, the first bright band ($m=0$) corresponds to $nt=\frac{\lambda}{4}$.

Since $\frac{x_1}{x_2}=\frac{t_1}{t_2}$

we have $x_2=x_1\left(\frac{t_2}{t_1}\right)=x_1\left(\frac{\lambda_2}{\lambda_1}\right)=(3.00\text{ cm})\left(\frac{680\text{ nm}}{420\text{ nm}}\right)=\boxed{4.86\text{ cm}}$.

(b) $t_1=\frac{\lambda_1}{4n}=\frac{420\text{ nm}}{4(1.33)}=\boxed{78.9\text{ nm}}$

$t_2=\frac{\lambda_2}{4n}=\frac{680\text{ nm}}{4(1.33)}=\boxed{128\text{ nm}}$

(c) $\theta\approx\tan\theta=\frac{t_1}{x_1}=\frac{78.9\text{ nm}}{3.00\text{ cm}}=\boxed{2.63\times10^{-6}\text{ rad}}$

***P37.68** Depth = one-quarter of the wavelength in plastic.

$$t=\frac{\lambda}{4n}=\frac{780\text{ nm}}{4(1.50)}=\boxed{130\text{ nm}}$$

**P37.69** $2h\sin\theta=\left(m+\frac{1}{2}\right)\lambda$ bright

$2h\left(\frac{\Delta y}{2L}\right)=\frac{1}{2}\lambda$ so $h=\frac{L\lambda}{2\Delta y}=\frac{(2.00\text{ m})\left(606\times10^{-9}\text{ m}\right)}{2\left(1.2\times10^{-3}\text{ m}\right)}=\boxed{0.505\text{ mm}}$

***P37.70** Represent the light radiated from each slit to point $P$ as a phasor. The two have equal amplitudes $E$. Since intensity is proportional to amplitude squared, they add to amplitude $\sqrt{3}E$. Then $\cos\theta=\frac{\sqrt{3}\,E/2}{E}$, $\theta=30°$. Next, the obtuse angle between the two phasors is $180-30-30=120°$, and $\phi=180-120°=60°$. The phase difference between the two phasors is caused by the path difference $\delta=\overline{SS_2}-\overline{SS_1}$ according to $\frac{\delta}{\lambda}=\frac{\phi}{360°}$, $\delta=\lambda\frac{60°}{360°}=\frac{\lambda}{6}$. Then

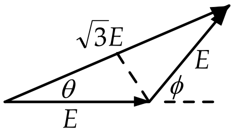

**FIG. P37.70**

$$\sqrt{L^2+d^2}-L=\frac{\lambda}{6}$$

$$L^2+d^2=L^2+\frac{2L\lambda}{6}+\frac{\lambda^2}{36}$$

The last term is negligible

$$d=\left(\frac{2L\lambda}{6}\right)^{1/2}=\sqrt{\frac{2(1.2\text{ m})620\times10^{-9}\text{ m}}{6}}=\boxed{0.498\text{ mm}}.$$

**P37.71** Superposing the two vectors, $E_R=\left|\mathbf{E}_1+\mathbf{E}_2\right|$

$$E_R=\left|\mathbf{E}_1+\mathbf{E}_2\right|=\sqrt{\left(E_0+\frac{E_0}{3}\cos\phi\right)^2+\left(\frac{E_0}{3}\sin\phi\right)^2}=\sqrt{E_0^2+\frac{2}{3}E_0^2\cos\phi+\frac{E_0^2}{9}\cos^2\phi+\frac{E_0^2}{9}\sin^2\phi}$$

$$E_R=\sqrt{\frac{10}{9}E_0^2+\frac{2}{3}E_0^2\cos\phi}$$

Since intensity is proportional to the square of the amplitude,

$$I=\frac{10}{9}I_{\max}+\frac{2}{3}I_{\max}\cos\phi.$$

Using the trigonometric identity $\cos\phi=2\cos^2\frac{\phi}{2}-1$, this becomes

$$I=\frac{10}{9}I_{\max}+\frac{2}{3}I_{\max}\left(2\cos^2\frac{\phi}{2}-1\right)=\frac{4}{9}I_{\max}+\frac{4}{3}I_{\max}\cos^2\frac{\phi}{2},$$

or $\boxed{I=\frac{4}{9}I_{\max}\left(1+3\cos^2\frac{\phi}{2}\right)}$.

## ANSWERS TO EVEN PROBLEMS

| | |
|---|---|
| **P37.2** | 515 nm |
| **P37.4** | (a) 36.2°; (b) 5.08 cm; (c) 508 THz |
| **P37.6** | maxima at 0°, 29.1°, 76.3°;<br>minima at 14.1° and 46.8° |
| **P37.8** | 36.2 cm |
| **P37.10** | 641 |
| **P37.12** | 6.33 mm/s |
| **P37.14** | see the solution |
| **P37.16** | (a) 1.29 rad; (b) 99.8 nm |
| **P37.18** | 0.968 |
| **P37.20** | (a) see the solution; (b) 9.00 |
| **P37.22** | (a) $2.88E_0$ at 0.349 rad;<br>(b) $2.00E_0$ at $\frac{\pi}{3}$ rad; (c) 0;<br>(d) $E_0$ at $\frac{3\pi}{2}$ rad |
| **P37.24** | see the solution |
| **P37.26** | $x_1 - x_2 = \left(m - \frac{1}{48}\right)\lambda$ where<br>$m = 0, 1, 2, 3, \ldots$ |
| **P37.28** | see the solution |
| **P37.30** | 612 nm |
| **P37.32** | 512 nm |
| **P37.34** | 96.2 nm |
| **P37.36** | (a) 238 nm; (b) $\lambda$ increase; (c) 328 nm |
| **P37.38** | 1.31 |
| **P37.40** | 1.20 mm |
| **P37.42** | 449 nm; blue |
| **P37.44** | (a) see the solution; (b) 2.74 m |
| **P37.46** | number of antinodes = number of constructive interference zones = 1 plus 2 times the greatest positive integer $\leq \frac{d}{\lambda}$<br>number of nodes = number of destructive interference zones = 2 times the greatest positive integer $< \left(\frac{d}{\lambda} + \frac{1}{2}\right)$ |
| **P37.48** | $\frac{\lambda}{2(n-1)}$ |
| **P37.50** | 5.00 $\text{km}^2$ |
| **P37.52** | 2.50 mm |
| **P37.54** | 113 |
| **P37.56** | 115 nm |
| **P37.58** | (a) see the solution; (b) 266 nm |
| **P37.60** | see the solution |
| **P37.62** | see the solution |
| **P37.64** | (a) 14.7 $\mu$m; (b) 1.53 cm; (c) –16.0 m |
| **P37.66** | $7.99 \sin(\omega t + 4.44 \text{ rad})$ |
| **P37.68** | 130 nm |
| **P37.70** | 0.498 mm |

# 38

# Diffraction Patterns and Polarization

**CHAPTER OUTLINE**

38.1 Introduction to Diffraction Patterns
38.2 Diffraction Patterns from Narrow Slits
38.3 Resolution of Single-Slit and Circular Apertures
38.4 The Diffraction Grating
38.5 Diffraction of X-Rays by Crystals
38.6 Polarization of Light Waves

## ANSWERS TO QUESTIONS

**Q38.1** Audible sound has wavelengths on the order of meters or centimeters, while visible light has a wavelength on the order of half a micrometer. In this world of breadbox-sized objects, $\frac{\lambda}{a}$ is large for sound, and sound diffracts around behind walls with doorways. But $\frac{\lambda}{a}$ is a tiny fraction for visible light passing ordinary-size objects or apertures, so light changes its direction by only very small angles when it diffracts.

Another way of phrasing the answer: We can see by a small angle around a small obstacle or around the edge of a small opening. The side fringes in Figure 38.1 and the Arago spot in the center of Figure 38.3 show this diffraction. We cannot always hear around corners. Out-of-doors, away from reflecting surfaces, have someone a few meters distant face away from you and whisper. The high-frequency, short-wavelength, information-carrying components of the sound do not diffract around his head enough for you to understand his words.

**Q38.2** The wavelength of light is extremely small in comparison to the dimensions of your hand, so the diffraction of light around an obstacle the size of your hand is totally negligible. However, sound waves have wavelengths that are comparable to the dimensions of the hand or even larger. Therefore, significant diffraction of sound waves occurs around hand-sized obstacles.

**Q38.3** If you are using an extended light source, the gray area at the edge of the shadow is the penumbra. A bug looking up from there would see the light source partly but not entirely blocked by the book. If you use a point source of light, hold it and the book motionless, and look at very small angles out from the geometrical edge of the shadow, you may see a series of bright and dark bands produced by diffraction of light at the straight edge, as shown in the diagram.

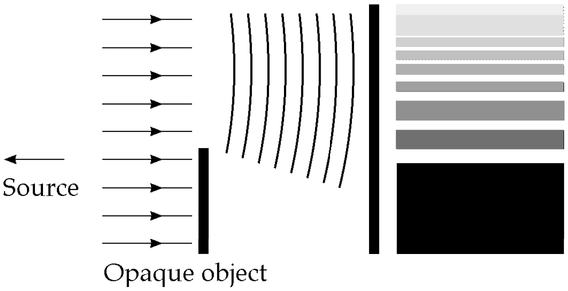

**FIG. Q38.3**

**Q38.4** An AM radio wave has wavelength on the order of $\frac{3\times10^8\ \text{m/s}}{1\times10^6\ \text{s}^{-1}} \sim 300\ \text{m}$. This is large compared to the width of the mouth of a tunnel, so the AM radio waves can reflect from the surrounding ground as if the hole were not there. (In the same way, a metal screen forming the dish of a radio telescope can reflect radio waves as if it were solid, and a hole-riddled screen in the door of a microwave oven keeps the microwaves inside.) The wave does not "see" the hole. Very little of the radio wave energy enters the tunnel, and the AM radio signal fades. An FM radio wave has wavelength a hundred times smaller, on the order of a few meters. This is smaller than the size of the tunnel opening, so the wave can readily enter the opening. (On the other hand, the long wavelength of AM radio waves lets them diffract more around obstacles. Long-wavelength waves can change direction more in passing hills or large buildings, so in some experiments FM fades more than AM.)

**Q38.5** The intensity of the light coming through the slit decreases, as you would expect. The central maximum increases in width as the width of the slit decreases. In the condition $\sin\theta = \frac{\lambda}{a}$ for destructive interference on each side of the central maximum, $\theta$ increases as $a$ decreases.

**Q38.6** It is shown in the correct orientation. If the horizontal width of the opening is equal to or less than the wavelength of the sound, then the equation $a\sin\theta = (1)\lambda$ has the solution $\theta = 90°$, or has no solution. The central diffraction maximum covers the whole seaward side. If the vertical height of the opening is large compared to the wavelength, then the angle in $a\sin\theta = (1)\lambda$ will be small, and the central diffraction maximum will form a thin horizontal sheet.

**Q38.7** The speaker is mounted incorrectly—it should be rotated by 90°. The speaker is mounted with its narrower dimension vertical. That means that the sound will diffract more vertically than it does horizontally. Mounting the speaker so that its thinner dimension is horizontal will give more diffraction spreading in the horizontal plane, broadcasting "important" information to the troops, instead of to the birds in the air and the worms in the ground, as the speaker was mounted in the movie.

**Q38.8** We apply the equation $\theta_m = \frac{1.22\lambda}{D}$ for the resolution of a circular aperture, the pupil of your eye. Suppose your dark-adapted eye has pupil diameter $D = 5\ \text{mm}$. An average wavelength for visible light is $\lambda = 550\ \text{nm}$. Suppose the headlights are 2 m apart and the car is a distance $L$ away. Then $\theta_m = \frac{2\ \text{m}}{L} = 1.22\times1.1\times10^{-4}$ so $L \sim 10\ \text{km}$. The actual distance is less than this because the variable-temperature air between you and the car makes the light refract unpredictably. The headlights twinkle like stars.

**Q38.9** Consider incident light nearly parallel to the horizontal ruler. Suppose it scatters from bumps at distance $d$ apart to produce a diffraction pattern on a vertical wall a distance $L$ away. At a point of height $y$, where $\theta = \frac{y}{L}$ gives the scattering angle $\theta$, the character of the interference is determined by the shift $\delta$ between beams scattered by adjacent bumps, where $\delta = \frac{d}{\cos\theta} - d$. Bright spots appear for $\delta = m\lambda$, where $0, 1, 2, 3, \ldots$. For small $\theta$, these equations combine and reduce to $m\lambda = \frac{y_m^2 d}{2L^2}$. Measurement of the heights $y_m$ of bright spots allows calculation of the wavelength of the light.

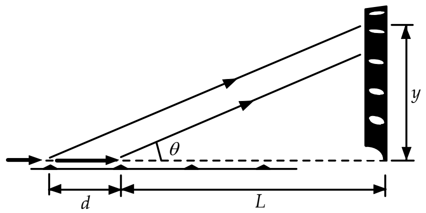

**FIG. Q38.9**

**Q38.10** Yes, but no diffraction effects are observed because the separation distance between adjacent ribs is so much greater than the wavelength of x-rays. Diffraction does not limit the resolution of an x-ray image. Diffraction might sometimes limit the resolution of an ultrasonogram.

**Q38.11** Vertical. Glare, as usually encountered when driving or boating, is horizontally polarized. Reflected light is polarized in the same plane as the reflecting surface. As unpolarized light hits a shiny horizontal surface, the atoms on the surface absorb and then reemit the light energy as a reflection. We can model the surface as containing conduction electrons free to vibrate easily along the surface, but not to move easily out of surface. The light emitted from a vibrating electron is partially or completely polarized along the plane of vibration, thus horizontally.

**Q38.12** The earth has an atmosphere, while the moon does not. The nitrogen and oxygen molecules in the earth's atmosphere are of the right size to scatter short-wavelength (blue) light especially well, while there is nothing surrounding the moon to scatter light.

**Q38.13** The little particles of dust diffusely reflect light from the light beam. Note that this is not necessarily *scattering*. Scattering is a resonance phenomenon—as when the $O_2$ and $N_2$ molecules in our atmosphere scatter blue light more than red. In general, light is visible when it enters your eye. Your eyes and brain are well prepared to make you think on a subconscious level that you can 'see' where light is coming from or sometimes 'see' light going past you, but really you see only light entering your eye.

**Q38.14** Light from the sky is partially polarized. Light from the blue sky that is polarized at 90° to the polarization axis of the glasses will be blocked, making the sky look darker as compared to the clouds.

**Q38.15** First think about the glass without a coin and about one particular point $P$ on the screen. We can divide up the area of the glass into ring-shaped zones centered on the line joining $P$ and the light source, with successive zones contributing alternately in-phase and out-of-phase with the light that takes the straight-line path to $P$. These Fresnel zones have nearly equal areas. An outer zone contributes only slightly less to the total wave disturbance at $P$ than does the central circular zone. Now insert the coin. If $P$ is in line with its center, the coin will block off the light from some particular number of zones. The first unblocked zone around its circumference will send light to $P$ with significant amplitude. Zones farther out will predominantly interfere destructively with each other, and the Arago spot is bright. Slightly off the axis there is nearly complete destructive interference, so most of the geometrical shadow is dark. A bug on the screen crawling out past the edge of the geometrical shadow would in effect see the central few zones coming out of eclipse. As the light from them interferes alternately constructively and destructively, the bug moves through bright and dark fringes on the screen. The diffraction pattern is shown in Figure 38.3 in the text.

**Q38.16** Since obsidian glass is opaque, a standard method of measuring incidence and refraction angles and using Snell's Law is ineffective. Reflect unpolarized light from the horizontal surface of the obsidian through a vertically polarized filter. Change the angle of incidence until you observe that none of the reflected light is transmitted through the filter. This means that the reflected light is completely horizontally polarized, and that the incidence and reflection angles are the polarization angle. The tangent of the polarization angle is the index of refraction of the obsidian.

**Q38.17** The fine hair blocks off light that would otherwise go through a fine slit and produce a diffraction pattern on a distant screen. The width of the central maximum in the pattern is inversely proportional to the distance across the slit. When the hair is in place, it subtracts the same diffraction pattern from the projected disk of laser light. The hair produces a diffraction minimum that crosses the bright circle on the screen. The width of the minimum is inversely proportional to the diameter of the hair. The central minimum is flanked by narrower maxima and minima. Measure the width $2y$ of the central minimum between the maxima bracketing it, and use Equation 38.1 in the form $\frac{y}{L}=\frac{\lambda}{a}$ to find the width $a$ of the hair.

**Q38.18** The condition for constructive interference is that the three radio signals arrive at the city in phase. We know the speed of the waves (it is the speed of light $c$), the angular bearing $\theta$ of the city east of north from the broadcast site, and the distance $d$ between adjacent towers. The wave from the westernmost tower must travel an extra distance $2d\sin\theta$ to reach the city, compared to the signal from the eastern tower. For each cycle of the carrier wave, the western antenna would transmit first, the center antenna after a time delay $\frac{d\sin\theta}{c}$, and the eastern antenna after an additional equal time delay.

## SOLUTIONS TO PROBLEMS

### Section 38.1 **Introduction to Diffraction Patterns**

### Section 38.2 **Diffraction Patterns from Narrow Slits**

**P38.1** $$\sin\theta=\frac{\lambda}{a}=\frac{6.328\times10^{-7}}{3.00\times10^{-4}}=2.11\times10^{-3}$$

$$\frac{y}{1.00\text{ m}}=\tan\theta\approx\sin\theta=\theta\ \text{ (for small }\theta)$$

$$2y=\boxed{4.22\text{ mm}}$$

**P38.2** The positions of the first-order minima are $\frac{y}{L}\approx\sin\theta=\pm\frac{\lambda}{a}$. Thus, the spacing between these two minima is $\Delta y=2\left(\frac{\lambda}{a}\right)L$ and the wavelength is

$$\lambda=\left(\frac{\Delta y}{2}\right)\left(\frac{a}{L}\right)=\left(\frac{4.10\times10^{-3}\text{ m}}{2}\right)\left(\frac{0.550\times10^{-3}\text{ m}}{2.06\text{ m}}\right)=\boxed{547\text{ nm}}.$$

**P38.3** $\frac{y}{L}\approx\sin\theta=\frac{m\lambda}{a}$ $\qquad\Delta y=3.00\times10^{-3}\text{ nm}$

$\Delta m=3-1=2$ and $a=\frac{\Delta m\lambda L}{\Delta y}$

$$a=\frac{2\left(690\times10^{-9}\text{ m}\right)(0.500\text{ m})}{\left(3.00\times10^{-3}\text{ m}\right)}=\boxed{2.30\times10^{-4}\text{ m}}$$

**P38.4** For destructive interference,

$$\sin\theta = m\frac{\lambda}{a} = \frac{\lambda}{a} = \frac{5.00\text{ cm}}{36.0\text{ cm}} = 0.139$$

and $\theta = 7.98°$

$$\frac{d}{L} = \tan\theta$$

gives $d = L\tan\theta = (6.50\text{ m})\tan 7.98° = 0.912\text{ m}$

$$d = \boxed{91.2\text{ cm}}\,.$$

**P38.5** If the speed of sound is 340 m/s, $\lambda = \frac{v}{f} = \frac{340\text{ m/s}}{650\text{ s}^{-1}} = 0.523\text{ m}$.

Diffraction minima occur at angles described by $a\sin\theta = m\lambda$

$(1.10\text{ m})\sin\theta_1 = 1(0.523\text{ m})$ $\theta_1 = 28.4°$

$(1.10\text{ m})\sin\theta_2 = 2(0.523\text{ m})$ $\theta_2 = 72.0°$

$(1.10\text{ m})\sin\theta_3 = 3(0.523\text{ m})$ $\theta_3$ nonexistent

Maxima appear straight ahead at $\boxed{0°}$ and left and right at an angle given approximately by

$(1.10\text{ m})\sin\theta_x = 1.5(0.523\text{ m})$ $\theta_x\ \boxed{\approx 46°}$.

There is no solution to $a\sin\theta = 2.5\lambda$, so our answer is already complete, with $\boxed{\text{three}}$ sound maxima.

**P38.6** (a) $\sin\theta = \frac{y}{L} = \frac{m\lambda}{a}$

Therefore, for first minimum, $m = 1$ and

$$L = \frac{ay}{m\lambda} = \frac{(7.50\times10^{-4}\text{ m})(8.50\times10^{-4}\text{ m})}{(1)(587.5\times10^{-9}\text{ m})} = \boxed{1.09\text{ m}}\,.$$

(b) $w = 2y_1$ yields $y_1 = 0.850\text{ mm}$

$$w = 2(0.850\times10^{-3}\text{ m}) = \boxed{1.70\text{ mm}}$$

***P38.7** The rectangular patch on the wall is wider than it is tall. The aperture will be taller than it is wide. For horizontal spreading we have

$$\tan\theta_{\text{width}} = \frac{y_{\text{width}}}{L} = \frac{0.110\text{ m}/2}{4.5\text{ m}} = 0.012\,2$$

$$a_{\text{width}}\sin\theta_{\text{width}} = 1\lambda$$

$$a_{\text{width}} = \frac{632.8\times10^{-9}\text{ m}}{0.012\,2} = \boxed{5.18\times10^{-5}\text{ m}}$$

For vertical spreading, similarly

$$\tan\theta_{\text{height}} = \frac{0.006\text{ m}/2}{4.5\text{ m}} = 0.000\,667$$

$$a_{\text{height}} = \frac{1\lambda}{\sin\theta_h} = \frac{632.8\times10^{-9}\text{ m}}{0.000\,667} = \boxed{9.49\times10^{-4}\text{ m}}$$

**P38.8** Equation 38.1 states that $\sin\theta = \frac{m\lambda}{a}$, where $m = \pm1, \pm2, \pm3, \ldots$. The requirement for $m = 1$ is from an analysis of the extra path distance traveled by ray 1 compared to ray 3 in Figure 38.5. This extra distance must be equal to $\frac{\lambda}{2}$ for destructive interference. When the source rays approach the slit at an angle $\beta$, there is a distance added to the path difference (of ray 1 compared to ray 3) of $\frac{a}{2}\sin\beta$ Then, for destructive interference,

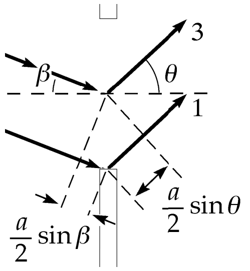

**FIG. P38.8**

$$\frac{a}{2}\sin\beta + \frac{a}{2}\sin\theta = \frac{\lambda}{2} \quad\text{so}\quad \sin\theta = \frac{\lambda}{a} - \sin\beta.$$

Dividing the slit into 4 parts leads to the 2nd order minimum: $\sin\theta = \frac{2\lambda}{a} - \sin\beta.$

Dividing the slit into 6 parts gives the third order minimum: $\sin\theta = \frac{3\lambda}{a} - \sin\beta.$

Generalizing, we obtain the condition for the *m*th order minimum: $\sin\theta = \frac{m\lambda}{a} - \sin\beta.$

**P38.9** $$\sin\theta \approx \frac{y}{L} = \frac{4.10\times10^{-3}\text{ m}}{1.20\text{ m}}$$

$$\frac{\beta}{2} = \frac{\pi a\sin\theta}{\lambda} = \frac{\pi\left(4.00\times10^{-4}\text{ m}\right)}{546.1\times10^{-9}\text{ m}}\left(\frac{4.10\times10^{-3}\text{ m}}{1.20\text{ m}}\right) = 7.86\text{ rad}$$

$$\frac{I}{I_{\max}} = \left[\frac{\sin(\beta/2)}{\beta/2}\right]^2 = \left[\frac{\sin(7.86)}{7.86}\right]^2 = \boxed{1.62\times10^{-2}}$$

**38.10** (a) Double-slit interference maxima are at angles given by $d\sin\theta = m\lambda$.

For $m = 0$, $\theta_0 = \boxed{0°}$.

For $m = 1$, $(2.80\ \mu\text{m})\sin\theta = 1(0.501\,5\ \mu\text{m})$: $\theta_1 = \sin^{-1}(0.179) = \boxed{10.3°}$.

Similarly, for $m = 2, 3, 4, 5$ and 6, $\theta_2 = \boxed{21.0°}$, $\theta_3 = \boxed{32.5°}$, $\theta_4 = \boxed{45.8°}$,

$\theta_5 = \boxed{63.6°}$, and $\theta_6 = \sin^{-1}(1.07) =$ nonexistent.

Thus, there are $5 + 5 + 1 = \boxed{\text{11 directions for interference maxima}}$.

(b) We check for missing orders by looking for single-slit diffraction minima, at $a\sin\theta = m\lambda$.

For $m = 1$, $(0.700\ \mu\text{m})\sin\theta = 1(0.501\,5\ \mu\text{m})$ and $\theta_1 = 45.8°$.

Thus, there is no bright fringe at this angle. There are only $\boxed{\text{nine bright fringes}}$, at $\boxed{\theta = 0°,\ \pm 10.3°,\ \pm 21.0°,\ \pm 32.5°,\ \text{and}\ \pm 63.6°}$.

(c) $$I = I_{\max}\left[\frac{\sin(\pi a\sin\theta/\lambda)}{\pi\sin\theta/\lambda}\right]^2$$

At $\theta = 0°$, $\dfrac{\sin\theta}{\theta} \to 1$ and $\dfrac{I}{I_{\max}} \to \boxed{1.00}$.

At $\theta = 10.3°$, $\dfrac{\pi a\sin\theta}{\lambda} = \dfrac{\pi(0.700\ \mu\text{m})\sin 10.3°}{0.501\,5\ \mu\text{m}} = 0.785\ \text{rad} = 45.0°$

$$\frac{I}{I_{\max}} = \left[\frac{\sin 45.0°}{0.785}\right]^2 = \boxed{0.811}.$$

Similarly, at $\theta = 21.0°$, $\dfrac{\pi a\sin\theta}{\lambda} = 1.57\ \text{rad} = 90.0°$ and $\dfrac{I}{I_{\max}} = \boxed{0.405}$.

At $\theta = 32.5°$, $\dfrac{\pi a\sin\theta}{\lambda} = 2.36\ \text{rad} = 135°$ and $\dfrac{I}{I_{\max}} = \boxed{0.090\,1}$.

At $\theta = 63.6°$, $\dfrac{\pi a\sin\theta}{\lambda} = 3.93\ \text{rad} = 225°$ and $\dfrac{I}{I_{\max}} = \boxed{0.032\,4}$.

## Section 38.3 **Resolution of Single-Slit and Circular Apertures**

**P38.11** $\sin\theta = \dfrac{\lambda}{a} = \dfrac{5.00 \times 10^{-7}\ \text{m}}{5.00 \times 10^{-4}} = \boxed{1.00 \times 10^{-3}\ \text{rad}}$

**P38.12** $\theta_{\min} = \frac{y}{L} = 1.22\frac{\lambda}{D}$

$$y = \frac{(1.22)(5.00\times10^{-7})(0.030\,0)}{7.00\times10^{-3}} = \boxed{2.61\ \mu\text{m}}$$

$y$ = radius of star-image
$L$ = length of eye
$\lambda$ = 500 nm
$D$ = pupil diameter
$\theta$ = half angle

**P38.13** Undergoing diffraction from a circular opening, the beam spreads into a cone of half-angle

$$\theta_{\min} = 1.22\frac{\lambda}{D} = 1.22\left(\frac{632.8\times10^{-9}\ \text{m}}{0.005\,00\ \text{m}}\right) = 1.54\times10^{-4}\ \text{rad}\,.$$

The radius of the beam ten kilometers away is, from the definition of radian measure,

$$r_{\text{beam}} = \theta_{\min}\left(1.00\times10^{4}\ \text{m}\right) = 1.544\ \text{m}$$

and its diameter is $d_{\text{beam}} = 2r_{\text{beam}} = \boxed{3.09\ \text{m}}$.

***P38.14** When you are at the maximum range, the elves' eyes will be resolved by Rayleigh's criterion:

$$\frac{d}{L} = \theta_{\min} = 1.22\frac{\lambda}{D}$$

$$\frac{0.100\ \text{m}}{L} = 1.22\frac{660\times10^{-9}\ \text{m}}{7\times10^{-3}\ \text{m}} = 1.15\times10^{-4}$$

$$L = \frac{0.1\ \text{m}}{1.15\times10^{-4}} = \boxed{869\ \text{m}}$$

***P38.15** By Rayleigh's criterion: $\theta_{\min} = \frac{d}{L} = 1.22\frac{\lambda}{D}$, where $\theta_{\min}$ is the smallest angular separation of two objects for which they are resolved by an aperture of diameter $D$, $d$ is the separation of the two objects, and $L$ is the maximum distance of the aperture from the two objects at which they can be resolved.

Two objects can be resolved if their angular separation is greater than $\theta_{\min}$. Thus, $\theta_{\min}$ should be as small as possible. Therefore, the smaller of the two given wavelengths is easier to resolve, i.e. $\boxed{\text{violet}}$.

$$L = \frac{Dd}{1.22\lambda} = \frac{\left(5.20\times10^{-3}\ \text{m}\right)\left(2.80\times10^{-2}\ \text{m}\right)}{1.22\lambda} = \frac{1.193\times10^{-4}\ \text{m}^2}{\lambda}$$

Thus $L = 186$ m for $\lambda = 640$ nm, and $L = 271$ m for $\lambda = 440$ nm. The viewer can resolve adjacent tubes of violet in the range $\boxed{186\ \text{m to }271\ \text{m}}$, but cannot resolve adjacent tubes of red in this range.

**P38.16** $\theta_{\min} = 1.22\frac{\lambda}{D} = \frac{d}{L}$ $\qquad 1.22\left(\frac{5.80\times10^{-7}\ \text{m}}{4.00\times10^{-3}\ \text{m}}\right) = \frac{d}{1.80\ \text{mi}}\left(\frac{1\ \text{mi}}{1\,609\ \text{m}}\right)$ $\qquad d = \boxed{0.512\ \text{m}}$

The shortening of the wavelength inside the patriot's eye does not change the answer.

**P38.17** By Rayleigh's criterion, two dots separated center-to-center by 2.00 mm would overlap

when $\theta_{\min} = \frac{d}{L} = 1.22\frac{\lambda}{D}$.

Thus, $L = \frac{dD}{1.22\lambda} = \frac{\left(2.00\times10^{-3}\text{ m}\right)\left(4.00\times10^{-3}\text{ m}\right)}{1.22\left(500\times10^{-9}\text{ m}\right)} = \boxed{13.1\text{ m}}$.

**P38.18** $D = 1.22\frac{\lambda}{\theta_{\min}} = \frac{1.22\left(5.00\times10^{-7}\right)}{1.00\times10^{-5}}\text{ m} = \boxed{6.10\text{ cm}}$

***P38.19** The concave mirror of the spy satellite is probably about 2 m in diameter, and is surely not more than 5 m in diameter. That is the size of the largest piece of glass successfully cast to a precise shape, for the mirror of the Hale telescope on Mount Palomar. If the spy satellite had a larger mirror, its manufacture could not be kept secret, and it would be visible from the ground. Outer space is probably closer than your state capitol, but the satellite is surely above 200-km altitude, for reasonably low air friction. We find the distance between barely resolvable objects at a distance of 200 km, seen in yellow light through a 5-m aperture:

$$\frac{y}{L} = 1.22\frac{\lambda}{D}$$

$$y = (200\,000\text{ m})(1.22)\left(\frac{6\times10^{-7}\text{ m}}{5\text{ m}}\right) = 3\text{ cm}$$

(Considering atmospheric seeing caused by variations in air density and temperature, the distance between barely resolvable objects is more like $(200\,000\text{ m})(1\text{ s})\left(\frac{1°}{3\,600\text{ s}}\right)\left(\frac{\pi\text{ rad}}{180°}\right) = 97\text{ cm}$.) Thus the snooping spy satellite cannot see the difference between III and II or IV on a license plate. It cannot count coins spilled on a sidewalk, much less read the date on them.

**P38.20** $1.22\frac{\lambda}{D} = \frac{d}{L}$ $\qquad\lambda = \frac{c}{f} = 0.020\,0\text{ m}$

$D = 2.10\text{ m}$ $\qquad L = 9\,000\text{ m}$

$$d = 1.22\frac{(0.020\,0\text{ m})(9\,000\text{ m})}{2.10\text{ m}} = \boxed{105\text{ m}}$$

**P38.21** $\theta_{\min} = 1.22\frac{\lambda}{D} = 1.22\frac{(2.00\text{ m})}{(10.0\text{ m})} = \boxed{0.244\text{ rad} = 14.0°}$

**P38.22** $L = 88.6\times10^{9}\text{ m}$, $D = 0.300\text{ m}$, $\lambda = 590\times10^{-9}\text{ m}$

(a) $1.22\frac{\lambda}{D} = \theta_{\min} = \boxed{2.40\times10^{-6}\text{ rad}}$

(b) $d = \theta_{\min}L = \boxed{213\text{ km}}$

## Section 38.4 **The Diffraction Grating**

**P38.23** $d=\frac{1.00\text{ cm}}{2\,000}=\frac{1.00\times10^{-2}\text{ m}}{2\,000}=5.00\ \mu\text{m}$

$$\sin\theta=\frac{m\lambda}{d}=\frac{1\left(640\times10^{-9}\text{ m}\right)}{5.00\times10^{-6}\text{ m}}=0.128\qquad\theta=\boxed{7.35°}$$

**P38.24** The principal maxima are defined by

$d\sin\theta=m\lambda$  $m=0,\,1,\,2,\,\ldots.$

For $m=1$,  $\lambda=d\sin\theta$

where $\theta$ is the angle between the central ($m=0$) and the first order ($m=1$) maxima. The value of $\theta$ can be determined from the information given about the distance between maxima and the grating-to-screen distance. From the figure,

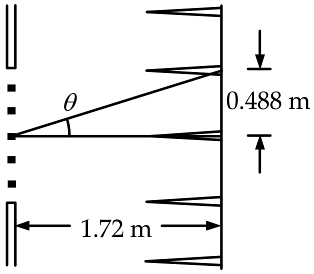

**FIG. P38.24**

$$\tan\theta=\frac{0.488\text{ m}}{1.72\text{ m}}=0.284$$

so  $\theta=15.8°$

and  $\sin\theta=0.273$.

The distance between grating "slits" equals the reciprocal of the number of grating lines per centimeter

$$d=\frac{1}{5\,310\text{ cm}^{-1}}=1.88\times10^{-4}\text{ cm}=1.88\times10^{3}\text{ nm}.$$

The wavelength is  $\lambda=d\sin\theta=\left(1.88\times10^{3}\text{ nm}\right)(0.273)=\boxed{514\text{ nm}}$.

**P38.25** The grating spacing is  $d=\frac{1.00\times10^{-2}\text{ m}}{4\,500}=2.22\times10^{-6}\text{ m}$.

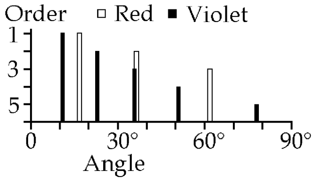

**FIG. P38.25**

In the 1st-order spectrum, diffraction angles are given by

$\sin\theta=\frac{\lambda}{d}$:  $\sin\theta_1=\frac{656\times10^{-9}\text{ m}}{2.22\times10^{-6}\text{ m}}=0.295$

so that for red  $\theta_1=17.17°$

and for violet  $\sin\theta_2=\frac{434\times10^{-9}\text{ m}}{2.22\times10^{-6}\text{ m}}=0.195$

so that  $\theta_2=11.26°$.

The angular separation is in first-order,  $\Delta\theta=17.17°-11.26°=\boxed{5.91°}$.

In the second-order spectrum,  $\Delta\theta=\sin^{-1}\left(\frac{2\lambda_1}{d}\right)-\sin^{-1}\left(\frac{2\lambda_2}{d}\right)=\boxed{13.2°}$.

Again, in the third order,  $\Delta\theta=\sin^{-1}\left(\frac{3\lambda_1}{d}\right)-\sin^{-1}\left(\frac{3\lambda_2}{d}\right)=\boxed{26.5°}$.

Since the red does not appear in the fourth-order spectrum, the answer is complete.

**P38.26** $\sin\theta = 0.350:$ $\quad d = \frac{\lambda}{\sin\theta} = \frac{632.8\text{ nm}}{0.350} = 1.81\times 10^3\text{ nm}$

Line spacing $= \boxed{1.81\ \mu\text{m}}$

**P38.27** (a) $d = \frac{1}{3\,660\text{ lines/cm}} = 2.732\times 10^{-4}\text{ cm} = 2.732\times 10^{-6}\text{ m} = 2\,732\text{ nm}$

$\lambda = \frac{d\sin\theta}{m}:$ At $\theta = 10.09°$ $\quad\lambda = \boxed{478.7\text{ nm}}$

At $\theta = 13.71°$, $\quad\lambda = \boxed{647.6\text{ nm}}$

At $\theta = 14.77°$, $\quad\lambda = \boxed{696.6\text{ nm}}$

(b) $d = \frac{\lambda}{\sin\theta_1}$ and $2\lambda = d\sin\theta_2$ so $\sin\theta_2 = \frac{2\lambda}{d} = \frac{2\lambda}{\lambda/\sin\theta_1} = 2\sin\theta_1$.

Therefore, if $\theta_1 = 10.09°$ then $\sin\theta_2 = 2\sin(10.09°)$ gives $\theta_2 = \boxed{20.51°}$.

Similarly, for $\theta_1 = 13.71°$, $\theta_2 = \boxed{28.30°}$ and for $\theta_1 = 14.77°$, $\theta_2 = \boxed{30.66°}$.

**P38.28** $\sin\theta = \frac{m\lambda}{d}$

Therefore, taking the ends of the visible spectrum to be $\lambda_v = 400\text{ nm}$ and $\lambda_r = 750\text{ nm}$, the ends the different order spectra are:

End of second order: $\quad\sin\theta_{2r} = \frac{2\lambda_r}{d} = \frac{1\,500\text{ nm}}{d}$.

Start of third order: $\quad\sin\theta_{3v} = \frac{3\lambda_v}{d} = \frac{1\,200\text{ nm}}{d}$.

Thus, it is seen that $\boxed{\theta_{2r} > \theta_{3v}\text{ and these orders must overlap}}$ regardless of the value of the grating spacing *d*.

**P38.29** (a) From Equation 38.12, $R = Nm$ where $\quad N = (3\,000\text{ lines/cm})(4.00\text{ cm}) = 1.20\times 10^4\text{ lines}$.

In the 1st order, $\quad R = (1)\left(1.20\times 10^4\text{ lines}\right) = \boxed{1.20\times 10^4}$.

In the 2nd order, $\quad R = (2)\left(1.20\times 10^4\text{ lines}\right) = \boxed{2.40\times 10^4}$.

In the 3rd order, $\quad R = (3)\left(1.20\times 10^4\text{ lines}\right) = \boxed{3.60\times 10^4}$.

(b) From Equation 38.11, $\quad R = \frac{\lambda}{\Delta\lambda}:$

In the 3rd order, $\quad\Delta\lambda = \frac{\lambda}{R} = \frac{400\text{ nm}}{3.60\times 10^4} = 0.011\,1\text{ nm} = \boxed{11.1\text{ pm}}$.

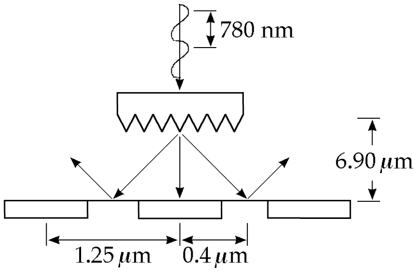

**FIG. P38.30**

***P38.30** For a side maximum, $\tan\theta = \frac{y}{L} = \frac{0.4\ \mu\text{m}}{6.9\ \mu\text{m}}$

$$\theta = 3.32°$$

$d\sin\theta = m\lambda$ $\quad d = \frac{(1)\left(780\times10^{-9}\ \text{m}\right)}{\sin 3.32°} = 13.5\ \mu\text{m}.$

The number of grooves per millimeter $= \frac{1\times10^{-3}\ \text{m}}{13.5\times10^{-6}\ \text{m}} = \boxed{74.2}$.

**P38.31** (a) $Nm = \frac{\lambda}{\Delta\lambda}$ $\quad N(1) = \frac{531.7\ \text{nm}}{0.19\ \text{nm}} = \boxed{2\,800}$

(b) $\frac{1.32\times10^{-2}\ \text{m}}{2\,800} = \boxed{4.72\ \mu\text{m}}$

**P38.32** $d = \frac{1}{4\,200/\text{cm}} = 2.38\times10^{-6}\ \text{m} = 2\,380\ \text{nm}.$

$d\sin\theta = m\lambda$ or $\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right)$ and $y = L\tan\theta = L\tan\left[\sin^{-1}\left(\frac{m\lambda}{d}\right)\right]$.

Thus, $\Delta y = L\left\{\tan\left[\sin^{-1}\left(\frac{m\lambda_2}{d}\right)\right] - \tan\left[\sin^{-1}\left(\frac{m\lambda_1}{d}\right)\right]\right\}.$

For $m = 1$, $\Delta y = (2.00\ \text{m})\left\{\tan\left[\sin^{-1}\left(\frac{589.6}{2\,380}\right)\right] - \tan\left[\sin^{-1}\left(\frac{589}{2\,380}\right)\right]\right\} = 0.554\ \text{mm}.$

For $m = 2$, $\Delta y = (2.00\ \text{m})\left\{\tan\left[\sin^{-1}\left(\frac{2(589.6)}{2\,380}\right)\right] - \tan\left[\sin^{-1}\left(\frac{2(589)}{2\,380}\right)\right]\right\} = 1.54\ \text{mm}.$

For $m = 3$, $\Delta y = (2.00\ \text{m})\left\{\tan\left[\sin^{-1}\left(\frac{3(589.6)}{2\,380}\right)\right] - \tan\left[\sin^{-1}\left(\frac{3(589)}{2\,380}\right)\right]\right\} = 5.04\ \text{mm}.$

Thus, the observed order must be $\boxed{m = 2}$.

**P38.33** $d=\dfrac{1.00\times10^{-3}\text{ m/mm}}{250\text{ lines/mm}}=4.00\times10^{-6}\text{ m}=4\,000\text{ nm}$ $\quad d\sin\theta=m\lambda\Rightarrow m=\dfrac{d\sin\theta}{\lambda}$

(a) The number of times a complete order is seen is the same as the number of orders in which the long wavelength limit is visible.

$$m_{\max}=\frac{d\sin\theta_{\max}}{\lambda}=\frac{(4\,000\text{ nm})\sin 90.0°}{700\text{ nm}}=5.71$$

or $\boxed{\text{5 orders is the maximum}}$.

(b) The highest order in which the violet end of the spectrum can be seen is:

$$m_{\max}=\frac{d\sin\theta_{\max}}{\lambda}=\frac{(4\,000\text{ nm})\sin 90.0°}{400\text{ nm}}=10.0$$

or $\boxed{\text{10 orders in the short-wavelength region}}$.

***P38.34** (a) The several narrow parallel slits make a diffraction grating. The zeroth- and first- order maxima are separated according to

$d\sin\theta=(1)\lambda \qquad \sin\theta=\dfrac{\lambda}{d}=\dfrac{632.8\times10^{-9}\text{ m}}{1.2\times10^{-3}\text{ m}}$

$\theta=\sin^{-1}(0.000\,527)=0.000\,527\text{ rad}$

$y=L\tan\theta=(1.40\text{ m})(0.000\,527)=\boxed{0.738\text{ mm}}$.

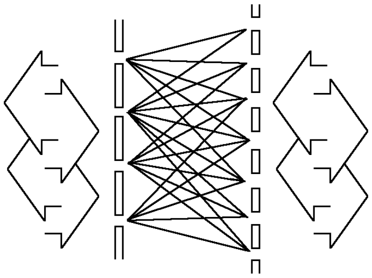

FIG. P38.34

(b) Many equally spaced transparent lines appear on the film. It is itself a diffraction grating. When the same light is sent through the film, it produces interference maxima separated according to

$d\sin\theta=(1)\lambda \qquad \sin\theta=\dfrac{\lambda}{d}=\dfrac{632.8\times10^{-9}\text{ m}}{0.738\times10^{-3}\text{ m}}=0.000\,857$

$y=L\tan\theta=(1.40\text{ m})(0.000\,857)=1.20\text{ mm}$

An image of the original set of slits appears on the screen. If the screen is removed, light diverges from the real images with the same wave fronts reconstructed as the original slits produced. Reasoning from the mathematics of Fourier transforms, Gabor showed that light diverging from any object, not just a set of slits, could be used. In the picture, the slits or maxima on the left are separated by 1.20 mm. The slits or maxima on the right are separated by 0.738 mm. The length difference between any pair of lines is an integer number of wavelengths. Light can be sent through equally well toward the right or toward the left.

## Section 38.5 Diffraction of X-Rays by Crystals

**P38.35** $2d\sin\theta=m\lambda$: $\quad \lambda=\dfrac{2d\sin\theta}{m}=\dfrac{2(0.353\times10^{-9}\text{ m})\sin 7.60°}{1}=9.34\times10^{-11}\text{ m}=\boxed{0.093\,4\text{ nm}}$

**P38.36** $2d\sin\theta = m\lambda \Rightarrow d = \dfrac{m\lambda}{2\sin\theta} = \dfrac{(1)(0.129\text{ nm})}{2\sin(8.15°)} = \boxed{0.455\text{ nm}}$

**P38.37** $2d\sin\theta = m\lambda$: $\sin\theta = \dfrac{m\lambda}{2d} = \dfrac{1\left(0.140\times10^{-9}\text{ m}\right)}{2\left(0.281\times10^{-9}\text{ m}\right)} = 0.249$

and $\boxed{\theta = 14.4°}$

**P38.38** $\sin\theta_m = \dfrac{m\lambda}{2d}$: $\sin 12.6° = \dfrac{1\lambda}{2d} = 0.218$

$\sin\theta_2 = \dfrac{2\lambda}{2d} = 2(0.218)$ so $\theta_2 = 25.9°$

$\boxed{\text{Three}}$ other orders appear: $\theta_3 = \sin^{-1}(3\times0.218) = 40.9°$

$\theta_4 = \sin^{-1}(4\times0.218) = 60.8°$

$\theta_5 = \sin^{-1}(5\times0.218) = \text{nonexistent}$

**P38.39** Figure 38.27 of the text shows the situation.

$2d\sin\theta = m\lambda$ or $\lambda = \dfrac{2d\sin\theta}{m}$

$m=1$: $\lambda_1 = \dfrac{2(2.80\text{ m})\sin 80.0°}{1} = \boxed{5.51\text{ m}}$

$m=2$: $\lambda_2 = \dfrac{2(2.80\text{ m})\sin 80.0°}{2} = \boxed{2.76\text{ m}}$

$m=3$: $\lambda_3 = \dfrac{2(2.80\text{ m})\sin 80.0°}{3} = \boxed{1.84\text{ m}}$

## Section 38.6 **Polarization of Light Waves**

**P38.40** The average value of the cosine-squared function is one-half, so the first polarizer transmits $\dfrac{1}{2}$ the light. The second transmits $\cos^2 30.0° = \dfrac{3}{4}$.

$I_f = \dfrac{1}{2}\times\dfrac{3}{4}I_i = \boxed{\dfrac{3}{8}I_i}$

**P38.41** $I = I_{\max}\cos^2\theta \quad\Rightarrow\quad \theta = \cos^{-1}\sqrt{\dfrac{I}{I_{\max}}}$

(a) $\dfrac{I}{I_{\max}} = \dfrac{1}{3.00} \quad\Rightarrow\quad \theta = \cos^{-1}\sqrt{\dfrac{1}{3.00}} = \boxed{54.7°}$

(b) $\dfrac{I}{I_{\max}} = \dfrac{1}{5.00} \quad\Rightarrow\quad \theta = \cos^{-1}\sqrt{\dfrac{1}{5.00}} = \boxed{63.4°}$

(c) $\dfrac{I}{I_{\max}} = \dfrac{1}{10.0} \quad\Rightarrow\quad \theta = \cos^{-1}\sqrt{\dfrac{1}{10.0}} = \boxed{71.6°}$

**P38.42** (a) $\theta_1 = 20.0°$, $\theta_2 = 40.0°$, $\theta_3 = 60.0°$

$$I_f = I_i \cos^2(\theta_1 - 0°)\cos^2(\theta_2 - \theta_1)\cos^2(\theta_3 - \theta_2)$$

$$I_f = (10.0\text{ units})\cos^2(20.0°)\cos^2(20.0°)\cos^2(20.0°)$$

$$= \boxed{6.89\text{ units}}$$

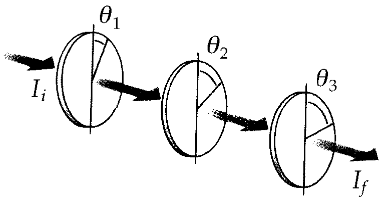

FIG. P38.42

(b) $\theta_1 = 0°$, $\theta_2 = 30.0°$, $\theta_3 = 60.0°$

$$I_f = (10.0\text{ units})\cos^2(0°)\cos^2(30.0°)\cos^2(30.0°) = \boxed{5.63\text{ units}}$$

**P38.43** By Brewster's law, $n = \tan\theta_p = \tan(48.0°) = \boxed{1.11}$.

***P38.44** (a) At incidence, $n_1 \sin\theta_1 = n_2 \sin\theta_2$ and $\theta_1' = \theta_1$. For complete polarization of the reflected light,

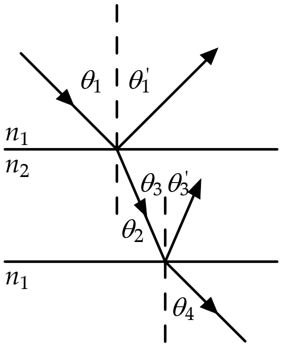

FIG. P38.44(a)

$$(90 - \theta_1') + (90 - \theta_2) = 90°$$

$$\theta_1' + \theta_2 = 90 = \theta_1 + \theta_2$$

Then $n_1 \sin\theta_1 = n_2 \sin(90 - \theta_1) = n_2 \cos\theta_1$

$$\frac{\sin\theta_1}{\cos\theta_1} = \frac{n_2}{n_1} = \tan\theta_1$$

At the bottom surface, $\theta_3 = \theta_2$ because the normals to the surfaces of entry and exit are parallel.

Then $n_2 \sin\theta_3 = n_1 \sin\theta_4$ and $\theta_3' = \theta_3$

$n_2 \sin\theta_2 = n_1 \sin\theta_4$ and $\theta_4 = \theta_1$

The condition for complete polarization of the reflected light is

$90 - \theta_3' + 90 - \theta_4 = 90°$ $\quad\theta_2 + \theta_1 = 90$

This is the same as the condition for $\theta_1$ to be Brewster's angle at the top surface.

(b) We consider light moving in a plane perpendicular to the line where the surfaces of the prism meet at the unknown angle $\Phi$. We require

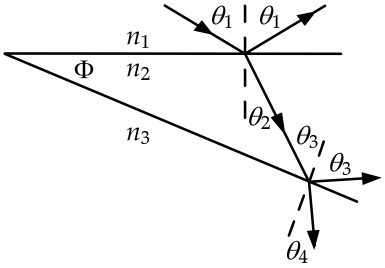

FIG. P38.44(b)

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\theta_1 + \theta_2 = 90°$$

So $n_1 \sin(90 - \theta_2) = n_2 \sin\theta_2$ $\quad \dfrac{n_1}{n_2} = \tan\theta_2$

And $n_2 \sin\theta_3 = n_3 \sin\theta_4$ $\quad \theta_3 + \theta_4 = 90°$

$n_2 \sin\theta_3 = n_3 \cos\theta_3$ $\quad \tan\theta_3 = \dfrac{n_3}{n_2}$

In the triangle made by the faces of the prism and the ray in the prism,

$$\Phi + 90 + \theta_2 + (90 - \theta_3) = 180.$$

So one particular apex angle is required, and it is

$$\Phi = \theta_3 - \theta_2 = \boxed{\tan^{-1}\left(\frac{n_3}{n_2}\right) - \tan^{-1}\left(\frac{n_1}{n_2}\right)}.$$

Here a negative result is to be interpreted as meaning the same as a positive result.

**P38.45** $\sin\theta_c = \dfrac{1}{n}$ or $n = \dfrac{1}{\sin\theta_c} = \dfrac{1}{\sin 34.4^\circ} = 1.77$.

Also, $\tan\theta_p = n$. Thus, $\theta_p = \tan^{-1}(n) = \tan^{-1}(1.77) = \boxed{60.5^\circ}$.

**P38.46** $\sin\theta_c = \dfrac{1}{n}$ and $\tan\theta_p = n$

Thus, $\sin\theta_c = \dfrac{1}{\tan\theta_p}$ or $\boxed{\cot\theta_p = \sin\theta_c}$.

**P38.47** Complete polarization occurs at Brewster's angle $\tan\theta_p = 1.33$ $\theta_p = 53.1^\circ$.

Thus, the Moon is $\boxed{36.9^\circ}$ above the horizon.

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## Additional Problems

**P38.48** For incident unpolarized light of intensity $I_{\max}$:

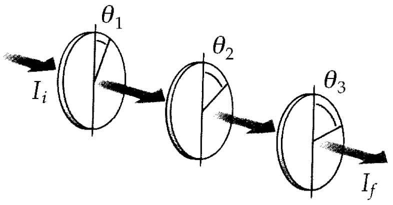

**FIG. P38.48**

After transmitting 1st disk: $I = \dfrac{1}{2}I_{\max}$.

After transmitting 2nd disk: $I = \dfrac{1}{2}I_{\max}\cos^2\theta$.

After transmitting 3rd disk: $I = \dfrac{1}{2}I_{\max}\cos^2\theta\cos^2(90^\circ - \theta)$.

where the angle between the first and second disk is $\theta = \omega t$.

Using trigonometric identities $\cos^2\theta = \dfrac{1}{2}(1+\cos 2\theta)$

and $\cos^2(90^\circ - \theta) = \sin^2\theta = \dfrac{1}{2}(1-\cos 2\theta)$

we have $I = \dfrac{1}{2}I_{\max}\left[\dfrac{(1+\cos 2\theta)}{2}\right]\left[\dfrac{(1-\cos 2\theta)}{2}\right]$

$$I = \frac{1}{8}I_{\max}\left(1-\cos^2 2\theta\right) = \frac{1}{8}I_{\max}\left(\frac{1}{2}\right)(1-\cos 4\theta).$$

Since $\theta = \omega t$, the intensity of the emerging beam is given by $\boxed{I = \dfrac{1}{16}I_{\max}(1-4\omega t)}$.

**P38.49** Let the first sheet have its axis at angle $\theta$ to the original plane of polarization, and let each further sheet have its axis turned by the same angle.

The first sheet passes intensity $I_{\max}\cos^2\theta$.

The second sheet passes $I_{\max}\cos^4\theta$

and the $n$th sheet lets through $I_{\max}\cos^{2n}\theta \ge 0.90 I_{\max}$ where $\theta = \dfrac{45°}{n}$.

Try different integers to find $\cos^{2\times 5}\left(\dfrac{45°}{5}\right) = 0.885$ $\cos^{2\times 6}\left(\dfrac{45°}{6}\right) = 0.902$.

(a) So $n = \boxed{6}$

(b) $\theta = \boxed{7.50°}$

**P38.50** Consider vocal sound moving at 340 m/s and of frequency 3 000 Hz. Its wavelength is

$$\lambda = \frac{v}{f} = \frac{340\ \text{m/s}}{3\,000\ \text{Hz}} = 0.113\ \text{m}.$$

If your mouth, for horizontal dispersion, behaves similarly to a slit 6.00 cm wide, then $a\sin\theta = m\lambda$ predicts no diffraction minima. You are a nearly isotropic source of this sound. It spreads out from you nearly equally in all directions. On the other hand, if you use a megaphone with width 60.0 cm at its wide end, then $a\sin\theta = m\lambda$ predicts the first diffraction minimum at

$$\theta = \sin^{-1}\left(\frac{m\lambda}{a}\right) = \sin^{-1}\left(\frac{0.113\ \text{m}}{0.600\ \text{m}}\right) = 10.9°.$$

This suggests that the sound is radiated mostly toward the front into a diverging beam of angular diameter only about 20°. With less sound energy wasted in other directions, more is available for your intended auditors. We could check that a distant observer to the side or behind you receives less sound when a megaphone is used.

**P38.51** The first minimum is at $a\sin\theta = (1)\lambda$.

This has no solution if $\dfrac{\lambda}{a} > 1$.

or if $a < \lambda = \boxed{632.8\ \text{nm}}$.

**P38.52** $x = 1.22\dfrac{\lambda}{d}D = 1.22\left(\dfrac{5.00\times 10^{-7}\ \text{m}}{5.00\times 10^{-3}\ \text{m}}\right)\left(250\times 10^{3}\ \text{m}\right) = \boxed{30.5\ \text{m}}$

$D = 250\times 10^{3}\ \text{m}$
$\lambda = 5.00\times 10^{-7}\ \text{m}$
$d = 5.00\times 10^{-3}\ \text{m}$

**P38.53** $d = \dfrac{1}{400\text{ mm}^{-1}} = 2.50\times10^{-6}\text{ m}$

(a) $d\sin\theta = m\lambda$ $\qquad$ $\theta_a = \sin^{-1}\left(\dfrac{2\times541\times10^{-9}\text{ m}}{2.50\times10^{-6}\text{ m}}\right) = \boxed{25.6°}$

(b) $\lambda = \dfrac{541\times10^{-9}\text{ m}}{1.33} = 4.07\times10^{-7}\text{ m}$ $\qquad$ $\theta_b = \sin^{-1}\left(\dfrac{2\times4.07\times10^{-7}\text{ m}}{2.50\times10^{-6}\text{ m}}\right) = \boxed{19.0°}$

(c) $d\sin\theta_a = 2\lambda$ $\qquad$ $d\sin\theta_b = \dfrac{2\lambda}{n}$

$n\sin\theta_b = (1)\sin\theta_a$

**P38.54** (a) $\lambda = \dfrac{v}{f}$: $\qquad$ $\lambda = \dfrac{3.00\times10^{8}\text{ m/s}}{1.40\times10^{9}\text{ s}^{-1}} = 0.214\text{ m}$

$\theta_{\min} = 1.22\dfrac{\lambda}{D}$: $\theta_{\min} = 1.22\left(\dfrac{0.214\text{ m}}{3.60\times10^{4}\text{ m}}\right) = \boxed{7.26\ \mu\text{rad}}$

$\theta_{\min} = 7.26\ \mu\text{rad}\left(\dfrac{180\times60\times60\text{ s}}{\pi}\right) = \boxed{1.50\text{ arc seconds}}$

(b) $\theta_{\min} = \dfrac{d}{L}$: $\qquad$ $d = \theta_{\min}L = (7.26\times10^{-6}\text{ rad})(26\,000\text{ ly}) = \boxed{0.189\text{ ly}}$

(c) $\theta_{\min} = 1.22\dfrac{\lambda}{D}$ $\quad$ $\theta_{\min} = 1.22\left(\dfrac{500\times10^{-9}\text{ m}}{12.0\times10^{-3}\text{ m}}\right) = \boxed{50.8\ \mu\text{rad}}$ (10.5 seconds of arc)

(d) $d = \theta_{\min}L = (50.8\times10^{-6}\text{ rad})(30.0\text{ m}) = 1.52\times10^{-3}\text{ m} = \boxed{1.52\text{ mm}}$

**P38.55** With a grazing angle of 36.0°, the angle of incidence is 54.0°

$\tan\theta_p = n = \tan54.0° = 1.38$.

In the liquid, $\lambda_n = \dfrac{\lambda}{n} = \dfrac{750\text{ nm}}{1.38} = \boxed{545\text{ nm}}$.

***P38.56** (a) Bragg's law applies to the space lattice of melanin rods. Consider the planes $d = 0.25\ \mu\text{m}$ apart. For light at near-normal incidence, strong reflection happens for the wavelength given by $2d\sin\theta = m\lambda$. The longest wavelength reflected strongly corresponds to $m = 1$:

$2(0.25\times10^{-6}\text{ m})\sin90° = 1\lambda$ $\qquad$ $\lambda = 500\text{ nm}$. This is the blue-green color.

(b) For light incident at grazing angle 60°, $2d\sin\theta = m\lambda$ gives $1\lambda = 2(0.25\times10^{-6}\text{ m})\sin60° = 433\text{ nm}$. This is violet.

(c) Your two eyes receive light reflected from the feather at different angles, so they receive light incident at different angles and containing different colors reinforced by constructive interference.

*continued on next page*

(d) The longest wavelength that can be reflected with extra strength by these melanin rods is the one we computed first, 500 nm blue-green.

(e) If the melanin rods were farther apart (say 0.32 $\mu$m) they could reflect red with constructive interference.

**P38.57** (a) $d\sin\theta = m\lambda$

$$\text{or } d = \frac{m\lambda}{\sin\theta} = \frac{3\left(500\times10^{-9}\text{ m}\right)}{\sin 32.0°} = 2.83\ \mu\text{m}$$

$$\text{Therefore, lines per unit length } = \frac{1}{d} = \frac{1}{2.83\times10^{-6}\text{ m}}$$

$$\text{or lines per unit length } = 3.53\times10^{5}\text{ m}^{-1} = \boxed{3.53\times10^{3}\text{ cm}^{-1}}.$$

(b) $$\sin\theta = \frac{m\lambda}{d} = \frac{m\left(500\times10^{-9}\text{ m}\right)}{2.83\times10^{-6}\text{ m}} = m(0.177)$$

For $\sin\theta \le 1.00$, we must have $m(0.177) \le 1.00$

or $m \le 5.66$.

Therefore, the highest order observed is $m = 5$.

Total number of primary maxima observed is $2m+1 = \boxed{11}$.

**P38.58** For the air-to-water interface,

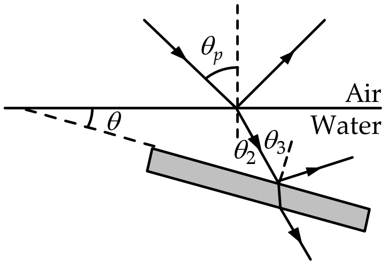

FIG. P38.58

$$\tan\theta_p = \frac{n_{\text{water}}}{n_{\text{air}}} = \frac{1.33}{1.00} \qquad \theta_p = 53.1°$$

and $$(1.00)\sin\theta_p = (1.33)\sin\theta_2$$

$$\theta_2 = \sin^{-1}\left(\frac{\sin 53.1°}{1.33}\right) = 36.9°.$$

For the water-to-glass interface, $\tan\theta_p = \tan\theta_3 = \dfrac{n_{\text{glass}}}{n_{\text{water}}} = \dfrac{1.50}{1.33}$ so

$\theta_3 = 48.4°$.

The angle between surfaces is $\theta = \theta_3 - \theta_2 = \boxed{11.5°}$.

***P38.59** A central maximum and side maxima in seven orders of interference appear. If the seventh order is just at 90°,

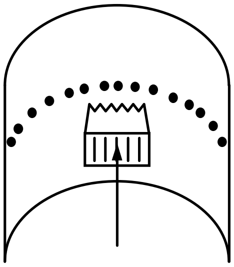

FIG. P38.59

$$d\sin\theta = m\lambda \qquad d1 = 7\left(654\times10^{-9}\text{ m}\right) \qquad d = 4.58\ \mu\text{m}.$$

If the seventh order is at less than 90°, the eighth order might be nearly ready to appear according to

$$d1 = 8\left(654\times10^{-9}\text{ m}\right) \qquad d = 5.23\ \mu\text{m}.$$

Thus $\boxed{4.58\ \mu\text{m} < d < 5.23\ \mu\text{m}}$.

**P38.60** (a) We require $\theta_{\min} = 1.22\dfrac{\lambda}{D} = \dfrac{\text{radius of diffraction disk}}{L} = \dfrac{D}{2L}$.

Then $\boxed{D^2 = 2.44\lambda L}$.

(b) $D = \sqrt{2.44\left(500\times10^{-9}\text{ m}\right)(0.150\text{ m})} = \boxed{428\ \mu\text{m}}$

**P38.61** The limiting resolution between lines $\theta_{\min} = 1.22\dfrac{\lambda}{D} = 1.22\dfrac{\left(550\times10^{-9}\text{ m}\right)}{\left(5.00\times10^{-3}\text{ m}\right)} = 1.34\times10^{-4}\text{ rad}$.

Assuming a picture screen with vertical dimension $\ell$, the minimum viewing distance for no visible lines is found from $\theta_{\min} = \dfrac{\ell/485}{L}$. The desired ratio is then

$$\frac{L}{\ell} = \frac{1}{485\theta_{\min}} = \frac{1}{485\left(1.34\times10^{-4}\text{ rad}\right)} = \boxed{15.4}.$$

**P38.62** (a) Applying Snell's law gives $n_2\sin\phi = n_1\sin\theta$. From the sketch, we also see that:

$\theta+\phi+\beta=\pi$, or $\phi=\pi-(\theta+\beta)$.

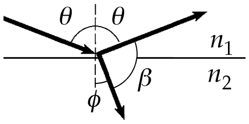

FIG. P38.62(a)

Using the given identity: $\sin\phi = \sin\pi\cos(\theta+\beta) - \cos\pi\sin(\theta+\beta)$,

which reduces to: $\sin\phi = \sin(\theta+\beta)$.

Applying the identity again: $\sin\phi = \sin\theta\cos\beta + \cos\theta\sin\beta$.

Snell's law then becomes: $n_2(\sin\theta\cos\beta+\cos\theta\sin\beta) = n_1\sin\theta$

or (after dividing by $\cos\theta$): $n_2(\tan\theta\cos\beta+\sin\beta) = n_1\tan\theta$.

Solving for $\tan\theta$ gives: $\boxed{\tan\theta = \dfrac{n_2\sin\beta}{n_1 - n_2\cos\beta}}$.

(b) If $\beta = 90.0°$, $n_1 = 1.00$, and $n_2 = n$, the above result becomes:

$\tan\theta = \dfrac{n(1.00)}{1.00-0}$, or $n = \tan\theta$, which is Brewster's law.

**P38.63** (a) From Equation 38.1, $\theta = \sin^{-1}\left(\dfrac{m\lambda}{a}\right)$.

In this case $m=1$ and $\lambda = \dfrac{c}{f} = \dfrac{3.00\times10^{8}\text{ m/s}}{7.50\times10^{9}\text{ Hz}} = 4.00\times10^{-2}\text{ m}$.

Thus, $\theta = \sin^{-1}\left(\dfrac{4.00\times10^{-2}\text{ m}}{6.00\times10^{-2}\text{ m}}\right) = \boxed{41.8°}$.

*continued on next page*

(b) From Equation 38.4, $\frac{I}{I_{\max}} = \left[\frac{\sin(\beta/2)}{\beta/2}\right]^2$ where $\beta = \frac{2\pi a \sin\theta}{\lambda}$.

When $\theta = 15.0°$, $\beta = \frac{2\pi(0.060\,0 \text{ m})\sin 15.0°}{0.040\,0 \text{ m}} = 2.44 \text{ rad}$

and $\frac{I}{I_{\max}} = \left[\frac{\sin(1.22 \text{ rad})}{1.22 \text{ rad}}\right]^2 = \boxed{0.593}$.

(c) $\sin\theta = \frac{\lambda}{a}$ so $\theta = 41.8°$:

This is the minimum angle subtended by the two sources at the slit. Let $\alpha$ be the half angle between the sources, each a distance $\ell = 0.100$ m from the center line and a distance $L$ from the slit plane. Then,

$L = \ell \cot\alpha = (0.100 \text{ m})\cot\left(\frac{41.8°}{2}\right) = \boxed{0.262 \text{ m}}$.

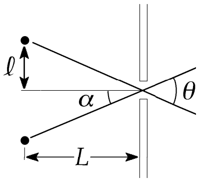

**FIG. P38.63(c)**

**P38.64** $\frac{I}{I_{\max}} = \frac{1}{2}\left(\cos^2 45.0°\right)\left(\cos^2 45.0°\right) = \boxed{\frac{1}{8}}$

**P38.65** $d\sin\theta = m\lambda$

and, differentiating, $d(\cos\theta)d\theta = m d\lambda$

or $d\sqrt{1-\sin^2\theta}\,\Delta\theta \approx m\Delta\lambda$

$d\sqrt{1-\frac{m^2\lambda^2}{d^2}}\,\Delta\theta \approx m\Delta\lambda$

so $\boxed{\Delta\theta \approx \frac{\Delta\lambda}{\sqrt{\left(d^2/m^2\right)-\lambda^2}}}$.

***P38.66** (a) The angles of bright beams diffracted from the grating are given by $(d)\sin\theta = m\lambda$. The angular dispersion is defined as the derivative $\frac{d\theta}{d\lambda}$: $(d)\cos\theta\frac{d\theta}{d\lambda} = m$ $\frac{d\theta}{d\lambda} = \frac{m}{d\cos\theta}$

(b) For the average wavelength 578 nm,

$d\sin\theta = m\lambda$ $\frac{0.02 \text{ m}}{8\,000}\sin\theta = 2\left(578 \times 10^{-9} \text{ m}\right)$

$\theta = \sin^{-1}\frac{2 \times 578 \times 10^{-9} \text{ m}}{2.5 \times 10^{-6} \text{ m}} = 27.5°$

The separation angle between the lines is

$\Delta\theta = \frac{d\theta}{d\lambda}\Delta\lambda = \frac{m}{d\cos\theta}\Delta\lambda = \frac{2}{2.5 \times 10^{-6} \text{ m}\cos 27.5°}2.11 \times 10^{-9} \text{ m}$

$= 0.001\,90 = 0.001\,90 \text{ rad} = 0.001\,90 \text{ rad}\left(\frac{180°}{\pi \text{ rad}}\right) = \boxed{0.109°}$

***P38.67** (a) Constructive interference of light of wavelength $\lambda$ on the screen is described by $d\sin\theta = m\lambda$ where $\tan\theta = \frac{y}{L}$ so $\sin\theta = \frac{y}{\sqrt{L^2+y^2}}$. Then $(d)y\left(L^2+y^2\right)^{-1/2} = m\lambda$. Differentiating with respect to $y$ gives

$$d1\left(L^2+y^2\right)^{-1/2} + (d)y\left(-\frac{1}{2}\right)\left(L^2+y^2\right)^{-3/2}(0+2y) = m\frac{d\lambda}{dy}$$

$$\frac{d}{\left(L^2+y^2\right)^{1/2}} - \frac{(d)y^2}{\left(L^2+y^2\right)^{3/2}} = m\frac{d\lambda}{dy} = \frac{(d)L^2+(d)y^2-(d)y^2}{\left(L^2+y^2\right)^{3/2}}$$

$$\frac{d\lambda}{dy} = \frac{(d)L^2}{m\left(L^2+y^2\right)^{3/2}}$$

(b) Here $d\sin\theta = m\lambda$ gives $\frac{10^{-2}\text{ m}}{8\,000}\sin\theta = 1\left(550\times10^{-9}\text{ m}\right)$, $\theta = \sin^{-1}\left(\frac{0.55\times10^{-6}\text{ m}}{1.25\times10^{-6}\text{ m}}\right) = 26.1°$

$y = L\tan\theta = 2.40\text{ m}\tan 26.1° = 1.18\text{ m}$

Now $\frac{d\lambda}{dy} = \frac{dL^2}{m\left(L^2+y^2\right)^{3/2}} = \frac{1.25\times10^{-6}\text{ m}(2.40\text{ m})^2}{1\left((2.4\text{ m})^2+(1.18\text{ m})^2\right)^{3/2}} = 3.77\times10^{-7} = \boxed{3.77\text{ nm/cm}}$.

**P38.68** For a diffraction grating, the locations of the principal maxima for wavelength $\lambda$ are given by $\sin\theta = \frac{m\lambda}{d} \approx \frac{y}{L}$. The grating spacing may be expressed as $d = \frac{a}{N}$ where $a$ is the width of the grating and $N$ is the number of slits. Thus, the screen locations of the maxima become $y = \frac{NLm\lambda}{a}$. If two nearly equal wavelengths are present, the difference in the screen locations of corresponding maxima is

$$\Delta y = \frac{NLm(\Delta\lambda)}{a}.$$

For a single slit of width $a$, the location of the first diffraction minimum is $\sin\theta = \frac{\lambda}{a} \approx \frac{y}{L}$, or $y = \left(\frac{L}{a}\right)\lambda$. If the two wavelengths are to be just resolved by Rayleigh's criterion, $y = \Delta y$ from above. Therefore,

$\left(\frac{L}{a}\right)\lambda = \frac{NLm(\Delta\lambda)}{a}$ or the resolving power of the grating is $\boxed{R \equiv \frac{\lambda}{\Delta\lambda} = Nm}$.

**P38.69** (a) The E and O rays, in phase at the surface of the plate, will have a phase difference

$$\theta = \left(\frac{2\pi}{\lambda}\right)\delta$$

after traveling distance *d* through the plate. Here $\delta$ is the difference in the *optical path* lengths of these rays. The optical path length between two points is the product of the actual path length *d* and the index of refraction. Therefore,

$$\delta = |dn_O - dn_E|.$$

The absolute value is used since $\frac{n_O}{n_E}$ may be more or less than unity. Therefore,

$$\theta = \left(\frac{2\pi}{\lambda}\right)|dn_O - dn_E| = \boxed{\left(\frac{2\pi}{\lambda}\right)d|n_O - n_E|}.$$

(b) $$d = \frac{\lambda\theta}{2\pi|n_O - n_E|} = \frac{(550\times10^{-9}\text{ m})(\pi/2)}{2\pi|1.544 - 1.553|} = 1.53\times10^{-5}\text{ m} = \boxed{15.3\ \mu\text{m}}$$

**P38.70** (a) From Equation 38.4, $$\frac{I}{I_{\max}} = \left[\frac{\sin(\beta/2)}{\beta/2}\right]^2.$$

If we define $\phi \equiv \frac{\beta}{2}$ this becomes $$\frac{I}{I_{\max}} = \left[\frac{\sin\phi}{\phi}\right]^2.$$

Therefore, when $\frac{I}{I_{\max}} = \frac{1}{2}$ we must have $\frac{\sin\phi}{\phi} = \frac{1}{\sqrt{2}}$, or $\boxed{\sin\phi = \frac{\phi}{\sqrt{2}}}$.

(b) Let $y_1 = \sin\phi$ and $y_2 = \frac{\phi}{\sqrt{2}}$.

A plot of $y_1$ and $y_2$ in the range $1.00 \le \phi \le \frac{\pi}{2}$ is shown to the right.

The solution to the transcendental equation is found to be $\boxed{\phi = 1.39\text{ rad}}$.

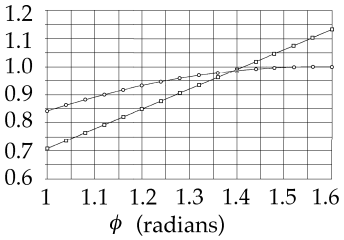

**FIG. P38.70(b)**

(c) $$\beta = \frac{2\pi a\sin\theta}{\lambda} = 2\phi$$

gives $$\sin\theta = \left(\frac{\phi}{\pi}\right)\frac{\lambda}{a} = 0.443\frac{\lambda}{a}.$$

If $\frac{\lambda}{a}$ is small, then $\theta \approx 0.443\frac{\lambda}{a}$.

This gives the half-width, measured away from the maximum at $\theta = 0$. The pattern is symmetric, so the full width is given by

$$\Delta\theta = 0.443\frac{\lambda}{a} - \left(-0.443\frac{\lambda}{a}\right) = \boxed{\frac{0.886\lambda}{a}}.$$

**P38.71**

| $\phi$ | $\sqrt{2}\sin\phi$ | |
|---|---|---|
| 1 | 1.19 | bigger than $\phi$ |
| 2 | 1.29 | smaller than $\phi$ |
| 1.5 | 1.41 | smaller |
| 1.4 | 1.394 | |
| 1.39 | 1.391 | bigger |
| 1.395 | 1.392 | |
| 1.392 | 1.391 7 | smaller |
| 1.391 5 | 1.391 54 | bigger |
| 1.391 52 | 1.391 55 | bigger |
| 1.391 6 | 1.391 568 | smaller |
| 1.391 58 | 1.391 563 | |
| 1.391 57 | 1.391 561 | |
| 1.391 56 | 1.391 558 | |
| 1.391 559 | 1.391 557 8 | |
| 1.391 558 | 1.391 557 5 | |
| 1.391 557 | 1.391 557 3 | |
| 1.391 557 4 | 1.391 557 4 | |

We get the answer to seven digits after 17 steps. Clever guessing, like using the value of $\sqrt{2}\sin\phi$ as the next guess for $\phi$, could reduce this to around 13 steps.

**P38.72** In $I = I_{\max}\left[\dfrac{\sin(\beta/2)}{\beta/2}\right]^2$ find $\dfrac{dI}{d\beta} = I_{\max}\left(\dfrac{2\sin(\beta/2)}{\beta/2}\right)\left[\dfrac{(\beta/2)\cos(\beta/2)(1/2)-\sin(\beta/2)(1/2)}{(\beta/2)^2}\right]$

and require that it be zero. The possibility $\sin\left(\dfrac{\beta}{2}\right)=0$ locates all of the minima and the central maximum, according to

$\dfrac{\beta}{2}=0,\ \pi,\ 2\pi,\ \ldots;$ $\qquad \beta=\dfrac{2\pi a\sin\theta}{\lambda}=0,\ 2\pi,\ 4\pi,\ \ldots\ ;\ a\sin\theta=0,\ \lambda,\ 2\lambda,\ \ldots.$

The side maxima are found from $\dfrac{\beta}{2}\cos\left(\dfrac{\beta}{2}\right)-\sin\left(\dfrac{\beta}{2}\right)=0$, or $\tan\left(\dfrac{\beta}{2}\right)=\dfrac{\beta}{2}$.

This has solutions $\boxed{\dfrac{\beta}{2}=4.493\,4}$, $\boxed{\dfrac{\beta}{2}=7.725\,3}$, and others, giving

(a) $\pi a\sin\theta=4.493\,4\lambda$ $\qquad \boxed{a\sin\theta=1.430\,3\lambda}$

(b) $\pi a\sin\theta=7.725\,3\lambda$ $\qquad \boxed{a\sin\theta=2.459\,0\lambda}$

**P38.73** The first minimum in the single-slit diffraction pattern occurs at

$$\sin\theta = \frac{\lambda}{a} \approx \frac{y_{\min}}{L}.$$

Thus, the slit width is given by

$$a = \frac{\lambda L}{y_{\min}}.$$

For a minimum located at $y_{\min} = 6.36\text{ mm} \pm 0.08\text{ mm}$, the width is

$$a = \frac{\left(632.8\times10^{-9}\text{ m}\right)(1.00\text{ m})}{6.36\times10^{-3}\text{ m}} = \boxed{99.5\ \mu\text{m} \pm 1\%}.$$

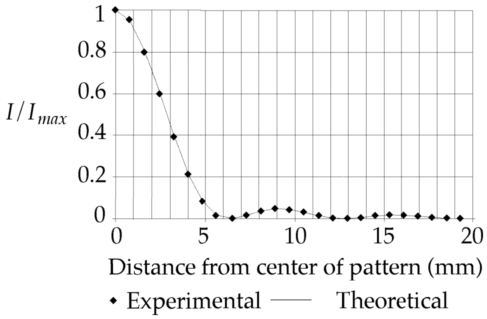

**FIG. P38.73**

## ANSWERS TO EVEN PROBLEMS

**P38.2** 547 nm

**P38.4** 91.2 cm

**P38.6** (a) 1.09 m; (b) 1.70 mm

**P38.8** see the solution

**P38.10** (a) 0°, 10.3°, 21.0°, 32.5°, 45.8°, 63.6°; (b) nine bright fringes at 0° and on either side at 10.3°, 21.0°, 32.5°, and 63.6°; (c) 1.00, 0.811, 0.405, 0.090 1, 0.032 4

**P38.12** 2.61 $\mu$m

**P38.14** 869 m

**P38.16** 0.512 m

**P38.18** 6.10 cm

**P38.20** 105 m

**P38.22** (a) 2.40 $\mu$rad; (b) 213 km

**P38.24** 514 nm

**P38.26** 1.81 $\mu$m

**P38.28** see the solution

**P38.30** 74.2 grooves/mm

**P38.32** 2

**P38.34** (a) 0.738 mm; (b) see the solution

**P38.36** 0.455 nm

**P38.38** 3

**P38.40** $\frac{3}{8}$

**P38.42** (a) 6.89 units; (b) 5.63 units

**P38.44** (a) see the solution; (b) For light confined to a plane, yes. $\left|\tan^{-1}\left(\frac{n_3}{n_2}\right) - \tan^{-1}\left(\frac{n_1}{n_2}\right)\right|$

**P38.46** see the solution

**P38.48** see the solution

**P38.50** see the solution

**P38.52** 30.5 m

**P38.54** (a) 1.50 sec; (b) 0.189 ly; (c) 10.5 sec; (d) 1.52 mm

**P38.56** see the solution

**P38.58** 11.5°

**P38.60** (a) see the solution; (b) 428 $\mu$m

**P38.62** see the solution

**P38.64** $\frac{1}{8}$

**P38.66** (a) see the solution; (b) 0.109°

**P38.68** see the solution

**P38.70** (a) see the solution; (b) $\phi = 1.39\ \text{rad}$;
(c) see the solution

**P38.72** (a) $a \sin\theta = 1.430\,3\lambda$; (b) $a \sin\theta = 2.459\,0\lambda$

# 39

# Relativity

## CHAPTER OUTLINE

39.1 The Principle of Galilean Relativity
39.2 The Michelson-Morley Experiment
39.3 Einstein's Principle of Relativity
39.4 Consequences of the Special Theory of Relativity
39.5 The Lorentz Transformation Equations
39.6 The Lorentz Velocity Transformation Equations
39.7 Relativistic Linear Momentum and the Relativistic Form of Newton's Laws
39.8 Relativistic Energy
39.9 Mass and Energy
39.10 The General Theory of Relativity

## ANSWERS TO QUESTIONS

**Q39.1** The speed of light $c$ and the speed $v$ of their relative motion.

**Q39.2** An ellipsoid. The dimension in the direction of motion would be measured to be scrunched in.

**Q39.3** No. The principle of relativity implies that nothing can travel faster than the speed of light in a *vacuum*, which is 300 Mm/s. The electron would emit light in a conical shock wave of Cerenkov radiation.

**Q39.4** The clock in orbit runs slower. No, they are not synchronized. Although they both tick at the same rate after return, a time difference has developed between the two clocks.

**Q39.5** Suppose a railroad train is moving past you. One way to measure its length is this: You mark the tracks at the cowcatcher forming the front of the moving engine at 9:00:00 AM, while your assistant marks the tracks at the back of the caboose at the same time. Then you find the distance between the marks on the tracks with a tape measure. You and your assistant must make the marks simultaneously in your frame of reference, for otherwise the motion of the train would make its length different from the distance between marks.

**Q39.6** (a) Yours does.

(b) His does.

(c) If the velocity of relative motion is constant, both observers have equally valid views.

**Q39.7** Get a *Mr. Tompkins* book by George Gamow for a wonderful fictional exploration of this question. Driving home in a hurry, you push on the gas pedal not to increase your speed by very much, but rather to make the blocks get shorter. Big Doppler shifts in wave frequencies make red lights look green as you approach them and make car horns and car radios useless. High-speed transportation is very expensive, requiring huge fuel purchases. And it is dangerous, as a speeding car can knock down a building. Having had breakfast at home, you return hungry for lunch, but you find you have missed dinner. There is a five-day delay in transmission when you watch the Olympics in Australia on live television. It takes ninety-five years for sunlight to reach Earth. We cannot see the Milky Way; the fireball of the Big Bang surrounds us at the distance of Rigel or Deneb.

**Q39.8** Nothing physically unusual. An observer riding on the clock does not think that you are *really* strange, either.

**Q39.9** By a curved line. This can be seen in the middle of Speedo's world-line in Figure 39.12, where he turns around and begins his trip home.

**Q39.10** According to $\mathbf{p} = \gamma m\mathbf{u}$, doubling the speed $u$ will make the momentum of an object increase by the factor $2\left[\frac{c^2 - u^2}{c^2 - 4u^2}\right]^{1/2}$.

**Q39.11** As the object approaches the speed of light, its kinetic energy grows without limit. It would take an infinite investment of work to accelerate the object to the speed of light.

**Q39.12** There is no upper limit on the momentum of an electron. As more energy $E$ is fed into the object without limit, its speed approaches the speed of light and its momentum approaches $\frac{E}{c}$.

**Q39.13** Recall that when a spring of force constant $k$ is compressed or stretched from its relaxed position a distance $x$, it stores elastic potential energy $U = \frac{1}{2}kx^2$. According to the special theory of relativity, any change in the total energy of the system is equivalent to a change in the mass of the system. Therefore, the mass of a compressed or stretched spring is greater than the mass of a relaxed spring by an amount $\frac{U}{c^2}$. The fractional change is typically unobservably small for a mechanical spring.

**Q39.14** You see no change in your reflection at any speed you can attain. You cannot attain the speed of light, for that would take an infinite amount of energy.

**Q39.15** Quasar light moves at three hundred million meters per second, just like the light from a firefly at rest.

**Q39.16** A photon transports energy. The relativistic equivalence of mass and energy means that is enough to give it momentum.

**Q39.17** Any physical theory must agree with experimental measurements within some domain. Newtonian mechanics agrees with experiment for objects moving slowly compared to the speed of light. Relativistic mechanics agrees with experiment for objects at all speeds. Thus the two theories must and do agree with each other for ordinary nonrelativistic objects. Both statements given in the question are formally correct, but the first is clumsily phrased. It seems to suggest that relativistic mechanics applies only to fast-moving objects.

**Q39.18** The point of intersection moves to the right. To state the problem precisely, let us assume that each of the two cards moves toward the other parallel to the long dimension of the picture, with velocity of magnitude $v$. The point of intersection moves to the right at speed $\frac{2v}{\tan\phi} = 2v\cot\phi$, where $\phi$ is the small angle between the cards. As $\phi$ approaches zero, $\cot\phi$ approaches infinity. Thus the point of intersection can move with a speed faster than $c$ if $v$ is sufficiently large and $\phi$ sufficiently small. For example, take $v = 500\ \text{m/s}$ and $\phi = 0.000\,19°$. If you are worried about holding the cards steady enough to be sure of the angle, cut the edge of one card along a curve so that the angle will necessarily be sufficiently small at some place along the edge.

Let us assume the spinning flashlight is at the center of a grain elevator, forming a circular screen of radius $R$. The linear speed of the spot on the screen is given by $v = \omega R$, where $\omega$ is the angular speed of rotation of the flashlight. With sufficiently large $\omega$ and $R$, the speed of the spot moving on the screen can exceed $c$.

*continued on next page*

Neither of these examples violates the principle of relativity. Both cases are describing a point of intersection: in the first case, the intersection of two cards and in the second case, the intersection of a light beam with a screen. A point of intersection is not made of matter so it has no mass, and hence no energy. A bug momentarily at the intersection point could yelp, take a bite out of one card, or reflect the light. None of these actions would result in communication reaching another bug so soon as the intersection point reaches him. The second bug would have to wait for sound or light to travel across the distance between the first bug and himself, to get the message.

As a child, the author used an Erector set to build a superluminal speed generator using the intersecting-cards method. Can you get a visible dot to run across a computer screen faster than light? Want'a see it again?

**Q39.19** In this case, both the relativistic and Galilean treatments would yield the same result: it is that the experimentally observed speed of one car with respect to the other is the sum of the speeds of the cars.

**Q39.20** The hotter object has more energy per molecule than the cooler one. The equivalence of energy and mass predicts that each molecule of the hotter object will, on average, have a larger mass than those in the cooler object. This implies that given the same net applied force, the cooler object would have a larger acceleration than the hotter object would experience. In a controlled experiment, the difference will likely be too small to notice.

**Q39.21** Special relativity describes inertial reference frames: that is, reference frames that are not accelerating. General relativity describes all reference frames.

**Q39.22** The downstairs clock runs more slowly because it is closer to the Earth and hence in a stronger gravitational field than the upstairs clock.

**Q39.23** The ants notice that they have a stronger sense of being pushed outward when they venture closer to the rim of the merry-go-round. If they wish, they can call this the effect of a stronger gravitational field produced by some mass concentration toward the edge of the disk. An ant named Albert figures out that the strong gravitational field makes measuring rods contract when they are near the rim of the disk. He shows that this effect precisely accounts for the discrepancy.

## SOLUTIONS TO PROBLEMS

### Section 39.1 The Principle of Galilean Relativity

**P39.1** In the rest frame,

$$p_i = m_1 v_{1i} + m_2 v_{2i} = (2\,000\text{ kg})(20.0\text{ m/s}) + (1\,500\text{ kg})(0\text{ m/s}) = 4.00\times10^4\text{ kg}\cdot\text{m/s}$$

$$p_f = (m_1 + m_2)v_f = (2\,000\text{ kg} + 1\,500\text{ kg})v_f$$

Since $p_i = p_f$, $\qquad v_f = \dfrac{4.00\times10^4\text{ kg}\cdot\text{m/s}}{2\,000\text{ kg}+1\,500\text{ kg}} = 11.429\text{ m/s}$.

In the moving frame, these velocities are all reduced by +10.0 m/s.

$$v'_{1i} = v_{1i} - v' = 20.0\text{ m/s} - (+10.0\text{ m/s}) = 10.0\text{ m/s}$$

$$v'_{2i} = v_{2i} - v' = 0\text{ m/s} - (+10.0\text{ m/s}) = -10.0\text{ m/s}$$

$$v'_f = 11.429\text{ m/s} - (+10.0\text{ m/s}) = 1.429\text{ m/s}$$

Our initial momentum is then

$$p'_i = m_1 v'_{1i} + m_2 v'_{2i} = (2\,000\text{ kg})(10.0\text{ m/s}) + (1\,500\text{ kg})(-10.0\text{ m/s}) = 5\,000\text{ kg}\cdot\text{m/s}$$

and our final momentum is

$$p'_f = (2\,000\text{ kg} + 1\,500\text{ kg})v'_f = (3\,500\text{ kg})(1.429\text{ m/s}) = 5\,000\text{ kg}\cdot\text{m/s}.$$

**P39.2** (a) $v = v_T + v_B = \boxed{60.0\ \text{m/s}}$

(b) $v = v_T - v_B = \boxed{20.0\ \text{m/s}}$

(c) $v = \sqrt{v_T^2 + v_B^2} = \sqrt{20^2 + 40^2} = \boxed{44.7\ \text{m/s}}$

**P39.3** The first observer watches some object accelerate under applied forces. Call the instantaneous velocity of the object $\mathbf{v}_1$. The second observer has constant velocity $\mathbf{v}_{21}$ relative to the first, and measures the object to have velocity $\mathbf{v}_2 = \mathbf{v}_1 - \mathbf{v}_{21}$.

The second observer measures an acceleration of $\mathbf{a}_2 = \frac{d\mathbf{v}_2}{dt} = \frac{d\mathbf{v}_1}{dt}$.

This is the same as that measured by the first observer. In this nonrelativistic case, they measure the same forces as well. Thus, the second observer also confirms that $\sum \mathbf{F} = m\mathbf{a}$.

**P39.4** The laboratory observer notes Newton's second law to hold: $\mathbf{F}_1 = m\mathbf{a}_1$

(where the subscript 1 implies the measurement was made in the laboratory frame of reference). The observer in the accelerating frame measures the acceleration of the mass as $\mathbf{a}_2 = \mathbf{a}_1 - \mathbf{a}'$

(where the subscript 2 implies the measurement was made in the accelerating frame of reference, and the primed acceleration term is the acceleration of the accelerated frame with respect to the laboratory frame of reference). If Newton's second law held for the accelerating frame, that observer would then find valid the relation

$\mathbf{F}_2 = m\mathbf{a}_2$ or $\mathbf{F}_1 = m\mathbf{a}_2$

(since $\mathbf{F}_1 = \mathbf{F}_2$ and the mass is unchanged in each). But, instead, the accelerating frame observer will find that $\mathbf{F}_2 = m\mathbf{a}_2 - m\mathbf{a}'$ which is *not* Newton's second law.

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## Section 39.2 **The Michelson-Morley Experiment**

## Section 39.3 **Einstein's Principle of Relativity**

## Section 39.4 **Consequences of the Special Theory of Relativity**

**P39.5** $L = L_p\sqrt{1 - \frac{v^2}{c^2}}$ $\quad v = c\sqrt{1 - \left(\frac{L}{L_p}\right)^2}$

Taking $L = \frac{L_p}{2}$ where $L_p = 1.00$ m gives $\quad v = c\sqrt{1 - \left(\frac{L_p/2}{L_p}\right)^2} = c\sqrt{1 - \frac{1}{4}} = \boxed{0.866c}$.

**P39.6** $\Delta t = \frac{\Delta t_p}{\left[1 - (v/c)^2\right]^{1/2}}$ so $\quad v = c\left[1 - \left(\frac{\Delta t_p}{\Delta t}\right)^2\right]^{1/2}$.

For $\Delta t = 2\Delta t_p$ $\quad v = c\left[1 - \left(\frac{\Delta t_p}{2\Delta t_p}\right)^2\right]^{1/2} = c\left[1 - \frac{1}{4}\right]^{1/2} = \boxed{0.866c}$.

**P39.7** (a) $$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} = \frac{1}{\sqrt{1-(0.500)^2}} = \frac{2}{\sqrt{3}}$$

The time interval between pulses as measured by the Earth observer is

$$\Delta t = \gamma \Delta t_p = \frac{2}{\sqrt{3}}\left(\frac{60.0\text{ s}}{75.0}\right) = 0.924\text{ s}.$$

Thus, the Earth observer records a pulse rate of $\frac{60.0\text{ s/min}}{0.924\text{ s}} = \boxed{64.9/\text{min}}$.

(b) At a relative speed $v = 0.990c$, the relativistic factor $\gamma$ increases to 7.09 and the pulse rate recorded by the Earth observer decreases to $\boxed{10.6/\text{min}}$. That is, the life span of the astronaut (reckoned by the duration of the total number of his heartbeats) is much longer as measured by an Earth clock than by a clock aboard the space vehicle.

***P39.8** (a) The $0.8c$ and the 20 ly are measured in the Earth frame,

so in this frame, $$\Delta t = \frac{x}{v} = \frac{20\text{ ly}}{0.8c} = \frac{20\text{ ly}}{0.8c}\,\frac{1c}{1\text{ ly/yr}} = \boxed{25.0\text{ yr}}.$$

(b) We see a clock on the meteoroid moving, so we do not measure proper time; that clock measures proper time.

$$\Delta t = \gamma\Delta t_p: \qquad \Delta t_p = \frac{\Delta t}{\gamma} = \frac{25.0\text{ yr}}{1\big/\sqrt{1-v^2/c^2}} = 25.0\text{ yr}\sqrt{1-0.8^2} = 25.0\text{ yr}(0.6) = \boxed{15.0\text{ yr}}$$

(c) Method one: We measure the 20 ly on a stick stationary in our frame, so it is proper length. The tourist measures it to be contracted to

$$L = \frac{L_p}{\gamma} = \frac{20\text{ ly}}{1\big/\sqrt{1-0.8^2}} = \frac{20\text{ ly}}{1.667} = \boxed{12.0\text{ ly}}.$$

Method two: The tourist sees the Earth approaching at $0.8c$

$$(0.8\text{ ly/yr})(15\text{ yr}) = \boxed{12.0\text{ ly}}.$$

Not only do distances and times differ between Earth and meteoroid reference frames, but within the Earth frame apparent distances differ from actual distances. As we have interpreted it, the 20-lightyear actual distance from the Earth to the meteoroid at the time of discovery must be a calculated result, different from the distance measured directly. Because of the finite maximum speed of information transfer, the astronomer sees the meteoroid as it was years previously, when it was much farther away. Call its apparent distance $d$. The time required for light to reach us from the newly-visible meteoroid is the lookback time $t = \frac{d}{c}$.

The astronomer calculates that the meteoroid has approached to be 20 ly away as it moved with constant velocity throughout the lookback time. We can work backwards to reconstruct her calculation:

$$d = 20\text{ ly} + 0.8ct = 20\text{ ly} + \frac{0.8cd}{c}$$

$$0.2d = 20\text{ ly}$$

$$d = 100\text{ ly}$$

Thus in terms of direct observation, the meteoroid we see covers 100 ly in only 25 years. Such an apparent superluminal velocity is actually observed for some jets of material emanating from quasars, because they happen to be pointed nearly toward the Earth. If we can watch events unfold on the meteoroid, we see them slowed by relativistic time dilation, but also greatly speeded up by the Doppler effect.

**P39.9** $\Delta t = \gamma \Delta t_p = \dfrac{\Delta t_p}{\sqrt{1 - v^2/c^2}}$ so $\Delta t_p = \left(\sqrt{1 - \dfrac{v^2}{c^2}}\right)\Delta t \cong \left(1 - \dfrac{v^2}{2c^2}\right)\Delta t$

and $\Delta t - \Delta t_p = \left(\dfrac{v^2}{2c^2}\right)\Delta t$.

If $v = 1\,000 \text{ km/h} = \dfrac{1.00 \times 10^6 \text{ m}}{3\,600 \text{ s}} = 277.8 \text{ m/s}$

then $\dfrac{v}{c} = 9.26 \times 10^{-7}$

and $\left(\Delta t - \Delta t_p\right) = \left(4.28 \times 10^{-13}\right)(3\,600 \text{ s}) = 1.54 \times 10^{-9} \text{ s} = \boxed{1.54 \text{ ns}}$.

**P39.10** For $\dfrac{v}{c} = 0.990$, $\gamma = 7.09$

(a) The muon's lifetime as measured in the Earth's rest frame is

$$\Delta t = \frac{4.60 \text{ km}}{0.990c}$$

and the lifetime measured in the muon's rest frame is

$$\Delta t_p = \frac{\Delta t}{\gamma} = \frac{1}{7.09}\left[\frac{4.60 \times 10^3 \text{ m}}{0.990\left(3.00 \times 10^8 \text{ m/s}\right)}\right] = \boxed{2.18 \ \mu\text{s}}.$$

(b) $L = L_p\sqrt{1 - \left(\dfrac{v}{c}\right)^2} = \dfrac{L_p}{\gamma} = \dfrac{4.60 \times 10^3 \text{ m}}{7.09} = \boxed{649 \text{ m}}$

**P39.11** The spaceship is measured by the Earth observer to be length-contracted to

$L = L_p\sqrt{1 - \dfrac{v^2}{c^2}}$ or $L^2 = L_p^2\left(1 - \dfrac{v^2}{c^2}\right)$.

Also, the contracted length is related to the time required to pass overhead by:

$L = vt$ or $L^2 = v^2t^2 = \dfrac{v^2}{c^2}(ct)^2$.

Equating these two expressions gives $L_p^2 - L_p^2\dfrac{v^2}{c^2} = (ct)^2\dfrac{v^2}{c^2}$

or $\left[L_p^2 + (ct)^2\right]\dfrac{v^2}{c^2} = L_p^2$.

Using the given values: $L_p = 300 \text{ m}$ and $t = 7.50 \times 10^{-7} \text{ s}$

this becomes $\left(1.41 \times 10^5 \text{ m}^2\right)\dfrac{v^2}{c^2} = 9.00 \times 10^4 \text{ m}^2$

giving $v = \boxed{0.800c}$.

**P39.12** (a) The spaceship is measured by Earth observers to be of length $L$, where

$$L = L_p\sqrt{1-\frac{v^2}{c^2}} \qquad \text{and} \qquad L = v\Delta t$$

$$v\Delta t = L_p\sqrt{1-\frac{v^2}{c^2}} \qquad \text{and} \qquad v^2\Delta t^2 = L_p^2\left(1-\frac{v^2}{c^2}\right).$$

Solving for $v$, $\quad v^2\left(\Delta t^2 + \frac{L_p^2}{c^2}\right) = L_p^2 \qquad \boxed{v = \frac{cL_p}{\sqrt{c^2\Delta t^2 + L_p^2}}}$.

(b) The tanks move nonrelativistically, so we have $v = \frac{300\text{ m}}{75\text{ s}} = \boxed{4.00\text{ m/s}}$.

(c) For the data in problem 11,

$$v = \frac{c(300\text{ m})}{\sqrt{\left(3\times10^8\text{ m/s}\right)^2\left(0.75\times10^{-6}\text{ s}\right)^2 + (300\text{ m})^2}} = \frac{c(300\text{ m})}{\sqrt{225^2+300^2}\text{ m}} = 0.800c$$

in agreement with problem 11. For the data in part (b),

$$v = \frac{c(300\text{ m})}{\sqrt{\left(3\times10^8\text{ m/s}\right)^2(75\text{ s})^2 + (300\text{ m})^2}} = \frac{c(300\text{ m})}{\sqrt{\left(2.25\times10^{10}\right)^2+300^2}\text{ m}} = 1.33\times10^{-8}c = 4.00\text{ m/s}$$

in agreement with part (b).

**P39.13** We find Cooper's speed: $\quad \frac{GMm}{r^2} = \frac{mv^2}{r}$.

Solving, $\quad v = \left[\frac{GM}{(R+h)}\right]^{1/2} = \left[\frac{\left(6.67\times10^{-11}\right)\left(5.98\times10^{24}\right)}{\left(6.37\times10^6 + 0.160\times10^6\right)}\right]^{1/2} = 7.82\text{ km/s}$.

Then the time period of one orbit, $\quad T = \frac{2\pi(R+h)}{v} = \frac{2\pi\left(6.53\times10^6\right)}{7.82\times10^3} = 5.25\times10^3\text{ s}$.

(a) The time difference for 22 orbits is $\Delta t - \Delta t_p = (\gamma-1)\Delta t_p = \left[\left(1-\frac{v^2}{c^2}\right)^{-1/2} - 1\right](22T)$

$$\Delta t - \Delta t_p \approx \left(1+\frac{1}{2}\frac{v^2}{c^2}-1\right)(22T) = \frac{1}{2}\left(\frac{7.82\times10^3\text{ m/s}}{3\times10^8\text{ m/s}}\right)^2 22\left(5.25\times10^3\text{ s}\right) = \boxed{39.2\ \mu\text{s}}.$$

(b) For one orbit, $\Delta t - \Delta t_p = \frac{39.2\ \mu\text{s}}{22} = 1.78\ \mu\text{s}$. The press report is $\boxed{\text{accurate to one digit}}$.

**P39.14** $\gamma = \frac{1}{\sqrt{1-\left(v^2/c^2\right)}} = 1.01 \qquad$ so $\qquad \boxed{v = 0.140c}$

**P39.15** (a) Since your ship is identical to his, and you are at rest with respect to your own ship, its length is $\boxed{20.0\text{ m}}$.

(b) His ship is in motion relative to you, so you measure its length contracted to $\boxed{19.0\text{ m}}$.

(c) We have $L = L_p\sqrt{1-\dfrac{v^2}{c^2}}$

from which $\dfrac{L}{L_p} = \dfrac{19.0\text{ m}}{20.0\text{ m}} = 0.950 = \sqrt{1-\dfrac{v^2}{c^2}}$ and $\boxed{v = 0.312c}$.

***P39.16** In the Earth frame, Speedo's trip lasts for a time

$$\Delta t = \frac{\Delta x}{v} = \frac{20.0\text{ ly}}{0.950\text{ ly/yr}} = 21.05\text{ yr}.$$

Speedo's age advances only by the proper time interval

$$\Delta t_p = \frac{\Delta t}{\gamma} = 21.05\text{ yr}\sqrt{1-0.95^2} = 6.574\text{ yr during his trip.}$$

Similarly for Goslo,

$$\Delta t_p = \frac{\Delta x}{v}\sqrt{1-\frac{v^2}{c^2}} = \frac{20.0\text{ ly}}{0.750\text{ ly/yr}}\sqrt{1-0.75^2} = 17.64\text{ yr}.$$

While Speedo has landed on Planet X and is waiting for his brother, he ages by

$$\frac{20.0\text{ ly}}{0.750\text{ ly/yr}} - \frac{20.0\text{ ly}}{0.950\text{ ly/yr}} = 5.614\text{ yr}.$$

Then $\boxed{\text{Goslo}}$ ends up older by $17.64\text{ yr} - (6.574\text{ yr} + 5.614\text{ yr}) = \boxed{5.45\text{ yr}}$.

**P39.17** (a) $\Delta t = \gamma\Delta t_p = \dfrac{\Delta t_p}{\sqrt{1-(v/c)^2}} = \dfrac{15.0\text{ yr}}{\sqrt{1-(0.700)^2}} = \boxed{21.0\text{ yr}}$

(b) $d = v(\Delta t) = [0.700c](21.0\text{ yr}) = [(0.700)(1.00\text{ ly/yr})](21.0\text{ yr}) = \boxed{14.7\text{ ly}}$

(c) The astronauts see Earth flying out the back window at $0.700c$:

$d = v(\Delta t_p) = [0.700c](15.0\text{ yr}) = [(0.700)(1.00\text{ ly/yr})](15.0\text{ yr}) = \boxed{10.5\text{ ly}}$

(d) Mission control gets signals for 21.0 yr while the battery is operating, and then for 14.7 years after the battery stops powering the transmitter, 14.7 ly away:

$21.0\text{ yr} + 14.7\text{ yr} = \boxed{35.7\text{ yr}}$

**P39.18** The orbital speed of the Earth is as described by $\sum F = ma$ : $\dfrac{Gm_S m_E}{r^2} = \dfrac{m_E v^2}{r}$

$$v = \sqrt{\frac{Gm_S}{r}} = \sqrt{\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.99\times10^{30}\ \text{kg}\right)}{1.496\times10^{11}\ \text{m}}} = 2.98\times10^{4}\ \text{m/s}\,.$$

The maximum frequency received by the extraterrestrials is

$$f_{\text{obs}} = f_{\text{source}}\sqrt{\frac{1+v/c}{1-v/c}} = \left(57.0\times10^{6}\ \text{Hz}\right)\sqrt{\frac{1+\left(2.98\times10^{4}\ \text{m/s}\right)/\left(3.00\times10^{8}\ \text{m/s}\right)}{1-\left(2.98\times10^{4}\ \text{m/s}\right)/\left(3.00\times10^{8}\ \text{m/s}\right)}} = 57.005\,66\times10^{6}\ \text{Hz}\,.$$

The minimum frequency received is

$$f_{\text{obs}} = f_{\text{source}}\sqrt{\frac{1-v/c}{1+v/c}} = \left(57.0\times10^{6}\ \text{Hz}\right)\sqrt{\frac{1-\left(2.98\times10^{4}\ \text{m/s}\right)/\left(3.00\times10^{8}\ \text{m/s}\right)}{1+\left(2.98\times10^{4}\ \text{m/s}\right)/\left(3.00\times10^{8}\ \text{m/s}\right)}} = 56.994\,34\times10^{6}\ \text{Hz}\,.$$

The difference, which lets them figure out the speed of our planet, is

$$(57.005\,66 - 56.994\,34)\times10^{6}\ \text{Hz} = \boxed{1.13\times10^{4}\ \text{Hz}}\,.$$

**P39.19** (a) Let $f_c$ be the frequency as seen by the car. Thus, $f_c = f_{\text{source}}\sqrt{\dfrac{c+v}{c-v}}$

and, if $f$ is the frequency of the reflected wave, $f = f_c\sqrt{\dfrac{c+v}{c-v}}$.

Combining gives $\boxed{f = f_{\text{source}}\dfrac{(c+v)}{(c-v)}}$.

(b) Using the above result, $f(c-v) = f_{\text{source}}(c+v)$

which gives $(f - f_{\text{source}})c = (f + f_{\text{source}})v \approx 2f_{\text{source}}v$.

The beat frequency is then $f_{\text{beat}} = f - f_{\text{source}} = \dfrac{2f_{\text{source}}v}{c} = \boxed{\dfrac{2v}{\lambda}}$.

(c) $$f_{\text{beat}} = \frac{(2)(30.0\ \text{m/s})\left(10.0\times10^{9}\ \text{Hz}\right)}{3.00\times10^{8}\ \text{m/s}} = \frac{(2)(30.0\ \text{m/s})}{(0.030\,0\ \text{m})} = 2\,000\ \text{Hz} = \boxed{2.00\ \text{kHz}}$$

$$\lambda = \frac{c}{f_{\text{source}}} = \frac{3.00\times10^{8}\ \text{m/s}}{10.0\times10^{9}\ \text{Hz}} = 3.00\ \text{cm}$$

(d) $v = \dfrac{f_{\text{beat}}\lambda}{2}$ so $\Delta v = \dfrac{\Delta f_{\text{beat}}\lambda}{2} = \dfrac{(5\ \text{Hz})(0.030\,0\ \text{m})}{2} = \boxed{0.075\,0\ \text{m/s} \approx 0.2\ \text{mi/h}}$

**P39.20** (a) When the source moves away from an observer, the observed frequency is

$$f_{\text{obs}} = f_{\text{source}}\left(\frac{c-v_s}{c+v_s}\right)^{1/2} \quad \text{where } v_s = v_{\text{source}}.$$

When $v_s << c$, the binomial expansion gives

$$\left(\frac{c-v_s}{c+v_s}\right)^{1/2} = \left[1-\left(\frac{v_s}{c}\right)\right]^{1/2}\left[1+\left(\frac{v_s}{c}\right)\right]^{-1/2} \approx \left(1-\frac{v_s}{2c}\right)\left(1-\frac{v_s}{2c}\right) \approx \left(1-\frac{v_s}{c}\right).$$

So, $$f_{\text{obs}} \approx f_{\text{source}}\left(1-\frac{v_s}{c}\right).$$

The observed wavelength is found from $c = \lambda_{\text{obs}} f_{\text{obs}} = \lambda f_{\text{source}}$:

$$\lambda_{\text{obs}} = \frac{\lambda f_{\text{source}}}{f_{\text{obs}}} \approx \frac{\lambda f_{\text{source}}}{f_{\text{source}}(1-v_s/c)} = \frac{\lambda}{1-v_s/c}$$

$$\Delta\lambda = \lambda_{\text{obs}} - \lambda = \lambda\left(\frac{1}{1-v_s/c}-1\right) = \lambda\left(\frac{v_s/c}{1-v_s/c}\right)$$

Since $1-\frac{v_s}{c} \approx 1$, $$\boxed{\frac{\Delta\lambda}{\lambda} \approx \frac{v_{\text{source}}}{c}}.$$

(b) $$v_{\text{source}} = c\left(\frac{\Delta\lambda}{\lambda}\right) = c\left(\frac{20.0 \text{ nm}}{397 \text{ nm}}\right) = \boxed{0.050\,4c}$$

***P39.21** For the light as observed

$$f_{\text{obs}} = \frac{c}{\lambda_{\text{obs}}} = \sqrt{\frac{1+v/c}{1-v/c}}\, f_{\text{source}} = \sqrt{\frac{1+v/c}{1-v/c}}\, \frac{c}{\lambda_{\text{source}}}$$

$$\sqrt{\frac{1+v/c}{1-v/c}} = \frac{\lambda_{\text{source}}}{\lambda_{\text{obs}}} = \frac{650 \text{ nm}}{520 \text{ nm}} \qquad \frac{1+v/c}{1-v/c} = 1.25^2 = 1.562$$

$$1+\frac{v}{c} = 1.562 - 1.562\frac{v}{c} \qquad \frac{v}{c} = \frac{0.562}{2.562} = 0.220$$

$$v = \boxed{0.220c} = 6.59\times10^7 \text{ m/s}$$

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## Section 39.5 **The Lorentz Transformation Equations**

***P39.22** Let Suzanne be fixed in reference from S and see the two light-emission events with coordinates $x_1 = 0$, $t_1 = 0$, $x_2 = 0$, $t_2 = 3\ \mu\text{s}$. Let Mark be fixed in reference frame S′ and give the events coordinate $x_1' = 0$, $t_1' = 0$, $t_2' = 9\ \mu\text{s}$.

(a) Then we have

$$t_2' = \gamma\left(t_2 - \frac{v}{c^2}x_2\right)$$

$$9\ \mu\text{s} = \frac{1}{\sqrt{1-v^2/c^2}}(3\ \mu\text{s} - 0) \qquad \sqrt{1-\frac{v^2}{c^2}} = \frac{1}{3}$$

$$\frac{v^2}{c^2} = \frac{8}{9} \qquad \boxed{v = 0.943c}$$

(b) $$x_2' = \gamma(x_2 - vt_2) = 3\left(0 - 0.943c \times 3\times10^{-6}\text{ s}\right)\left(\frac{3\times10^8 \text{ m/s}}{c}\right) = \boxed{2.55\times10^3 \text{ m}}$$

**P39.23** $$\gamma = \frac{1}{\sqrt{1-v^2/c^2}} = \frac{1}{\sqrt{1-0.995^2}} = 10.0$$

We are also given: $L_1 = 2.00\text{ m}$, and $\theta = 30.0°$ (both measured in a reference frame moving relative to the rod).

Thus, $L_{1x} = L_1\cos\theta_1 = (2.00\text{ m})(0.867) = 1.73\text{ m}$

and $L_{1y} = L_1\sin\theta_1 = (2.00\text{ m})(0.500) = 1.00\text{ m}$

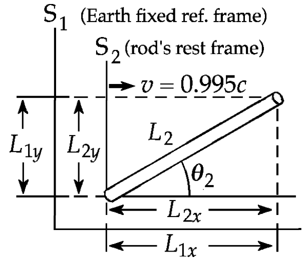

FIG. P39.23

$L_{2x}$ is a proper length, related to $L_{1x}$ by $L_{1x} = \dfrac{L_{2x}}{\gamma}$.

Therefore, $L_{2x} = 10.0L_{1x} = 17.3\text{ m}$

and $L_{2y} = L_{1y} = 1.00\text{ m}$.

(Lengths perpendicular to the motion are unchanged).

(a) $L_2 = \sqrt{(L_{2x})^2 + (L_{2y})^2}$ gives $\boxed{L_2 = 17.4\text{ m}}$

(b) $\theta_2 = \tan^{-1}\dfrac{L_{2y}}{L_{2x}}$ gives $\boxed{\theta_2 = 3.30°}$

***P39.24** Einstein's reasoning about lightning striking the ends of a train shows that the moving observer sees the event toward which she is moving, event B, as occurring first. The *S*-frame coordinates of the events we may take as $(x=0,\ y=0,\ z=0,\ t=0)$ and $(x=100\text{ m},\ y=0,\ z=0,\ t=0)$. Then the coordinates in $S'$ are given by the Lorentz transformation. Event A is at $(x'=0,\ y'=0,\ z'=0,\ t'=0)$. The time of event B is

$$t' = \gamma\left(t - \frac{v}{c^2}x\right) = \frac{1}{\sqrt{1-0.8^2}}\left(0 - \frac{0.8c}{c^2}(100\text{ m})\right) = 1.667\left(-\frac{80\text{ m}}{3\times10^8\text{ m/s}}\right) = -4.44\times10^{-7}\text{ s}.$$

The time elapsing before A occurs is $\boxed{444\text{ ns}}$.

**P39.25** (a) From the Lorentz transformation, the separations between the blue-light and red-light events are described by

$$\Delta x' = \gamma(\Delta x - v\Delta t) \qquad 0 = \gamma\left[2.00\text{ m} - v\left(8.00\times10^{-9}\text{ s}\right)\right]$$

$$v = \frac{2.00\text{ m}}{8.00\times10^{-9}\text{ s}} = \boxed{2.50\times10^8\text{ m/s}} \qquad \gamma = \frac{1}{\sqrt{1-\left(2.50\times10^8\text{ m/s}\right)^2\Big/\left(3.00\times10^8\text{ m/s}\right)^2}} = 1.81.$$

(b) Again from the Lorentz transformation, $x' = \gamma(x - vt)$:

$$x' = 1.81\left[3.00\text{ m} - \left(2.50\times10^8\text{ m/s}\right)\left(1.00\times10^{-9}\text{ s}\right)\right]$$

$$x' = \boxed{4.97\text{ m}}.$$

(c) $t' = \gamma\left(t - \dfrac{v}{c^2}x\right)$:

$$t' = 1.81\left[1.00\times10^{-9}\text{ s} - \frac{\left(2.50\times10^8\text{ m/s}\right)}{\left(3.00\times10^8\text{ m/s}\right)^2}(3.00\text{ m})\right]$$

$$t' = \boxed{-1.33\times10^{-8}\text{ s}}$$

## Section 39.6 **The Lorentz Velocity Transformation Equations**

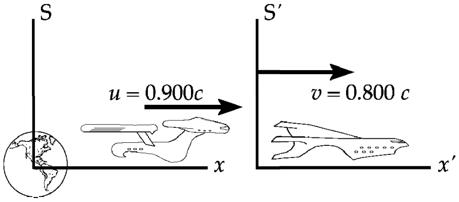

**FIG. P39.26**

**P39.26** $u_x =$ Enterprise velocity

$v =$ Klingon velocity

From Equation 39.16

$$u'_x = \frac{u_x - v}{1 - u_x v/c^2} = \frac{0.900c - 0.800c}{1-(0.900)(0.800)} = \boxed{0.357c}\,.$$

**P39.27**

$$u'_x = \frac{u_x - v}{1 - u_x v/c^2} = \frac{-0.750c - 0.750c}{1-(-0.750)(0.750)} = \boxed{-0.960c}$$

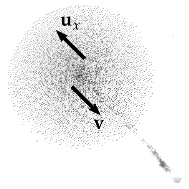

**FIG. P39.27**

***P39.28** Let frame $S$ be the Earth frame of reference. Then $v = -0.7c$.

The components of the velocity of the first spacecraft are $u_x = (0.6c)\cos 50° = 0.386c$

and $u_y = (0.6c)\sin 50° = 0.459c$.

As measured from the $S'$ frame of the second spacecraft,

$$u'_x = \frac{u_x - v}{1 - u_x v/c^2} = \frac{0.386c - (-0.7c)}{1 - \left[(0.386c)(-0.7c)/c^2\right]} = \frac{1.086c}{1.27} = 0.855c$$

$$u'_y = \frac{u_y}{\gamma\left(1 - u_x v/c^2\right)} = \frac{0.459c\sqrt{1-(0.7)^2}}{1-(0.386)(-0.7)} = \frac{0.459c(0.714)}{1.27} = 0.258c$$

The magnitude of $\mathbf{u}'$ is $\sqrt{(0.855c)^2 + (0.285c)^2} = \boxed{0.893c}$

and its direction is at $\tan^{-1}\dfrac{0.258c}{0.855c} = \boxed{16.8° \text{ above the } x'\text{-axis}}$.

## Section 39.7 **Relativistic Linear Momentum and the Relativistic Form of Newton's Laws**

**P39.29** (a) $p = \gamma mu$; for an electron moving at 0.010 0$c$,

$$\gamma = \frac{1}{\sqrt{1-(u/c)^2}} = \frac{1}{\sqrt{1-(0.010\,0)^2}} = 1.000\,05 \approx 1.00.$$

Thus,

$$p = 1.00\left(9.11\times10^{-31}\text{ kg}\right)(0.010\,0)\left(3.00\times10^{8}\text{ m/s}\right)$$

$$p = \boxed{2.73\times10^{-24}\text{ kg}\cdot\text{m/s}}\,.$$

(b) Following the same steps as used in part (a),
we find at $0.500c$, $\gamma = 1.15$ and $p = \boxed{1.58\times10^{-22}\text{ kg}\cdot\text{m/s}}$.

(c) At $0.900c$, $\gamma = 2.29$ and $p = \boxed{5.64\times10^{-22}\text{ kg}\cdot\text{m/s}}$.

**P39.30** Using the relativistic form, $p=\dfrac{mu}{\sqrt{1-(u/c)^2}}=\gamma mu$

we find the difference $\Delta p$ from the classical momentum, $mu$: $\Delta p=\gamma mu-mu=(\gamma-1)mu$.

(a) The difference is 1.00% when $(\gamma-1)mu=0.010\,0\gamma mu$: $\gamma=\dfrac{1}{0.990}=\dfrac{1}{\sqrt{1-(u/c)^2}}$

thus $1-\left(\dfrac{u}{c}\right)^2=(0.990)^2$, and $u=\boxed{0.141c}$.

(b) The difference is 10.0% when $(\gamma-1)mu=0.100\gamma mu$: $\gamma=\dfrac{1}{0.900}=\dfrac{1}{\sqrt{1-(u/c)^2}}$

thus $1-\left(\dfrac{u}{c}\right)^2=(0.900)^2$ and $u=\boxed{0.436c}$.

**P39.31** $\dfrac{p-mu}{mu}=\dfrac{\gamma mu-mu}{mu}=\gamma-1$: $\gamma-1=\dfrac{1}{\sqrt{1-(u/c)^2}}-1\approx1+\dfrac{1}{2}\left(\dfrac{u}{c}\right)^2-1=\dfrac{1}{2}\left(\dfrac{u}{c}\right)^2$

$$\frac{p-mu}{mu}=\frac{1}{2}\left(\frac{90.0\text{ m/s}}{3.00\times10^8\text{ m/s}}\right)^2=\boxed{4.50\times10^{-14}}$$

**P39.32** $p=\dfrac{mu}{\sqrt{1-(u/c)^2}}$ becomes $1-\dfrac{u^2}{c^2}=\dfrac{m^2u^2}{p^2}$

which gives: $1=u^2\left(\dfrac{m^2}{p^2}+\dfrac{1}{c^2}\right)$

or $c^2=u^2\left(\dfrac{m^2c^2}{p^2}+1\right)$ and $\boxed{u=\dfrac{c}{\sqrt{\left(m^2c^2/p^2\right)+1}}}$.

**P39.33** Relativistic momentum of the system of fragments must be conserved. For total momentum to be zero after as it was before, we must have, with subscript 2 referring to the heavier fragment, and subscript 1 to the lighter, $p_2=p_1$

or $$\gamma_2m_2u_2=\gamma_1m_1u_1=\frac{2.50\times10^{-28}\text{ kg}}{\sqrt{1-(0.893)^2}}\times(0.893c)$$

or $$\frac{\left(1.67\times10^{-27}\text{ kg}\right)u_2}{\sqrt{1-(u_2/c)^2}}=\left(4.960\times10^{-28}\text{ kg}\right)c.$$

Proceeding to solve, we find $$\left(\frac{1.67\times10^{-27}}{4.960\times10^{-28}}\frac{u_2}{c}\right)^2=1-\frac{u_2^2}{c^2}$$

$$12.3\frac{u_2^2}{c^2}=1\text{ and }u_2=\boxed{0.285c}.$$

## Section 39.8 **Relativistic Energy**

**P39.34** $\Delta E = (\gamma_1 - \gamma_2)mc^2$

For an electron, $mc^2 = 0.511\text{ MeV}$

(a) $$\Delta E = \left(\sqrt{\frac{1}{(1-0.810)}} - \sqrt{\frac{1}{(1-0.250)}}\right)mc^2 = \boxed{0.582\text{ MeV}}$$

(b) $$\Delta E = \left(\sqrt{\frac{1}{1-(0.990)^2}} - \sqrt{\frac{1}{1-0.810}}\right)mc^2 = \boxed{2.45\text{ MeV}}$$

**P39.35** $$\sum W = K_f - K_i = \left(\frac{1}{\sqrt{1-(v_f/c)^2}} - 1\right)mc^2 - \left(\frac{1}{\sqrt{1-(v_i/c)^2}}\right)mc^2$$

or $$\sum W = \left(\frac{1}{\sqrt{1-(v_f/c)^2}} - \frac{1}{\sqrt{1-(v_i/c)^2}}\right)mc^2$$

(a) $$\sum W = \left(\frac{1}{\sqrt{1-(0.750)^2}} - \frac{1}{\sqrt{1-(0.500)^2}}\right)\left(1.673\times10^{-27}\text{ kg}\right)\left(2.998\times10^{8}\text{ m/s}\right)^2$$

$$\sum W = \boxed{5.37\times10^{-11}\text{ J}}$$

(b) $$\sum W = \left(\frac{1}{\sqrt{1-(0.995)^2}} - \frac{1}{\sqrt{1-(0.500)^2}}\right)\left(1.673\times10^{-27}\text{ kg}\right)\left(2.998\times10^{8}\text{ m/s}\right)^2$$

$$\sum W = \boxed{1.33\times10^{-9}\text{ J}}$$

**P39.36** The relativistic kinetic energy of an object of mass $m$ and speed $u$ is $K_r = \left(\frac{1}{\sqrt{1-u^2/c^2}} - 1\right)mc^2$.

For $u = 0.100c$, $$K_r = \left(\frac{1}{\sqrt{1-0.0100}} - 1\right)mc^2 = 0.005\,038mc^2.$$

The classical equation $K_c = \frac{1}{2}mu^2$ gives $$K_c = \frac{1}{2}m(0.100c)^2 = 0.005\,000mc^2$$

different by $$\frac{0.005\,038 - 0.005\,000}{0.005\,038} = 0.751\%.$$

For still smaller speeds the agreement will be still better.

**P39.37** $E=\gamma mc^2=2mc^2$ or $\gamma=2$.

Thus, $\frac{u}{c}=\sqrt{1-\left(\frac{1}{\gamma}\right)^2}=\frac{\sqrt{3}}{2}$ or $u=\frac{c\sqrt{3}}{2}$.

The momentum is then $p=\gamma mu=2m\left(\frac{c\sqrt{3}}{2}\right)=\left(\frac{mc^2}{c}\right)\sqrt{3}$

$$p=\left(\frac{938.3\text{ MeV}}{c}\right)\sqrt{3}=\boxed{1.63\times 10^3\ \frac{\text{MeV}}{c}}.$$

**P39.38** (a) Using the classical equation, $K=\frac{1}{2}mv^2=\frac{1}{2}(78.0\text{ kg})\left(1.06\times 10^5\text{ m/s}\right)^2=\boxed{4.38\times 10^{11}\text{ J}}$.

(b) Using the relativistic equation, $K=\left(\frac{1}{\sqrt{1-(v/c)^2}}-1\right)mc^2$.

$$K=\left[\frac{1}{\sqrt{1-\left(1.06\times 10^5/2.998\times 10^8\right)^2}}-1\right](78.0\text{ kg})\left(2.998\times 10^8\text{ m/s}\right)^2=\boxed{4.38\times 10^{11}\text{ J}}$$

When $\frac{v}{c}<<1$, the binomial series expansion gives $\left[1-\left(\frac{v}{c}\right)^2\right]^{-1/2}\approx 1+\frac{1}{2}\left(\frac{v}{c}\right)^2$.

Thus, $\left[1-\left(\frac{v}{c}\right)^2\right]^{-1/2}-1\approx\frac{1}{2}\left(\frac{v}{c}\right)^2$.

and the relativistic expression for kinetic energy becomes $K\approx\frac{1}{2}\left(\frac{v}{c}\right)^2 mc^2=\frac{1}{2}mv^2$. That is, in the limit of speeds much smaller than the speed of light, the relativistic and classical expressions yield the same results.

**P39.39** (a) $E_R=mc^2=\left(1.67\times 10^{-27}\text{ kg}\right)\left(2.998\times 10^8\text{ m/s}\right)^2=1.50\times 10^{-10}\text{ J}=\boxed{938\text{ MeV}}$

(b) $E=\gamma mc^2=\frac{1.50\times 10^{-10}\text{ J}}{\left[1-(0.950c/c)^2\right]^{1/2}}=4.81\times 10^{-10}\text{ J}=\boxed{3.00\times 10^3\text{ MeV}}$

(c) $K=E-mc^2=4.81\times 10^{-10}\text{ J}-1.50\times 10^{-10}\text{ J}=3.31\times 10^{-10}\text{ J}=\boxed{2.07\times 10^3\text{ MeV}}$

**P39.40** The relativistic density is

$$\frac{E_R}{c^2V}=\frac{mc^2}{c^2V}=\frac{m}{V}=\frac{m}{\left(L_p\right)\left(L_p\right)\left[L_p\sqrt{1-(u/c)^2}\right]}=\frac{8.00\text{ g}}{(1.00\text{ cm})^3\sqrt{1-(0.900)^2}}=\boxed{18.4\text{ g/cm}^3}.$$

**P39.41** We must conserve both energy and relativistic momentum of the system of fragments. With subscript 1 referring to the $0.868c$ particle and subscript 2 to the $0.987c$ particle,

$$\gamma_1 = \frac{1}{\sqrt{1-(0.868)^2}} = 2.01 \text{ and } \gamma_2 = \frac{1}{\sqrt{1-(0.987)^2}} = 6.22 .$$

Conservation of energy gives $E_1 + E_2 = E_{\text{total}}$

which is $\gamma_1 m_1 c^2 + \gamma_2 m_2 c^2 = m_{\text{total}} c^2$

or $2.01 m_1 + 6.22 m_2 = 3.34 \times 10^{-27}\text{ kg}$.

This reduces to: $m_1 + 3.09 m_2 = 1.66 \times 10^{-27}\text{ kg}$. (1)

Since the final momentum of the system must equal zero, $p_1 = p_2$

gives $\gamma_1 m_1 u_1 = \gamma_2 m_2 u_2$

or $(2.01)(0.868c) m_1 = (6.22)(0.987c) m_2$

which becomes $m_1 = 3.52 m_2$. (2)

Solving (1) and (2) simultaneously, $m_1 = \boxed{8.84 \times 10^{-28}\text{ kg}}$ and $m_2 = \boxed{2.51 \times 10^{-28}\text{ kg}}$.

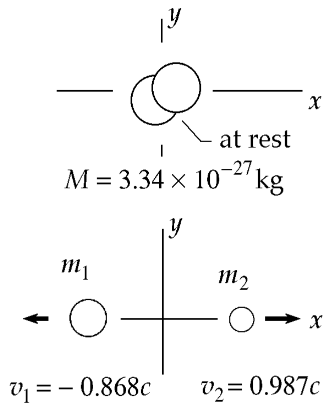

FIG. P39.41

***P39.42** Energy conservation: $\frac{1}{\sqrt{1-0^2}} 1\,400\text{ kg}c^2 + \frac{900\text{ kg}c^2}{\sqrt{1-0.85^2}} = \frac{Mc^2}{\sqrt{1-v^2/c^2}}$

$$3\,108\text{ kg}\sqrt{1-\frac{v^2}{c^2}} = M .$$

Momentum conservation: $0 + \frac{900\text{ kg}(0.85c)}{\sqrt{1-0.85^2}} = \frac{Mv}{\sqrt{1-v^2/c^2}}$

$$1\,452\text{ kg}\sqrt{1-\frac{v^2}{c^2}} = \frac{Mv}{c} .$$

(a) Dividing gives $\frac{v}{c} = \frac{1\,452}{3\,108} = 0.467$ $\boxed{v = 0.467c}$.

(b) Now by substitution $3\,108\text{ kg}\sqrt{1-0.467^2} = \boxed{M = 2.75 \times 10^3\text{ kg}}$.

**P39.43** $E = \gamma mc^2$ $p = \gamma mu$

$E^2 = \left(\gamma mc^2\right)^2$ $p^2 = (\gamma mu)^2$

$$E^2 - p^2c^2 = \left(\gamma mc^2\right)^2 - (\gamma mu)^2 c^2 = \gamma^2\left(\left(mc^2\right)^2 - (mc)^2 u^2\right) = \left(mc^2\right)^2\left(1-\frac{u^2}{c^2}\right)\left(1-\frac{u^2}{c^2}\right)^{-1} = \left(mc^2\right)^2$$

Q.E.D.

**P39.44** (a) $q(\Delta V) = K = (\gamma - 1) m_e c^2$

Thus, $\gamma = \frac{1}{\sqrt{1-(u/c)^2}} = 1 + \frac{q(\Delta V)}{m_e c^2}$ from which $\boxed{u = 0.302c}$.

(b) $K = (\gamma - 1) m_e c^2 = q(\Delta V) = \left(1.60 \times 10^{-19}\text{ C}\right)\left(2.50 \times 10^4\text{ J/C}\right) = \boxed{4.00 \times 10^{-15}\text{ J}}$

**P39.45** (a) $E = \gamma mc^2 = 20.0\text{ GeV}$ with $mc^2 = 0.511\text{ MeV}$ for electrons. Thus,

$$\gamma = \frac{20.0 \times 10^9\text{ eV}}{0.511 \times 10^6\text{ eV}} = \boxed{3.91 \times 10^4}.$$

(b) $\gamma = \dfrac{1}{\sqrt{1-(u/c)^2}} = 3.91 \times 10^4$ from which $\boxed{u = 0.999\,999\,999\,7c}$

(c) $$L = L_p\sqrt{1-\left(\frac{u}{c}\right)^2} = \frac{L_p}{\gamma} = \frac{3.00 \times 10^3\text{ m}}{3.91 \times 10^4} = 7.67 \times 10^{-2}\text{ m} = \boxed{7.67\text{ cm}}$$

***P39.46** (a) $$\mathscr{P} = \frac{\text{energy}}{\Delta t} = \frac{2\text{ J}}{100 \times 10^{-15}\text{ s}} = \boxed{2.00 \times 10^{13}\text{ W}}$$

(b) The kinetic energy of one electron with $v = 0.999\,9c$ is

$$(\gamma - 1)mc^2 = \left(\frac{1}{\sqrt{1-0.999\,9^2}} - 1\right)9.11 \times 10^{-31}\text{ kg}\left(3 \times 10^8\text{ m/s}\right)^2 = 69.7\left(8.20 \times 10^{-14}\text{ J}\right)$$
$$= 5.72 \times 10^{-12}\text{ J}$$

Then we require $\dfrac{0.01}{100}2\text{ J} = N\left(5.72 \times 10^{-12}\text{ J}\right)$

$$N = \frac{2 \times 10^{-4}\text{ J}}{5.72 \times 10^{-12}\text{ J}} = \boxed{3.50 \times 10^7}.$$

**P39.47** Conserving total momentum of the decaying particle system, $p_{\text{before decay}} = p_{\text{after decay}} = 0$

$$p_v = p_\mu = \gamma m_\mu u = \gamma(207m_e)u.$$

Conservation of mass-energy for the system gives $E_\mu + E_v = E_\pi$: $\gamma m_\mu c^2 + p_v c = m_\pi c^2$

$$\gamma(207m_e) + \frac{p_v}{c} = 273m_e.$$

Substituting from the momentum equation above, $\gamma(207m_e) + \gamma(207m_e)\dfrac{u}{c} = 273m_e$

or $\gamma\left(1+\dfrac{u}{c}\right) = \dfrac{273}{207} = 1.32$: $\dfrac{1+u/c}{1-u/c} = 1.74$ $\dfrac{u}{c} = 0.270.$

Then, $K_\mu = (\gamma - 1)m_\mu c^2 = (\gamma - 1)207\left(m_e c^2\right)$: $K_\mu = \left(\dfrac{1}{\sqrt{1-(0.270)^2}} - 1\right)207(0.511\text{ MeV})$

$$K_\mu = \boxed{4.08\text{ MeV}}.$$

Also, $E_v = E_\pi - E_\mu$: $E_v = m_\pi c^2 - \gamma m_\mu c^2 = (273 - 207\gamma)m_e c^2$

$$E_v = \left(273 - \frac{207}{\sqrt{1-(0.270)^2}}\right)(0.511\text{ MeV})$$

$$E_v = \boxed{29.6\text{ MeV}}$$

***P39.48** Let observer A hold the unprimed reference frame, with $u_1 = \frac{3c}{4}$ and $u_2 = -\frac{3c}{4}$. Let observer B be at rest in the primed frame with $u_1' = 0 = \frac{u_1 - v}{1 - u_1 v/c^2}$ $\qquad v = u_1 = \frac{3c}{4}$.

(a) Then $u_2' = \frac{u_2 - v}{1 - u_2 v/c^2} = \frac{-3c/4 - 3c/4}{1 - (-3c/4)(+3c/4)} = \frac{-1.5c}{1 + 9/16}$

speed $= |u_2'| = \frac{3c/2}{25/16} = \frac{24}{25}c = \boxed{0.960c}$.

(b) In the unprimed frame the objects, each of mass $m$, together have energy $\gamma mc^2 + \gamma mc^2 = 2\frac{mc^2}{\sqrt{1 - 0.75^2}} = 3.02mc^2$.

In the primed frame the energy is $\frac{mc^2}{\sqrt{1 - 0^2}} + \frac{mc^2}{\sqrt{1 - 0.96^2}} = 4.57mc^2$, greater by $\frac{4.57mc^2}{3.02mc^2} = \boxed{\text{1.51 times greater as measured by observer B}}$.

## Section 39.9 **Mass and Energy**

**P39.49** Let a 0.3-kg flag be run up a flagpole 7 m high.

We put into it energy $\qquad mgh = 0.3\text{ kg}\left(9.8\text{ m/s}^2\right)7\text{ m} \approx 20\text{ J}$.

So we put into it extra mass $\qquad \Delta m = \frac{E}{c^2} = \frac{20\text{ J}}{\left(3 \times 10^8\text{ m/s}\right)^2} = 2 \times 10^{-16}\text{ kg}$

for a fractional increase of $\qquad \frac{2 \times 10^{-16}\text{ kg}}{0.3\text{ kg}}\boxed{\sim 10^{-15}}$.

**P39.50** $E = 2.86 \times 10^5\text{ J}$. Also, the mass-energy relation says that $E = mc^2$.

Therefore, $\qquad m = \frac{E}{c^2} = \frac{2.86 \times 10^5\text{ J}}{\left(3.00 \times 10^8\text{ m/s}\right)^2} = \boxed{3.18 \times 10^{-12}\text{ kg}}$.

No, a mass loss of this magnitude (out of a total of 9.00 g) $\boxed{\text{could not be detected}}$.

**P39.51** $\Delta m = \frac{E}{c^2} = \frac{\mathcal{P}\Delta t}{c^2} = \frac{0.800\left(1.00 \times 10^9\text{ J/s}\right)(3.00\text{ yr})\left(3.16 \times 10^7\text{ s/yr}\right)}{\left(3.00 \times 10^8\text{ m/s}\right)^2} = \boxed{0.842\text{ kg}}$

**P39.52** $\Delta m = \frac{E}{c^2} = \frac{mc(\Delta T)}{c^2} = \frac{\rho Vc(\Delta T)}{c^2} = \frac{\left(1\,030\text{ kg/m}^3\right)\left(1.40 \times 10^9\right)\left(10^3\text{ m}\right)^3(4\,186\text{ J/kg}\cdot{}^\circ\text{C})(10.0\text{ }^\circ\text{C})}{\left(3.00 \times 10^8\text{ m/s}\right)^2}$

$\Delta m = \boxed{6.71 \times 10^8\text{ kg}}$

**P39.53** $\mathscr{P}=\frac{dE}{dt}=\frac{d\left(mc^2\right)}{dt}=c^2\frac{dm}{dt}=3.77\times10^{26}\text{ W}$

Thus, $\frac{dm}{dt}=\frac{3.77\times10^{26}\text{ J/s}}{\left(3.00\times10^8\text{ m/s}\right)^2}=\boxed{4.19\times10^9\text{ kg/s}}$

**P39.54** $2m_ec^2=1.02\text{ MeV}$ $E_\gamma\geq\boxed{1.02\text{ MeV}}$

## Section 39.10 **The General Theory of Relativity**

***P39.55** (a) For the satellite $\sum F=ma$: $\frac{GM_Em}{r^2}=\frac{mv^2}{r}=\frac{m}{r}\left(\frac{2\pi r}{T}\right)^2$

$$GM_ET^2=4\pi^2r^3$$

$$r=\left(\frac{6.67\times10^{-11}\text{ N}\cdot\text{m}^2\left(5.98\times10^{24}\text{ kg}\right)\left(43\,080\text{ s}\right)^2}{\text{kg}^2\,4\pi^2}\right)^{1/3}$$

$$r=\boxed{2.66\times10^7\text{ m}}$$

(b) $v=\frac{2\pi r}{T}=\frac{2\pi\left(2.66\times10^7\text{ m}\right)}{43\,080\text{ s}}=\boxed{3.87\times10^3\text{ m/s}}$

(c) The small fractional decrease in frequency received is equal in magnitude to the fractional increase in period of the moving oscillator due to time dilation:

$$\text{fractional change in }f=-(\gamma-1)=-\left(\frac{1}{\sqrt{1-\left(3.87\times10^3/3\times10^8\right)^2}}-1\right)$$

$$=1-\left(1-\frac{1}{2}\left[-\left(\frac{3.87\times10^3}{3\times10^8}\right)^2\right]\right)=\boxed{-8.34\times10^{-11}}$$

(d) The orbit altitude is large compared to the radius of the Earth, so we must use $U_g=-\frac{GM_Em}{r}$.

$$\Delta U_g=-\frac{6.67\times10^{-11}\text{ Nm}^2\left(5.98\times10^{24}\text{ kg}\right)m}{\text{kg}^2\;2.66\times10^7\text{ m}}+\frac{6.67\times10^{-11}\text{ Nm}\left(5.98\times10^{24}\text{ kg}\right)m}{\text{kg}\;6.37\times10^6\text{ m}}$$

$$=4.76\times10^7\text{ J/kg}\,m$$

$$\frac{\Delta f}{f}=\frac{\Delta U_g}{mc^2}=\frac{4.76\times10^7\text{ m}^2/\text{s}^2}{\left(3\times10^8\text{ m/s}\right)^2}=\boxed{+5.29\times10^{-10}}$$

(e) $-8.34\times10^{-11}+5.29\times10^{-10}=\boxed{+4.46\times10^{-10}}$

## Additional Problems

**P39.56** (a) $d_{\text{earth}} = vt_{\text{earth}} = v\gamma\, t_{\text{astro}}$ so $\left(2.00\times10^{6}\text{ yr}\right)c = v\dfrac{1}{\sqrt{1-v^2/c^2}}30.0\text{ yr}$

$$\sqrt{1-\frac{v^2}{c^2}} = \left(\frac{v}{c}\right)\left(1.50\times10^{-5}\right) \qquad 1-\frac{v^2}{c^2} = \frac{v^2\left(2.25\times10^{-10}\right)}{c^2}$$

$$1 = \frac{v^2}{c^2}\left(1+2.25\times10^{-10}\right) \quad \text{so} \quad \frac{v}{c} = \left(1+2.25\times10^{-10}\right)^{-1/2} = 1-\frac{1}{2}\left(2.25\times10^{-10}\right)$$

$$\boxed{\frac{v}{c} = 1-1.12\times10^{-10}}$$

(b) $$K = \left(\frac{1}{\sqrt{1-v^2/c^2}}-1\right)mc^2 = \left(\frac{2.00\times10^{6}\text{ yr}}{30\text{ yr}}-1\right)(1\,000)(1\,000\text{ kg})\left(3\times10^{8}\text{ m/s}\right)^2 = \boxed{6.00\times10^{27}\text{ J}}$$

(c) $$6.00\times10^{27}\text{ J} = 6.00\times10^{27}\text{ J}\left(\frac{13¢}{\text{kWh}}\right)\left(\frac{\text{k}}{10^3}\right)\left(\frac{\text{W}\cdot\text{s}}{\text{J}}\right)\left(\frac{\text{h}}{3\,600\text{ s}}\right) = \boxed{\$2.17\times10^{20}}$$

**P39.57** (a) $10^{13}\text{ MeV} = (\gamma-1)m_pc^2$ so $\gamma = 10^{10}$

$v_p \approx c$ $\qquad t' = \dfrac{t}{\gamma} = \dfrac{10^5\text{ yr}}{10^{10}} = 10^{-5}\text{ yr} \sim \boxed{10^2\text{ s}}$

(b) $d' = ct'\ \boxed{\sim10^{8}\text{ km}}$

**P39.58** (a) When $K_e = K_p$, $\quad m_ec^2(\gamma_e-1) = m_pc^2(\gamma_p-1)$.

In this case, $\quad m_ec^2 = 0.511\text{ MeV}$, $m_pc^2 = 938\text{ MeV}$

and $\quad \gamma_e = \left[1-(0.750)^2\right]^{-1/2} = 1.511\,9$.

Substituting, $$\gamma_p = 1+\frac{m_ec^2(\gamma_e-1)}{m_pc^2} = 1+\frac{(0.511\text{ MeV})(1.511\,9-1)}{938\text{ MeV}} = 1.000\,279$$

but $$\gamma_p = \frac{1}{\left[1-\left(u_p/c\right)^2\right]^{1/2}}.$$

Therefore, $\quad u_p = c\sqrt{1-\gamma_p^{-2}} = \boxed{0.023\,6c}$.

(b) When $p_e = p_p$ $\quad \gamma_pm_pu_p = \gamma_em_eu_e$ or $\gamma_pu_p = \dfrac{\gamma_em_eu_e}{m_p}$.

Thus, $$\gamma_pu_p = \frac{(1.511\,9)\left(0.511\text{ MeV}/c^2\right)(0.750c)}{938\text{ MeV}/c^2} = 6.177\,2\times10^{-4}c$$

and $$\frac{u_p}{c} = 6.177\,2\times10^{-4}\sqrt{1-\left(\frac{u_p}{c}\right)^2}$$

which yields $\quad u_p = \boxed{6.18\times10^{-4}c} = 185\text{ km/s}$.

**P39.59** (a) Since Mary is in the same reference frame, $S'$, as Ted, she measures the ball to have the same speed Ted observes, namely $|u'_x| = \boxed{0.800c}$.

(b) $$\Delta t' = \frac{L_p}{|u'_x|} = \frac{1.80\times10^{12}\ \text{m}}{0.800\left(3.00\times10^{8}\ \text{m/s}\right)} = \boxed{7.50\times10^{3}\ \text{s}}$$

(c) $$L = L_p\sqrt{1-\frac{v^2}{c^2}} = \left(1.80\times10^{12}\ \text{m}\right)\sqrt{1-\frac{(0.600c)^2}{c^2}} = \boxed{1.44\times10^{12}\ \text{m}}$$

Since $v = 0.600c$ and $u'_x = -0.800c$, the velocity Jim measures for the ball is

$$u_x = \frac{u'_x + v}{1+u'_x v/c^2} = \frac{(-0.800c)+(0.600c)}{1+(-0.800)(0.600)} = \boxed{-0.385c}.$$

(d) Jim measures the ball and Mary to be initially separated by $1.44\times10^{12}$ m. Mary's motion at 0.600*c* and the ball's motion at 0.385*c* nibble into this distance from both ends. The gap closes at the rate $0.600c + 0.385c = 0.985c$, so the ball and catcher meet after a time

$$\Delta t = \frac{1.44\times10^{12}\ \text{m}}{0.985\left(3.00\times10^{8}\ \text{m/s}\right)} = \boxed{4.88\times10^{3}\ \text{s}}.$$

***P39.60** (a) The charged battery stores energy $E = \mathscr{P}t = (1.20\ \text{J/s})(50\ \text{min})(60\ \text{s/min}) = 3\,600\ \text{J}$

so its mass excess is $$\Delta m = \frac{E}{c^2} = \frac{3\,600\ \text{J}}{\left(3\times10^{8}\ \text{m/s}\right)^2} = \boxed{4.00\times10^{-14}\ \text{kg}}.$$

(b) $$\frac{\Delta m}{m} = \frac{4.00\times10^{-14}\ \text{kg}}{25\times10^{-3}\ \text{kg}} = \boxed{1.60\times10^{-12}}$$ too small to measure.

**P39.61** $$\frac{\Delta mc^2}{mc^2} = \frac{4(938.78\ \text{MeV}) - 3\,728.4\ \text{MeV}}{4(938.78\ \text{MeV})}\times100\% = \boxed{0.712\%}$$

***P39.62** The energy of the first fragment is given by $E_1^2 = p_1^2c^2 + \left(m_1c^2\right)^2 = (1.75\ \text{MeV})^2 + (1.00\ \text{MeV})^2$

$E_1 = 2.02\ \text{MeV}$.

For the second, $E_2^2 = (2.00\ \text{MeV})^2 + (1.50\ \text{MeV})^2$ $\qquad E_2 = 2.50\ \text{MeV}$.

(a) Energy is conserved, so the unstable object had $E = 4.52\ \text{MeV}$. Each component of momentum is conserved, so the original object moved with $p^2 = p_x^2 + p_y^2 = \left(\frac{1.75\ \text{MeV}}{c}\right)^2 + \left(\frac{2.00\ \text{MeV}}{c}\right)^2$. Then for it

$$(4.52\ \text{MeV})^2 = (1.75\ \text{MeV})^2 + (2.00\ \text{MeV})^2 + \left(mc^2\right)^2 \qquad \boxed{m = \frac{3.65\ \text{MeV}}{c^2}}.$$

(b) Now $E = \gamma mc^2$ gives $4.52\ \text{MeV} = \dfrac{1}{\sqrt{1-v^2/c^2}}3.65\ \text{MeV}$ $\qquad 1-\dfrac{v^2}{c^2} = 0.654$, $\boxed{v = 0.589c}$.

**P39.63** (a) Take the spaceship as the primed frame, moving toward the right at $v = +0.600c$.

Then $u'_x = +0.800c$, and $$u_x = \frac{u'_x + v}{1 + (u'_x v)/c^2} = \frac{0.800c + 0.600c}{1 + (0.800)(0.600)} = \boxed{0.946c}.$$

(b) $L = \dfrac{L_p}{\gamma}$: $$L = (0.200 \text{ ly})\sqrt{1 - (0.600)^2} = \boxed{0.160 \text{ ly}}$$

(c) The aliens observe the 0.160-ly distance closing because the probe nibbles into it from one end at 0.800*c* and the Earth reduces it at the other end at 0.600*c*.

Thus, $$\text{time} = \frac{0.160 \text{ ly}}{0.800c + 0.600c} = \boxed{0.114 \text{ yr}}.$$

(d) $K = \left(\dfrac{1}{\sqrt{1 - u^2/c^2}} - 1\right)mc^2$: $$K = \left(\frac{1}{\sqrt{1 - (0.946)^2}} - 1\right)(4.00 \times 10^5 \text{ kg})(3.00 \times 10^8 \text{ m/s})^2$$

$$K = \boxed{7.50 \times 10^{22} \text{ J}}$$

**P39.64** In this case, the proper time is $T_0$ (the time measured by the students on a clock at rest relative to them). The dilated time measured by the professor is: $\Delta t = \gamma T_0$

where $\Delta t = T + t$ Here *T* is the time she waits before sending a signal and *t* is the time required for the signal to reach the students.

Thus, we have: $$T + t = \gamma T_0. \qquad (1)$$

To determine the travel time *t*, realize that the distance the students will have moved beyond the professor before the signal reaches them is: $d = v(T + t)$.

The time required for the signal to travel this distance is: $t = \dfrac{d}{c} = \left(\dfrac{v}{c}\right)(T + t)$.

Solving for *t* gives: $t = \dfrac{(v/c)T}{1 - (v/c)}$.

Substituting this into equation (1) yields: $T + \dfrac{(v/c)T}{1 - (v/c)} = \gamma T_0$

or $\dfrac{T}{1 - v/c} = \gamma T_0$.

Then $$T = T_0 \frac{1 - (v/c)}{\sqrt{1 - (v^2/c^2)}} = T_0 \frac{1 - (v/c)}{\sqrt{[1 + (v/c)][1 - (v/c)]}} = \boxed{T_0 \sqrt{\frac{1 - (v/c)}{1 + (v/c)}}}.$$

**P39.65** Look at the situation from the instructors' viewpoint since they are at rest relative to the clock, and hence measure the proper time. The Earth moves with velocity $v = -0.280c$ relative to the instructors while the students move with a velocity $u' = -0.600c$ relative to Earth. Using the velocity addition equation, the velocity of the students relative to the instructors (and hence the clock) is:

$$u = \frac{v+u'}{1+vu'/c^2} = \frac{(-0.280c)-(0.600c)}{1+(-0.280c)(-0.600c)/c^2} = -0.753c \text{ (students relative to clock).}$$

(a) With a proper time interval of $\Delta t_p = 50.0\text{ min}$, the time interval measured by the students is:

$$\Delta t = \gamma\,\Delta t_p \qquad \text{with} \qquad \gamma = \frac{1}{\sqrt{1-(0.753c)^2/c^2}} = 1.52\,.$$

Thus, the students measure the exam to last $T = 1.52(50.0\text{ min}) = \boxed{76.0\text{ minutes}}$.

(b) The duration of the exam as measured by observers on Earth is:

$$\Delta t = \gamma\,\Delta t_p \text{ with } \gamma = \frac{1}{\sqrt{1-(0.280c)^2/c^2}} \text{ so } T = 1.04(50.0\text{ min}) = \boxed{52.1\text{ minutes}}\,.$$

**P39.66** The energy which arrives in one year is

$$E = \mathcal{P}\,\Delta t = \left(1.79\times10^{17}\text{ J/s}\right)\left(3.16\times10^{7}\text{ s}\right) = 5.66\times10^{24}\text{ J}.$$

Thus, $$m = \frac{E}{c^2} = \frac{5.66\times10^{24}\text{ J}}{\left(3.00\times10^{8}\text{ m/s}\right)^2} = \boxed{6.28\times10^{7}\text{ kg}}\,.$$

**P39.67** The observer measures the proper length of the tunnel, 50.0 m, but measures the train contracted to length

$$L = L_p\sqrt{1-\frac{v^2}{c^2}} = 100\text{ m}\sqrt{1-(0.950)^2} = 31.2\text{ m}$$

shorter than the tunnel by $50.0 - 31.2 = \boxed{18.8\text{ m}}$ so $\boxed{\text{it is completely within the tunnel.}}$.

**P39.68** If the energy required to remove a mass $m$ from the surface is equal to its rest energy $mc^2$,

then $$\frac{GM_s m}{R_g} = mc^2$$

and $$R_g = \frac{GM_s}{c^2} = \frac{\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.99\times10^{30}\text{ kg}\right)}{\left(3.00\times10^{8}\text{ m/s}\right)^2}$$

$$R_g = 1.47\times10^{3}\text{ m} = \boxed{1.47\text{ km}}\,.$$

**P39.69** (a) At any speed, the momentum of the particle is given by

$$p = \gamma mu = \frac{mu}{\sqrt{1-(u/c)^2}}.$$

Since $F = qE = \frac{dp}{dt}$:

$$qE = \frac{d}{dt}\left[mu\left(1-\frac{u^2}{c^2}\right)^{-1/2}\right]$$

$$qE = m\left(1-\frac{u^2}{c^2}\right)^{-1/2}\frac{du}{dt} + \frac{1}{2}mu\left(1-\frac{u^2}{c^2}\right)^{-3/2}\left(\frac{2u}{c^2}\right)\frac{du}{dt}.$$

So

$$\frac{qE}{m} = \frac{du}{dt}\left[\frac{1-u^2/c^2+u^2/c^2}{\left(1-u^2/c^2\right)^{3/2}}\right]$$

and

$$\boxed{a = \frac{du}{dt} = \frac{qE}{m}\left(1-\frac{u^2}{c^2}\right)^{3/2}}.$$

(b) For $u$ small compared to $c$, the relativistic expression reduces to the classical $a = \frac{qE}{m}$. As $u$ approaches $c$, the acceleration approaches zero, so that the object can never reach the speed of light.

(c)
$$\int_0^u \frac{du}{\left(1-u^2/c^2\right)^{3/2}} = \int_{t=0}^{t} \frac{qE}{m}dt \qquad \boxed{u = \frac{qEct}{\sqrt{m^2c^2+q^2E^2t^2}}}$$

$$x = \int_0^t u\,dt = qEc\int_0^t \frac{t\,dt}{\sqrt{m^2c^2+q^2E^2t^2}} \qquad \boxed{x = \frac{c}{qE}\left(\sqrt{m^2c^2+q^2E^2t^2}-mc\right)}$$

**P39.70** (a) An observer at rest relative to the mirror sees the light travel a distance $D = 2d - x$, where $x = vt_S$ is the distance the ship moves toward the mirror in time $t_S$. Since this observer agrees that the speed of light is $c$, the time for it to travel distance $D$ is

$$t_S = \frac{D}{c} = \frac{2d - vt_S}{c} \qquad t_S = \boxed{\frac{2d}{c+v}}.$$

(b) The observer in the rocket measures a length-contracted initial distance to the mirror of

$$L = d\sqrt{1-\frac{v^2}{c^2}}$$

and the mirror moving toward the ship at speed $v$. Thus, he measures the distance the light travels as $D = 2(L-y)$ where $y = \frac{vt}{2}$ is the distance the mirror moves toward the ship before the light reflects from it. This observer also measures the speed of light to be $c$, so the time for it to travel distance $D$ is:

$$t = \frac{D}{c} = \frac{2}{c}\left[d\sqrt{1-\frac{v^2}{c^2}}-\frac{vt}{2}\right] \text{ so } (c+v)t = \frac{2d}{c}\sqrt{(c+v)(c-v)} \text{ or } t = \boxed{\frac{2d}{c}\sqrt{\frac{c-v}{c+v}}}.$$

***P39.71** Take the two colliding protons as the system

$E_1 = K + mc^2$ $\qquad E_2 = mc^2$

$E_1^2 = p_1^2c^2 + m^2c^4$ $\qquad p_2 = 0$.

In the final state, $E_f = K_f + Mc^2$: $\qquad E_f^2 = p_f^2c^2 + M^2c^4$.

By energy conservation, $E_1 + E_2 = E_f$, so

$$E_1^2 + 2E_1E_2 + E_2^2 = E_f^2$$

$$p_1^2c^2 + m^2c^4 + 2\left(K + mc^2\right)mc^2 + m^2c^4$$

$$= p_f^2c^2 + M^2c^4$$

By conservation of momentum, $\qquad p_1 = p_f$.

Then $\qquad M^2c^4 = 2Kmc^2 + 4m^2c^4 = \dfrac{4Km^2c^4}{2mc^2} + 4m^2c^4$

$$Mc^2 = 2mc^2\sqrt{1 + \frac{K}{2mc^2}}.$$

By contrast, for colliding beams we have

In the original state, $\qquad E_1 = K + mc^2$

$$E_2 = K + mc^2.$$

In the final state, $\qquad E_f = Mc^2$

$E_1 + E_2 = E_f$: $\qquad K + mc^2 + K + mc^2 = Mc^2$

$$Mc^2 = 2mc^2\left(1 + \frac{K}{2mc^2}\right).$$

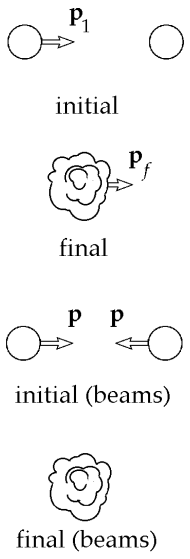

**FIG. P39.71**

***P39.72** Conservation of momentum $\gamma mu$:

$$\frac{mu}{\sqrt{1 - u^2/c^2}} + \frac{m(-u)}{3\sqrt{1 - u^2/c^2}} = \frac{Mv_f}{\sqrt{1 - v_f^2/c^2}} = \frac{2mu}{3\sqrt{1 - u^2/c^2}}.$$

Conservation of energy $\gamma mc^2$:

$$\frac{mc^2}{\sqrt{1 - u^2/c^2}} + \frac{mc^2}{3\sqrt{1 - u^2/c^2}} = \frac{Mc^2}{\sqrt{1 - v_f^2/c^2}} = \frac{4mc^2}{3\sqrt{1 - u^2/c^2}}.$$

To start solving we can divide: $v_f = \dfrac{2u}{4} = \dfrac{u}{2}$. Then

$$\frac{M}{\sqrt{1 - u^2/4c^2}} = \frac{4m}{3\sqrt{1 - u^2/c^2}} = \frac{M}{(1/2)\sqrt{4 - u^2/c^2}}$$

$$\boxed{M = \frac{2m\sqrt{4 - u^2/c^2}}{3\sqrt{1 - u^2/c^2}}}$$

Note that when $v \ll c$, this reduces to $M = \dfrac{4m}{3}$, in agreement with the classical result.

**P39.73** (a) $L_0^2 = L_{0x}^2 + L_{0y}^2$ and $L^2 = L_x^2 + L_y^2$.

The motion is in the $x$ direction: $L_y = L_{0y} = L_0 \sin\theta_0$

$$L_x = L_{0x}\sqrt{1-\left(\frac{v}{c}\right)^2} = (L_0 \cos\theta_0)\sqrt{1-\left(\frac{v}{c}\right)^2}.$$

Thus,

$$L^2 = L_0^2 \cos^2\theta_0\left[1-\left(\frac{v}{c}\right)^2\right] + L_0^2 \sin^2\theta_0 = L_0^2\left[1-\left(\frac{v}{c}\right)^2 \cos^2\theta_0\right]$$

or

$$\boxed{L = L_0\left[1-\left(\frac{v}{c}\right)^2 \cos^2\theta_0\right]^{1/2}}.$$

(b) $$\tan\theta = \frac{L_y}{L_x} = \frac{L_{0y}}{L_{0x}\sqrt{1-(v/c)^2}} = \boxed{\gamma \tan\theta_0}$$

**P39.74** (b) Consider a hermit who lives on an asteroid halfway between the Sun and Tau Ceti, stationary with respect to both. Just as our spaceship is passing him, he also sees the blast waves from both explosions. Judging both stars to be stationary, this observer concludes that $\boxed{\text{the two stars blew up simultaneously}}$.

(a) We in the spaceship moving past the hermit do not calculate the explosions to be simultaneous. We measure the distance we have traveled from the Sun as

$$L = L_p\sqrt{1-\left(\frac{v}{c}\right)^2} = (6.00 \text{ ly})\sqrt{1-(0.800)^2} = 3.60 \text{ ly}.$$

We see the Sun flying away from us at $0.800c$ while the light from the Sun approaches at $1.00c$. Thus, the gap between the Sun and its blast wave has opened at $1.80c$, and the time we calculate to have elapsed since the Sun exploded is

$$\frac{3.60 \text{ ly}}{1.80c} = 2.00 \text{ yr}.$$

We see Tau Ceti as moving toward us at $0.800c$, while its light approaches at $1.00c$, only $0.200c$ faster. We measure the gap between that star and its blast wave as 3.60 ly and growing at $0.200c$. We calculate that it must have been opening for

$$\frac{3.60 \text{ ly}}{0.200c} = 18.0 \text{ yr}$$

and conclude that $\boxed{\text{Tau Ceti exploded 16.0 years before the Sun}}$.

**P39.75** Since the total momentum is zero before decay, it is necessary that after the decay

$$p_{\text{nucleus}} = p_{\text{photon}} = \frac{E_\gamma}{c} = \frac{14.0\text{ keV}}{c}.$$

Also, for the recoiling nucleus, $E^2 = p^2c^2 + \left(mc^2\right)^2$ with $mc^2 = 8.60 \times 10^{-9}\text{ J} = 53.8\text{ GeV}$.

Thus, $\left(mc^2 + K\right)^2 = (14.0\text{ keV})^2 + \left(mc^2\right)^2$ or $\left(1 + \frac{K}{mc^2}\right)^2 = \left(\frac{14.0\text{ keV}}{mc^2}\right)^2 + 1$.

So $1 + \frac{K}{mc^2} = \sqrt{1 + \left(\frac{14.0\text{ keV}}{mc^2}\right)^2} \approx 1 + \frac{1}{2}\left(\frac{14.0\text{ keV}}{mc^2}\right)^2$ (Binomial Theorem)

and $K \approx= \frac{(14.0\text{ keV})^2}{2mc^2} = \frac{\left(14.0 \times 10^3\text{ eV}\right)^2}{2\left(53.8 \times 10^9\text{ eV}\right)} = \boxed{1.82 \times 10^{-3}\text{ eV}}$.

**P39.76** Take $m = 1.00\text{ kg}$.

The classical kinetic energy is $K_c = \frac{1}{2}mu^2 = \frac{1}{2}mc^2\left(\frac{u}{c}\right)^2 = \left(4.50 \times 10^{16}\text{ J}\right)\left(\frac{u}{c}\right)^2$

and the actual kinetic energy is $K_r = \left(\frac{1}{\sqrt{1-(u/c)^2}} - 1\right)mc^2 = \left(9.00 \times 10^{16}\text{ J}\right)\left(\frac{1}{\sqrt{1-(u/c)^2}} - 1\right)$.

| $\frac{u}{c}$ | $K_c$ (J) | $K_r$ (J) |
|---|---|---|
| 0.000 | 0.000 | 0.000 |
| 0.100 | $0.045 \times 10^{16}$ | $0.0453 \times 10^{16}$ |
| 0.200 | $0.180 \times 10^{16}$ | $0.186 \times 10^{16}$ |
| 0.300 | $0.405 \times 10^{16}$ | $0.435 \times 10^{16}$ |
| 0.400 | $0.720 \times 10^{16}$ | $0.820 \times 10^{16}$ |
| 0.500 | $1.13 \times 10^{16}$ | $1.39 \times 10^{16}$ |
| 0.600 | $1.62 \times 10^{16}$ | $2.25 \times 10^{16}$ |
| 0.700 | $2.21 \times 10^{16}$ | $3.60 \times 10^{16}$ |
| 0.800 | $2.88 \times 10^{16}$ | $6.00 \times 10^{16}$ |
| 0.900 | $3.65 \times 10^{16}$ | $11.6 \times 10^{16}$ |
| 0.990 | $4.41 \times 10^{16}$ | $54.8 \times 10^{16}$ |

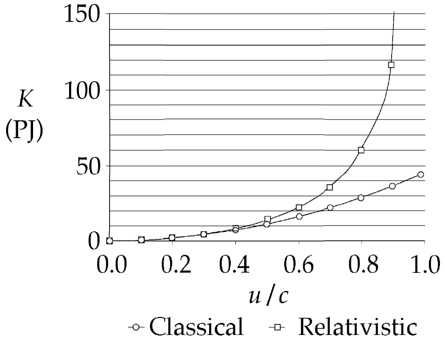

**FIG. P39.76**

$K_c = 0.990K_r$, when $\frac{1}{2}\left(\frac{u}{c}\right)^2 = 0.990\left[\frac{1}{\sqrt{1-(u/c)^2}} - 1\right]$, yielding $u = \boxed{0.115c}$.

Similarly, $K_c = 0.950K_r$ when $u = \boxed{0.257c}$

and $K_c = 0.500K_r$ when $u = \boxed{0.786c}$.

## ANSWERS TO EVEN PROBLEMS

| | |
|---|---|
| **P39.2** | (a) $60.0\ \text{m/s}$; (b) $20.0\ \text{m/s}$; (c) $44.7\ \text{m/s}$ |
| **P39.4** | see the solution |
| **P39.6** | $0.866c$ |
| **P39.8** | (a) 25.0 yr; (b) 15.0 yr; (c) 12.0 ly |
| **P39.10** | (a) $2.18\ \mu\text{s}$; (b) The moon sees the planet surface moving 649 m up toward it. |
| **P39.12** | (a) $\dfrac{cL_p}{\sqrt{c^2\Delta t^2+L_p^2}}$; (b) $4.00\ \text{m/s}$;<br>(c) see the solution |
| **P39.14** | $v=0.140c$ |
| **P39.16** | 5.45 yr, Goslo is older |
| **P39.18** | 11.3 kHz |
| **P39.20** | (a) see the solution; (b) $0.050\,4c$ |
| **P39.22** | (a) $0.943c$; (b) 2.55 km |
| **P39.24** | B occurred 444 ns before A |
| **P39.26** | $0.357c$ |
| **P39.28** | $0.893c$ at $16.8°$ above the $x'$-axis |
| **P39.30** | (a) $0.141c$; (b) $0.436c$ |
| **P39.32** | see the solution |
| **P39.34** | (a) $0.582\ \text{MeV}$; (b) $2.45\ \text{MeV}$ |
| **P39.36** | see the solution |
| **P39.38** | (a) $438\ \text{GJ}$; (b) $438\ \text{GJ}$ |
| **P39.40** | $18.4\ \text{g/cm}^3$ |
| **P39.42** | (a) $0.467c$; (b) $2.75\times10^3\ \text{kg}$ |
| **P39.44** | (a) $0.302c$; (b) 4.00 fJ |
| **P39.46** | (a) 20.0 TW; (b) $3.50\times10^7$ electrons |
| **P39.48** | (a) $0.960c$; (b) 1.51 times greater as measured by B. |
| **P39.50** | $3.18\times10^{-12}\ \text{kg}$, not detectable |
| **P39.52** | $6.71\times10^8\ \text{kg}$ |
| **P39.54** | 1.02 MeV |
| **P39.56** | (a) $\dfrac{v}{c}=1-1.12\times10^{-10}$; (b) $6.00\times10^{27}\ \text{J}$;<br>(c) $\$2.17\times10^{20}$ |
| **P39.58** | (a) $0.023\,6c$; (b) $6.18\times10^{-4}c$ |
| **P39.60** | (a) $4.00\times10^{-14}\ \text{kg}$; (b) $1.60\times10^{-12}$ |
| **P39.62** | (a) $\dfrac{3.65\ \text{MeV}}{c^2}$; (b) $v=0.589c$ |
| **P39.64** | see the solution |
| **P39.66** | $6.28\times10^7\ \text{kg}$ |
| **P39.68** | 1.47 km |
| **P39.70** | (a) $\dfrac{2d}{c+v}$; (b) $\dfrac{2d}{c}\sqrt{\dfrac{c-v}{c+v}}$ |
| **P39.72** | $M=\dfrac{2m}{3}\sqrt{\dfrac{4-u^2/c^2}{1-u^2/c^2}}$ |
| **P39.74** | (a) Tau Ceti exploded 16.0 yr before the Sun; (b) they exploded simultaneously |
| **P39.76** | see the solution, $0.115c$, $0.257c$, $0.786c$ |

# 40

# Introduction to Quantum Physics

**CHAPTER OUTLINE**

40.1 Blackbody Radiation and Planck's Hypothesis
40.2 The Photoelectric Effect
40.3 The Compton Effect
40.4 Photons and Electromagnetic Waves
40.5 The Wave Properties of Particles
40.6 The Quantum Particle
40.7 The Double-Slit Experiment Revisited
40.8 The Uncertainty Principle

## ANSWERS TO QUESTIONS

**Q40.1** Planck made two new assumptions: (1) molecular energy is quantized and (2) molecules emit or absorb energy in discrete irreducible packets. These assumptions contradict the classical idea of energy as continuously divisible. They also imply that an atom must have a definite structure—it cannot just be a soup of electrons orbiting the nucleus.

**Q40.2** The first flaw is that the Rayleigh–Jeans law predicts that the intensity of short wavelength radiation emitted by a blackbody approaches infinity as the wavelength decreases. This is known as the *ultraviolet catastrophe*. The second flaw is the prediction much more power output from a black-body than is shown experimentally. The intensity of radiation from the blackbody is given by the area under the red $I(\lambda, T)$ *vs.* $\lambda$ curve in Figure 40.5 in the text, not by the area under the blue curve.

Planck's Law dealt with both of these issues and brought the theory into agreement with the experimental data by adding an exponential term to the denominator that depends on $\frac{1}{\lambda}$. This both keeps the predicted intensity from approaching infinity as the wavelength decreases and keeps the area under the curve finite.

**Q40.3** Our eyes are not able to detect all frequencies of energy. For example, all objects that are above 0 K in temperature emit electromagnetic radiation in the infrared region. This describes *everything* in a dark room. We are only able to see objects that emit or reflect electromagnetic radiation in the visible portion of the spectrum.

**Q40.4** Most stars radiate nearly as blackbodies. Vega has a higher surface temperature than Arcturus. Vega radiates most intensely at shorter wavelengths.

**Q40.5** No. The second metal may have a larger work function than the first, in which case the incident photons may not have enough energy to eject photoelectrons.

**Q40.6** Comparing Equation 40.9 with the slope-intercept form of the equation for a straight line, $y = mx + b$, we see that the slope in Figure 40.11 in the text is Planck's constant $h$ and that the $y$ intercept is $-\phi$, the negative of the work function. If a different metal were used, the slope would remain the same but the work function would be different, Thus, data for different metals appear as parallel lines on the graph.

**Q40.7** Wave theory predicts that the photoelectric effect should occur at any frequency, provided the light intensity is high enough. However, as seen in the photoelectric experiments, the light must have a sufficiently high frequency for the effect to occur.

**Q40.8** The stopping voltage measures the kinetic energy of the most energetic photoelectrons. Each of them has gotten its energy from a single photon. According to Planck's $E = hf$, the photon energy depends on the frequency of the light. The intensity controls only the number of photons reaching a unit area in a unit time.

**Q40.9** Let's do some quick calculations and see: 1.62 MHz is the highest frequency in the commercial AM band. From the relationship between the energy and the frequency, $E = hf$, the energy available from such a wave would be $1.07 \times 10^{-27}$ J, or 6.68 neV. That is 9 orders of magnitude too small to eject electrons from the metal. The only thing this student could gain from this experiment is a hefty fine and a long jail term from the FCC. To get on the order of a few eV from this experiment, she would have to broadcast at a minimum frequency of 250 Thz, which is in the infrared region.

**Q40.10** No. If an electron breaks free from an atom absorbing a photon, we say the atom is ionized. Ionization typically requires energy of several eV. As with the photoelectric effect in a solid metal, the light must have a sufficiently high frequency for a photon energy that is large enough. The gas can absorb energy from longer-wavelength light as it gains more internal energy of random motion of whole molecules.

**Q40.11** Ultraviolet light has shorter wavelength and higher photon energy than visible light.

**Q40.12** (c) UV light has the highest frequency of the three, and hence each photon delivers more energy to a skin cell. This explains why you can become sunburned on a cloudy day: clouds block visible light and infrared, but not much ultraviolet. You usually do not become sunburned through window glass, even though you can see the visible light from the Sun coming through the window, because the glass absorbs much of the ultraviolet and reemits it as infrared.

**Q40.13** The Compton effect describes the *scattering* of photons from electrons, while the photoelectric effect predicts the ejection of electrons due to the *absorption* of photons by a material.

**Q40.14** In developing a theory in accord with experimental evidence, Compton assumed that photons exhibited clear particle-like behavior, and that both energy and momentum are conserved in electron-photon interactions. Photons had previously been thought of as bits of waves.

**Q40.15** The x-ray photon transfers some of its energy to the electron. Thus, its frequency must decrease.

**Q40.16** A few photons would only give a few dots of exposure, apparently randomly scattered.

**Q40.17** Light has both classical-wave and classical-particle characteristics. In single- and double-slit experiments light behaves like a wave. In the photoelectric effect light behaves like a particle. Light may be characterized as an electromagnetic wave with a particular wavelength or frequency, yet at the same time light may be characterized as a stream of photons, each carrying a discrete energy, $hf$. Since light displays *both* wave and particle characteristics, perhaps it would be fair to call light a "wavicle". It is customary to call a photon a quantum particle, different from a classical particle.

**Q40.18** An electron has both classical-wave and classical-particle characteristics. In single- and double-slit diffraction and interference experiments, electrons behave like classical waves. An electron has mass and charge. It carries kinetic energy and momentum in parcels of definite size, as classical particles do. At the same time it has a particular wavelength and frequency. Since an electron displays characteristics of both classical waves and classical particles, it is neither a classical wave nor a classical particle. It is customary to call it a quantum particle, but another invented term, such as "wavicle", could serve equally well.

**Q40.19** The discovery of electron diffraction by Davisson and Germer was a fundamental advance in our understanding of the motion of material particles. Newton's laws fail to properly describe the motion of an object with small mass. It moves as a wave, not as a classical particle. Proceeding from this recognition, the development of quantum mechanics made possible describing the motion of electrons in atoms; understanding molecular structure and the behavior of matter at the atomic scale, including electronics, photonics, and engineered materials; accounting for the motion of nucleons in nuclei; and studying elementary particles.

**Q40.20** If we set $\frac{p^2}{2m} = q\Delta V$, which is the same for both particles, then we see that the electron has the smaller momentum and therefore the longer wavelength $\left(\lambda = \frac{h}{p}\right)$.

**Q40.21** Any object of macroscopic size—including a grain of dust—has an undetectably small wavelength and does not exhibit quantum behavior.

**Q40.22** A particle is represented by a wave packet of nonzero width. The width necessarily introduces uncertainty in the position of the particle. The width of the wave packet can be reduced toward zero only by adding waves of all possible wavelengths together. Doing this, however, results in loss of all information about the momentum and, therefore, the speed of the particle.

**Q40.23** The *intensity* of electron waves in some small region of space determines the *probability* that an electron will be found in that region.

**Q40.24** The wavelength of violet light is on the order of $\frac{1}{2}$ $\mu$m, while the de Broglie wavelength of an electron can be 4 orders of magnitude smaller. Would your height be measured more precisely with an unruled meter stick or with one engraved with divisions down to $\frac{1}{10}$ mm?

**Q40.25** The spacing between repeating structures on the surface of the feathers or scales is on the order of 1/2 the wavelength of light. An optical microscope would not have the resolution to see such fine detail, while an electron microscope can. The electrons can have much shorter wavelength.

**Q40.26** (a) The slot is blacker than any black material or pigment. Any radiation going in through the hole will be absorbed by the walls or the contents of the box, perhaps after several reflections. Essentially none of that energy will come out through the hole again. Figure 40.1 in the text shows this effect if you imagine the beam getting weaker at each reflection.

*continued on next page*

(b) The open slots between the glowing tubes are brightest. When you look into a slot, you receive direct radiation emitted by the wall on the far side of a cavity enclosed by the fixture; and you also receive radiation that was emitted by other sections of the cavity wall and has bounced around a few or many times before escaping through the slot. In Figure 40.1 in the text, reverse all of the arrowheads and imagine the beam getting stronger at each reflection. Then the figure shows the extra efficiency of a cavity radiator. Here is the conclusion of Kirchhoff's thermodynamic argument: ... energy radiated. A poor reflector—a good absorber—avoids rising in temperature by being an efficient emitter. Its emissivity is equal to its absorptivity: $e = a$ . The slot in the box in part (a) of the question is a black body with reflectivity zero and absorptivity 1, so it must also be the most efficient possible radiator, to avoid rising in temperature above its surroundings in thermal equilibrium. Its emissivity in Stefan's law is $100\% = 1$, higher than perhaps 0.9 for black paper, 0.1 for light-colored paint, or 0.04 for shiny metal. Only in this way can the material objects underneath these different surfaces maintain equal temperatures after they come to thermal equilibrium and continue to exchange energy by electromagnetic radiation. By considering one blackbody facing another, Kirchhoff proved logically that the material forming the walls of the cavity made no difference to the radiation. By thinking about inserting color filters between two cavity radiators, he proved that the spectral distribution of blackbody radiation must be a universal function of wavelength, the same for all materials and depending only on the temperature. Blackbody radiation is a fundamental connection between the matter and the energy that physicists had previously studied separately.

## SOLUTIONS TO PROBLEMS

### Section 40.1 **Blackbody Radiation and Planck's Hypothesis**

**P40.1** $$T = \frac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{560\times10^{-9}\ \text{m}} = \boxed{5.18\times10^3\ \text{K}}$$

**P40.2** (a) $$\lambda_{\max} = \frac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{T} \sim \frac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{10^4\ \text{K}}\boxed{\sim10^{-7}\ \text{m}} \qquad \boxed{\text{ultraviolet}}$$

(b) $$\lambda_{\max} \sim \frac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{10^7\ \text{K}}\boxed{\sim10^{-10}\ \text{m}} \quad \boxed{\gamma-\text{ray}}$$

**P40.3** Planck's radiation law gives intensity-per-wavelength. Taking $E$ to be the photon energy and $n$ to be the number of photons emitted each second, we multiply by area and wavelength range to have energy-per-time leaving the hole:

$$\mathcal{P} = \frac{2\pi hc^2(\lambda_2-\lambda_1)\pi(d/2)^2}{[(\lambda_1+\lambda_2)/2]^5\left(e^{2hc/[(\lambda_1+\lambda_2)k_BT]}-1\right)} = En = nhf \qquad \text{where} \qquad E = hf = \frac{2hc}{\lambda_1+\lambda_2}$$

$$n = \frac{\mathcal{P}}{E} = \frac{8\pi^2cd^2(\lambda_2-\lambda_1)}{(\lambda_1+\lambda_2)^4\left(e^{2hc/[(\lambda_1+\lambda_2)k_BT]}-1\right)}$$

$$= \frac{8\pi^2(3.00\times10^8\ \text{m/s})(5.00\times10^{-5}\ \text{m})^2(1.00\times10^{-9}\ \text{m})}{(1\,001\times10^{-9}\ \text{m})^4\left(e^{[2(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^8\ \text{m/s})]/[(1\,001\times10^{-9}\ \text{m})(1.38\times10^{-23}\ \text{J/K})(7.50\times10^3\ \text{K})]}-1\right)}$$

$$n = \frac{5.90\times10^{16}/\text{s}}{\left(e^{3.84}-1\right)} = \boxed{1.30\times10^{15}/\text{s}}$$

**P40.4** (a) $\mathcal{P} = eA\sigma T^4 = 1\left(20.0\times10^{-4}\ \text{m}^2\right)\left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)\left(5\,000\ \text{K}\right)^4 = \boxed{7.09\times10^4\ \text{W}}$

(b) $\lambda_{\max}T = \lambda_{\max}\left(5\,000\ \text{K}\right) = 2.898\times10^{-3}\ \text{m}\cdot\text{K} \Rightarrow \lambda_{\max} = \boxed{580\ \text{nm}}$

(c) We compute: $\dfrac{hc}{k_\text{B}T} = \dfrac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^8\ \text{m/s}\right)}{\left(1.38\times10^{-23}\ \text{J/K}\right)\left(5\,000\ \text{K}\right)} = 2.88\times10^{-6}\ \text{m}$

The power per wavelength interval is $\mathcal{P}(\lambda) = AI(\lambda) = \dfrac{2\pi hc^2A}{\lambda^5\left[\exp\left(hc/\lambda k_\text{B}T\right)-1\right]}$, and

$$2\pi hc^2A = 2\pi\left(6.626\times10^{-34}\right)\left(3.00\times10^8\right)^2\left(20.0\times10^{-4}\right) = 7.50\times10^{-19}\ \text{J}\cdot\text{m}^4/\text{s}$$

$$\mathcal{P}(580\ \text{nm}) = \frac{7.50\times10^{-19}\ \text{J}\cdot\text{m}^4/\text{s}}{\left(580\times10^{-9}\ \text{m}\right)^5\left[\exp(2.88\ \mu\text{m}/0.580\ \mu\text{m})-1\right]} = \frac{1.15\times10^{13}\ \text{J/m}\cdot\text{s}}{e^{4.973}-1}$$

$$= \boxed{7.99\times10^{10}\ \text{W/m}}$$

(d)–(i) The other values are computed similarly:

| | $\lambda$ | $\dfrac{hc}{\lambda k_B T}$ | $e^{hc/\lambda k_B T}-1$ | $\dfrac{2\pi hc^2 A}{\lambda^5}$ | $\mathcal{P}(\lambda)$, W/m |
|---|---|---|---|---|---|
| (d) | 1.00 nm | 2882.6 | $7.96\times10^{1251}$ | $7.50\times10^{26}$ | $9.42\times10^{-1226}$ |
| (e) | 5.00 nm | 576.5 | $2.40\times10^{250}$ | $2.40\times10^{23}$ | $1.00\times10^{-227}$ |
| (f) | 400 nm | 7.21 | 1347 | $7.32\times10^{13}$ | $5.44\times10^{10}$ |
| (c) | 580 nm | 4.97 | 143.5 | $1.15\times10^{13}$ | $7.99\times10^{10}$ |
| (g) | 700 nm | 4.12 | 60.4 | $4.46\times10^{12}$ | $7.38\times10^{10}$ |
| (h) | 1.00 mm | 0.00288 | 0.00289 | $7.50\times10^{-4}$ | 0.260 |
| (i) | 10.0 cm | $2.88\times10^{-5}$ | $2.88\times10^{-5}$ | $7.50\times10^{-14}$ | $2.60\times10^{-9}$ |

(j) We approximate the area under the $\mathcal{P}(\lambda)$ versus $\lambda$ curve, between 400 nm and 700 nm, as two trapezoids:

$$\mathcal{P} = \frac{\left[(5.44+7.99)\times10^{10}\ \text{W/m}\right]\left[(580-400)\times10^{-9}\ \text{m}\right]}{2}$$

$$+\frac{\left[(7.99+7.38)\times10^{10}\ \text{W/m}\right]\left[(700-580)\times10^{-9}\ \text{m}\right]}{2}$$

$\mathcal{P} = 2.13\times10^4\ \text{W}$ so the power radiated as visible light is $\boxed{\text{approximately 20 kW}}$.

**P40.5** (a) $\mathcal{P} = eA\sigma T^4$, so

$$T = \left(\frac{\mathcal{P}}{eA\sigma}\right)^{1/4} = \left[\frac{3.77\times10^{26}\ \text{W}}{1\left[4\pi\left(6.96\times10^8\ \text{m}\right)^2\right]\left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)}\right]^{1/4} = \boxed{5.75\times10^3\ \text{K}}$$

(b) $\lambda_{\max} = \dfrac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{T} = \dfrac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{5.75\times10^3\ \text{K}} = 5.04\times10^{-7}\ \text{m} = \boxed{504\ \text{nm}}$

**P40.6** $$E = hf = \frac{hc}{\lambda} = \frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{589.3\times10^{-9}\ \text{m}} = 3.37\times10^{-19}\ \text{J/photon}$$

$$n = \frac{\mathcal{P}}{E} = \frac{10.0\ \text{J/s}}{3.37\times10^{-19}\ \text{J/photon}} = \boxed{2.96\times10^{19}\ \text{photons/s}}$$

**P40.7** (a) $$E = hf = \left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(620\times10^{12}\ \text{s}^{-1}\right)\left(\frac{1.00\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \boxed{2.57\ \text{eV}}$$

(b) $$E = hf = \left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.10\times10^{9}\ \text{s}^{-1}\right)\left(\frac{1.00\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \boxed{1.28\times10^{-5}\ \text{eV}}$$

(c) $$E = hf = \left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(46.0\times10^{6}\ \text{s}^{-1}\right)\left(\frac{1.00\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \boxed{1.91\times10^{-7}\ \text{eV}}$$

(d) $$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\ \text{m/s}}{620\times10^{12}\ \text{Hz}} = 4.84\times10^{-7}\ \text{m} = \boxed{484\ \text{nm, visible light (blue)}}$$

$$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\ \text{m/s}}{3.10\times10^{9}\ \text{Hz}} = 9.68\times10^{-2}\ \text{m} = \boxed{9.68\ \text{cm, radio wave}}$$

$$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\ \text{m/s}}{46.0\times10^{6}\ \text{Hz}} = \boxed{6.52\ \text{m, radio wave}}$$

**P40.8** Energy of a single 500-nm photon:

$$E_\gamma = hf = \frac{hc}{\lambda} = \frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{\left(500\times10^{-9}\ \text{m}\right)} = 3.98\times10^{-19}\ \text{J}.$$

The energy entering the eye each second

$$E = \mathcal{P}\Delta t = IA\Delta t = \left(4.00\times10^{-11}\ \text{W/m}^2\right)\left[\frac{\pi}{4}\left(8.50\times10^{-3}\ \text{m}\right)^2\right](1.00\ \text{s}) = 2.27\times10^{-15}\ \text{J}.$$

The number of photons required to yield this energy

$$n = \frac{E}{E_\gamma} = \frac{2.27\times10^{-15}\ \text{J}}{3.98\times10^{-19}\ \text{J/photon}} = \boxed{5.71\times10^{3}\ \text{photons}}.$$

**P40.9** Each photon has an energy $E = hf = \left(6.626\times10^{-34}\right)\left(99.7\times10^{6}\right) = 6.61\times10^{-26}\ \text{J}$.

This implies that there are $\dfrac{150\times10^{3}\ \text{J/s}}{6.61\times10^{-26}\ \text{J/photon}} = \boxed{2.27\times10^{30}\ \text{photons/s}}$.

**P40.10** We take $\theta = 0.030\,0$ radians. Then the pendulum's total energy is

$$E = mgh = mg(L - L\cos\theta)$$

$$E = (1.00\text{ kg})\left(9.80\text{ m/s}^2\right)(1.00 - 0.999\,5) = 4.41\times10^{-3}\text{ J}$$

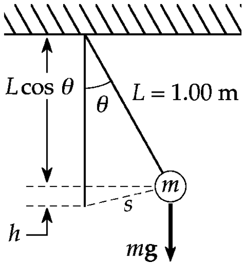

**FIG. P40.10**

The frequency of oscillation is $f = \dfrac{\omega}{2\pi} = \dfrac{1}{2\pi}\sqrt{\dfrac{g}{L}} = 0.498\text{ Hz}$.

The energy is quantized, $E = nhf$.

Therefore, $n = \dfrac{E}{hf} = \dfrac{4.41\times10^{-3}\text{ J}}{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(0.498\text{ s}^{-1}\right)}$

$$= \boxed{1.34\times10^{31}}$$

**P40.11** The radiation wavelength of $\lambda' = 500$ nm that is observed by observers on Earth is not the true wavelength, $\lambda$, emitted by the star because of the Doppler effect. The true wavelength is related to the observed wavelength using:

$$\frac{c}{\lambda'} = \frac{c}{\lambda}\sqrt{\frac{1-(v/c)}{1+(v/c)}}: \qquad \lambda = \lambda'\sqrt{\frac{1-(v/c)}{1+(v/c)}} = (500\text{ nm})\sqrt{\frac{1-(0.280)}{1+(0.280)}} = 375\text{ nm}.$$

The temperature of the star is given by $\lambda_{\max}T = 2.898\times10^{-3}\text{ m}\cdot\text{K}$:

$$T = \frac{2.898\times10^{-3}\text{ m}\cdot\text{K}}{\lambda_{\max}}: \qquad T = \frac{2.898\times10^{-3}\text{ m}\cdot\text{K}}{375\times10^{-9}} = \boxed{7.73\times10^{3}\text{ K}}.$$

**P40.12** Planck's radiation law is $I(\lambda, T) = \dfrac{2\pi hc^2}{\lambda^5\left(e^{hc/\lambda k_B T} - 1\right)}$.

Using the series expansion $e^x = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \ldots$.

Planck's law reduces to $I(\lambda, T) = \dfrac{2\pi hc^2}{\lambda^5\left[\left(1 + hc/\lambda k_B T + \ldots\right) - 1\right]} \approx \dfrac{2\pi hc^2}{\lambda^5\left(hc/\lambda k_B T\right)} = \dfrac{2\pi c k_B T}{\lambda^4}$

which is the Rayleigh-Jeans law, for very long wavelengths.

## Section 40.2 The Photoelectric Effect

**P40.13** (a) $\lambda_c = \dfrac{hc}{\phi} = \dfrac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^8\text{ m/s}\right)}{(4.20\text{ eV})\left(1.60\times10^{-19}\text{ J/eV}\right)} = \boxed{296\text{ nm}}$

$$f_c = \frac{c}{\lambda_c} = \frac{3.00\times10^8\text{ m/s}}{296\times10^{-9}\text{ m}} = \boxed{1.01\times10^{15}\text{ Hz}}$$

(b) $\dfrac{hc}{\lambda} = \phi + e\Delta V_S$: $\dfrac{\left(6.626\times10^{-34}\right)\left(3.00\times10^8\right)}{180\times10^{-9}} = (4.20\text{ eV})\left(1.60\times10^{-19}\text{ J/eV}\right) + \left(1.60\times10^{-19}\right)\Delta V_S$

Therefore, $\boxed{\Delta V_S = 2.71\text{ V}}$

**P40.14** $K_{\text{max}} = \frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}\left(9.11\times10^{-31}\right)\left(4.60\times10^5\right)^2 = 9.64\times10^{-20}\text{ J} = 0.602\text{ eV}$

(a) $\phi = E - K_{\text{max}} = \dfrac{1\,240\text{ eV}\cdot\text{nm}}{625\text{ nm}} - 0.602\text{ nm} = \boxed{1.38\text{ eV}}$

(b) $f_c = \dfrac{\phi}{h} = \dfrac{1.38\text{ eV}}{6.626\times10^{-34}\text{ J}\cdot\text{s}}\left(\dfrac{1.60\times10^{-19}\text{ J}}{1\text{ eV}}\right) = \boxed{3.34\times10^{14}\text{ Hz}}$

**P40.15** (a) $\lambda_c = \dfrac{hc}{\phi}$ Li: $\lambda_c = \dfrac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^8\text{ m/s}\right)}{(2.30\text{ eV})\left(1.60\times10^{-19}\text{ J/eV}\right)} = 540\text{ nm}$

Be: $\lambda_c = \dfrac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^8\text{ m/s}\right)}{(3.90\text{ eV})\left(1.60\times10^{-19}\text{ J/eV}\right)} = 318\text{ nm}$

Hg: $\lambda_c = \dfrac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^8\text{ m/s}\right)}{(4.50\text{ eV})\left(1.60\times10^{-19}\text{ J/eV}\right)} = 276\text{ nm}$

$\lambda < \lambda_c$ for photo current. $\boxed{\text{Thus, only lithium will exhibit the photoelectric effect.}}$

(b) For lithium, $\dfrac{hc}{\lambda} = \phi + K_{\text{max}}$

$$\frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^8\text{ m/s}\right)}{400\times10^{-9}\text{ m}} = (2.30\text{ eV})\left(1.60\times10^{-19}\right) + K_{\text{max}}$$

$$K_{\text{max}} = 1.29\times10^{-19}\text{ J} = \boxed{0.808\text{ eV}}$$

**P40.16** From condition (i), $hf = e\left(\Delta V_{S1}\right) + \phi_1$ and $hf = e\left(\Delta V_{S2}\right) + \phi_2$

$\left(\Delta V_{S1}\right) = \left(\Delta V_{S2}\right) + 1.48\text{ V}$.

Then $\phi_2 - \phi_1 = 1.48\text{ eV}$.

From condition (ii), $hf_{c1} = \phi_1 = 0.600hf_{c2} = 0.600\phi_2$

$\phi_2 - 0.600\phi_2 = 1.48\text{ eV}$

$\boxed{\phi_2 = 3.70\text{ eV}}$ $\boxed{\phi_1 = 2.22\text{ eV}}$.

**P40.17** (a) $e\Delta V_S = \dfrac{hc}{\lambda} - \phi \rightarrow \phi = \dfrac{1\,240\text{ nm}\cdot\text{eV}}{546.1\text{ nm}} - 0.376\text{ eV} = \boxed{1.90\text{ eV}}$

(b) $e\Delta V_S = \dfrac{hc}{\lambda} - \phi = \dfrac{1\,240\text{ nm}\cdot\text{eV}}{587.5\text{ nm}} - 1.90\text{ eV} \rightarrow \boxed{\Delta V_S = 0.216\text{ V}}$

**P40.18** The energy needed is $E = 1.00 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$.

The energy absorbed in time interval $\Delta t$ is $E = \mathcal{P}\Delta t = IA\Delta t$

so $$\Delta t = \frac{E}{IA} = \frac{1.60 \times 10^{-19} \text{ J}}{\left(500 \text{ J/s}\cdot\text{m}^2\right)\left[\pi\left(2.82 \times 10^{-15} \text{ m}\right)^2\right]} = 1.28 \times 10^7 \text{ s} = \boxed{148 \text{ days}}.$$

The gross failure of the classical theory of the photoelectric effect contrasts with the success of quantum mechanics.

**P40.19** Ultraviolet photons will be absorbed to knock electrons out of the sphere with maximum kinetic energy $K_{\max} = hf - \phi$,

or $$K_{\max} = \frac{\left(6.626 \times 10^{-34} \text{ J}\cdot\text{s}\right)\left(3.00 \times 10^8 \text{ m/s}\right)}{200 \times 10^{-9} \text{ m}}\left(\frac{1.00 \text{ eV}}{1.60 \times 10^{-19} \text{ J}}\right) - 4.70 \text{ eV} = 1.51 \text{ eV}.$$

The sphere is left with positive charge and so with positive potential relative to $V = 0$ at $r = \infty$. As its potential approaches 1.51 V, no further electrons will be able to escape, but will fall back onto the sphere. Its charge is then given by

$$V = \frac{k_e Q}{r} \quad \text{or} \quad Q = \frac{rV}{k_e} = \frac{\left(5.00 \times 10^{-2} \text{ m}\right)\left(1.51 \text{ N}\cdot\text{m/C}\right)}{8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2} = \boxed{8.41 \times 10^{-12} \text{ C}}.$$

**P40.20** (a) By having the photon source move toward the metal, the incident photons are Doppler shifted to higher frequencies, and hence, higher energy.

(b) If $v = 0.280c$, $$f' = f\sqrt{\frac{1 + v/c}{1 - v/c}} = \left(7.00 \times 10^{14}\right)\sqrt{\frac{1.28}{0.720}} = 9.33 \times 10^{14} \text{ Hz}.$$

Therefore, $$\phi = \left(6.626 \times 10^{-34} \text{ J}\cdot\text{s}\right)\left(9.33 \times 10^{14} \text{ Hz}\right) = 6.18 \times 10^{-19} \text{ J} = \boxed{3.87 \text{ eV}}.$$

(c) At $v = 0.900c$, $f = 3.05 \times 10^{15} \text{ Hz}$

and $$K_{\max} = hf - \phi = \left(6.626 \times 10^{-34} \text{ J}\cdot\text{s}\right)\left(3.05 \times 10^{15} \text{ Hz}\right)\left(\frac{1.00 \text{ eV}}{1.60 \times 10^{-19} \text{ J}}\right) - 3.87 \text{ eV} = \boxed{8.78 \text{ eV}}.$$

## Section 40.3 **The Compton Effect**

**P40.21** $$E = \frac{hc}{\lambda} = \frac{\left(6.626 \times 10^{-34} \text{ J}\cdot\text{s}\right)\left(3.00 \times 10^8 \text{ m/s}\right)}{700 \times 10^{-9} \text{ m}} = 2.84 \times 10^{-19} \text{ J} = \boxed{1.78 \text{ eV}}$$

$$p = \frac{h}{\lambda} = \frac{6.626 \times 10^{-34} \text{ J}\cdot\text{s}}{700 \times 10^{-9} \text{ m}} = \boxed{9.47 \times 10^{-28} \text{ kg}\cdot\text{m/s}}$$

**P40.22** (a) $\Delta\lambda = \frac{h}{m_e c}(1-\cos\theta)$: $\Delta\lambda = \frac{6.626\times10^{-34}}{\left(9.11\times10^{-31}\right)\left(3.00\times10^{8}\right)}(1-\cos 37.0°) = \boxed{4.88\times10^{-13}\text{ m}}$

(b) $E_0 = \frac{hc}{\lambda_0}$: $\left(300\times10^{3}\text{ eV}\right)\left(1.60\times10^{-19}\text{ J/eV}\right) = \frac{\left(6.626\times10^{-34}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{\lambda_0}$

$$\lambda_0 = 4.14\times10^{-12}\text{ m}$$

and $\lambda' = \lambda_0 + \Delta\lambda = 4.63\times10^{-12}\text{ m}$

$$E' = \frac{hc}{\lambda'} = \frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{4.63\times10^{-12}\text{ m}} = 4.30\times10^{-14}\text{ J} = \boxed{268\text{ keV}}$$

(c) $K_e = E_0 - E' = 300\text{ keV} - 268.5\text{ keV} = \boxed{31.5\text{ keV}}$

**P40.23** With $K_e = E'$, $K_e = E_0 - E'$ gives $E' = E_0 - E'$

$E' = \frac{E_0}{2}$ and $\lambda' = \frac{hc}{E'}$ $\qquad \lambda' = \frac{hc}{E_0/2} = 2\frac{hc}{E_0} = 2\lambda_0$

$\lambda' = \lambda_0 + \lambda_C(1-\cos\theta)$ $\qquad 2\lambda_0 = \lambda_0 + \lambda_C(1-\cos\theta)$

$$1-\cos\theta = \frac{\lambda_0}{\lambda_C} = \frac{0.001\,60}{0.002\,43} \qquad \theta = \boxed{70.0°}$$

**P40.24** This is Compton scattering through 180°:

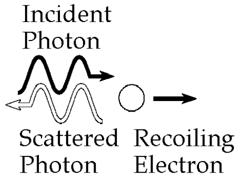

FIG. P40.24

$$E_0 = \frac{hc}{\lambda_0} = \frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{\left(0.110\times10^{-9}\text{ m}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)} = 11.3\text{ keV}$$

$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\theta) = \left(2.43\times10^{-12}\text{ m}\right)(1-\cos 180°) = 4.86\times10^{-12}\text{ m}$$

$\lambda' = \lambda_0 + \Delta\lambda = 0.115\text{ nm}$ so $\qquad E' = \frac{hc}{\lambda'} = 10.8\text{ keV}$.

By conservation of momentum for the photon-electron system, $\frac{h}{\lambda_0}\hat{\mathbf{i}} = \frac{h}{\lambda'}\left(-\hat{\mathbf{i}}\right) + p_e\hat{\mathbf{i}}$

and $\qquad p_e = h\left(\frac{1}{\lambda_0} + \frac{1}{\lambda'}\right)$

$$p_e = \left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(\frac{\left(3.00\times10^{8}\text{ m/s}\right)/c}{1.60\times10^{-19}\text{ J/eV}}\right)\left(\frac{1}{0.110\times10^{-9}\text{ m}} + \frac{1}{0.115\times10^{-9}\text{ m}}\right) = \boxed{\frac{22.1\text{ keV}}{c}}.$$

By conservation of system energy, $\qquad 11.3\text{ keV} = 10.8\text{ keV} + K_e$

so that $\qquad \boxed{K_e = 478\text{ eV}}$.

Check: $E^2 = p^2c^2 + m_e^2c^4$ or $\qquad \left(m_ec^2 + K_e\right)^2 = (pc)^2 + \left(m_ec^2\right)^2$

$$(511\text{ keV} + 0.478\text{ keV})^2 = (22.1\text{ keV})^2 + (511\text{ keV})^2$$

$$2.62\times10^{11} = 2.62\times10^{11}$$

**P40.25** (a) Conservation of momentum in the $x$ direction gives: $p_\gamma = p'_\gamma \cos\theta + p_e \cos\phi$

or since $\theta = \phi$, $$\frac{h}{\lambda_0} = \left(p_e + \frac{h}{\lambda'}\right)\cos\theta. \qquad [1]$$

Conservation of momentum in the $y$ direction gives: $0 = p'_\gamma \sin\theta - p_e \sin\theta$,

which (neglecting the trivial solution $\theta = 0$) gives: $$p_e = p'_\gamma = \frac{h}{\lambda'}. \qquad [2]$$

Substituting [2] into [1] gives: $\frac{h}{\lambda_0} = \frac{2h}{\lambda'}\cos\theta$, or $$\lambda' = 2\lambda_0 \cos\theta. \qquad [3]$$

Then the Compton equation is $$\lambda' - \lambda_0 = \frac{h}{m_e c}(1 - \cos\theta)$$

giving $$2\lambda_0 \cos\theta - \lambda_0 = \frac{h}{m_e c}(1 - \cos\theta)$$

or $$2\cos\theta - 1 = \frac{hc}{\lambda_0}\frac{1}{m_e c^2}(1 - \cos\theta).$$

Since $E_\gamma = \frac{hc}{\lambda_0}$, this may be written as: $$2\cos\theta - 1 = \left(\frac{E_\gamma}{m_e c^2}\right)(1 - \cos\theta)$$

which reduces to: $$\left(2 + \frac{E_\gamma}{m_e c^2}\right)\cos\theta = 1 + \frac{E_\gamma}{m_e c^2}$$

or $\cos\theta = \frac{m_e c^2 + E_\gamma}{2m_e c^2 + E_\gamma} = \frac{0.511\text{ MeV} + 0.880\text{ MeV}}{1.02\text{ MeV} + 0.880\text{ MeV}} = 0.732$ so that $\boxed{\theta = \phi = 43.0^\circ}$.

(b) Using Equation (3): $E'_\gamma = \frac{hc}{\lambda'} = \frac{hc}{\lambda_0(2\cos\theta)} = \frac{E_\gamma}{2\cos\theta} = \frac{0.880\text{ MeV}}{2\cos 43.0^\circ} = 0.602\text{ MeV} = \boxed{602\text{ keV}}$.

Then, $p'_\gamma = \frac{E'_\gamma}{c} = \frac{0.602\text{ MeV}}{c} = \boxed{3.21 \times 10^{-22}\ \text{kg}\cdot\text{m/s}}$.

(c) From Equation (2), $p_e = p'_\gamma = \frac{0.602\text{ MeV}}{c} = \boxed{3.21 \times 10^{-22}\ \text{kg}\cdot\text{m/s}}$.

From energy conservation: $K_e = E_\gamma - E'_\gamma = 0.880\text{ MeV} - 0.602\text{ MeV} = 0.278\text{ MeV} = \boxed{278\text{ keV}}$.

**P40.26** The energy of the incident photon is $E_0 = p_\gamma c = \dfrac{hc}{\lambda_0}$.

(a) Conserving momentum in the $x$ direction gives

$$p_\lambda = p_e \cos\phi + p'_\gamma \cos\theta, \text{ or since } \phi = \theta,\ \frac{E_0}{c} = \left(p_e + p'_\gamma\right)\cos\theta. \quad [1]$$

Conserving momentum in the $y$ direction (with $\phi = 0$) yields

$$0 = p'_\gamma \sin\theta - p_e \sin\theta, \text{ or } p_e = p'_\gamma = \frac{h}{\lambda'}. \quad [2]$$

Substituting Equation [2] into Equation [1] gives

$$\frac{E_0}{c} = \left(\frac{h}{\lambda'} + \frac{h}{\lambda'}\right)\cos\theta, \text{ or } \lambda' = \frac{2hc}{E_0}\cos\theta. \quad [3]$$

By the Compton equation, $\lambda' - \lambda_0 = \dfrac{h}{m_e c}(1 - \cos\theta)$, $\quad \dfrac{2hc}{E_0}\cos\theta - \dfrac{2hc}{E_0} = \dfrac{h}{m_e c}(1 - \cos\theta)$

which reduces to $\quad \left(2m_e c^2 + E_0\right)\cos\theta = m_e c^2 + E_0$.

Thus, $\quad \boxed{\phi = \theta = \cos^{-1}\left(\dfrac{m_e c^2 + E_0}{2m_e c^2 + E_0}\right)}$.

(b) From Equation [3], $\quad \lambda' = \dfrac{2hc}{E_0}\cos\theta = \dfrac{2hc}{E_0}\left(\dfrac{m_e c^2 + E_0}{2m_e c^2 + E_0}\right)$.

Therefore, $\quad E'_\gamma = \dfrac{hc}{\lambda'} = \dfrac{hc}{(2hc/E_0)\left(m_e c^2 + E_0\right)/\left(2m_e c^2 + E_0\right)} = \boxed{\dfrac{E_0}{2}\left(\dfrac{2m_e c^2 + E_0}{m_e c^2 + E_0}\right)}$,

and $\quad p'_\gamma = \dfrac{E'_\gamma}{c} = \boxed{\dfrac{E_0}{2c}\left(\dfrac{2m_e c^2 + E_0}{m_e c^2 + E_0}\right)}$.

(c) From conservation of energy, $K_e = E_0 - E'_\gamma = E_0 - \dfrac{E_0}{2}\left(\dfrac{2m_e c^2 + E_0}{m_e c^2 + E_0}\right)$

or $\quad K_e = \dfrac{E_0}{2}\left(\dfrac{2m_e c^2 + 2E_0 - 2m_e c^2 - E_0}{m_e c^2 + E_0}\right) = \boxed{\dfrac{E_0^2}{2\left(m_e c^2 + E_0\right)}}$.

Finally, from Equation (2), $p_e = p'_\gamma = \boxed{\dfrac{E_0}{2c}\left(\dfrac{2m_e c^2 + E_0}{m_e c^2 + E_0}\right)}$.

***P40.27** The electron's kinetic energy is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}9.11\times10^{-31}\ \text{kg}\left(2.18\times10^{6}\ \text{m/s}\right)^2 = 2.16\times10^{-18}\ \text{J}.$$

This is the energy lost by the photon, $hf_0 - hf'$

$\frac{hc}{\lambda_0} - \frac{hc}{\lambda'} = 2.16\times10^{-18}\ \text{J}$. We also have

$$\lambda' - \lambda_0 = \frac{h}{m_e c}(1-\cos\theta) = \frac{6.63\times10^{-34}\ \text{Js s}}{9.11\times10^{-31}\ \text{kg}\left(3\times10^{8}\ \text{m}\right)}(1-\cos 17.4°)$$

$$\lambda' = \lambda_0 + 1.11\times10^{-13}\ \text{m}$$

(a) Combining the equations by substitution,

$$\frac{1}{\lambda_0} - \frac{1}{\lambda_0 + 0.111\ \text{pm}} = \frac{2.16\times10^{-18}\ \text{J s}}{6.63\times10^{-34}\ \text{Js}\left(3\times10^{8}\ \text{m}\right)} = 1.09\times10^{7}/\text{m}$$

$$\frac{\lambda_0 + 0.111\ \text{pm} - \lambda_0}{\lambda_0^2 + \lambda_0(0.111\ \text{pm})} = 1.09\times10^{7}/\text{m}$$

$$0.111\ \text{pm} = \left(1.09\times10^{7}/\text{m}\right)\lambda_0^2 + 1.21\times10^{-6}\lambda_0$$

$$1.09\times10^{7}\lambda_0^2 + 1.21\times10^{-6}\ \text{m}\lambda_0 - 1.11\times10^{-13}\ \text{m}^2 = 0$$

$$\lambda_0 = \frac{-1.21\times10^{-6}\ \text{m} \pm \sqrt{\left(1.21\times10^{-6}\ \text{m}\right)^2 - 4\left(1.09\times10^{7}\right)\left(-1.11\times10^{-13}\ \text{m}^2\right)}}{2\left(1.09\times10^{7}\right)}$$

only the positive answer is physical: $\lambda_0 = \boxed{1.01\times10^{-10}\ \text{m}}$.

(b) Then $\lambda' = 1.01\times10^{-10}\ \text{m} + 1.11\times10^{-13}\ \text{m} = 1.01\times10^{-10}\ \text{m}$.
Conservation of momentum in the transverse direction:

$$0 = \frac{h}{\lambda'}\sin\theta - \gamma m_e v\sin\phi$$

$$\frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{1.01\times10^{-10}\ \text{m}}\sin 17.4° = \frac{9.11\times10^{-31}\ \text{kg}\left(2.18\times10^{6}\ \text{m/s}\right)\sin\phi}{\sqrt{1-\left(2.18\times10^{6}/3\times10^{8}\right)^2}}$$

$1.96\times10^{-24} = 1.99\times10^{-24}\sin\phi \qquad \phi = \boxed{81.1°}$

**P40.28** (a) Thanks to Compton we have four equations in the unknowns $\phi$, $v$, and $\lambda'$:

$$\frac{hc}{\lambda_0}=\frac{hc}{\lambda'}+\gamma m_e c^2-m_e c^2 \qquad \text{(energy conservation)} \qquad [1]$$

$$\frac{h}{\lambda_0}=\frac{h}{\lambda'}\cos 2\phi+\gamma m_e v\cos\phi \qquad \text{(momentum in } x \text{ direction)} \qquad [2]$$

$$0=\frac{h}{\lambda'}\sin 2\phi-\gamma m_e v\sin\phi \qquad \text{(momentum in } y \text{ direction)} \qquad [3]$$

$$\lambda'-\lambda_0=\frac{h}{m_e c}(1-\cos 2\phi) \qquad \text{(Compton equation).} \qquad [4]$$

Using $\sin 2\phi=2\sin\phi\cos\phi$ in Equation [3] gives $\gamma m_e v=\frac{2h}{\lambda'}\cos\phi$.

Substituting this into Equation [2] and using $\cos 2\phi=2\cos^2\phi-1$ yields

$$\frac{h}{\lambda_0}=\frac{h}{\lambda'}\left(2\cos^2\phi-1\right)+\frac{2h}{\lambda'}\cos^2\phi=\frac{h}{\lambda'}\left(4\cos^2\phi-1\right),$$

or $\quad \lambda'=4\lambda_0\cos^2\phi-\lambda_0$. [5]

Substituting the last result into the Compton equation gives

$$4\lambda_0\cos^2\phi-2\lambda_0=\frac{h}{m_e c}\left[1-\left(2\cos^2\phi-1\right)\right]=2\frac{hc}{m_e c^2}\left(1-\cos^2\phi\right).$$

With the substitution $\lambda_0=\frac{hc}{E_0}$, this reduces to

$$\cos^2\phi=\frac{m_e c^2+E_0}{2m_e c^2+E_0}=\frac{1+x}{2+x} \text{ where } x\equiv\frac{E_0}{m_e c^2}.$$

For $x=\frac{0.700\text{ MeV}}{0.511\text{ MeV}}=1.37$, this gives $\phi=\cos^{-1}\sqrt{\frac{1+x}{2+x}}=\boxed{33.0°}$.

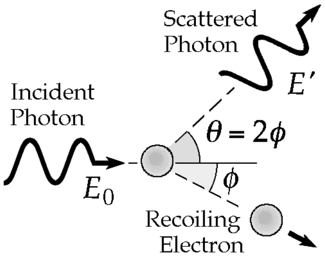

FIG. P40.28(a)

(b) From Equation [5], $\lambda'=\lambda_0\left(4\cos^2\phi-1\right)=\lambda_0\left[4\left(\frac{1+x}{2+x}\right)-1\right]=\lambda_0\left(\frac{2+3x}{2+x}\right)$.

Then, Equation [1] becomes

$$\frac{hc}{\lambda_0}=\frac{hc}{\lambda_0}\left(\frac{2+x}{2+3x}\right)+\gamma m_e c^2-m_e c^2 \text{ or } \frac{E_0}{m_e c^2}-\frac{E_0}{m_e c^2}\left(\frac{2+x}{2+3x}\right)+1=\gamma.$$

Thus, $\gamma=1+x-x\left(\frac{2+x}{2+3x}\right)$, and with $x=1.37$ we get $\gamma=1.614$.

Therefore, $\frac{v}{c}=\sqrt{1-\gamma^{-2}}=\sqrt{1-0.384}=0.785$ or $v=\boxed{0.785c}$.

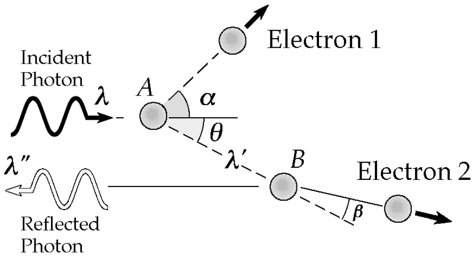

FIG. P40.29

**P40.29** $$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

$$\lambda'' - \lambda' = \frac{h}{m_e c}\left[1 - \cos(\pi - \theta)\right]$$

$$\lambda'' - \lambda = \frac{h}{m_e c} - \frac{h}{m_e c}\cos(\pi - \theta) + \frac{h}{m_e c} - \frac{h}{m_e c}\cos\theta$$

Now $\cos(\pi - \theta) = -\cos\theta$, so $\lambda'' - \lambda = 2\frac{h}{m_e c} = \boxed{0.004\,86 \text{ nm}}$.

**P40.30** Maximum energy loss appears as maximum increase in wavelength, which occurs for scattering angle 180°. Then $\Delta\lambda = (1 - \cos 180°)\left(\frac{h}{mc}\right) = \frac{2h}{mc}$ where $m$ is the mass of the target particle. The fractional energy loss is

$$\frac{E_0 - E'}{E_0} = \frac{hc/\lambda_0 - hc/\lambda'}{hc/\lambda_0} = \frac{\lambda' - \lambda_0}{\lambda'} = \frac{\Delta\lambda}{\lambda_0 + \Delta\lambda} = \frac{2h/mc}{\lambda_0 + 2h/mc}.$$

Further, $\lambda_0 = \frac{hc}{E_0}$, so $\frac{E_0 - E'}{E_0} = \frac{2h/mc}{hc/E_0 + 2h/mc} = \frac{2E_0}{mc^2 + 2E_0}$.

(a) For scattering from a free electron, $mc^2 = 0.511$ MeV, so

$$\frac{E_0 - E'}{E_0} = \frac{2(0.511 \text{ MeV})}{0.511 \text{ MeV} + 2(0.511 \text{ MeV})} = \boxed{0.667}.$$

(b) For scattering from a free proton, $mc^2 = 938$ MeV, and

$$\frac{E_0 - E'}{E_0} = \frac{2(0.511 \text{ MeV})}{938 \text{ MeV} + 2(0.511 \text{ MeV})} = \boxed{0.001\,09}.$$

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## Section 40.4 **Photons and Electromagnetic Waves**

***P40.31** With photon energy 10.0 eV $= hf$

$$f = \frac{10.0\left(1.6 \times 10^{-19} \text{ J}\right)}{6.63 \times 10^{-34} \text{ J} \cdot \text{s}} = 2.41 \times 10^{15} \text{ Hz}.$$

Any electromagnetic wave with frequency higher than $2.41 \times 10^{15}$ Hz counts as ionizing radiation. This includes far ultraviolet light, x-rays, and gamma rays.

***P40.32** The photon energy is $E=\frac{hc}{\lambda}=\frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}\left(3\times10^{8}\ \text{m/s}\right)}{633\times10^{-9}\ \text{m}}=3.14\times10^{-19}\ \text{J}$. The power carried by the beam is $\left(2\times10^{18}\ \text{photons/s}\right)\left(3.14\times10^{-19}\ \text{J/photon}\right)=0.628\ \text{W}$. Its intensity is the average Poynting vector $I=S_{\text{av}}=\frac{\mathscr{P}}{\pi r^2}=\frac{0.628\ \text{W}(4)}{\pi\left(1.75\times10^{-3}\ \text{m}\right)^2}=2.61\times10^{5}\ \text{W/m}^2$.

(a) $S_{\text{av}}=\frac{1}{\mu_0}E_{\text{rms}}B_{\text{rms}}\sin 90°=\frac{1}{\mu_0}\frac{E_{\text{max}}}{\sqrt{2}}\frac{B_{\text{max}}}{\sqrt{2}}$. Also $E_{\text{max}}=B_{\text{max}}c$. So $S_{\text{av}}=\frac{E_{\text{max}}^2}{2\mu_0 c}$.

$$E_{\text{max}}=\left(2\mu_0 cS_{\text{av}}\right)^{1/2}=\left(2\left(4\pi\times10^{-7}\ \text{Tm/A}\right)\left(3\times10^{8}\ \text{m/s}\right)\left(2.61\times10^{5}\ \text{W/m}^2\right)\right)^{1/2}$$

$$=\boxed{1.40\times10^{4}\ \text{N/C}}$$

$$B_{\text{max}}=\frac{1.40\times10^{4}\ \text{N/C}}{3\times10^{8}\ \text{m/s}}=\boxed{4.68\times10^{-5}\ \text{T}}$$

(b) Each photon carries momentum $\frac{E}{c}$. The beam transports momentum at the rate $\frac{\mathscr{P}}{c}$. It imparts momentum to a perfectly reflecting surface at the rate

$$\frac{2\mathscr{P}}{c}=\text{force}=\frac{2(0.628\ \text{W})}{3\times10^{8}\ \text{m/s}}=\boxed{4.19\times10^{-9}\ \text{N}}.$$

(c) The block of ice absorbs energy $mL=\mathscr{P}\Delta t$ melting

$$m=\frac{\mathscr{P}\Delta t}{L}=\frac{0.628\ \text{W}(1.5\times3\,600\ \text{s})}{3.33\times10^{5}\ \text{J/kg}}=\boxed{1.02\times10^{-2}\ \text{kg}}.$$

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## Section 40.5 **The Wave Properties of Particles**

**P40.33** $$\lambda=\frac{h}{p}=\frac{h}{mv}=\frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{\left(1.67\times10^{-27}\ \text{kg}\right)\left(1.00\times10^{6}\ \text{m/s}\right)}=\boxed{3.97\times10^{-13}\ \text{m}}$$

**P40.34** (a)
$$\frac{p^2}{2m}=(50.0)\left(1.60\times10^{-19}\ \text{J}\right)$$
$$p=3.81\times10^{-24}\ \text{kg}\cdot\text{m/s}$$
$$\lambda=\frac{h}{p}=\boxed{0.174\ \text{nm}}$$

(b)
$$\frac{p^2}{2m}=\left(50.0\times10^{3}\right)\left(1.60\times10^{-19}\ \text{J}\right)$$
$$p=1.20\times10^{-22}\ \text{kg}\cdot\text{m/s}$$
$$\lambda=\frac{h}{p}=5.49\times10^{-12}\ \text{m}$$

The relativistic answer is slightly more precise:

$$\lambda=\frac{h}{p}=\frac{hc}{\left[\left(mc^2+K\right)^2-m^2c^4\right]^{1/2}}=\boxed{5.37\times10^{-12}\ \text{m}}.$$

**P40.35** (a) Electron: $\lambda = \frac{h}{p}$ and $K = \frac{1}{2}m_e v^2 = \frac{m_e^2 v^2}{2m_e} = \frac{p^2}{2m_e}$ so $p = \sqrt{2m_e K}$

and

$$\lambda = \frac{h}{\sqrt{2m_e K}} = \frac{6.626 \times 10^{-34}\ \text{J}\cdot\text{s}}{\sqrt{2\left(9.11 \times 10^{-31}\ \text{kg}\right)(3.00)\left(1.60 \times 10^{-19}\ \text{J}\right)}}$$

$$\lambda = 7.09 \times 10^{-10}\ \text{m} = \boxed{0.709\ \text{nm}}.$$

(b) Photon: $\lambda = \frac{c}{f}$ and $E = hf$ so $f = \frac{E}{h}$

and

$$\lambda = \frac{hc}{E} = \frac{\left(6.626 \times 10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00 \times 10^{8}\ \text{m/s}\right)}{3\left(1.60 \times 10^{-19}\ \text{J}\right)} = 4.14 \times 10^{-7}\ \text{m} = \boxed{414\ \text{nm}}.$$

**P40.36** (a) The wavelength of a non-relativistic particle of mass $m$ is given by $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}$ where the kinetic energy $K$ is in joules. If the neutron kinetic energy $K_n$ is given in electron volts, its kinetic energy in joules is $K = \left(1.60 \times 10^{-19}\ \text{J/eV}\right)K_n$ and the equation for the wavelength becomes

$$\lambda = \frac{h}{\sqrt{2mK}} = \frac{6.626 \times 10^{-34}\ \text{J}\cdot\text{s}}{\sqrt{2\left(1.67 \times 10^{-27}\ \text{kg}\right)\left(1.60 \times 10^{-19}\ \text{J/eV}\right)K_n}} = \boxed{\frac{2.87 \times 10^{-11}}{\sqrt{K_n}}\ \text{m}}$$

where $K_n$ is expressed in electron volts.

(b) If $K_n = 1.00\ \text{keV} = 1\,000\ \text{eV}$, then $\lambda = \frac{2.87 \times 10^{-11}}{\sqrt{1\,000}}\ \text{m} = 9.07 \times 10^{-13}\ \text{m} = \boxed{907\ \text{fm}}.$

**P40.37** (a) $\lambda \sim 10^{-14}\ \text{m}$ or less. $p = \frac{h}{\lambda} \sim \frac{6.6 \times 10^{-34}\ \text{J}\cdot\text{s}}{10^{-14}\ \text{m}} = 10^{-19}\ \text{kg}\cdot\text{m/s}$ or more.

The energy of the electron is $E = \sqrt{p^2c^2 + m_e^2c^4} \sim \sqrt{\left(10^{-19}\right)^2\left(3 \times 10^{8}\right)^2 + \left(9 \times 10^{-31}\right)^2\left(3 \times 10^{8}\right)^4}$

or $E \sim 10^{-11}\ \text{J} \sim 10^{8}\ \text{eV}$ or more,

so that $K = E - m_ec^2 \sim 10^{8}\ \text{eV} - \left(0.5 \times 10^{6}\ \text{eV}\right)\boxed{\sim 10^{8}\ \text{eV}}$ or more.

(b) The electric potential energy of the electron-nucleus system would be

$$U_e = \frac{k_eq_1q_2}{r} \sim \frac{\left(9 \times 10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(10^{-19}\ \text{C}\right)(-e)}{10^{-14}\ \text{m}} \sim -10^{5}\ \text{eV}.$$

With its $K + U_e >> 0$, $\boxed{\text{the electron would immediately escape the nucleus}}$.

**P40.38** From the condition for Bragg reflection,

$$m\lambda = 2d\sin\theta = 2d\cos\left(\frac{\phi}{2}\right).$$

But $d = a\sin\left(\frac{\phi}{2}\right)$

where $a$ is the lattice spacing.

Thus, with $m=1$, $\lambda = 2a\sin\left(\frac{\phi}{2}\right)\cos\left(\frac{\phi}{2}\right) = a\sin\phi$

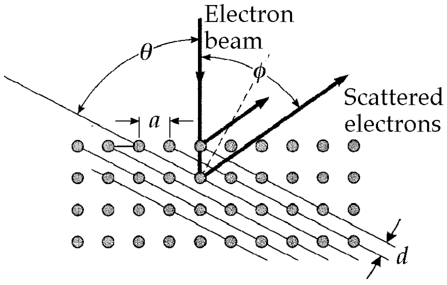

FIG. P40.38

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_eK}} \qquad \lambda = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{\sqrt{2\left(9.11\times10^{-31}\ \text{kg}\right)\left(54.0\times1.60\times10^{-19}\ \text{J}\right)}} = 1.67\times10^{-10}\ \text{m}.$$

Therefore, the lattice spacing is $a = \frac{\lambda}{\sin\phi} = \frac{1.67\times10^{-10}\ \text{m}}{\sin 50.0°} = 2.18\times10^{-10} = \boxed{0.218\ \text{nm}}$.

**P40.39** (a) $E^2 = p^2c^2 + m^2c^4$

with $E = hf$, $p = \frac{h}{\lambda}$ and $mc = \frac{h}{\lambda_C}$

so $h^2f^2 = \frac{h^2c^2}{\lambda^2} + \frac{h^2c^2}{\lambda_C^2}$ and $\left(\frac{f}{c}\right)^2 = \frac{1}{\lambda^2} + \frac{1}{\lambda_C^2}$ (Eq. 1).

(b) For a photon $\frac{f}{c} = \frac{1}{\lambda}$.

The third term $\frac{1}{\lambda_C}$ in Equation 1 for electrons and other massive particles shows that

$\boxed{\text{they will always have a different frequency from photons of the same wavelength}}$.

***P40.40** For the massive particle, $K = (\gamma-1)mc^2$ and $\lambda_m = \frac{h}{p} = \frac{h}{\gamma mv}$. For the photon (which we represent as $\gamma$), $E = K$ and $\lambda_\gamma = \frac{c}{f} = \frac{ch}{E} = \frac{ch}{K} = \frac{ch}{(\gamma-1)mc^2}$. Then the ratio is $\frac{\lambda_\gamma}{\lambda_m} = \frac{ch\gamma mv}{(\gamma-1)mc^2h} = \frac{\gamma}{\gamma-1}\frac{v}{c}$.

(a) $$\frac{\lambda_\gamma}{\lambda_m} = \frac{1(0.9)}{\sqrt{1-0.9^2}\left[\left(1/\sqrt{1-0.9^2}\right)-1\right]} = \boxed{1.60}$$

(b) $$\frac{\lambda_\gamma}{\lambda_m} = \frac{1(0.001)}{\sqrt{1-(0.001)^2}\left[\left(1/\sqrt{1-(0.001)^2}\right)-1\right]} = \boxed{2.00\times10^3}$$

(c) As $\frac{v}{c}\to1$, $\gamma\to\infty$ and $\gamma-1$ becomes nearly equal to $\gamma$. Then $\frac{\lambda_\gamma}{\lambda_m}\to\frac{\gamma}{\gamma}1 = \boxed{1}$.

(d) As $\frac{v}{c}\to0$, $\left(1-\frac{v^2}{c^2}\right)^{-1/2} - 1 \approx 1-\left(-\frac{1}{2}\right)\frac{v^2}{c^2} - 1 = \frac{1}{2}\frac{v^2}{c^2}$ and $\frac{\lambda_\gamma}{\lambda_m}\to 1\frac{v/c}{(1/2)\left(v^2/c^2\right)} = \frac{2c}{v}\to\boxed{\infty}$.

**P40.41** $\lambda=\frac{h}{p}$ $\qquad p=\frac{h}{\lambda}=\frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{1.00\times10^{-11}\text{ m}}=6.63\times10^{-23}\text{ kg}\cdot\text{m/s}$

(a) electrons: $K_e=\frac{p^2}{2m_e}=\frac{\left(6.63\times10^{-23}\right)^2}{2\left(9.11\times10^{-31}\right)}\text{ J}=15.1\text{ keV}$

The relativistic answer is more precisely correct:

$$K_e=\sqrt{p^2c^2+m_e^2c^4}-m_ec^2=\boxed{14.9\text{ keV}}.$$

(b) photons: $E_\gamma=pc=\left(6.63\times10^{-23}\right)\left(3.00\times10^8\right)=\boxed{124\text{ keV}}$

**P40.42** (a) The wavelength of the student is $\lambda=\frac{h}{p}=\frac{h}{mv}$. If $w$ is the width of the diffracting aperture,

then we need $\qquad w\le10.0\lambda=10.0\left(\frac{h}{mv}\right)$

so that $\qquad v\le10.0\frac{h}{mw}=10.0\left(\frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{(80.0\text{ kg})(0.750\text{ m})}\right)=\boxed{1.10\times10^{-34}\text{ m/s}}.$

(b) Using $\Delta t=\frac{d}{v}$ we get: $\qquad \Delta t\ge\frac{0.150\text{ m}}{1.10\times10^{-34}\text{ m/s}}=\boxed{1.36\times10^{33}\text{ s}}.$

(c) $\boxed{\text{No}}$. The minimum time to pass through the door is over $10^{15}$ times the age of the Universe.

## Section 40.6 The Quantum Particle

***P40.43** $E=K=\frac{1}{2}mu^2=hf$ and $\lambda=\frac{h}{mu}$.

$$v_{\text{phase}}=f\lambda=\frac{mu^2}{2h}\frac{h}{mu}=\boxed{\frac{u}{2}=v_{\text{phase}}}.$$

This is different from the speed $u$ at which the particle transports mass, energy, and momentum.

***P40.44** As a bonus, we begin by proving that the phase speed $v_p=\frac{\omega}{k}$ is not the speed of the particle.

$$v_p=\frac{\omega}{k}=\frac{\sqrt{p^2c^2+m^2c^4}\hbar}{\hbar\gamma mv}=\frac{\sqrt{\gamma^2m^2v^2c^2+m^2c^4}}{\sqrt{\gamma^2m^2v^2}}=c\sqrt{1+\frac{c^2}{\gamma^2v^2}}=c\sqrt{1+\frac{c^2}{v^2}\left(1-\frac{v^2}{c^2}\right)}=c\sqrt{1+\frac{c^2}{v^2}-1}=\frac{c^2}{v}$$

In fact, the phase speed is larger than the speed of light. A point of constant phase in the wave function carries no mass, no energy, and no information.

Now for the group speed:

*continued on next page*

$$v_g=\frac{d\omega}{dk}=\frac{d\hbar\omega}{d\hbar k}=\frac{dE}{dp}=\frac{d}{dp}\sqrt{m^2c^4+p^2c^2}$$

$$v_g=\frac{1}{2}\left(m^2c^4+p^2c^2\right)^{-1/2}\left(0+2pc^2\right)=\sqrt{\frac{p^2c^4}{p^2c^2+m^2c^4}}$$

$$v_g=c\sqrt{\frac{\gamma^2m^2v^2}{\gamma^2m^2v^2+m^2c^2}}=c\sqrt{\frac{v^2/\left(1-v^2/c^2\right)}{v^2/\left(1-v^2/c^2\right)+c^2}}=c\sqrt{\frac{v^2/\left(1-v^2/c^2\right)}{\left(v^2+c^2-v^2\right)/\left(1-v^2/c^2\right)}}=v$$

It is this speed at which mass, energy, and momentum are transported.

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## Section 40.7 The Double-Slit Experiment Revisited

**P40.45** (a) $\lambda=\frac{h}{mv}=\frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{\left(1.67\times10^{-27}\text{ kg}\right)(0.400\text{ m/s})}=\boxed{9.92\times10^{-7}\text{ m}}$

(b) For destructive interference in a multiple-slit experiment, $d\sin\theta=\left(m+\frac{1}{2}\right)\lambda$, with $m=0$ for the first minimum.

Then, $\theta=\sin^{-1}\left(\frac{\lambda}{2d}\right)=0.028\,4°$

so $\frac{y}{L}=\tan\theta$ $y=L\tan\theta=(10.0\text{ m})(\tan 0.028\,4°)=\boxed{4.96\text{ mm}}$.

(c) We cannot say the neutron passed through one slit. We can only say it passed through the slits.

**P40.46** Consider the first bright band away from the center:

$d\sin\theta=m\lambda$ $\left(6.00\times10^{-8}\text{ m}\right)\sin\left(\tan^{-1}\left[\frac{0.400}{200}\right]\right)=(1)\lambda=1.20\times10^{-10}\text{ m}$

$\lambda=\frac{h}{m_ev}$ so $m_ev=\frac{h}{\lambda}$

and $K=\frac{1}{2}m_ev^2=\frac{m_e^2v^2}{2m_e}=\frac{h^2}{2m_e\lambda^2}=e\Delta V$

$\Delta V=\frac{h^2}{2em_e\lambda^2}$ $\Delta V=\frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)^2}{2\left(1.60\times10^{-19}\text{ C}\right)\left(9.11\times10^{-31}\text{ kg}\right)\left(1.20\times10^{-10}\text{ m}\right)^2}=\boxed{105\text{ V}}$.

***P40.47** We find the speed of each electron from energy conservation in the firing process:

$$0=K_f+U_f=\frac{1}{2}mv^2-\text{eV}$$

$$v=\sqrt{\frac{2eV}{m}}=\sqrt{\frac{2\times1.6\times10^{-19}\text{ C}(45\text{ V})}{9.11\times10^{-31}\text{ kg}}}=3.98\times10^{6}\text{ m/s}$$

The time of flight is $\Delta t=\frac{\Delta x}{v}=\frac{0.28\text{ m}}{3.98\times10^6\text{ m/s}}=7.04\times10^{-8}\text{ s}$. The current when electrons are 28 cm apart is $I=\frac{q}{t}=\frac{e}{\Delta t}=\frac{1.6\times10^{-19}\text{ C}}{7.04\times10^{-8}\text{ s}}=\boxed{2.27\times10^{-12}\text{ A}}$.

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## Section 40.8 The Uncertainty Principle

**P40.48** (a) $\Delta p \Delta x = m\Delta v \Delta x \geq \frac{\hbar}{2}$ so $\Delta v \geq \frac{h}{4\pi m\Delta x} = \frac{2\pi\,\text{J}\cdot\text{s}}{4\pi(2.00\text{ kg})(1.00\text{ m})} = \boxed{0.250\text{ m/s}}$.

(b) The duck might move by $(0.25\text{ m/s})(5\text{ s}) = 1.25\text{ m}$. With original position uncertainty of 1.00 m, we can think of $\Delta x$ growing to $1.00\text{ m} + 1.25\text{ m} = \boxed{2.25\text{ m}}$.

**P40.49** For the electron, $\Delta p = m_e \Delta v = (9.11\times10^{-31}\text{ kg})(500\text{ m/s})(1.00\times10^{-4}) = 4.56\times10^{-32}\text{ kg}\cdot\text{m/s}$

$$\Delta x = \frac{h}{4\pi\,\Delta p} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{4\pi(4.56\times10^{-32}\text{ kg}\cdot\text{m/s})} = \boxed{1.16\text{ mm}}.$$

For the bullet, $\Delta p = m\Delta v = (0.020\,0\text{ kg})(500\text{ m/s})(1.00\times10^{-4}) = 1.00\times10^{-3}\text{ kg}\cdot\text{m/s}$

$$\Delta x = \frac{h}{4\pi\,\Delta p} = \boxed{5.28\times10^{-32}\text{ m}}.$$

**P40.50** $\frac{\Delta y}{x} = \frac{\Delta p_y}{p_x}$ and $d\Delta p_y \geq \frac{h}{4\pi}$.

Eliminate $\Delta p_y$ and solve for $x$.

$$x = 4\pi p_x(\Delta y)\frac{d}{h}: \qquad x = 4\pi(1.00\times10^{-3}\text{ kg})(100\text{ m/s})(1.00\times10^{-2}\text{ m})\frac{(2.00\times10^{-3}\text{ m})}{(6.626\times10^{-34}\text{ J}\cdot\text{s})}$$

The answer, $x = \boxed{3.79\times10^{28}\text{ m}}$, is 190 times greater than the diameter of the Universe!

**P40.51** With $\Delta x = 2\times10^{-15}$ m m, the uncertainty principle requires $\Delta p_x \geq \frac{\hbar}{2\Delta x} = 2.6\times10^{-20}\text{ kg}\cdot\text{m/s}$.

The average momentum of the particle bound in a stationary nucleus is zero. The uncertainty in momentum measures the root-mean-square momentum, so we take $p_{rms} \approx 3\times10^{-20}\text{ kg}\cdot\text{m/s}$ . For an electron, the non-relativistic approximation $p = m_e v$ would predict $v \approx 3\times10^{10}\text{ m/s}$, while $v$ cannot be greater than $c$.

Thus, a better solution would be $E = \left[(m_e c^2)^2 + (pc)^2\right]^{1/2} \approx 56\text{ MeV} = \gamma\, m_e c^2$

$$\gamma \approx 110 = \frac{1}{\sqrt{1 - v^2/c^2}} \qquad \text{so} \qquad v \approx 0.999\,96c.$$

For a proton, $v = \frac{p}{m}$ gives $v = 1.8\times10^{7}\text{ m/s}$, less than one-tenth the speed of light.

***P40.52** (a) $K = \frac{1}{2}mv^2 = \frac{(mv)^2}{2m} = \frac{p^2}{2m}$

(b) To find the minimum kinetic energy, think of the minimum momentum uncertainty, and maximum position uncertainty of $10^{-15}\text{ m} = \Delta x$. We model the proton as moving along a straight line with $\Delta p \Delta x = \frac{\hbar}{2}$, $\Delta p = \frac{\hbar}{2\Delta x}$. The average momentum is zero. The average squared momentum is equal to the squared uncertainty:

$$K = \frac{p^2}{2m} = \frac{(\Delta p)^2}{2m} = \frac{\hbar^2}{4(\Delta x)^2 2m} = \frac{h^2}{32\pi^2(\Delta x)^2 m} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})^2}{32\pi^2(10^{-15}\text{ m})^2 1.67\times10^{-27}\text{ kg}} = 8.33\times10^{-13}\text{ J}$$

$$= \boxed{5.21\text{ MeV}}$$

**P40.53** (a) At the top of the ladder, the woman holds a pellet inside a small region $\Delta x_i$. Thus, the uncertainty principle requires her to release it with typical horizontal momentum $\Delta p_x = m\Delta v_x = \frac{\hbar}{2\Delta x_i}$. It falls to the floor in a travel time given by $H = 0 + \frac{1}{2}gt^2$ as $t = \sqrt{\frac{2H}{g}}$, so the total width of the impact points is

$$\Delta x_f = \Delta x_i + (\Delta v_x)t = \Delta x_i + \left(\frac{\hbar}{2m\Delta x_i}\right)\sqrt{\frac{2H}{g}} = \Delta x_i + \frac{A}{\Delta x_i}$$

where $A = \frac{\hbar}{2m}\sqrt{\frac{2H}{g}}$.

To minimize $\Delta x_f$, we require $\frac{d(\Delta x_f)}{d(\Delta x_i)} = 0$ or $1 - \frac{A}{\Delta x_i^2} = 0$

so $\Delta x_i = \sqrt{A}$.

The minimum width of the impact points is

$$(\Delta x_f)_{\min} = \left(\Delta x_i + \frac{A}{\Delta x_i}\right)\Bigg|_{\Delta x_i = \sqrt{A}} = 2\sqrt{A} = \boxed{\sqrt{\frac{2\hbar}{m}}\left(\frac{2H}{g}\right)^{1/4}}.$$

(b) $$(\Delta x_f)_{\min} = \left[\frac{2(1.054\,6\times10^{-34}\ \text{J}\cdot\text{s})}{5.00\times10^{-4}\ \text{kg}}\right]^{1/2}\left[\frac{2(2.00\ \text{m})}{9.80\ \text{m/s}^2}\right]^{1/4} = \boxed{5.19\times10^{-16}\ \text{m}}$$

## Additional Problems

**P40.54** $\Delta V_S = \left(\frac{h}{e}\right)f - \frac{\phi}{e}$

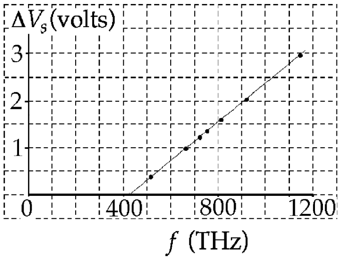

**FIG. P40.54**

From two points on the graph $0 = \left(\frac{h}{e}\right)(4.1\times10^{14}\ \text{Hz}) - \frac{\phi}{e}$

and $3.3\ \text{V} = \left(\frac{h}{e}\right)(12\times10^{14}\ \text{Hz}) - \frac{\phi}{e}$.

Combining these two expressions we find:

(a) $\phi = \boxed{1.7\ \text{eV}}$

(b) $\frac{h}{e} = \boxed{4.2\times10^{-15}\ \text{V}\cdot\text{s}}$

(c) At the cutoff wavelength $\frac{hc}{\lambda_c} = \phi = \left(\frac{h}{e}\right)\frac{ec}{\lambda_c}$

$$\lambda_c = (4.2\times10^{-15}\ \text{V}\cdot\text{s})(1.6\times10^{-19}\ \text{C})\frac{(3\times10^8\ \text{m/s})}{(1.7\ \text{eV})(1.6\times10^{-19}\ \text{J/eV})} = \boxed{730\ \text{nm}}$$

**P40.55** We want an Einstein plot of $K_{\max}$ versus $f$

| $\lambda$, nm | $f$, $10^{14}$ Hz | $K_{\max}$, eV |
|---|---|---|
| 588 | 5.10 | 0.67 |
| 505 | 5.94 | 0.98 |
| 445 | 6.74 | 1.35 |
| 399 | 7.52 | 1.63 |

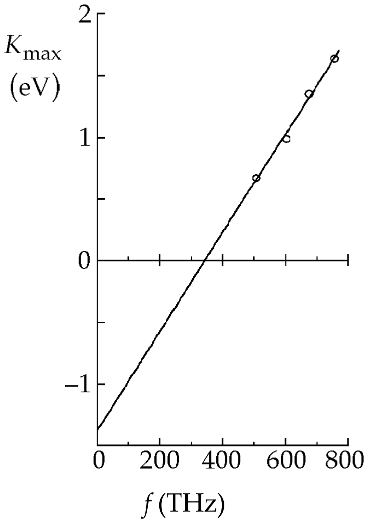

FIG. P40.55

(a) $$\text{slope} = \frac{0.402\text{ eV}}{10^{14}\text{ Hz}} \pm 8\%$$

(b) $$e\Delta V_S = hf - \phi$$

$$h = (0.402)\left(\frac{1.60\times10^{-19}\text{ J}\cdot\text{s}}{10^{14}}\right) = \boxed{6.4\times10^{-34}\text{ J}\cdot\text{s}\pm 8\%}$$

(c) $$K_{\max} = 0$$

at $f \approx 344\times10^{12}$ Hz

$$\phi = hf = 2.32\times10^{-19}\text{ J} = \boxed{1.4\text{ eV}}$$

**P40.56** From the path the electrons follow in the magnetic field, the maximum kinetic energy is seen to be:

$$K_{\max} = \frac{e^2B^2R^2}{2m_e}.$$

From the photoelectric equation, $K_{\max} = hf - \phi = \frac{hc}{\lambda} - \phi$.

Thus, the work function is $\phi = \frac{hc}{\lambda} - K_{\max} = \boxed{\frac{hc}{\lambda} - \frac{e^2B^2R^2}{2m_e}}$.

**P40.57** $$\Delta\lambda = \frac{h}{m_pc}(1-\cos\theta) = \frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)}{\left(1.67\times10^{-27}\text{ kg}\right)\left(3.00\times10^{8}\text{ m/s}\right)}(0.234) = 3.09\times10^{-16}\text{ m}$$

$$\lambda_0 = \frac{hc}{E_0} = \frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{(200\text{ MeV})\left(1.60\times10^{-13}\text{ J/MeV}\right)} = 6.20\times10^{-15}\text{ m}$$

$$\lambda' = \lambda_0 + \Delta\lambda = 6.51\times10^{-15}\text{ m}$$

(a) $$E_\gamma = \frac{hc}{\lambda'} = \boxed{191\text{ MeV}}$$

(b) $$K_p = \boxed{9.20\text{ MeV}}$$

**P40.58** Isolate the terms involving $\phi$ in Equations 40.13 and 40.14. Square and add to eliminate $\phi$.

$$h^2\left[\frac{1}{\lambda_0^2}+\frac{1}{\lambda'^2}-\frac{2\cos\theta}{\lambda_0\lambda'}\right]=\gamma^2 m_e^2 v^2$$

Solve for $\frac{v^2}{c^2}=\frac{b}{\left(b+c^2\right)}$:

$$b=\frac{h^2}{m_e^2}\left[\frac{1}{\lambda_0^2}+\frac{1}{\lambda'^2}-\frac{2\cos\theta}{\lambda_0\lambda'}\right].$$

Substitute into Eq. 40.12:

$$1+\left(\frac{h}{m_e c}\right)\left[\frac{1}{\lambda_0}-\frac{1}{\lambda'}\right]=\gamma=\left(1-\frac{b}{b+c^2}\right)^{-1/2}=\sqrt{\frac{c^2+b}{c^2}}.$$

Square each side:

$$c^2+\frac{2hc}{m_e}\left[\frac{1}{\lambda_0}-\frac{1}{\lambda'}\right]+\frac{h^2}{m_e^2}\left[\frac{1}{\lambda_0}-\frac{1}{\lambda'}\right]^2=c^2+\left(\frac{h^2}{m_e^2}\right)\left[\frac{1}{\lambda_0^2}+\frac{1}{\lambda'^2}-\frac{2\cos\theta}{\lambda_0\lambda'}\right].$$

From this we get Eq. 40.11:

$$\lambda'-\lambda_0=\left(\frac{h}{m_e c}\right)[1-\cos\theta].$$

**P40.59** Show that if all of the energy of a photon is transmitted to an electron, momentum will not be conserved.

Energy:

$$\frac{hc}{\lambda_0}=\frac{hc}{\lambda'}+K_e=m_e c^2(\gamma-1)\text{ if }\frac{hc}{\lambda'}=0 \tag{1}$$

Momentum:

$$\frac{h}{\lambda_0}=\frac{h}{\lambda'}+\gamma\, m_e v=\gamma\, m_e v\text{ if }\lambda'=\infty \tag{2}$$

From (1),

$$\gamma=\frac{h}{\lambda_0 m_e c}+1 \tag{3}$$

$$v=c\sqrt{1-\left(\frac{\lambda_0 m_e c}{h+\lambda_0 m_e c}\right)^2} \tag{4}$$

Substitute (3) and (4) into (2) and show the inconsistency:

$$\frac{h}{\lambda_0}=\left(1+\frac{h}{\lambda_0 m_e c}\right)m_e c\sqrt{1-\left(\frac{\lambda_0 m_e c}{h+\lambda_0 m_e c}\right)^2}=\frac{\lambda_0 m_e c+h}{\lambda_0}\sqrt{\frac{h(h+2\lambda_0 m_e c)}{(h+\lambda_0 m_e c)^2}}=\frac{h}{\lambda_0}\sqrt{\frac{h+2\lambda_0 m_e c}{h}}.$$

This is impossible, so all of the energy of a photon cannot be transmitted to an electron.

**P40.60** Begin with momentum expressions: $p=\frac{h}{\lambda}$, and $p=\gamma\, mv=\gamma\, mc\left(\frac{v}{c}\right)$.

Equating these expressions,

$$\gamma\left(\frac{v}{c}\right)=\left(\frac{h}{mc}\right)\frac{1}{\lambda}=\frac{\lambda_C}{\lambda}.$$

Thus,

$$\frac{(v/c)^2}{1-(v/c)^2}=\left(\frac{\lambda_C}{\lambda}\right)^2$$

or

$$\left(\frac{v}{c}\right)^2=\left(\frac{\lambda_C}{\lambda}\right)^2-\left(\frac{\lambda_C}{\lambda}\right)^2\left(\frac{v}{c}\right)^2$$

$$\frac{v^2}{c^2}=\frac{(\lambda_C/\lambda)^2}{1+(\lambda_C/\lambda)^2}=\frac{1}{(\lambda/\lambda_C)^2+1}$$

giving

$$\boxed{v=\frac{c}{\sqrt{1+(\lambda/\lambda_C)^2}}}.$$

**P40.61** (a) Starting with Planck's law, $I(\lambda,\,T)=\dfrac{2\pi hc^2}{\lambda^5\left[e^{hc/\lambda k_\mathrm{B}T}-1\right]}$

the total power radiated per unit area $\displaystyle\int_0^\infty I(\lambda,\,T)d\lambda=\int_0^\infty \frac{2\pi hc^2}{\lambda^5\left[e^{hc/\lambda k_\mathrm{B}T}-1\right]}d\lambda$.

Change variables by letting $x=\dfrac{hc}{\lambda k_\mathrm{B}T}$

and $dx=-\dfrac{hc\,d\lambda}{k_\mathrm{B}T\lambda^2}$.

Note that as $\lambda$ varies from $0\to\infty$, $x$ varies from $\infty\to 0$.

Then $\displaystyle\int_0^\infty I(\lambda,\,T)d\lambda=-\frac{2\pi k_\mathrm{B}^4T^4}{h^3c^2}\int_\infty^0\frac{x^3}{\left(e^x-1\right)}dx=\frac{2\pi k_\mathrm{B}^4T^4}{h^3c^2}\left(\frac{\pi^4}{15}\right)$.

Therefore, $\boxed{\displaystyle\int_0^\infty I(\lambda,\,T)d\lambda=\left(\frac{2\pi^5k_\mathrm{B}^4}{15h^3c^2}\right)T^4=\sigma T^4}$.

(b) From part (a), $\sigma=\dfrac{2\pi^5k_\mathrm{B}^4}{15h^3c^2}=\dfrac{2\pi^5\left(1.38\times10^{-23}\ \mathrm{J/K}\right)^4}{15\left(6.626\times10^{-34}\ \mathrm{J\cdot s}\right)^3\left(3.00\times10^8\ \mathrm{m/s}\right)^2}$

$\sigma=\boxed{5.67\times10^{-8}\ \mathrm{W/m^2\cdot K^4}}$.

**P40.62** Planck's law states $I(\lambda,\,T)=\dfrac{2\pi hc^2}{\lambda^5\left[e^{hc/\lambda k_\mathrm{B}T}-1\right]}=2\pi hc^2\lambda^{-5}\left[e^{hc/\lambda k_\mathrm{B}T}-1\right]^{-1}$.

To find the wavelength at which this distribution has a maximum, compute

$$\frac{dI}{d\lambda}=2\pi hc^2\left\{-5\lambda^{-6}\left[e^{hc/\lambda k_\mathrm{B}T}-1\right]^{-1}-\lambda^{-5}\left[e^{hc/\lambda k_\mathrm{B}T}-1\right]^{-2}e^{hc/\lambda k_\mathrm{B}T}\left(-\frac{hc}{\lambda^2k_\mathrm{B}T}\right)\right\}=0$$

$$\frac{dI}{d\lambda}=\frac{2\pi hc^2}{\lambda^6\left[e^{hc/\lambda k_\mathrm{B}T}-1\right]}\left\{-5+\frac{hc}{\lambda k_\mathrm{B}T}\frac{e^{hc/\lambda k_\mathrm{B}T}}{\left[e^{hc/\lambda k_\mathrm{B}T}-1\right]}\right\}=0$$

Letting $x=\dfrac{hc}{\lambda k_\mathrm{B}T}$, the condition for a maximum becomes $\dfrac{xe^x}{e^x-1}=5$.

We zero in on the solution to this transcendental equation by iterations as shown in the table below. The solution is found to be

| $x$ | $xe^x/\left(e^x-1\right)$ |
|---|---|
| 4.000 00 | 4.074 629 4 |
| 4.500 00 | 4.550 552 1 |
| 5.000 00 | 5.033 918 3 |
| 4.900 00 | 4.936 762 0 |
| 4.950 00 | 4.985 313 0 |
| 4.975 00 | 5.009 609 0 |
| 4.963 00 | 4.997 945 2 |
| 4.969 00 | 5.003 776 7 |
| 4.966 00 | 5.000 860 9 |
| 4.964 50 | 4.999 403 0 |
| 4.965 50 | 5.000 374 9 |
| 4.965 00 | 4.999 889 0 |
| 4.965 25 | 5.000 132 0 |
| 4.965 13 | 5.000 015 3 |
| 4.965 07 | 4.999 957 0 |
| 4.965 10 | 4.999 986 2 |
| 4.965 115 | 5.000 000 8 |
| | |

$x=\dfrac{hc}{\lambda_\mathrm{max}k_\mathrm{B}T}=4.965\,115$ and $\lambda_\mathrm{max}T=\dfrac{hc}{4.965\,115k_\mathrm{B}}$.

*continued on next page*

Thus, $\lambda_{\max}T = \dfrac{\left(6.626\,075\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(2.997\,925\times10^{8}\ \text{m/s}\right)}{4.965\,115\left(1.380\,658\times10^{-23}\ \text{J/K}\right)} = \boxed{2.897\,755\times10^{-3}\ \text{m}\cdot\text{K}}$.

This result is very close to Wien's experimental value of $\lambda_{\max}T = 2.898\times10^{-3}\ \text{m}\cdot\text{K}$ for this constant.

**P40.63** (a) Planck's radiation law predicts maximum intensity at a wavelength $\lambda_{\max}$ we find from

$$\frac{dI}{d\lambda} = 0 = \frac{d}{d\lambda}\left\{2\pi hc^2\lambda^{-5}\left[e^{(hc/\lambda k_BT)}-1\right]^{-1}\right\}$$

$$0 = 2\pi hc^2\lambda^{-5}(-1)\left[e^{(hc/\lambda k_BT)}-1\right]^{-2}e^{(hc/\lambda k_BT)}\left(\frac{-hc}{\lambda^2k_BT}\right)+2\pi hc^2(-5)\lambda^{-6}\left[e^{(hc/\lambda k_BT)}-1\right]^{-1}$$

or

$$\frac{-hce^{(hc/\lambda k_BT)}}{\lambda^7k_BT\left[e^{(hc/\lambda k_BT)}-1\right]^2}+\frac{5}{\lambda^6\left[e^{(hc/\lambda k_BT)}-1\right]}=0$$

which reduces to

$$5\left(\frac{\lambda k_BT}{hc}\right)\left[e^{(hc/\lambda k_BT)}-1\right]=e^{(hc/\lambda k_BT)}.$$

Define $x = \dfrac{hc}{\lambda k_BT}$. Then we require $5e^x-5=xe^x$.

Numerical solution of this transcendental equation gives $x = 4.965$ to four digits. So $\lambda_{\max} = \dfrac{hc}{4.965k_BT}$, in agreement with Wien's law.

The intensity radiated over all wavelengths is $\displaystyle\int_0^\infty I(\lambda,T)d\lambda = A+B = \int_0^\infty\frac{2\pi hc^2d\lambda}{\lambda^5\left[e^{(hc/\lambda k_BT)}-1\right]}$.

Again, define $x = \dfrac{hc}{\lambda k_BT}$ so $\lambda = \dfrac{hc}{xk_BT}$ and $d\lambda = -\dfrac{hc}{x^2k_BT}dx$.

Then, $\displaystyle A+B = \int_{x=\infty}^{0}\frac{-2\pi hc^2x^5k_B^5T^5hcdx}{h^5c^5x^2k_BT\left(e^x-1\right)} = \frac{2\pi k_B^4T^4}{h^3c^2}\int_0^\infty\frac{x^3dx}{\left(e^x-1\right)}$.

The integral is tabulated as $\dfrac{\pi^4}{15}$, so (in agreement with Stefan's law) $A+B = \dfrac{2\pi^5k_B^4T^4}{15h^3c^2}$.

The intensity radiated over wavelengths shorter than $\lambda_{\max}$ is

$$\int_0^{\lambda_{\max}} I(\lambda,T)d\lambda = A = \int_0^{\lambda_{\max}}\frac{2\pi hc^2d\lambda}{\lambda^5\left[e^{(hc/\lambda k_BT)}-1\right]}.$$

With $x = \dfrac{hc}{\lambda k_BT}$, this similarly becomes $\displaystyle A = \frac{2\pi k_B^4T^4}{h^3c^2}\int_{4.965}^{\infty}\frac{x^3dx}{e^x-1}$.

So the fraction of power or of intensity radiated at wavelengths shorter than $\lambda_{\max}$ is

$$\frac{A}{A+B} = \frac{\left(2\pi k_B^4T^4/h^3c^2\right)\left[\pi^4/15-\int_0^{4.965}\left[x^3dx/\left(e^x-1\right)\right]\right]}{2\pi^5\,k_B^4T^4/15h^3c^2} = \boxed{1-\frac{15}{\pi^4}\int_0^{4.965}\frac{x^3dx}{e^x-1}}.$$

*continued on next page*

(b) Here are some sample values of the integrand, along with a sketch of the curve:

| $x$ | $x^3\left(e^x-1\right)^{-1}$ |
|---|---|
| 0.000 | 0.00 |
| 0.100 | $9.51\times10^{-3}$ |
| 0.200 | $3.61\times10^{-2}$ |
| 1.00 | 0.582 |
| 2.00 | 1.25 |
| 3.00 | 1.42 |
| 4.00 | 1.19 |
| 4.90 | 0.883 |
| 4.965 | 0.860 |

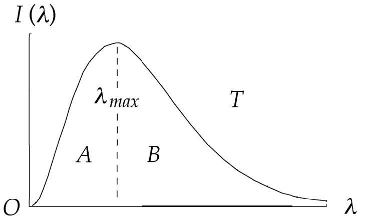

**FIG. P40.63(b)**

Approximating the integral by trapezoids gives $\frac{A}{A+B}\approx1-\frac{15}{\pi^4}(4.870)=\boxed{0.250\,1}$.

**P40.64** $p=mv=\sqrt{2mE}=\sqrt{2\left(1.67\times10^{-27}\text{ kg}\right)(0.040\,0\text{ eV})\left(1.60\times10^{-19}\text{ J/eV}\right)}$

$\lambda=\frac{h}{mv}=1.43\times10^{-10}\text{ m}=\boxed{0.143\text{ nm}}$

This is of the same order of magnitude as the spacing between atoms in a crystal so diffraction should appear.

**P40.65** $\lambda_C=\frac{h}{m_ec}$ and $\lambda=\frac{h}{p}$: $\frac{\lambda_C}{\lambda}=\frac{h/m_ec}{h/p}=\frac{p}{m_ec}$;

$E^2=c^2p^2+\left(m_ec^2\right)^2$: $p=\sqrt{\frac{E^2}{c^2}-\left(m_ec\right)^2}$

$$\frac{\lambda_C}{\lambda}=\frac{1}{m_ec}\sqrt{\frac{E^2}{c^2}-\left(m_ec\right)^2}=\sqrt{\frac{1}{\left(m_ec\right)^2}\left[\frac{E^2}{c^2}-\left(m_ec\right)^2\right]}=\sqrt{\left(\frac{E}{m_ec^2}\right)^2-1}$$

**P40.66** (a) $mgy_i=\frac{1}{2}mv_f^2$

$v_f=\sqrt{2gy_i}=\sqrt{2\left(9.80\text{ m/s}^2\right)(50.0\text{ m})}=31.3\text{ m/s}$

$\lambda=\frac{h}{mv}=\frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{(75.0\text{ kg})(31.3\text{ m/s})}=\boxed{2.82\times10^{-37}\text{ m}}$ (not observable)

(b) $\Delta E\Delta t\geq\frac{\hbar}{2}$

so $\Delta E\geq\frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{4\pi\left(5.00\times10^{-3}\text{ s}\right)}=\boxed{1.06\times10^{-32}\text{ J}}$

(c) $\frac{\Delta E}{E}=\frac{1.06\times10^{-32}\text{ J}}{(75.0\text{ kg})\left(9.80\text{ m/s}^2\right)(50.0\text{ m})}=\boxed{2.87\times10^{-35}\%}$

**P40.67** From the uncertainty principle $\Delta E\Delta t \geq \frac{\hbar}{2}$

or $\Delta\left(mc^2\right)\Delta t = \frac{\hbar}{2}$.

Therefore,

$$\frac{\Delta m}{m} = \frac{h}{4\pi c^2(\Delta t)m} = \frac{h}{4\pi(\Delta t)E_R}$$

$$\frac{\Delta m}{m} = \frac{6.626\times10^{-34}\ \mathrm{J\cdot s}}{4\pi\left(8.70\times10^{-17}\ \mathrm{s}\right)(135\ \mathrm{MeV})}\left(\frac{1\ \mathrm{MeV}}{1.60\times10^{-13}\ \mathrm{J}}\right) = \boxed{2.81\times10^{-8}}.$$

**P40.68** $\Delta\lambda = \frac{h}{m_e c}(1-\cos\theta) = \lambda' - \lambda_0$

$$E' = \frac{hc}{\lambda'} = \frac{hc}{\lambda_0 + \Delta\lambda} = hc\left[\lambda_0 + \frac{h}{m_e c}(1-\cos\theta)\right]^{-1}$$

$$E' = \frac{hc}{\lambda_0}\left[1 + \frac{hc}{m_e c^2\lambda_0}(1-\cos\theta)\right]^{-1}$$

$$E' = \frac{hc}{\lambda_0}\left[1 + \frac{hc}{m_e c^2\lambda_0}(1-\cos\theta)\right]^{-1} = E_0\left[1 + \frac{E_0}{m_e c^2}(1-\cos\theta)\right]^{-1}$$

**P40.69** (a) The light is unpolarized. It contains both horizontal and vertical field oscillations.

(b) The interference pattern appears, but with diminished overall intensity.

(c) The results are the same in each case.

(d) The interference pattern appears and disappears as the polarizer turns, with alternately increasing and decreasing contrast between the bright and dark fringes. The intensity on the screen is precisely zero at the center of a dark fringe four times in each revolution, when the filter axis has turned by 45°, 135°, 225°, and 315° from the vertical.

(e) Looking at the overall light energy arriving at the screen, we see a low-contrast interference pattern. After we sort out the individual photon runs into those for trial 1, those for trial 2, and those for trial 3, we have the original results replicated: The runs for trials 1 and 2 form the two blue graphs in Figure 40.24 in the text, and the runs for trial 3 build up the red graph.

**P40.70** Let $u'$ represent the final speed of the electron and let $\gamma' = \left(1 - \frac{u'^2}{c^2}\right)^{-1/2}$. We must eliminate $\beta$ and $u'$ from the three conservation equations:

$$\frac{hc}{\lambda_0} + \gamma\, m_e c^2 = \frac{hc}{\lambda'} + \gamma' m_e c^2 \qquad [1]$$

$$\frac{h}{\lambda_0} + \gamma m_e u - \frac{h}{\lambda'}\cos\theta = \gamma' m_e u' \cos\beta \qquad [2]$$

$$\frac{h}{\lambda'}\sin\theta = \gamma' m_e u' \sin\beta \qquad [3]$$

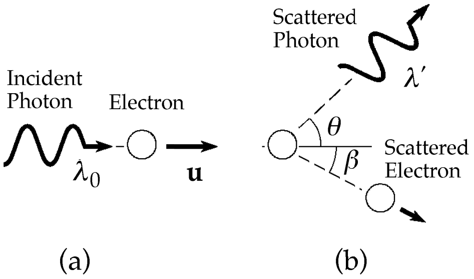

**FIG. P40.70**

Square Equations [2] and [3] and add:

$$\frac{h^2}{\lambda_0^2} + \gamma^2 m_e^2 u^2 + \frac{h^2}{\lambda'^2} + \frac{2h\gamma\, m_e u}{\lambda_0} - \frac{2h^2\cos\theta}{\lambda_0\lambda'} - \frac{2h\gamma\, m_e u\cos\theta}{\lambda'} = \gamma'^2 m_e^2 u'^2$$

$$\frac{h^2}{\lambda_0^2} + \frac{h^2}{\lambda'^2} + \gamma^2 m_e^2 u^2 + \frac{2h\gamma\, m_e u}{\lambda_0} - \frac{2h\gamma\, m_e u\cos\theta}{\lambda'} - \frac{2h^2\cos\theta}{\lambda_0\lambda'} = \frac{m_e^2 u'^2}{1 - u'^2/c^2}$$

Call the left-hand side $b$. Then $b - \frac{bu'^2}{c^2} = m_e^2 u'^2$ and $u'^2 = \frac{b}{m_e^2 + b/c^2} = \frac{c^2 b}{m_e^2 c^2 + b}$.

Now square Equation [1] and substitute to eliminate $\gamma'$:

$$\frac{h^2}{\lambda_0^2} + \gamma^2 m_e^2 c^2 + \frac{h^2}{\lambda'^2} + \frac{2h\gamma\, m_e c}{\lambda_0} - \frac{2h^2}{\lambda_0\lambda'} - \frac{2h\gamma\, m_e c}{\lambda'} = \frac{m_e^2 c^2}{1 - u'^2/c^2} = m_e^2 c^2 + b.$$

So we have $\frac{h^2}{\lambda_0^2} + \frac{h^2}{\lambda'^2} + \gamma^2 m_e^2 c^2 + \frac{2h\gamma\, m_e c}{\lambda_0} - \frac{2h\gamma\, m_e c}{\lambda'} - \frac{2h^2}{\lambda_0\lambda'}$

$$= m_e c^2 + \frac{h^2}{\lambda_0^2} + \frac{h^2}{\lambda'^2} + \gamma^2 m_e^2 u^2 + \frac{2h\gamma\, m_e u}{\lambda_0} - \frac{2h\gamma\, m_e u\cos\theta}{\lambda'} - \frac{2h^2\cos\theta}{\lambda_0\lambda'}$$

Multiply through by $\frac{\lambda_0\lambda'}{m_e^2 c^2}$

$$\lambda_0\lambda'\gamma^2 + \frac{2h\lambda'\gamma}{m_e c} - \frac{2h\lambda_0\gamma}{m_e c} - \frac{2h^2}{m_e^2 c^2} = \lambda_0\lambda' + \frac{\lambda_0\lambda'\gamma^2 u^2}{m_e c^2} + \frac{2h\lambda' u\gamma}{m_e c^2} - \frac{2h\gamma\lambda_0 u\cos\theta}{m_e c^2} - \frac{2h^2\cos\theta}{m_e^2 c^2}$$

$$\lambda_0\lambda'\left(\gamma^2 - 1 - \frac{\gamma^2 u^2}{c^2}\right) + \frac{2h\gamma\lambda'}{m_e c}\left(1 - \frac{u}{c}\right) = \frac{2h\gamma\lambda_0}{m_e c}\left(1 - \frac{u\cos\theta}{c}\right) + \frac{2h^2}{m_e^2 c^2}(1 - \cos\theta)$$

The first term is zero. Then $\lambda' = \lambda_0\left(\frac{1 - (u\cos\theta)/c}{1 - u/c}\right) + \frac{h\gamma^{-1}}{m_e c}\left(\frac{1}{1 - u/c}\right)(1 - \cos\theta).$

Since $\gamma^{-1} = \sqrt{1 - \left(\frac{u}{c}\right)^2} = \sqrt{\left(1 - \frac{u}{c}\right)\left(1 + \frac{u}{c}\right)}$

this result may be written as $\boxed{\lambda' = \lambda_0\left(\frac{1 - (u\cos\theta)/c}{1 - u/c}\right) + \frac{h}{m_e c}\sqrt{\frac{1 + u/c}{1 - u/c}}(1 - \cos\theta)}$.

It shows a specific combination of what looks like a Doppler shift and a Compton shift. This problem is about the same as the first problem in Albert Messiah's graduate text on quantum mechanics.

## ANSWERS TO EVEN PROBLEMS

| | |
|---|---|
| **P40.2** | (a) $\sim 10^{-7}$ m ultraviolet;<br>(b) $\sim 10^{-10}$ m gamma ray |
| **P40.4** | (a) 70.9 kW; (b) 580 nm;<br>(c) $7.99 \times 10^{10}$ W/m;<br>(d) $9.42 \times 10^{-1226}$ W/m;<br>(e) $1.00 \times 10^{-227}$ W/m; (f) $5.44 \times 10^{10}$ W/m;<br>(g) $7.38 \times 10^{10}$ W/m; (h) 0.260 W/m;<br>(i) $2.60 \times 10^{-9}$ W/m; (j) 20 kW |
| **P40.6** | $2.96 \times 10^{19}$ photons/s |
| **P40.8** | $5.71 \times 10^{3}$ photons |
| **P40.10** | $1.34 \times 10^{31}$ |
| **P40.12** | see the solution |
| **P40.14** | (a) 1.38 eV; (b) 334 THz |
| **P40.16** | Metal one: 2.22 eV, Metal two: 3.70 eV |
| **P40.18** | 148 d, the classical theory is a gross failure |
| **P40.20** | (a) The incident photons are Doppler shifted to higher frequencies, and hence, higher energy; (b) 3.87 eV; (c) 8.78 eV |
| **P40.22** | (a) 488 fm; (b) 268 keV; (c) 31.5 keV |
| **P40.24** | $p_e = \dfrac{22.1\text{ keV}}{c}$; $K_e = 478$ eV |
| **P40.26** | (a) $\cos^{-1}\left(\dfrac{m_e c^2 + E_0}{2m_e c^2 + E_0}\right)$;<br>(b) $E'_\gamma = \dfrac{E_0}{2}\left(\dfrac{2m_e c^2 + E_0}{m_e c^2 + E_0}\right)$,<br>$p'_\gamma = \dfrac{E_0}{2c}\left(\dfrac{2m_e c^2 + E_0}{m_e c^2 + E_0}\right)$;<br>(c) $K_e = \dfrac{E_0^2}{2\left(m_e c^2 + E_0\right)}$,<br>$p_e = \dfrac{E_0}{2c}\left(\dfrac{2m_e c^2 + E_0}{m_e c^2 + E_0}\right)$ |
| **P40.28** | (a) 33.0°; (b) $0.785c$ |
| **P40.30** | (a) 0.667; (b) 0.001 09 |
| **P40.32** | (a) 14.0 kV/m, 46.8 $\mu$T; (b) 4.19 nN;<br>(c) 10.2 g |
| **P40.34** | (a) 0.174 nm; (b) 5.37 pm or 5.49 pm ignoring relativistic correction |
| **P40.36** | (a) see the solution; (b) 907 fm |
| **P40.38** | 0.218 nm |
| **P40.40** | (a) 1.60; (b) $2.00 \times 10^{3}$; (c) 1; (d) $\infty$ |
| **P40.42** | (a) $1.10 \times 10^{-34}$ m/s; (b) $1.36 \times 10^{33}$ s; (c) no |
| **P40.44** | see the solution |
| **P40.46** | 105 V |
| **P40.48** | (a) 0.250 m/s; (b) 2.25 m |
| **P40.50** | $3.79 \times 10^{28}$ m, much larger than the diameter of the observable Universe |
| **P40.52** | (a) see the solution; (b) 5.21 MeV |
| **P40.54** | (a) 1.7 eV; (b) $4.2 \times 10^{-15}$ V·s; (c) 730 nm |
| **P40.56** | $\dfrac{hc}{\lambda} - \dfrac{e^2 B^2 R^2}{2m_e}$ |
| **P40.58** | see the solution |
| **P40.60** | see the solution |
| **P40.62** | see the solution |
| **P40.64** | 0.143 nm, comparable to the distance between atoms in a crystal, so diffraction can be observed |
| **P40.66** | (a) $2.82 \times 10^{-37}$ m; (b) $1.06 \times 10^{-32}$ J;<br>(c) $2.87 \times 10^{-35}$% |
| **P40.68** | see the solution |
| **P40.70** | see the solution |

# 41

# Quantum Mechanics

## CHAPTER OUTLINE

41.1 An Interpretation of Quantum Mechanics
41.2 A Particle in a Box
41.3 The Particle Under Boundary Conditions
41.4 The Schrödinger Equation
41.5 A Particle in a Well of Finite Height
41.6 Tunneling Through a Potential Energy Barrier
41.7 The Scanning Tunneling Microscope
41.8 The Simple Harmonic Oscillator

## ANSWERS TO QUESTIONS

**Q41.1** A particle's wave function represents its state, containing all the information there is about its location and motion. The squared absolute value of its wave function tells where we would classically think of the particle as a spending most its time. $|\Psi|^2$ is the probability distribution function for the position of the particle.

**Q41.2** The motion of the quantum particle does not consist of moving through successive points. The particle has no definite position. It can sometimes be found on one side of a node and sometimes on the other side, but never at the node itself. There is no contradiction here, for the quantum particle is moving as a wave. It is not a classical particle. In particular, the particle does not speed up to infinite speed to cross the node.

**Q41.3** Consider a particle bound to a restricted region of space. If its minimum energy were zero, then the particle could have zero momentum and zero uncertainty in its momentum. At the same time, the uncertainty in its position would not be infinite, but equal to the width of the region. In such a case, the uncertainty product $\Delta x \Delta p_x$ would be zero, violating the uncertainty principle. This contradiction proves that the minimum energy of the particle is not zero.

**Q41.4** The reflected amplitude decreases as $U$ decreases. The amplitude of the reflected wave is proportional to the reflection coefficient, $R$, which is $1-T$, where $T$ is the transmission coefficient as given in equation 41.20. As $U$ decreases, $C$ decreases as predicted by equation 41.21, $T$ increases, and $R$ decreases.

**Q41.5** Consider the Heisenberg uncertainty principle. It implies that electrons initially moving at the same speed and accelerated by an electric field through the same distance *need not* all have the same measured speed after being accelerated. Perhaps the philosopher could have said "it is necessary for the very existence of science that the same conditions always produce the same results within the uncertainty of the measurements."

**Q41.6** In quantum mechanics, particles are treated as wave functions, not classical particles. In classical mechanics, the kinetic energy is never negative. That implies that $E \geq U$. Treating the particle as a wave, the Schrödinger equation predicts that there is a nonzero probability that a particle can tunnel through a barrier—a region in which $E < U$.

**Q41.7** Consider Figure 41.8, (a) and (b) in the text. In the square well with infinitely high walls, the particle's simplest wave function has strict nodes separated by the length $L$ of the well. The particle's wavelength is $2L$, its momentum $\frac{h}{2L}$, and its energy $\frac{p^2}{2m}=\frac{h^2}{8mL^2}$. Now in the well with walls of only finite height, the wave function has nonzero amplitude at the walls. The wavelength is longer. The particle's momentum in its ground state is smaller, and its energy is less.

**Q41.8** Quantum mechanically, the lowest kinetic energy possible for any bound particle is greater than zero. The following is a proof: If its minimum energy were zero, then the particle could have zero momentum and zero uncertainty in its momentum. At the same time, the uncertainty in its position would not be infinite, but equal to the width of the region in which it is restricted to stay. In such a case, the uncertainty product $\Delta x \Delta p_x$ would be zero, violating the uncertainty principle. This contradiction proves that the minimum energy of the particle is not zero. Any harmonic oscillator can be modeled as a particle or collection of particles in motion; thus it cannot have zero energy.

**Q41.9** As Newton's laws are the rules which a particle of large mass follows in its motion, so the Schrödinger equation describes the motion of a quantum particle, a particle of small or large mass. In particular, the states of atomic electrons are confined-wave states with wave functions that are solutions to the Schrödinger equation.

## SOLUTIONS TO PROBLEMS

### Section 41.1 An Interpretation of Quantum Mechanics

**P41.1** (a) $\psi(x)=Ae^{i(5.00\times10^{10}x)}=A\cos\left(5\times10^{10}x\right)+Ai\sin\left(5\times10^{10}x\right)=A\cos(kx)+Ai\sin(kx)$ goes through a full cycle when $x$ changes by $\lambda$ and when $kx$ changes by $2\pi$. Then $k\lambda=2\pi$ where $k=5.00\times10^{10}\text{ m}^{-1}=\frac{2\pi}{\lambda}$. Then $\lambda=\frac{2\pi\text{ m}}{\left(5.00\times10^{10}\right)}=\boxed{1.26\times10^{-10}\text{ m}}$.

(b) $$p=\frac{h}{\lambda}=\frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{1.26\times10^{-10}\text{ m}}=\boxed{5.27\times10^{-24}\text{ kg}\cdot\text{m/s}}$$

(c) $m_e=9.11\times10^{-31}\text{ kg}$

$$K=\frac{m_e^2v^2}{2m_e}=\frac{p^2}{2m}=\frac{\left(5.27\times10^{-24}\text{ kg}\cdot\text{m/s}\right)^2}{\left(2\times9.11\times10^{-31}\text{ kg}\right)}=1.52\times10^{-17}\text{ J}=\frac{1.52\times10^{-17}\text{ J}}{1.60\times10^{-19}\text{ J/eV}}=\boxed{95.5\text{ eV}}$$

**P41.2** Probability $$P=\int_{-a}^{a}|\psi(x)|^2=\int_{-a}^{a}\frac{a}{\pi\left(x^2+a^2\right)}dx=\left(\frac{a}{\pi}\right)\left(\frac{1}{a}\right)\tan^{-1}\left(\frac{x}{a}\right)\Bigg|_{-a}^{a}$$

$$P=\frac{1}{\pi}\left[\tan^{-1}1-\tan^{-1}(-1)\right]=\frac{1}{\pi}\left[\frac{\pi}{4}-\left(-\frac{\pi}{4}\right)\right]=\boxed{\frac{1}{2}}$$

## Section 41.2 **A Particle in a Box**

**P41.3** $E_1 = 2.00 \text{ eV} = 3.20 \times 10^{-19} \text{ J}$

For the ground-state, $E_1 = \dfrac{h^2}{8m_eL^2}$.

(a) $$L = \frac{h}{\sqrt{8m_eE_1}} = 4.34 \times 10^{-10} \text{ m} = \boxed{0.434 \text{ nm}}$$

(b) $$\Delta E = E_2 - E_1 = 4\left(\frac{h^2}{8m_eL^2}\right) - \left(\frac{h^2}{8m_eL^2}\right) = \boxed{6.00 \text{ eV}}$$

**P41.4** For an electron wave to "fit" into an infinitely deep potential well, an integral number of half-wavelengths must equal the width of the well.

$$\frac{n\lambda}{2} = 1.00 \times 10^{-9} \text{ m} \qquad \text{so} \qquad \lambda = \frac{2.00 \times 10^{-9}}{n} = \frac{h}{p}$$

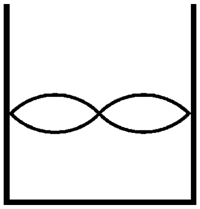

**FIG. P41.4**

(a) Since $$K = \frac{p^2}{2m_e} = \frac{\left(h^2/\lambda^2\right)}{2m_e} = \frac{h^2}{2m_e}\frac{n^2}{\left(2 \times 10^{-9}\right)^2} = \left(0.377n^2\right) \text{ eV}$$

For $K \approx 6$ eV $\boxed{n = 4}$

(b) With $n = 4$, $\boxed{K = 6.03 \text{ eV}}$

**P41.5** (a) We can draw a diagram that parallels our treatment of standing mechanical waves. In each state, we measure the distance $d$ from one node to another (N to N), and base our solution upon that:

Since $$d_{\text{N to N}} = \frac{\lambda}{2} \text{ and } \lambda = \frac{h}{p}$$

$$p = \frac{h}{\lambda} = \frac{h}{2d}.$$

Next, $$K = \frac{p^2}{2m_e} = \frac{h^2}{8m_ed^2} = \frac{1}{d^2}\left[\frac{\left(6.626 \times 10^{-34} \text{ J}\cdot\text{s}\right)^2}{8\left(9.11 \times 10^{-31} \text{ kg}\right)}\right].$$

Evaluating, $$K = \frac{6.02 \times 10^{-38} \text{ J}\cdot\text{m}^2}{d^2} \qquad K = \frac{3.77 \times 10^{-19} \text{ eV}\cdot\text{m}^2}{d^2}.$$

In state 1, $d = 1.00 \times 10^{-10}$ m $K_1 = 37.7$ eV.

In state 2, $d = 5.00 \times 10^{-11}$ m $K_2 = 151$ eV.

In state 3, $d = 3.33 \times 10^{-11}$ m $K_3 = 339$ eV.

In state 4, $d = 2.50 \times 10^{-11}$ m $K_4 = 603$ eV.

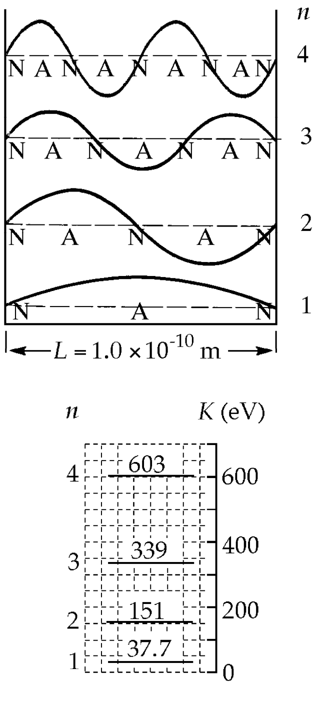

**FIG. P41.5**

*continued on next page*

(b) When the electron falls from state 2 to state 1, it puts out energy

$$E = 151\text{ eV} - 37.7\text{ eV} = 113\text{ eV} = hf = \frac{hc}{\lambda}$$

into emitting a photon of wavelength

$$\lambda = \frac{hc}{E} = \frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{(113\text{ eV})\left(1.60\times10^{-19}\text{ J/eV}\right)} = 11.0\text{ nm}.$$

The wavelengths of the other spectral lines we find similarly:

| Transition | $4\to3$ | $4\to2$ | $4\to1$ | $3\to2$ | $3\to1$ | $2\to1$ |
|---|---|---|---|---|---|---|
| $E$(eV) | 264 | 452 | 565 | 188 | 302 | 113 |
| $\lambda$(nm) | 4.71 | 2.75 | 2.20 | 6.60 | 4.12 | 11.0 |

**P41.6** $\lambda = 2D$ for the lowest energy state

$$K = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} = \frac{h^2}{8mD^2} = \frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)^2}{8\left[4\left(1.66\times10^{-27}\text{ kg}\right)\right]\left(1.00\times10^{-14}\text{ m}\right)^2} = 8.27\times10^{-14}\text{ J} = \boxed{0.517\text{ MeV}}$$

$$p = \frac{h}{\lambda} = \frac{h}{2D} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{2\left(1.00\times10^{-14}\text{ m}\right)} = \boxed{3.31\times10^{-20}\text{ kg}\cdot\text{m/s}}$$

**P41.7** $\Delta E = \frac{hc}{\lambda} = \left(\frac{h^2}{8m_eL^2}\right)\left[2^2-1^2\right] = \frac{3h^2}{8m_eL^2}$

$$L = \sqrt{\frac{3h\lambda}{8m_ec}} = 7.93\times10^{-10}\text{ m} = \boxed{0.793\text{ nm}}$$

**P41.8** $\Delta E = \frac{hc}{\lambda} = \left(\frac{h^2}{8m_eL^2}\right)\left[2^2-1^2\right] = \frac{3h^2}{8m_eL^2}$

so $L = \boxed{\sqrt{\frac{3h\lambda}{8m_ec}}}$

**P41.9** The confined proton can be described in the same way as a standing wave on a string. At level 1, the node-to-node distance of the standing wave is $1.00\times10^{-14}$ m, so the wavelength is twice this distance: $\frac{h}{p} = 2.00\times10^{-14}$ m.

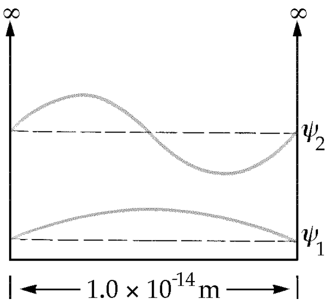

FIG. P41.9

The proton's kinetic energy is

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} = \frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)^2}{2\left(1.67\times10^{-27}\text{ kg}\right)\left(2.00\times10^{-14}\text{ m}\right)^2}$$

$$= \frac{3.29\times10^{-13}\text{ J}}{1.60\times10^{-19}\text{ J/eV}} = 2.05\text{ MeV}$$

*continued on next page*

In the first excited state, level 2, the node-to-node distance is half as long as in state 1. The momentum is two times larger and the energy is four times larger: $K = 8.22\text{ MeV}$.

The proton has mass, has charge, moves slowly compared to light in a standing wave state, and stays inside the nucleus. When it falls from level 2 to level 1, its energy change is

$2.05\text{ MeV} - 8.22\text{ MeV} = -6.16\text{ MeV}$.

Therefore, we know that a photon (a traveling wave with no mass and no charge) is emitted at the speed of light, and that it has an energy of $\boxed{+6.16\text{ MeV}}$.

Its frequency is $$f = \frac{E}{h} = \frac{\left(6.16\times10^{6}\text{ eV}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)}{6.626\times10^{-34}\text{ J}\cdot\text{s}} = 1.49\times10^{21}\text{ Hz}.$$

And its wavelength is $$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\text{ m/s}}{1.49\times10^{21}\text{ s}^{-1}} = \boxed{2.02\times10^{-13}\text{ m}}.$$

This is a gamma ray, according to the electromagnetic spectrum chart in Chapter 34.

**P41.10** The ground state energy of a particle (mass *m*) in a 1-dimensional box of width *L* is $E_1 = \dfrac{h^2}{8mL^2}$.

(a) For a proton $\left(m = 1.67\times10^{-27}\text{ kg}\right)$ in a 0.200-nm wide box:

$$E_1 = \frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)^2}{8\left(1.67\times10^{-27}\text{ kg}\right)\left(2.00\times10^{-10}\text{ m}\right)^2} = 8.22\times10^{-22}\text{ J} = \boxed{5.13\times10^{-3}\text{ eV}}.$$

(b) For an electron $\left(m = 9.11\times10^{-31}\text{ kg}\right)$ in the same size box:

$$E_1 = \frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)^2}{8\left(9.11\times10^{-31}\text{ kg}\right)\left(2.00\times10^{-10}\text{ m}\right)^2} = 1.51\times10^{-18}\text{ J} = \boxed{9.41\text{ eV}}.$$

(c) The electron has a much higher energy because it is much less massive.

**P41.11** $$E_n = \left(\frac{h^2}{8mL^2}\right)n^2$$

$$E_1 = \frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)^2}{8\left(1.67\times10^{-27}\text{ kg}\right)\left(2.00\times10^{-14}\text{ m}\right)^2} = 8.21\times10^{-14}\text{ J}$$

$E_1 = \boxed{0.513\text{ MeV}}$ $\quad E_2 = 4E_1 = \boxed{2.05\text{ MeV}}$ $\quad E_3 = 9E_1 = \boxed{4.62\text{ MeV}}$

$\boxed{\text{Yes}}$, the energy differences are $\sim 1\text{ MeV}$, which is a typical energy for a $\gamma$-ray photon.

***P41.12** (a) The energies of the confined electron are $E_n = \frac{h^2}{8m_eL^2}n^2$. Its energy gain in the quantum jump from state 1 to state 4 is $\frac{h^2}{8m_eL^2}\left(4^2 - 1^2\right)$ and this is the photon energy:

$$\frac{h^2 15}{8m_eL^2} = hf = \frac{hc}{\lambda}. \text{ Then } 8m_ecL^2 = 15h\lambda \text{ and } \boxed{L = \left(\frac{15h\lambda}{8m_ec}\right)^{1/2}}.$$

(b) Let $\lambda'$ represent the wavelength of the photon emitted: $\frac{hc}{\lambda'} = \frac{h^2}{8m_eL^2}4^2 - \frac{h^2}{8m_eL^2}2^2 = \frac{12h^2}{8m_eL^2}$.

$$\text{Then } \frac{hc}{\lambda}\frac{\lambda'}{hc} = \frac{h^2 15\left(8m_eL^2\right)}{8m_eL^2 12h^2} = \frac{5}{4} \text{ and } \boxed{\lambda' = 1.25\lambda}.$$

## Section 41.3 The Particle Under Boundary Conditions

## Section 41.4 The Schrödinger Equation

**P41.13** We have $\psi = Ae^{i(kx-\omega t)}$ and $\frac{\partial^2\psi}{\partial x^2} = -k^2\psi$.

Schrödinger's equation: $\frac{\partial^2\psi}{\partial x^2} = -k^2\psi = -\frac{2m}{\hbar^2}(E-U)\psi$.

Since $k^2 = \frac{(2\pi)^2}{\lambda^2} = \frac{(2\pi p)^2}{h^2} = \frac{p^2}{\hbar^2}$ and $E - U = \frac{p^2}{2m}$.

$\boxed{\text{Thus this equation balances.}}$

**P41.14** $\psi(x) = A\cos kx + B\sin kx$ $\qquad$ $\frac{\partial\psi}{\partial x} = -kA\sin kx + kB\cos kx$

$\frac{\partial^2\psi}{\partial x^2} = -k^2A\cos kx - k^2B\sin kx$ $\qquad$ $-\frac{2m}{\hbar}(E-U)\psi = -\frac{2mE}{\hbar^2}(A\cos kx + B\sin kx)$

Therefore the Schrödinger equation is satisfied if

$$\frac{\partial^2\psi}{\partial x^2} = \left(-\frac{2m}{\hbar^2}\right)(E-U)\psi \quad \text{or} \quad -k^2(A\cos kx + B\sin kx) = \left(-\frac{2mE}{\hbar^2}\right)(A\cos kx + B\sin kx).$$

This is true as an identity (functional equality) for all $x$ if $\boxed{E = \frac{\hbar^2k^2}{2m}}$.

***P41.15** (a) With $\psi(x) = A\sin(kx)$

$$\frac{d}{dx}A\sin kx = Ak\cos kx \quad \text{and} \quad \frac{d^2}{dx^2}\psi = -Ak^2\sin kx.$$

$$\text{Then} \quad -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = +\frac{\hbar^2k^2}{2m}A\sin kx = \frac{h^2\left(4\pi^2\right)}{4\pi^2\left(\lambda^2\right)(2m)}\psi = \frac{p^2}{2m}\psi = \frac{m^2v^2}{2m}\psi = \frac{1}{2}mv^2\psi = K\psi.$$

(b) With $\psi(x) = A\sin\left(\frac{2\pi x}{\lambda}\right) = A\sin kx$, the proof given in part (a) applies again.

**P41.16** (a) $$\langle x\rangle=\int_0^L x\frac{2}{L}\sin^2\left(\frac{2\pi x}{L}\right)dx=\frac{2}{L}\int_0^L x\left(\frac{1}{2}-\frac{1}{2}\cos\frac{4\pi x}{L}\right)dx$$

$$\langle x\rangle=\frac{1}{L}\frac{x^2}{2}\bigg|_0^L-\frac{1}{L}\frac{L^2}{16\pi^2}\left[\frac{4\pi x}{L}\sin\frac{4\pi x}{L}+\cos\frac{4\pi x}{L}\right]_0^L=\boxed{\frac{L}{2}}$$

(b) $$\text{Probability}=\int_{0.490L}^{0.510L}\frac{2}{L}\sin^2\left(\frac{2\pi x}{L}\right)dx=\left[\frac{1}{L}x-\frac{1}{L}\frac{L}{4\pi}\sin\frac{4\pi x}{L}\right]_{0.490L}^{0.510L}$$

$$\text{Probability}=0.020-\frac{1}{4\pi}(\sin 2.04\pi-\sin 1.96\pi)=\boxed{5.26\times10^{-5}}$$

(c) $$\text{Probability}\left[\frac{x}{L}-\frac{1}{4\pi}\sin\frac{4\pi x}{L}\right]_{0.240L}^{0.260L}=\boxed{3.99\times10^{-2}}$$

(d) In the $n=2$ graph in Figure 41.4 (b), it is more probable to find the particle either near $x=\frac{L}{4}$ or $x=\frac{3L}{4}$ than at the center, where the probability density is zero.

Nevertheless, the symmetry of the distribution means that the average position is $\frac{L}{2}$.

**P41.17** Normalization requires

$$\int_{\text{all space}}|\psi|^2dx=1 \quad\text{or}\quad \int_0^L A^2\sin^2\left(\frac{n\pi x}{L}\right)dx=1$$

$$\int_0^L A^2\sin^2\left(\frac{n\pi x}{L}\right)dx=A^2\left(\frac{L}{2}\right)=1 \quad\text{or}\quad \boxed{A=\sqrt{\frac{2}{L}}}.$$

**P41.18** The desired probability is $$P=\int_0^{L/4}|\psi|^2dx=\frac{2}{L}\int_0^{L/4}\sin^2\left(\frac{2\pi x}{L}\right)dx$$

where $$\sin^2\theta=\frac{1-\cos2\theta}{2}.$$

Thus, $$P=\left(\frac{x}{L}-\frac{1}{4\pi}\sin\frac{4\pi x}{L}\right)\bigg|_0^{L/4}=\left(\frac{1}{4}-0-0+0\right)=\boxed{0.250}.$$

**P41.19** In $0\le x\le L$, the argument $\frac{2\pi x}{L}$ of the sine function ranges from 0 to $2\pi$. The probability density $\left(\frac{2}{L}\right)\sin^2\left(\frac{2\pi x}{L}\right)$ reaches maxima at $\sin\theta=1$ and $\sin\theta=-1$ at

$$\frac{2\pi x}{L}=\frac{\pi}{2}\text{ and }\frac{2\pi x}{L}=\frac{3\pi}{2}.$$

$\therefore$ The most probable positions of the particle are at $\boxed{\text{at } x=\frac{L}{4}\text{ and } x=\frac{3L}{4}}$.

***P41.20** (a) Probability $= \int_0^{\ell} |\psi_1|^2 dx = \frac{2}{L}\int_0^{\ell} \sin^2\left(\frac{\pi x}{L}\right) dx = \frac{1}{L}\int_0^{\ell}\left[1-\cos\left(\frac{2\pi x}{L}\right)\right]dx$

$$= \frac{1}{L}\left[x - \frac{L}{2\pi}\sin\left(\frac{2\pi x}{L}\right)\right]_0^{\ell} = \boxed{\frac{\ell}{L} - \frac{1}{2\pi}\sin\left(\frac{2\pi \ell}{L}\right)}$$

(b)

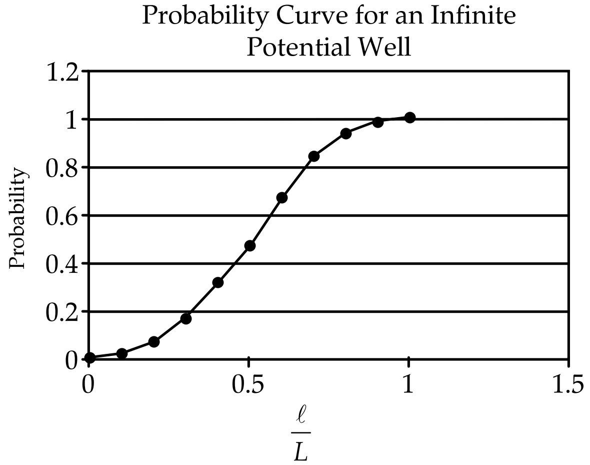

**FIG. P41.20(b)**

(c) The probability of finding the particle between $x = 0$ and $x = \ell$ is $\frac{2}{3}$, and between $x = \ell$ and $x = L$ is $\frac{1}{3}$.

Thus, $\int_0^{\ell} |\psi_1|^2 dx = \frac{2}{3}$

$$\therefore \frac{\ell}{L} - \frac{1}{2\pi}\sin\left(\frac{2\pi \ell}{L}\right) = \frac{2}{3}, \qquad \text{or} \qquad u - \frac{1}{2\pi}\sin 2\pi u = \frac{2}{3}.$$

This equation for $\frac{\ell}{L}$ can be solved by homing in on the solution with a calculator, the result being $\frac{\ell}{L} = 0.585$, or $\ell = \boxed{0.585L}$ to three digits.

**P41.21** (a) The probability is $P=\int_0^{L/3}|\psi|^2dx=\int_0^{L/3}\frac{2}{L}\sin^2\left(\frac{\pi x}{L}\right)dx=\frac{2}{L}\int_0^{L/3}\left(\frac{1}{2}-\frac{1}{2}\cos\frac{2\pi x}{L}\right)dx$

$$P=\left(\frac{x}{L}-\frac{1}{2\pi}\sin\frac{2\pi x}{L}\right)\Bigg|_0^{L/3}=\left(\frac{1}{3}-\frac{1}{2\pi}\sin\frac{2\pi}{3}\right)=\left(\frac{1}{3}-\frac{\sqrt{3}}{4\pi}\right)=\boxed{0.196}.$$

(b) The probability density is symmetric about $x=\frac{L}{2}$. Thus, the probability of finding the particle between $x=\frac{2L}{3}$ and $x=L$ is the same 0.196. Therefore, the probability of finding it in the range $\frac{L}{3}\le x\le\frac{2L}{3}$ is $P=1.00-2(0.196)=\boxed{0.609}$.

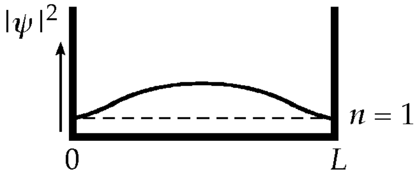

FIG. P41.21(b)

(c) Classically, the electron moves back and forth with constant speed between the walls, and the probability of finding the electron is the same for all points between the walls. Thus, the classical probability of finding the electron in any range equal to one-third of the available space is $P_{\text{classical}}=\boxed{\frac{1}{3}}$.

**P41.22** (a) $\psi_1(x)=\sqrt{\frac{2}{L}}\cos\left(\frac{\pi x}{L}\right)$; $P_1(x)=|\psi_1(x)|^2=\frac{2}{L}\cos^2\left(\frac{\pi x}{L}\right)$

$\psi_2(x)=\sqrt{\frac{2}{L}}\sin\left(\frac{2\pi x}{L}\right)$; $P_2(x)=|\psi_2(x)|^2=\frac{2}{L}\sin^2\left(\frac{2\pi x}{L}\right)$

$\psi_3(x)=\sqrt{\frac{2}{L}}\cos\left(\frac{3\pi x}{L}\right)$; $P_3(x)=|\psi_3(x)|^2=\frac{2}{L}\cos^2\left(\frac{3\pi x}{L}\right)$

(b)

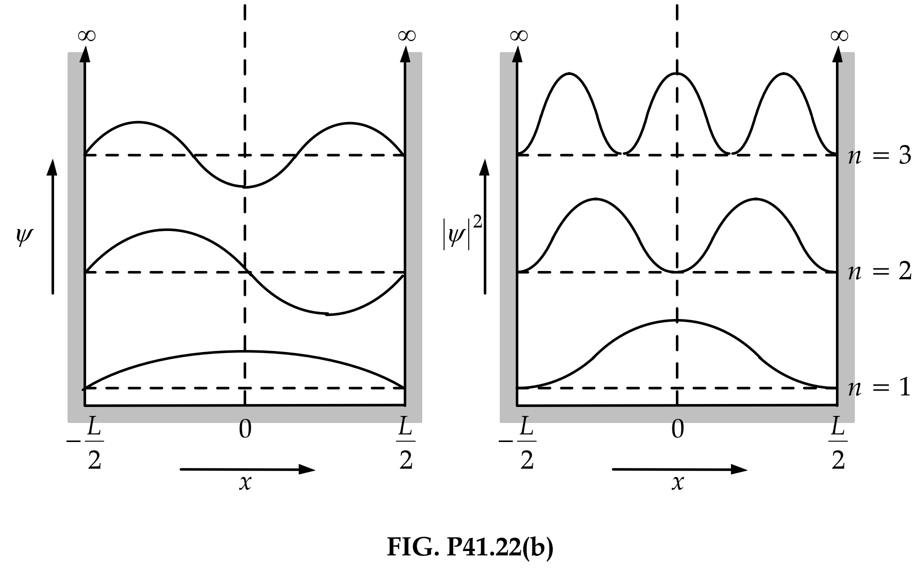

FIG. P41.22(b)

**P41.23** Problem 43 in Chapter 16 helps students to understand how to draw conclusions from an identity

(a) $$\psi(x)=A\left(1-\frac{x^2}{L^2}\right) \qquad \frac{d\psi}{dx}=-\frac{2Ax}{L^2} \qquad \frac{d^2\psi}{dx^2}=-\frac{2A}{L}$$

Schrödinger's equation $$\frac{d^2\psi}{dx^2}=-\frac{2m}{\hbar^2}(E-U)\psi$$

becomes $$-\frac{2A}{L^2}=-\frac{2m}{\hbar^2}EA\left(1-\frac{x^2}{L^2}\right)+\frac{2m}{\hbar^2}\frac{\left(-\hbar^2x^2\right)A\left(1-x^2/L^2\right)}{mL^2\left(L^2-x^2\right)}$$

$$-\frac{1}{L^2}=-\frac{mE}{\hbar^2}+\frac{mEx^2}{\hbar^2L^2}-\frac{x^2}{L^4}.$$

This will be true for all $x$ if both $$\frac{1}{L^2}=\frac{mE}{\hbar^2}$$

and $$\frac{mE}{\hbar^2L^2}-\frac{1}{L^4}=0$$

both these conditions are satisfied for a particle of energy $\boxed{E=\dfrac{\hbar^2}{L^2m}}$.

(b) For normalization, $$1=\int_{-L}^{L}A^2\left(1-\frac{x^2}{L^2}\right)^2dx=A^2\int_{-L}^{L}\left(1-\frac{2x^2}{L^2}+\frac{x^4}{L^4}\right)dx$$

$$1=A^2\left[x-\frac{2x^3}{3L^2}+\frac{x^5}{5L^4}\right]_{-L}^{L}=A^2\left[L-\frac{2}{3}L+\frac{L}{5}+L-\frac{2}{3}L+\frac{L}{5}\right]=A^2\left(\frac{16L}{15}\right) \qquad \boxed{A=\sqrt{\frac{15}{16L}}}.$$

(c) $$P=\int_{-L/3}^{L/3}\psi^2dx=\frac{15}{16L}\int_{-L/3}^{L/3}\left(1-\frac{2x^2}{L^2}+\frac{x^4}{L^4}\right)dx=\frac{15}{16L}\left[x-\frac{2x^3}{3L^2}+\frac{x^5}{5L^5}\right]_{-L/3}^{L/3}=\frac{30}{16L}\left[\frac{L}{3}-\frac{2L}{81}+\frac{L}{1\,215}\right]$$

$$P=\frac{47}{81}=\boxed{0.580}$$

**P41.24** (a) Setting the total energy $E$ equal to zero and rearranging the Schrödinger equation to isolate the potential energy function gives

$$U(x)=\left(\frac{\hbar^2}{2m}\right)\frac{1}{\psi}\frac{d^2\psi}{dx^2}.$$

If $$\psi(x)=Axe^{-x^2/L^2}.$$

Then $$\frac{d^2\psi}{dx^2}=\left(4Ax^3-6AxL^2\right)\frac{e^{-x^2/L^2}}{L^4}$$

or $$\frac{d^2\psi}{dx^2}=\frac{\left(4x^2-6L^2\right)}{L^4}\psi(x)$$

and $$\boxed{U(x)=\frac{\hbar^2}{2mL^2}\left(\frac{4x^2}{L^2}-6\right)}.$$

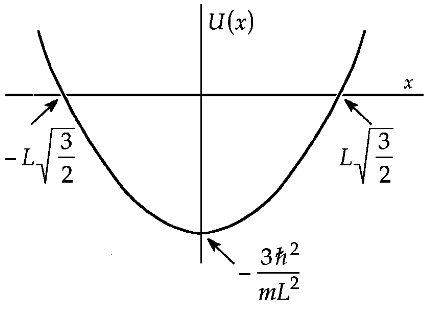

**FIG. P41.24(b)**

(b) $\boxed{\text{See the figure to the right.}}$

## Section 41.5 **A Particle in a Well of Finite Height**

**P41.25** (a) $\boxed{\text{See figure to the right.}}$

(b) The wavelength of the transmitted wave traveling to the left is the same as the original wavelength, which equals $\boxed{2L}$.

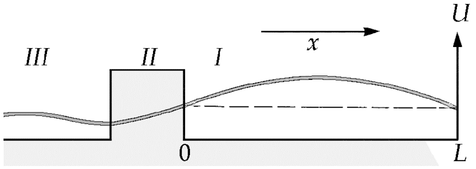

**FIG. P41.25(a)**

**P41.26**

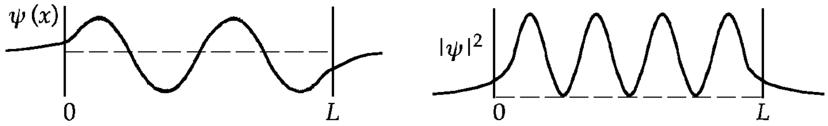

**FIG. P41.26**

## Section 41.6 **Tunneling Through a Potential Energy Barrier**

**P41.27** $T = e^{-2CL}$ where $C = \dfrac{\sqrt{2m(U-E)}}{\hbar}$

$$2CL = \frac{2\sqrt{2\left(9.11\times10^{-31}\right)\left(8.00\times10^{-19}\right)}}{1.055\times10^{-34}}\left(2.00\times10^{-10}\right) = 4.58$$

(a) $T = e^{-4.58} = \boxed{0.010\,3}$, a 1% chance of transmission.

(b) $R = 1 - T = \boxed{0.990}$, a 99% chance of reflection.

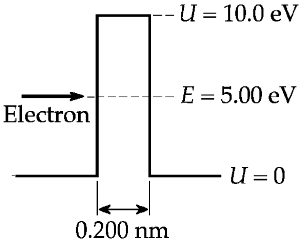

**FIG. P41.27**

**P41.28** $$C = \frac{\sqrt{2\left(9.11\times10^{-31}\right)(5.00-4.50)\left(1.60\times10^{-19}\right)}\ \text{kg}\cdot\text{m/s}}{1.055\times10^{-34}\ \text{J}\cdot\text{s}} = 3.62\times10^{9}\ \text{m}^{-1}$$

$$T = e^{-2CL} = \exp\left[-2\left(3.62\times10^{9}\ \text{m}^{-1}\right)\left(950\times10^{-12}\ \text{m}\right)\right] = \exp(-6.88)$$

$$T = \boxed{1.03\times10^{-3}}$$

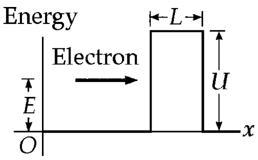

**FIG. P41.28**

**P41.29** From problem 28, $C = 3.62\times10^{9}\ \text{m}^{-1}$

$$10^{-6} = \exp\left[-2\left(3.62\times10^{9}\ \text{m}^{-1}\right)L\right].$$

Taking logarithms, $-13.816 = -2\left(3.62\times10^{9}\ \text{m}^{-1}\right)L$.

New $L = 1.91$ nm

$\Delta L = 1.91\ \text{nm} - 0.950\ \text{nm} = \boxed{0.959\ \text{nm}}$.

***P41.30** The original tunneling probability is $T = e^{-2CL}$ where

$$C = \frac{\left(2m(U-E)\right)^{1/2}}{\hbar} = \frac{2\pi\left(2\times 9.11\times 10^{-31}\text{ kg}(20-12)1.6\times 10^{-19}\text{ J}\right)^{1/2}}{6.626\times 10^{-34}\text{ J}\cdot\text{s}} = 1.448\,1\times 10^{10}\text{ m}^{-1}.$$

The photon energy is $hf = \frac{hc}{\lambda} = \frac{1\,240\text{ eV}\cdot\text{nm}}{546\text{ nm}} = 2.27\text{ eV}$, to make the electron's new kinetic energy $12 + 2.27 = 14.27\text{ eV}$ and its decay coefficient inside the barrier

$$C' = \frac{2\pi\left(2\times 9.11\times 10^{-31}\text{ kg}(20-14.27)1.6\times 10^{-19}\text{ J}\right)^{1/2}}{6.626\times 10^{-34}\text{ J}\cdot\text{s}} = 1.225\,5\times 10^{10}\text{ m}^{-1}.$$

Now the factor of increase in transmission probability is

$$\frac{e^{-2C'L}}{e^{-2CL}} = e^{2L(C-C')} = e^{2\times 10^{-9}\text{ m}\times 0.223\times 10^{10}\text{ m}^{-1}} = e^{4.45} = \boxed{85.9}.$$

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## Section 41.7 **The Scanning Tunneling Microscope**

**P41.31** With the wave function proportional to $e^{-CL}$, the transmission coefficient and the tunneling current are proportional to $|\psi|^2$, to $e^{-2CL}$.

Then, $$\frac{I(0.500\text{ nm})}{I(0.515\text{ nm})} = \frac{e^{-2(10.0/\text{nm})(0.500\text{ nm})}}{e^{-2(10.0/\text{nm})(0.515\text{ nm})}} = e^{20.0(0.015)} = \boxed{1.35}.$$

**P41.32** With transmission coefficient $e^{-2CL}$, the fractional change in transmission is

$$\frac{e^{-2(10.0/\text{nm})L} - e^{-2(10.0/\text{nm})(L+0.002\,00\text{ nm})}}{e^{-2(10.0/\text{nm})L}} = 1 - e^{-20.0(0.002\,00)} = 0.0392 = \boxed{3.92\%}.$$

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## Section 41.8 **The Simple Harmonic Oscillator**

**P41.33** $\psi = Be^{-(m\omega/2\hbar)x^2}$ so $\frac{d\psi}{dx} = -\left(\frac{m\omega}{\hbar}\right)x\psi$ and $\frac{d^2\psi}{dx^2} = \left(\frac{m\omega}{\hbar}\right)^2 x^2\psi + \left(-\frac{m\omega}{\hbar}\right)\psi$.

Substituting into Equation 41.13 gives $\left(\frac{m\omega}{\hbar}\right)^2 x^2\psi + \left(-\frac{m\omega}{\hbar}\right)\psi = -\left(\frac{2mE}{\hbar^2}\right)\psi + \left(\frac{m\omega}{\hbar}\right)^2 x^2\psi$

which is satisfied provided that $E = \frac{\hbar\omega}{2}$.

**P41.34** Problem 43 in Chapter 16 helps students to understand how to draw conclusions from an identity.

$\psi = Axe^{-bx^2}$ so $$\frac{d\psi}{dx} = Ae^{-bx^2} - 2bx^2 Ae^{-bx^2}$$

and $$\frac{d^2\psi}{dx^2} = -2bxAe^{-bx^2} - 4bxAe^{-bx^2} + 4b^2x^3e^{-bx^2} = -6b\psi + 4b^2x^2\psi.$$

Substituting into Equation 41.13, $$-6b\psi + 4b^2x^2\psi = -\left(\frac{2mE}{\hbar}\right)\psi + \left(\frac{m\omega}{\hbar}\right)^2 x^2\psi.$$

For this to be true as an identity, it must be true for all values of $x$.

So we must have both $$-6b = -\frac{2mE}{\hbar^2} \text{ and } 4b^2 = \left(\frac{m\omega}{\hbar}\right)^2.$$

(a) Therefore $$\boxed{b = \frac{m\omega}{2\hbar}}$$

(b) and $$E = \frac{3b\hbar^2}{m} = \boxed{\frac{3}{2}\hbar\omega}.$$

(c) The wave function is that of the $\boxed{\text{first excited state}}$.

**P41.35** The longest wavelength corresponds to minimum photon energy, which must be equal to the spacing between energy levels of the oscillator:

$$\frac{hc}{\lambda} = \hbar\omega = \hbar\sqrt{\frac{k}{m}} \text{ so } \lambda = 2\pi c\sqrt{\frac{m}{k}} = 2\pi\left(3.00\times10^8 \text{ m/s}\right)\left(\frac{9.11\times10^{-31}\text{ kg}}{8.99\text{ N/m}}\right)^{1/2} = \boxed{600\text{ nm}}.$$

**P41.36** (a) With $\psi = Be^{-(m\omega/2\hbar)x^2}$, the normalization condition $\int_{\text{all } x} |\psi|^2 dx = 1$

becomes $$1 = \int_{-\infty}^{\infty} B^2 e^{-2(m\omega/2\hbar)x^2} dx = 2B^2 \int_0^{\infty} e^{-(m\omega/\hbar)x^2} dx = 2B^2 \frac{1}{2}\sqrt{\frac{\pi}{m\omega/\hbar}}$$

where Table B.6 in Appendix B was used to evaluate the integral.

Thus, $1 = B^2\sqrt{\frac{\pi\hbar}{m\omega}}$ and $\boxed{B = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}}$.

(b) For small $\delta$, the probability of finding the particle in the range $-\frac{\delta}{2} < x < \frac{\delta}{2}$ is

$$\int_{-\delta/2}^{\delta/2} |\psi|^2 dx = \delta|\psi(0)|^2 = \delta B^2 e^{-0} = \boxed{\delta\left(\frac{m\omega}{\pi\hbar}\right)^{1/2}}.$$

***P41.37** (a) For the center of mass to be fixed, $m_1v_1+m_2v_2=0$. Then $v=|v_1|+|v_2|=|v_1|+\frac{m_1}{m_2}|v_1|=\frac{m_2+m_1}{m_2}|v_1|$ and $|v_1|=\frac{m_2v}{m_1+m_2}$. Similarly, $v=\frac{m_2}{m_1}|v_2|+|v_2|$ and $|v_2|=\frac{m_1v}{m_1+m_2}$. Then

$$\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2+\frac{1}{2}kx^2=\frac{1}{2}\frac{m_1m_2^2v^2}{(m_1+m_2)^2}+\frac{1}{2}\frac{m_2m_1^2v^2}{(m_1+m_2)^2}+\frac{1}{2}kx^2$$

$$=\frac{1}{2}\frac{m_1m_2(m_1+m_2)}{(m_1+m_2)^2}v^2+\frac{1}{2}kx^2=\frac{1}{2}\mu v^2+\frac{1}{2}kx^2$$

(b) $\frac{d}{dx}\left(\frac{1}{2}\mu v^2+\frac{1}{2}kx^2\right)=0$ because energy is constant

$$0=\frac{1}{2}\mu 2v\frac{dv}{dx}+\frac{1}{2}k2x=\mu\frac{dx}{dt}\frac{dv}{dx}+kx=\mu\frac{dv}{dt}+kx.$$

Then $\mu a=-kx$, $a=-\frac{kx}{\mu}$. This is the condition for simple harmonic motion, that the acceleration of the equivalent particle be a negative constant times the excursion from equilibrium. By identification with $a=-\omega^2x$, $\omega=\sqrt{\frac{k}{\mu}}=2\pi f$ and $\boxed{f=\frac{1}{2\pi}\sqrt{\frac{k}{\mu}}}$.

**P41.38** (a) With $\langle x\rangle=0$ and $\langle p_x\rangle=0$, the average value of $x^2$ is $(\Delta x)^2$ and the average value of $p_x^2$ is $(\Delta p_x)^2$. Then $\Delta x\ge\frac{\hbar}{2\Delta p_x}$ requires

$$E\ge\frac{p_x^2}{2m}+\frac{k}{2}\frac{\hbar^2}{4p_x^2}=\boxed{\frac{p_x^2}{2m}+\frac{k\hbar^2}{8p_x^2}}.$$

(b) To minimize this as a function of $p_x^2$, we require $\frac{dE}{dp_x^2}=0=\frac{1}{2m}+\frac{k\hbar^2}{8}(-1)\frac{1}{p_x^4}$.

Then $\frac{k\hbar^2}{8p_x^4}=\frac{1}{2m}$ so $p_x^2=\left(\frac{2mk\hbar^2}{8}\right)^{1/2}=\frac{\hbar\sqrt{mk}}{2}$

and $E\ge\frac{\hbar\sqrt{mk}}{2(2m)}+\frac{k\hbar^2 2}{8\hbar\sqrt{mk}}=\frac{\hbar}{4}\sqrt{\frac{k}{m}}+\frac{\hbar}{4}\sqrt{\frac{k}{m}}$

$$E_{\min}=\frac{\hbar}{2}\sqrt{\frac{k}{m}}=\boxed{\frac{\hbar\omega}{2}}.$$

## Additional Problems

**P41.39** Suppose the marble has mass 20 g. Suppose the wall of the box is 12 cm high and 2 mm thick. While it is inside the wall,

$$U = mgy = (0.02\text{ kg})\left(9.8\text{ m/s}^2\right)(0.12\text{ m}) = 0.023\,5\text{ J}$$

and $$E = K = \frac{1}{2}mv^2 = \frac{1}{2}(0.02\text{ kg})(0.8\text{ m/s})^2 = 0.006\,4\text{ J}.$$

Then $$C = \frac{\sqrt{2m(U-E)}}{\hbar} = \frac{\sqrt{2(0.02\text{ kg})(0.017\,1\text{ J})}}{1.055\times10^{-34}\text{ J}\cdot\text{s}} = 2.5\times10^{32}\text{ m}^{-1}$$

and the transmission coefficient is

$$e^{-2CL} = e^{-2\left(2.5\times10^{32}\right)\left(2\times10^{-3}\right)} = e^{-10\times10^{29}} = e^{-2.30\left(4.3\times10^{29}\right)} = 10^{-4.3\times10^{29}} = \boxed{\sim10^{-10^{30}}}.$$

**P41.40** (a) $\lambda = 2L = \boxed{2.00\times10^{-10}\text{ m}}$

(b) $$p = \frac{h}{\lambda} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{2.00\times10^{-10}\text{ m}} = \boxed{3.31\times10^{-24}\text{ kg}\cdot\text{m/s}}$$

(c) $$E = \frac{p^2}{2m} = \boxed{0.172\text{ eV}}$$

**P41.41** (a) $\boxed{\text{See the figure.}}$ (b) $\boxed{\text{See the figure.}}$

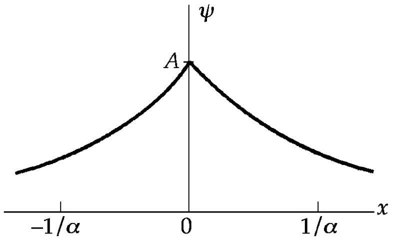

FIG. P41.41(a)

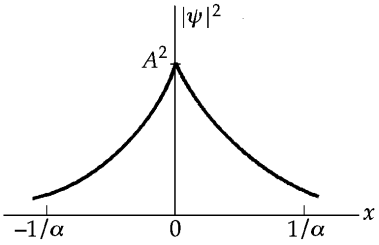

FIG. P41.41(b)

(c) $\psi$ is continuous and $\psi \to 0$ as $x \to \pm\infty$. The function can be normalized. It describes a particle bound near $x = 0$.

(d) Since $\psi$ is symmetric,

$$\int_{-\infty}^{\infty}|\psi|^2\,dx = 2\int_{0}^{\infty}|\psi|^2\,dx = 1$$

or $$2A^2\int_0^{\infty}e^{-2\alpha x}\,dx = \left(\frac{2A^2}{-2\alpha}\right)\left(e^{-\infty}-e^0\right) = 1.$$

This gives $\boxed{A = \sqrt{\alpha}}$.

(e) $$P_{(-1/2\alpha)\to(1/2\alpha)} = 2\left(\sqrt{a}\right)^2\int_{x=0}^{1/2\alpha}e^{-2\alpha x}\,dx = \left(\frac{2\alpha}{-2\alpha}\right)\left(e^{-2\alpha/2\alpha}-1\right) = \left(1-e^{-1}\right) = \boxed{0.632}$$

**P41.42** (a) Use Schrödinger's equation

$$\frac{\partial^2\psi}{\partial x^2}=-\frac{2m}{\hbar^2}(E-U)\psi$$

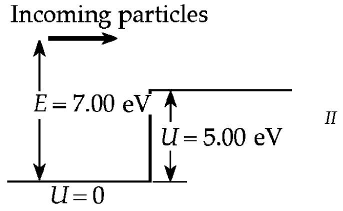

**FIG. P41.42(a)**

with solutions

$$\psi_1=Ae^{ik_1x}+Be^{-ik_1x} \qquad \text{[region } I\text{]}$$

$$\psi_2=Ce^{ik_2x} \qquad \text{[region } II\text{]}.$$

Where $k_1=\dfrac{\sqrt{2mE}}{\hbar}$

and $k_2=\dfrac{\sqrt{2m(E-U)}}{\hbar}$.

Then, matching functions and derivatives at $x=0$

$(\psi_1)_0=(\psi_2)_0$ gives $A+B=C$

and $\left(\dfrac{d\psi_1}{dx}\right)_0=\left(\dfrac{d\psi_2}{dx}\right)_0$ gives $k_1(A-B)=k_2C$.

Then $B=\dfrac{1-k_2/k_1}{1+k_2/k_1}A$

and $C=\dfrac{2}{1+k_2/k_1}A$.

Incident wave $Ae^{ikx}$ reflects $Be^{-ikx}$, with probability $R=\dfrac{B^2}{A^2}=\dfrac{(1-k_2/k_1)^2}{(1+k_2/k_1)^2}=\boxed{\dfrac{(k_1-k_2)^2}{(k_1+k_2)^2}}$.

(b) With $E=7.00$ eV

and $U=5.00$ eV

$$\frac{k_2}{k_1}=\sqrt{\frac{E-U}{E}}=\sqrt{\frac{2.00}{7.00}}=0.535.$$

The reflection probability is $R=\dfrac{(1-0.535)^2}{(1+0.535)^2}=\boxed{0.092\,0}$.

The probability of transmission is $T=1-R=\boxed{0.908}$.

**P41.43** $$R = \frac{(k_1 - k_2)^2}{(k_1 + k_2)^2} = \frac{(1 - k_2/k_1)^2}{(1 + k_2/k_1)^2}$$

$$\frac{\hbar^2 k^2}{2m} = E - U \text{ for constant } U$$

$$\frac{\hbar^2 k_1^2}{2m} = E \text{ since } U = 0 \qquad (1)$$

$$\frac{\hbar^2 k_2^2}{2m} = E - U \qquad (2)$$

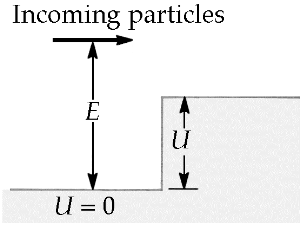

**FIG. P41.43**

Dividing (2) by (1), $\frac{k_2^2}{k_1^2} = 1 - \frac{U}{E} = 1 - \frac{1}{2} = \frac{1}{2}$ so $\frac{k_2}{k_1} = \frac{1}{\sqrt{2}}$

and therefore, $R = \frac{(1 - 1/\sqrt{2})^2}{(1 + 1/\sqrt{2})^2} = \frac{(\sqrt{2} - 1)^2}{(\sqrt{2} + 1)^2} = \boxed{0.029\ 4}$.

**P41.44** (a) The wave functions and probability densities are the same as those shown in the two lower curves in Figure 41.4 of the textbook.

(b) $$P_1 = \int_{0.150 \text{ nm}}^{0.350 \text{ nm}} |\psi_1|^2 dx = \left(\frac{2}{1.00 \text{ nm}}\right) \int_{0.150}^{0.350} \sin^2\left(\frac{\pi x}{1.00 \text{ nm}}\right) dx$$

$$= (2.00/\text{nm}) \left[\frac{x}{2} - \frac{1.00 \text{ nm}}{4\pi} \sin\left(\frac{2\pi x}{1.00 \text{ nm}}\right)\right]_{0.150 \text{ nm}}^{0.350 \text{ nm}}$$

In the above result we used $\int \sin^2 ax dx = \left(\frac{x}{2}\right) - \left(\frac{1}{4a}\right) \sin(2ax)$.

Therefore, $P_1 = (1.00/\text{nm}) \left[x - \frac{1.00 \text{ nm}}{2\pi} \sin\left(\frac{2\pi x}{1.00 \text{ nm}}\right)\right]_{0.150 \text{ nm}}^{0.350 \text{ nm}}$

$$P_1 = (1.00/\text{nm}) \left\{0.350 \text{ nm} - 0.150 \text{ nm} - \frac{1.00 \text{ nm}}{2\pi} [\sin(0.700\pi) - \sin(0.300\pi)]\right\} = \boxed{0.200}.$$

(c) $$P_2 = \frac{2}{1.00} \int_{0.150}^{0.350} \sin^2\left(\frac{2\pi x}{1.00}\right) dx = 2.00 \left[\frac{x}{2} - \frac{1.00}{8\pi} \sin\left(\frac{4\pi x}{1.00}\right)\right]_{0.150}^{0.350}$$

$$P_2 = 1.00 \left[x - \frac{1.00}{4\pi} \sin\left(\frac{4\pi x}{1.00}\right)\right]_{0.150}^{0.350} = 1.00 \left\{(0.350 - 0.150) - \frac{1.00}{4\pi} [\sin(1.40\pi) - \sin(0.600\pi)]\right\}$$

$$= \boxed{0.351}$$

(d) Using $E_n = \frac{n^2 h^2}{8mL^2}$, we find that $E_1 = \boxed{0.377 \text{ eV}}$ and $E_2 = \boxed{1.51 \text{ eV}}$.

**P41.45** (a) $$f=\frac{E}{h}=\frac{(1.80\text{ eV})}{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)}\left(\frac{1.60\times10^{-19}\text{ J}}{1.00\text{ eV}}\right)=\boxed{4.34\times10^{14}\text{ Hz}}$$

(b) $$\lambda=\frac{c}{f}=\frac{3.00\times10^{8}\text{ m/s}}{4.34\times10^{14}\text{ Hz}}=6.91\times10^{-7}\text{ m}=\boxed{691\text{ nm}}$$

(c) $$\Delta E\Delta t\geq\frac{\hbar}{2}\text{ so }\Delta E\geq\frac{\hbar}{2\Delta t}=\frac{h}{4\pi(\Delta t)}=\frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{4\pi\left(2.00\times10^{-6}\text{ s}\right)}=2.64\times10^{-29}\text{ J}=\boxed{1.65\times10^{-10}\text{ eV}}$$

***P41.46** (a) Taking $L_x=L_y=L$, we see that the expression for $E$ becomes

$$E=\frac{h^2}{8m_eL^2}\left(n_x^2+n_y^2\right).$$

For a normalizable wave function describing a particle, neither $n_x$ nor $n_y$ can be zero. The ground state, corresponding to $n_x=n_y=1$, has an energy of

$$E_{1,\,1}=\frac{h^2}{8m_eL^2}\left(1^2+1^2\right)=\boxed{\frac{h^2}{4m_eL^2}}.$$

The first excited state, corresponding to either $n_x=2$, $n_y=1$ or $n_x=1$, $n_y=2$, has an energy

$$E_{2,\,1}=E_{1,\,2}=\frac{h^2}{8m_eL^2}\left(2^2+1^2\right)=\boxed{\frac{5h^2}{8m_eL^2}}.$$

The second excited state, corresponding to $n_x=2$, $n_y=2$ has an energy of

$$E_{2,\,2}=\frac{h^2}{8m_eL^2}\left(2^2+2^2\right)=\boxed{\frac{h^2}{m_eL^2}}.$$

Finally, the third excited state, corresponding to either $n_x=1$, $n_y=3$ or $n_x=3$, $n_x=1$, has an energy

$$E_{1,\,3}=E_{3,\,1}=\frac{h^2}{8m_eL^2}\left(1^2+3^2\right)=\boxed{\frac{5h^2}{4m_eL^2}}.$$

(b) The energy difference between the second excited state and the ground state is given by

$$\Delta E=E_{2,\,2}-E_{1,\,1}=\frac{h^2}{m_eL^2}-\frac{h^2}{4m_eL^2}$$
$$=\boxed{\frac{3h^2}{4m_eL^2}}.$$

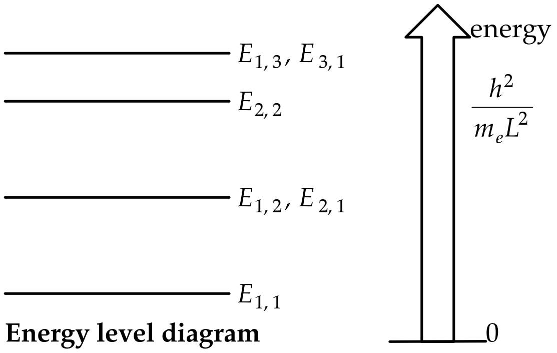

**FIG. P41.46(b)**

**P41.47** $\langle x^2\rangle = \int_{-\infty}^{\infty} x^2|\psi|^2\,dx$

For a one-dimensional box of width $L$, $\psi_n = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right)$.

Thus, $\langle x^2\rangle = \frac{2}{L}\int_0^L x^2\sin^2\left(\frac{n\pi x}{L}\right)dx = \boxed{\frac{L^2}{3} - \frac{L^2}{2n^2\pi^2}}$ (from integral tables).

**P41.48** (a) $\int_{-\infty}^{\infty}|\psi|^2\,dx = 1$ becomes

$$A^2\int_{-L/4}^{L/4}\cos^2\left(\frac{2\pi x}{L}\right)dx = A^2\left(\frac{L}{2\pi}\right)\left[\frac{\pi x}{L} + \frac{1}{4}\sin\left(\frac{4\pi x}{L}\right)\right]_{-L/4}^{L/4} = A^2\left(\frac{L}{2\pi}\right)\left[\frac{\pi}{2}\right] = 1$$

or $A^2 = \frac{4}{L}$ and $\boxed{A = \frac{2}{\sqrt{L}}}$.

(b) The probability of finding the particle between 0 and $\frac{L}{8}$ is

$$\int_0^{L/8}|\psi|^2\,dx = A^2\int_0^{L/8}\cos^2\left(\frac{2\pi x}{L}\right)dx = \frac{1}{4} + \frac{1}{2\pi} = \boxed{0.409}.$$

**P41.49** For a particle with wave function

$\psi(x) = \sqrt{\frac{2}{a}}e^{-x/a}$ for $x > 0$

and 0 for $x < 0$.

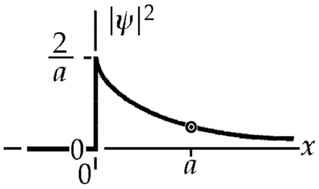

FIG. P41.49

(a) $|\psi(x)|^2 = 0$, $x < 0$ and $|\psi^2(x)| = \frac{2}{a}e^{-2x/a}$, $x > 0$

(b) $\text{Prob}(x < 0) = \int_{-\infty}^{0}|\psi(x)|^2\,dx = \int_{-\infty}^{0}(0)dx = \boxed{0}$

(c) Normalization $\int_{-\infty}^{\infty}|\psi(x)|^2\,dx = \int_{-\infty}^{0}|\psi|^2\,dx + \int_0^{\infty}|\psi|^2\,dx = 1$

$$\int_{-\infty}^{0}0dx + \int_0^{\infty}\left(\frac{2}{a}\right)e^{-2x/a}dx = 0 - e^{-2x/a}\Big|_0^{\infty} = -\left(e^{-\infty} - 1\right) = 1$$

$$\text{Prob}(0 < x < a) = \int_0^a|\psi|^2\,dx = \int_0^a\left(\frac{2}{a}\right)e^{-2x/a}dx = -e^{-2x/a}\Big|_0^a = 1 - e^{-2} = \boxed{0.865}$$

**P41.50** (a) The requirement that $\frac{n\lambda}{2}=L$ so $p=\frac{h}{\lambda}=\frac{nh}{2L}$ is still valid.

$$E=\sqrt{(pc)^2+\left(mc^2\right)^2}\Rightarrow E_n=\sqrt{\left(\frac{nhc}{2L}\right)^2+\left(mc^2\right)^2}$$

$$K_n=E_n-mc^2=\boxed{\sqrt{\left(\frac{nhc}{2L}\right)^2+\left(mc^2\right)^2}-mc^2}$$

(b) Taking $L=1.00\times10^{-12}$ m, $m=9.11\times10^{-31}$ kg, and $n=1$, we find $K_1=\boxed{4.69\times10^{-14}\text{ J}}$.

Nonrelativistic, $E_1=\frac{h^2}{8mL^2}=\frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)^2}{8\left(9.11\times10^{-31}\text{ kg}\right)\left(1.00\times10^{-12}\text{ m}\right)^2}=6.02\times10^{-14}\text{ J}$.

Comparing this to $K_1$, we see that this value is too large by $\boxed{28.6\%}$.

**P41.51** (a) $U=\frac{e^2}{4\pi\epsilon_0 d}\left[-1+\frac{1}{2}-\frac{1}{3}+\left(-1+\frac{1}{2}\right)+(-1)\right]=\frac{(-7/3)e^2}{4\pi\epsilon_0 d}=\boxed{-\frac{7k_ee^2}{3d}}$

(b) From Equation 41.12, $K=2E_1=\frac{2h^2}{8m_e\left(9d^2\right)}=\boxed{\frac{h^2}{36m_ed^2}}$.

(c) $E=U+K$ and $\frac{dE}{dd}=0$ for a minimum: $\frac{7k_ee^2}{3d^2}-\frac{h^2}{18m_ed^3}=0$

$$d=\frac{3h^2}{(7)\left(18k_ee^2m_e\right)}=\frac{h^2}{42m_ek_ee^2}=\frac{\left(6.626\times10^{-34}\right)^2}{(42)\left(9.11\times10^{-31}\right)\left(8.99\times10^9\right)\left(1.60\times10^{-19}\text{ C}\right)^2}=\boxed{0.049\,9\text{ nm}}.$$

(d) Since the lithium spacing is $a$, where $Na^3=V$, and the density is $\frac{Nm}{V}$, where $m$ is the mass of one atom, we get:

$$a=\left(\frac{Vm}{Nm}\right)^{1/3}=\left(\frac{m}{\text{density}}\right)^{1/3}=\left(\frac{1.66\times10^{-27}\text{ kg}\times7}{530\text{ kg}}\right)^{1/3}\text{ m}=2.80\times10^{-10}\text{ m}=\boxed{0.280\text{ nm}}$$

(5.62 times larger than *c*).

**P41.52** (a) $\psi = Bxe^{-(m\omega/2\hbar)x^2}$

$$\frac{d\psi}{dx} = Be^{-(m\omega/2\hbar)x^2} + Bx\left(-\frac{m\omega}{2\hbar}\right)2xe^{-(m\omega/2\hbar)x^2} = Be^{-(m\omega/2\hbar)x^2} - B\left(\frac{m\omega}{\hbar}\right)x^2e^{-(m\omega/2\hbar)x^2}$$

$$\frac{d^2\psi}{dx^2} = Bx\left(-\frac{m\omega}{\hbar}\right)xe^{-(m\omega/2\hbar)x^2} - B\left(\frac{m\omega}{\hbar}\right)2xe^{-(m\omega/2\hbar)x^2} - B\left(\frac{m\omega}{\hbar}\right)x^2\left(-\frac{m\omega}{\hbar}\right)xe^{-(m\omega/2\hbar)x^2}$$

$$\frac{d^2\psi}{dx^2} = -3B\left(\frac{m\omega}{\hbar}\right)xe^{-(m\omega/2\hbar)x^2} + B\left(\frac{m\omega}{\hbar}\right)^2x^3e^{-(m\omega/2\hbar)x^2}$$

Substituting into the Schrödinger Equation (41.13), we have

$$-3B\left(\frac{m\omega}{\hbar}\right)xe^{-(m\omega/2\hbar)x^2} + B\left(\frac{m\omega}{\hbar}\right)^2x^3e^{-(m\omega/2\hbar)x^2} = -\frac{2mE}{\hbar^2}Bxe^{-(m\omega/2\hbar)x^2} + \left(\frac{m\omega}{\hbar}\right)^2x^2Bxe^{-(m\omega/2\hbar)x^2}.$$

This is true if $-3\omega = -\frac{2E}{\hbar}$; it is true if $\boxed{E = \frac{3\hbar\omega}{2}}$.

(b) We never find the particle at $\boxed{x = 0}$ because $\psi = 0$ there.

(c) $\psi$ is maximized if $\frac{d\psi}{dx} = 0 = 1 - x^2\left(\frac{m\omega}{\hbar}\right)$, which is true at $\boxed{x = \pm\sqrt{\frac{\hbar}{m\omega}}}$.

(d) We require $\int_{-\infty}^{\infty}|\psi|^2\,dx = 1$:

$$1 = \int_{-\infty}^{\infty}B^2x^2e^{-(m\omega/\hbar)x^2}dx = 2B^2\int x^2e^{-(m\omega/\hbar)x^2}dx = 2B^2\frac{1}{4}\sqrt{\frac{\pi}{(m\omega/\hbar)^3}} = \frac{B^2}{2}\frac{\pi^{1/2}\hbar^{3/2}}{(m\omega)^{3/2}}.$$

Then $B = \frac{2^{1/2}}{\pi^{1/4}}\left(\frac{m\omega}{\hbar}\right)^{3/4} = \boxed{\left(\frac{4m^3\omega^3}{\pi\hbar^3}\right)^{1/4}}$.

(e) At $x = 2\sqrt{\frac{\hbar}{m\omega}}$, the potential energy is $\frac{1}{2}m\omega^2x^2 = \frac{1}{2}m\omega^2\left(\frac{4\hbar}{m\omega}\right) = 2\hbar\omega$. This is larger than the total energy $\frac{3\hbar\omega}{2}$, so there is $\boxed{\text{zero}}$ classical probability of finding the particle here.

(f) Probability $= |\psi|^2\,dx = \left(Bxe^{-(m\omega/2\hbar)x^2}\right)^2\delta = \delta B^2x^2e^{-(m\omega/\hbar)x^2}$

Probability $= \delta\frac{2}{\pi^{1/2}}\left(\frac{m\omega}{\hbar}\right)^{3/2}\left(\frac{4\hbar}{m\omega}\right)e^{-(m\omega/\hbar)4(\hbar/m\omega)} = \boxed{8\delta\left(\frac{m\omega}{\hbar\pi}\right)^{1/2}e^{-4}}$

**P41.53** (a) $$\int_0^L |\psi|^2\,dx = 1: \qquad A^2\int_0^L\left[\sin^2\left(\frac{\pi x}{L}\right)+16\sin^2\left(\frac{2\pi x}{L}\right)+8\sin\left(\frac{\pi x}{L}\right)\sin\left(\frac{2\pi x}{L}\right)\right]dx=1$$

$$A^2\left[\left(\frac{L}{2}\right)+16\left(\frac{L}{2}\right)+8\int_0^L \sin\left(\frac{\pi x}{L}\right)\sin\left(\frac{2\pi x}{L}\right)dx\right]=1$$

$$A^2\left[\frac{17L}{2}+16\int_0^L \sin^2\left(\frac{\pi x}{L}\right)\cos\left(\frac{\pi x}{L}\right)dx\right]=A^2\left[\frac{17L}{2}+\frac{16L}{3\pi}\sin^3\left(\frac{\pi x}{L}\right)\Bigg|_{x=0}^{x=L}\right]=1$$

$A^2=\dfrac{2}{17L}$, so the normalization constant is $\boxed{A=\sqrt{\dfrac{2}{17L}}}$.

(b) $$\int_{-a}^{a} |\psi|^2\,dx = 1: \qquad \int_{-a}^{a}\left[|A|^2\cos^2\left(\frac{\pi x}{2a}\right)+|B|^2\sin^2\left(\frac{\pi x}{a}\right)+2|A||B|\cos\left(\frac{\pi x}{2a}\right)\sin\left(\frac{\pi x}{a}\right)\right]dx=1$$

The first two terms are $|A|^2 a$ and $|B|^2 a$. The third term is:

$$2|A||B|\int_{-a}^{a}\cos\left(\frac{\pi x}{2a}\right)\left[2\sin\left(\frac{\pi x}{2a}\right)\cos\left(\frac{\pi x}{2a}\right)\right]dx=4|A||B|\int_{-a}^{a}\cos^2\left(\frac{\pi x}{2a}\right)\sin\left(\frac{\pi x}{2a}\right)dx$$

$$=\frac{8a|A||B|}{3\pi}\cos^3\left(\frac{\pi x}{2a}\right)\Bigg|_{-a}^{a}=0$$

so that $a\left(|A|^2+|B|^2\right)=1$, giving $\boxed{|A|^2+|B|^2=\dfrac{1}{a}}$.

***P41.54** (a) $\langle x\rangle_0=\displaystyle\int_{-\infty}^{\infty} x\left(\frac{a}{\pi}\right)^{1/2} e^{-ax^2}\,dx=\boxed{0}$, since the integrand is an odd function of *x*.

(b) $\langle x\rangle_1=\displaystyle\int_{-\infty}^{\infty} x\left(\frac{4a^3}{\pi}\right)^{1/2} x^2e^{-ax^2}\,dx=\boxed{0}$, since the integrand is an odd function of *x*.

(c) $$\langle x\rangle_{01}=\int_{-\infty}^{\infty} x\frac{1}{2}(\psi_0+\psi_1)^2\,dx=\frac{1}{2}\langle x\rangle_0+\frac{1}{2}\langle x\rangle_1+\int_{-\infty}^{\infty} x\psi_0(x)\psi_1(x)\,dx$$

The first two terms are zero, from (a) and (b). Thus:

$$\langle x\rangle_{01}=\int_{-\infty}^{\infty} x\left(\frac{a}{\pi}\right)^{1/4} e^{-ax^2/2}\left(\frac{4a^3}{\pi}\right)^{1/4} xe^{-ax^2/2}\,dx=2\left(\frac{2a^2}{\pi}\right)^{1/2}\int_0^{\infty} x^2e^{-ax^2}\,dx$$

$$=2\left(\frac{2a^2}{\pi}\right)^{1/2}\frac{1}{4}\left(\frac{\pi}{a^3}\right)^{1/2}, \text{ from Table B.6}$$

$$=\boxed{\frac{1}{\sqrt{2a}}}$$

**P41.55** With one slit open $P_1 = |\psi_1|^2$ or $P_2 = |\psi_2|^2$.

With both slits open, $P = |\psi_1 + \psi_2|^2$.

At a maximum, the wave functions are in phase $P_{\max} = \left(|\psi_1| + |\psi_2|\right)^2$.

At a minimum, the wave functions are out of phase $P_{\min} = \left(|\psi_1| - |\psi_2|\right)^2$.

Now $\dfrac{P_1}{P_2} = \dfrac{|\psi_1|^2}{|\psi_2|^2} = 25.0$, so $\dfrac{|\psi_1|}{|\psi_2|} = 5.00$

and $\dfrac{P_{\max}}{P_{\min}} = \dfrac{\left(|\psi_1| + |\psi_2|\right)^2}{\left(|\psi_1| - |\psi_2|\right)^2} = \dfrac{\left(5.00|\psi_2| + |\psi_2|\right)^2}{\left(5.00|\psi_2| - |\psi_2|\right)^2} = \dfrac{(6.00)^2}{(4.00)^2} = \dfrac{36.0}{16.0} = \boxed{2.25}$.

## ANSWERS TO EVEN PROBLEMS

**P41.2** $\dfrac{1}{2}$

**P41.4** (a) 4; (b) 6.03 eV

**P41.6** 0.517 MeV, $3.31 \times 10^{-20}$ kg·m/s

**P41.8** $\left(\dfrac{3h\lambda}{8m_e c}\right)^{1/2}$

**P41.10** (a) 5.13 meV; (b) 9.41 eV; (c) The much smaller mass of the electron requires it to have much more energy to have the same momentum.

**P41.12** (a) $\left(\dfrac{15h\lambda}{8m_e c}\right)^{1/2}$; (b) $1.25\lambda$

**P41.14** see the solution; $\dfrac{\hbar^2 k^2}{2m}$

**P41.16** (a) $\dfrac{L}{2}$; (b) $5.26 \times 10^{-5}$; (c) $3.99 \times 10^{-2}$; (d) see the solution

**P41.18** 0.250

**P41.20** (a) $\dfrac{\ell}{L} - \dfrac{1}{2\pi}\sin\left(\dfrac{2\pi\ell}{L}\right)$; (b) see the solution; (c) $0.585L$

**P41.22** (a) $\psi_1(x) = \sqrt{\dfrac{2}{L}}\cos\left(\dfrac{\pi x}{L}\right)$;
$P_1(x) = \dfrac{2}{L}\cos^2\left(\dfrac{\pi x}{L}\right)$;
$\psi_2(x) = \sqrt{\dfrac{2}{L}}\sin\left(\dfrac{2\pi x}{L}\right)$;
$P_2(x) = \dfrac{2}{L}\sin^2\left(\dfrac{2\pi x}{L}\right)$;
$\psi_3(x) = \sqrt{\dfrac{2}{L}}\cos\left(\dfrac{3\pi x}{L}\right)$;
$P_3(x) = \dfrac{2}{L}\cos^2\left(\dfrac{3\pi x}{L}\right)$;
(b) see the solution

**P41.24** (a) $\dfrac{\hbar^2}{2mL^2}\left(\dfrac{4x^2}{L^2} - 6\right)$; (b) see the solution

**P41.26** see the solution

**P41.28** $1.03 \times 10^{-3}$

**P41.30** 85.9

**P41.32** 3.92%

**P41.34** (a) see the solution; $b = \dfrac{m\omega}{2\hbar}$; (b) $E = \dfrac{3}{2}\hbar\omega$; (c) first excited state

**P41.36** (a) $B=\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}$; (b) $\delta\left(\frac{m\omega}{\pi\hbar}\right)^{1/2}$

**P41.38** see the solution

**P41.40** (a) $2.00\times10^{-10}$ m; (b) $3.31\times10^{-24}$ $\text{kg}\cdot\text{m/s}$; (c) 0.172 eV

**P41.42** (a) see the solution; (b) 0.092 0, 0.908

**P41.44** (a) see the solution; (b) 0.200; (c) 0.351; (d) 0.377 eV, 1.51 eV

**P41.46** (a) $\frac{h^2}{4m_eL^2}, \frac{5h^2}{8m_eL^2}, \frac{h^2}{m_eL^2}, \frac{5h^2}{4m_eL^2}$; (b) see the solution, $\frac{3h^2}{4m_eL^2}$

**P41.48** (a) $\frac{2}{\sqrt{L}}$; (b) 0.409

**P41.50** (a) $\sqrt{\left(\frac{nhc}{2L}\right)^2+m^2c^4}-mc^2$; (b) 46.9 fJ; 28.6%

**P41.52** (a) $\frac{3\hbar\omega}{2}$; (b) $x=0$; (c) $\pm\sqrt{\frac{\hbar}{m\omega}}$; (d) $\left(\frac{4m^3\omega^3}{\pi\hbar^3}\right)^{1/4}$; (e) 0; (f) $8\delta\left(\frac{m\omega}{\hbar\pi}\right)^{1/2}e^{-4}$

**P41.54** (a) 0; (b) 0; (c) $(2a)^{-1/2}$

# 42

# Atomic Physics

**CHAPTER OUTLINE**

42.1 Atomic Spectra of Gases
42.2 Early Models of the Atom
42.3 Bohr's Model of the Hydrogen Atom
42.4 The Quantum Model of the Hydrogen Atom
42.5 The Wave Functions of Hydrogen
42.6 Physical Interpretation of the Quantum Numbers
42.7 The Exclusion Principle and the Periodic Table
42.8 More on Atomic Spectra: Visible and X-ray
42.9 Spontaneous and Stimulated Transitions
42.10 Lasers

## ANSWERS TO QUESTIONS

**Q42.1** Neon signs emit light in a bright-line spectrum, rather than in a continuous spectrum. There are many discrete wavelengths which correspond to transitions among the various energy levels of the neon atom. This also accounts for the particular color of the light emitted from a neon sign. You can see the separate colors if you look at a section of the sign through a diffraction grating, or at its reflection in a compact disk. A spectroscope lets you read their wavelengths.

**Q42.2** One assumption is natural from the standpoint of classical physics: The electron feels an electric force of attraction to the nucleus, causing the centripetal acceleration to hold it in orbit. The other assumptions are in sharp contrast to the behavior of ordinary-size objects: The electron's angular momentum must be one of a set of certain special allowed values. During the time when it is in one of these quantized orbits, the electron emits no electromagnetic radiation. The atom radiates a photon when the electron makes a quantum jump from one orbit to a lower one.

**Q42.3** If an electron moved like a hockey puck, it could have any arbitrary frequency of revolution around an atomic nucleus. If it behaved like a charge in a radio antenna, it would radiate light with frequency equal to its own frequency of oscillation. Thus, the electron in hydrogen atoms would emit a continuous spectrum, electromagnetic waves of all frequencies smeared together.

**Q42.4** (a) Yes—provided that the energy of the photon is *precisely* enough to put the electron into one of the allowed energy states. Strangely—more precisely non-classically—enough, if the energy of the photon is not sufficient to put the electron into a particular excited energy level, the photon will not interact with the atom at all!

(b) Yes—a photon of any energy greater than 13.6 eV will ionize the atom. Any "extra" energy will go into kinetic energy of the newly liberated electron.

**Q42.5** An atomic electron does not possess enough kinetic energy to escape from its electrical attraction to the nucleus. Positive ionization energy must be injected to pull the electron out to a very large separation from the nucleus, a condition for which we define the energy of the atom to be zero. The atom is a bound system. All this is summarized by saying that the total energy of an atom is negative.

**Q42.6** From Equations 42.7, 42.8 and 42.9, we have $-|E| = -\frac{k_e e^2}{2r} = +\frac{k_e e^2}{2r} - \frac{k_e e^2}{r} = K + U_e$. Then $K = |E|$ and $U_e = -2|E|$.

**Q42.7** Bohr modeled the electron as moving in a perfect circle, with zero uncertainity in its radial coordinate. Then its radial velocity is always zero with zero uncertainty. Bohr's theory violates the uncertainty principle by making the uncertainty product $\Delta r \Delta p_r$ be zero, less than the minimum allowable $\frac{\hbar}{2}$.

**Q42.8** Fundamentally, three quantum numbers describe an orbital wave function because we live in three-dimensional space. They arise mathematically from boundary conditions on the wave function, expressed as a product of a function of *r*, a function of *θ*, and a function of *ϕ*.

**Q42.9** Bohr's theory pictures the electron as moving in a flat circle like a classical particle described by $\sum F = ma$. Schrödinger's theory pictures the electron as a cloud of probability amplitude in the three-dimensional space around the hydrogen nucleus, with its motion described by a wave equation. In the Bohr model, the ground-state angular momentum is $1\hbar$; in the Schrödinger model the ground-state angular momentum is zero. Both models predict that the electron's energy is limited to discrete energy levels, given by $\frac{-13.606 \text{ eV}}{n^2}$ with $n = 1, 2, 3$.

**Q42.10** The term *electron cloud* refers to the unpredictable location of an electron around an atomic nucleus. It is a cloud of probability amplitude. An electron in an *s* subshell has a spherically symmetric probability distribution. Electrons in *p*, *d*, and *f* subshells have directionality to their distribution. The shape of these electron clouds influences how atoms form molecules and chemical compounds.

**Q42.11** The direction of the magnetic moment due to an orbiting charge is given by the right hand rule, but assumes a *positive* charge. Since the electron is negatively charged, its magnetic moment is in the opposite direction to its angular momentum.

**Q42.12** Practically speaking, no. Ions have a net charge and the magnetic force $q(\mathbf{v} \times \mathbf{B})$ would deflect the beam, making it difficult to separate the atoms with different orientations of magnetic moments.

**Q42.13** The deflecting force on an atom with a magnetic moment is proportional to the *gradient* of the magnetic field. Thus, atoms with oppositely directed magnetic moments would be deflected in *opposite* directions in an inhomogeneous magnetic field.

**Q42.14** If the exclusion principle were not valid, the elements and their chemical behavior would be grossly different because every electron would end up in the lowest energy level of the atom. All matter would be nearly alike in its chemistry and composition, since the shell structures of all elements would be identical. Most materials would have a much higher density. The spectra of atoms and molecules would be very simple, and there would be very little color in the world.

**Q42.15** The Stern-Gerlach experiment with hydrogen atoms shows that the component of an electron's spin angular momentum along an applied magnetic field can have only one of two allowed values. So does electron spin resonance on atoms with one unpaired electron.

**Q42.16** The three elements have similar electronic configurations. Each has filled inner shells plus one electron in an *s* orbital. Their single outer electrons largely determine their chemical interactions with other atoms.

**Q42.17** When a photon interacts with an atom, the atom's orbital angular momentum changes, thus the photon must carry orbital angular momentum. Since the allowed transitions of an atom are restricted to a change in angular momentum of $\Delta\ell = \pm 1$, the photon must have spin 1.

**Q42.18** In a neutral helium atom, one electron can be modeled as moving in an electric field created by the nucleus and the other electron. According to Gauss's law, if the electron is above the ground state it moves in the electric field of a net charge of $+2e - 1e = +1e$ We say the nuclear charge is *screened* by the inner electron. The electron in a $\text{He}^+$ ion moves in the field of the unscreened nuclear charge of 2 protons. Then the potential energy function for the electron is about double that of one electron in the neutral atom.

**Q42.19** At low density, the gas consists of essentially separate atoms. As the density increases, the atoms interact with each other. This has the effect of giving different atoms levels at slightly different energies, at any one instant. The collection of atoms can then emit photons in lines or bands, narrower or wider, depending on the density.

**Q42.20** An atom is a quantum system described by a wave function. The electric force of attraction to the nucleus imposes a constraint on the electrons. The physical constraint implies mathematical boundary conditions on the wave functions, with consequent quantization so that only certain wave functions are allowed to exist. The Schrödinger equation assigns a definite energy to each allowed wave function. Each wave function is spread out in space, describing an electron with no definite position. If you like analogies, think of a classical standing wave on a string fixed at both ends. Its position is spread out to fill the whole string, but its frequency is one of a certain set of quantized values.

**Q42.21** Each of the electrons must have at least one quantum number different from the quantum numbers of each of the other electrons. They can differ (in $m_s$) by being spin-up or spin-down. They can also differ (in $\ell$) in angular momentum and in the general shape of the wave function. Those electrons with $\ell = 1$ can differ (in $m_\ell$) in orientation of angular momentum—look at Figure Q42.21.

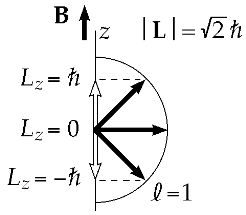

**FIG. Q42.21**

**Q42.22** The Mosely graph shows that the reciprocal square root of the wavelength of $K_\alpha$ characteristic x-rays is a linear function of atomic number. Then measuring this wavelength for a new chemical element reveals its location on the graph, including its atomic number.

**Q42.23** No. Laser light is collimated. The energy generally travels in the same direction. The intensity of a laser beam stays remarkably constant, independent of the distance it has traveled.

**Q42.24** Stimulated emission coerces atoms to emit photons along a specific axis, rather than in the random directions of spontaneously emitted photons. The photons that are emitted through stimulation can be made to accumulate over time. The fraction allowed to escape constitutes the intense, collimated, and coherent laser beam. If this process relied solely on spontaneous emission, the emitted photons would not exit the laser tube or crystal in the same direction. Neither would they be coherent with one another.

**Q42.25** (a) The terms "I define" and "this part of the universe" seem vague, in contrast to the precision of the rest of the statement. But the statement is true in the sense of being experimentally verifiable. The way to test the orientation of the magnetic moment of an electron is to apply a magnetic field to it. When that is done for any electron, it has precisely a 50% chance of being either spin-up or spin-down. Its spin magnetic moment vector must make one of two allowed angles with the applied magnetic field. They are given by $\cos\theta = \frac{S_z}{S} = \frac{1/2}{\sqrt{3}/2}$ and $\cos\theta = \frac{-1/2}{\sqrt{3}/2}$. You can calculate as many digits of the two angles allowed by "space quantization" as you wish.

(b) This statement may be true. There is no reason to suppose that an ant can comprehend the cosmos, and no reason to suppose that a human can comprehend all of it. Our experience with macroscopic objects does not prepare us to understand quantum particles. On the other hand, what seems strange to us now may be the common knowledge of tomorrow. Looking back at the past 150 years of physics, great strides in understanding the Universe—from the quantum to the galactic scale—have been made. Think of trying to explain the photoelectric effect using Newtonian mechanics. What seems strange sometimes just has an underlying structure that has not yet been described fully. On the other hand still, it has been demonstrated that a "hidden-variable" theory, that would model quantum uncertainty as caused by some determinate but fluctuating quantity, cannot agree with experiment.

## SOLUTIONS TO PROBLEMS

### Section 42.1 **Atomic Spectra of Gases**

**P42.1** (a) Lyman series

$$\frac{1}{\lambda} = R\left(1 - \frac{1}{n_i^2}\right) \qquad n_i = 2, 3, 4, \ldots$$

$$\frac{1}{\lambda} = \frac{1}{94.96 \times 10^{-9}} = \left(1.097 \times 10^7\right)\left(1 - \frac{1}{n_i^2}\right) \qquad \boxed{n_i = 5}$$

(b) Paschen series:

$$\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n_i^2}\right) \qquad n_i = 4, 5, 6, \ldots$$

The shortest wavelength for this series corresponds to $n_i = \infty$ for ionization

$$\frac{1}{\lambda} = 1.097 \times 10^7\left(\frac{1}{9} - \frac{1}{n_i^2}\right)$$

For $n_i = \infty$, this gives $\lambda = 820$ nm

This is larger than 94.96 nm, so this wave length $\boxed{\text{cannot be associated with the Paschen series}}$.

Balmer series:

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n_i^2}\right) \qquad n_i = 3, 4, 5, \ldots$$

$$\frac{1}{\lambda} = 1.097 \times 10^7\left(\frac{1}{4} - \frac{1}{n_i^2}\right) \qquad \text{with } n_i = \infty \text{ for ionization, } \lambda_{\min} = 365 \text{ nm}$$

Once again the shorter given wavelength $\boxed{\text{cannot be associated with the Balmer series}}$.

**P42.2** (a) $\lambda_{\min} = \dfrac{hc}{E_{\max}}$

Lyman ($n_f = 1$): $\lambda_{\min} = \dfrac{hc}{|E_1|} = \dfrac{1\,240\text{ eV}\cdot\text{nm}}{13.6\text{ eV}} = \boxed{91.2\text{ nm}}$ (Ultraviolet)

Balmer ($n_f = 2$): $\lambda_{\min} = \dfrac{hc}{|E_2|} = \dfrac{1\,240\text{ eV}\cdot\text{nm}}{(1/4)13.6\text{ eV}} = \boxed{365\text{ nm}}$ (UV)

Paschen ($n_f = 3$): $\lambda_{\min} = \ldots = 3^2(91.2\text{ nm}) = \boxed{821\text{ nm}}$ (Infrared)

Bracket ($n_f = 4$): $\lambda_{\min} = \ldots = 4^2(91.2\text{ nm}) = \boxed{1\,460\text{ nm}}$ (IR)

(b) $E_{\max} = \dfrac{hc}{\lambda_{\min}}$

Lyman: $E_{\max} = \boxed{13.6\text{ eV}}\left(=|E_1|\right)$

Balmer: $E_{\max} = \boxed{3.40\text{ eV}}\left(=|E_2|\right)$

Paschen: $E_{\max} = \boxed{1.51\text{ eV}}\left(=|E_3|\right)$

Brackett: $E_{\max} = \boxed{0.850\text{ eV}}\left(=|E_4|\right)$

## Section 42.2 **Early Models of the Atom**

**P42.3** (a) For a classical atom, the centripetal acceleration is

$$a = \frac{v^2}{r} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2 m_e}$$

$$E = -\frac{e^2}{4\pi\epsilon_0 r} + \frac{m_e v^2}{2} = -\frac{e^2}{8\pi\epsilon_0 r}$$

so $$\frac{dE}{dt} = \frac{e^2}{8\pi\epsilon_0 r^2}\frac{dr}{dt} = \frac{-1}{6\pi\epsilon_0}\frac{e^2 a^2}{c^3} = \frac{-e^2}{6\pi\epsilon_0 c^3}\left(\frac{e^2}{4\pi\epsilon_0 r^2 m_e}\right)^2.$$

Therefore, $$\frac{dr}{dt} = -\frac{e^4}{12\pi^2\epsilon_0^2 r^2 m_e^2 c^3}.$$

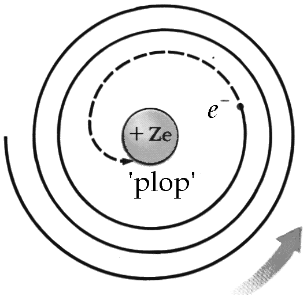

**FIG. P42.3**

(b) $$-\int_{2.00\times10^{-10}\text{ m}}^{0} 12\pi^2\epsilon_0^2 r^2 m_e^2 c^3\,dr = e^4\int_0^T dt \qquad \frac{12\pi^2\epsilon_0^2 m_e^2 c^3}{e^4}\left.\frac{r^3}{3}\right|_0^{2.00\times10^{-10}} = T = \boxed{8.46\times10^{-10}\text{ s}}$$

Since atoms last a lot longer than 0.8 ns, the classical laws (fortunately!) do not hold for systems of atomic size.

**P42.4** (a) The point of closest approach is found when

$$E = K + U = 0 + \frac{k_e q_\alpha q_{\text{Au}}}{r}$$

or $$r_{\min} = \frac{k_e(2e)(79e)}{E}$$

$$r_{\min} = \frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(158)\left(1.60\times10^{-19}\ \text{C}\right)^2}{(4.00\ \text{MeV})\left(1.60\times10^{-13}\ \text{J/MeV}\right)} = \boxed{5.68\times10^{-14}\ \text{m}}.$$

(b) The maximum force exerted on the alpha particle is

$$F_{\max} = \frac{k_e q_\alpha q_{\text{Au}}}{r_{\min}^2} = \frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(158)\left(1.60\times10^{-19}\ \text{C}\right)^2}{\left(5.68\times10^{-14}\ \text{m}\right)^2} = \boxed{11.3\ \text{N}}$$ away from the nucleus.

## Section 42.3 Bohr's Model of the Hydrogen Atom

**P42.5** (a) $$v_1 = \sqrt{\frac{k_e e^2}{m_e r_1}}$$

where $r_1 = (1)^2 a_0 = 0.005\,29\ \text{nm} = 5.29\times10^{-11}\ \text{m}$

$$v_1 = \sqrt{\frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)^2}{\left(9.11\times10^{-31}\ \text{kg}\right)\left(5.29\times10^{-11}\ \text{m}\right)}} = \boxed{2.19\times10^{6}\ \text{m/s}}$$

(b) $$K_1 = \frac{1}{2}m_e v_1^2 = \frac{1}{2}\left(9.11\times10^{-31}\ \text{kg}\right)\left(2.19\times10^{6}\ \text{m/s}\right)^2 = 2.18\times10^{-18}\ \text{J} = \boxed{13.6\ \text{eV}}$$

(c) $$U_1 = -\frac{k_e e^2}{r_1} = -\frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)^2}{5.29\times10^{-11}\ \text{m}} = -4.35\times10^{-18}\ \text{J} = \boxed{-27.2\ \text{eV}}$$

**P42.6** $$\Delta E = (13.6\ \text{eV})\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$$

Where for $\Delta E > 0$ we have absorption and for $\Delta E < 0$ we have emission.

(i) for $n_i = 2$ and $n_f = 5$, $\Delta E = 2.86\ \text{eV}$ (absorption)

(ii) for $n_i = 5$ and $n_f = 3$, $\Delta E = -0.967\ \text{eV}$ (emission)

(iii) for $n_i = 7$ and $n_f = 4$, $\Delta E = -0.572\ \text{eV}$ (emission)

(iv) for $n_i = 4$ and $n_f = 7$, $\Delta E = 0.572\ \text{eV}$ (absorption)

(a) $E = \frac{hc}{\lambda}$ so the shortest wavelength is emitted in transition $\boxed{\text{ii}}$.

(b) The atom gains most energy in transition $\boxed{\text{i}}$.

(c) The atom loses energy in transitions $\boxed{\text{ii and iii}}$.

**P42.7** (a) $r_2^2 = (0.052\,9\text{ nm})(2)^2 = \boxed{0.212\text{ nm}}$

(b) $$m_e v_2 = \sqrt{\frac{m_e k_e e^2}{r_2}} = \sqrt{\frac{\left(9.11\times10^{-31}\text{ kg}\right)\left(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\text{ C}\right)^2}{0.212\times10^{-9}\text{ m}}}$$

$$m_e v_2 = \boxed{9.95\times10^{-25}\text{ kg}\cdot\text{m/s}}$$

(c) $$L_2 = m_e v_2 r_2 = \left(9.95\times10^{-25}\text{ kg}\cdot\text{m/s}\right)\left(0.212\times10^{-9}\text{ m}\right) = \boxed{2.11\times10^{-34}\text{ kg}\cdot\text{m}^2/\text{s}}$$

(d) $$K_2 = \frac{1}{2}m_e v_2^2 = \frac{(m_e v_2)^2}{2m_e} = \frac{\left(9.95\times10^{-25}\text{ kg}\cdot\text{m/s}\right)^2}{2\left(9.11\times10^{-31}\text{ kg}\right)} = 5.43\times10^{-19}\text{ J} = \boxed{3.40\text{ eV}}$$

(e) $$U_2 = -\frac{k_e e^2}{r_2} = -\frac{\left(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\text{ C}\right)^2}{0.212\times10^{-9}\text{ m}} = -1.09\times10^{-18}\text{ J} = \boxed{-6.80\text{ eV}}$$

(f) $E_2 = K_2 + U_2 = 3.40\text{ eV} - 6.80\text{ eV} = \boxed{-3.40\text{ eV}}$

**P42.8** We use $E_n = \frac{-13.6\text{ eV}}{n^2}$.

To ionize the atom when the electron is in the $n^{\text{th}}$ level,

it is necessary to add an amount of energy given by $E = -E_n = \frac{13.6\text{ eV}}{n^2}$.

(a) Thus, in the ground state where $n = 1$, we have $\boxed{E = 13.6\text{ eV}}$.

(b) In the $n = 3$ level, $E = \frac{13.6\text{ eV}}{9} = \boxed{1.51\text{ eV}}$.

**P42.9** (b) $$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = \left(1.097\times10^{7}\text{ m}^{-1}\right)\left(\frac{1}{2^2} - \frac{1}{6^2}\right) \text{ so } \boxed{\lambda = 410\text{ nm}}$$

(a) $$E = \frac{hc}{\lambda} = \frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{410\times10^{-9}\text{ m}} = 4.85\times10^{-19}\text{ J} = \boxed{3.03\text{ eV}}$$

(c) $$f = \frac{c}{\lambda} = \frac{3.00\times10^{8}}{410\times10^{-9}} = \boxed{7.32\times10^{14}\text{ Hz}}$$

**P42.10** Starting with $\frac{1}{2}m_e v^2 = \frac{k_e e^2}{2r}$

we have $v^2 = \frac{k_e e^2}{m_e r}$

and using $r_n = \frac{n^2\hbar^2}{m_e k_e e^2}$

gives $v_n^2 = \frac{k_e e^2}{m_e\left(n^2\hbar^2/m_e k_e e^2\right)}$

or $v_n = \frac{k_e e^2}{n\hbar}$.

**P42.11** Each atom gives up its kinetic energy in emitting a photon,

so $$\frac{1}{2}mv^2 = \frac{hc}{\lambda} = \frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{\left(1.216\times10^{-7}\text{ m}\right)} = 1.63\times10^{-18}\text{ J}$$

$$v = \boxed{4.42\times10^{4}\text{ m/s}}.$$

**P42.12** The batch of excited atoms must make these six transitions to get back to state one: $2\to1$, and also $3\to2$ and $3\to1$, and also $4\to3$ and $4\to2$ and $4\to1$. Thus, the incoming light must have just enough energy to produce the $1\to4$ transition. It must be the third line of the Lyman series in the absorption spectrum of hydrogen. The absorbing atom changes from energy

$$E_i = -\frac{13.6\text{ eV}}{1^2} = -13.6\text{ eV} \text{ to } E_f = -\frac{13.6\text{ eV}}{4^2} = -0.850\text{ eV},$$

so the incoming photons have wavelength

$$\lambda = \frac{hc}{E_f - E_i} = \frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{-0.850\text{ eV}-(-13.6\text{ eV})}\left(\frac{1.00\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) = 9.75\times10^{-8}\text{ m} = \boxed{97.5\text{ nm}}.$$

**P42.13** (a) The energy levels of a hydrogen-like ion whose charge number is $Z$ are given by

$$E_n = (-13.6\text{ eV})\frac{Z^2}{n^2}.$$

Thus for Helium $(Z = 2)$, the energy levels are

$$\boxed{E_n = -\frac{54.4\text{ eV}}{n^2} \quad n = 1, 2, 3, \dots}.$$

(b) For $\text{He}^+$, $Z = 2$, so we see that the ionization energy (the energy required to take the electron from the $n = 1$ to the $n = \infty$ state) is

$$E = E_\infty - E_1 = 0 - \frac{(-13.6\text{ eV})(2)^2}{(1)^2} = \boxed{54.4\text{ eV}}.$$

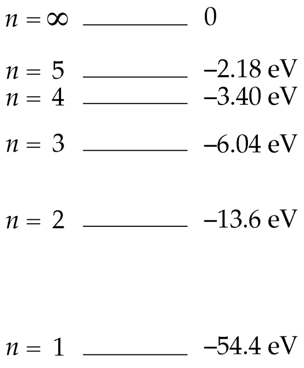

**FIG. P42.13**

***P42.14** (a) $\frac{1}{\lambda}=Z^2R_H\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right)$. The shortest wavelength, $\lambda_s$, corresponds to $n_i=\infty$, and the longest wavelength, $\lambda_\ell$, to $n_i=n_f+1$.

$$\frac{1}{\lambda_s}=\frac{Z^2R_H}{n_f^2} \qquad (1)$$

$$\frac{1}{\lambda_\ell}=Z^2R_H\left[\frac{1}{n_f^2}-\frac{1}{(n_f+1)^2}\right]=\frac{Z^2R_H}{n_f^2}\left[1-\left(\frac{n_f}{n_f+1}\right)^2\right] \qquad (2)$$

Divide (1) and (2): $\frac{\lambda_s}{\lambda_\ell}=1-\left(\frac{n_f}{n_f+1}\right)^2$

$$\therefore \frac{n_f}{n_f+1}=\sqrt{1-\frac{\lambda_s}{\lambda_\ell}}=\sqrt{1-\frac{22.8\text{ nm}}{63.3\text{ nm}}}=0.800 \qquad \therefore n_f=4$$

From (1): $Z=\sqrt{\frac{n_f^2}{\lambda_sR_H}}=\sqrt{\frac{4^2}{(22.8\times10^{-9}\text{ m})(1.097\times10^{7}\text{ m}^{-1})}}=8.00$.

Hence $\boxed{\text{the ion is O}^{7+}}$.

(b) $\lambda=\left\{(7.020\,8\times10^{8}\text{ m}^{-1})\left[\frac{1}{4^2}-\frac{1}{(4+k)^2}\right]\right\}^{-1}$, $k=1, 2, 3, \ldots$

Setting $k=2, 3, 4$ gives $\lambda=\boxed{41.0\text{ nm, }33.8\text{ nm, }30.4\text{ nm}}$.

**P42.15** (a) The speed of the moon in its orbit is $v=\frac{2\pi r}{T}=\frac{2\pi(3.84\times10^{8}\text{ m})}{2.36\times10^{6}\text{ s}}=1.02\times10^{3}\text{ m/s}$.

So, $L=mvr=(7.36\times10^{22}\text{ kg})(1.02\times10^{3}\text{ m/s})(3.84\times10^{8}\text{ m})=\boxed{2.89\times10^{34}\text{ kg}\cdot\text{m}^2/\text{s}}$.

(b) We have $L=n\hbar$

or $n=\frac{L}{\hbar}=\frac{2.89\times10^{34}\text{ kg}\cdot\text{m}^2/\text{s}}{1.055\times10^{-34}\text{ J}\cdot\text{s}}=\boxed{2.74\times10^{68}}$.

(c) We have $n\hbar=L=mvr=m\left(\frac{GM_e}{r}\right)^{1/2}r$,

so $r=\frac{\hbar^2}{m^2GM_e}n^2=Rn^2$ and $\frac{\Delta r}{r}=\frac{(n+1)^2R-n^2R}{n^2R}=\frac{2n+1}{n^2}$

which is approximately equal to $\frac{2}{n}=\boxed{7.30\times10^{-69}}$.

## Section 42.4 **The Quantum Model of the Hydrogen Atom**

**P42.16** The reduced mass of positronium is **less** than hydrogen, so the photon energy will be **less** for positronium than for hydrogen. This means that the wavelength of the emitted photon will be **longer** than 656.3 nm. On the other hand, helium has about the same reduced mass but more charge than hydrogen, so its transition energy will be **larger**, corresponding to a wavelength **shorter** than 656.3 nm.

All the factors in the given equation are constant for this problem except for the reduced mass and the nuclear charge. Therefore, the wavelength corresponding to the energy difference for the transition can be found simply from the ratio of mass and charge variables.

For hydrogen, $\mu = \frac{m_p m_e}{m_p + m_e} \approx m_e$. The photon energy is $\Delta E = E_3 - E_2$.

Its wavelength is $\lambda = 656.3 \text{ nm}$, where $\lambda = \frac{c}{f} = \frac{hc}{\Delta E}$.

(a) For positronium, $\mu = \frac{m_e m_e}{m_e + m_e} = \frac{m_e}{2}$

so the energy of each level is one half as large as in hydrogen, which we could call "protonium". The photon energy is inversely proportional to its wavelength , so for positronium,

$$\lambda_{32} = 2(656.3 \text{ nm}) = \boxed{1.31\ \mu\text{m}} \text{ (in the infrared region).}$$

(b) For He$^+$, $\mu \approx m_e$, $q_1 = e$, and $q_2 = 2e$,

so the transition energy is $2^2 = 4$ times larger than hydrogen.

Then, $\lambda_{32} = \left(\frac{656}{4}\right) \text{nm} = \boxed{164 \text{ nm}}$ (in the ultraviolet region).

**P42.17** (a) $\Delta x \Delta p \geq \frac{\hbar}{2}$ so if $\Delta x = r$, $\Delta p \geq \boxed{\frac{\hbar}{2r}}$.

(b) Choosing $\Delta p \approx \frac{\hbar}{r}$, $K = \frac{p^2}{2m_e} \approx \frac{(\Delta p)^2}{2m_e} = \boxed{\frac{\hbar^2}{2m_e r^2}}$

$U = \frac{-k_e e^2}{r}$, so $E = K + U \approx \boxed{\frac{\hbar^2}{2m_e r^2} - \frac{k_e e^2}{r}}$.

(c) To minimize $E$,

$$\frac{dE}{dr} = -\frac{\hbar^2}{m_e r^3} + \frac{k_e e^2}{r^2} = 0 \to r = \boxed{\frac{\hbar^2}{m_e k_e e^2} = a_0} \text{ (the Bohr radius).}$$

Then, $E = \frac{\hbar^2}{2m_e}\left(\frac{m_e k_e e^2}{\hbar^2}\right)^2 - k_e e^2 \left(\frac{m_e k_e e^2}{\hbar^2}\right) = -\frac{m_e k_e^2 e^4}{2\hbar^2} = \boxed{-13.6 \text{ eV}}$.

## Section 42.5 **The Wave Functions of Hydrogen**

**P42.18** $\psi_{1s}(r)=\dfrac{1}{\sqrt{\pi a_0^3}}e^{-r/a_0}$ (Eq. 42.22)

$P_{1s}(r)=\dfrac{4r^2}{a_0^3}e^{-2r/a_0}$ (Eq. 42.25)

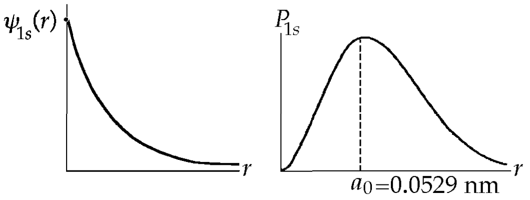

**FIG. P42.18**

**P42.19** (a) $$\int|\psi|^2dV=4\pi\int_0^\infty|\psi|^2r^2dr=4\pi\left(\frac{1}{\pi a_0^3}\right)\int_0^\infty r^2e^{-2r/a_0}dr$$

Using integral tables, $\displaystyle\int|\psi|^2dV=-\frac{2}{a_0^2}\left[e^{-2r/a_0}\left(r^2+a_0r+\frac{a_0^2}{2}\right)\right]_0^\infty=\left(-\frac{2}{a_0^2}\right)\left(-\frac{a_0^2}{2}\right)=\boxed{1}$

so the wave function as given is normalized.

(b) $$P_{a_0\,2\to3a_0\,2}=4\pi\int_{a_0/2}^{3a_0/2}|\psi|^2r^2dr=4\pi\left(\frac{1}{\pi a_0^3}\right)\int_{a_0/2}^{3a_0/2}r^2e^{-2r/a_0}dr$$

Again, using integral tables,

$$P_{a_0\,2\to3a_0\,2}=-\frac{2}{a_0^2}\left[e^{-2r/a_0}\left(r^2+a_0r+\frac{a_0^2}{2}\right)\right]_{a_0/2}^{3a_0/2}=-\frac{2}{a_0^2}\left[e^{-3}\left(\frac{17a_0^2}{4}\right)-e^{-1}\left(\frac{5a_0^2}{4}\right)\right]=\boxed{0.497}.$$

**P42.20** $\psi=\dfrac{1}{\sqrt{3}}\dfrac{1}{(2a_0)^{3/2}}\dfrac{r}{a_0}e^{-r/2a_0}$

so $\qquad P_r=4\pi r^2\left|\psi^2\right|=4\pi r^2\dfrac{r^2}{24a_0^5}e^{-r/a_0}.$

Set $\qquad \dfrac{dP}{dr}=\dfrac{4\pi}{24a_0^5}\left[4r^3e^{-r/a_0}+r^4\left(-\dfrac{1}{a_0}\right)e^{-r/a_0}\right]=0.$

Solving for $r$, this is a maximum at $\boxed{r=4a_0}$.

**P42.21** $\psi=\dfrac{1}{\sqrt{\pi a_0^3}}e^{-r/a_0}$ $\qquad \dfrac{2}{r}\dfrac{d\psi}{dr}=\dfrac{-2}{r\sqrt{\pi a_0^5}}e^{-r/a_0}=-\dfrac{2}{ra_0}\psi$

$\dfrac{d^2\psi}{dr^2}=\dfrac{1}{\sqrt{\pi a_0^7}}e^{-r/a_0}=\dfrac{1}{a_0^2}\psi$ $\qquad -\dfrac{\hbar^2}{2m_e}\left(\dfrac{1}{a_0^2}-\dfrac{2}{ra_0}\right)\psi-\dfrac{e^2}{4\pi\in_0 r}\psi=E\psi$

But $\qquad a_0=\dfrac{\hbar^2(4\pi\in_0)}{m_ee^2}$

so $\qquad -\dfrac{e^2}{8\pi\in_0 a_0}=E$

or $\qquad \boxed{E=-\dfrac{k_ee^2}{2a_0}}.$

This is true, so the Schrödinger equation is satisfied.

**P42.22** The hydrogen ground-state radial probability density is

$$P(r)=4\pi r^2\left|\psi_{1s}\right|^2=\frac{4r^2}{a_0^3}\exp\left(-\frac{2r}{a_0}\right).$$

The number of observations at $2a_0$ is, by proportion

$$N=1\,000\frac{P(2a_0)}{P(a_0/2)}=1\,000\frac{(2a_0)^2}{(a_0/2)^2}\frac{e^{-4a_0/a_0}}{e^{-a_0/a_0}}=1\,000(16)e^{-3}=\boxed{797\text{ times}}.$$

## Section 42.6 **Physical Interpretation of the Quantum Numbers**

*Note*: Problems 17 and 25 in Chapter 29 and Problem 68 in Chapter 30 can be assigned with this section.

**P42.23** (a) In the 3*d* subshell, $n=3$ and $\ell=2$,

we have

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $n$ | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| $\ell$ | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| $m_\ell$ | +2 | +2 | +1 | +1 | 0 | 0 | –1 | –1 | –2 | –2 |
| $m_s$ | +1/2 | –1/2 | +1/2 | –1/2 | +1/2 | –1/2 | +1/2 | –1/2 | +1/2 | –1/2 |

(A total of 10 states)

(b) In the 3*p* subshell, $n=3$ and $\ell=1$,

we have

| | | | | | | |
|---|---|---|---|---|---|---|
| $n$ | 3 | 3 | 3 | 3 | 3 | 3 |
| $\ell$ | 1 | 1 | 1 | 1 | 1 | 1 |
| $m_\ell$ | +1 | +1 | +0 | +0 | –1 | –1 |
| $m_s$ | +1/2 | –1/2 | +1/2 | –1/2 | +1/2 | –1/2 |

(A total of 6 states)

**P42.24** (a) For the *d* state, $\ell=2$, $L=\boxed{\sqrt{6}\hbar}=2.58\times10^{-34}\text{ J}\cdot\text{s}$.

(b) For the *f* state, $\ell=3$, $L=\sqrt{\ell(\ell+1)}\hbar=\boxed{\sqrt{12}\hbar}=3.65\times10^{-34}\text{ J}\cdot\text{s}$.

**P42.25** $L=\sqrt{\ell(\ell+1)}\hbar$:

$$4.714\times10^{-34}=\sqrt{\ell(\ell+1)}\left(\frac{6.626\times10^{-34}}{2\pi}\right)$$

$$\ell(\ell+1)=\frac{\left(4.714\times10^{-4}\right)^2(2\pi)^2}{\left(6.626\times10^{-34}\right)^2}=1.998\times10^{1}\approx20=4(4+1)$$

so $\boxed{\ell=4}$.

**P42.26** The 5th excited state has $n = 6$, energy $\dfrac{-13.6 \text{ eV}}{36} = -0.378 \text{ eV}$.

The atom loses this much energy: $\dfrac{hc}{\lambda} = \dfrac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^{8} \text{ m/s})}{(1\,090 \times 10^{-9} \text{ m})(1.60 \times 10^{-19} \text{ J/eV})} = 1.14 \text{ eV}$

to end up with energy $-0.378 \text{ eV} - 1.14 \text{ eV} = -1.52 \text{ eV}$

which is the energy in state 3: $-\dfrac{13.6 \text{ eV}}{3^3} = -1.51 \text{ eV}$.

While $n = 3$, $\ell$ can be as large as 2, giving angular momentum $\sqrt{\ell(\ell+1)}\hbar = \boxed{\sqrt{6}\hbar}$.

**P42.27** (a) $n = 1$: For $n = 1$, $\ell = 0$, $m_\ell = 0$, $m_s = \pm\dfrac{1}{2}$

| $n$ | $\ell$ | $m_\ell$ | $m_s$ |
|---|---|---|---|
| 1 | 0 | 0 | –1/2 |
| 1 | 0 | 0 | +1/2 |

Yields 2 sets; $2n^2 = 2(1)^2 = \boxed{2}$

(b) $n = 2$: For $n = 2$,

we have

| $n$ | $\ell$ | $m_\ell$ | $m_s$ |
|---|---|---|---|
| 2 | 0 | 0 | ±1/2 |
| 2 | 1 | –1 | ±1/2 |
| 2 | 1 | 0 | ±1/2 |
| 2 | 1 | 1 | ±1/2 |

yields 8 sets; $2n^2 = 2(2)^2 = \boxed{8}$

Note that the number is twice the number of $m_\ell$ values. Also, for each $\ell$ there are $(2\ell+1)$ different $m_\ell$ values. Finally, $\ell$ can take on values ranging from 0 to $n-1$.

So the general expression is $\text{number} = \sum_{0}^{n-1} 2(2\ell+1)$.

The series is an arithmetic progression: $2 + 6 + 10 + 14\ldots$

the sum of which is $\text{number} = \dfrac{n}{2}\left[2a + (n-1)d\right]$

where $a = 2$, $d = 4$: $\text{number} = \dfrac{n}{2}\left[4 + (n-1)4\right] = 2n^2$.

(c) $n = 3$: $2(1) + 2(3) + 2(5) = 2 + 6 + 10 = 18$ $2n^2 = 2(3)^2 = \boxed{18}$

(d) $n = 4$: $2(1) + 2(3) + 2(5) + 2(7) = 32$ $2n^2 = 2(4)^2 = \boxed{32}$

(e) $n = 5$: $32 + 2(9) = 32 + 18 = 50$ $2n^2 = 2(5)^2 = \boxed{50}$

**P42.28** For a 3*d* state, $n = 3$ and $\ell = 2$.

Therefore, $L = \sqrt{\ell(\ell+1)}\hbar = \boxed{\sqrt{6}\hbar} = 2.58 \times 10^{-34}\ \text{J} \cdot \text{s}$

$m_\ell$ can have the values –2, –1, 0, 1, and 2

so $\boxed{L_z \text{ can have the values } -2\hbar,\ -\hbar,\ 0,\ \hbar \text{ and } 2\hbar}$.

Using the relation $\cos\theta = \dfrac{L_z}{L}$

we find the possible values of $\theta$ $\boxed{145°,\ 114°,\ 90.0°,\ 65.9°,\ \text{and } 35.3°}$.

**P42.29** (a) Density of a proton: $\rho = \dfrac{m}{V} = \dfrac{1.67 \times 10^{-27}\ \text{kg}}{(4/3)\pi\left(1.00 \times 10^{-15}\ \text{m}\right)^3} = \boxed{3.99 \times 10^{17}\ \text{kg/m}^3}$.

(b) Size of model electron: $r = \left(\dfrac{3m}{4\pi\rho}\right)^{1/3} = \left(\dfrac{3\left(9.11 \times 10^{-31}\ \text{kg}\right)}{4\pi\left(3.99 \times 10^{17}\ \text{kg/m}^3\right)}\right)^{1/3} = \boxed{8.17 \times 10^{-17}\ \text{m}}$.

(c) Moment of inertia: $I = \dfrac{2}{5}mr^2 = \dfrac{2}{5}\left(9.11 \times 10^{-31}\ \text{kg}\right)\left(8.17 \times 10^{-17}\ \text{m}\right)^2 = 2.43 \times 10^{-63}\ \text{kg} \cdot \text{m}^2$

$$L_z = I\omega = \frac{\hbar}{2} = \frac{Iv}{r}.$$

Therefore, $v = \dfrac{\hbar r}{2I} = \dfrac{\left(6.626 \times 10^{-34}\ \text{J} \cdot \text{s}\right)\left(8.17 \times 10^{-17}\ \text{m}\right)}{2\pi\left(2 \times 2.43 \times 10^{-63}\ \text{kg} \cdot \text{m}^2\right)} = \boxed{1.77 \times 10^{12}\ \text{m/s}}$.

(d) This is $\boxed{5.91 \times 10^3 \text{ times larger}}$ than the speed of light.

**P42.30** In the N shell, $n = 4$. For $n = 4$, $\ell$ can take on values of 0, 1, 2, and 3. For each value of $\ell$, $m_\ell$ can be $-\ell$ to $\ell$ in integral steps. Thus, the maximum value for $m_\ell$ is 3. Since $L_z = m_\ell \hbar$, the maximum value for $L_z$ is $L_z = \boxed{3\hbar}$.

**P42.31** The 3*d* subshell has $\ell = 2$, and $n = 3$. Also, we have $s = 1$.

Therefore, we can have $\boxed{n = 3,\ \ell = 2;\ m_\ell = -2,\ -1,\ 0,\ 1,\ 2;\ s = 1;\ \text{and } m_s = -1,\ 0,\ 1}$

leading to the following table:

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $n$ | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| $\ell$ | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| $m_\ell$ | –2 | –2 | –2 | –1 | –1 | –1 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 2 |
| $s$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $m_s$ | –1 | 0 | 1 | –1 | 0 | 1 | –1 | 0 | 1 | –1 | 0 | 1 | –1 | 0 | 1 |

## Section 42.7 **The Exclusion Principle and the Periodic Table**

**P42.32** (a) $\boxed{1s^2 2s^2 2p^4}$

(b) For the 1*s* electrons, $n = 1$, $\ell = 0$, $m_\ell = 0$, $m_s = +\frac{1}{2}$ and $-\frac{1}{2}$.

For the two 2*s* electrons, $n = 2$, $\ell = 0$, $m_\ell = 0$, $m_s = +\frac{1}{2}$ and $-\frac{1}{2}$.

For the four 2*p* electrons, $n = 2$; $\ell = 1$; $m_\ell = -1$, 0, or 1; and $m_s = +\frac{1}{2}$ or $-\frac{1}{2}$.

**P42.33** The $\boxed{4s\text{ subshell fills first}}$, for potassium and calcium, before the 3*d* subshell starts to fill for scandium through zinc. Thus, we would first suppose that $[\mathrm{Ar}]3d^4 4s^2$ would have lower energy than $[\mathrm{Ar}]3d^5 4s^1$. But the latter has more unpaired spins, six instead of four, and Hund's rule suggests that this could give the latter configuration lower energy. In fact it must, for $[\mathrm{Ar}]3d^5 4s^1$ is the ground state for chromium.

**P42.34** Electronic configuration: Sodium to Argon

$\left[1s^2 2s^2 2p^6\right]$ $+3s^1$ → $\mathrm{Na}^{11}$

$+3s^2$ → $\mathrm{Mg}^{12}$

$+3s^2 3p^1$ → $\mathrm{Al}^{13}$

$+3s^2 3p^2$ → $\mathrm{Si}^{14}$

$+3s^2 3p^3$ → $\mathrm{P}^{15}$

$+3s^2 3p^4$ → $\mathrm{S}^{16}$

$+3s^2 3p^5$ → $\mathrm{Cl}^{17}$

$+3s^2 3p^6$ → $\mathrm{Ar}^{18}$

$\left[1s^2 2s^2 2p^6 3s^2 3p^6\right]4s^1$ → $\mathrm{K}^{19}$

***P42.35** In the table of electronic configurations in the text, or on a periodic table, we look for the element whose last electron is in a 3*p* state and which has three electrons outside a closed shell. Its electron configuration then ends in $3s^2 3p^1$. The element is $\boxed{\text{aluminum}}$.

**P42.36** (a) For electron one and also for electron two, $n = 3$ and $\ell = 1$. The possible states are listed here in columns giving the other quantum numbers:

| | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| electron | $m_\ell$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| one | $m_s$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $-\frac{1}{2}$ | $-\frac{1}{2}$ | $-\frac{1}{2}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
| electron | $m_\ell$ | 1 | 0 | 0 | –1 | –1 | 1 | 0 | 0 | –1 | –1 | 1 | 1 | 0 | –1 | –1 |
| two | $m_s$ | $-\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ |

| | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| electron | $m_\ell$ | 0 | 0 | 0 | 0 | 0 | –1 | –1 | –1 | –1 | –1 | –1 | –1 | –1 | –1 | –1 |
| one | $m_s$ | $-\frac{1}{2}$ | $-\frac{1}{2}$ | $-\frac{1}{2}$ | $-\frac{1}{2}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $-\frac{1}{2}$ | $-\frac{1}{2}$ | $-\frac{1}{2}$ | $-\frac{1}{2}$ |
| electron | $m_\ell$ | 1 | 1 | 0 | –1 | –1 | 1 | 1 | 0 | 0 | –1 | 1 | 1 | 0 | 0 | –1 |
| two | $m_s$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ |

There are thirty allowed states, since electron one can have any of three possible values for $m_\ell$ for both spin up and spin down, amounting to six states, and the second electron can have any of the other five states.

(b) Were it not for the exclusion principle, there would be $\boxed{36}$ possible states, six for each electron independently.

**P42.37** (a)

| $n + \ell$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| subshell | $1s$ | $2s$ | $2p, 3s$ | $3p, 4s$ | $3d, 4p, 5s$ | $4d, 5p, 6s$ | $4f, 5d, 6p, 7s$ |

(b)

| | | |
|---|---|---|
| $Z = 15$: | Filled subshells: | $1s$, $2s$, $2p$, $3s$<br>(12 electrons) |
| | Valence subshell: | 3 electrons in $3p$ subshell |
| | Prediction: | Valence $= +3$ or $-5$ |
| | Element is phosphorus, | Valence $= +3$ or $-5$ (Prediction correct) |
| $Z = 47$: | Filled subshells: | $1s$, $2s$, $2p$, $3s$, $3p$, $4s$, $3d$, $4p$, $5s$<br>(38 electrons) |
| | Outer subshell: | 9 electrons in $4d$ subshell |
| | Prediction: | Valence $= -1$ |
| | Element is silver, | (Prediction fails) Valence is $+1$ |
| $Z = 86$: | Filled subshells: | $1s$, $2s$, $2p$, $3s$, $3p$, $4s$, $3d$, $4p$, $5s$, $4d$, $5p$, $6s$, $4f$, $5d$, $6p$<br>(86 electrons) |
| | Prediction | Outer subshell is full: inert gas |
| | Element is radon, inert | (Prediction correct) |

**P42.38** Listing subshells in the order of filling, we have for element 110,

$1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10} 4p^6 5s^2 4d^{10} 5p^6 6s^2 4f^{14} 5d^{10} 6p^6 7s^2 5f^{14} 6d^8$.

In order of increasing principal quantum number, this is

$\boxed{1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 4f^{14} 5s^2 5p^6 5d^{10} 5f^{14} 6s^2 6p^6 6d^8 7s^2}$.

***P42.39** In the ground state of sodium, the outermost electron is in an *s* state. This state is spherically symmetric, so it generates no magnetic field by orbital motion, and has the same energy no matter whether the electron is spin-up or spin-down. The energies of the states $3p\uparrow$ and $3p\downarrow$ above 3*s* are $hf_1 = \frac{hc}{\lambda}$ and $hf_2 = \frac{hc}{\lambda_2}$.

The energy difference is

$$2\mu_B B = hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$$

so $$B = \frac{hc}{2\mu_B}\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) = \frac{\left(6.63 \times 10^{-34}\ \text{J}\cdot\text{s}\right)\left(3 \times 10^{8}\ \text{m/s}\right)}{2\left(9.27 \times 10^{-24}\ \text{J/T}\right)}\left(\frac{1}{588.995 \times 10^{-9}\ \text{m}} - \frac{1}{589.592 \times 10^{-9}\ \text{m}}\right)$$

$$B = \boxed{18.4\ \text{T}}.$$

## Section 42.8 More on Atomic Spectra: Visible and X-ray

**P42.40** (a) $\boxed{n = 3,\ \ell = 0,\ m_\ell = 0}$

$\boxed{n = 3,\ \ell = 1,\ m_\ell = -1,\ 0,\ 1}$

For $\boxed{n = 3,\ \ell = 2,\ m_\ell = -2,\ -1,\ 0,\ 1,\ 2}$

(b) $\psi_{300}$ corresponds to $E_{300} = -\frac{Z^2 E_0}{n^2} = -\frac{2^2(13.6)}{3^2} = \boxed{-6.05\ \text{eV}}$.

$\psi_{31-1}$, $\psi_{310}$, $\psi_{311}$ have the same energy since *n* is the same.

$\psi_{32-2}$, $\psi_{32-1}$, $\psi_{320}$, $\psi_{321}$, $\psi_{322}$ have the same energy since *n* is the same.

All states are degenerate.

**P42.41** $E = \frac{hc}{\lambda} = e\Delta V$ : $$\frac{\left(6.626 \times 10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00 \times 10^{8}\ \text{m/s}\right)}{\left(10.0 \times 10^{-9}\ \text{m}\right)} = \left(1.60 \times 10^{-19}\right)\Delta V$$

$$\Delta V = \boxed{124\ \text{V}}$$

**P42.42** Some electrons can give all their kinetic energy $K_e = e\Delta V$ to the creation of a single photon of x-radiation, with

$$hf = \frac{hc}{\lambda} = e\Delta V$$

$$\lambda = \frac{hc}{e\Delta V} = \frac{\left(6.626\,1 \times 10^{-34}\ \text{J}\cdot\text{s}\right)\left(2.997\,9 \times 10^{8}\ \text{m/s}\right)}{\left(1.602\,2 \times 10^{-19}\ \text{C}\right)\Delta V} = \boxed{\frac{1\,240\ \text{nm}\cdot\text{V}}{\Delta V}}$$

**P42.43** Following Example 42.9

$$E_\gamma = \frac{3}{4}(42-1)^2(13.6\text{ eV}) = 1.71\times10^4\text{ eV} = 2.74\times10^{-15}\text{ J}$$

$$f = 4.14\times10^{18}\text{ Hz}$$

and

$$\lambda = \boxed{0.072\,5\text{ nm}}.$$

**P42.44**

$$E = \frac{hc}{\lambda} = \frac{1\,240\text{ eV}\cdot\text{nm}}{\lambda} = \frac{1.240\text{ keV}\cdot\text{nm}}{\lambda}$$

For $\lambda_1 = 0.018\,5\text{ nm}$, $E = 67.11\text{ keV}$

$\lambda_2 = 0.020\,9\text{ nm}$, $E = 59.4\text{ keV}$

$\lambda_3 = 0.021\,5\text{ nm}$, $E = 57.7\text{ keV}$

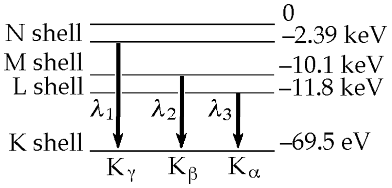

**FIG. P42.44**

The ionization energy for the K shell is 69.5 keV, so the ionization energies for the other shells are:

$\boxed{\text{L shell} = 11.8\text{ keV}}$ $\boxed{\text{M shell} = 10.1\text{ keV}}$ $\boxed{\text{N shell} = 2.39\text{ keV}}$.

**P42.45** The $K_\beta$ x-rays are emitted when there is a vacancy in the ($n = 1$) K shell and an electron from the ($n = 3$) M shell falls down to fill it. Then this electron is shielded by nine electrons originally and by one in its final state.

$$\frac{hc}{\lambda} = -\frac{13.6(Z-9)^2}{3^2}\text{ eV} + \frac{13.6(Z-1)^2}{1^2}\text{ eV}$$

$$\frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^8\text{ m/s})}{(0.152\times10^{-9}\text{ m})(1.60\times10^{-19}\text{ J/eV})} = (13.6\text{ eV})\left(-\frac{Z^2}{9} + \frac{18Z}{9} - \frac{81}{9} + Z^2 - 2Z + 1\right)$$

$$8.17\times10^3\text{ eV} = (13.6\text{ eV})\left(\frac{8Z^2}{9} - 8\right)$$

so

$$601 = \frac{8Z^2}{9} - 8$$

and

$$Z = 26 \qquad \boxed{\text{Iron}}.$$

## Section 42.9 Spontaneous and Stimulated Transitions

## Section 42.10 Lasers

**P42.46** The photon energy is

$$E_4 - E_3 = (20.66 - 18.70)\text{ eV} = 1.96\text{ eV} = \frac{hc}{\lambda}$$

$$\lambda = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^8\text{ m/s})}{1.96(1.60\times10^{-19}\text{ J})} = \boxed{633\text{ nm}}.$$

**P42.47**

$$f = \frac{E}{h} = \frac{0.117\text{ eV}}{6.630\times10^{-34}\text{ J}\cdot\text{s}}\left(\frac{1.60\times10^{-19}\text{ C}}{e}\right)\left(\frac{1\text{ J}}{1\text{ V}\cdot\text{C}}\right) = \boxed{2.82\times10^{13}\text{ s}^{-1}}$$

$$\lambda = \frac{c}{f} = \frac{3.00\times10^8\text{ m/s}}{2.82\times10^{13}\text{ s}^{-1}} = \boxed{10.6\ \mu\text{m}},\ \boxed{\text{infrared}}$$

**P42.48** (a) $$I=\frac{\left(3.00\times10^{-3}\text{ J}\right)}{\left(1.00\times10^{-9}\text{ s}\right)\left[\pi\left(15.0\times10^{-6}\text{ m}\right)^{2}\right]}=\boxed{4.24\times10^{15}\text{ W/m}^{2}}$$

(b) $$\left(3.00\times10^{-3}\text{ J}\right)\frac{\left(0.600\times10^{-9}\text{ m}\right)^{2}}{\left(30.0\times10^{-6}\text{ m}\right)^{2}}=\boxed{1.20\times10^{-12}\text{ J}}=7.50\text{ MeV}$$

**P42.49** $$E=\mathscr{P}\Delta t=\left(1.00\times10^{6}\text{ W}\right)\left(1.00\times10^{-8}\text{ s}\right)=0.010\,0\text{ J}$$

$$E_{\gamma}=hf=\frac{hc}{\lambda}=\frac{\left(6.626\times10^{-34}\right)\left(3.00\times10^{8}\right)}{694.3\times10^{-9}}\text{ J}=2.86\times10^{-19}\text{ J}$$

$$N=\frac{E}{E_{\gamma}}=\frac{0.010\,0}{2.86\times10^{-19}}=\boxed{3.49\times10^{16}\text{ photons}}$$

***P42.50** (a) $$\frac{N_3}{N_2}=\frac{N_g e^{-E_3/(k_B\cdot300\text{ K})}}{N_g e^{-E_2/(k_B\cdot300\text{ K})}}=e^{-(E_3-E_2)/(k_B\cdot300\text{ K})}=e^{-hc/\lambda(k_B\cdot300\text{ K})}$$

where $\lambda$ is the wavelength of light radiated in the $3\to2$ transition.

$$\frac{N_3}{N_2}=e^{-\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3\times10^{8}\text{ m/s}\right)/\left(632.8\times10^{-9}\text{ m}\right)\left(1.38\times10^{-23}\text{ J/K}\right)(300\text{ K})}$$

$$\frac{N_3}{N_2}=e^{-75.9}=\boxed{1.07\times10^{-33}}$$

(b) $$\frac{N_u}{N_\ell}=e^{-(E_u-E_\ell)/k_BT}$$

where the subscript $u$ refers to an upper energy state and the subscript $\ell$ to a lower energy state.

Since $E_u-E_\ell=E_{\text{photon}}=\dfrac{hc}{\lambda}$ $\qquad \dfrac{N_u}{N_\ell}=e^{-hc/\lambda k_BT}$.

Thus, we require $\qquad 1.02=e^{-hc/\lambda k_BT}$

or $$\ln(1.02)=-\frac{\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3\times10^{8}\text{ m/s}\right)}{\left(632.8\times10^{-9}\text{ m}\right)\left(1.38\times10^{23}\text{ J/K}\right)T}$$

$$T=-\frac{2.28\times10^{4}}{\ln(1.02)}=\boxed{-1.15\times10^{6}\text{ K}}.$$

A negative-temperature state is not achieved by cooling the system below 0 K, but by heating it above $T=\infty$, for as $T\to\infty$ the populations of upper and lower states approach equality.

(c) Because $E_u-E_\ell>0$, and in any real equilibrium state $T>0$,

$e^{-(E_u-E_\ell)/k_BT}<1$ and $N_u<N_\ell$.

Thus, a population inversion cannot happen in thermal equilibrium.

***P42.51** (a) The light in the cavity is incident perpendicularly on the mirrors, although the diagram shows a large angle of incidence for clarity. We ignore the variation of the index of refraction with wavelength. To minimize reflection at a vacuum wavelength of 632.8 nm, the net phase difference between rays (1) and (2) should be 180°. There is automatically a 180° shift in one of the two rays upon reflection, so the extra distance traveled by ray (2) should be one whole wavelength:

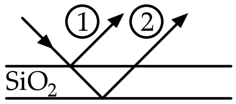

**FIG. P42.51**

$$2t = \frac{\lambda}{n}$$

$$t = \frac{\lambda}{2n} = \frac{632.8 \text{ nm}}{2(1.458)} = \boxed{217 \text{ nm}}$$

(b) The total phase difference should be 360°, including contributions of 180° by reflection and 180° by extra distance traveled

$$2t = \frac{\lambda}{2n}$$

$$t = \frac{\lambda}{4n} = \frac{543 \text{ nm}}{4(1.458)} = \boxed{93.1 \text{ nm}}$$

## Additional Problems

***P42.52** (a) Using the same procedure that was used in the Bohr model of the hydrogen atom, we apply Newton's second law to the Earth. We simply replace the Coulomb force by the gravitational force exerted by the Sun on the Earth and find

$$G\frac{M_S M_E}{r^2} = M_E \frac{v^2}{r} \qquad (1)$$

where $v$ is the orbital speed of the Earth. Next, we apply the postulate that angular momentum of the Earth is quantized in multiples of $\hbar$:

$$M_E vr = n\hbar \qquad (n = 1, 2, 3, \ldots).$$

Solving for $v$ gives

$$v = \frac{n\hbar}{M_E r}. \qquad (2)$$

Substituting (2) into (1), we find

$$r = \frac{n^2\hbar^2}{GM_S M_E^2}. \qquad (3)$$

*continued on next page*

(b) Solving (3) for $n$ gives

$$n=\sqrt{GM_S r}\,\frac{M_E}{\hbar}\,. \qquad (4)$$

Taking $M_S=1.99\times10^{30}$ kg, and $M_E=5.98\times10^{24}$ kg, $r=1.496\times10^{11}$ m, $G=6.67\times10^{-11}$ $\text{Nm}^2/\text{kg}^2$, and $\hbar=1.055\times10^{-34}$ Js, we find

$$n=\boxed{2.53\times10^{74}}\,.$$

(c) We can use (3) to determine the radii for the orbits corresponding to the quantum numbers $n$ and $n+1$:

$$r_n=\frac{n^2\hbar^2}{GM_SM_E^2} \qquad \text{and} \qquad r_{n+1}=\frac{(n+1)^2\hbar^2}{GM_SM_E^2}\,.$$

Hence, the separation between these two orbits is

$$\Delta r=\frac{\hbar^2}{GM_SM_E^2}\left[(n+1)^2-n^2\right]=\frac{\hbar^2}{GM_SM_E^2}(2n+1)\,.$$

Since $n$ is very large, we can neglect the number 1 in the parentheses and express the separation as

$$\Delta r\approx\frac{\hbar^2}{GM_SM_E^2}(2n)=\boxed{1.18\times10^{-63}\ \text{m}}\,.$$

This number is *much smaller* than the radius of an atomic nucleus $\left(\sim10^{-15}\ \text{m}\right)$, so the distance between quantized orbits of the Earth is too small to observe.

***P42.53** (a) $$\Delta E=\frac{e\hbar B}{m_e}=\frac{1.60\times10^{-19}\ \text{C}\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)(5.26\ \text{T})}{2\pi\left(9.11\times10^{-31}\ \text{kg}\right)}\left(\frac{\text{N}\cdot\text{s}}{\text{T}\cdot\text{C}\cdot\text{m}}\right)\left(\frac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}\right)=9.75\times10^{-23}\ \text{J}$$

$$=\boxed{609\ \mu\text{eV}}$$

(b) $$k_BT=\left(1.38\times10^{-23}\ \text{J/K}\right)\left(80\times10^{-3}\ \text{K}\right)=1.10\times10^{-24}\ \text{J}=\boxed{6.90\ \mu\text{eV}}$$

(c) $$f=\frac{\Delta E}{h}=\frac{9.75\times10^{-23}\ \text{J}}{6.63\times10^{-34}\ \text{J}\cdot\text{s}}=\boxed{1.47\times10^{11}\ \text{Hz}}$$

$$\lambda=\frac{c}{f}=\frac{3\times10^{8}\ \text{m/s}}{1.47\times10^{11}\ \text{Hz}}=\boxed{2.04\times10^{-3}\ \text{m}}$$

***P42.54** (a) $$\text{Probability}=\int_r^\infty P_{1s}(r')dr'=\frac{4}{a_0^3}\int_r^\infty r'^2e^{-2r'/a_0}dr'=\left[-\left(\frac{2r'^2}{a_0^2}+\frac{2r'}{a_0}+1\right)e^{-2r'/a_0}\right]_r^\infty,$$

using integration by parts, or Example 42.5

$$=\boxed{\left(\frac{2r^2}{a_0^2}+\frac{2r}{a_0}+1\right)e^{-2r/a_0}}$$

*continued on next page*

(b)

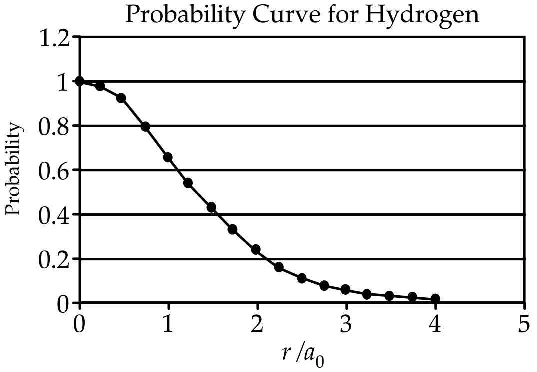

**FIG. P42.66**

(c) The probability of finding the electron inside or outside the sphere of radius $r$ is $\frac{1}{2}$.

$$\therefore\left(\frac{2r^2}{a_0^2}+\frac{2r}{a_0}+1\right)e^{-2r/a_0}=\frac{1}{2}\text{ or }z^2+2z+2=e^z\text{ where }z=\frac{2r}{a_0}$$

One can home in on a solution to this transcendental equation for $r$ on a calculator, the result being $r=\boxed{1.34a_0}$ to three digits.

**P42.55** Let $r$ represent the distance between the electron and the positron. The two move in a circle of radius $\frac{r}{2}$ around their center of mass with opposite velocities. The total angular momentum of the electron-positron system is quantized to according to

$$L_n=\frac{mvr}{2}+\frac{mvr}{2}=n\hbar$$

where $n=1,\ 2,\ 3,\ \dots$ .

For each particle, $\sum F=ma$ expands to $\frac{k_e e^2}{r^2}=\frac{mv^2}{r/2}$.

We can eliminate $v=\frac{n\hbar}{mr}$ to find $\frac{k_e e^2}{r}=\frac{2mn^2\hbar^2}{m^2r^2}$.

So the separation distances are $r=\frac{2n^2\hbar^2}{mk_e e^2}=2a_0n^2=\boxed{\left(1.06\times10^{-10}\text{ m}\right)n^2}$.

The orbital radii are $\frac{r}{2}=a_0n^2$, the same as for the electron in hydrogen.

The energy can be calculated from $E=K+U=\frac{1}{2}mv^2+\frac{1}{2}mv^2-\frac{k_e e^2}{r}$.

Since $mv^2=\frac{k_e e^2}{2r}$, $E=\frac{k_e e^2}{2r}-\frac{k_e e^2}{r}=-\frac{k_e e^2}{2r}=\frac{-k_e e^2}{4a_0n^2}=\boxed{-\frac{6.80\text{ eV}}{n^2}}$.

**P42.56** (a) The energy difference between these two states is equal to the energy that is absorbed.

Thus, $E=E_2-E_1=\frac{(-13.6\text{ eV})}{4}-\frac{(-13.6\text{ eV})}{1}=10.2\text{ eV}=\boxed{1.63\times10^{-18}\text{ J}}$.

(b) $E=\frac{3}{2}k_BT$ or $T=\frac{2E}{3k_B}=\frac{2\left(1.63\times10^{-18}\text{ J}\right)}{3\left(1.38\times10^{-23}\text{ J/K}\right)}=\boxed{7.88\times10^4\text{ K}}$.

**P42.57** $hf = \Delta E = \dfrac{4\pi^2 m k_e^2 e^4}{2h^2}\left(\dfrac{1}{(n-1)^2} - \dfrac{1}{n^2}\right)$

$$f = \frac{2\pi^2 m k_e^2 e^4}{h^3}\left(\frac{2n-1}{(n-1)^2 n^2}\right)$$

As $n$ approaches infinity, we have $f$ approaching $\dfrac{2\pi^2 m k_e^2 e^4}{h^3}\dfrac{2}{n^3}$

The classical frequency is $f = \dfrac{v}{2\pi r} = \dfrac{1}{2\pi}\sqrt{\dfrac{k_e e^2}{m}}\dfrac{1}{r^{3/2}}$

where $r = \dfrac{n^2 h^2}{4\pi m k_e e^2}$

Using this equation to eliminate $r$ from the expression for $f$, $f = \dfrac{2\pi^2 m k_e^2 e^4}{h^3}\dfrac{2}{n^3}$

**P42.58** (a) The energy of the ground state is: $E_1 = -\dfrac{hc}{\lambda_{\text{series limit}}} = -\dfrac{1\,240\text{ eV}\cdot\text{nm}}{152.0\text{ nm}} = \boxed{-8.16\text{ eV}}$.

From the wavelength of the Lyman $\alpha$ line: $E_2 - E_1 = \dfrac{hc}{\lambda} = \dfrac{1\,240\text{ nm}\cdot\text{eV}}{202.6\text{ nm}} = 6.12\text{ eV}$

$$E_2 = E_1 + 6.12\text{ eV} = \boxed{-2.04\text{ eV}}.$$

The wavelength of the Lyman $\beta$ line gives: $E_3 - E_1 = \dfrac{1\,240\text{ nm}\cdot\text{eV}}{170.9\text{ nm}} = 7.26\text{ eV}$

so $E_3 = \boxed{-0.902\text{ eV}}$.

Next, using the Lyman $\gamma$ line gives: $E_4 - E_1 = \dfrac{1\,240\text{ nm}\cdot\text{eV}}{162.1\text{ nm}} = 7.65\text{ eV}$

and $E_4 = \boxed{-0.508\text{ eV}}$.

From the Lyman $\delta$ line, $E_5 - E_1 = \dfrac{1\,240\text{ nm}\cdot\text{eV}}{158.3\text{ nm}} = 7.83\text{ eV}$

so $E_5 = \boxed{-0.325\text{ eV}}$.

(b) For the Balmer series, $\dfrac{hc}{\lambda} = E_i - E_2$, or $\lambda = \dfrac{1\,240\text{ nm}\cdot\text{eV}}{E_i - E_2}$.

For the $\alpha$ line, $E_i = E_3$ and so $\lambda_a = \dfrac{1\,240\text{ nm}\cdot\text{eV}}{(-0.902\text{ eV}) - (-2.04\text{ eV})} = \boxed{1\,090\text{ nm}}$.

Similarly, the wavelengths of the $\beta$ line, $\gamma$ line, and the short wavelength limit are found to be: $\boxed{811\text{ nm}}$, $\boxed{724\text{ nm}}$, and $\boxed{609\text{ nm}}$.

*continued*

(c) Computing 60.0% of the wavelengths of the spectral lines shown on the energy-level diagram gives:

$0.600(202.6\text{ nm}) = \boxed{122\text{ nm}}$, $0.600(170.9\text{ nm}) = \boxed{103\text{ nm}}$, $0.600(162.1\text{ nm}) = \boxed{97.3\text{ nm}}$, $0.600(158.3\text{ nm}) = \boxed{95.0\text{ nm}}$, and $0.600(152.0\text{ nm}) = \boxed{91.2\text{ nm}}$

These are seen to be the wavelengths of the $\alpha$, $\beta$, $\gamma$, and $\delta$ lines as well as the short wavelength limit for the Lyman series in Hydrogen.

(d) The observed wavelengths could be the result of Doppler shift when the source moves away from the Earth. The required speed of the source is found from

$$\frac{f'}{f} = \frac{\lambda}{\lambda'} = \sqrt{\frac{1-(v/c)}{1+(v/c)}} = 0.600 \qquad \text{yielding} \qquad \boxed{v = 0.471c}.$$

**P42.59** The wave function for the 2*s* state is given by Eq. 42.26: $\psi_{2s}(r) = \frac{1}{4\sqrt{2\pi}}\left(\frac{1}{a_0}\right)^{3/2}\left[2-\frac{r}{a_0}\right]e^{-r/2a_0}$.

(a) Taking $r = a_0 = 0.529 \times 10^{-10}\text{ m}$

we find $$\psi_{2s}(a_0) = \frac{1}{4\sqrt{2\pi}}\left(\frac{1}{0.529\times10^{-10}\text{ m}}\right)^{3/2}[2-1]e^{-1/2} = \boxed{1.57\times10^{14}\text{ m}^{-3/2}}.$$

(b) $$|\psi_{2s}(a_0)|^2 = \left(1.57\times10^{14}\text{ m}^{-3/2}\right)^2 = \boxed{2.47\times10^{28}\text{ m}^{-3}}$$

(c) Using Equation 42.24 and the results to (b) gives $P_{2s}(a_0) = 4\pi a_0^2|\psi_{2s}(a_0)|^2 = \boxed{8.69\times10^{8}\text{ m}^{-1}}$.

***P42.60** From Figure 42.20, a typical ionization energy is 8 eV. For internal energy to ionize most of the atoms we require

$$\frac{3}{2}k_BT = 8\text{ eV}: \qquad T = \frac{2\times8\left(1.60\times10^{-19}\text{ J}\right)}{3\left(1.38\times10^{-23}\text{ J/K}\right)}\boxed{\sim\text{ between }10^4\text{ K and }10^5\text{ K}}.$$

**P42.61** (a) $$\left(3.00\times10^{8}\text{ m/s}\right)\left(14.0\times10^{-12}\text{ s}\right) = \boxed{4.20\text{ mm}}$$

(b) $$E = \frac{hc}{\lambda} = 2.86\times10^{-19}\text{ J}$$

$$N = \frac{3.00\text{ J}}{2.86\times10^{-19}\text{ J}} = \boxed{1.05\times10^{19}\text{ photons}}$$

(c) $$V = (4.20\text{ mm})\left[\pi(3.00\text{ mm})^2\right] = 119\text{ mm}^3$$

$$n = \frac{1.05\times10^{19}}{119} = \boxed{8.82\times10^{16}\text{ mm}^{-3}}$$

**P42.62** (a) The length of the pulse is $\Delta L = \boxed{c\Delta t}$.

(b) The energy of each photon is $E_\gamma = \dfrac{hc}{\lambda}$ so $N = \dfrac{E}{E_\gamma} = \boxed{\dfrac{E\lambda}{hc}}$.

(c) $V = \Delta L \pi \dfrac{d^2}{4}$ $\quad n = \dfrac{N}{V} = \boxed{\left(\dfrac{4}{c\Delta t \pi d^2}\right)\left(\dfrac{E\lambda}{hc}\right)}$

***P42.63** The fermions are described by the exclusion principle. Two of them, one spin-up and one spin-down, will be in the ground energy level, with

$$d_{\text{NN}} = L = \frac{1}{2}\lambda,\ \lambda = 2L = \frac{h}{p},\text{ and } p = \frac{h}{2L} \qquad K = \frac{1}{2}mv^2 = \frac{p^2}{2m} = \frac{h^2}{8mL^2}.$$

The third must be in the next higher level, with

$$d_{\text{NN}} = \frac{L}{2} = \frac{\lambda}{2},\ \lambda = L,\text{ and } p = \frac{h}{L} \qquad K = \frac{p^2}{2m} = \frac{h^2}{2mL^2}.$$

The total energy is then $\dfrac{h^2}{8mL^2} + \dfrac{h^2}{8mL^2} + \dfrac{h^2}{2mL^2} = \boxed{\dfrac{3h^2}{4mL^2}}$.

**P42.64** $\Delta z = \dfrac{at^2}{2} = \dfrac{1}{2}\left(\dfrac{F_z}{m_0}\right)t^2 = \dfrac{\mu_z(dB_z/dz)}{2m_0}\left(\dfrac{\Delta x}{v}\right)^2$ and $\mu_z = \dfrac{e\hbar}{2m_e}$

$$\frac{dB_z}{dz} = \frac{2m_0(\Delta z)v^2(2m_e)}{\Delta x^2 e\hbar} = \frac{2(108)\left(1.66\times10^{-27}\text{ kg}\right)\left(10^{-3}\text{ m}\right)\left(10^4\text{ m}^2/\text{s}^2\right)\left(2\times9.11\times10^{-31}\text{ kg}\right)}{\left(1.00\text{ m}^2\right)\left(1.60\times10^{-19}\text{ C}\right)\left(1.05\times10^{-34}\text{ J}\cdot\text{s}\right)}$$

$$\frac{dB_z}{dz} = \boxed{0.389\text{ T/m}}$$

**P42.65** We use $\psi_{2s}(r) = \dfrac{1}{4}\left(2\pi a_0^3\right)^{-1/2}\left(2 - \dfrac{r}{a_0}\right)e^{-r/2a_0}$.

By Equation 42.24, $P(r) = 4\pi r^2\psi^2 = \dfrac{1}{8}\left(\dfrac{r^2}{a_0^3}\right)\left(2 - \dfrac{r}{a_0}\right)^2 e^{-r/a_0}$.

(a) $$\frac{dP(r)}{dr} = \frac{1}{8}\left[\frac{2r}{a_0^3}\left(2 - \frac{r}{a_0}\right)^2 - \frac{2r^2}{a_0^3}\left(\frac{1}{a_0}\right)\left(2 - \frac{r}{a_0}\right) - \frac{r^2}{a_0^3}\left(2 - \frac{r}{a_0}\right)^2\left(\frac{1}{a_0}\right)\right]e^{-r/a_0} = 0$$

or $$\frac{1}{8}\left(\frac{r}{a_0^3}\right)\left(2 - \frac{r}{a_0}\right)\left[2\left(2 - \frac{r}{a_0}\right) - \frac{2r}{a_0} - \frac{r}{a_0}\left(2 - \frac{r}{a_0}\right)\right]e^{-r/a_0} = 0.$$

The roots of $\dfrac{dP}{dr} = 0$ at $r = 0$, $r = 2a_0$ and $r = \infty$ are minima with $P(r) = 0$.

*continued*

Therefore we require $$[\ldots..]=4-\left(\frac{6r}{a_0}\right)+\left(\frac{r}{a_0}\right)^2=0$$

with solutions $$r=\left(3\pm\sqrt{5}\right)a_0.$$

We substitute the last two roots into $P(r)$ to determine the most probable value:

When $r=\left(3-\sqrt{5}\right)a_0=0.763\,9a_0$, $$P(r)=\frac{0.051\,9}{a_0}.$$

When $r=\left(3+\sqrt{5}\right)a_0=5.236a_0$, $$P(r)=\frac{0.191}{a_0}.$$

Therefore, the most probable value of *r* is $$\left(3+\sqrt{5}\right)a_0=\boxed{5.236a_0}.$$

(b) $$\int_0^\infty P(r)dr=\int_0^\infty\frac{1}{8}\left(\frac{r^2}{a_0^3}\right)\left(2-\frac{r}{a_0}\right)^2e^{-r/a_0}dr$$

Let $u=\frac{r}{a_0}$, $dr=a_0du$,

$$\int_0^\infty P(r)dr=\int_0^\infty\frac{1}{8}u^2\left(4-4u+u^2\right)e^{-u}dr=\int_0^\infty\frac{1}{8}\left(u^4-4u^3+4u^2\right)e^{-u}du=-\frac{1}{8}\left(u^4+4u^2+8u+8\right)e^{-u}\Big|_0^\infty=1$$

$\boxed{\text{This is as required for normalization.}}$

**P42.66** $$E=\frac{hc}{\lambda}=\frac{1\,240\text{ eV}\cdot\text{nm}}{\lambda}=\Delta E$$

$\lambda_1=310\text{ nm}$, so $\Delta E_1=4.00\text{ eV}$

$\lambda_2=400\text{ nm}$, $\Delta E_2=3.10\text{ eV}$

$\lambda_3=1\,378\text{ nm}$, $\Delta E_3=0.900\text{ eV}$

and the ionization energy $=4.10\text{ eV}$.

The energy level diagram having the fewest levels and consistent with these energies is shown at the right.

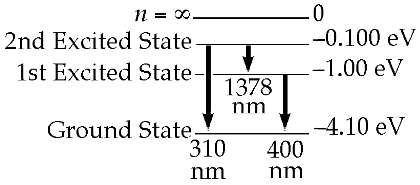

**FIG. P42.66**

**P42.67** With one vacancy in the K shell, excess energy

$$\Delta E\approx-(Z-1)^2(13.6\text{ eV})\left(\frac{1}{2^2}-\frac{1}{1^2}\right)=5.40\text{ keV}.$$

We suppose the outermost 4*s* electron is shielded by 22 electrons inside its orbit:

$$E_{\text{ionization}}\approx\frac{2^2(13.6\text{ eV})}{4^2}=3.40\text{ eV}.$$

Note the experimental ionization energy is 6.76 eV.

$$K=\Delta E-E_{\text{ionization}}\approx\boxed{5.39\text{ keV}}.$$

**P42.68** (a) The configuration we may model as $\boxed{\text{SN}}\ \boxed{\text{NS}}$ has higher energy than $\boxed{\text{SN}}\ \boxed{\text{SN}}$. The higher energy state has antiparallel magnetic moments, so it has $\boxed{\text{parallel spins}}$ of the oppositely charged particles.

(b) $E=\dfrac{hc}{\lambda}=9.42\times10^{-25}\ \text{J}=\boxed{5.89\ \mu\text{eV}}$

(c) $\Delta E\Delta t\approx\dfrac{\hbar}{2}$ so $\Delta E\approx\dfrac{1.055\times10^{-34}\ \text{J}\cdot\text{s}}{2\left(10^{7}\ \text{yr}\right)\left(3.16\times10^{7}\ \text{s/yr}\right)}\left(\dfrac{1.00\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right)=\boxed{1.04\times10^{-30}\ \text{eV}}$

**P42.69** $$P=\int_{2.50a_0}^{\infty}\frac{4r^2}{a_0^3}e^{-2r/a_0}dr=\frac{1}{2}\int_{5.00}^{\infty}z^2e^{-z}dz\ \text{ where } z\equiv\frac{2r}{a_0}$$

$$P=-\frac{1}{2}\left(z^2+2z+2\right)e^{-z}\Big|_{5.00}^{\infty}=-\frac{1}{2}[0]+\frac{1}{2}(25.0+10.0+2.00)e^{-5}=\left(\frac{37}{2}\right)(0.006\,74)=\boxed{0.125}$$

**P42.70** (a) One molecule's share of volume

Al: $V=\dfrac{\text{mass per molecule}}{\text{density}}=\left(\dfrac{27.0\ \text{g/mol}}{6.02\times10^{23}\ \text{molecules/mol}}\right)\left(\dfrac{1.00\times10^{-6}\ \text{m}^3}{2.70\ \text{g}}\right)=1.66\times10^{-29}\ \text{m}^3$

$\sqrt[3]{V}=\boxed{2.55\times10^{-10}\ \text{m}\sim10^{-1}\ \text{nm}}$.

U: $V=\left(\dfrac{238\ \text{g}}{6.02\times10^{23}\ \text{molecules}}\right)\left(\dfrac{1.00\times10^{-6}\ \text{m}^3}{18.9\ \text{g}}\right)=2.09\times10^{-29}\ \text{m}^3$

$\sqrt[3]{V}=\boxed{2.76\times10^{-10}\ \text{m}\sim10^{-1}\ \text{nm}}$.

(b) The outermost electron in any atom sees the nuclear charge screened by all the electrons below it. If we can visualize a single outermost electron, it moves in the electric field of net charge, $+Ze-(Z-1)e=+e$, the charge of a single proton, as felt by the electron in hydrogen. So the Bohr radius sets the scale for the outside diameter of every atom. An innermost electron, on the other hand, sees the nuclear charge unscreened, and the scale size of its (K-shell) orbit is $\dfrac{a_0}{Z}$.

**P42.71** $\Delta E=2\mu_{\text{B}}B=hf$

so $2\left(9.27\times10^{-24}\ \text{J/T}\right)(0.350\ \text{T})=\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)f$

and $f=\boxed{9.79\times10^{9}\ \text{Hz}}$.

**P42.72** $\psi_{25} = \frac{1}{4}(2\pi)^{-1/2}\left(\frac{1}{a_0}\right)^{3/2}\left(2-\frac{r}{a_0}\right)e^{-r/2a_0} = A\left(2-\frac{r}{a_0}\right)e^{-r/2a_0}$ $\qquad \frac{d\psi}{dr} = Ae^{-r/2a_0}\left(-\frac{2}{a_0}+\frac{r}{2a_0^2}\right)$

$$\frac{d^2\psi}{dr^2} = \left(\frac{Ae^{-r/2a_0}}{a_0^2}\right)\left(\frac{3}{2}-\frac{r}{4a_0}\right)$$

Substituting into Schrödinger's equation and dividing by $Ae^{-r/2a_0}$, we will have a solution if

$$-\frac{5}{4}\frac{\hbar^2}{m_e a_0^2}+\frac{k_e e^2}{a_0}+\frac{\hbar^2 r}{8m_e a_0^3}+\frac{2\hbar^2}{m_e a_0 r}-\frac{2k_e e^2}{r} = 2E-\frac{Er}{a_0}.$$

Now with $a_0 = \frac{\hbar^2}{m_e e^2 k_e}$, this reduces to

$$-\frac{m_e e^4 k_e^2}{8\hbar^2}\left(2-\frac{r}{a_0}\right) = E\left(2-\frac{r}{a_0}\right).$$

This is true, so $\psi_{25}$ is a solution to the Schrödinger equation, provided $E = \frac{1}{4}E_1 = -3.40\text{ eV}$.

**P42.73** (a) Suppose the atoms move in the $+x$ direction. The absorption of a photon by an atom is a completely inelastic collision, described by

$$mv_i\hat{\mathbf{i}} + \frac{h}{\lambda}\left(-\hat{\mathbf{i}}\right) = mv_f\hat{\mathbf{i}} \qquad \text{so} \qquad v_f - v_i = -\frac{h}{m\lambda}.$$

This happens promptly every time an atom has fallen back into the ground state, so it happens every $10^{-8}\text{ s} = \Delta t$. Then,

$$a = \frac{v_f - v_i}{\Delta t} = -\frac{h}{m\lambda\Delta t} \sim -\frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{\left(10^{-25}\text{ kg}\right)\left(500\times10^{-9}\text{ m}\right)\left(10^{-8}\text{ s}\right)} \sim \boxed{-10^6\text{ m/s}^2}.$$

(b) With constant average acceleration,

$$v_f^2 = v_i^2 + 2a\Delta x \qquad 0 \sim \left(10^3\text{ m/s}\right)^2 + 2\left(-10^6\text{ m/s}^2\right)\Delta x$$

$$\text{so} \qquad \Delta x \sim \frac{\left(10^3\text{ m/s}\right)^2}{10^6\text{ m/s}^2}\boxed{\sim 1\text{ m}}.$$

**P42.74** $\left\langle\frac{1}{r}\right\rangle = \int_0^\infty \frac{4r^2}{a_0^3}e^{-2r/a_0}\frac{1}{r}dr = \frac{4}{a_0^3}\int_0^\infty re^{-(2/a_0)r}dr = \frac{4}{a_0^3}\frac{1}{(2/a_0)^2} = \boxed{\frac{1}{a_0}}$

We compare this to $\frac{1}{\langle r\rangle} = \frac{1}{3a_0/2} = \frac{2}{3a_0}$, and find that the average reciprocal value is $\boxed{\text{NOT}}$ the reciprocal of the average value.

## ANSWERS TO EVEN PROBLEMS

**P42.2** (a) 91.2 nm, 365 nm, 821 nm, 1.46 $\mu$m; (b) 13.6 eV, 3.40 eV, 1.51 eV, 0.850 eV

**P42.4** (a) 56.8 fm; (b) 11.3 N away from the nucleus

**P42.6** (a) ii; (b) i; (c) ii and iii

**P42.8** (a) 13.6 eV; (b)1.51 eV

**P42.10** see the solution

**P42.12** 97.5 nm

**P42.14** (a) $O^{7+}$; (b) 41.0 nm, 33.8 nm, 30.4 nm

**P42.16** (a) 1.31 $\mu$m; (b) 164 nm

**P42.18** see the solution

**P42.20** $4a_0$

**P42.22** 797 times

**P42.24** (a) $\sqrt{6}\hbar = 2.58 \times 10^{-34}$ J·s; (b) $\sqrt{12}\hbar = 3.65 \times 10^{-34}$ J·s

**P42.26** $\sqrt{6}\hbar$

**P42.28** $\sqrt{6}\hbar$; $-2\hbar$, $-\hbar$, 0, $\hbar$, $2\hbar$; 145°, 114°, 90.0°, 65.9°, 35.3°

**P42.30** $3\hbar$

**P42.32** (a) $1s^2 2s^2 2p^4$;

(b)

| $n$ | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 |
|---|---|---|---|---|---|---|---|---|
| $\ell$ | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| $m_\ell$ | 0 | 0 | 0 | 0 | 1 | 1 | 0 | $-1$ |
| $m_s$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |

**P42.34** see the solution

**P42.36** (a) see the solution; (b) 36 states instead of 30

**P42.38** $1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 4f^{14} 5s^2 5p^6 5d^{10} 5f^{14} 6s^2 6p^6 6d^8 7s^2$

**P42.40** (a) $\ell = 0$ with $m_\ell = 0$; $\ell = 1$ with $m_\ell = 1, 0,$ or 1; and $\ell = 2$ with $m_\ell = -2, -1, 0, 1, 2$; (b) –6.05 eV

**P42.42** see the solution

**P42.44** L shell 11.8 keV, M shell 10.1 keV, N shell 2.39 keV, see the solution

**P42.46** see the solution

**P42.48** (a) 4.24 $\text{PW/m}^2$; (b) 1.20 pJ = 7.50 MeV

**P42.50** (a) $1.07 \times 10^{-33}$; (b) $-1.15 \times 10^6$ K; (c) negative temperatures do not describe systems in thermal equilibrium

**P42.52** (a) see the solution; (b) $2.53 \times 10^{74}$; (c) $1.18 \times 10^{-63}$ m, unobservably small

**P42.54** (a) Probability $= \left( \frac{2r^2}{a_0^2} + \frac{2r}{a_0} + 1 \right) e^{-2r/a_0}$; (b) see the solution; (c) $1.34a_0$

**P42.56** (a) 10.2 eV = 1.63 aJ; (b) $7.88 \times 10^4$ K

**P42.58** (a) –8.16 eV, –2.04 eV, –0.902 eV, –0.508 eV, –0.325 eV; (b) 1 090 nm, 811 nm, 724 nm, 609 nm; (c) see the solution; (d) The spectrum could be that of hydrogen, Doppler shifted by motion away from us at speed $0.471c$.

**P42.60** between $10^4$ K and $10^5$ K

**P42.62** (a) $c\Delta t$; (b) $\frac{E\lambda}{hc}$; (c) $\frac{4E\lambda}{\Delta t \pi d^2 hc^2}$

**P42.64** 0.389 T/m

**P42.66** Energy levels at 0, –0.100 eV, –1.00 eV, and –4.10 eV

**P42.68** (a) parallel spins; (b) 5.89 $\mu$eV; (c) $1.04 \times 10^{-30}$ eV

**P42.70** (a) diameter $\sim 10^{-1}$ nm for both; (b) A K-shell electron moves in an orbit with size on the order of $\frac{a_0}{Z}$

**P42.72** see the solution

**P42.74** $\frac{1}{a_0}$, no

# 43

# Molecules and Solids

**CHAPTER OUTLINE**

43.1 Molecular Bonds
43.2 Energy States and Spectra of Molecules
43.3 Bonding in Solids
43.4 Free-Electron Theory of Metals
43.5 Band Theory of Solids
43.6 Electrical Conduction in Metals, Insulators, and Semiconductors
43.7 Semiconductor Devices
43.8 Superconductivity

## ANSWERS TO QUESTIONS

**Q43.1** Rotational, vibrational and electronic (as discussed in Chapter 42) are the three major forms of excitation. Rotational energy for a diatomic molecule is on the order of $\frac{\hbar^2}{2I}$, where $I$ is the moment of inertia of the molecule. A typical value for a small molecule is on the order of $1\text{ meV} = 10^{-3}\text{ eV}$. Vibrational energy is on the order of $hf$, where $f$ is the vibration frequency of the molecule. A typical value is on the order of 0.1 eV. Electronic energy depends on the state of an electron in the molecule and is on the order of a few eV. The rotational energy can be zero, but neither the vibrational nor the electronic energy can be zero.

**Q43.2** The Pauli exclusion principle limits the number of electrons in the valence band of a metal, as no two electrons can occupy the same state. If the valence band is full, additional electrons must be in the conduction band, and the material can be a good conductor. For further discussion, see Q43.3.

**Q43.3** The conductive properties of a material depend on the electron population of the conduction band of the material. If the conduction band is empty and a full valence band lies below the conduction band by an energy gap of a few eV, then the material will be an insulator. Electrons will be unable to move easily through the material in response to an applied electric field. If the conduction band is partly full, states are accessible to electrons accelerated by an electric field, and the material is a good conductor. If the energy gap between a full valence band and an empty conduction band is comparable to the thermal energy $k_B T$, the material is a semiconductor.

**Q43.4** Thermal excitation increases the vibrational energy of the molecules. It makes the crystal lattice less orderly. We can expect it to increase the width of both the valence band and the conduction band, to decrease the gap between them.

**Q43.5** First consider electric conduction in a metal. The number of conduction electrons is essentially fixed. They conduct electricity by having drift motion in an applied electric field superposed on their random thermal motion. At higher temperature, the ion cores vibrate more and scatter more efficiently the conduction electrons flying among them. The mean time between collisions is reduced. The electrons have time to develop only a lower drift speed. The electric current is reduced, so we see the resistivity increasing with temperature.

Now consider an intrinsic semiconductor. At absolute zero its valence band is full and its conduction band is empty. It is an insulator, with very high resistivity. As the temperature increases, more electrons are promoted to the conduction band, leaving holes in the valence band. Then both electrons and holes move in response to an applied electric field. Thus we see the resistivity decreasing as temperature goes up.

**Q43.6** In a metal, there is no energy gap between the valence and conduction bands, or the conduction band is partly full even at absolute zero in temperature. Thus an applied electric field is able to inject a tiny bit of energy into an electron to promote it to a state in which it is moving through the metal as part of an electric current. In an insulator, there is a large energy gap between a full valence band and an empty conduction band. An applied electric field is unable to give electrons in the valence band enough energy to jump across the gap into the higher energy conduction band. In a semiconductor, the energy gap between valence and conduction bands is smaller than in an insulator. At absolute zero the valence band is full and the conduction band is empty, but at room temperature thermal energy has promoted some electrons across the gap. Then there are some mobile holes in the valence band as well as some mobile electrons in the conduction band.

**Q43.7** Ionic bonds are ones between oppositely charged ions. A simple model of an ionic bond is the electrostatic attraction of a negatively charged latex balloon to a positively charged Mylar balloon.

Covalent bonds are ones in which atoms share electrons. Classically, two children playing a short-range game of catch with a ball models a covalent bond. On a quantum scale, the two atoms are sharing a wave function, so perhaps a better model would be two children using a single hula hoop.

Van der Waals bonds are weak electrostatic forces: the dipole-dipole force is analogous to the attraction between the opposite poles of two bar magnets, the dipole—induced dipole force is similar to a bar magnet attracting an iron nail or paper clip, and the dispersion force is analogous to an alternating-current electromagnet attracting a paper clip.

A hydrogen atom in a molecule is not ionized, but its electron can spend more time elsewhere than it does in the hydrogen atom. The hydrogen atom can be a location of net positive charge, and can weakly attract a zone of negative charge in another molecule.

**Q43.8** Ionically bonded solids are generally poor electric conductors, as they have no free electrons. While they are transparent in the visible spectrum, they absorb infrared radiation. Physically, they form stable, hard crystals with high melting temperatures.

**Q43.9** Covalently bonded solids are generally poor conductors, as they form structures in which the atoms share several electrons in the outer shell, leaving no room for conducting electrons. Depending on the structure of the solid, they are usually very hard and have high melting points.

**Q43.10** Metals are good conductors, as the atoms have many free electrons in the conduction band. Metallic bonds allow the mixing of different metals to form alloys. Metals are opaque to visible light, and can be highly reflective. A metal can bend under stress instead of fracturing like ionically and covalently bonded crystals. The physical properties vary greatly depending on the composition.

**Q43.11** The energy of the photon is given to the electron. The energy of a photon of visible light is sufficient to promote the electron from the lower-energy valence band to the higher-energy conduction band. This results in the additional electron in the conduction band and an additional hole—the energy state that the electron used to occupy—in the valence band.

**Q43.12** Along with arsenic (As), any other element in group V, such as phosphorus (P), antimony (Sb), and bismuth (Bi), would make good donor atoms. Each has 5 valence electrons. Any element in group III would make good acceptor atoms, such as boron (B), aluminum, (Al), gallium (Ga), and indium (In). They all have only 3 valence electrons.

**Q43.13** The two assumptions in the free-electron theory are that the conduction electrons are not bound to any particular atom, and that the nuclei of the atoms are fixed in a lattice structure. In this model, it is the "soup" of free electrons that are conducted through metals. The energy band model is more comprehensive than the free-electron theory. The energy band model includes an account of the more tightly bound electrons as well as the conduction electrons. It can be developed into a theory of the structure of the crystal and its mechanical and thermal properties.

**Q43.14** A molecule containing two atoms of ${}^2\mathrm{H}$, deuterium, has twice the mass of a molecule containing two atoms of ordinary hydrogen ${}^1\mathrm{H}$. The atoms have the same electronic structure, so the molecules have the same interatomic spacing, and the same spring constant. Then the moment of inertia of the double-deuteron is twice as large and the rotational energies one-half as large as for ordinary hydrogen. Each vibrational energy level for $\mathrm{D}_2$ is $\frac{1}{\sqrt{2}}$ times that of $\mathrm{H}_2$.

**Q43.15** Rotation of a diatomic molecule involves less energy than vibration. Absorption of microwave photons, of frequency $\sim 10^{11}$ Hz, excites rotational motion, while absorption of infrared photons, of frequency $\sim 10^{13}$ Hz, excites vibration in typical simple molecules.

**Q43.16** Yes. A material can absorb a photon of energy greater than the energy gap, as an electron jumps into a higher energy state. If the photon does not have enough energy to raise the energy of the electron by the energy gap, then the photon will not be absorbed.

**Q43.17** From the rotational spectrum of a molecule, one can easily calculate the moment of inertia of the molecule using Equation 43.7 in the text. Note that with this method, only the spacing between adjacent energy levels needs to be measured. From the moment of inertia, the size of the molecule can be calculated, provided that the structure of the molecule is known.

## SOLUTIONS TO PROBLEMS

### Section 43.1 Molecular Bonds

**P43.1** (a)

$$F = \frac{q^2}{4\pi \epsilon_0 r^2} = \frac{\left(1.60\times 10^{-19}\right)^2\left(8.99\times 10^9\right)}{\left(5.00\times 10^{-10}\right)^2}\ \mathrm{N} = \boxed{0.921\times 10^{-9}\ \mathrm{N}} \text{ toward the other ion.}$$

(b)

$$U = \frac{-q^2}{4\pi \epsilon_0 r} = -\frac{\left(1.60\times 10^{-19}\right)^2\left(8.99\times 10^9\right)}{5.00\times 10^{-10}}\ \mathrm{J} \approx \boxed{-2.88\ \mathrm{eV}}$$

**P43.2** We are told $\mathrm{K}+\mathrm{Cl}+0.7\text{ eV}\to\mathrm{K}^{+}+\mathrm{Cl}^{-}$

and $\mathrm{Cl}+\mathrm{e}^{-}\to\mathrm{Cl}^{-}+3.6\text{ eV}$

or $\mathrm{Cl}^{-}\to\mathrm{Cl}+\mathrm{e}^{-}-3.6\text{ eV}$.

By substitution, $\mathrm{K}+\mathrm{Cl}+0.7\text{ eV}\to\mathrm{K}^{+}+\mathrm{Cl}+\mathrm{e}^{-}-3.6\text{ eV}$

$\mathrm{K}+4.3\text{ eV}\to\mathrm{K}^{+}+\mathrm{e}^{-}$

or the ionization energy of potassium is $\boxed{4.3\text{ eV}}$.

**P43.3** (a) Minimum energy of the molecule is found from

$$\frac{dU}{dr}=-12Ar^{-13}+6Br^{-7}=0\text{ yielding }\boxed{r_0=\left[\frac{2A}{B}\right]^{1/6}}.$$

(b) $$E=U\big|_{r=\infty}-U\big|_{r=r_0}=0-\left[\frac{A}{4A^2/B^2}-\frac{B}{2A/B}\right]=-\left[\frac{1}{4}-\frac{1}{2}\right]\frac{B^2}{A}=\boxed{\frac{B^2}{4A}}$$

This is also the equal to the binding energy, the amount of energy given up by the two atoms as they come together to form a molecule.

(c) $$r_0=\left[\frac{2\left(0.124\times10^{-120}\text{ eV}\cdot\text{m}^{12}\right)}{1.488\times10^{-60}\text{ eV}\cdot\text{m}^{6}}\right]^{1/6}=7.42\times10^{-11}\text{ m}=\boxed{74.2\text{ pm}}$$

$$E=\frac{\left(1.488\times10^{-60}\text{ eV}\cdot\text{m}^{6}\right)^2}{4\left(0.124\times10^{-120}\text{ eV}\cdot\text{m}^{12}\right)}=\boxed{4.46\text{ eV}}$$

***P43.4** (a) We add the reactions $\mathrm{K}+4.34\text{ eV}\to\mathrm{K}^{+}+\mathrm{e}^{-}$

and $\mathrm{I}+\mathrm{e}^{-}\to\mathrm{I}^{-}+3.06\text{ eV}$

to obtain $\mathrm{K}+\mathrm{I}\to\mathrm{K}^{+}+\mathrm{I}^{-}+(4.34-3.06)\text{ eV}$.

The activation energy is $\boxed{1.28\text{ eV}}$.

(b) $$\frac{dU}{dr}=\frac{4\in}{\sigma}\left[-12\left(\frac{\sigma}{r}\right)^{13}+6\left(\frac{\sigma}{r}\right)^{7}\right]$$

At $r=r_0$ we have $\frac{dU}{dr}=0$. Here $\left(\frac{\sigma}{r_0}\right)^{13}=\frac{1}{2}\left(\frac{\sigma}{r_0}\right)^{7}$

$$\frac{\sigma}{r_0}=2^{-1/6}\qquad\sigma=2^{-1/6}(0.305)\text{ nm}=\boxed{0.272\text{ nm}=\sigma}.$$

Then also

$$U(r_0)=4\in\left[\left(\frac{2^{-1/6}r_0}{r_0}\right)^{12}-\left(\frac{2^{-1/6}r_0}{r_0}\right)^{6}\right]+E_a=4\in\left[\frac{1}{4}-\frac{1}{2}\right]+E_a=-\in+E_a$$

$$\in=E_a-U(r_0)=1.28\text{ eV}+3.37\text{ eV}=\boxed{4.65\text{ eV}=\in}$$

*continued on next page*

(c) $$F(r) = -\frac{dU}{dr} = \frac{4\in}{\sigma}\left[12\left(\frac{\sigma}{r}\right)^{13} - 6\left(\frac{\sigma}{r}\right)^{7}\right]$$

To find the maximum force we calculate $\frac{dF}{dr} = \frac{4\in}{\sigma^2}\left[-156\left(\frac{\sigma}{r}\right)^{14} + 42\left(\frac{\sigma}{r}\right)^{8}\right] = 0$ when

$$\frac{\sigma}{r_{\text{rupture}}} = \left(\frac{42}{156}\right)^{1/6}$$

$$F_{\max} = \frac{4(4.65\text{ eV})}{0.272\text{ nm}}\left[12\left(\frac{42}{156}\right)^{13/6} - 6\left(\frac{42}{156}\right)^{7/6}\right] = -41.0\text{ eV/nm} = -41.0\,\frac{1.6\times10^{-19}\text{ Nm}}{10^{-9}\text{ m}}$$
$$= -6.55\text{ nN}$$

Therefore the applied force required to rupture the molecule is $\boxed{+6.55\text{ nN}}$ away from the center.

(d) $$U(r_0+s) = 4\in\left[\left(\frac{\sigma}{r_0+s}\right)^{12} - \left(\frac{\sigma}{r_0+s}\right)^{6}\right] + E_a = 4\in\left[\left(\frac{2^{-1/6}r_0}{r_0+s}\right)^{12} - \left(\frac{2^{-1/6}}{r_0+s}\right)^{6}\right] + E_a$$

$$= 4\in\left[\frac{1}{4}\left(1+\frac{s}{r_0}\right)^{-12} - \frac{1}{2}\left(1+\frac{s}{r_0}\right)^{-6}\right] + E_a$$

$$= 4\in\left[\frac{1}{4}\left(1-12\frac{s}{r_0}+78\frac{s^2}{r_0^2}-\ldots\right) - \frac{1}{2}\left(1-6\frac{s}{r_0}+21\frac{s^2}{r_0^2}-\ldots\right)\right] + E_a$$

$$= \in - 12\in\frac{s}{r_0} + 78\in\frac{s^2}{r_0^2} - 2\in + 12\in\frac{s}{r_0} - 42\in\frac{s^2}{r_0^2} + E_a + \ldots$$

$$= -\in + E_a + 0\left(\frac{s}{r_0}\right) + 36\in\frac{s^2}{r_0^2} + \ldots$$

$$U(r_0+s) \cong U(r_0) + \frac{1}{2}ks^2$$

where $k = \frac{72\in}{r_0^2} = \frac{72(4.65\text{ eV})}{(0.305\text{ nm})^2} = 3\,599\text{ eV/nm}^2 = \boxed{576\text{ N/m}}$.

**P43.5** At the boiling or condensation temperature, $k_BT \approx 10^{-3}\text{ eV} = 10^{-3}\left(1.6\times10^{-19}\text{ J}\right)$

$$T \approx \frac{1.6\times10^{-22}\text{ J}}{1.38\times10^{-23}\text{ J/K}}\boxed{\sim 10\text{ K}}.$$

## Section 43.2 **Energy States and Spectra of Molecules**

**P43.6** $$\mu=\frac{m_1m_2}{m_1+m_2}=\frac{132.9(126.9)}{132.9+126.9}\left(1.66\times10^{-27}\text{ kg}\right)=1.08\times10^{-25}\text{ kg}$$

$$I=\mu r^2=\left(1.08\times10^{-25}\text{ kg}\right)\left(0.127\times10^{-9}\text{ m}\right)^2=1.74\times10^{-45}\text{ kg}\cdot\text{m}^2$$

(a) $$E=\frac{1}{2}I\omega^2=\frac{(I\omega)^2}{2I}=\frac{J(J+1)\hbar^2}{2I}$$

$J=0$ gives $E=0$

$$J=1\text{ gives }E=\frac{\hbar^2}{I}=\frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)^2}{4\pi^2\left(1.74\times10^{-45}\text{ kg}\cdot\text{m}^2\right)}=6.41\times10^{-24}\text{ J}=\boxed{40.0\ \mu\text{eV}}$$

$$hf=6.41\times10^{-24}\text{ J}-0\text{ to }f=\boxed{9.66\times10^{9}\text{ Hz}}$$

(b) $$f=\frac{E_1}{h}=\frac{\hbar^2}{hI}=\frac{h}{4\pi^2\mu r^2}\propto r^{-2}\qquad\boxed{\text{If } r \text{ is 10\% too small, } f \text{ is 20\% too large.}}$$

**P43.7** For the HCl molecule in the $J=1$ rotational energy level, we are given $r_0=0.127\,5\text{ nm}$.

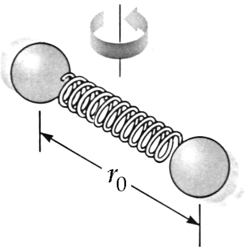

**FIG. P43.7**

$$E_{\text{rot}}=\frac{\hbar^2}{2I}J(J+1)$$

Taking $J=1$, we have $$E_{\text{rot}}=\frac{\hbar^2}{I}=\frac{1}{2}I\omega^2\text{ or }\omega=\sqrt{\frac{2\hbar^2}{I^2}}=\sqrt{2}\frac{\hbar}{I}.$$

The moment of inertia of the molecule is given by Equation 43.3.

$$I=\mu r_0^2=\left(\frac{m_1m_2}{m_1+m_2}\right)r_0^2$$

$$I=\left[\frac{(1\text{ u})(35\text{ u})}{1\text{ u}+35\text{ u}}\right]r_0^2=(0.972\text{ u})\left(1.66\times10^{-27}\text{ kg/u}\right)\left(1.275\times10^{-10}\text{ m}\right)^2=2.62\times10^{-47}\text{ kg}\cdot\text{m}^2.$$

Therefore, $$\omega=\sqrt{2}\frac{\hbar}{I}=\sqrt{2}\frac{1.055\times10^{-34}\text{ J}\cdot\text{s}}{2.62\times10^{-47}\text{ kg}\cdot\text{m}^2}=\boxed{5.69\times10^{12}\text{ rad/s}}.$$

**P43.8** $$hf=\Delta E=\frac{\hbar^2}{2I}\left[2(2+1)\right]-\frac{\hbar^2}{2I}\left[1(1+1)\right]=\frac{\hbar^2}{2I}(4)$$

$$I=\frac{4(h/2\pi)^2}{2hf}=\frac{h}{2\pi^2f}=\frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{2\pi^2\left(2.30\times10^{11}\text{ Hz}\right)}=\boxed{1.46\times10^{-46}\text{ kg}\cdot\text{m}^2}$$

**P43.9** $I = m_1r_1^2 + m_2r_2^2$ where $m_1r_1 = m_2r_2$ and $r_1 + r_2 = r$.

Then $r_1 = \dfrac{m_2r_2}{m_1}$ so $\dfrac{m_2r_2}{m_1} + r_2 = r$ and $r_2 = \dfrac{m_1 r}{m_1 + m_2}$.

Also, $r_2 = \dfrac{m_1r_1}{m_2}$. Thus, $r_1 + \dfrac{m_1r_1}{m_2} = r$ and $r_1 = \dfrac{m_2 r}{m_1 + m_2}$

$$I = m_1\frac{m_2^2r^2}{(m_1+m_2)^2} + \frac{m_2m_1^2r^2}{(m_1+m_2)^2} = \frac{m_1m_2r^2(m_2+m_1)}{(m_1+m_2)^2} = \frac{m_1m_2r^2}{m_1+m_2} = \boxed{\mu r^2}.$$

**P43.10** (a) $$\mu = \frac{22.99(35.45)}{(22.99+35.45)}\left(1.66\times10^{-27}\text{ kg}\right) = 2.32\times10^{-26}\text{ kg}$$

$$I = \mu r^2 = \left(2.32\times10^{-26}\text{ kg}\right)\left(0.280\times10^{-9}\text{ m}\right)^2 = \boxed{1.81\times10^{-45}\text{ kg}\cdot\text{m}^2}$$

(b) $$\frac{hc}{\lambda} = \frac{\hbar^2}{2I}2(2+1) - \frac{\hbar^2}{2I}1(1+1) = \frac{3\hbar^2}{I} - \frac{\hbar^2}{I} = \frac{2\hbar^2}{I} = \frac{2h^2}{4\pi^2 I}$$

$$\lambda = \frac{c4\pi^2 I}{2h} = \frac{\left(3.00\times10^8\text{ m/s}\right)4\pi^2\left(1.81\times10^{-45}\text{ kg}\cdot\text{m}^2\right)}{2\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)} = \boxed{1.62\text{ cm}}$$

**P43.11** The energy of a rotational transition is $\Delta E = \left(\dfrac{\hbar^2}{I}\right)J$ where $J$ is the rotational quantum number of the higher energy state (see Equation 43.7). We do not know $J$ from the data. However,

$$\Delta E = \frac{hc}{\lambda} = \frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^8\text{ m/s}\right)}{\lambda}\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right).$$

For each observed wavelength,

| $\lambda$ (mm) | $\Delta E$ (eV) |
|---|---|
| 0.120 4 | 0.010 32 |
| 0.096 4 | 0.012 88 |
| 0.080 4 | 0.015 44 |
| 0.069 0 | 0.018 00 |
| 0.060 4 | 0.020 56 |

The $\Delta E$'s consistently increase by 0.002 56 eV. $E_1 = \dfrac{\hbar^2}{I} = 0.002\,56\text{ eV}$

and $I = \dfrac{\hbar^2}{E_1} = \dfrac{\left(1.055\times10^{-34}\text{ J}\cdot\text{s}\right)^2}{(0.002\,56\text{ eV})}\left(\dfrac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) = \boxed{2.72\times10^{-47}\text{ kg}\cdot\text{m}^2}$.

For the HCl molecule, the internuclear radius is $r = \sqrt{\dfrac{I}{\mu}} = \sqrt{\dfrac{2.72\times10^{-47}}{1.62\times10^{-27}}}\text{ m} = 0.130\text{ nm}.$

**P43.12** (a) Minimum amplitude of vibration of HI is

$$\frac{1}{2}kA^2 = \frac{1}{2}hf: \quad A = \sqrt{\frac{hf}{k}} = \sqrt{\frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(6.69\times10^{13}/\text{s})}{320\ \text{N/m}}} = 1.18\times10^{-11}\ \text{m} = \boxed{0.011\,8\ \text{nm}}.$$

(b) For HF,
$$A = \sqrt{\frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(8.72\times10^{13}/\text{s})}{970\ \text{N/m}}} = 7.72\times10^{-12}\ \text{m} = \boxed{0.007\,72\ \text{nm}}.$$

Since HI has the smaller *k*, it is more weakly bound.

**P43.13** $\mu = \dfrac{m_1 m_2}{m_1 + m_2} = \dfrac{35}{36}\times1.66\times10^{-27}\ \text{kg} = 1.61\times10^{-27}\ \text{kg}$

$$\Delta E_{\text{vib}} = \hbar\sqrt{\frac{k}{\mu}} = (1.055\times10^{-34})\sqrt{\frac{480}{1.61\times10^{-27}}} = 5.74\times10^{-20}\ \text{J} = \boxed{0.358\ \text{eV}}$$

**P43.14** (a) The reduced mass of the $O_2$ is $\mu = \dfrac{(16\ \text{u})(16\ \text{u})}{(16\ \text{u})+(16\ \text{u})} = 8\ \text{u} = 8(1.66\times10^{-27}\ \text{kg}) = 1.33\times10^{-26}\ \text{kg}$.

The moment of inertia is then $I = \mu r^2 = (1.33\times10^{-26}\ \text{kg})(1.20\times10^{-10}\ \text{m})^2 = 1.91\times10^{-46}\ \text{kg}\cdot\text{m}^2$.

The rotational energies are $E_{\text{rot}} = \dfrac{\hbar^2}{2I}J(J+1) = \dfrac{(1.055\times10^{-34}\ \text{J}\cdot\text{s})^2}{2(1.91\times10^{-46}\ \text{kg}\cdot\text{m}^2)}J(J+1)$.

Thus $E_{\text{rot}} = (2.91\times10^{-23}\ \text{J})J(J+1)$.

And for $J = 0, 1, 2,$ $E_{\text{rot}} = \boxed{0,\ 3.64\times10^{-4}\ \text{eV},\ 1.09\times10^{-3}\ \text{eV}}$.

(b)
$$E_{\text{vib}} = \left(v+\frac{1}{2}\right)\hbar\sqrt{\frac{k}{\mu}} = \left(v+\frac{1}{2}\right)(1.055\times10^{-34}\ \text{J}\cdot\text{s})\sqrt{\frac{1\,177\ \text{N/m}}{8(1.66\times10^{-27}\ \text{kg})}}$$

$$E_{\text{vib}} = \left(v+\frac{1}{2}\right)(3.14\times10^{-20}\ \text{J})\left(\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \left(v+\frac{1}{2}\right)(0.196\ \text{eV})$$

For $v = 0, 1, 2,$ $\boxed{E_{\text{vib}} = 0.098\,2\ \text{eV},\ 0.295\ \text{eV},\ 0.491\ \text{eV}}$.

**P43.15** In Benzene, the carbon atoms are each 0.110 nm from the axis and each hydrogen atom is $(0.110+0.100\text{ nm})=0.210\text{ nm}$ from the axis. Thus, $I=\sum mr^2$:

$$I=6\left(1.99\times10^{-26}\text{ kg}\right)\left(0.110\times10^{-9}\text{ m}\right)^2+6\left(1.67\times10^{-27}\text{ kg}\right)\left(0.210\times10^{-9}\text{ m}\right)^2=1.89\times10^{-45}\text{ kg}\cdot\text{m}^2.$$

The allowed rotational energies are then

$$E_{\text{rot}}=\frac{\hbar^2}{2I}J(J+1)=\frac{\left(1.055\times10^{-34}\text{ J}\cdot\text{s}\right)^2}{2\left(1.89\times10^{-45}\text{ kg}\cdot\text{m}^2\right)}J(J+1)=\left(2.95\times10^{-24}\text{ J}\right)J(J+1)=\left(18.4\times10^{-6}\text{ eV}\right)J(J+1)$$

$$E_{\text{rot}}=\boxed{(18.4\ \mu\text{eV})J(J+1)\text{ where }J=0,\ 1,\ 2,\ 3,\ \ldots}$$

The first five of these allowed energies are: $E_{\text{rot}}=0,\ 36.9\ \mu\text{eV},\ 111\ \mu\text{eV},\ 221\ \mu\text{eV},\text{ and }369\ \mu\text{eV}$.

***P43.16** We carry extra digits through the solution because part (c) involves the subtraction of two close numbers. The longest wavelength corresponds to the smallest energy difference between the rotational energy levels. It is between $J=0$ and $J=1$, namely $\dfrac{\hbar^2}{I}$

$\lambda=\dfrac{hc}{\Delta E_{\min}}=\dfrac{hc}{\hbar^2/I}=\dfrac{4\pi^2Ic}{h}$. If $\mu$ is the reduced mass, then

$$I=\mu r^2=\mu\left(0.127\,46\times10^{-9}\text{ m}\right)^2=\left(1.624\,605\times10^{-20}\text{ m}^2\right)\mu$$

$$\therefore\lambda=\frac{4\pi^2\left(1.624\,605\times10^{-20}\text{ m}^2\right)\mu\left(2.997\,925\times10^8\text{ m/s}\right)}{6.626\,075\times10^{-34}\text{ J}\cdot\text{s}}=\left(2.901\,830\times10^{23}\text{ m/kg}\right)\mu \qquad (1)$$

(a) $$\mu_{35}=\frac{(1.007\,825\text{u})(34.968\,853\text{u})}{1.007\,825\text{u}+34.968\,853\text{u}}=0.979\,593\text{u}=1.626\,653\times10^{-27}\text{ kg}$$

From (1): $\lambda_{35}=\left(2.901\,830\times10^{23}\text{ m/kg}\right)\left(1.626\,653\times10^{-27}\text{ kg}\right)=\boxed{472\ \mu\text{m}}$

(b) $$\mu_{37}=\frac{(1.007\,825\text{u})(36.965\,903\text{u})}{1.007\,825\text{u}+36.965\,903\text{u}}=0.981\,077\text{u}=1.629\,118\times10^{-27}\text{ kg}$$

From (1): $\lambda_{37}=\left(2.901\,830\times10^{23}\text{ m/kg}\right)\left(1.629\,118\times10^{-27}\text{ kg}\right)=\boxed{473\ \mu\text{m}}$

(c) $\lambda_{37}-\lambda_{35}=472.742\,4\ \mu\text{m}-472.027\,0\ \mu\text{m}=\boxed{0.715\ \mu\text{m}}$

**P43.17** $hf=\dfrac{h^2}{4\pi^2I}J$ where the rotational transition is from $J-1$ to $J$,

where $f=6.42\times10^{13}\text{ Hz}$ and $I=1.46\times10^{-46}\text{ kg}\cdot\text{m}^2$ from Example 43.1.

$$J=\frac{4\pi^2If}{h}=\frac{4\pi^2\left(1.46\times10^{-46}\text{ kg}\cdot\text{m}^2\right)\left(6.42\times10^{13}/\text{s}\right)}{6.626\times10^{-34}\text{ J}\cdot\text{s}}=\boxed{558}$$

***P43.18** We find an average spacing between peaks by counting 22 gaps between $7.96\times10^{13}$ Hz and $9.24\times10^{13}$ Hz:

$$\Delta f=\frac{(9.24-7.96)10^{13}\text{ Hz}}{22}=0.058\,2\times10^{13}\text{ Hz}=\frac{1}{h}\left(\frac{h^2}{4\pi^2 I}\right)$$

$$I=\frac{h}{4\pi^2\Delta f}=\frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{4\pi^2 5.82\times10^{11}/\text{s}}=\boxed{2.9\times10^{-47}\text{ kg}\cdot\text{m}^2}$$

***P43.19** We carry extra digits through the solution because the given wavelengths are close together.

(a) $$E_{vJ}=\left(v+\frac{1}{2}\right)hf+\frac{\hbar^2}{2I}J(J+1)$$

$$\therefore E_{00}=\frac{1}{2}hf,\qquad E_{11}=\frac{3}{2}hf+\frac{\hbar^2}{I},\ E_{02}=\frac{1}{2}hf+\frac{3\hbar^2}{I}$$

$$\therefore E_{11}-E_{00}=hf+\frac{\hbar^2}{I}=\frac{hc}{\lambda}=\frac{\left(6.626\,075\times10^{-34}\text{ J}\cdot\text{s}\right)\left(2.997\,925\times10^{8}\text{ m/s}\right)}{2.211\,2\times10^{-6}\text{ m}}$$

$$\therefore hf+\frac{\hbar^2}{I}=8.983\,573\times10^{-20}\text{ J}\qquad(1)$$

$$E_{11}-E_{02}=hf-\frac{2\hbar^2}{I}=\frac{hc}{\lambda}=\frac{\left(6.626\,075\times10^{-34}\text{ J}\cdot\text{s}\right)\left(2.997\,925\times10^{8}\text{ m/s}\right)}{2.405\,4\times10^{-6}\text{ m}}$$

$$\therefore hf-\frac{2\hbar^2}{I}=8.258\,284\times10^{-20}\text{ J}\qquad(2)$$

Subtract (2) from (1): $\dfrac{3\hbar^2}{I}=7.252\,89\times10^{-21}$ J

$$\therefore I=\frac{3\left(1.054\,573\times10^{-34}\text{ J}\cdot\text{s}\right)^2}{7.252\,89\times10^{-21}\text{ J}}=\boxed{4.60\times10^{-48}\text{ kg}\cdot\text{m}^2}$$

(b) From (1):

$$f=\frac{8.983\,573\times10^{-20}\text{ J}}{6.626\,075\times10^{-34}\text{ J}\cdot\text{s}}-\frac{\left(1.054\,573\times10^{-34}\text{ J}\cdot\text{s}\right)^2}{\left(4.600\,060\times10^{-48}\text{ kg}\cdot\text{m}^2\right)\left(6.626\,075\times10^{-34}\text{ J}\cdot\text{s}\right)}$$
$$=\boxed{1.32\times10^{14}\text{ Hz}}$$

(c) $I=\mu r^2$, where $\mu$ is the reduced mass:

$$\mu=\frac{1}{2}m_H=\frac{1}{2}(1.007\,825\text{u})=8.367\,669\times10^{-28}\text{ kg}.$$

So $r=\sqrt{\dfrac{I}{\mu}}=\sqrt{\dfrac{4.600\,060\times10^{-48}\text{ kg}\cdot\text{m}^2}{8.367\,669\times10^{-28}\text{ kg}}}=\boxed{0.0741\text{ nm}}$.

**P43.20** The emission energies are the same as the absorption energies, but the final state must be below $(v=1,\ J=0)$. The transition must satisfy $\Delta J = \pm 1$, so it must end with $J=1$. To be lower in energy, it must be $(v=0,\ J=1)$. The emitted photon energy is therefore

$$hf_{\text{photon}} = \left(E_{\text{vib}}\big|_{v=1} + E_{\text{rot}}\big|_{J=0}\right) - \left(E_{\text{vib}}\big|_{v=0} + E_{\text{rot}}\big|_{J=1}\right) = \left(E_{\text{vib}}\big|_{v=1} - E_{\text{vib}}\big|_{v=0}\right) - \left(E_{\text{rot}}\big|_{J=1} - E_{\text{rot}}\big|_{J=0}\right)$$

$$hf_{\text{photon}} = hf_{\text{vib}} - hf_{\text{rot}}$$

Thus, $f_{\text{photon}} = f_{\text{vib}} - f_{\text{rot}} = 6.42\times10^{13}\ \text{Hz} - 1.15\times10^{11}\ \text{Hz} = \boxed{6.41\times10^{13}\ \text{Hz}}$.

**P43.21** The moment of inertia about the molecular axis is $I_x = \frac{2}{5}mr^2 + \frac{2}{5}mr^2 = \frac{4}{5}m\left(2.00\times10^{-15}\ \text{m}\right)^2$.

The moment of inertia about a perpendicular axis is $I_y = m\left(\frac{R}{2}\right)^2 + m\left(\frac{R}{2}\right)^2 = \frac{m}{2}\left(2.00\times10^{-10}\ \text{m}\right)^2$.

The allowed rotational energies are $E_{\text{rot}} = \left(\frac{\hbar^2}{2I}\right)J(J+1)$, so the energy of the first excited state is $E_1 = \frac{\hbar^2}{I}$. The ratio is therefore

$$\frac{E_{1,x}}{E_{1,y}} = \frac{\left(\hbar^2/I_x\right)}{\left(\hbar^2/I_y\right)} = \frac{I_y}{I_x} = \frac{(1/2)m\left(2.00\times10^{-10}\ \text{m}\right)^2}{(4/5)m\left(2.00\times10^{-15}\ \text{m}\right)^2} = \frac{5}{8}\left(10^5\right)^2 = \boxed{6.25\times10^9}.$$

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Section 43.3 **Bonding in Solids**

**P43.22** Consider a cubical salt crystal of edge length 0.1 mm.

The number of atoms is $\left(\frac{10^{-4}\ \text{m}}{0.261\times10^{-9}\ \text{m}}\right)^3 \boxed{\sim10^{17}}$.

This number of salt crystals would have volume $\left(10^{-4}\ \text{m}\right)^3\left(\frac{10^{-4}\ \text{m}}{0.261\times10^{-9}\ \text{m}}\right)^3 \boxed{\sim10^5\ \text{m}^3}$.

If it is cubic, it has edge length 40 m.

**P43.23** $$U = -\frac{\alpha k_e e^2}{r_0}\left(1-\frac{1}{m}\right) = -(1.747\,6)\left(8.99\times10^9\right)\frac{\left(1.60\times10^{-19}\right)^2}{\left(0.281\times10^{-9}\right)}\left(1-\frac{1}{8}\right) = -1.25\times10^{-18}\ \text{J} = \boxed{-7.84\ \text{eV}}$$

**P43.24** Visualize a K$^+$ ion at the center of each shaded cube, a Cl$^-$ ion at the center of each white one.

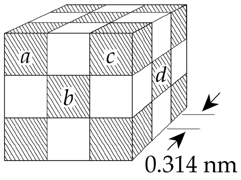


FIG. P43.24

The distance *ab* is $\sqrt{2}(0.314\ \text{nm}) = \boxed{0.444\ \text{nm}}$.

Distance *ac* is $2(0.314\ \text{nm}) = \boxed{0.628\ \text{nm}}$.

Distance *ad* is $\sqrt{2^2+\left(\sqrt{2}\right)^2}(0.314\ \text{nm}) = \boxed{0.769\ \text{nm}}$.

**P43.25** $$U=-\frac{k_ee^2}{r}-\frac{k_ee^2}{r}+\frac{k_ee^2}{2r}+\frac{k_ee^2}{2r}-\frac{k_ee^2}{3r}-\frac{k_ee^2}{3r}+\frac{k_ee^2}{4r}+\frac{k_ee^2}{4r}-\dots$$

$$=-\frac{2k_ee^2}{r}\left(1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\dots\right)$$

But, $\ln(1+x)=1-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\dots$

so, $U=-\frac{2k_ee^2}{r}\ln 2$, or $\boxed{U=-k_e\alpha\frac{e^2}{r}\text{ where }\alpha=2\ln 2}$.

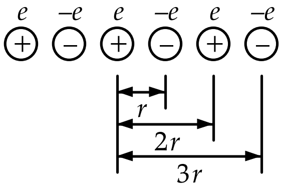


FIG. P43.25

Section 43.4 **Free-Electron Theory of Metals**

Section 43.5 **Band Theory of Solids**

**P43.26** $$E_F=\frac{h^2}{2m}\left(\frac{3n_e}{8\pi}\right)^{2/3}=\left[\frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})^2}{2(9.11\times10^{-31}\text{ kg})(1.60\times10^{-19}\text{ J/eV})}\right]\left(\frac{3}{8\pi}\right)^{2/3}n_e^{2/3}$$

$E_F=(3.65\times10^{-19})n_e^{2/3}$ eV with $n$ measured in electrons/m$^3$.

**P43.27** The density of conduction electrons $n$ is given by $E_F=\frac{h^2}{2m}\left(\frac{3n_e}{8\pi}\right)^{2/3}$

or $$n_e=\frac{8\pi}{3}\left(\frac{2mE_F}{h^2}\right)^{3/2}=\frac{8\pi}{3}\frac{\left[2(9.11\times10^{-31}\text{ kg})(5.48)(1.60\times10^{-19}\text{ J})\right]^{3/2}}{(6.626\times10^{-34}\text{ J}\cdot\text{s})^3}=5.80\times10^{28}\text{ m}^{-3}.$$

The number-density of silver atoms is

$$n_{\text{Ag}}=(10.6\times10^3\text{ kg/m}^3)\left(\frac{1\text{ atom}}{108\text{ u}}\right)\left(\frac{1\text{ u}}{1.66\times10^{-27}\text{ kg}}\right)=5.91\times10^{28}\text{ m}^{-3}.$$

So an average atom contributes $\frac{5.80}{5.91}=\boxed{0.981\text{ electron to the conduction band}}$.

**P43.28** (a) $\frac{1}{2}mv^2=7.05$ eV

$$v=\sqrt{\frac{2(7.05\text{ eV})(1.60\times10^{-19}\text{ J/eV})}{9.11\times10^{-31}\text{ kg}}}=\boxed{1.57\times10^6\text{ m/s}}$$

(b) $\boxed{\text{Larger than }10^{-4}\text{ m/s by ten orders of magnitude.}}$ However, the energy of an electron at room temperature is typically $k_BT=\frac{1}{40}$ eV.

**P43.29** For sodium, $M = 23.0\ \text{g/mol}$ and $\rho = 0.971\ \text{g/cm}^3$.

(a) $$n_e = \frac{N_A \rho}{M} = \frac{\left(6.02\times10^{23}\ \text{electrons/mol}\right)\left(0.971\ \text{g/cm}^3\right)}{23.0\ \text{g/mol}}$$

$$n_e = 2.54\times10^{22}\ \text{electrons/cm}^3 = \boxed{2.54\times10^{28}\ \text{electrons/m}^3}$$

(b) $$E_F = \left(\frac{h^2}{2m}\right)\left(\frac{3n_e}{8\pi}\right)^{2/3} = \frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)^2}{2\left(9.11\times10^{-31}\ \text{kg}\right)}\left[\frac{3\left(2.54\times10^{28}\ \text{m}^{-3}\right)}{8\pi}\right]^{2/3} = 5.05\times10^{-19}\ \text{J} = \boxed{3.15\ \text{eV}}$$

**P43.30** The melting point of silver is 1 234 K. Its Fermi energy at 300 K is 5.48 eV. The approximate fraction of electrons excited is

$$\frac{k_B T}{E_F} = \frac{\left(1.38\times10^{-23}\ \text{J/K}\right)(1\,234\ \text{K})}{(5.48\ \text{eV})\left(1.60\times10^{-19}\ \text{J/eV}\right)} \approx \boxed{2\%}.$$

**P43.31** Taking $E_F = 5.48\ \text{eV}$ for sodium at 800 K,

$$f = \left[e^{(E-E_F)/k_BT} + 1\right]^{-1} = 0.950$$

$$e^{(E-E_F)/k_BT} = \frac{1}{0.950} - 1 = 0.052\,6$$

$$\frac{E-E_F}{k_BT} = \ln(0.052\,6) = -2.94$$

$$E - E_F = -2.94\frac{\left(1.38\times10^{-23}\right)(800)\ \text{J}}{1.60\times10^{-19}\ \text{J/eV}} = -0.203\ \text{eV} \text{ or } \boxed{E = 5.28\ \text{eV}}.$$

**P43.32** $d = 1.00\ \text{mm}$, so $V = \left(1.00\times10^{-3}\ \text{m}\right)^3 = 1.00\times10^{-9}\ \text{m}^3$.

The **density of states** is $$g(E) = CE^{1/2} = \frac{8\sqrt{2}\pi m^{3/2}}{h^3}E^{1/2}$$

or $$g(E) = \frac{8\sqrt{2}\pi\left(9.11\times10^{-31}\ \text{kg}\right)^{3/2}}{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)^3}\sqrt{(4.00\ \text{eV})\left(1.60\times10^{-19}\ \text{J/eV}\right)}$$

$$g(E) = 8.50\times10^{46}\ \text{m}^{-3}\cdot\text{J}^{-1} = 1.36\times10^{28}\ \text{m}^{-3}\cdot\text{eV}^{-1}.$$

So, the total number of electrons is

$$N = [g(E)](\Delta E)V = \left(1.36\times10^{28}\ \text{m}^{-3}\cdot\text{eV}^{-1}\right)(0.025\,0\ \text{eV})\left(1.00\times10^{-9}\ \text{m}^3\right) = \boxed{3.40\times10^{17}\ \text{electrons}}.$$

**P43.33** $E_{\text{av}} = \dfrac{1}{n_e}\int_0^\infty EN(E)dE$

At $T = 0$, $N(E) = 0$ for $E > E_F$;.

Since $f(E) = 1$ for $E < E_{EF}$ and $f(E) = 0$ for $E > E_F$, we can take $N(E) = CE^{1/2} = \dfrac{8\sqrt{2}\pi m_e^{3/2}}{h^3}E^{1/2}$

$$E_{\text{av}} = \frac{1}{n_e}\int_0^{E_F} CE^{3/2}dE = \frac{C}{n_e}\int_0^{E_F} E^{3/2}dE = \frac{2C}{5n_e}E_F^{5/2}.$$

But from Equation 43.24, $\dfrac{C}{n_e} = \dfrac{3}{2}E_F^{-3/2}$, so that $E_{\text{av}} = \left(\dfrac{2}{5}\right)\left(\dfrac{3}{2}E_F^{-3/2}\right)E_F^{5/2} = \boxed{\dfrac{3}{5}E_F}$.

**P43.34** Consider first the wave function in $x$. At $x = 0$ and $x = L$, $\psi = 0$.

Therefore, $\sin k_x L = 0$ and $k_x L = \pi, 2\pi, 3\pi, \ldots$.

Similarly, $\sin k_y L = 0$ and $k_y L = \pi, 2\pi, 3\pi, \ldots$

$\sin k_z L = 0$ and $k_z L = \pi, 2\pi, 3\pi, \ldots$

$$\psi = A\sin\left(\frac{n_x \pi x}{L}\right)\sin\left(\frac{n_y \pi y}{L}\right)\sin\left(\frac{n_z \pi z}{L}\right).$$

From $\dfrac{\partial^2\psi}{\partial x^2} + \dfrac{\partial^2\psi}{\partial y^2} + \dfrac{\partial^2\psi}{\partial z^2} = \dfrac{2m_e}{\hbar^2}(U - E)\psi$, we have inside the box, where $U = 0$,

$$\left(-\frac{n_x^2\pi^2}{L^2} - \frac{n_y^2\pi^2}{L^2} - \frac{n_z^2\pi^2}{L^2}\right)\psi = \frac{2m_e}{\hbar^2}(-E)\psi \qquad \boxed{E = \frac{\hbar^2\pi^2}{2m_e L^2}\left(n_x^2 + n_y^2 + n_z^2\right) \qquad n_x, n_y, n_z = 1, 2, 3, \ldots}.$$

Outside the box we require $\psi = 0$.

The minimum energy state inside the box is $n_x = n_y = n_z = 1$, with $E = \dfrac{3\hbar^2\pi^2}{2m_e L^2}$

**P43.35** (a) The density of states at energy $E$ is $g(E) = CE^{1/2}$.

Hence, the required ratio is $\dfrac{g(8.50\text{ eV})}{g(7.00\text{ eV})} = \dfrac{C(8.50)^{1/2}}{C(7.00)^{1/2}} = \boxed{1.10}$.

(b) From Eq. 43.22, the number of occupied states having energy $E$ is

$$N(E) = \frac{CE^{1/2}}{e^{(E-E_F)/k_BT} + 1}.$$

Hence, the required ratio is $\dfrac{N(8.50\text{ eV})}{N(7.00\text{ eV})} = \dfrac{(8.50)^{1/2}}{(7.00)^{1/2}}\left[\dfrac{e^{(7.00-7.00)/k_BT} + 1}{e^{(8.50-7.00)/k_BT} + 1}\right]$.

At $T = 300$ K, $k_BT = 4.14 \times 10^{-21}$ J $= 0.025\,9$ eV, $\dfrac{N(8.50\text{ eV})}{N(7.00\text{ eV})} = \dfrac{(8.50)^{1/2}}{(7.00)^{1/2}}\left[\dfrac{2.00}{e^{(1.50)/0.025\,9} + 1}\right]$.

And $\dfrac{N(8.50\text{ eV})}{N(7.00\text{ eV})} = \boxed{1.55 \times 10^{-25}}$.

Comparing this result with that from part (a), we conclude that very few states with $E > E_F$ are occupied.

## Section 43.6 **Electrical Conduction in Metals, Insulators, and Semiconductors**

**P43.36** (a) $E_g = 1.14 \text{ eV}$ for Si

$$hf = 1.14 \text{ eV} = (1.14 \text{ eV})\left(1.60 \times 10^{-19} \text{ J/eV}\right) = 1.82 \times 10^{-19} \text{ J so } f \ge \boxed{2.75 \times 10^{14} \text{ Hz}}$$

(b) $$c = \lambda f;\ \lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{2.75 \times 10^{14} \text{ Hz}} = 1.09 \times 10^{-6} \text{ m} = \boxed{1.09\ \mu\text{m}} \text{ (in the infrared region)}$$

**P43.37** Photons of energy greater than 2.42 eV will be absorbed. This means wavelength shorter than

$$\lambda = \frac{hc}{E} = \frac{\left(6.626 \times 10^{-34} \text{ J}\cdot\text{s}\right)\left(3.00 \times 10^8 \text{ m/s}\right)}{2.42 \times 1.60 \times 10^{-19} \text{ J}} = 514 \text{ nm}.$$

All the hydrogen Balmer lines except for the red line at 656 nm will be absorbed.

**P43.38** $$E_g = \frac{hc}{\lambda} = \frac{\left(6.626 \times 10^{-34} \text{ J}\cdot\text{s}\right)\left(3.00 \times 10^8 \text{ m/s}\right)}{650 \times 10^{-9} \text{ m}} \text{ J} \approx \boxed{1.91 \text{ eV}}$$

**P43.39** If $\lambda \le 1.00 \times 10^{-6}$ m, then photons of sunlight have energy

$$E \ge \frac{hc}{\lambda_{\max}} = \frac{\left(6.626 \times 10^{-34} \text{ J}\cdot\text{s}\right)\left(3.00 \times 10^8 \text{ m/s}\right)}{1.00 \times 10^{-6} \text{ m}} \left(\frac{1 \text{ eV}}{1.60 \times 10^{-19} \text{ J}}\right) = 1.24 \text{ eV}.$$

Thus, the energy gap for the collector material should be $\boxed{E_g \le 1.24 \text{ eV}}$. Since Si has an energy gap $E_g \approx 1.14 \text{ eV}$, it will absorb radiation of this energy and greater. Therefore, $\boxed{\text{Si is acceptable}}$ as a material for a solar collector.

**P43.40** If the photon energy is 5.5 eV or higher, the diamond window will absorb. Here,

$$(hf)_{\max} = \frac{hc}{\lambda_{\min}} = 5.5 \text{ eV}: \qquad \lambda_{\min} = \frac{hc}{5.5 \text{ eV}} = \frac{\left(6.626 \times 10^{-34} \text{ J}\cdot\text{s}\right)\left(3.00 \times 10^8 \text{ m/s}\right)}{(5.5 \text{ eV})\left(1.60 \times 10^{-19} \text{ J/eV}\right)}$$

$$\lambda_{\min} = 2.26 \times 10^{-7} \text{ m} = \boxed{226 \text{ nm}}.$$

***P43.41** In the Bohr model we replace $k_e$ by $\frac{k_e}{\kappa}$ and $m_e$ by $m^*$. Then the radius of the first Bohr orbit, $a_0 = \frac{\hbar^2}{m_e k_e e^2}$ in hydrogen, changes to

$$a' = \frac{\hbar^2 \kappa}{m^* k_e e^2} = \left(\frac{m_e}{m^*}\right)\kappa \frac{\hbar^2}{m_e k_e e^2} = \left(\frac{m_e}{m^*}\right)\kappa\, a_0 = \left(\frac{m_e}{0.220 m_e}\right) 11.7(0.052\,9 \text{ nm}) = \boxed{2.81 \text{ nm}}.$$

The energy levels are in hydrogen $E_n = -\frac{k_e e^2}{2a_0}\frac{1}{n^2}$ and here

$$E_n' = -\frac{k_e e^2}{\kappa 2a'}\frac{1}{n^2} = -\frac{k_e e^2}{\kappa 2\left(m_e/m^*\right)\kappa\, a_0} = -\left(\frac{m^*}{m_e}\right)\frac{E_n}{\kappa^2}$$

For $n = 1$, $E_1' = -0.220 \frac{13.6 \text{ eV}}{11.7^2} = \boxed{-0.021\,9 \text{ eV}}$.

## Section 43.7 **Semiconductor Devices**

**P43.42** $I = I_0\left(e^{e(\Delta V)/k_BT} - 1\right)$. Thus, $e^{e(\Delta V)/k_BT} = 1 + \dfrac{I}{I_0}$

and $\Delta V = \dfrac{k_BT}{e}\ln\left(1 + \dfrac{I}{I_0}\right)$.

At $T = 300\text{ K}$, $\Delta V = \dfrac{\left(1.38\times10^{-23}\text{ J/K}\right)(300\text{ K})}{1.60\times10^{-19}\text{ C}}\ln\left(1+\dfrac{I}{I_0}\right) = (25.9\text{ mV})\ln\left(1+\dfrac{I}{I_0}\right)$.

(a) If $I = 9.00I_0$, $\Delta V = (25.9\text{ mV})\ln(10.0) = \boxed{59.5\text{ mV}}$.

(b) If $I = -0.900I_0$, $\Delta V = (25.9\text{ mV})\ln(0.100) = \boxed{-59.5\text{ mV}}$.

The basic idea behind a semiconductor device is that a large current or charge can be controlled by a small control voltage.

**P43.43** The voltage across the diode is about 0.6 V. The voltage drop across the resistor is $(0.025\text{ A})(150\ \Omega) = 3.75\text{ V}$. Thus, $\varepsilon - 0.6\text{ V} - 3.8\text{ V} = 0$ and $\varepsilon = \boxed{4.4\text{ V}}$.

**P43.44** First, we evaluate $I_0$ in $I = I_0\left(e^{e(\Delta V)/k_BT} - 1\right)$, given that $I = 200\text{ mA}$ when $\Delta V = 100\text{ mV}$ and $T = 300\text{ K}$.

$$\frac{e(\Delta V)}{k_BT} = \frac{\left(1.60\times10^{-19}\text{ C}\right)(0.100\text{ V})}{\left(1.38\times10^{-23}\text{ J/K}\right)(300\text{ K})} = 3.86 \text{ so } I_0 = \frac{I}{e^{e(\Delta V)/k_BT} - 1} = \frac{200\text{ mA}}{e^{3.86} - 1} = 4.28\text{ mA}$$

If $\Delta V = -100\text{ mV}$, $\dfrac{e(\Delta V)}{k_BT} = -3.86$; and the current will be

$$I = I_0\left(e^{e(\Delta V)/k_BT} - 1\right) = (4.28\text{ mA})\left(e^{-3.86} - 1\right) = \boxed{-4.19\text{ mA}}.$$

***P43.45** (a) The currents to be plotted are

$$I_D = \left(10^{-6}\text{ A}\right)\left(e^{\Delta V/0.025\text{ V}} - 1\right),$$

$$I_W = \frac{2.42\text{ V} - \Delta V}{745\ \Omega}$$

The two graphs intersect at $\Delta V = 0.200\text{ V}$. The currents are then

$$I_D = \left(10^{-6}\text{ A}\right)\left(e^{0.200\text{ V}/0.025\text{ V}} - 1\right)$$
$$= 2.98\text{ mA}$$

$$I_W = \frac{2.42\text{ V} - 0.200\text{ V}}{745\ \Omega} = 2.98\text{ mA}.$$ They agree to three digits.

$$\therefore I_D = I_W = \boxed{2.98\text{ mA}}$$

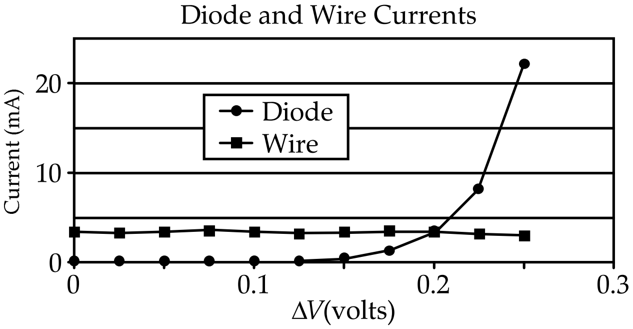


**FIG. P43.45**

(b) $$\frac{\Delta V}{I_D} = \frac{0.200\text{ V}}{2.98\times10^{-3}\text{ A}} = \boxed{67.1\ \Omega}$$

(c) $$\frac{d(\Delta V)}{dI_D} = \left[\frac{dI_D}{d(\Delta V)}\right]^{-1} = \left[\frac{10^{-6}\text{ A}}{0.025\text{ V}}e^{0.200\text{ V}/0.025\text{ V}}\right]^{-1} = \boxed{8.39\ \Omega}$$

## Section 43.8 Superconductivity

**P43.46** (a) $\boxed{\text{See the figure at right.}}$

(b) For a surface current around the outside of the cylinder as shown,

$$B = \frac{N\mu_0 I}{\ell} \text{ or } NI = \frac{B\ell}{\mu_0} = \frac{(0.540 \text{ T})\left(2.50\times10^{-2}\text{ m}\right)}{\left(4\pi\times10^{-7}\right)\text{ T}\cdot\text{m/A}} = \boxed{10.7\text{ kA}}.$$

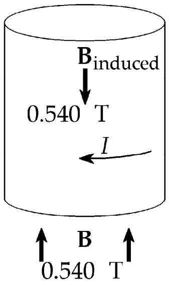


**FIG. P43.46**

**P43.47** By Faraday's law (Equation 32.1), $\frac{\Delta\Phi_B}{\Delta t} = L\frac{\Delta I}{\Delta t} = A\frac{\Delta B}{\Delta t}$.

Thus, $$\Delta I = \frac{A(\Delta B)}{L} = \frac{\pi(0.010\,0\text{ m})^2(0.020\,0\text{ T})}{3.10\times10^{-8}\text{ H}} = \boxed{203\text{ A}}.$$

The direction of the induced current is such as to maintain the $B$-field through the ring.

**P43.48** (a) $\Delta V = IR$

If $R = 0$, then $\Delta V = 0$, even when $I \neq 0$.

(b) The graph shows a direct proportionality.

$$\text{Slope} = \frac{1}{R} = \frac{\Delta I}{\Delta V} = \frac{(155 - 57.8)\text{ mA}}{(3.61 - 1.356)\text{ mV}} = 43.1\ \Omega^{-1}$$

$$R = \boxed{0.023\,2\ \Omega}$$

(c) Expulsion of magnetic flux and therefore fewer current-carrying paths could explain the decrease in current.

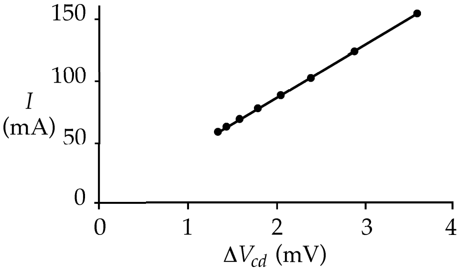


**FIG. P43.48**

## Additional Problems

**P43.49** (a) Since the interatomic potential is the same for both molecules, the spring constant is the same.

Then $f = \frac{1}{2\pi}\sqrt{\frac{k}{\mu}}$ where $\mu_{12} = \frac{(12\text{ u})(16\text{ u})}{12\text{ u} + 16\text{ u}} = 6.86\text{ u}$ and $\mu_{14} = \frac{(14\text{ u})(16\text{ u})}{14\text{ u} + 16\text{ u}} = 7.47\text{ u}$.

Therefore,

$$f_{14} = \frac{1}{2\pi}\sqrt{\frac{k}{\mu_{14}}} = \frac{1}{2\pi}\sqrt{\frac{k}{\mu_{12}}\left(\frac{\mu_{12}}{\mu_{14}}\right)} = f_{12}\sqrt{\frac{\mu_{12}}{\mu_{14}}} = \left(6.42\times10^{13}\text{ Hz}\right)\sqrt{\frac{6.86\text{ u}}{7.47\text{ u}}} = \boxed{6.15\times10^{13}\text{ Hz}}.$$

*continued on next page*

(b) The equilibrium distance is the same for both molecules.

$$I_{14} = \mu_{14} r^2 = \left(\frac{\mu_{14}}{\mu_{12}}\right)\mu_{12} r^2 = \left(\frac{\mu_{14}}{\mu_{12}}\right) I_{12}$$

$$I_{14} = \left(\frac{7.47\ \text{u}}{6.86\ \text{u}}\right)\left(1.46\times10^{-46}\ \text{kg}\cdot\text{m}^2\right) = \boxed{1.59\times10^{-46}\ \text{kg}\cdot\text{m}^2}$$

(c) The molecule can move to the $(v=1,\ J=9)$ state or to the $(v=1,\ J=11)$ state. The energy it can absorb is either

$$\Delta E = \frac{hc}{\lambda} = \left[\left(1+\frac{1}{2}\right)hf_{14} + 9(9+1)\frac{\hbar^2}{2I_{14}}\right] - \left[\left(0-\frac{1}{2}\right)hf_{14} + 10(10+1)\frac{\hbar^2}{2I_{14}}\right],$$

or $$\Delta E = \frac{hc}{\lambda} = \left[\left(1+\frac{1}{2}\right)hf_{14} + 11(11+1)\frac{\hbar^2}{2I_{14}}\right] - \left[\left(0+\frac{1}{2}\right)hf_{14} + 10(10+1)\frac{\hbar^2}{2I_{14}}\right].$$

The wavelengths it can absorb are then

$$\lambda = \frac{c}{f_{14} - 10\hbar/(2\pi I_{14})} \text{ or } \lambda = \frac{c}{f_{14} + 11\hbar/(2\pi I_{14})}.$$

These are: $$\lambda = \frac{3.00\times10^8\ \text{m/s}}{6.15\times10^{13}\ \text{Hz} - \left[10\left(1.055\times10^{-34}\ \text{J}\cdot\text{s}\right)\right]/\left[2\pi\left(1.59\times10^{-46}\ \text{kg}\cdot\text{m}^2\right)\right]} = \boxed{4.96\ \mu\text{m}}$$

and $$\lambda = \frac{3.00\times10^8\ \text{m/s}}{6.15\times10^{13}\ \text{Hz} + \left[11\left(1.055\times10^{-34}\ \text{J}\cdot\text{s}\right)\right]/\left[2\pi\left(1.59\times10^{-46}\ \text{kg}\cdot\text{m}^2\right)\right]} = \boxed{4.79\ \mu\text{m}}.$$

**P43.50** For the $N_2$ molecule, $k = 2\,297\ \text{N/m}$, $m = 2.32\times10^{-26}\ \text{kg}$, $r = 1.20\times10^{-10}\ \text{m}$, $\mu = \frac{m}{2}$

$$\omega = \sqrt{\frac{k}{\mu}} = 4.45\times10^{14}\ \text{rad/s}, \qquad I = \mu r^2 = \left(1.16\times10^{-26}\ \text{kg}\right)\left(1.20\times10^{-10}\ \text{m}\right)^2 = 1.67\times10^{-46}\ \text{kg}\cdot\text{m}^2.$$

For a rotational state sufficient to allow a transition to the first exited vibrational state,

$$\frac{\hbar^2}{2I}J(J+1) = \hbar\omega \text{ so } J(J+1) = \frac{2I\omega}{\hbar} = \frac{2\left(1.67\times10^{-46}\right)\left(4.45\times10^{14}\right)}{1.055\times10^{-34}} = 1\,410.$$

Thus $\boxed{J=37}$.

**P43.51** $\Delta E_{\text{max}} = 4.5\ \text{eV} = \left(v+\frac{1}{2}\right)\hbar\omega$ so $$\frac{(4.5\ \text{eV})\left(1.6\times10^{-19}\ \text{J/eV}\right)}{\left(1.055\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(8.28\times10^{14}\ \text{s}^{-1}\right)} \ge \left(v+\frac{1}{2}\right)$$

$8.25 > 7.5$ $\boxed{v=7}$

**P43.52** With 4 van der Waal bonds per atom pair or 2 electrons per atom, the total energy of the solid is

$$E = 2\left(1.74\times10^{-23}\ \text{J/atom}\right)\left(\frac{6.02\times10^{23}\ \text{atoms}}{4.00\ \text{g}}\right) = \boxed{5.23\ \text{J/g}}.$$

**P43.53** The total potential energy is given by Equation 43.17: $U_{\text{total}} = -\alpha\frac{k_e e^2}{r} + \frac{B}{r^m}$.

The total potential energy has its minimum value $U_0$ at the equilibrium spacing, $r = r_0$. At this point, $\left.\frac{dU}{dr}\right|_{r=r_0} = 0$,

or
$$\left.\frac{dU}{dr}\right|_{r=r_0} = \frac{d}{dr}\left(-\alpha\frac{k_e e^2}{r} + \frac{B}{r^m}\right)\Bigg|_{r=r_0} = \alpha\frac{k_e e^2}{r_0^2} - \frac{mB}{r_0^{m+1}} = 0.$$

Thus,
$$B = \alpha\frac{k_e e^2}{m}r_0^{m-1}.$$

Substituting this value of $B$ into $U_{\text{total}}$,
$$U_0 = -\alpha\frac{k_e e^2}{r_0} + \alpha\frac{k_e e^2}{m}r_0^{m-1}\left(\frac{1}{r_0^m}\right) = \boxed{-\alpha\frac{k_e e^2}{r_0}\left(1 - \frac{1}{m}\right)}.$$

**P43.54** Suppose it is a harmonic-oscillator potential well. Then, $\frac{1}{2}hf + 4.48\text{ eV} = \frac{3}{2}hf + 3.96\text{ eV}$ is the depth of the well below the dissociation point. We see $hf = 0.520\text{ eV}$, so the depth of the well is

$$\frac{1}{2}hf + 4.48\text{ eV} = \frac{1}{2}(0.520\text{ eV}) + 4.48\text{ eV} = \boxed{4.74\text{ eV}}.$$

**P43.55** (a) For equilibrium, $\frac{dU}{dx} = 0$: $\frac{d}{dx}\left(Ax^{-3} - Bx^{-1}\right) = -3Ax^{-4} + Bx^{-2} = 0$

$x \to \infty$ describes one equilibrium position, but the stable equilibrium position is at $3Ax_0^{-2} = B$.

$$x_0 = \sqrt{\frac{3A}{B}} = \sqrt{\frac{3\left(0.150\text{ eV}\cdot\text{nm}^3\right)}{3.68\text{ eV}\cdot\text{nm}}} = \boxed{0.350\text{ nm}}$$

(b) The depth of the well is given by $U_0 = U|_{x=x_0} = \frac{A}{x_0^3} - \frac{B}{x_0} = \frac{AB^{3/2}}{3^{3/2}A^{3/2}} - \frac{BB^{1/2}}{3^{1/2}A^{1/2}}$

$$U_0 = U|_{x=x_0} = -\frac{2B^{3/2}}{3^{3/2}A^{1/2}} = -\frac{2(3.68\text{ eV}\cdot\text{nm})^{3/2}}{3^{3/2}\left(0.150\text{ eV}\cdot\text{nm}^3\right)^{1/2}} = \boxed{-7.02\text{ eV}}.$$

(c) $F_x = -\frac{dU}{dx} = 3Ax^{-4} - Bx^{-2}$

To find the maximum force, we determine finite $x_m$ such that $\left.\frac{dF}{dx}\right|_{x=x_m} = 0$.

Thus, $\left[-12Ax^{-5} + 2Bx^{-3}\right]_{x=x_m} = 0$ so that $x_m = \left(\frac{6A}{B}\right)^{1/2}$.

Then
$$F_{\max} = 3A\left(\frac{B}{6A}\right)^2 - B\left(\frac{B}{6A}\right) = -\frac{B^2}{12A} = -\frac{(3.68\text{ eV}\cdot\text{nm})^2}{12\left(0.150\text{ eV}\cdot\text{nm}^3\right)}$$

or
$$F_{\max} = -7.52\text{ eV/nm}\left(\frac{1.60\times10^{-19}\text{ J}}{1\text{ eV}}\right)\left(\frac{1\text{ nm}}{10^{-9}\text{ m}}\right) = -1.20\times10^{-9}\text{ N} = \boxed{-1.20\text{ nN}}.$$

**P43.56** (a) For equilibrium, $\frac{dU}{dx}=0$: $\frac{d}{dx}\left(Ax^{-3}-Bx^{-1}\right)=-3Ax^{-4}+Bx^{-2}=0$

$x\to\infty$ describes one equilibrium position, but the stable equilibrium position is at

$3Ax_0^{-2}=B$ or $\boxed{x_0=\sqrt{\frac{3A}{B}}}$.

(b) The depth of the well is given by $U_0=U\big|_{x=x_0}=\frac{A}{x_0^3}-\frac{B}{x_0}=\frac{AB^{3/2}}{3^{3/2}A^{3/2}}-\frac{BB^{1/2}}{3^{1/2}A^{1/2}}=\boxed{-2\sqrt{\frac{B^3}{27A}}}$.

(c) $F_x=-\frac{dU}{dx}=3Ax^{-4}-Bx^{-2}$

To find the maximum force, we determine finite $x_m$ such that

$\left.\frac{dF_x}{dx}\right|_{x=x_m}=\left[-12Ax^{-5}+2Bx^{-3}\right]_{x=x_0}=0$ then $F_{\max}=3A\left(\frac{B}{6A}\right)^2-B\left(\frac{B}{6A}\right)=\boxed{-\frac{B^2}{12A}}$.

**P43.57** (a) At equilibrium separation, $r=r_e$, $\left.\frac{dU}{dr}\right|_{r=r_e}=-2aB\left[e^{-a(r_e-r_0)}-1\right]e^{-a(r_e-r_0)}=0$.

We have neutral equilibrium as $r_e\to\infty$ and stable equilibrium at $e^{-a(r_e-r_0)}=1$,

or $r_e=\boxed{r_0}$.

(b) At $r=r_0$, $U=0$. As $r\to\infty$, $U\to B$. The depth of the well is $\boxed{B}$.

(c) We expand the potential in a Taylor series about the equilibrium point:

$$U(r)\approx U(r_0)+\left.\frac{dU}{dr}\right|_{r=r_0}(r-r_0)+\frac{1}{2}\left.\frac{d^2U}{dr^2}\right|_{r=r_0}(r-r_0)^2$$

$$U(r)\approx 0+0+\frac{1}{2}(-2Ba)\left[-ae^{-2(r-r_0)}-ae^{-(r-r_0)}\left(e^{-2(r-r_0)}-1\right)\right]_{r=r_0}(r-r_0)^2\approx Ba^2(r-r_0)^2$$

This is of the form $\frac{1}{2}kx^2=\frac{1}{2}k(r-r_0)^2$

for a simple harmonic oscillator with $k=2Ba^2$.

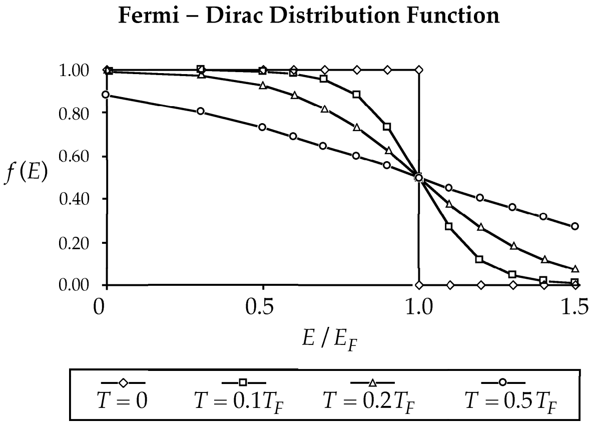
Then the molecule vibrates with frequency $f=\frac{1}{2\pi}\sqrt{\frac{k}{\mu}}=\frac{a}{2\pi}\sqrt{\frac{2B}{\mu}}=\boxed{\frac{a}{\pi}\sqrt{\frac{B}{2\mu}}}$.

(d) The zero-point energy is $\frac{1}{2}\hbar\omega=\frac{1}{2}hf=\frac{ha}{\pi}\sqrt{\frac{B}{8\mu}}$.

Therefore, to dissociate the molecule in its ground state requires energy $\boxed{B-\frac{ha}{\pi}\sqrt{\frac{B}{8\mu}}}$.

**P43.58**

| | $T=0$ | | $T=0.1T_F$ | | $T=0.2T_F$ | | $T=0.5T_F$ | |
|---|---|---|---|---|---|---|---|---|
| $\frac{E}{E_F}$ | $e^{[(E/E_F)-1](T_F/T)}$ | $f(E)$ | $e^{[(E/E_F)-1](T_F/T)}$ | $f(E)$ | $e^{[(E/E_F)-1](T_F/T)}$ | $f(E)$ | $e^{[(E/E_F)-1](T_F/T)}$ | $f(E)$ |
| 0 | $e^{-\infty}$ | 1.00 | $e^{-10.0}$ | 1.000 | $e^{-5.00}$ | 0.993 | $e^{-2.00}$ | 0.881 |
| 0.500 | $e^{-\infty}$ | 1.00 | $e^{-5.00}$ | 0.993 | $e^{-2.50}$ | 0.924 | $e^{-1.00}$ | 0.731 |
| 0.600 | $e^{-\infty}$ | 1.00 | $e^{-4.00}$ | 0.982 | $e^{-2.00}$ | 0.881 | $e^{-0.800}$ | 0.690 |
| 0.700 | $e^{-\infty}$ | 1.00 | $e^{-3.00}$ | 0.953 | $e^{-1.50}$ | 0.818 | $e^{-0.600}$ | 0.646 |
| 0.800 | $e^{-\infty}$ | 1.00 | $e^{-2.00}$ | 0.881 | $e^{-1.00}$ | 0.731 | $e^{-0.400}$ | 0.599 |
| 0.900 | $e^{-\infty}$ | 1.00 | $e^{-1.00}$ | 0.731 | $e^{-0.500}$ | 0.622 | $e^{-0.200}$ | 0.550 |
| 1.00 | $e^{0}$ | 0.500 | $e^{0}$ | 0.500 | $e^{0}$ | 0.500 | $e^{0}$ | 0.500 |
| 1.10 | $e^{+\infty}$ | 0.00 | $e^{1.00}$ | 0.269 | $e^{0.500}$ | 0.378 | $e^{0.200}$ | 0.450 |
| 1.20 | $e^{+\infty}$ | 0.00 | $e^{2.00}$ | 0.119 | $e^{1.00}$ | 0.269 | $e^{0.400}$ | 0.401 |
| 1.30 | $e^{+\infty}$ | 0.00 | $e^{3.00}$ | 0.047 4 | $e^{1.50}$ | 0.182 | $e^{0.600}$ | 0.354 |
| 1.40 | $e^{+\infty}$ | 0.00 | $e^{4.00}$ | 0.018 0 | $e^{2.00}$ | 0.119 | $e^{0.800}$ | 0.310 |
| 1.50 | $e^{+\infty}$ | 0.00 | $e^{5.00}$ | 0.006 69 | $e^{2.50}$ | 0.075 9 | $e^{1.00}$ | 0.269 |

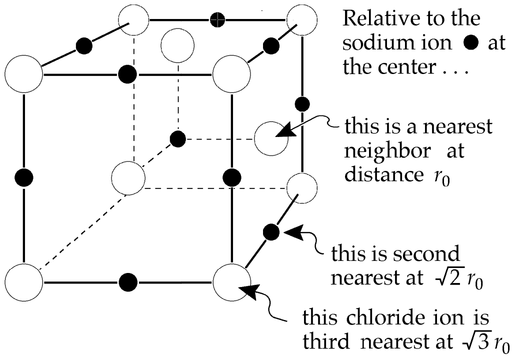


**Fermi – Dirac Distribution Function**

FIG. P43.58

**P43.59** (a) There are 6 $Cl^-$ ions at distance $r = r_0$. The contribution of these ions to the electrostatic potential energy is $\frac{-6k_e e^2}{r_0}$.

There are 12 $Na^+$ ions at distance $r = \sqrt{2}r_0$. Their contribution to the electrostatic potential energy is $\frac{+12k_e e^2}{\sqrt{2}r_0}$. Next, there are 8 $Cl^-$ ions at distance $r = \sqrt{3}r_0$. These contribute a term of $\frac{-8k_e e^2}{\sqrt{3}r_0}$ to the electrostatic potential energy.



Relative to the sodium ion ● at the center . . .
this is a nearest neighbor at distance $r_0$
this is second nearest at $\sqrt{2}\,r_0$
this chloride ion is third nearest at $\sqrt{3}\,r_0$

FIG. P43.59

To three terms, the electrostatic potential energy is:

$$U = \left(-6 + \frac{12}{\sqrt{2}} - \frac{8}{\sqrt{3}}\right)\frac{k_e e^2}{r_0} = -2.13\frac{k_e e^2}{r_0} \text{ or } \boxed{U = -\alpha\frac{k_e e^2}{r_0} \text{ with } \alpha = 2.13}.$$

*continued on next page*

(b) The fourth term consists of 6 $\text{Na}^+$ at distance $r = 2r_0$. Thus, to four terms,

$$U = (-2.13 + 3)\frac{k_e e^2}{r_0} = 0.866\frac{k_e e^2}{r_0}.$$

So we see that the electrostatic potential energy is not even attractive to 4 terms, and that the infinite series $\boxed{\text{does not converge rapidly}}$ when groups of atoms corresponding to nearest neighbors, next-nearest neighbors, etc. are added together.

## ANSWERS TO EVEN PROBLEMS

| | |
|---|---|
| **P43.2** | 4.3 eV |
| **P43.4** | (a) 1.28 eV; (b) $\sigma = 0.272$ nm, $\in = 4.65$ eV;<br>(c) 6.55 nN; (d) 576 N/m |
| **P43.6** | (a) 40.0 $\mu$eV, 9.66 GHz;<br>(b) If *r* is 10% too small, *f* is 20% too large. |
| **P43.8** | $1.46 \times 10^{-46}\ \text{kg} \cdot \text{m}^2$ |
| **P43.10** | (a) $1.81 \times 10^{-45}\ \text{kg} \cdot \text{m}^2$; (b) 1.62 cm |
| **P43.12** | (a) 11.8 pm; (b) 7.72 pm; HI is more loosely bound |
| **P43.14** | (a) 0, 364 $\mu$eV, 1.09 meV;<br>(b) 98.2 meV, 295 meV, 491 meV |
| **P43.16** | (a) 472 $\mu$m; (b) 473 $\mu$m; (c) 0.715 $\mu$m |
| **P43.18** | $2.9 \times 10^{-47}\ \text{kg} \cdot \text{m}^2$ |
| **P43.20** | only 64.1 THz |
| **P43.22** | (a) $\sim 10^{17}$; (b) $\sim 10^5\ \text{m}^3$ |
| **P43.24** | (a) 0.444 nm, 0.628 nm, 0.769 nm |
| **P43.26** | see the solution |
| **P43.28** | (a) 1.57 Mm/s;<br>(b) larger by 10 orders of magnitude |
| **P43.30** | 2% |
| **P43.32** | $3.40 \times 10^{17}$ electrons |
| **P43.34** | see the solution |
| **P43.36** | (a) 275 THz; (b) 1.09 $\mu$m |
| **P43.38** | 1.91 eV |
| **P43.40** | 226 nm |
| **P43.42** | (a) 59.5 mV; (b) –59.5 mV |
| **P43.44** | 4.19 mA |
| **P43.46** | (a) see the solution; (b) 10.7 kA |
| **P43.48** | see the solution |
| **P43.50** | 37 |
| **P43.52** | 5.23 J/g |
| **P43.54** | 4.74 eV |
| **P43.56** | (a) $x_0 = \sqrt{\frac{3A}{B}}$; (b) $-2\sqrt{\frac{B^3}{27A}}$; (c) $-\frac{B^2}{12A}$ |
| **P43.58** | see the solution |

# 44

# Nuclear Structure

### CHAPTER OUTLINE

44.1 Some Properties of Nuclei
44.2 Nuclear Binding Energy
44.3 Nuclear Models
44.4 Radioactivity
44.5 The Decay Processes
44.6 Natural Radioactivity
44.7 Nuclear Reactions
44.8 Nuclear Magnetic Resonance and Magnetic Resonance Imagining

## ANSWERS TO QUESTIONS

**Q44.1** Because of electrostatic repulsion between the positively-charged nucleus and the $+2e$ alpha particle. To drive the $\alpha$-particle into the nucleus would require extremely high kinetic energy.

**Q44.2** There are 86 protons and 136 neutrons in the nucleus $^{222}_{86}\mathrm{Rn}$. For the atom to be neutral, there must be 86 electrons orbiting the nucleus—the same as the number of protons.

**Q44.3** All of these isotopes have the same number of protons in the nucleus. Neutral atoms have the same number of electrons. Isotopes only differ in the number of neutrons in the nucleus.

**Q44.4** Nuclei with more nucleons than bismuth-209 are unstable because the electrical repulsion forces among all of the protons is stronger than the nuclear attractive force between nucleons.

**Q44.5** The nuclear force favors the formation of neutron-proton pairs, so a stable nucleus cannot be too far away from having equal numbers of protons and neutrons. This effect sets the upper boundary of the zone of stability on the neutron-proton diagram. All of the protons repel one another electrically, so a stable nucleus cannot have too many protons. This effect sets the lower boundary of the zone of stability.

**Q44.6** Nucleus $Y$ will be more unstable. The nucleus with the higher binding energy requires more energy to be disassembled into its constituent parts.

**Q44.7** Extra neutrons are required to overcome the increasing electrostatic repulsion of the protons. The neutrons participate in the net attractive effect of the nuclear force, but feel no Coulomb repulsion.

**Q44.8** In the liquid-drop model the nucleus is modeled as a drop of liquid. The nucleus is treated as a whole to determine its binding energy and behavior. The shell model differs completely from the liquid-drop model, as it utilizes quantum states of the individual nucleons to describe the structure and behavior of the nucleus. Like the electrons that orbit the nucleus, each nucleon has a spin state to which the Pauli exclusion principle applies. Unlike the electrons, for protons and neutrons the spin and orbital motions are strongly coupled.

**Q44.9** The liquid drop model gives a simpler account of a nuclear fission reaction, including the energy released and the probable fission product nuclei. The shell model predicts magnetic moments by necessarily describing the spin and orbital angular momentum states of the nucleons. Again, the shell model wins when it comes to predicting the spectrum of an excited nucleus, as the quantum model allows only quantized energy states, and thus only specific transitions.

**Q44.10** $^{4}\text{He}$, $^{16}\text{O}$, $^{40}\text{Ca}$, and $^{208}\text{Pb}$.

**Q44.11** If one half the number of radioactive nuclei decay in one year, then one half the remaining number will decay in the second year. Three quarters of the original nuclei will be gone, and one quarter will remain.

**Q44.12** The statement is false. Both patterns show monotonic decrease over time, but with very different shapes. For radioactive decay, maximum activity occurs at time zero. Cohorts of people now living will be dying most rapidly perhaps forty years from now. Everyone now living will be dead within less than two centuries, while the mathematical model of radioactive decay tails off exponentially forever. A radioactive nucleus never gets old. It has constant probability of decay however long it has existed.

**Q44.13** Since the samples are of the same radioactive isotope, their half-lives are the same. When prepared, sample A has twice the activity (number of radioactive decays per second) of sample B. After 5 half-lives, the activity of sample A is decreased by a factor of $2^5$, and after 5 half-lives the activity of sample B is decreased by a factor of $2^5$. So after 5 half-lives, the ratio of activities is still 2:1.

**Q44.14** After one half-life, one half the radioactive atoms have decayed. After the second half-life, one half of the remaining atoms have decayed. Therefore $\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$ of the original radioactive atoms have decayed after two half-lives.

**Q44.15** The motion of a molecule through space does not affect anything inside the nucleus of an atom of the molecule. The half-life of a nucleus is based on nuclear stability which, as discussed in Questions 44.4 and Q44.5, is predominantly determined by Coulomb repulsion and nuclear forces, not molecular motion.

**Q44.16** Long-lived progenitors at the top of each of the three natural radioactive series are the sources of our radium. As an example, thorium-232 with a half-life of 14 Gyr produces radium-228 and radium-224 at stages in its series of decays, shown in Figure 44.17.

**Q44.17** A free neutron decays into a proton plus an electron and an antineutrino. This implies that a proton is more stable than a neutron, and in particular the proton has lower mass. Therefore a proton cannot decay into a neutron plus a positron and a neutrino. This reaction satisfies every conservation law except for energy conservation.

**Q44.18** A neutrino has spin $\frac{1}{2}$ while a photon has spin 1. A neutrino interacts by the weak interaction while a photon is a quantum of the electromagnetic interaction.

**Q44.19** Let us consider the carbon-14 decay used in carbon dating. ${}^{14}_{6}\text{C}\rightarrow{}^{14}_{7}\text{N}+\text{e}^{-}+\bar{\nu}$. The carbon-14 atom has 6 protons in the nucleus. The nitrogen-14 atom has 7 protons in the nucleus, but the additional + charge from the extra proton is canceled by the – charge of the ejected electron. Since charge is conserved in this (and every) reaction, the antineutrino must have zero charge.

Similarly, when nitrogen-12 decays into carbon-12, the nucleus of the carbon atom has one fewer protons, but the change in charge of the nucleus is balanced by the positive charge of the ejected positron. Again according to charge conservation, the neutrino must have no charge.

**Q44.20** An electron has spin quantum number $\frac{1}{2}$. When a nucleus undergoes beta decay, an electron and antineutrino are ejected. With all nucleons paired, in their ground states the carbon-14, nitrogen-14, nitrogen-12, and carbon-12 nuclei have zero net angular momentum. Angular momentum is conserved in any process in an isolated system and in particular in the beta-decays of carbon-14 and nitrogen-12. Conclusion: the neutrino must have spin quantum number $\frac{1}{2}$, so that its $z$-component of angular momentum can be just $\frac{\hbar}{2}$ or $-\frac{\hbar}{2}$. A proton and a neutron have spin quantum number 1. For conservation of angular momentum in the beta-decay of a free neutron, an antineutrino must have spin quantum number $\frac{1}{2}$.

**Q44.21** The alpha particle and the daughter nucleus carry equal amounts of momentum in opposite directions. Since kinetic energy can be written as $\frac{p^2}{2m}$, the small-mass alpha particle has much more of the decay energy than the recoiling nucleus.

**Q44.22** Bullet and rifle carry equal amounts of momentum $p$. With a much smaller mass $m$, the bullet has much more kinetic energy $K = \frac{p^2}{2m}$. The daughter nucleus and alpha particle have equal momenta and the massive daughter nucleus, like the rifle, has a very small share of the energy released.

**Q44.23** Yes. The daughter nucleus can be left in its ground state or sometimes in one of a set of excited states. If the energy carried by the alpha particle is mysteriously low, the daughter nucleus can quickly emit the missing energy in a gamma ray.

**Q44.24** In a heavy nucleus each nucleon is strongly bound to its momentary neighbors. Even if the nucleus could step down in energy by shedding an individual proton or neutron, one individual nucleon is never free to escape. Instead, the nucleus can decay when two protons and two neutrons, strongly bound to one another but not to their neighbors, happen momentarily to have a lot of kinetic energy, to lie at the surface of the nucleus, to be headed outward, and to tunnel successfully through the potential energy barrier they encounter.

**Q44.25** From $\sum F = ma$, or $qvB = \frac{mv^2}{r}$, or $qBr = mv$, a charged particle fired into a magnetic field is deflected into a path with radius proportional to its momentum. If they have equal kinetic energies $K$, the much greater mass $m$ of the alpha particle gives it more momentum $mv = \sqrt{2mK}$ than an electron. Thus the electron undergoes greater deflection. This conclusion remains true if one or both particles are moving relativistically.

**Q44.26** The alpha particle stops in the wood, while many beta particles can make it through to deposit some or all of their energy in the film emulsion.

**Q44.27** The reaction energy is the amount of energy released as a result of a nuclear reaction. Equation 44.28 in the text implies that the reaction energy is (initial mass – final mass)$c^2$. The $Q$-value is taken as positive for an exothermic reaction.

**Q44.28** Carbon-14 is produced when carbon-12 is bombarded by cosmic rays. Both carbon-12 and carbon-14 combine with oxygen to form the atmospheric $CO_2$ that plants absorb in respiration. When the plant dies, the carbon-14 is no longer replenished and decays at a known rate. Since carbon-14 is a beta-emitter, one only needs to compare the activity of a living plant to the activity of the sample to determine its age, since the activity of a radioactive sample exponentially decreases in time.

**Q44.29** The samples would have started with more carbon-14 than we first thought. We would increase our estimates of their ages.

**Q44.30** There are two factors that determine the uncertainty on dating an old sample. The first is the fact that the activity level decreases exponentially in time. After a long enough period of time, the activity will approach background radiation levels, making precise dating difficult. Secondly, the ratio of carbon-12 to carbon-14 in the atomsphere can vary over long periods of time, and this effect contributes additional uncertainty.

**Q44.31** An $\alpha$-particle is a helium nucleus: ${}_{2}^{4}\mathrm{He}$

A $\beta$-particle is an electron or a positron: either $\mathrm{e}^{-}$ or $\mathrm{e}^{+}$.

A $\gamma$-ray is a high-energy photon emitted when a nucleus makes a downward transition between two states.

**Q44.32** $I_z$ may have 6 values for $I=\frac{5}{2}$, namely $\frac{5}{2}, \frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}$, and $-\frac{5}{2}$. Seven $I_z$ values are possible for $I=3$.

**Q44.33** The frequency increases linearly with the magnetic field strength.

**Q44.34** The decay of a radioactive nucleus at one particular moment instead of at another instant cannot be predicted and has no cause. Natural events are not just like a perfect clockworks. In history, the idea of a determinate mechanical Universe arose temporarily from an unwarranted wild extrapolation of Isaac Newton's account of planetary motion. Before Newton's time [really you can blame Pierre Simon de Laplace] and again now, no one thought of natural events as just like a perfect row of falling dominos. We can and do use the word "cause" more loosely to describe antecedent enabling events. One gear turning another is intelligible. So is the process of a hot dog getting toasted over a campfire, even though random molecular motion is at the essence of that process. In summary, we say that the future is not determinate. All natural events have causes in the ordinary sense of the word, but not necessarily in the contrived sense of a cause operating infallibly and predictably in a way that can be calculated. We have better reason now than ever before to think of the Universe as intelligible. First describing natural events, and second determining their causes form the basis of science, including physics but also scientific medicine and scientific bread-baking. The evidence alone of the past hundred years of discoveries in physics, finding causes of natural events from the photoelectric effect to x-rays and jets emitted by black holes, suggests that human intelligence is a good tool for figuring out how things go. Even without organized science, humans have always been searching for the causes of natural events, with explanations ranging from "the will of the gods" to Schrödinger's equation. We depend on the principle that things are intelligible as we make significant strides towards understanding the Universe. To hope that our search is not futile is the best part of human nature.

## SOLUTIONS TO PROBLEMS

### Section 44.1 **Some Properties of Nuclei**

**P44.1** An iron nucleus (in hemoglobin) has a few more neutrons than protons, but in a typical water molecule there are eight neutrons and ten protons.

So protons and neutrons are nearly equally numerous in your body, each contributing mass (say) 35 kg:

$$35\text{ kg}\left(\frac{1\text{ nucleon}}{1.67\times10^{-27}\text{ kg}}\right)\boxed{\sim10^{28}\text{ protons}}$$

and $\boxed{\sim10^{28}\text{ neutrons}}$.

The electron number is precisely equal to the proton number, $\boxed{\sim10^{28}\text{ electrons}}$.

**P44.2** $\frac{1}{2}mv^2=q\Delta V$ and $\frac{mv^2}{r}=qvB$

$2m\Delta V=qr^2B^2$: $r=\sqrt{\frac{2m\Delta V}{qB^2}}=\sqrt{\frac{2(1\,000\text{ V})}{\left(1.60\times10^{-19}\text{ C}\right)(0.200\text{ T})^2}}\sqrt{m}$

$$r=\left(5.59\times10^{11}\text{ m}/\sqrt{\text{kg}}\right)\sqrt{m}$$

(a) For $^{12}\text{C}$, $m=12$ u and $r=\left(5.59\times10^{11}\text{ m}/\sqrt{\text{kg}}\right)\sqrt{12\left(1.66\times10^{-27}\text{ kg}\right)}$

$$r=0.078\,9\text{ m}=\boxed{7.89\text{ cm}}.$$

For $^{13}\text{C}$: $r=\left(5.59\times10^{11}\text{ m}/\sqrt{\text{kg}}\right)\sqrt{13\left(1.66\times10^{-27}\text{ kg}\right)}$

$$r=0.082\,1\text{ m}=\boxed{8.21\text{ cm}}.$$

(b) With $r_1=\sqrt{\frac{2m_1\Delta V}{qB^2}}$ and $r_2=\sqrt{\frac{2m_2\Delta V}{qB^2}}$

the ratio gives $\boxed{\frac{r_1}{r_2}=\sqrt{\frac{m_1}{m_2}}}$

$$\frac{r_1}{r_2}=\frac{7.89\text{ cm}}{8.21\text{ cm}}=0.961$$

and $\sqrt{\frac{m_1}{m_2}}=\sqrt{\frac{12\text{ u}}{13\text{ u}}}=0.961$

so they do agree.

**P44.3** (a) $$F = k_e\frac{Q_1Q_2}{r^2} = \left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{(2)(6)\left(1.60\times10^{-19}\ \text{C}\right)^2}{\left(1.00\times10^{-14}\ \text{m}\right)^2} = \boxed{27.6\ \text{N}}$$

(b) $$a = \frac{F}{m} = \frac{27.6\ \text{N}}{6.64\times10^{-27}\ \text{kg}} = \boxed{4.17\times10^{27}\ \text{m/s}^2}\ \text{away from the nucleus.}$$

(c) $$U = k_e\frac{Q_1Q_2}{r} = \left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{(2)(6)\left(1.60\times10^{-19}\ \text{C}\right)^2}{\left(1.00\times10^{-14}\ \text{m}\right)} = 2.76\times10^{-13}\ \text{J} = \boxed{1.73\ \text{MeV}}$$

**P44.4** $E_\alpha = 7.70\ \text{MeV}$

(a) $$d_{\min} = \frac{4k_eZe^2}{mv^2} = \frac{2k_eZe^2}{E_\alpha} = \frac{2\left(8.99\times10^{9}\right)(79)\left(1.60\times10^{-19}\right)^2}{7.70\left(1.60\times10^{-13}\right)} = 29.5\times10^{-15}\ \text{m} = \boxed{29.5\ \text{fm}}$$

(b) The de Broglie wavelength of the $\alpha$ is

$$\lambda = \frac{h}{m_\alpha v_\alpha} = \frac{h}{\sqrt{2m_\alpha E_\alpha}} = \frac{6.626\times10^{-34}}{\sqrt{2\left(6.64\times10^{-27}\right)7.70\left(1.60\times10^{-13}\right)}} = 5.18\times10^{-15}\ \text{m} = \boxed{5.18\ \text{fm}}.$$

(c) Since $\boxed{\lambda \text{ is much less than the distance of closest approach}}$, the $\alpha$ may be considered a particle.

**P44.5** (a) The initial kinetic energy of the alpha particle must equal the electrostatic potential energy at the distance of closest approach.

$$K_i = U_f = \frac{k_eqQ}{r_{\min}}$$

$$r_{\min} = \frac{k_eqQ}{K_i} = \frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(2)(79)\left(1.60\times10^{-19}\ \text{C}\right)^2}{(0.500\ \text{MeV})\left(1.60\times10^{-13}\ \text{J/MeV}\right)} = \boxed{4.55\times10^{-13}\ \text{m}}$$

(b) Since $K_i = \frac{1}{2}m_\alpha v_i^2 = \frac{k_eqQ}{r_{\min}}$,

$$v_i = \sqrt{\frac{2k_eqQ}{m_\alpha r_{\min}}} = \sqrt{\frac{2\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(2)(79)\left(1.60\times10^{-19}\ \text{C}\right)^2}{(4.00\ \text{u})\left(1.66\times10^{-27}\ \text{kg/u}\right)\left(3.00\times10^{-13}\ \text{m}\right)}} = \boxed{6.04\times10^{6}\ \text{m/s}}.$$

**P44.6** It must start with kinetic energy equal to $K_i = U_f = \dfrac{k_e qQ}{r_f}$. Here $r_f$ stands for the sum of the radii of the ${}^{4}_{2}\text{He}$ and ${}^{197}_{79}\text{Au}$ nuclei, computed as

$$r_f = r_0 A_1^{1/3} + r_0 A_2^{1/3} = \left(1.20\times10^{-15}\ \text{m}\right)\left(4^{1/3} + 197^{1/3}\right) = 8.89\times10^{-15}\ \text{m}.$$

Thus, $K_i = U_f = \dfrac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(2)(79)\left(1.60\times10^{-19}\ \text{C}\right)^2}{8.89\times10^{-15}\ \text{m}} = 4.09\times10^{-12}\ \text{J} = \boxed{25.6\ \text{MeV}}$.

**P44.7** (a) $r = r_0 A^{1/3} = \left(1.20\times10^{-15}\ \text{m}\right)(4)^{1/3} = \boxed{1.90\times10^{-15}\ \text{m}}$

(b) $r = r_0 A^{1/3} = \left(1.20\times10^{-15}\ \text{m}\right)(238)^{1/3} = \boxed{7.44\times10^{-15}\ \text{m}}$

**P44.8** From $r = r_0 A^{1/3}$, the radius of uranium is $r_{\text{U}} = r_0(238)^{1/3}$.

Thus, if $r = \dfrac{1}{2} r_{\text{U}}$ then $r_0 A^{1/3} = \dfrac{1}{2} r_0 (238)^{1/3}$

from which $\boxed{A = 30}$.

**P44.9** The number of nucleons in a star of two solar masses is

$$A = \frac{2\left(1.99\times10^{30}\ \text{kg}\right)}{1.67\times10^{-27}\ \text{kg/nucleon}} = 2.38\times10^{57}\ \text{nucleons}.$$

Therefore $r = r_0 A^{1/3} = \left(1.20\times10^{-15}\ \text{m}\right)\left(2.38\times10^{57}\right)^{1/3} = \boxed{16.0\ \text{km}}$.

**P44.10** $V = \dfrac{4}{3}\pi r^4 = \dfrac{4}{3}\pi(0.021\,5\ \text{m})^3 = 4.16\times10^{-5}\ \text{m}^3$

We take the nuclear density from Example 44.2

$$m = \rho V = \left(2.3\times10^{17}\ \text{kg/m}^3\right)\left(4.16\times10^{-5}\ \text{m}^3\right) = 9.57\times10^{12}\ \text{kg}$$

and $F = G\dfrac{m_1 m_2}{r^2} = \left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\dfrac{\left(9.57\times10^{12}\ \text{kg}\right)^2}{(1.00\ \text{m})^2}$

$F = \boxed{6.11\times10^{15}\ \text{N}}$ toward the other ball.

**P44.11** The stable nuclei that correspond to magic numbers are:

$Z$ magic: ${}_{2}\text{He}$ ${}_{8}\text{O}$ ${}_{20}\text{Ca}$ ${}_{28}\text{Ni}$ ${}_{50}\text{Sn}$ ${}_{82}\text{Pb}$

An artificially produced nucleus with $Z = 126$ might be more stable than other nuclei with lower values for $Z$, since this number of protons is magic.

$N$ magic: ${}^{3}_{1}\text{T}$, ${}^{4}_{2}\text{He}$, ${}^{15}_{7}\text{N}$, ${}^{16}_{8}\text{O}$, ${}^{37}_{17}\text{Cl}$, ${}^{39}_{19}\text{K}$, ${}^{40}_{20}\text{Ca}$, ${}^{51}_{23}\text{V}$, ${}^{52}_{24}\text{Cr}$, ${}^{88}_{38}\text{Sr}$,
${}^{89}_{39}\text{Y}$, ${}^{90}_{40}\text{Zr}$, ${}^{136}_{54}\text{Xe}$, ${}^{138}_{56}\text{Ba}$, ${}^{139}_{57}\text{La}$, ${}^{140}_{58}\text{Ce}$, ${}^{141}_{59}\text{Pr}$, ${}^{142}_{60}\text{Nd}$, ${}^{208}_{82}\text{Pb}$, ${}^{209}_{83}\text{Bi}$,
${}^{210}_{84}\text{Po}$

***P44.12** (a) For even $Z$, even $N$, even $A$, the list begins ${}^{4}_{2}\mathrm{He}$, ${}^{12}_{6}\mathrm{C}$, and ends ${}^{194}_{78}\mathrm{Pt}$, ${}^{196}_{78}\mathrm{Pt}$, ${}^{202}_{80}\mathrm{Hg}$, ${}^{208}_{82}\mathrm{Pb}$, containing $\boxed{48}$ isotopes.

(b) The whole even $Z$, odd $N$, odd $A$ list is ${}^{9}_{4}\mathrm{Be}$, ${}^{129}_{54}\mathrm{Xe}$, ${}^{195}_{78}\mathrm{Pt}$, with $\boxed{3}$ entries.

(c) The odd $Z$, even $N$, odd $A$ list has $\boxed{46}$ entries, represented as ${}^{1}_{1}\mathrm{H}$, ${}^{7}_{3}\mathrm{Li}$, …, ${}^{203}_{81}\mathrm{Tl}$, ${}^{205}_{81}\mathrm{Tl}$, ${}^{209}_{83}\mathrm{Bi}$.

(d) The odd $Z$, odd $N$, even $A$ list has $\boxed{1}$ entry, ${}^{14}_{7}\mathrm{N}$. Do not be misled into thinking that nature favors nuclei with even numbers of neutrons. The form of the question here forces a count with essentially equal numbers of odd-$Z$ and even-$Z$ nuclei. If we counted all of the stable nuclei we would find many even-even isotopes but also lots of even-$Z$ odd-$N$ nuclei and odd-$Z$ even-$N$ nuclei; we would find roughly equal numbers of these two kinds of odd-$A$ nuclei. A nucleus with one odd neutron is no more likely to be unstable than a nucleus with one odd proton.

With the arbitrary 25% abundance standard, we can note that most elements have a single predominant isotope. Ni, Cu, Zn, Ga, Ge, Pd, Ag, Os, Ir, and Pt form a compact patch on the periodic table and have two common isotopes, as do some others. Tungsten is the only element with three isotopes over 25% abundance.

***P44.13** (a) $Z_1 = 8Z_2 \qquad N_1 = 5N_2$

$N_1 + Z_1 = 6(N_1 + Z_2)$ and $N_1 = Z_1 + 4$

Thus: $$N_1 + Z_1 = 6\left(\frac{1}{5}N_1 + \frac{1}{8}Z_1\right)$$

$$\therefore N_1 = \frac{5}{4}Z_1$$

$$\therefore Z_1 + 4 = \frac{5}{4}Z_1$$

$$\therefore Z_1 = 16$$

$$N_1 = Z_1 + 4 = 20,\ A_1 = Z_1 + N_1 = 36$$

$$N_2 = \frac{1}{5}N_1 = 4,\ Z_2 = \frac{1}{8}Z_1 = 2,\ A_2 = Z_2 + N_2 = 6$$

Hence: ${}^{36}_{16}\mathrm{S}$ and ${}^{6}_{2}\mathrm{He}$.

(b) ${}^{6}_{2}\mathrm{He}$ is unstable. Two neutrons must be removed to make it stable $\left({}^{4}_{2}\mathrm{He}\right)$.

## Section 44.2 Nuclear Binding Energy

**P44.14** Using atomic masses as given in Table A.3,

(a) For ${}^{2}_{1}\mathrm{H}$:
$$\frac{-2.014\,102+1(1.008\,665)+1(1.007\,825)}{2}$$
$$\frac{E_b}{A}=(0.001\,194\text{ u})\left(\frac{931.5\text{ MeV}}{\text{u}}\right)=\boxed{1.11\text{ MeV/nucleon}}.$$

(b) For ${}^{4}_{2}\mathrm{He}$:
$$\frac{2(1.008\,665)+2(1.007\,825)-4.002\,603}{4}$$
$$\frac{E_b}{A}=0.007\,59\text{ u}c^2=\boxed{7.07\text{ MeV/nucleon}}.$$

(c) For ${}^{56}_{26}\mathrm{Fe}$:
$$30(1.008\,665)+26(1.007\,825)-55.934\,942=0.528\text{ u}$$
$$\frac{E_b}{A}=\frac{0.528}{56}=0.009\,44\text{ u}c^2=\boxed{8.79\text{ MeV/nucleon}}.$$

(d) For ${}^{238}_{92}\mathrm{U}$:
$$146(1.008\,665)+92(1.007\,825)-238.050\,783=1.934\,2\text{ u}$$
$$\frac{E_b}{A}=\frac{1.934\,2}{238}=0.008\,13\text{ u}c^2=\boxed{7.57\text{ MeV/nucleon}}.$$

**P44.15**
$$\Delta M=Zm_{\mathrm{H}}+Nm_{\mathrm{n}}-M \qquad \frac{\mathrm{BE}}{A}=\frac{\Delta M(931.5)}{A}$$

| Nuclei | $Z$ | $N$ | $M$ in u | $\Delta M$ in u | $\frac{\mathrm{BE}}{A}$ in MeV |
|---|---|---|---|---|---|
| ${}^{55}\mathrm{Mn}$ | 25 | 30 | 54.938 050 | 0.517 5 | 8.765 |
| ${}^{56}\mathrm{Fe}$ | 26 | 30 | 55.934 942 | 0.528 46 | 8.790 |
| ${}^{59}\mathrm{Co}$ | 27 | 32 | 58.933 200 | 0.555 35 | 8.768 |

$\therefore {}^{56}\mathrm{Fe}$ has a greater $\frac{\mathrm{BE}}{A}$ than its neighbors. This tells us finer detail than is shown in Figure 44.5.

**P44.16** Use Equation 44.2.

The ${}^{23}_{11}\mathrm{Na}$, $\frac{E_b}{A}=8.11\text{ MeV/nucleon}$

and for ${}^{23}_{12}\mathrm{Mg}$, $\frac{E_b}{A}=7.90\text{ MeV/nucleon}$.

The binding energy per nucleon is greater for ${}^{23}_{11}\mathrm{Na}$ by $\boxed{0.210\text{ MeV}}$. (There is less proton repulsion in $\mathrm{Na}^{23}$.)

**P44.17** (a) The neutron-to-proton ratio $\frac{A-Z}{Z}$ is greatest for $\boxed{{}^{139}_{55}\mathrm{Cs}}$ and is equal to 1.53.

(b) $\boxed{{}^{139}\mathrm{La}}$ has the largest binding energy per nucleon of 8.378 MeV.

(c) ${}^{139}\mathrm{Cs}$ with a mass of 138.913 u. We locate the nuclei carefully on Figure 44.4, the neutron–proton plot of stable nuclei. $\boxed{\text{Cesium}}$ appears to be farther from the center of the zone of stability. Its instability means extra energy and extra mass.

**P44.18** (a) The radius of the ${}^{40}\text{Ca}$ nucleus is: $R = r_0 A^{1/3} = \left(1.20\times10^{-15}\ \text{m}\right)(40)^{1/3} = 4.10\times10^{-15}\ \text{m}$.

The energy required to overcome electrostatic repulsion is

$$U = \frac{3k_eQ^2}{5R} = \frac{3\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left[20\left(1.60\times10^{-19}\ \text{C}\right)\right]^2}{5\left(4.10\times10^{-15}\ \text{m}\right)} = 1.35\times10^{-11}\ \text{J} = \boxed{84.1\ \text{MeV}}.$$

(b) The binding energy of ${}^{40}_{20}\text{Ca}$ is

$$E_b = \left[20(1.007\,825\ \text{u}) + 20(1.008\,665\ \text{u}) - 39.962\,591\ \text{u}\right](931.5\ \text{MeV/u}) = \boxed{342\ \text{MeV}}.$$

(c) The nuclear force is so strong that the binding energy greatly exceeds the minimum energy needed to overcome electrostatic repulsion.

**P44.19** The binding energy of a nucleus is $E_b(\text{MeV}) = \left[ZM(\text{H}) + Nm_n - M\left({}^{A}_{Z}\text{X}\right)\right](931.494\ \text{MeV/u})$.

For ${}^{15}_{8}\text{O}$: $E_b = \left[8(1.007\,825\ \text{u}) + 7(1.008\,665\ \text{u}) - 15.003\,065\ \text{u}\right](931.494\ \text{MeV/u}) = 111.96\ \text{MeV}$.

For ${}^{15}_{7}\text{N}$: $E_b = \left[7(1.007\,825\ \text{u}) + 8(1.008\,665\ \text{u}) - 15.000\,109\ \text{u}\right](931.494\ \text{MeV/u}) = 115.49\ \text{MeV}$.

Therefore, $\boxed{\text{the binding energy of } {}^{15}_{7}\text{N is larger by 3.54 MeV}}$.

**P44.20** Removal of a neutron from ${}^{43}_{20}\text{Ca}$ would result in the residual nucleus, ${}^{42}_{20}\text{Ca}$. If the required separation energy is $S_n$, the overall process can be described by

$$\text{mass}\left({}^{43}_{20}\text{Ca}\right) + S_n = \text{mass}\left({}^{42}_{20}\text{Ca}\right) + \text{mass}(\text{n})$$

$$S_n = (41.958\,618 + 1.008\,665 - 42.958\,767)\ \text{u} = (0.008\,516\ \text{u})(931.5\ \text{MeV/u}) = \boxed{7.93\ \text{MeV}}.$$

## Section 44.3 **Nuclear Models**

**P44.21** $\Delta E_b = E_{bf} - E_{bi}$

For $A = 200$, $\dfrac{E_b}{A} = 7.4\ \text{MeV}$

so $E_{bi} = 200(7.4\ \text{MeV}) = 1\,480\ \text{MeV}$.

For $A \approx 100$, $\dfrac{E_b}{A} = 8.4\ \text{MeV}$

so $E_{bf} = 2(100)(8.4\ \text{MeV}) = 1\,680\ \text{MeV}$.

$\Delta E_b = E_{bf} - E_{bi}$: $E_b = 1\,680\ \text{MeV} - 1\,480\ \text{MeV} = \boxed{200\ \text{MeV}}$

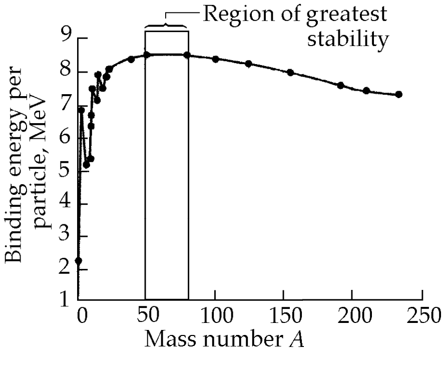


FIG. P44.21

**P44.22** (a) The first term overstates the importance of volume and the second term *subtracts* this overstatement.

(b) For spherical volume $\frac{(4/3)\pi R^3}{4\pi R^2} = \boxed{\frac{R}{3}}$. For cubical volume $\frac{R^3}{6R^2} = \boxed{\frac{R}{6}}$.

The maximum binding energy or lowest state of energy is achieved by building "nearly" spherical nuclei.

**P44.23** (a) "Volume" term: $E_1 = C_1 A = (15.7\text{ MeV})(56) = 879\text{ MeV}$.

"Surface" term: $E_2 = -C_2 A^{2/3} = -(17.8\text{ MeV})(56)^{2/3} = -260\text{ MeV}$.

"Coulomb" term: $E_3 = -C_3 \frac{Z(Z-1)}{A^{1/3}} = -(0.71\text{ MeV})\frac{(26)(25)}{(56)^{1/3}} = -121\text{ MeV}$.

"Asymmetry" term: $E_4 = C_4 \frac{(A-2Z)^2}{A} = -(23.6\text{ MeV})\frac{(56-52)^2}{56} = -6.74\text{ MeV}$.

$\boxed{E_b = 491\text{ MeV}}$

(b) $\frac{E_1}{E_b} = 179\%$; $\frac{E_2}{E_b} = -53.0\%$, $\frac{E_3}{E_b} = -24.6\%$; $\frac{E_4}{E_b} = -1.37\%$

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## Section 44.4 Radioactivity

**P44.24** $R = R_0 e^{-\lambda t} = (6.40\text{ mCi})e^{-(\ln 2/8.04\text{ d})(40.2\text{ d})} = (6.40\text{ mCi})\left(e^{-\ln 2}\right)^5 = (6.40\text{ mCi})\left(\frac{1}{2^5}\right) = \boxed{0.200\text{ mCi}}$

**P44.25** $\frac{dN}{dt} = -\lambda N$

so $\lambda = \frac{1}{N}\left(-\frac{dN}{dt}\right) = \left(1.00 \times 10^{-15}\right)\left(6.00 \times 10^{11}\right) = 6.00 \times 10^{-4}\text{ s}^{-1}$

$T_{1/2} = \frac{\ln 2}{\lambda} = \boxed{1.16 \times 10^3\text{ s}}\ (= 19.3\text{ min})$

**P44.26** $R = \lambda N = \left(\frac{\ln 2}{5.27\text{ yr}}\right)\left(\frac{1.00\text{ g}}{59.93\text{ g/mol}}\right)\left(6.02 \times 10^{23}\right)$

$R = \left(1.32 \times 10^{21}\text{ decays/yr}\right)\left(\frac{1\text{ yr}}{3.16 \times 10^7\text{ s}}\right) = \boxed{4.18 \times 10^{13}\text{ Bq}}$

**P44.27** (a) From $R = R_0 e^{-\lambda t}$,

$\lambda = \frac{1}{t}\ln\left(\frac{R_0}{R}\right) = \left(\frac{1}{4.00\text{ h}}\right)\ln\left(\frac{10.0}{8.00}\right) = 5.58 \times 10^{-2}\text{ h}^{-1} = \boxed{1.55 \times 10^{-5}\text{ s}^{-1}}$

$T_{1/2} = \frac{\ln 2}{\lambda} = \boxed{12.4\text{ h}}$

(b) $N_0 = \frac{R_0}{\lambda} = \frac{10.0 \times 10^{-3}\text{ Ci}}{1.55 \times 10^{-5}/\text{s}}\left(\frac{3.70 \times 10^{10}/\text{s}}{1\text{ Ci}}\right) = \boxed{2.39 \times 10^{13}\text{ atoms}}$

(c) $R = R_0 e^{-\lambda t} = (10.0\text{ mCi})\exp\left(-5.58 \times 10^{-2} \times 30.0\right) = \boxed{1.88\text{ mCi}}$

**P44.28** $R = R_0 e^{-\lambda t}$ where $\lambda = \dfrac{\ln 2}{26.0\text{ h}} = 0.026\,6\text{ h}^{-1}$

$\dfrac{R}{R_0} = 0.100 = e^{-\lambda t}$ so $\ln(0.100) = -\lambda t$

$2.30 = \left(\dfrac{0.026\,6}{\text{h}}\right)t$ $t = \boxed{86.4\text{ h}}$

**P44.29** The number of nuclei which decay during the interval will be $N_1 - N_2 = N_0\left(e^{-\lambda t_1} - e^{-\lambda t_2}\right)$.

First we find $l$: $\lambda = \dfrac{\ln 2}{T_{1/2}} = \dfrac{0.693}{64.8\text{ h}} = 0.010\,7\text{ h}^{-1} = 2.97 \times 10^{-6}\text{ s}^{-1}$

and $N_0 = \dfrac{R_0}{\lambda} = \dfrac{(40.0\ \mu\text{Ci})\left(3.70 \times 10^4\text{ s}^{-1}/\mu\text{Ci}\right)}{2.97 \times 10^{-6}\text{ s}^{-1}} = 4.98 \times 10^{11}\text{ nuclei}$.

Substituting these values, $N_1 - N_2 = \left(4.98 \times 10^{11}\right)\left[e^{-\left(0.010\,7\text{ h}^{-1}\right)(10.0\text{ h})} - e^{-\left(0.010\,7\text{ h}^{-1}\right)(12.0\text{ h})}\right]$.

Hence, the number of nuclei decaying during the interval is $N_1 - N_2 = \boxed{9.47 \times 10^9\text{ nuclei}}$.

**P44.30** The number of nuclei which decay during the interval will be $N_1 - N_2 = N_0\left(e^{-\lambda t_1} - e^{-\lambda t_2}\right)$.

First we find $\lambda$: $\lambda = \dfrac{\ln 2}{T_{1/2}}$

so $e^{-\lambda t} = e^{\ln 2\left(-t/T_{1/2}\right)} = 2^{-t/T_{1/2}}$

and $N_0 = \dfrac{R_0}{\lambda} = \dfrac{R_0 T_{1/2}}{\ln 2}$.

Substituting in these values $N_1 - N_2 = \dfrac{R_0 T_{1/2}}{\ln 2}\left(e^{-\lambda t_1} - e^{-\lambda t_2}\right) = \boxed{\dfrac{R_0 T_{1/2}}{\ln 2}\left(2^{-t_1/T_{1/2}} - 2^{-t_2/T_{1/2}}\right)}$.

***P44.31** We have all this information: $N_x(0) = 2.50 N_y(0)$

$N_x(3\text{d}) = 4.20 N_y(3\text{d})$

$N_x(0)e^{-\lambda_x 3\text{d}} = 4.20 N_y(0) e^{-\lambda_y 3\text{d}} = 4.20 \dfrac{N_x(0)}{2.50} e^{-\lambda_y 3\text{d}}$

$e^{3\text{d}\lambda_x} = \dfrac{2.5}{4.2} e^{3\text{d}\lambda_y}$

$3\text{d}\lambda_x = \ln\left(\dfrac{2.5}{4.2}\right) + 3\text{d}\lambda_y$

$3\text{d}\dfrac{0.693}{T_{1/2\,x}} = \ln\left(\dfrac{2.5}{4.2}\right) + 3\text{d}\dfrac{0.693}{1.60\text{ d}} = 0.781$

$T_{1/2\,x} = \boxed{2.66\text{ d}}$

***P44.32** (a) $\frac{dN_2}{dt} =$ rate of change of $N_2$

$=$ rate of production of $N_2$ – rate of decay of $N_2$

$=$ rate of decay of $N_1$ – rate of decay of $N_2$

$= \lambda_1 N_1 - \lambda_2 N_2$

(b) From the trial solution

$$N_2(t) = \frac{N_{10}\lambda_1}{\lambda_1 - \lambda_2}\left(e^{-\lambda_2 t} - e^{-\lambda_1 t}\right)$$

$$\therefore \frac{dN_2}{dt} = \frac{N_{10}\lambda_1}{\lambda_1 - \lambda_2}\left(-\lambda_2 e^{-\lambda_2 t} + \lambda_1 e^{-\lambda_1 t}\right) \qquad (1)$$

$$\begin{aligned}\therefore \frac{dN_2}{dt} + \lambda_2 N_2 &= \frac{N_{10}\lambda_1}{\lambda_1 - \lambda_2}\left(-\lambda_2 e^{-\lambda_2 t} + \lambda_1 e^{-\lambda_1 t} + \lambda_2 e^{-\lambda_2 t} - \lambda_2 e^{-\lambda_1 t}\right)\\ &= \frac{N_{10}\lambda_1}{\lambda_1 - \lambda_2}(\lambda_1 - \lambda_2)e^{-\lambda_1 t}\\ &= \lambda_1 N_1\end{aligned}$$

So $\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2$ as required.

(c) The functions to be plotted are

$$N_1(t) = 1\,000 e^{-\left(0.223\,6\ \text{min}^{-1}\right)t}$$

$$N_2(t) = 1\,130.8\left[e^{-\left(0.223\,6\ \text{min}^{-1}\right)t} - e^{-\left(0.025\,9\ \text{min}^{-1}\right)t}\right]$$

From the graph: $t_m \approx \boxed{10.9\ \text{min}}$

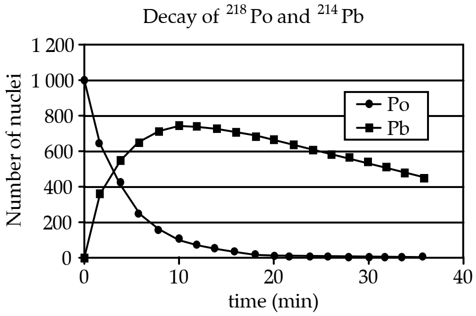

**FIG. P44.32(c)**

(d) From (1), $\frac{dN_2}{dt} = 0$ if $\lambda_2 e^{-\lambda_2 t} = \lambda_1 e^{-\lambda_1 t}$. $\therefore e^{(\lambda_1 - \lambda_2)t} = \frac{\lambda_1}{\lambda_2}$. Thus, $t = \boxed{t_m = \frac{\ln(\lambda_1/\lambda_2)}{\lambda_1 - \lambda_2}}$.

With $\lambda_1 = 0.223\,6\ \text{min}^{-1}$, $\lambda_2 = 0.025\,9\ \text{min}^{-1}$, this formula gives $t_m = \boxed{10.9\ \text{min}}$, in agreement with the result of part (c).

## Section 44.5 **The Decay Processes**

**P44.33** $Q = \left(M_{\text{U-238}} - M_{\text{Th-234}} - M_{\text{He-4}}\right)(931.5\ \text{MeV/u})$

$Q = (238.050\,783 - 234.043\,596 - 4.002\,603)\ \text{u}(931.5\ \text{MeV/u}) = \boxed{4.27\ \text{MeV}}$

**P44.34** (a) A gamma ray has zero charge and it contains no protons or neutrons. So for a gamma ray $Z = 0$ and $A = 0$. Keeping the total values of $Z$ and $A$ for the system conserved then requires $Z = 28$ and $A = 65$ for $X$. With this atomic number it must be nickel, and the nucleus must be in an exited state, so it is ${}^{65}_{28}\text{Ni}^*$.

(b) $\alpha = {}^4_2\text{He}$ has $Z = 2$ and $A = 4$

so for $X$ we require $Z = 84 - 2 = 82$

for Pb and $A = 215 - 4 = 211$, $X = {}^{211}_{82}\text{Pb}$.

(c) A positron $e^+ = {}^0_1 e$ has charge the same as a nucleus with $Z = 1$. A neutrino ${}^0_0\nu$ has no charge. Neither contains any protons or neutrons. So $X$ must have by conservation $Z = 26 + 1 = 27$. It is Co. And $A = 55 + 0 = 55$. It is ${}^{55}_{27}\text{Co}$.

Similar reasoning about balancing the sums of $Z$ and $A$ across the reaction reveals:

(d) ${}^{0}_{-1}e$

(e) ${}^1_1\text{H}$ (or p). Note that this process is a nuclear reaction, rather than radioactive decay. We can solve it from the same principles, which are fundamentally conservation of charge and conservation of baryon number.

**P44.35**

$$N_C = \left(\frac{0.021\,0\text{ g}}{12.0\text{ g/mol}}\right)\left(6.02\times 10^{23}\text{ molecules/mol}\right)$$

$\left(N_C = 1.05\times 10^{21}\text{ carbon atoms}\right)$ of which 1 in $7.70\times 10^{11}$ is a ${}^{14}\text{C}$ atom

$$(N_0)_{\text{C-14}} = 1.37\times 10^9,\ \lambda_{\text{C-14}} = \frac{\ln 2}{5\,730\text{ yr}} = 1.21\times 10^{-4}\text{ yr}^{-1} = 3.83\times 10^{-12}\text{ s}^{-1}$$

$$R = \lambda N = \lambda N_0 e^{-\lambda t}$$

At $t = 0$, $R_0 = \lambda N_0 = \left(3.83\times 10^{-12}\text{ s}^{-1}\right)\left(1.37\times 10^9\right)\left[\frac{7(86\,400\text{ s})}{1\text{ week}}\right] = 3.17\times 10^3\text{ decays/week}$.

At time $t$, $R = \frac{837}{0.88} = 951\text{ decays/week}$.

Taking logarithms, $\ln\frac{R}{R_0} = -\lambda t$ so $t = \frac{-1}{\lambda}\ln\left(\frac{R}{R_0}\right)$

$$t = \frac{-1}{1.21\times 10^{-4}\text{ yr}^{-1}}\ln\left(\frac{951}{3.17\times 10^3}\right) = \boxed{9.96\times 10^3\text{ yr}}.$$

***P44.36** $N = N_0 e^{-\lambda t}$ $\left|\frac{dN}{dt}\right| = R = \left|-\lambda N_0 e^{-\lambda t}\right| = R_0 e^{-\lambda t}$

$e^{-\lambda t} = \frac{R}{R_0}$ $e^{\lambda t} = \frac{R_0}{R}$ $\lambda t = \ln\left(\frac{R_0}{R}\right) = \frac{\ln 2}{T_{1/2}}t$ $t = T_{1/2}\frac{\ln(R_0/R)}{\ln 2}$

If $R = 0.13\text{ Bq}$, $t = 5\,730\text{ yr}\frac{\ln(0.25/0.13)}{0.693} = 5\,406\text{ yr}$.

If $R = 0.11\text{ Bq}$, $t = 5\,730\text{ yr}\frac{\ln(0.25/0.11)}{0.693} = 6\,787\text{ yr}$.

The range is most clearly written as $\boxed{\text{between 5 400 yr and 6 800 yr}}$, without understatement.

**P44.37** $^{3}_{1}\text{H nucleus} \to {}^{3}_{2}\text{He nucleus} + \text{e}^{-} + \overline{\nu}$

becomes $^{3}_{1}\text{H nucleus} + \text{e}^{-} \to {}^{3}_{2}\text{He nucleus} + 2\text{e}^{-} + \overline{\nu}$.

Ignoring the slight difference in ionization energies,

we have $^{3}_{1}\text{H atom} \to {}^{3}_{2}\text{He atom} + \overline{\nu}$

$$3.016\,049 \text{ u} = 3.016\,029 \text{ u} + 0 + \frac{Q}{c^2}$$

$$Q = (3.016\,049 \text{ u} - 3.016\,029 \text{ u})(931.5 \text{ MeV/u}) = 0.018\,6 \text{ MeV} = \boxed{18.6 \text{ keV}}$$

**P44.38** (a) For $\text{e}^{+}$ decay,

$$Q = (M_X - M_Y - 2m_e)c^2 = [39.962\,591 \text{ u} - 39.963\,999 \text{ u} - 2(0.000\,549 \text{ u})](931.5 \text{ MeV/u})$$
$$Q = -2.33 \text{ MeV}$$

Since $Q < 0$, the decay $\boxed{\text{cannot occur}}$ spontaneously.

(b) For alpha decay,

$$Q = (M_X - M_\alpha - M_Y)c^2 = [91.905\,287 \text{ u} - 4.002\,603 \text{ u} - 93.905\,088 \text{ u}](931.5 \text{ MeV/u})$$
$$Q = -2.24 \text{ MeV}$$

Since $Q < 0$, the decay $\boxed{\text{cannot occur}}$ spontaneously.

(c) For alpha decay,

$$Q = (M_X - M_\alpha - M_Y)c^2 = [143.910\,083 \text{ u} - 4.002\,603 \text{ u} - 139.905\,434 \text{ u}](931.5 \text{ MeV/u})$$
$$Q = 1.91 \text{ MeV}$$

Since $Q > 0$, the decay $\boxed{\text{can occur}}$ spontaneously.

**P44.39** (a) $\boxed{\text{e}^{-} + \text{p} \to \text{n} + \nu}$

(b) For nuclei, $^{15}\text{O} + \text{e}^{-} \to {}^{15}\text{N} + \nu$.

Add seven electrons to both sides to obtain $\boxed{^{15}_{8}\text{O atom} \to {}^{15}_{7}\text{N atom} + \nu}$.

(c) From Table A.3, $m\left(^{15}\text{O}\right) = m\left(^{15}\text{N}\right) + \frac{Q}{c^2}$

$$\Delta m = 15.003\,065 \text{ u} - 15.000\,109 \text{ u} = 0.002\,956 \text{ u}$$

$$Q = (931.5 \text{ MeV/u})(0.002\,956 \text{ u}) = \boxed{2.75 \text{ MeV}}$$

## Section 44.6 **Natural Radioactivity**

**P44.40** (a) Let $N$ be the number of ${}^{238}\text{U}$ nuclei and $N'$ be ${}^{206}\text{Pb}$ nuclei.

Then $N = N_0 e^{-\lambda t}$ and $N_0 = N + N'$ so $N = (N + N')e^{-\lambda t}$ or $e^{\lambda t} = 1 + \frac{N'}{N}$.

Taking logarithms, $\lambda t = \ln\left(1 + \frac{N'}{N}\right)$ where $\lambda = \frac{\ln 2}{T_{1/2}}$.

Thus, $t = \left(\frac{T_{1/2}}{\ln 2}\right)\ln\left(1 + \frac{N'}{N}\right)$.

If $\frac{N}{N'} = 1.164$ for the ${}^{238}\text{U} \rightarrow {}^{206}\text{Pb}$ chain with $T_{1/2} = 4.47 \times 10^9$ yr, the age is:

$$t = \left(\frac{4.47 \times 10^9 \text{ yr}}{\ln 2}\right)\ln\left(1 + \frac{1}{1.164}\right) = \boxed{4.00 \times 10^9 \text{ yr}}.$$

(b) From above, $e^{\lambda t} = 1 + \frac{N'}{N}$. Solving for $\frac{N}{N'}$ gives $\frac{N}{N'} = \frac{e^{-\lambda t}}{1 - e^{-\lambda t}}$.

With $t = 4.00 \times 10^9$ yr and $T_{1/2} = 7.04 \times 10^8$ yr for the ${}^{235}\text{U} \rightarrow {}^{207}\text{Pb}$ chain,

$$\lambda t = \left(\frac{\ln 2}{T_{1/2}}\right)t = \frac{(\ln 2)\left(4.00 \times 10^9 \text{ yr}\right)}{7.04 \times 10^8 \text{ yr}} = 3.938 \text{ and } \boxed{\frac{N}{N'} = 0.019\,9}.$$

With $t = 4.00 \times 10^9$ yr and $T_{1/2} = 1.41 \times 10^{10}$ yr for the ${}^{232}\text{Th} \rightarrow {}^{208}\text{Pb}$ chain,

$$\lambda t = \frac{(\ln 2)\left(4.00 \times 10^9 \text{ yr}\right)}{1.41 \times 10^{10} \text{ yr}} = 0.196\,6 \text{ and } \boxed{\frac{N}{N'} = 4.60}.$$

**P44.41**

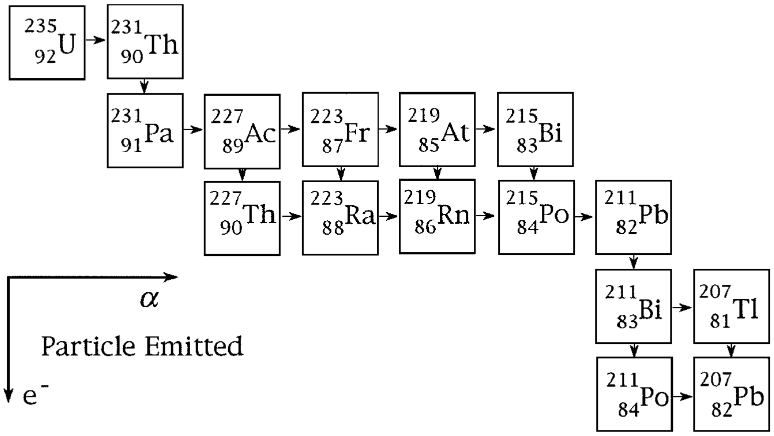

**FIG. P44.41**

**P44.42** (a) $$4.00\ \text{pCi/L} = \left(\frac{4.00\times10^{-12}\ \text{Ci}}{1\ \text{L}}\right)\left(\frac{3.70\times10^{10}\ \text{Bq}}{1\ \text{Ci}}\right)\left(\frac{1.00\times10^{3}\ \text{L}}{1\ \text{m}^3}\right) = \boxed{148\ \text{Bq/m}^3}$$

(b) $$N = \frac{R}{\lambda} = R\left(\frac{T_{1/2}}{\ln 2}\right) = \left(148\ \text{Bq/m}^3\right)\left(\frac{3.82\ \text{d}}{\ln 2}\right)\left(\frac{86\,400\ \text{s}}{1\ \text{d}}\right) = \boxed{7.05\times10^{7}\ \text{atoms/m}^3}$$

(c) $$\text{mass} = \left(7.05\times10^{7}\ \text{atoms/m}^3\right)\left(\frac{1\ \text{mol}}{6.02\times10^{23}\ \text{atoms}}\right)\left(\frac{222\ \text{g}}{1\ \text{mol}}\right) = 2.60\times10^{-14}\ \text{g/m}^3$$

Since air has a density of $1.20\ \text{kg/m}^3$, the fraction consisting of radon is

$$\text{fraction} = \frac{2.60\times10^{-14}\ \text{g/m}^3}{1\,200\ \text{g/m}^3} = \boxed{2.17\times10^{-17}}.$$

***P44.43** (a) Let $x$, $y$ denote the half-lives of the nuclei X, Y.

$$\frac{R_X}{R_Y} = \frac{R_0 e^{-\lambda_X t}}{R_0 e^{-\lambda_Y t}} = e^{-(0.685\ \text{h})(\ln 2)(1/x - 1/y)} = 1.04, \text{ which gives}$$

$$\frac{1}{x} - \frac{1}{y} = -0.082\,603\,69\ \text{h}^{-1}. \qquad (1)$$

From the data: $x - y = 77.2\ \text{h}$. (2)

Substitute (2) into (1): $$\frac{1}{x} - \frac{1}{x - 77.2\ \text{h}} = -0.082\,603\,69\ \text{h}^{-1}.$$

This reduces to the quadratic equation

$$x^2 - 77.2x - 934.6 = 0$$

which has solutions: $x = 87.84\ \text{h}$ or $-10.64\ \text{h}$.

Thus: $x = T_{1/2,\ \text{X}} = 87.84\ \text{h} = \boxed{3.66\ \text{days}}$ is the only physical root.

From (2): $y = T_{1/2,\ \text{Y}} = 87.84\ \text{h} - 77.2\ \text{h} = \boxed{10.6\ \text{h}}$.

(b) From Table A.3, X is $\boxed{^{224}\text{Ra}}$ and Y is $\boxed{^{212}\text{Pb}}$.

(c) From Figure 44.18, $^{224}\text{Ra}$ decays to $^{212}\text{Pb}$ by $\boxed{\text{three}}$ successive alpha-decays.

**P44.44** Number remaining: $N = N_0 e^{-(\ln 2)t/T_{1/2}}$.

Fraction remaining: $\dfrac{N}{N_0} = e^{-\lambda t} = e^{-(\ln 2)t/T_{1/2}}$.

(a) With $T_{1/2} = 3.82\ \text{d}$ and $t = 7.00\ \text{d}$, $\dfrac{N}{N_0} = e^{-(\ln 2)(7.00)/(3.82)} = \boxed{0.281}$.

(b) When $t = 1.00\ \text{yr} = 365.25\ \text{d}$, $\dfrac{N}{N_0} = e^{-(\ln 2)(365.25)/(3.82)} = \boxed{1.65\times10^{-29}}$.

(c) $\boxed{\text{Radon is continuously created}}$ as one daughter in the series of decays starting from the long-lived isotope $^{238}\text{U}$.

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## Section 44.7 **Nuclear Reactions**

**P44.45** $Q=\left[M_{^{27}\text{Al}}+M_{\alpha}-M_{^{30}\text{P}}-m_n\right]c^2$

$Q=\left[26.981\,539+4.002\,603-29.978\,314-1.008\,665\right]\text{u}(931.5\ \text{MeV/u})=\boxed{-2.64\ \text{MeV}}$

**P44.46** (a) For $X$, $A=24+1-4=21$

and $Z=12+0-2=10$, so $X$ is $\boxed{^{21}_{10}\text{Ne}}$.

(b) $A=235+1-90-2=144$

and $Z=92+0-38-0=54$, so $X$ is $\boxed{^{144}_{54}\text{Xe}}$.

(c) $A=2-2=0$

and $Z=2-1=+1$, so $X$ must be a positron.

As it is ejected, so is a neutrino: $\boxed{\text{X}={}^{0}_{1}\text{e}^{+}}$ and $\boxed{\text{X}'={}^{0}_{0}\nu}$.

**P44.47** (a) $^{197}_{79}\text{Au}+{}^{1}_{0}\text{n}\rightarrow{}^{198}_{79}\text{Au}^{*}\rightarrow{}^{198}_{80}\text{Hg}+{}^{0}_{-1}\text{e}+\overline{\nu}$

(b) Consider adding 79 electrons:

$^{197}_{79}\text{Au atom}+{}^{1}_{0}\text{n}\rightarrow{}^{198}_{80}\text{Hg atom}+\overline{\nu}+Q$

$Q=\left[M_{^{197}\text{Au}}+m_n-M_{^{198}\text{Hg}}\right]c^2$

$Q=\left[196.966\,552+1.008\,665-197.966\,752\right]\text{u}(931.5\ \text{MeV/u})=\boxed{7.89\ \text{MeV}}$

**P44.48** Neglect recoil of product nucleus, (i.e., do not require momentum conservation for the system of colliding particles). The energy balance gives $K_{\text{emerging}}=K_{\text{incident}}+Q$. To find $Q$:

$$Q=\left[\left(M_{\text{H}}+M_{\text{Al}}\right)-\left(M_{\text{Si}}+m_n\right)\right]c^2$$

$$Q=\left[\left(1.007\,825+26.981\,539\right)-\left(26.986\,705+1.008\,665\right)\right]\text{u}(931.5\ \text{MeV/u})=-5.59\ \text{MeV}$$

Thus, $K_{\text{emerging}}=6.61\ \text{MeV}-5.59\ \text{MeV}=\boxed{1.02\ \text{MeV}}$.

**P44.49** $^{9}_{4}\text{Be}+1.665\ \text{MeV}\rightarrow{}^{8}_{4}\text{Be}+{}^{1}_{0}\text{n}$, so $M_{^{8}_{4}\text{Be}}=M_{^{9}_{4}\text{Be}}-\dfrac{Q}{c^2}-m_n$

$$M_{^{8}_{4}\text{Be}}=9.012\,182\ \text{u}-\frac{(-1.665\ \text{MeV})}{931.5\ \text{MeV/u}}-1.008\,665\ \text{u}=\boxed{8.005\,3\ \text{u}}$$

$^{9}_{4}\text{Be}+{}^{1}_{0}\text{n}\rightarrow{}^{10}_{4}\text{Be}+6.812\ \text{MeV}$, so $M_{^{10}_{4}\text{Be}}=M_{^{9}_{4}\text{Be}}+m_n-\dfrac{Q}{c^2}$

$$M_{^{10}_{4}\text{Be}}=9.012\,182\ \text{u}+1.008\,665\ \text{u}-\frac{6.812\ \text{MeV}}{931.5\ \text{MeV/u}}=\boxed{10.013\,5\ \text{u}}$$

**P44.50** (a) $^{10}_{5}\mathrm{B} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{13}_{6}\mathrm{C} + {}^{1}_{1}\mathrm{H}$

The product nucleus is $\boxed{{}^{13}_{6}\mathrm{C}}$.

(b) $^{13}_{6}\mathrm{C} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{10}_{5}\mathrm{B} + {}^{4}_{2}\mathrm{He}$

The product nucleus is $\boxed{{}^{10}_{5}\mathrm{B}}$.

**P44.51** $^{236}_{92}\mathrm{U} \rightarrow {}^{90}_{37}\mathrm{Rb} + {}^{143}_{55}\mathrm{Cs} + 3{}^{1}_{0}\mathrm{n}$,

so $Q = \left[M_{^{236}_{92}\mathrm{U}} - M_{^{90}_{37}\mathrm{Rb}} - M_{^{143}_{55}\mathrm{Cs}} - 3m_n\right]c^2$

From Table A.3,

$Q = \left[236.045\,562 - 89.914\,809 - 142.927\,330 - 3(1.008\,665)\right]\mathrm{u}(931.5\ \mathrm{MeV/u}) = \boxed{165\ \mathrm{MeV}}$.

## Section 44.8 **Nuclear Magnetic Resonance and Magnetic Resonance Imagining**

**P44.52**

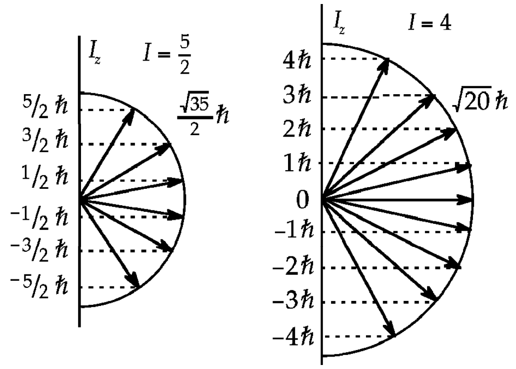

**FIG. P44.52**

**P44.53** (a) $$f_n = \frac{|2\mu B|}{h} = \frac{2(1.913\,5)(5.05\times10^{-27}\ \mathrm{J/T})(1.00\ \mathrm{T})}{6.626\times10^{-34}\ \mathrm{J\cdot s}} = \boxed{29.2\ \mathrm{MHz}}$$

(b) $$f_p = \frac{2(2.792\,8)(5.05\times10^{-27}\ \mathrm{J/T})(1.00\ \mathrm{T})}{6.626\times10^{-34}\ \mathrm{J\cdot s}} = \boxed{42.6\ \mathrm{MHz}}$$

(c) In the Earth's magnetic field,

$$f_p = \frac{2(2.792\,8)(5.05\times10^{-27})(50.0\times10^{-6})}{6.626\times10^{-34}} = \boxed{2.13\ \mathrm{kHz}}.$$

## Additional Problems

***P44.54** (a) With $m_n$ and $v_n$ as the mass and speed of the neutrons, Eq. 9.23 of the text becomes, after making appropriate notational changes, for the two collisions $v_1 = \left(\dfrac{2m_n}{m_n + m_1}\right)v_n$,

$$v_2 = \left(\frac{2m_n}{m_n + m_2}\right)v_n$$

$$\therefore (m_n + m_2)v_2 = (m_n + m_1)v_1 = 2m_n v_n$$

$$\therefore m_n(v_2 - v_1) = m_1 v_1 - m_2 v_2$$

$$\therefore m_n = \frac{m_1 v_1 - m_2 v_2}{v_2 - v_1}$$

(b)
$$m_n = \frac{(1\text{ u})(3.30\times10^7\text{ m/s}) - (14\text{ u})(4.70\times10^6\text{ m/s})}{4.70\times10^6\text{ m/s} - 3.30\times10^7\text{ m/s}} = \boxed{1.16\text{ u}}$$

**P44.55** (a)
$$Q = \left[M_{^9\text{Be}} + M_{^4\text{He}} - M_{^{12}\text{C}} - m_n\right]c^2$$

$$Q = [9.012\,182\text{ u} + 4.002\,603\text{ u} - 12.000\,000\text{ u} - 1.008\,665\text{ u}](931.5\text{ MeV/u}) = \boxed{5.70\text{ MeV}}$$

(b)
$$Q = \left[2M_{^2\text{H}} - M_{^3\text{He}} - m_n\right]$$

$$Q = [2(2.014\,102) - 3.016\,029 - 1.008\,665]\text{ u}(931.5\text{ MeV/u}) = \boxed{3.27\text{ MeV (exothermic)}}$$

**P44.56** (a) At threshold, the particles have no kinetic energy relative to each other. That is, they move like two particles that have suffered a perfectly inelastic collision. Therefore, in order to calculate the reaction threshold energy, we can use the results of a perfectly inelastic collision. Initially, the projectile $M_a$ moves with velocity $v_a$ while the target $M_X$ is at rest. We have from momentum conservation for the projectile-target system:

$$M_a v_a = (M_a + M_X)v_c.$$

The initial energy is: $E_i = \dfrac{1}{2}M_a v_a^2$.

The final kinetic energy is:

$$E_f = \frac{1}{2}(M_a + M_X)v_c^2 = \frac{1}{2}(M_a + M_X)\left[\frac{M_a v_a}{M_a + M_X}\right]^2 = \left[\frac{M_a}{M_a + M_X}\right]E_i.$$

From this, we see that $E_f$ is always less than $E_i$ and the change in energy, $E_f - E_i$, is given by

$$E_f - E_i = \left[\frac{M_a}{M_a + M_X} - 1\right]E_i = -\left[\frac{M_X}{M_a + M_X}\right]E_i.$$

This loss of kinetic energy in the isolated system corresponds to an increase in mass-energy during the reaction. Thus, the absolute value of this kinetic energy change is equal to $-Q$ (remember that $Q$ is negative in an endothermic reaction). The initial kinetic energy $E_i$ is the threshold energy $E_{th}$.

Therefore,
$$-Q = \left[\frac{M_X}{M_a + M_X}\right]E_{th}$$

or
$$E_{th} = -Q\left[\frac{M_X + M_a}{M_X}\right] = \boxed{-Q\left[1 + \frac{M_a}{M_X}\right]}.$$

*continued on next page*

(b) First, calculate the $Q$-value for the reaction: $Q=\left[M_{\text{N-14}}+M_{\text{He-4}}-M_{\text{O-17}}-M_{\text{H-1}}\right]c^2$

$$Q=\left[14.003\,074+4.002\,603-16.999\,132-1.007\,825\right]\text{u}(931.5\ \text{MeV/u})=-1.19\ \text{MeV}.$$

Then, $E_{th}=-Q\left[\dfrac{M_X+M_a}{M_X}\right]=-(-1.19\ \text{MeV})\left[1+\dfrac{4.002\,603}{14.003\,074}\right]=\boxed{1.53\ \text{MeV}}$.

**P44.57** ${}^1_1\text{H}+{}^7_3\text{Li}\rightarrow{}^7_4\text{Be}+{}^1_0\text{n}$

$$Q=\left[(M_{\text{H}}+M_{\text{Li}})-(M_{\text{Be}}+M_{\text{n}})\right](931.5\ \text{MeV/u})$$

$$Q=\left[(1.007\,825\ \text{u}+7.016\,004\ \text{u})-(7.016\,929\ \text{u}+1.008\,665\ \text{u})\right](931.5\ \text{MeV/u})$$

$$Q=\left(-1.765\times10^{-3}\ \text{u}\right)(931.5\ \text{MeV/u})=-1.644\ \text{MeV}$$

Thus, $KE_{\min}=\left(1+\dfrac{m_{\text{incident projectile}}}{m_{\text{target nucleus}}}\right)|Q|=\left(1+\dfrac{1.007\,825}{7.016\,004}\right)(1.644\ \text{MeV})=\boxed{1.88\ \text{MeV}}$.

**P44.58** (a) $N_0=\dfrac{\text{mass}}{\text{mass per atom}}=\dfrac{1.00\ \text{kg}}{(239.05\ \text{u})\left(1.66\times10^{-27}\ \text{kg/u}\right)}=\boxed{2.52\times10^{24}}$

(b) $\lambda=\dfrac{\ln2}{T_{1/2}}=\dfrac{\ln2}{\left(2.412\times10^4\ \text{yr}\right)\left(3.156\times10^7\ \text{s/yr}\right)}=9.106\times10^{-13}\ \text{s}^{-1}$

$$R_0=\lambda N_0=\left(9.106\times10^{-13}\ \text{s}^{-1}\right)\left(2.52\times10^{24}\right)=\boxed{2.29\times10^{12}\ \text{Bq}}$$

(c) $R=R_0e^{-\lambda t}$, so $t=\dfrac{-1}{\lambda}\ln\left(\dfrac{R}{R_0}\right)=\dfrac{1}{\lambda}\ln\left(\dfrac{R_0}{R}\right)$

$$t=\frac{1}{9.106\times10^{-13}\ \text{s}^{-1}}\ln\left(\frac{2.29\times10^{12}\ \text{Bq}}{0.100\ \text{Bq}}\right)=3.38\times10^{13}\ \text{s}\left(\frac{1\ \text{yr}}{3.156\times10^7\ \text{s}}\right)=\boxed{1.07\times10^6\ \text{yr}}$$

**P44.59** (a) ${}^{57}_{27}\text{Co}\rightarrow{}^{57}_{26}\text{Fe}+{}^{0}_{+1}\text{e}+{}^{0}_{0}\nu$

The $Q$-value for this positron emission is $Q=\left[M_{^{57}\text{Co}}-M_{^{57}\text{Fe}}-2m_e\right]c^2$.

$$Q=\left[56.936\,296-56.935\,399-2(0.000\,549)\right]\text{u}(931.5\ \text{MeV/u})=-0.187\ \text{MeV}$$

Since $Q<0$, this reaction $\boxed{\text{cannot spontaneously occur}}$.

(b) ${}^{14}_{6}\text{C}\rightarrow{}^{14}_{7}\text{N}+{}^{0}_{-1}\text{e}+{}^{0}_{0}\overline{\nu}$

The $Q$-value for this $\text{e}^-$ decay is $Q=\left[M_{^{14}\text{C}}-M_{^{14}\text{N}}\right]c^2$.

$$Q=\left[14.003\,242-14.003\,074\right]\text{u}(931.5\ \text{MeV/u})=0.156\ \text{MeV}=156\ \text{keV}$$

Since $Q>0$, the decay $\boxed{\text{can spontaneously occur}}$.

(c) The energy released in the reaction of (b) is shared by the electron and neutrino. Thus, $\boxed{K_e\ \text{can range from zero to 156 keV}}$.

**P44.60** (a) $r = r_0 A^{1/3} = 1.20 \times 10^{-15} A^{1/3}\ \text{m}$.

When $A = 12$, $r = \boxed{2.75 \times 10^{-15}\ \text{m}}$.

(b) $$F = \frac{k_e (Z-1)e^2}{r^2} = \frac{\left(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(Z-1)\left(1.60 \times 10^{-19}\ \text{C}\right)^2}{r^2}$$

When $Z = 6$ and $r = 2.75 \times 10^{-15}\ \text{m}$, $F = \boxed{152\ \text{N}}$.

(c) $$U = \frac{k_e q_1 q_2}{r} = \frac{k_e (Z-1)e^2}{r} = \frac{\left(8.99 \times 10^9\right)(Z-1)\left(1.6 \times 10^{-19}\right)^2}{r}$$

When $Z = 6$ and $r = 2.75 \times 10^{-15}\ \text{m}$, $U = 4.19 \times 10^{-13}\ \text{J} = \boxed{2.62\ \text{MeV}}$.

(d) $A = 238$; $Z = 92$, $r = \boxed{7.44 \times 10^{-15}\ \text{m}}$ $F = \boxed{379\ \text{N}}$

and $U = 2.82 \times 10^{-12}\ \text{J} = \boxed{17.6\ \text{MeV}}$.

**P44.61** (a) Because the reaction $\text{p} \to \text{n} + \text{e}^+ + \nu$ would violate the law of $\boxed{\text{conservation of energy}}$

$m_p = 1.007\,276\ \text{u}$ $m_n = 1.008\,665\ \text{u}$ $m_{\text{e}^+} = 5.49 \times 10^{-4}\ \text{u}$.

Note that $m_n + m_{\text{e}^+} > m_p$.

(b) The $\boxed{\text{required energy can come from the electrostatic repulsion}}$ of protons in the nucleus.

(c) Add seven electrons to both sides of the reaction for nuclei ${}^{13}_{7}\text{N} \to {}^{13}_{6}\text{C} + \text{e}^+ + \nu$

to obtain the reaction for neutral atoms ${}^{13}_{7}\text{N atom} \to {}^{13}_{6}\text{C atom} + \text{e}^+ + \text{e}^- + \nu$

$$Q = c^2\left[m\left({}^{13}\text{N}\right) - m\left({}^{13}\text{C}\right) - m_{\text{e}^+} - m_{\text{e}^-} - m_\nu\right]$$

$$Q = (931.5\ \text{MeV/u})\left[13.005\,739 - 13.003\,355 - 2\left(5.49 \times 10^{-4}\right) - 0\right]\text{u}$$

$$Q = (931.5\ \text{MeV/u})\left(1.286 \times 10^{-3}\ \text{u}\right) = \boxed{1.20\ \text{MeV}}$$

**P44.62** (a) A least-square fit to the graph yields:

$$\lambda = -\text{slope} = -\left(-0.250\ \text{h}^{-1}\right) = 0.250\ \text{h}^{-1}$$

and

$$\ln(\text{cpm})\big|_{t=0} = \text{intercept} = 8.30.$$

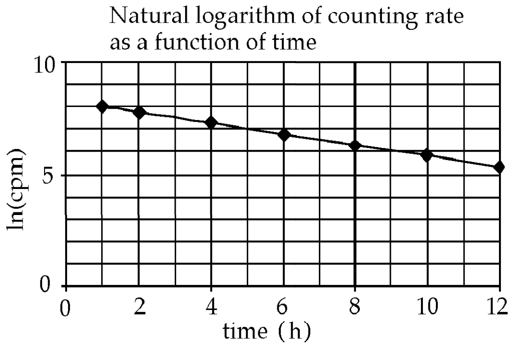

**FIG. P44.62**

(b) $$\lambda = 0.250\ \text{h}^{-1}\left(\frac{1\ \text{h}}{60.0\ \text{min}}\right) = \boxed{4.17 \times 10^{-3}\ \text{min}^{-1}}$$

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{4.17 \times 10^{-3}\ \text{min}^{-1}}$$

$$= 166\ \text{min} = \boxed{2.77\ \text{h}}$$

*continued on next page*

(c) From (a), intercept $=\ln(\text{cpm})_0 = 8.30$.

Thus, $(\text{cpm})_0 = e^{8.30}$ counts/min $= \boxed{4.02\times 10^3 \text{ counts/min}}$.

(d) $$N_0 = \frac{R_0}{\lambda} = \frac{1}{\lambda}\frac{(\text{cpm})_0}{\text{Eff}} = \frac{4.02\times 10^3 \text{ counts/min}}{\left(4.17\times 10^{-3}\text{ min}^{-1}\right)(0.100)} = \boxed{9.65\times 10^6 \text{ atoms}}$$

**P44.63** (a) The reaction is $^{145}_{61}\text{Pm} \rightarrow {}^{141}_{59}\text{Pr} + \alpha$

(b) $Q = (M_{\text{Pm}} - M_\alpha - M_{\text{Pr}})931.5 = (144.912\,744 - 4.002\,603 - 140.907\,648)931.5 = \boxed{2.32 \text{ MeV}}$

(c) The alpha and daughter have equal and opposite momenta $p_\alpha = p_d$

$$E_\alpha = \frac{p_\alpha^2}{2m_\alpha} \qquad E_d = \frac{p_d^2}{2m_d}$$

$$\frac{E_\alpha}{E_{\text{tot}}} = \frac{E_\alpha}{E_\alpha + E_d} = \frac{p_\alpha^2/2m_\alpha}{\left(p_\alpha^2/2m_\alpha\right)+\left(p_\alpha^2/2m_d\right)} = \frac{1/2m_\alpha}{(1/2m_\alpha)+(1/2m_d)} = \frac{m_d}{m_d+m_\alpha} = \frac{141}{141+4} = \boxed{97.2\%}$$ or 2.26 MeV.

This is carried away by the alpha.

**P44.64** (a) If $\Delta E$ is the energy difference between the excited and ground states of the nucleus of mass $M$, and $hf$ is the energy of the emitted photon, conservation of energy for the nucleus-photon system gives

$$\Delta E = hf + E_r. \qquad (1)$$

Where $E_r$ is the recoil energy of the nucleus, which can be expressed as

$$E_r = \frac{Mv^2}{2} = \frac{(Mv)^2}{2M}. \qquad (2)$$

Since system momentum must also be conserved, we have

$$Mv = \frac{hf}{c}. \qquad (3)$$

Hence, $E_r$ can be expresses as $E_r = \dfrac{(hf)^2}{2Mc^2}$.

When $hf << Mc^2$

we can make the approximation that $hf \approx \Delta E$

so $E_r \approx \boxed{\dfrac{(\Delta E)^2}{2Mc^2}}$.

(b) $E_r = \dfrac{(\Delta E)^2}{2Mc^2}$ where $\Delta E = 0.014\,4$ MeV

and $Mc^2 = (57 \text{ u})(931.5 \text{ MeV/u}) = 5.31\times 10^4$ MeV.

Therefore, $$E_r = \frac{\left(1.44\times 10^{-2}\text{ MeV}\right)^2}{2\left(5.31\times 10^4 \text{ MeV}\right)} = \boxed{1.94\times 10^{-3}\text{ eV}}.$$

**P44.65** (a) One liter of milk contains this many $^{40}\text{K}$ nuclei:

$$N = (2.00\text{ g})\left(\frac{6.02\times10^{23}\text{ nuclei/mol}}{39.1\text{ g/mol}}\right)\left(\frac{0.011\,7}{100}\right) = 3.60\times10^{18}\text{ nuclei}$$

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{1.28\times10^{9}\text{ yr}}\left(\frac{1\text{ yr}}{3.156\times10^{7}\text{ s}}\right) = 1.72\times10^{-17}\text{ s}^{-1}$$

$$R = \lambda N = \left(1.72\times10^{-17}\text{ s}^{-1}\right)\left(3.60\times10^{18}\right) = \boxed{61.8\text{ Bq}}$$

(b) For the iodine, $R = R_0e^{-\lambda t}$ with $\lambda = \dfrac{\ln 2}{8.04\text{ d}}$

$$t = \frac{1}{\lambda}\ln\left(\frac{R_0}{R}\right) = \frac{8.04\text{ d}}{\ln 2}\ln\left(\frac{2\,000}{61.8}\right) = \boxed{40.3\text{ d}}$$

**P44.66** (a) For cobalt–56,

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{77.1\text{ d}}\left(\frac{365.25\text{ d}}{1\text{ yr}}\right) = 3.28\text{ yr}^{-1}.$$

The elapsed time from July 1054 to July 2003 is 949 yr.

$$R = R_0e^{-\lambda t}$$

implies

$$\frac{R}{R_0} = e^{-\lambda t} = e^{-\left(3.28\text{ yr}^{-1}\right)(949\text{ yr})} = e^{-3\,116} = e^{-(\ln 10)1\,353} = \boxed{\sim 10^{-1\,353}}.$$

(b) For carbon–14,

$$\lambda = \frac{\ln 2}{5\,730\text{ yr}} = 1.21\times10^{-4}\text{ yr}^{-1}$$

$$\frac{R}{R_0} = e^{-\lambda t} = e^{-\left(1.21\times10^{-4}\text{ yr}^{-1}\right)(949\text{ yr})} = e^{-0.115} = \boxed{0.892}$$

**P44.67** We have $N_{235} = N_{0,\,235}e^{-\lambda_{235}t}$

and $N_{238} = N_{0,\,238}e^{-\lambda_{238}t}$

$$\frac{N_{235}}{N_{238}} = 0.007\,25 = e^{\left(-(\ln 2)t/T_{h,\,235}+(\ln 2)t/T_{h,\,238}\right)}.$$

Taking logarithms,

$$-4.93 = \left(-\frac{\ln 2}{0.704\times10^{9}\text{ yr}}+\frac{\ln 2}{4.47\times10^{9}\text{ yr}}\right)t$$

or

$$-4.93 = \left(-\frac{1}{0.704\times10^{9}\text{ yr}}+\frac{1}{4.47\times10^{9}\text{ yr}}\right)(\ln 2)t$$

$$t = \frac{-4.93}{\left(-1.20\times10^{-9}\text{ yr}^{-1}\right)\ln 2} = \boxed{5.94\times10^{9}\text{ yr}}.$$

**P44.68** (a) Add two electrons to both sides of the reaction to have it in energy terms:

$$4\,{}^1_1\text{H atom} \rightarrow {}^4_2\text{He atom} + Q \qquad Q = \Delta mc^2 = \left[4M_{{}^1_1\text{H}} - M_{{}^4_2\text{He}}\right]c^2$$

$$Q = \left[4(1.007\,825\text{ u}) - 4.002\,603\text{ u}\right](931.5\text{ MeV/u})\left(\frac{1.60\times10^{-13}\text{ J}}{1\text{ MeV}}\right) = \boxed{4.28\times10^{-12}\text{ J}}$$

(b) $$N = \frac{1.99\times10^{30}\text{ kg}}{1.67\times10^{-27}\text{ kg/atom}} = \boxed{1.19\times10^{57}\text{ atoms}} = 1.19\times10^{57}\text{ protons}$$

(c) The energy that could be created by this many protons in this reaction is:

$$\left(1.19\times10^{57}\text{ protons}\right)\left(\frac{4.28\times10^{-12}\text{ J}}{4\text{ protons}}\right) = 1.27\times10^{45}\text{ J}$$

$$\mathcal{P} = \frac{E}{\Delta t} \qquad \text{so} \qquad \Delta t = \frac{E}{\mathcal{P}} = \frac{1.27\times10^{45}\text{ J}}{3.77\times10^{26}\text{ W}} = 3.38\times10^{18}\text{ s} = \boxed{107\text{ billion years}}.$$

**P44.69** $E = -\mu\cdot\mathbf{B}$ so the energies are $E_1 = +\mu B$ and $E_2 = -\mu B$

$\mu = 2.792\,8\mu_n$ and $\mu_n = 5.05\times10^{-27}\text{ J/T}$

$$\Delta E = 2\mu B = 2(2.792\,8)\left(5.05\times10^{-27}\text{ J/T}\right)(12.5\text{ T}) = 3.53\times10^{-25}\text{ J} = \boxed{2.20\times10^{-6}\text{ eV}}.$$

**P44.70** (a) $$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{5.27\text{ yr}}\left(\frac{1\text{ yr}}{3.156\times10^7\text{ s}}\right) = 4.17\times10^{-9}\text{ s}^{-1}$$

$$t = 30.0\text{ months} = (2.50\text{ yr})\left(\frac{3.156\times10^7\text{ s}}{1\text{ yr}}\right) = 7.89\times10^7\text{ s}$$

$$R = R_0e^{-\lambda t} = (\lambda N_0)e^{-\lambda t}$$

$$\text{so } N_0 = \left(\frac{R}{\lambda}\right)e^{\lambda t} = \left[\frac{(10.0\text{ Ci})\left(3.70\times10^{10}\text{ Bq/Ci}\right)}{4.17\times10^{-9}\text{ s}^{-1}}\right]e^{\left(4.17\times10^{-9}\text{ s}^{-1}\right)\left(7.89\times10^7\text{ s}\right)}$$

$$N_0 = 1.23\times10^{20}\text{ nuclei}$$

$$\text{Mass} = \left(1.23\times10^{20}\text{ atoms}\right)\left(\frac{59.93\text{ g/mol}}{6.02\times10^{23}\text{ atoms/mol}}\right) = 1.23\times10^{-2}\text{ g} = \boxed{12.3\text{ mg}}$$

(b) We suppose that each decaying nucleus promptly puts out both a beta particle and two gamma rays, for

$$Q = (0.310 + 1.17 + 1.33)\text{MeV} = 2.81\text{ MeV}$$

$$\mathcal{P} = QR = (2.81\text{ MeV})\left(1.6\times10^{-13}\text{ J/MeV}\right)\left(3.70\times10^{11}\text{ s}^{-1}\right) = \boxed{0.166\text{ W}}$$

**P44.71** For an electric charge density $\rho = \dfrac{Ze}{(4/3)\pi R^3}$.

Using Gauss's Law inside the sphere,

$$E \cdot 4\pi r^2 = \frac{(4/3)\pi r^3}{\in_0} \frac{Ze}{(4/3)\pi R^3}: \qquad E = \frac{1}{4\pi \in_0} \frac{Zer}{R^3} \qquad (r \le R)$$

$$E = \frac{1}{4\pi \in_0} \frac{Ze}{r^2} \qquad (r \ge R)$$

We now find the electrostatic energy: $U = \displaystyle\int_{r=0}^{\infty} \frac{1}{2} \in_0 E^2 4\pi r^2 dr$

$$U = \frac{1}{2} \in_0 \int_0^R \left( \frac{1}{4\pi \in_0} \right)^2 \frac{Z^2 e^2 r^2}{R^6} 4\pi r^2 dr + \frac{1}{2} \in_0 \int_R^{\infty} \left( \frac{1}{4\pi \in_0} \right)^2 \frac{Z^2 e^2}{r^4} 4\pi r^2 dr = \frac{Z^2 e^2}{8\pi \in_0} \left[ \frac{R^5}{5R^6} + \frac{1}{R} \right]$$

$$= \boxed{\frac{3}{20} \frac{Z^2 e^2}{\pi \in_0 R}}$$

**P44.72** (a) For the electron capture, ${}^{93}_{43}\text{Tc} + {}^{0}_{-1}\text{e} \to {}^{93}_{42}\text{Mo} + \gamma$.

The disintegration energy is $Q = \left[ M_{{}^{93}\text{Tc}} - M_{{}^{93}\text{Mo}} \right] c^2$.

$$Q = [92.910\,2 - 92.906\,8]\text{u}(931.5\ \text{MeV/u}) = 3.17\ \text{MeV} > 2.44\ \text{MeV}$$

Electron capture is allowed $\boxed{\text{to all specified excited states}}$ in ${}^{93}_{42}\text{Mo}$.

For positron emission, ${}^{93}_{43}\text{Tc} \to {}^{93}_{42}\text{Mo} + {}^{0}_{+1}\text{e} + \gamma$.

The disintegration energy is $Q' = \left[ M_{{}^{93}\text{Tc}} - M_{{}^{93}\text{Mo}} - 2m_e \right] c^2$.

$$Q' = [92.910\,2 - 92.906\,8 - 2(0.000\,549)]\text{u}(931.5\ \text{MeV/u}) = 2.14\ \text{MeV}$$

Positron emission can reach

$\boxed{\text{the 1.35, 1.48, and 2.03 MeV states}}$

but there is insufficient energy to reach the 2.44 MeV state.

(b) The daughter nucleus in both forms of decay is $\boxed{{}^{93}_{42}\text{Mo}}$.

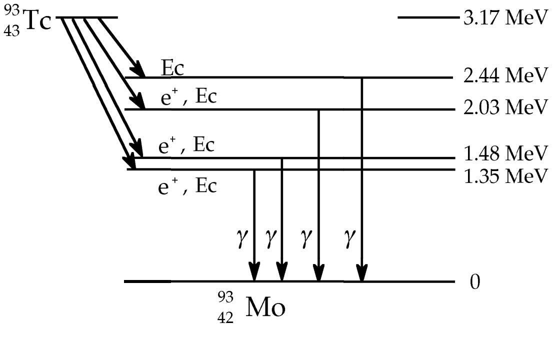

FIG. P44.72

**P44.73** $K = \frac{1}{2}mv^2$,

so $v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(0.040\,0\text{ eV})(1.60 \times 10^{-19}\text{ J/eV})}{1.67 \times 10^{-27}\text{ kg}}} = 2.77 \times 10^3\text{ m/s}$.

The time for the trip is $t = \frac{x}{v} = \frac{1.00 \times 10^4\text{ m}}{2.77 \times 10^3\text{ m/s}} = 3.61\text{ s}$.

The number of neutrons finishing the trip is given by $N = N_0 e^{-\lambda t}$.

The fraction decaying is $1 - \frac{N}{N_0} = 1 - e^{-(\ln 2)t/T_{1/2}} = 1 - e^{-(\ln 2)(3.61\text{ s}/624\text{ s})} = 0.004\,00 = \boxed{0.400\%}$.

**P44.74** (a) If we assume all the $^{87}\text{Sr}$ came from $^{87}\text{Rb}$,

then $N = N_0 e^{-\lambda t}$

yields $t = \frac{-1}{\lambda}\ln\left(\frac{N}{N_0}\right) = \frac{T_{1/2}}{\ln 2}\ln\left(\frac{N_0}{N}\right)$

where $N = N_{\text{Rb-87}}$

and $N_0 = N_{\text{Sr-87}} + N_{\text{Rb-87}}$

$$t = \frac{(4.75 \times 10^{10}\text{ yr})}{\ln 2}\ln\left(\frac{1.82 \times 10^{10} + 1.07 \times 10^9}{1.82 \times 10^{10}}\right) = \boxed{3.91 \times 10^9\text{ yr}}.$$

(b) It could be $\boxed{\text{no older}}$. The rock could be younger if some $^{87}\text{Sr}$ were originally present.

**P44.75** $R = R_0 \exp(-\lambda t)$ lets us write $\ln R = \ln R_0 - \lambda t$

which is the equation of a straight line with $|\text{slope}| = \lambda$.

The logarithmic plot shown in Figure P44.75 is fitted by $\ln R = 8.44 - 0.262t$.

If $t$ is measured in minutes, then decay constant $\lambda$ is 0.262 per minute. The half–life is

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{0.262/\text{min}} = \boxed{2.64\text{ min}}.$$

The reported half–life of $^{137}\text{Ba}$ is 2.55 min. The difference reflects experimental uncertainties.

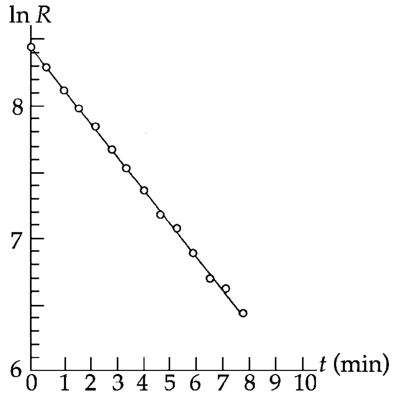

FIG. P44.75

## ANSWERS TO EVEN PROBLEMS

**P44.2** (a) 7.89 cm and 8.21 cm; (b) see the solution

**P44.4** (a) 29.5 fm; (b) 5.18 fm; (c) see the solution

**P44.6** 25.6 MeV

**P44.8** a nucleus such as $^{30}\text{Si}$ with $A = 30$

**P44.10** 6.11 PN toward the other ball

**P44.12** (a) 48; (b) 3; (c) 46; (d) 1

| | |
|---|---|
| **P44.14** | (a) $1.11\ \text{MeV/nucleon}$;<br>(b) $7.07\ \text{MeV/nucleon}$;<br>(c) $8.79\ \text{MeV/nucleon}$;<br>(d) $7.57\ \text{MeV/nucleon}$ |
| **P44.16** | $0.210\ \text{MeV}$ greater for $^{23}\text{Na}$ because it has less proton repulsion |
| **P44.18** | (a) $84.1\ \text{MeV}$; (b) $342\ \text{MeV}$; (c) The nuclear force of attraction dominates over electrical repulsion |
| **P44.20** | $7.93\ \text{MeV}$ |
| **P44.22** | (a) see the solution;<br>(b) $\frac{R}{3}$ and $\frac{R}{6}$; see the solution |
| **P44.24** | $0.200\ \text{mCi}$ |
| **P44.26** | $41.8\ \text{TBq}$ |
| **P44.28** | $86.4\ \text{h}$ |
| **P44.30** | $\frac{R_0 T_{1/2}}{\ln 2}\left(2^{-t_1/T_{1/2}} - 2^{-t_2/T_{1/2}}\right)$ |
| **P44.32** | (a) see the solution; (b) see the solution;<br>(c) see the solution; $10.9\ \text{min}$;<br>(d) $t_m = \frac{\ln(\lambda_1/\lambda_2)}{\lambda_1 - \lambda_2}$; yes |
| **P44.34** | (a) ${}^{65}_{28}\text{Ni}^*$; (b) ${}^{211}_{82}\text{Pb}$; (c) ${}^{55}_{27}\text{Co}$; (d) ${}^{\;\,0}_{-1}\text{e}$;<br>(e) ${}^{1}_{1}\text{H}$ |
| **P44.36** | between 5 400 yr and 6 800 yr |
| **P44.38** | (a) cannot occur; (b) cannot occur;<br>(c) can occur |
| **P44.40** | (a) $4.00\ \text{Gyr}$; (b) $0.019\,9$ and $4.60$ |
| **P44.42** | (a) $148\ \text{Bq/m}^3$; (b) $7.05 \times 10^7\ \text{atoms/m}^3$;<br>(c) $2.17 \times 10^{-17}$ |
| **P44.44** | (a) $0.281$; (b) $1.65 \times 10^{-29}$;<br>(c) see the solution |
| **P44.46** | (a) ${}^{21}_{10}\text{Ne}$; (b) ${}^{144}_{54}\text{Xe}$; (c) ${}^{0}_{1}\text{e}^+$ and ${}^{0}_{0}\nu$ |
| **P44.48** | $1.02\ \text{MeV}$ |
| **P44.50** | (a) ${}^{13}_{6}\text{C}$; (b) ${}^{10}_{5}\text{B}$ |
| **P44.52** | see the solution |
| **P44.54** | (a) see the solution; (b) 1.16 u |
| **P44.56** | (a) see the solution; (b) 1.53 MeV |
| **P44.58** | (a) $2.52 \times 10^{24}$; (b) $2.29\ \text{TBq}$; (c) $1.07\ \text{Myr}$ |
| **P44.60** | (a) $2.75\ \text{fm}$; (b) $152\ \text{N}$; (c) $2.62\ \text{MeV}$;<br>(d) $7.44\ \text{fm}$, $379\ \text{N}$, $17.6\ \text{MeV}$ |
| **P44.62** | (a) see the solution;<br>(b) $4.17 \times 10^{-3}\ \text{min}^{-1}$; $2.77\ \text{h}$;<br>(c) $4.02 \times 10^3\ \text{counts/min}$;<br>(d) $9.65 \times 10^6$ atoms |
| **P44.64** | (a) see the solution; (b) 1.94 meV |
| **P44.66** | (a) $\sim 10^{-1\,353}$; (b) $0.892$ |
| **P44.68** | (a) 4.28 pJ; (b) $1.19 \times 10^{57}$ atoms;<br>(c) 107 Gyr |
| **P44.70** | (a) $12.3\ \text{mg}$; (b) $0.166\ \text{W}$ |
| **P44.72** | (a) electron capture to all; positron emission to the $1.35\ \text{MeV}$, $1.48\ \text{MeV}$, and $2.03\ \text{MeV}$ states; (b) ${}^{93}_{42}\text{Mo}$; see the solution |
| **P44.74** | (a) $3.91\ \text{Gyr}$; (b) No older; it could be younger if some $^{87}\text{Sr}$ were originally present, contrary to our assumption. |

# 45

# Applications of Nuclear Physics

**CHAPTER OUTLINE**

45.1 Interactions Involving Neutrons
45.2 Nuclear Fission
45.3 Nuclear Reactors
45.4 Nuclear Fusion
45.5 Radiation Damage
45.6 Radiation Detectors
45.7 Uses of Radiation

## ANSWERS TO QUESTIONS

**Q45.1** A moderator is used to slow down neutrons released in the fission of one nucleus, so that they are likely to be absorbed by another nucleus to make it fission.

**Q45.2** The hydrogen nuclei in water molecules have mass similar to that of a neutron, so that they can efficiently rob a fast-moving neutron of kinetic energy as they scatter it. Once the neutron is slowed down, a hydrogen nucleus can absorb it in the reaction $\mathrm{n}+{}_1^1\mathrm{H}\rightarrow {}_1^2\mathrm{H}$.

**Q45.3** The excitation energy comes from the binding energy of the extra nucleon.

**Q45.4** The advantage of a fission reaction is that it can generate much more electrical energy per gram of fuel compared to fossil fuels. Also, fission reactors do not emit greenhouse gasses as combustion byproducts like fossil fuels—the only necessary environmental discharge is heat. The cost involved in producing fissile material is comparable to the cost of pumping, transporting and refining fossil fuel.

The disadvantage is that some of the products of a fission reaction are radioactive—and some of those have long half-lives. The other problem is that there will be a point at which enough fuel is spent that the fuel rods do not supply power economically and need to be replaced. The fuel rods are still radioactive after removal. Both the waste and the "spent" fuel rods present serious health and environmental hazards that can last for tens of thousands of years. Accidents and sabotage involving nuclear reactors can be very serious, as can accidents and sabotage involving fossil fuels.

**Q45.5** The products of fusion reactors are generally not themselves unstable, while fission reactions result in a chain of reactions which almost all have some unstable products.

**Q45.6** For the deuterium nuclei to fuse, they must be close enough to each other for the nuclear forces to overcome the Coulomb repulsion of the protons—this is why the ion density is a factor. The more time that the nuclei in a sample spend in close proximity, the more nuclei will fuse—hence the confinement time is a factor.

**Q45.7** In a fusion reaction, the main idea is to get the nuclear forces, which act over very short distances, to overcome the Coulomb repulsion of the protons. Tritium has one more neutron in the nucleus, and thus increases the nuclear force, decreasing the necessary kinetic energy to obtain D–T fusion as compared to D–D fusion.

**Q45.8** The biggest obstacle is power loss due to radiation. Remember that a high temperature must be maintained to keep the fuel in a reactive plasma state. If this kinetic energy is lost due to bremsstrahlung radiation, then the probability of nuclear fusion will decrease significantly. Additionally, each of the confinement techniques requires power input, thus raising the bar for sustaining a reaction in which the power output is greater than the power input.

**Q45.9** Fusion of light nuclei to a heavier nucleus releases energy. Fission of a heavy nucleus to lighter nuclei releases energy. Both processes are steps towards greater stability on the curve of binding energy, Figure 44.5. The energy release per nucleon is typically greater for fusion, and this process is harder to control.

**Q45.10** Advantages of fusion: high energy yield, no emission of greenhouse gases, fuel very easy to obtain, reactor can not go supercritical like a fission reactor, low amounts of radioactive waste.

Disadvantages: requires high energy input to sustain reaction, lithium and helium are scarce, neutrons released by reaction cause structural damage to reactor housing.

**Q45.11** The fusion fuel must be heated to a very high temperature. It must be contained at a sufficiently high density for a sufficiently long time to achieve a reasonable energy output.

**Q45.12** The first method uses magnetic fields to contain the plasma, reducing its contact with the walls of the container. This way, there is a reduction in heat loss to the environment, so that the reaction may be sustained over seconds.

The second method involves striking the fuel with high intensity, focused lasers from multiple directions, effectively imploding the fuel. This increases the internal pressure and temperature of the fuel to the point of ignition.

**Q45.13** No. What is critical in radiation safety is the type of radiation encountered. The curie is a measure of the rate of decay, not the products of the decay or of their energies.

**Q45.14** X-ray radiation can cause genetic damage in the developing fetus. If the damaged cells survive the radiation and reproduce, then the genetic errors will be replicated, potentially causing severe birth defects or death of the child.

**Q45.15** For each additional dynode, a larger applied voltage is needed, and hence a larger output from a power supply—"infinite" amplification would not be practical. Nor would it be desirable: the goal is to connect the tube output to a simple counter, so a massive pulse amplitude is not needed. If you made the detector sensitive to weaker and weaker signals, you would make it more and more sensitive to background noise.

**Q45.16** Sometimes the references are oblique indeed. Some must serve for more than one form of energy or mode of transfer. Here is one list:

kinetic: ocean currents
rotational kinetic: Earth turning
gravitational: water lifted up
elastic: Elastic energy is necessary for sound, listed below.
internal: by contrast to a chilly night; or in forging a chain
chemical: flames
sound: thunder
electrical transmission: lightning
electromagnetic radiation: heavens blazing; lightning
atomic electronic: In the blazing heavens, stars have different colors because of different predominant energy losses by atoms at their surfaces.
nuclear: The blaze of the heavens is produced by nuclear reactions in the cores of stars.

Remarkably, the word "energy" in this translation is an anachronism. Goethe wrote the song a few years before Thomas Young coined the term.

## SOLUTIONS TO PROBLEMS

### Section 45.1 Interactions Involving Neutrons

### Section 45.2 Nuclear Fission

***P45.1** The energy is

$$3.30\times10^{10}\text{ J}\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right)\left(\frac{1\text{ U - 235 nucleus}}{208\text{ MeV}}\right)\left(\frac{235\text{ g}}{6.02\times10^{23}\text{ nucleus}}\right)\left(\frac{\text{M}}{10^6}\right)=\boxed{0.387\text{ g}}\text{ of U - 235}.$$

**P45.2**

$$\Delta m=(m_n+M_{\text{U}})-(M_{\text{Zr}}+M_{\text{Te}}+3m_n)$$

$$\Delta m=(1.008\,665\text{ u}+235.043\,923\text{ u})-(97.912\,7\text{ u}+134.916\,5\text{ u}+3(1.008\,665\text{ u}))$$

$$\Delta m=0.197\,39\text{ u}=3.28\times10^{-28}\text{ kg}\qquad\text{so}\qquad Q=\Delta mc^2=2.95\times10^{-11}\text{ J}=\boxed{184\text{ MeV}}$$

**P45.3** Three different fission reactions are possible: $^{1}_{0}\text{n}+{}^{235}_{92}\text{U}\to{}^{90}_{38}\text{Sr}+{}^{144}_{54}\text{Xe}+2^{1}_{0}\text{n}$ $\boxed{^{144}_{54}\text{Xe}}$

$^{1}_{0}\text{n}+{}^{235}_{92}\text{U}\to{}^{90}_{38}\text{Sr}+{}^{143}_{54}\text{Xe}+3^{1}_{0}\text{n}$ $\boxed{^{143}_{54}\text{Xe}}$ $^{1}_{0}\text{n}+{}^{235}_{92}\text{U}\to{}^{90}_{38}\text{Sr}+{}^{142}_{54}\text{Xe}+4^{1}_{0}\text{n}$ $\boxed{^{142}_{54}\text{Xe}}$

**P45.4** $^{1}_{0}\text{n}+{}^{238}_{92}\text{U}\to{}^{239}_{92}\text{U}\to{}^{239}_{93}\text{Np}+\text{e}^-+\overline{\nu}$ $^{239}_{93}\text{Np}\to{}^{239}_{94}\text{Pu}+\text{e}^-+\overline{\nu}$

**P45.5** $^{1}_{0}\text{n}+{}^{232}_{90}\text{Th}\to{}^{233}_{90}\text{Th}\to{}^{233}_{91}\text{Pa}+\text{e}^-+\overline{\nu}$ $^{233}_{91}\text{Pa}\to{}^{233}_{92}\text{U}+\text{e}^-+\overline{\nu}$

**P45.6** (a)

$$Q=(\Delta m)c^2=\left[m_n+M_{\text{U235}}-M_{\text{Ba141}}-M_{\text{Kr92}}-3m_n\right]c^2$$

$$\Delta m=\left[(1.008\,665+235.043\,923)-(140.914\,4+91.926\,2+3\times1.008\,665)\right]\text{u}=0.185\,993\text{ u}$$

$$Q=(0.185\,993\text{ u})(931.5\text{ MeV/u})=\boxed{173\text{ MeV}}$$

(b)

$$f=\frac{\Delta m}{m_i}=\frac{0.185993\text{ u}}{236.05\text{ u}}=7.88\times10^{-4}=\boxed{0.078\,8\%}$$

***P45.7** (a) The initial mass is $1.007\,825\text{ u} + 11.009\,306\text{ u} = 12.017\,131\text{ u}$. The final mass is $3(4.002\,603\text{ u}) = 12.007\,809\text{ u}$. The rest mass annihilated is $\Delta m = 0.009\,322\text{ u}$. The energy created is $Q = \Delta mc^2 = 0.009\,322\text{ u}\left(\dfrac{931.5\text{ MeV}}{1\text{ u}}\right) = \boxed{8.68\text{ MeV}}$.

(b) The proton and the boron nucleus have positive charges. The colliding particles must have enough kinetic energy to approach very closely in spite of their electric repulsion.

**P45.8** If the electrical power output of 1 000 MW is 40.0% of the power derived from fission reactions, the power output of the fission process is

$$\frac{1\,000\text{ MW}}{0.400} = \left(2.50\times10^9\text{ J/s}\right)\left(8.64\times10^4\text{ s/d}\right) = 2.16\times10^{14}\text{ J/d}.$$

The number of fissions per day is $\left(2.16\times10^{14}\text{ J/d}\right)\left(\dfrac{1\text{ fission}}{200\times10^6\text{ eV}}\right)\left(\dfrac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) = 6.74\times10^{24}\text{ d}^{-1}$.

This also is the number of $^{235}\text{U}$ nuclei used, so the mass of $^{235}\text{U}$ used per day is

$$\left(6.74\times10^{24}\text{ nuclei/d}\right)\left(\frac{235\text{ g/mol}}{6.02\times10^{23}\text{ nuclei/mol}}\right) = 2.63\times10^3\text{ g/d} = \boxed{2.63\text{ kg/d}}.$$

In contrast, a coal-burning steam plant producing the same electrical power uses more than $6\times10^6\text{ kg/d}$ of coal.

**P45.9** The available energy to do work is 0.200 times the energy content of the fuel.

$$(1.00\text{ kg fuel})\left(0.034\,0\ ^{235}\text{U/fuel}\right)\left(\frac{1\,000\text{ g}}{1\text{ kg}}\right)\left(\frac{1\text{ mol}}{235\text{ g}}\right)\left(6.02\times10^{23}/\text{mol}\right)\left(\frac{(208)\left(1.60\times10^{-13}\text{ J}\right)}{\text{fission}}\right) = 2.90\times10^{12}\text{ J}$$

$$\left(2.90\times10^{12}\text{ J}\right)(0.200) = 5.80\times10^{11}\text{ J} = \left(1.00\times10^5\text{ N}\right)\Delta r$$

$$\Delta r = 5.80\times10^6\text{ m} = \boxed{5.80\text{ Mm}}$$

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## Section 45.3 **Nuclear Reactors**

**P45.10** (a) For a sphere: $V = \dfrac{4}{3}\pi r^3$ and $r = \left(\dfrac{3V}{4\pi}\right)^{1/3}$ so $\dfrac{A}{V} = \dfrac{4\pi r^2}{(4/3)\pi r^3} = \boxed{4.84V^{-1/3}}$.

(b) For a cube: $V = \ell^3$ and $\ell = V^{1/3}$ so $\dfrac{A}{V} = \dfrac{6\ell^2}{\ell^3} = \boxed{6V^{-1/3}}$.

(c) For a parallelepiped: $V = 2a^3$ and $a = \left(\dfrac{V}{2}\right)^{1/3}$ so $\dfrac{A}{V} = \dfrac{\left(2a^2+8a^2\right)}{2a^3} = \boxed{6.30V^{-1/3}}$.

(d) Therefore, the $\boxed{\text{sphere has the least leakage}}$ and the $\boxed{\text{parallelepiped has the greatest leakage}}$ for a given volume.

**P45.11** mass of $^{235}$U available $\approx (0.007)\left(10^9 \text{ metric tons}\right)\left(\dfrac{10^6 \text{ g}}{1 \text{ metric ton}}\right) = 7\times10^{12} \text{ g}$

number of nuclei $\approx \left(\dfrac{7\times10^{12} \text{ g}}{235 \text{ g/mol}}\right)\left(6.02\times10^{23} \text{ nuclei/mol}\right) = 1.8\times10^{34} \text{ nuclei}$

The energy available from fission (at 208 MeV/event) is

$E \approx \left(1.8\times10^{34} \text{ events}\right)(208 \text{ MeV/event})\left(1.60\times10^{-13} \text{ J/MeV}\right) = 6.0\times10^{23} \text{ J}$.

This would last for a time interval of

$$\Delta t = \frac{E}{\mathscr{P}} \approx \frac{6.0\times10^{23} \text{ J}}{7.0\times10^{12} \text{ J/s}} = \left(8.6\times10^{10} \text{ s}\right)\left(\frac{1 \text{ yr}}{3.16\times10^{7} \text{ s}}\right) \approx \boxed{3\,000 \text{ yr}}.$$

**P45.12** In one minute there are $\dfrac{60.0 \text{ s}}{1.20 \text{ ms}} = 5.00\times10^4 \text{ fissions}$.

So the rate increases by a factor of $(1.000\,25)^{50\,000} = \boxed{2.68\times10^5}$.

**P45.13** $\mathscr{P} = 10.0 \text{ MW} = 1.00\times10^7 \text{ J/s}$

If each decay delivers $1.00 \text{ MeV} = 1.60\times10^{-13} \text{ J}$, then the number of decays/s $= \boxed{6.25\times10^{19} \text{ Bq}}$.

## Section 45.4 **Nuclear Fusion**

**P45.14** (a) The $Q$ value for the D-T reaction is 17.59 MeV.

Specific energy content in fuel for D-T reaction:

$$\frac{(17.59 \text{ MeV})\left(1.60\times10^{-13} \text{ J/MeV}\right)}{(5 \text{ u})\left(1.66\times10^{-27} \text{ kg/u}\right)} = 3.39\times10^{14} \text{ J/kg}$$

$$r_{\text{DT}} = \frac{\left(3.00\times10^{9} \text{ J/s}\right)(3\,600 \text{ s/hr})}{\left(3.39\times10^{14} \text{ J/kg}\right)\left(10^{-3} \text{ kg/g}\right)} = \boxed{31.9 \text{ g/h burning of D and T}}.$$

(b) Specific energy content in fuel for D-D reaction: $Q = \dfrac{1}{2}(3.27+4.03) = 3.65 \text{ MeV}$ average of two $Q$ values

$$\frac{(3.65 \text{ MeV})\left(1.60\times10^{-13} \text{ J/MeV}\right)}{(4 \text{ u})\left(1.66\times10^{-27} \text{ kg/u}\right)} = 8.80\times10^{13} \text{ J/kg}$$

$$r_{\text{DD}} = \frac{\left(3.00\times10^{9} \text{ J/s}\right)(3\,600 \text{ s/hr})}{\left(8.80\times10^{13} \text{ J/kg}\right)\left(10^{-3} \text{ kg/g}\right)} = \boxed{122 \text{ g/h burning of D}}.$$

**P45.15** (a) At closest approach, the electrostatic potential energy equals the total energy $E$.

$$U_f = \frac{k_e(Z_1 e)(Z_2 e)}{r_{\min}} = E:$$

$$E = \frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.6\times10^{-19}\ \text{C}\right)^2 Z_1 Z_2}{1.00\times10^{-14}\ \text{m}} = \boxed{\left(2.30\times10^{-14}\ \text{J}\right)Z_1 Z_2}.$$

(b) For both the D-D and the D-T reactions, $Z_1 = Z_2 = 1$. Thus, the minimum energy required in both cases is

$$E = \left(2.30\times10^{-14}\ \text{J}\right)\left(\frac{1\ \text{MeV}}{1.60\times10^{-13}\ \text{J}}\right) = \boxed{144\ \text{keV}}.$$

Section 45.4 in the text gives more accurate values for the critical ignition temperatures, of about 52 keV for D-D fusion and 6 keV for D-T fusion. The nuclei can fuse by tunneling. A triton moves more slowly than a deuteron at a given temperature. Then D-T collisions last longer than D-D collisions and have much greater tunneling probabilities.

**P45.16** (a) $r_f = r_\text{D} + r_\text{T} = \left(1.20\times10^{-15}\ \text{m}\right)\left[(2)^{1/3}+(3)^{1/3}\right] = \boxed{3.24\times10^{-15}\ \text{m}}$

(b) $$U_f = \frac{k_e e^2}{r_f} = \frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)^2}{3.24\times10^{-15}\ \text{m}} = 7.10\times10^{-14}\ \text{J} = \boxed{444\ \text{keV}}$$

(c) Conserving momentum, $m_\text{D} v_i = (m_\text{D}+m_\text{T})v_f$, or $v_f = \left(\frac{m_\text{D}}{m_\text{D}+m_\text{T}}\right)v_i = \boxed{\frac{2}{5}v_i}$

(d) $K_i + U_i = K_f + U_f$: $$K_i + 0 = \frac{1}{2}(m_\text{D}+m_\text{T})v_f^2 + U_f = \frac{1}{2}(m_\text{D}+m_\text{T})\left(\frac{m_\text{D}}{m_\text{D}+m_\text{T}}\right)^2 v_i^2 + U_f$$

$$K_i + 0 = \left(\frac{m_\text{D}}{m_\text{D}+m_\text{T}}\right)\left(\frac{1}{2}m_\text{D}v_i^2\right) + U_f = \left(\frac{m_\text{D}}{m_\text{D}+m_\text{T}}\right)K_i + U_f$$

$\left(1-\frac{m_\text{D}}{m_\text{D}+m_\text{T}}\right)K_i = U_f$: $$K_i = U_f\left(\frac{m_\text{D}+m_\text{T}}{m_\text{T}}\right) = \frac{5}{3}(444\ \text{keV}) = \boxed{740\ \text{keV}}$$

(e) $\boxed{\text{Possibly by tunneling.}}$

**P45.17** (a) Average KE per particle is $\frac{3}{2}k_\text{B}T = \frac{1}{2}mv^2$.

Therefore, $$v_\text{rms} = \sqrt{\frac{3k_\text{B}T}{m}} = \sqrt{\frac{3\left(1.38\times10^{-23}\ \text{J/K}\right)\left(4.00\times10^{8}\ \text{K}\right)}{2\left(1.67\times10^{-27}\ \text{kg}\right)}} = \boxed{2.23\times10^{6}\ \text{m/s}}.$$

(b) $$t = \frac{x}{v} \sim \frac{0.1\ \text{m}}{10^6\ \text{m/s}} = \boxed{\sim10^{-7}\ \text{s}}$$

**P45.18** (a) $$V=\left(317\times10^{6}\ \text{mi}^{3}\right)\left(\frac{1\,609\ \text{m}}{1\ \text{mi}}\right)^{3}=1.32\times10^{18}\ \text{m}^{3}$$

$$m_{\text{water}}=\rho V=\left(10^{3}\ \text{kg/m}^{3}\right)\left(1.32\times10^{18}\ \text{m}^{3}\right)=1.32\times10^{21}\ \text{kg}$$

$$m_{\text{H}_2}=\left(\frac{M_{\text{H}_2}}{M_{\text{H}_2\text{O}}}\right)m_{\text{H}_2\text{O}}=\left(\frac{2.016}{18.015}\right)\left(1.32\times10^{21}\ \text{kg}\right)=1.48\times10^{20}\ \text{kg}$$

$$m_{\text{Deuterium}}=(0.030\,0\%)m_{\text{H}_2}=\left(0.030\,0\times10^{-2}\right)\left(1.48\times10^{20}\ \text{kg}\right)=4.43\times10^{16}\ \text{kg}$$

The number of deuterium nuclei in this mass is

$$N=\frac{m_{\text{Deuterium}}}{m_{\text{Deuteron}}}=\frac{4.43\times10^{16}\ \text{kg}}{(2.014\ \text{u})\left(1.66\times10^{-27}\ \text{kg/u}\right)}=1.33\times10^{43}.$$

Since two deuterium nuclei are used per fusion, ${}^{2}_{1}\text{H}+{}^{2}_{1}\text{H}\rightarrow{}^{4}_{2}\text{He}+Q$, the number of events is $\frac{N}{2}=6.63\times10^{42}$.

The energy released per event is

$$Q=\left[M_{{}^2\text{H}}+M_{{}^2\text{H}}-M_{{}^4\text{He}}\right]c^{2}=\left[2(2.014\,102)-4.002\,603\right]\text{u}(931.5\ \text{MeV/u})=23.8\ \text{MeV}.$$

The total energy available is then

$$E=\left(\frac{N}{2}\right)Q=\left(6.63\times10^{42}\right)(23.8\ \text{MeV})\left(\frac{1.60\times10^{-13}\ \text{J}}{1\ \text{MeV}}\right)=\boxed{2.53\times10^{31}\ \text{J}}.$$

(b) The time this energy could possibly meet world requirements is

$$\Delta t=\frac{E}{\mathscr{P}}=\frac{2.53\times10^{31}\ \text{J}}{100\left(7.00\times10^{12}\ \text{J/s}\right)}=\left(3.61\times10^{16}\ \text{s}\right)\left(\frac{1\ \text{yr}}{3.16\times10^{7}\ \text{s}}\right)=\boxed{1.14\times10^{9}\ \text{yr}}\sim 1\ \text{billion years}.$$

**P45.19** (a) Including both ions and electrons, the number of particles in the plasma is $N=2nV$ where $n$ is the ion density and $V$ is the volume of the container. Application of Equation 21.6 gives the total energy as

$$E=\frac{3}{2}Nk_{\text{B}}T=3nVk_{\text{B}}T=3\left(2.0\times10^{13}\ \text{cm}^{-3}\right)\left[\left(50\ \text{m}^{3}\right)\left(\frac{10^{6}\ \text{cm}^{3}}{1\ \text{m}^{3}}\right)\right]\left(1.38\times10^{-23}\ \text{J/K}\right)\left(4.0\times10^{8}\ \text{K}\right)$$

$$E=\boxed{1.7\times10^{7}\ \text{J}}$$

(b) From Table 20.2, the heat of vaporization of water is $L_v=2.26\times10^{6}\ \text{J/kg}$. The mass of water that could be boiled away is

$$m=\frac{E}{L_v}=\frac{1.7\times10^{7}\ \text{J}}{2.26\times10^{6}\ \text{J/kg}}=\boxed{7.3\ \text{kg}}.$$

**P45.20** (a) Lawson's criterion for the D-T reaction is $n\tau \geq 10^{14}\ \text{s/cm}^3$. For a confinement time of $\tau = 1.00\ \text{s}$, this requires a minimum ion density of $n = \boxed{10^{14}\ \text{cm}^{-3}}$.

(b) At the ignition temperature of $T = 4.5\times 10^7\ \text{K}$ and the ion density found above, the plasma pressure is

$$P = 2nk_{\text{B}}T = 2\left[\left(10^{14}\ \text{cm}^{-3}\right)\left(\frac{10^6\ \text{cm}^3}{1\ \text{m}^3}\right)\right]\left(1.38\times 10^{-23}\ \text{J/K}\right)\left(4.5\times 10^7\ \text{K}\right) = \boxed{1.24\times 10^5\ \text{J/m}^3}.$$

(c) The required magnetic energy density is then

$$u_B = \frac{B^2}{2\mu_0} \geq 10P = 10\left(1.24\times 10^5\ \text{J/m}^3\right) = 1.24\times 10^6\ \text{J/m}^3,$$

$$B \geq \sqrt{2\left(4\pi\times 10^{-7}\ \text{N/A}^2\right)\left(1.24\times 10^6\ \text{J/m}^3\right)} = \boxed{1.77\ \text{T}}.$$

**P45.21** Let the number of $^6\text{Li}$ atoms, each having mass 6.015 u, be $N_6$ while the number of $^7\text{Li}$ atoms, each with mass 7.016 u, is $N_7$.

Then, $N_6 = 7.50\%$ of $N_{\text{total}} = 0.075\,0(N_6 + N_7)$, or $N_7 = \left(\frac{0.925}{0.075\,0}\right)N_6$.

Also, total mass $= \left[N_6(6.015\ \text{u}) + N_7(7.016\ \text{u})\right]\left(1.66\times 10^{-27}\ \text{kg/u}\right) = 2.00\ \text{kg}$,

or $N_6\left[(6.015\ \text{u}) + \left(\frac{0.925}{0.075\,0}\right)(7.016\ \text{u})\right]\left(1.66\times 10^{-27}\ \text{kg/u}\right) = 2.00\ \text{kg}$.

This yields $N_6 = \boxed{1.30\times 10^{25}}$ as the number of $^6\text{Li}$ atoms and

$N_7 = \left(\frac{0.925}{0.075\,0}\right)\left(1.30\times 10^{25}\right) = \boxed{1.61\times 10^{26}}$ as the number of $^7\text{Li}$ atoms.

**P45.22** The number of nuclei in 1.00 metric ton of trash is

$$N = 1\,000\ \text{kg}(1\,000\ \text{g/kg})\frac{6.02\times 10^{23}\ \text{nuclei/mol}}{56.0\ \text{g/mol}} = 1.08\times 10^{28}\ \text{nuclei}.$$

At an average charge of 26.0 e/nucleus, $q = \left(1.08\times 10^{28}\right)(26.0)\left(1.60\times 10^{-19}\right) = 4.47\times 10^{10}\ \text{C}$.

Therefore $t = \frac{q}{I} = \frac{4.47\times 10^{10}}{1.00\times 10^6} = 4.47\times 10^4\ \text{s} = \boxed{12.4\ \text{h}}$.

## Section 45.5 **Radiation Damage**

**P45.23** $$N_0 = \frac{\text{mass present}}{\text{mass of nucleus}} = \frac{5.00\text{ kg}}{(89.9077\text{ u})(1.66\times10^{-27}\text{ kg/u})} = 3.35\times10^{25}\text{ nuclei}$$

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{29.1\text{ yr}} = 2.38\times10^{-2}\text{ yr}^{-1} = 4.52\times10^{-8}\text{ min}^{-1}$$

$$R_0 = \lambda N_0 = \left(4.52\times10^{-8}\text{ min}^{-1}\right)\left(3.35\times10^{25}\right) = 1.52\times10^{18}\text{ counts/min}$$

$$\frac{R}{R_0} = e^{-\lambda t} = \frac{10.0\text{ counts/min}}{1.52\times10^{18}\text{ counts/min}} = 6.60\times10^{-18}$$

and $$\lambda t = -\ln\left(6.60\times10^{-18}\right) = 39.6$$

giving $$t = \frac{39.6}{\lambda} = \frac{39.6}{2.38\times10^{-2}\text{ yr}^{-1}} = \boxed{1.66\times10^{3}\text{ yr}}.$$

**P45.24** Source: 100 mrad of 2-MeV $\gamma$-rays/h at a 1.00-m distance.

(a) For $\gamma$-rays, dose in rem = dose in rad.

Thus a person would have to stand $\boxed{\text{10.0 hours}}$ to receive 1.00 rem from a 100-mrad/h source.

(b) If the $\gamma$-radiation is emitted isotropically, the dosage rate falls off as $\frac{1}{r^2}$.

Thus a dosage 10.0 mrad/h would be received at a distance $r = \sqrt{10.0}\text{ m} = \boxed{3.16\text{ m}}$.

**P45.25** (a) The number of x-rays taken per year is

$n = (8\text{ x-ray/d})(5\text{ d/wk})(50\text{ wk/yr}) = 2.0\times10^{3}\text{ x-ray/yr}$.

The average dose per photograph is $$\frac{5.0\text{ rem/yr}}{2.0\times10^{3}\text{ x-ray/yr}} = \boxed{2.5\times10^{-3}\text{ rem/x-ray}}.$$

(b) The technician receives low-level background radiation at a rate of $0.13\text{ rem/yr}$. The dose of $5.0\text{ rem/yr}$ received as a result of the job is

$$\frac{5.0\text{ rem/yr}}{0.13\text{ rem/yr}} = \boxed{\text{38 times background levels}}.$$

**P45.26** (a) $I = I_0 e^{-\mu x}$, so $$x = \frac{1}{\mu}\ln\left(\frac{I_0}{I}\right)$$

With $\mu = 1.59\text{ cm}^{-1}$, the thickness when $I = \frac{I_0}{2}$ is $$x = \frac{1}{1.59\text{ cm}^{-1}}\ln(2) = \boxed{0.436\text{ cm}}.$$

(b) When $\frac{I_0}{I} = 1.00\times10^{4}$, $$x = \frac{1}{1.59\text{ cm}^{-1}}\ln\left(1.00\times10^{4}\right) = \boxed{5.79\text{ cm}}.$$

**P45.27** $1\text{ rad} = 10^{-2}\text{ J/kg} \qquad Q = mc\Delta T \qquad \mathcal{P}\Delta t = mc\Delta T$

$$\Delta t = \frac{mc\Delta T}{\mathcal{P}} = \frac{m(4\,186\text{ J/kg}\cdot{}^\circ\text{C})(50.0^\circ\text{C})}{(10)\left(10^{-2}\text{ J/kg}\cdot\text{s}\right)(m)} = \boxed{2.09\times10^{6}\text{ s}} \approx 24\text{ days!}$$

Note that power is the product of dose rate and mass.

**P45.28** $\dfrac{Q}{m} = \dfrac{\text{absorbed energy}}{\text{unit mass}} = (1\,000\text{ rad})\dfrac{10^{-2}\text{ J/kg}}{1\text{ rad}} = 10.0\text{ J/kg}$

The rise in body temperature is calculated from $Q = mc\Delta T$ where $c = 4\,186\text{ J/kg}$ for water and the human body

$$\Delta T = \frac{Q}{mc} = (10.0\text{ J/kg})\frac{1}{4\,186\text{ J/kg}\cdot{}^\circ\text{C}} = \boxed{2.39\times10^{-3}\,{}^\circ\text{C}}\ \text{(Negligible).}$$

**P45.29** If half of the 0.140-MeV gamma rays are absorbed by the patient, the total energy absorbed is

$$E = \frac{(0.140\text{ MeV})}{2}\left[\left(\frac{1.00\times10^{-8}\text{ g}}{98.9\text{ g/mol}}\right)\left(\frac{6.02\times10^{23}\text{ nuclei}}{1\text{ mol}}\right)\right] = 4.26\times10^{12}\text{ MeV}$$

$$E = \left(4.26\times10^{12}\text{ MeV}\right)\left(1.60\times10^{-13}\text{ J/MeV}\right) = 0.682\text{ J}$$

Thus, the dose received is $\quad \text{Dose} = \dfrac{0.682\text{ J}}{60.0\text{ kg}}\left(\dfrac{1\text{ rad}}{10^{-2}\text{ J/kg}}\right) = \boxed{1.14\text{ rad}}$.

**P45.30** The nuclei initially absorbed are $\quad N_0 = \left(1.00\times10^{-9}\text{ g}\right)\left(\dfrac{6.02\times10^{23}\text{ nuclei/mol}}{89.9\text{ g/mol}}\right) = 6.70\times10^{12}$.

The number of decays in time *t* is $\quad \Delta N = N_0 - N = N_0\left(1 - e^{-\lambda t}\right) = N_0\left(1 - e^{-(\ln 2)t/T_{1/2}}\right)$.

At the end of 1 year, $\quad \dfrac{t}{T_{1/2}} = \dfrac{1.00\text{ yr}}{29.1\text{ yr}} = 0.034\,4$

and $\quad \Delta N = N_0 - N = \left(6.70\times10^{12}\right)\left(1 - e^{-0.023\,8}\right) = 1.58\times10^{11}$.

The energy deposited is $\quad E = \left(1.58\times10^{11}\right)(1.10\text{ MeV})\left(1.60\times10^{-13}\text{ J/MeV}\right) = 0.027\,7\text{ J}$.

Thus, the dose received is $\quad \text{Dose} = \left(\dfrac{0.027\,7\text{ J}}{70.0\text{ kg}}\right) = \boxed{3.96\times10^{-4}\text{ J/kg}} = 0.039\,6\text{ rad}$.

## Section 45.6 **Radiation Detectors**

**P45.31** (a) $$\frac{E}{E_\beta} = \frac{(1/2)C(\Delta V)^2}{0.500\text{ MeV}} = \frac{(1/2)\left(5.00\times10^{-12}\text{ F}\right)\left(1.00\times10^{3}\text{ V}\right)^2}{(0.500\text{ MeV})\left(1.60\times10^{-13}\text{ J/MeV}\right)} = \boxed{3.12\times10^{7}}$$

(b) $$N = \frac{Q}{e} = \frac{C(\Delta V)}{e} = \frac{\left(5.00\times10^{-12}\text{ F}\right)\left(1.00\times10^{3}\text{ V}\right)}{1.60\times10^{-19}\text{ C}} = \boxed{3.12\times10^{10}\text{ electrons}}$$

**P45.32** (a) $E_I = 10.0\text{ eV}$ is the energy required to liberate an electron from a dynode. Let $n_i$ be the number of electrons incident upon a dynode, each having gained energy $e(\Delta V)$ as it was accelerated to this dynode. The number of electrons that will be freed from this dynode is $N_i = n_i e\dfrac{\Delta V}{E_I}$:

At the first dynode, $n_i = 1$ and $N_1 = \dfrac{(1)e(100\text{ V})}{10.0\text{ eV}} = \boxed{10^1\text{ electrons}}$.

(b) For the second dynode, $n_i = N_1 = 10^1$, so $N_2 = \dfrac{(10^1)e(100\text{ V})}{10.0\text{ eV}} = 10^2$.

At the third dynode, $n_i = N_2 = 10^2$ and $N_3 = \dfrac{(10^2)e(100\text{ V})}{10.0\text{ eV}} = 10^3$.

Observing the developing pattern, we see that the number of electrons incident on the seventh and last dynode is $n_7 = N_6 = \boxed{10^6}$.

(c) The number of electrons incident on the last dynode is $n_7 = 10^6$. The total energy these electrons deliver to that dynode is given by

$E = n_i e(\Delta V) = 10^6 e(700\text{ V} - 600\text{ V}) = \boxed{10^8\text{ eV}}$.

**P45.33** (a) The average time between slams is $60\text{ min}/38 = 1.6\text{ min}$. Sometimes, the actual interval is nearly zero. Perhaps about equally as often, it is $2 \times 1.6\text{ min}$. Perhaps about half as often, it is $4 \times 1.6\text{ min}$. Somewhere around $5 \times 1.6\text{ min} = \boxed{8.0\text{ min}}$, the chances of randomness producing so long a wait get slim, so such a long wait might likely be due to mischief.

(b) The midpoints of the time intervals are separated by 5.00 minutes. We use $R = R_0 e^{-\lambda t}$. Subtracting the background counts,

$$337 - 5(15) = \left[372 - 5(15)\right]e^{-\left(\ln 2/T_{1/2}\right)(5.00\text{ min})}$$

or $$\ln\left(\frac{262}{297}\right) = \ln(0.882) = -3.47\text{ min}/T_{1/2} \text{ which yields } T_{1/2} = \boxed{27.6\text{ min}}.$$

(c) As in the random events in part (a), we imagine a $\pm 5$ count counting uncertainty. The smallest likely value for the half-life is then given by

$$\ln\left(\frac{262 - 5}{297 + 5}\right) = -3.47\text{ min}/T_{1/2}\text{, or } \left(T_{1/2}\right)_{\text{min}} = 21.1\text{ min}.$$

The largest credible value is found from

$$\ln\left(\frac{262 + 5}{297 - 5}\right) = -3.47\text{ min}/T_{1/2}\text{, yielding } \left(T_{1/2}\right)_{\text{max}} = 38.8\text{ min}.$$

Thus, $$T_{1/2} = \left(\frac{38.8 + 21.1}{2}\right) \pm \left(\frac{38.8 - 21.1}{2}\right)\text{ min} = (30 \pm 9)\text{ min} = \boxed{30\text{ min} \pm 30\%}.$$

## Section 45.7 **Uses of Radiation**

**P45.34** The initial specific activity of $^{59}\text{Fe}$ in the steel,

$$(R/m)_0 = \frac{20.0\ \mu\text{Ci}}{0.200\ \text{kg}} = \frac{100\ \mu\text{Ci}}{\text{kg}}\left(\frac{3.70\times 10^4\ \text{Bq}}{1\ \mu\text{Ci}}\right) = 3.70\times 10^6\ \text{Bq/kg}.$$

After 1 000 h, $\quad \dfrac{R}{m} = \left(\dfrac{R}{m}\right)_0 e^{-\lambda t} = \left(3.70\times 10^6\ \text{Bq/kg}\right)e^{-\left(6.40\times 10^{-4}\ \text{h}^{-1}\right)(1\,000\ \text{h})} = 1.95\times 10^6\ \text{Bq/kg}.$

The activity of the oil, $\quad R_{\text{oil}} = \left(\dfrac{800}{60.0}\ \text{Bq/liter}\right)(6.50\ \text{liters}) = 86.7\ \text{Bq}.$

Therefore, $\quad m_{\text{in oil}} = \dfrac{R_{\text{oil}}}{(R/m)} = \dfrac{86.7\ \text{Bq}}{1.95\times 10^6\ \text{Bq/kg}} = 4.45\times 10^{-5}\ \text{kg}.$

So that wear rate is $\quad \dfrac{4.45\times 10^{-5}\ \text{kg}}{1\,000\ \text{h}} = \boxed{4.45\times 10^{-8}\ \text{kg/h}}.$

**P45.35** The half-life of $^{14}\text{O}$ is 70.6 s, so the decay constant is $\lambda = \dfrac{\ln 2}{T_{1/2}} = \dfrac{\ln 2}{70.6\ \text{s}} = 0.009\,82\ \text{s}^{-1}$.

The $^{14}\text{O}$ nuclei remaining after five min is $N = N_0 e^{-\lambda t} = \left(10^{10}\right)e^{-\left(0.009\,82\ \text{s}^{-1}\right)(300\ \text{s})} = 5.26\times 10^8$.

The number of these in one cubic centimeter of blood is

$$N' = N\left(\frac{1.00\ \text{cm}^3}{\text{total vol. of blood}}\right) = \left(5.26\times 10^8\right)\left(\frac{1.00\ \text{cm}^3}{2000\ \text{cm}^3}\right) = 2.63\times 10^5$$

and their activity is $\quad R = \lambda N' = \left(0.009\ 82\ \text{s}^{-1}\right)\left(2.63\times 10^5\right) = 2.58\times 10^3\ \text{Bq}\ \boxed{\sim 10^3\ \text{Bq}}.$

**P45.36** (a) The number of photons is $\dfrac{10^4\ \text{MeV}}{1.04\ \text{MeV}} = 9.62\times 10^3$. Since only 50% of the photons are detected, the number of $^{65}\text{Cu}$ nuclei decaying is twice this value, or $1.92\times 10^4$. In two half-lives, three-fourths of the original nuclei decay, so $\dfrac{3}{4}N_0 = 1.92\times 10^4$ and $N_0 = 2.56\times 10^4$. This is 1% of the $^{65}\text{Cu}$, so the number of $^{65}\text{Cu}$ is $2.56\times 10^6$ $\boxed{\sim 10^6}$.

(b) Natural copper is 69.17% $^{63}\text{Cu}$ and 30.83% $^{65}\text{Cu}$. Thus, if the sample contains $N_{\text{Cu}}$ copper atoms, the number of atoms of each isotope is

$N_{63} = 0.691\,7 N_{\text{Cu}}$ and $N_{65} = 0.308\,3 N_{\text{Cu}}$.

Therefore, $\dfrac{N_{63}}{N_{65}} = \dfrac{0.691\,7}{0.308\,3}$ or $N_{63} = \left(\dfrac{0.6917}{0.3083}\right)N_{65} = \left(\dfrac{0.6917}{0.3083}\right)\left(2.56\times 10^6\right) = 5.75\times 10^6$.

The total mass of copper present is then $\quad m_{\text{Cu}} = (62.93\ \text{u})N_{63} + (64.93\ \text{u})N_{65}$:

$$m_{\text{Cu}} = \left[(62.93)\left(5.75\times 10^6\right) + (64.93)\left(2.56\times 10^6\right)\right]\text{u}\ \left(1.66\times 10^{-24}\ \text{g/u}\right)$$

$$= 8.77\times 10^{-16}\ \text{g}\ \boxed{\sim 10^{-15}\ \text{g}}$$

**P45.37** (a) Starting with $N=0$ radioactive atoms at $t=0$, the rate of increase is (production – decay)

$$\frac{dN}{dt}=R-\lambda N \qquad \text{so} \qquad dN=(R-\lambda N)dt.$$

The variables are separable.

$$\int_0^N \frac{dN}{R-\lambda N}=\int_0^t dt: \qquad -\frac{1}{\lambda}\ln\left(\frac{R-\lambda N}{R}\right)=t$$

so $$\ln\left(\frac{R-\lambda N}{R}\right)=-\lambda t \qquad \text{and} \qquad \left(\frac{R-\lambda N}{R}\right)=e^{-\lambda t}.$$

Therefore, $$1-\frac{\lambda}{R}N=e^{-\lambda t} \qquad N=\boxed{\frac{R}{\lambda}\left(1-e^{-\lambda t}\right)}.$$

(b) The maximum number of radioactive nuclei would be $\boxed{\frac{R}{\lambda}}$.

## Additional Problems

**P45.38** (a) Suppose each $^{235}\text{U}$ fission releases 208 MeV of energy. Then, the number of nuclei that must have undergone fission is

$$N=\frac{\text{total release}}{\text{energy per nuclei}}=\frac{5\times10^{13}\ \text{J}}{(208\ \text{MeV})\left(1.60\times10^{-13}\ \text{J/MeV}\right)}=\boxed{1.5\times10^{24}\ \text{nuclei}}.$$

(b) $$\text{mass}=\left(\frac{1.5\times10^{24}\ \text{nuclei}}{6.02\times10^{23}\ \text{nuclei/mol}}\right)(235\ \text{g/mol})\approx\boxed{0.6\ \text{kg}}$$

**P45.39** (a) At $6\times10^8\ \text{K}$, the average kinetic energy of a carbon atom is

$$\frac{3}{2}k_BT=(1.5)\left(8.62\times10^{-5}\ \text{eV/K}\right)\left(6\times10^8\ \text{K}\right)=\boxed{8\times10^4\ \text{eV}}$$

Note that $6\times10^8\ \text{K}$ is about $6^2=36$ times larger than $1.5\times10^7\ \text{K}$, the core temperature of the Sun. This factor corresponds to the higher potential-energy barrier to carbon fusion compared to hydrogen fusion. It could be misleading to compare it to the temperature $\sim10^8\ \text{K}$ required for fusion in a low-density plasma in a fusion reactor.

(b) The energy released is

$$E=\left[2m\left(\text{C}^{12}\right)-m\left(\text{Ne}^{20}\right)-m\left(\text{He}^4\right)\right]c^2$$

$$E=(24.000\,000-19.992\,440-4.002\,603)(931.5)\ \text{MeV}=\boxed{4.62\ \text{MeV}}$$

In the second reaction,

$$E=\left[2m\left(\text{C}^{12}\right)-m\left(\text{Mg}^{24}\right)\right](931.5)\ \text{MeV/u}$$

$$E=(24.000\,000-23.985\,042)(931.5)\ \text{MeV}=\boxed{13.9\ \text{MeV}}$$

*continued on next page*

(c) The energy released is the energy of reaction of the number of carbon nuclei in a 2.00-kg sample, which corresponds to

$$\Delta E = \left(2.00\times10^{3}\text{ g}\right)\left(\frac{6.02\times10^{23}\text{ atoms/mol}}{12.0\text{ g/mol}}\right)\left(\frac{4.62\text{ MeV/fusion event}}{2\text{ nuclei/fusion event}}\right)\left(\frac{1\text{ kWh}}{2.25\times10^{19}\text{ MeV}}\right)$$

$$\Delta E = \frac{\left(1.00\times10^{26}\right)(4.62)}{2\left(2.25\times10^{19}\right)}\text{kWh} = \boxed{1.03\times10^{7}\text{ kWh}}$$

**P45.40** To conserve momentum, the two fragments must move in opposite directions with speeds $v_1$ and $v_2$ such that

$$m_1v_1 = m_2v_2 \qquad \text{or} \qquad v_2 = \left(\frac{m_1}{m_2}\right)v_1.$$

The kinetic energies after the break-up are then

$$K_1 = \frac{1}{2}m_1v_1^2 \qquad \text{and} \qquad K_2 = \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_2\left(\frac{m_1}{m_2}\right)^2 v_1^2 = \left(\frac{m_1}{m_2}\right)K_1.$$

The fraction of the total kinetic energy carried off by $m_1$ is $\frac{K_1}{K_1+K_2} = \frac{K_1}{K_1+(m_1/m_2)K_1} = \boxed{\frac{m_2}{m_1+m_2}}$

and the fraction carried off by $m_2$ is $1-\frac{m_2}{m_1+m_2} = \boxed{\frac{m_1}{m_1+m_2}}$.

***P45.41** (a) $Q = 236.045\,562\text{u}\,c^2 - 86.920\,711\text{u}\,c^2 - 148.934\,370\text{u}\,c^2 = 0.190\,481\text{u}\,c^2 = \boxed{177\text{ MeV}}$

Immediately after fission, this *Q*-value is the total kinetic energy of the fission products.

(b) $K_{\text{Br}} = \left(\frac{m_{\text{La}}}{m_{\text{Br}}+m_{\text{La}}}\right)Q$, from Problem 45.40.

$$= \left(\frac{149\text{ u}}{87\text{ u}+149\text{ u}}\right)(177.4\text{ MeV}) = \boxed{112\text{ MeV}}$$

$$K_{\text{La}} = Q - K_{\text{Br}} = 177.4\text{ MeV} - 112.0\text{ MeV} = \boxed{65.4\text{ MeV}}$$

(c) $$v_{\text{Br}} = \sqrt{\frac{2K_{\text{Br}}}{m_{\text{Br}}}} = \sqrt{\frac{2\left(112\times10^{6}\text{ eV}\right)\left(1.6\times10^{-19}\text{ J/eV}\right)}{(87\text{ u})\left(1.66\times10^{-27}\text{ kg/u}\right)}} = \boxed{1.58\times10^{7}\text{ m/s}}$$

$$v_{\text{La}} = \sqrt{\frac{2K_{\text{La}}}{m_{\text{La}}}} = \sqrt{\frac{2\left(65.4\times10^{6}\text{ eV}\right)\left(1.6\times10^{-19}\text{ J/eV}\right)}{(149\text{ u})\left(1.66\times10^{-27}\text{ kg/u}\right)}} = \boxed{9.20\times10^{6}\text{ m/s}}$$

**P45.42** For a typical $^{235}\text{U}$, $Q = 208\text{ MeV}$; and the initial mass is 235 u. Thus, the fractional energy loss is

$$\frac{Q}{mc^2} = \frac{208\text{ MeV}}{(235\text{ u})(931.5\text{ MeV/u})} = 9.50 \times 10^{-4} = \boxed{0.095\,0\%}.$$

For the D-T fusion reaction, $Q = 17.6\text{ MeV}$.

The initial mass is $m = (2.014\text{ u}) + (3.016\text{ u}) = 5.03\text{ u}$.

The fractional loss in this reaction is $\dfrac{Q}{mc^2} = \dfrac{17.6\text{ MeV}}{(5.03\text{ u})(931.5\text{ MeV/u})} = 3.75 \times 10^{-3} = \boxed{0.375\%}$

$$\frac{0.375\%}{0.0950\%} = 3.95 \text{ or } \boxed{\text{the fractional loss in D-T is about 4 times that in } ^{235}\text{U fission}}.$$

**P45.43** The decay constant is $\lambda = \dfrac{\ln 2}{T_{1/2}} = \dfrac{\ln 2}{(12.3\text{ yr})(3.16 \times 10^7\text{ s/yr})} = 1.78 \times 10^{-9}\text{ s}^{-1}$.

The tritium in the plasma decays at a rate of

$$R = \lambda N = \left(1.78 \times 10^{-9}\text{ s}^{-1}\right)\left[\left(\frac{2.00 \times 10^{14}}{\text{cm}^3}\right)\left(\frac{10^6\text{ cm}^3}{1\text{ m}^3}\right)\left(50.0\text{ m}^3\right)\right]$$

$$R = 1.78 \times 10^{13}\text{ Bq} = \left(1.78 \times 10^{13}\text{ Bq}\right)\left(\frac{1\text{ Ci}}{3.70 \times 10^{10}\text{ Bq}}\right) = \boxed{482\text{ Ci}}.$$

$\boxed{\text{The fission inventory is } \dfrac{4 \times 10^{10}\text{ Ci}}{482\text{ Ci}} \sim 10^8 \text{ times greater}}$ than this amount.

**P45.44** Momentum conservation: $0 = m_{\text{Li}}\mathbf{v}_{\text{Li}} + m_\alpha \mathbf{v}_\alpha$, or, $m_{\text{Li}}v_{\text{Li}} = m_\alpha v_\alpha$.

Thus,

$$K_{\text{Li}} = \frac{1}{2}m_{\text{Li}}v_{\text{Li}}^2 = \frac{1}{2}\frac{(m_{\text{Li}}v_{\text{Li}})^2}{m_{\text{Li}}} = \frac{(m_\alpha v_\alpha)^2}{2m_{\text{Li}}} = \left(\frac{m_\alpha^2}{2m_{\text{Li}}}\right)v_\alpha^2$$

$$K_{\text{Li}} = \left(\frac{(4.002\,6\text{ u})^2}{2(7.016\,0\text{ u})}\right)\left(9.25 \times 10^6\text{ m/s}\right)^2 = (1.14\text{ u})\left(9.25 \times 10^6\text{ m/s}\right)^2$$

$$K_{\text{Li}} = 1.14\left(1.66 \times 10^{-27}\text{ kg}\right)\left(9.25 \times 10^6\text{ m/s}\right)^2 = 1.62 \times 10^{-13}\text{ J} = \boxed{1.01\text{ MeV}}.$$

**P45.45** The complete fissioning of 1.00 gram of $\text{U}^{235}$ releases

$$Q = \frac{(1.00\text{ g})}{235\text{ grams/mol}}\left(6.02 \times 10^{23}\text{ atoms/mol}\right)(200\text{ MeV/fission})\left(1.60 \times 10^{-13}\text{ J/MeV}\right) = 8.20 \times 10^{10}\text{ J}.$$

If all this energy could be utilized to convert $m$ kilograms of 20.0°C water to 400°C steam (see Chapter 20 of text for values),

then $Q = mc_w\,\Delta T + mL_v + mc_s\,\Delta T$

$$Q = m\left[(4186\text{ J/kg }^\circ\text{C})(80.0\text{ }^\circ\text{C}) + 2.26 \times 10^6\text{ J/kg} + (2010\text{ J/kg }^\circ\text{C})(300\text{ }^\circ\text{C})\right].$$

Therefore $m = \dfrac{8.20 \times 10^{10}\text{ J}}{3.20 \times 10^6\text{ J/kg}} = \boxed{2.56 \times 10^4\text{ kg}}$.

**P45.46** When mass $m$ of $^{235}\text{U}$ undergoes complete fission, releasing 200 MeV per fission event, the total energy released is:

$$Q=\left(\frac{m}{235\ \text{g/mol}}\right)N_\text{A}(200\ \text{MeV})\ \text{where } N_\text{A}\ \text{is Avogadro's number.}$$

If all this energy could be utilized to convert a mass $m_w$ of liquid water at $T_c$ into steam at $T_h$, then,

$$Q=m_w\left[c_w(100°\text{C}-T_c)+L_v+c_s(T_h-100°\text{C})\right]$$

where $c_w$ is the specific heat of liquid water, $L_v$ is the latent heat of vaporization, and $c_s$ is the specific heat of steam. Solving for the mass of water converted gives

$$m_w=\frac{Q}{\left[c_w(100°\text{C}-T_c)+L_v+c_s(T_h-100°\text{C})\right]}=\boxed{\frac{mN_\text{A}(200\ \text{MeV})}{(235\ \text{g/mol})\left[c_w(100°\text{C}-T_c)+L_v+c_s(T_h-100°\text{C})\right]}}.$$

**P45.47** (a) The number of molecules in 1.00 liter of water (mass = 1 000 g) is

$$N=\left(\frac{1.00\times10^{3}\ \text{g}}{18.0\ \text{g/mol}}\right)\left(6.02\times10^{23}\ \text{molecules/mol}\right)=3.34\times10^{25}\ \text{molecules}.$$

The number of deuterium nuclei contained in these molecules is

$$N'=\left(3.34\times10^{25}\ \text{molecules}\right)\left(\frac{1\ \text{deuteron}}{3300\ \text{molecules}}\right)=1.01\times10^{22}\ \text{deuterons}.$$

Since 2 deuterons are consumed per fusion event, the number of events possible is $\frac{N'}{2}=5.07\times10^{21}$ reactions, and the energy released is

$$E_\text{fusion}=\left(5.07\times10^{21}\ \text{reactions}\right)(3.27\ \text{MeV/reaction})=1.66\times10^{22}\ \text{MeV}$$

$$E_\text{fusion}=\left(1.66\times10^{22}\ \text{MeV}\right)\left(1.60\times10^{-13}\ \text{J/MeV}\right)=\boxed{2.65\times10^{9}\ \text{J}}.$$

(b) In comparison to burning 1.00 liter of gasoline, the energy from the fusion of deuterium is

$$\frac{E_\text{fusion}}{E_\text{gasoline}}=\frac{2.65\times10^{9}\ \text{J}}{3.40\times10^{7}\ \text{J}}==\boxed{78.0\ \text{times larger}}.$$

**P45.48** (a) $\Delta V=4\pi r^2\Delta r=4\pi\left(14.0\times10^{3}\ \text{m}\right)^2(0.05\ \text{m})=1.23\times10^{8}\ \text{m}^3\ \boxed{\sim10^{8}\ \text{m}^3}$

(b) The force on the next layer is determined by atmospheric pressure.

$$W=P\Delta V=\left(1.013\times10^{5}\ \text{N/m}^2\right)\left(1.23\times10^{8}\ \text{m}^3\right)=1.25\times10^{13}\ \text{J}\ \boxed{\sim10^{13}\ \text{J}}$$

(c) $1.25\times10^{13}\ \text{J}=\frac{1}{10}(\text{yield})$, so yield $=1.25\times10^{14}\ \text{J}\ \boxed{\sim10^{14}\ \text{J}}$

(d) $$\frac{1.25\times10^{14}\ \text{J}}{4.2\times10^{9}\ \text{J/ton TNT}}=2.97\times10^{4}\ \text{ton TNT}\sim10^{4}\ \text{ton TNT}$$

or $\boxed{\sim10\ \text{kilotons}}$

**P45.49** (a) The thermal power transferred to the water is $\mathcal{P}_w = 0.970(\text{waste heat})$

$$\mathcal{P}_w = 0.970(3\,065 - 1\,000)\text{MW} = 2.00 \times 10^9 \text{ J/s}$$

$r_w$ is the mass of water heated per hour:

$$r_w = \frac{\mathcal{P}_w}{c(\Delta T)} = \frac{(2.00 \times 10^9 \text{ J/s})(3600 \text{ s/h})}{(4186 \text{ J/kg}\cdot{}^\circ\text{C})(3.50\ {}^\circ\text{C})} = \boxed{4.91 \times 10^8 \text{ kg/h}}.$$

The volume used per hour is $\dfrac{4.91 \times 10^8 \text{ kg/h}}{1.00 \times 10^3 \text{ kg/m}^3} = \boxed{4.91 \times 10^5 \text{ m}^3\text{/h}}$.

(b) The ${}^{235}\text{U}$ fuel is consumed at a rate $r_f = \left(\dfrac{3\,065 \times 10^6 \text{ J/s}}{7.80 \times 10^{10} \text{ J/g}}\right)\left(\dfrac{1 \text{ kg}}{1\,000 \text{ g}}\right)\left(\dfrac{3\,600 \text{ s}}{1 \text{ h}}\right) = \boxed{0.141 \text{ kg/h}}$.

**P45.50** The number of nuclei in 0.155 kg of ${}^{210}\text{Po}$ is

$$N_0 = \left(\frac{155 \text{ g}}{209.98 \text{ g/mol}}\right)(6.02 \times 10^{23} \text{ nuclei/mol}) = 4.44 \times 10^{23} \text{ nuclei}.$$

The half-life of ${}^{210}\text{Po}$ is 138.38 days, so the decay constant is given by

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{(138.38 \text{ d})(8.64 \times 10^4 \text{ s/d})} = 5.80 \times 10^{-8} \text{ s}^{-1}.$$

The initial activity is

$$R_0 = \lambda N_0 = (5.80 \times 10^{-8} \text{ s}^{-1})(4.44 \times 10^{23} \text{ nuclei}) = 2.58 \times 10^{16} \text{ Bq}.$$

The energy released in each ${}^{210}_{84}\text{Po} \to {}^{206}_{82}\text{Pb} + {}^{4}_{2}\text{He}$ reaction is

$$Q = \left[M_{{}^{210}_{84}\text{Po}} - M_{{}^{206}_{82}\text{Pb}} - M_{{}^{4}_{2}\text{He}}\right]c^2:$$

$$Q = [209.982\,857 - 205.974\,449 - 4.002\,603]\text{u}(931.5 \text{ MeV/u}) = 5.41 \text{ MeV}.$$

Thus, assuming a conversion efficiency of 1.00%, the initial power output of the battery is

$$\mathcal{P} = (0.010\,0)R_0Q = (0.010\,0)(2.58 \times 10^{16} \text{ decays/s})(5.41 \text{ MeV/decay})(1.60 \times 10^{-13} \text{ J/MeV}) = \boxed{223 \text{ W}}.$$

**P45.51** (a) $V = \ell^3 = \dfrac{m}{\rho}$, so $\ell = \left(\dfrac{m}{\rho}\right)^{1/3} = \left(\dfrac{70.0 \text{ kg}}{18.7 \times 10^3 \text{ kg/m}^3}\right)^{1/3} = \boxed{0.155 \text{ m}}$

(b) Add 92 electrons to both sides of the given nuclear reaction. Then it becomes ${}^{238}_{92}\text{U atom} \to 8\ {}^{4}_{2}\text{He atom} + {}^{206}_{82}\text{Pb atom} + Q_{\text{net}}$.

$$Q_{\text{net}} = \left[M_{{}^{238}_{92}\text{U}} - 8M_{{}^{4}_{2}\text{He}} - M_{{}^{206}_{82}\text{Pb}}\right]c^2 = [238.050\,783 - 8(4.002\,603) - 205.974\,449]\text{u}\ (931.5 \text{ MeV/u})$$

$$Q_{\text{net}} = \boxed{51.7 \text{ MeV}}$$

(c) If there is a single step of decay, the number of decays per time is the decay rate $R$ and the energy released in each decay is $Q$. Then the energy released per time is $\boxed{\mathcal{P} = QR}$. If there is a series of decays in steady state, the equation is still true, with $Q$ representing the net decay energy.

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(d) The decay rate for all steps in the radioactive series in steady state is set by the parent uranium:

$$N=\left(\frac{7.00\times10^4\ \text{g}}{238\ \text{g/mol}}\right)\left(6.02\times10^{23}\ \text{nuclei/mol}\right)=1.77\times10^{26}\ \text{nuclei}$$

$$\lambda=\frac{\ln 2}{T_{1/2}}=\frac{\ln 2}{4.47\times10^{9}\ \text{yr}}=1.55\times10^{-10}\ \frac{1}{\text{yr}}$$

$$R=\lambda N=\left(1.55\times10^{-10}\ \frac{1}{\text{yr}}\right)\left(1.77\times10^{26}\ \text{nuclei}\right)=2.75\times10^{16}\ \text{decays/yr},$$

so $\mathscr{P}=QR=(51.7\ \text{MeV})\left(2.75\times10^{16}\ \frac{1}{\text{yr}}\right)\left(1.60\times10^{-13}\ \text{J/MeV}\right)=\boxed{2.27\times10^{5}\ \text{J/yr}}$.

(e) dose in rem = dose in rad × RBE

$5.00\ \text{rem/yr}=(\text{dose in rad/yr})1.10$, giving $(\text{dose in rad/yr})=4.55\ \text{rad/yr}$

The allowed whole-body dose is then $(70.0\ \text{kg})(4.55\ \text{rad/yr})\left(\frac{10^{-2}\ \text{J/kg}}{1\ \text{rad}}\right)=\boxed{3.18\ \text{J/yr}}$.

**P45.52** $E_T\equiv E(\text{thermal})=\frac{3}{2}k_BT=0.039\ \text{eV}$

$E_T=\left(\frac{1}{2}\right)^n E$ where $n\equiv$ number of collisions, and $0.039=\left(\frac{1}{2}\right)^n\left(2.0\times10^{6}\right)$.

Therefore, $n=25.6=\boxed{26\ \text{collisions}}$.

**P45.53** Conservation of linear momentum and energy can be applied to find the kinetic energy of the neutron. We first suppose the particles are moving nonrelativistically.

The momentum of the alpha particle and that of the neutron must add to zero, so their velocities must be in opposite directions with magnitudes related by

$$m_n\mathbf{v}_n+m_\alpha\mathbf{v}_\alpha=0 \qquad \text{or} \qquad (1.008\,7\ \text{u})v_n=(4.002\,6\ \text{u})v_\alpha .$$

At the same time, their kinetic energies must add to 17.6 MeV

$$E=\frac{1}{2}m_nv_n^2+\frac{1}{2}m_\alpha v_\alpha^2=\frac{1}{2}(1.008\,7\ \text{u})v_n^2+\frac{1}{2}(4.002\,6)v_\alpha^2=17.6\ \text{MeV}.$$

Substitute $v_\alpha=0.252\,0v_n$: $E=(0.504\,35\ \text{u})v_n^2+(0.127\,10\ \text{u})v_n^2=17.6\ \text{MeV}\left(\frac{1\ \text{u}}{931.494\ \text{MeV}/c^2}\right)$

$$v_n=\sqrt{\frac{0.018\,9c^2}{0.631\,45}}=0.173c=5.19\times10^{7}\ \text{m/s}.$$

Since this speed is not too much greater than $0.1c$, we can get a reasonable estimate of the kinetic energy of the neutron from the classical equation,

$$K=\frac{1}{2}mv^2=\frac{1}{2}(1.008\,7\ \text{u})(0.173c)^2\left(\frac{931.494\ \text{MeV}/c^2}{\text{u}}\right)=14.1\ \text{MeV}.$$

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For a more accurate calculation of the kinetic energy, we should use relativistic expressions. Conservation of momentum gives

$$\gamma_n m_n \mathbf{v}_n + \gamma_\alpha m_\alpha \mathbf{v}_\alpha = 0 \qquad 1.008\,7\frac{v_n}{\sqrt{1-v_n^2/c^2}} = 4.002\,6\frac{v_\alpha}{\sqrt{1-v_\alpha^2/c^2}}$$

yielding $\dfrac{v_\alpha^2}{c^2} = \dfrac{v_n^2}{15.746c^2 - 14.746v_n^2}$.

Then $(\gamma_n - 1)m_n c^2 + (\gamma_\alpha - 1)m_\alpha c^2 = 17.6 \text{ MeV}$

and $v_n = 0.171c$, implying that $(\gamma_n - 1)m_n c^2 = \boxed{14.0 \text{ MeV}}$.

**P45.54** From Table A.3, the half-life of $^{32}\text{P}$ is 14.26 d. Thus, the decay constant is

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{14.26 \text{ d}} = 0.048\,6 \text{ d}^{-1} = 5.63\times10^{-7} \text{ s}^{-1}.$$

$$N_0 = \frac{R_0}{\lambda} = \frac{5.22\times10^6 \text{ decay/s}}{5.63\times10^{-7} \text{ s}^{-1}} = 9.28\times10^{12} \text{ nuclei}$$

At $t = 10.0$ days, the number remaining is

$$N = N_0 e^{-\lambda t} = \left(9.28\times10^{12} \text{ nuclei}\right)e^{-\left(0.048\,6 \text{ d}^{-1}\right)(10.0 \text{ d})} = 5.71\times10^{12} \text{ nuclei}$$

so the number of decays has been $N_0 - N = 3.57\times10^{12}$ and the energy released is

$$E = \left(3.57\times10^{12}\right)(700 \text{ keV})\left(1.60\times10^{-16} \text{ J/keV}\right) = 0.400 \text{ J}.$$

If this energy is absorbed by 100 g of tissue, the absorbed dose is

$$\text{Dose} = \left(\frac{0.400 \text{ J}}{0.100 \text{ kg}}\right)\left(\frac{1 \text{ rad}}{10^{-2} \text{ J/kg}}\right) = \boxed{400 \text{ rad}}.$$

**P45.55** (a) The number of Pu nuclei in 1.00 kg $= \dfrac{6.02\times10^{23} \text{ nuclei/mol}}{239.05 \text{ g/mol}}(1\,000 \text{ g})$.

The total energy $= \left(25.2\times10^{23} \text{ nuclei}\right)(200 \text{ MeV}) = 5.04\times10^{26} \text{ MeV}$

$$E = \left(5.04\times10^{26} \text{ MeV}\right)\left(4.44\times10^{-20} \text{ kWh/MeV}\right) = \boxed{2.24\times10^7 \text{ kWh}}$$

or 22 million kWh.

(b) $E = \Delta m c^2 = (3.016\,049 \text{ u} + 2.014\,102 \text{ u} - 4.002\,603 \text{ u} - 1.008\,665 \text{ u})(931.5 \text{ MeV/u})$

$E = \boxed{17.6 \text{ MeV for each D-T fusion}}$

(c) $E_n = (\text{Total number of D nuclei})(17.6)\left(4.44\times10^{-20}\right)$

$$E_n = \left(6.02\times10^{23}\right)\left(\frac{1\,000}{2.014}\right)(17.6)\left(4.44\times10^{-20}\right) = \boxed{2.34\times10^8 \text{ kWh}}$$

*continued on next page*

(d) $E_n$ = the number of C atoms in 1.00 kg × 4.20 eV

$$E_n=\left(\frac{6.02\times10^{26}}{12}\right)\left(4.20\times10^{-6}\text{ MeV}\right)\left(4.44\times10^{-20}\right)=\boxed{9.36\text{ kWh}}$$

(e) Coal is cheap at this moment in human history. We hope that safety and waste disposal problems can be solved so that nuclear energy can be affordable before scarcity drives up the price of fossil fuels.

**P45.56** Add two electrons to both sides of the given reaction.

Then $4\ {}_1^1\text{H atom}\rightarrow{}_2^4\text{He atom}+Q$

where $Q=(\Delta m)c^2=\left[4(1.007\,825)-4.002603\right]\text{u}(931.5\text{ MeV/u})=26.7\text{ MeV}$

or $Q=(26.7\text{ MeV})\left(1.60\times10^{-13}\text{ J/MeV}\right)=4.28\times10^{-12}\text{ J}$.

The proton fusion rate is then

$$\text{rate}=\frac{\text{power output}}{\text{energy per proton}}=\frac{3.77\times10^{26}\text{ J/s}}{\left(4.28\times10^{-12}\text{ J}\right)/(4\text{ protons})}=\boxed{3.53\times10^{38}\text{ protons/s}}.$$

**P45.57** (a) $Q_\text{I}=\left[M_\text{A}+M_\text{B}-M_\text{C}-M_\text{E}\right]c^2$, and $Q_\text{II}=\left[M_\text{C}+M_\text{D}-M_\text{F}-M_\text{G}\right]c^2$

$$Q_\text{net}=Q_\text{I}+Q_\text{II}=\left[M_\text{A}+M_\text{B}-M_\text{C}-M_\text{E}+M_\text{C}+M_\text{D}-M_\text{F}-M_\text{G}\right]c^2$$

$$Q_\text{net}=Q_\text{I}+Q_\text{II}=\left[M_\text{A}+M_\text{B}+M_\text{D}-M_\text{E}-M_\text{F}-M_\text{G}\right]c^2$$

Thus, reactions may be added. Any product like C used in a subsequent reaction does not contribute to the energy balance.

(b) Adding all five reactions gives

$${}_1^1\text{H}+{}_1^1\text{H}+{}_{-1}^{0}\text{e}+{}_1^1\text{H}+{}_1^1\text{H}+{}_{-1}^{0}\text{e}\rightarrow{}_2^4\text{He}+2\nu+Q_\text{net}$$

or $4\ {}_1^1\text{H}+2\ {}_{-1}^{0}\text{e}\rightarrow{}_2^4\text{He}+2\nu+Q_\text{net}$.

Adding two electrons to each side $4\ {}_1^1\text{H atom}\rightarrow{}_2^4\text{He atom}+Q_\text{net}$.

Thus, $Q_\text{net}=\left[4M_{{}_1^1\text{H}}-M_{{}_2^4\text{He}}\right]c^2=\left[4(1.007\,825)-4.002\,603\right]\text{u}(931.5\text{ MeV/u})=\boxed{26.7\text{ MeV}}$.

**P45.58** (a) The mass of the pellet is $m=\rho V=\left(0.200\ \text{g/cm}^3\right)\left[\frac{4\pi}{3}\left(\frac{1.50\times10^{-2}\ \text{cm}}{2}\right)^3\right]=3.53\times10^{-7}\ \text{g}$.

The pellet consists of equal numbers of $^2\text{H}$ and $^3\text{H}$ atoms, so the average molar mass is 2.50 and the total number of atoms is

$$N=\left(\frac{3.53\times10^{-7}\ \text{g}}{2.50\ \text{g/mol}}\right)\left(6.02\times10^{23}\ \text{atoms/mol}\right)=8.51\times10^{16}\ \text{atoms}\cdot$$

When the pellet is vaporized, the plasma will consist of $2N$particles ($N$ nuclei and $N$ electrons). The total energy delivered to the plasma is 1.00% of 200 kJ or 2.00 kJ. The temperature of the plasma is found from $E=(2N)\left(\frac{3}{2}k_{\text{B}}T\right)$ as

$$T=\frac{E}{3Nk_{\text{B}}}=\frac{2.00\times10^3\ \text{J}}{3\left(8.51\times10^{16}\right)\left(1.38\times10^{-23}\ \text{J/K}\right)}=\boxed{5.68\times10^8\ \text{K}}.$$

(b) Each fusion event uses 2 nuclei, so $\frac{N}{2}$ events will occur. The energy released will be

$$E=\left(\frac{N}{2}\right)Q=\left(\frac{8.51\times10^{16}}{2}\right)(17.59\ \text{MeV})\left(1.60\times10^{-13}\ \text{J/MeV}\right)=1.20\times10^5\ \text{J}=\boxed{120\ \text{kJ}}.$$

**P45.59** (a) The solar-core temperature of 15 MK gives particles enough kinetic energy to overcome the Coulomb-repulsion barrier to ${}_1^1\text{H}+{}_2^3\text{He}\to{}_2^4\text{He}+\text{e}^++\nu$, estimated as $\frac{k_e(e)(2e)}{r}$. The Coulomb barrier to Bethe's fifth and eight reactions is like $\frac{k_e(e)(7e)}{r}$, larger by $\frac{7}{2}$ times, so the required temperature can be estimated as $\frac{7}{2}\left(15\times10^6\ \text{K}\right)\approx\boxed{5\times10^7\ \text{K}}$.

(b) For $^{12}\text{C}+{}^1\text{H}\to{}^{13}\text{N}+Q$,

$Q_1=(12.000\,000+1.007\,825-13.005\,739)(931.5\ \text{MeV})=\boxed{1.94\ \text{MeV}}$

For the second step, add seven electrons to both sides to have: $^{13}\text{N atom}\to{}^{13}\text{C atom}+\text{e}^++\text{e}^-+Q$

$Q_2=\left[13.005\,739-13.003\,355-2(0.000\,549)\right](931.5\ \text{MeV})=\boxed{1.20\ \text{MeV}}$

$Q_3=Q_7=2(0.000\,549)(931.5\ \text{MeV})=\boxed{1.02\ \text{MeV}}$

$Q_4=\left[13.003\,355+1.007\,825-14.003\,074\right](931.5\ \text{MeV})=\boxed{7.55\ \text{MeV}}$

$Q_5=\left[14.003\,074+1.007\,825-15.003\,065\right](931.5\ \text{MeV})=\boxed{7.30\ \text{MeV}}$

$Q_6=\left[15.003\,065-15.000\,109-2(0.000\,549)\right](931.5\ \text{MeV})=\boxed{1.73\ \text{MeV}}$

$Q_8=\left[15.000\,109+1.007\,825-12-4.002\,603\right](931.5\ \text{MeV})=\boxed{4.97\ \text{MeV}}$

The sum is $\boxed{26.7\ \text{MeV}}$, the same as for the proton-proton cycle.

(c) Not all of the energy released appears as internal energy in the star. When a neutrino is created, it will likely fly directly out of the star without interacting with any other particle.

**P45.60** (a) $$\frac{I_2}{I_1}=\frac{I_0e^{-\mu_2 x}}{I_0e^{-\mu_1 x}}=\boxed{e^{-(\mu_2-\mu_1)x}}$$

(b) $$\frac{I_{50}}{I_{100}}=e^{-(5.40-41.0)(0.100)}=e^{3.56}=\boxed{35.2}$$

(c) $$\frac{I_{50}}{I_{100}}=e^{-(5.40-41.0)(1.00)}=e^{35.6}=\boxed{2.89\times 10^{15}}$$

Thus, a 1.00-cm aluminum plate has essentially removed the long-wavelength x-rays from the beam.

***P45.61** (a) The number of fissions ocurring in the zeroth, first, second, … $n$th generation is

$$N_0,\ N_0K,\ N_0K^2,\ \ldots,\ N_0K^n.$$

The total number of fissions that have ocurred up to and including the $n$th generation is

$$N=N_0+N_0K+N_0K^2+\ldots+N_0K^n=N_0\left(1+K+K^2+\ldots+K^n\right).$$

Note that the factoring of the difference of two squares, $a^2-1=(a+1)(a-1)$, can be generalized to a difference of two quantities to any power,

$$a^3-1=\left(a^2+a+1\right)(a-1)$$
$$a^{n+1}-1=\left(a^n+a^{n-1}+\ldots+a^2+a+1\right)(a-1).$$

Thus $$K^n+K^{n-1}+\ldots+K^2+K+1=\frac{K^{n+1}-1}{K-1}$$

and $$\boxed{N=N_0\frac{K^{n+1}-1}{K-1}}.$$

(b) The number of U-235 nuclei is

$$N=5.50\text{ kg}\left(\frac{1\text{ atom}}{235\text{ u}}\right)\left(\frac{1\text{ u}}{1.66\times 10^{-27}\text{ kg}}\right)=1.41\times 10^{25}\text{ nuclei}.$$

We solve the equation from part (a) for $n$, the number of generations:

$$\frac{N}{N_0}(K-1)=K^{n+1}-1$$

$$\frac{N}{N_0}(K-1)+1=K^n(K)$$

$$n\ln K=\ln\left(\frac{N(K-1)/N_0+1}{K}\right)=\ln\left(\frac{N(K-1)}{N_0}+1\right)-\ln K$$

$$n=\frac{\ln\left(N(K-1)/N_0+1\right)}{\ln K}-1=\frac{\ln\left(1.41\times 10^{25}(0.1)/10^{20}+1\right)}{\ln 1.1}-1=99.2$$

Therefore time must be alotted for 100 generations:

$$\Delta t_b=100\left(10\times 10^{-9}\text{ s}\right)=\boxed{1.00\times 10^{-6}\text{ s}}.$$

*continued on next page*

(c) $$v=\sqrt{\frac{B}{\rho}}=\sqrt{\frac{150\times10^{9}\ \text{N/m}^{2}}{18.7\times10^{3}\ \text{kg/m}^{3}}}=\boxed{2.83\times10^{3}\ \text{m/s}}$$

(d) $$V=\frac{4}{3}\pi r^{3}=\frac{m}{\rho}$$

$$r=\left(\frac{3m}{4\pi\rho}\right)^{1/3}=\left(\frac{3(5.5\ \text{kg})}{4\pi\left(18.7\times10^{3}\ \text{kg/m}^{3}\right)}\right)^{1/3}=4.13\times10^{-2}\ \text{m}$$

$$\Delta t_{d}=\frac{r}{v}=\frac{4.13\times10^{-2}\ \text{m}}{2.83\times10^{3}\ \text{m/s}}=\boxed{1.46\times10^{-5}\ \text{s}}$$

(e) 14.6 $\mu$s is greater than 1 $\mu$s, so the entire bomb can fission. The destructive energy released is

$$1.41\times10^{25}\ \text{nuclei}\left(\frac{200\times10^{6}\ \text{eV}}{\text{fissioning nucleus}}\right)\left(\frac{1.6\times10^{-19}\ \text{J}}{1\ \text{eV}}\right)=4.51\times10^{14}\ \text{J}=4.51\times10^{14}\ \text{J}\left(\frac{1\ \text{ton TNT}}{4.2\times10^{9}\ \text{J}}\right)$$
$$=1.07\times10^{5}\ \text{ton TNT}$$
$$=\boxed{107\ \text{kilotons of TNT}}$$

**What if?** If the bomb did not have an "initiator" to inject $10^{20}$ neutrons at the moment when the critical mass is assembled, the number of generations would be

$$n=\frac{\ln\left(1.41\times10^{25}\,(0.1)/1+1\right)}{\ln 1.1}-1=582\ \text{requiring}\ 583\left(10\times10^{-9}\ \text{s}\right)=5.83\ \mu\text{s}.$$

This time is not very short compared with 14.6 $\mu$s, so this bomb would likely release much less energy.

## ANSWERS TO EVEN PROBLEMS

| | |
|---|---|
| **P45.2** | 184 MeV |
| **P45.4** | see the solution |
| **P45.6** | (a) 173 MeV; (b) 0.078 8% |
| **P45.8** | 2.63 kg/d |
| **P45.10** | (a) $4.84V^{-1/3}$; (b) $6V^{-1/3}$; (c) $6.30V^{-1/3}$;<br>(d) the sphere has minimum loss and the parallelepiped maximum |
| **P45.12** | $2.68\times10^{5}$ |
| **P45.14** | (a) 31.9 g/h; (b) 122 g/h |
| **P45.16** | (a) 3.24 fm; (b) 444 keV; (c) $\frac{2}{5}v_i$;<br>(d) 740 keV; (e) possibly by tunneling |
| **P45.18** | (a) $2.53\times10^{31}$ J; (b) $1.14\times10^{9}$ yr |
| **P45.20** | (a) $10^{14}\ \text{cm}^{-3}$; (b) $1.24\times10^{5}\ \text{J/m}^{3}$;<br>(c) 1.77 T |
| **P45.22** | 12.4 h |
| **P45.24** | (a) 10.0 h; (b) 3.16 m |
| **P45.26** | (a) 0.436 cm; (b) 5.79 cm |
| **P45.28** | $2.39\times10^{-3}\,^{\circ}\text{C}$ |
| **P45.30** | $3.96\times10^{-4}$ J/kg |
| **P45.32** | (a) 10; (b) $10^{6}$; (c) $10^{8}$ eV |
| **P45.34** | $4.45\times10^{-8}$ kg/h |

| | |
|---|---|
| **P45.36** | (a) $\sim 10^{6}$; (b) $\sim 10^{-15}$ g |
| **P45.38** | (a) $1.5\times 10^{24}$; (b) 0.6 kg |
| **P45.40** | see the solution |
| **P45.42** | The fractional loss in D-T is about 4 times that in $^{235}$U fission |
| **P45.44** | 1.01 MeV |
| **P45.46** | $\dfrac{mN_{\mathrm{A}}(200\ \mathrm{MeV})}{(235\ \mathrm{g/mol})\left[\begin{array}{l} c_w(100^\circ\mathrm{C}-T_c)+L_v \\ +c_s(T_h-100^\circ\mathrm{C}) \end{array}\right]}$ |
| **P45.48** | (a) $\sim 10^{8}\ \mathrm{m}^3$; (b) $\sim 10^{13}$ J; (c) $\sim 10^{14}$ J;<br>(d) $\sim 10$ kilotons |
| **P45.50** | 223 W |
| **P45.52** | 26 collisions |
| **P45.54** | 400 rad |
| **P45.56** | $3.53\times 10^{38}$ protons/s |
| **P45.58** | (a) $5.68\times 10^{8}$ K; (b) 120 kJ |
| **P45.60** | (a) see the solution; (b) 35.2; (c) $2.89\times 10^{15}$ |

# 46

# Particle Physics and Cosmology

**CHAPTER OUTLINE**

46.1 The Fundamental Forces in Nature
46.2 Positrons and Other Antiparticles
46.3 Mesons and the Beginning of Particle Physics
46.4 Classification of Particles
46.5 Conservation Laws
46.6 Strange Particles and Strangeness
46.7 Making Elementary Particles and Measuring Their Properties
46.8 Finding Patterns in the Particles
46.9 Quarks
46.10 Multicolored Quarks
46.11 The Standard Model
46.12 The Cosmic Connection
46.13 Problems and Perspectives

## ANSWERS TO QUESTIONS

**Q46.1** Strong Force—Mediated by gluons.

Electromagnetic Force—Mediated by photons.

Weak Force—Mediated by $W^+$, $W^-$, and $Z^0$ bosons.

Gravitational Force—Mediated by gravitons.

**Q46.2** The production of a single gamma ray could not satisfy the law of conservation of momentum, which must hold true in this—and every—interaction.

**Q46.3** In the quark model, all hadrons are composed of smaller units called quarks. Quarks have a fractional electric charge and a baryon number of $\frac{1}{3}$. There are 6 types of quarks: up, down, strange, charmed, top, and bottom. Further, all *baryons* contain 3 quarks, and all *mesons* contain one quark and one anti-quark. *Leptons* are thought to be fundamental particles.

**Q46.4** Hadrons are massive particles with structure and size. There are two classes of hadron: mesons and baryons. Hadrons are composed of quarks. Hadrons interact via the strong force.

Leptons are light particles with no structure or size. It is believed that leptons are fundamental particles. Leptons interact via the weak force.

**Q46.5** Baryons are heavy hadrons with spin $\frac{1}{2}$ or $\frac{3}{2}$, are composed of three quarks, and have long lifetimes. Mesons are light hadrons with spin 0 or 1, are composed of a quark and an antiquark, and have short lifetimes.

**Q46.6** Resonances are hadrons. They decay into strongly interacting particles such as protons, neutrons, and pions, all of which are hadrons.

**Q46.7** The baryon number of a proton or neutron is one. Since baryon number is conserved, the baryon number of the kaon must be zero.

**Q46.8** Decays by the weak interaction typically take $10^{-10}$ s or longer to occur. This is slow in particle physics.

**Q46.9** The decays of the muon, tau, charged pion, kaons, neutron, lambda, charged sigmas, xis, and omega occur by the weak interaction. All have lifetimes longer than $10^{-13}$ s. Several produce neutrinos; none produce photons. Several violate strangeness conservation.

**Q46.10** The decays of the neutral pion, eta, and neutral sigma occur by the electromagnetic interaction. These are three of the shortest lifetimes in Table 46.2. All produce photons, which are the quanta of the electromagnetic force. All conserve strangeness.

**Q46.11** Yes, protons interact via the weak interaction; but the strong interaction predominates.

**Q46.12** You can think of a conservation law as a superficial regularity which we happen to notice, as a person who does not know the rules of chess might observe that one player's two bishops are always on squares of opposite colors. Alternatively, you can think of a conservation law as identifying some stuff of which the universe is made. In classical physics one can think of both matter and energy as fundamental constituents of the world. We buy and sell both of them. In classical physics you can also think of linear momentum, angular momentum, and electric charge as basic stuffs of which the universe is made. In relativity we learn that matter and energy are not conserved separately, but are both aspects of the conserved quantity *relativistic total energy*. Discovered more recently, four conservation laws appear equally general and thus equally fundamental: Conservation of baryon number, conservation of electron-lepton number, conservation of tau-lepton number, and conservation of muon-lepton number. Processes involving the strong force and the electromagnetic force follow conservation of strangeness, charm, bottomness, and topness, while the weak interaction can alter the total $S$, $C$, $B$ and $T$ quantum numbers of an isolated system.

**Q46.13** No. Antibaryons have baryon number –1, mesons have baryon number 0, and baryons have baryon number +1. The reaction cannot occur because it would not conserve baryon number, unless so much energy is available that a baryon-antibaryon pair is produced.

**Q46.14** The Standard Model consists of quantum chromodynamics (to describe the strong interaction) and the electroweak theory (to describe the electromagnetic and weak interactions). The Standard Model is our most comprehensive description of nature. It fails to unify the two theories it includes, and fails to include the gravitational force. It pictures matter as made of six quarks and six leptons, interacting by exchanging gluons, photons, and W and Z bosons.

**Q46.15** All baryons and antibaryons consist of three quarks. All mesons and antimesons consist of two quarks. Since quarks have spin quantum number $\frac{1}{2}$ and can be spin-up or spin-down, it follows that the three-quark baryons must have a half-integer spin, while the two-quark mesons must have spin 0 or 1.

**Q46.16** Each flavor of quark can have colors, designated as red, green and blue. Antiquarks are colored antired, antigreen, and antiblue. A baryon consists of three quarks, each having a different color. By analogy to additive color mixing we call it colorless. A meson consists of a quark of one color and antiquark with the corresponding anticolor, making it colorless as a whole.

**Q46.17** In 1961 Gell-Mann predicted the omega-minus particle, with quark composition sss. Its discovery in 1964 confirmed the quark theory.

**Q46.18** The $\Xi^-$ particle has, from Table 46.2, charge –*e*, spin $\frac{1}{2}$, $B=1$, $L_e=L_\mu=L_\tau=0$, and strangeness –2. All of these are described by its quark composition dss (Table 46.5). The properties of the quarks from Table 46.3 let us add up charge: $-\frac{1}{3}e-\frac{1}{3}e-\frac{1}{3}e=-e$; spin $+\frac{1}{2}-\frac{1}{2}+\frac{1}{2}=\frac{1}{2}$, supposing one of the quarks is spin-down relative to the other two; baryon number $\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=1$; lepton numbers, charm, bottomness, and topness zero; and strangeness $0-1-1=-2$.

**Q46.19** The electroweak theory of Glashow, Salam, and Weinberg predicted the $\mathrm{W}^+$, $\mathrm{W}^-$, and Z particles. Their discovery in 1983 confirmed the electroweak theory.

**Q46.20** Hubble determined experimentally that all galaxies outside the Local Group are moving away from us, with speed directly proportional to the distance of the galaxy from us.

**Q46.21** Before that time, the Universe was too hot for the electrons to remain in any sort of stable orbit around protons. The thermal motion of both protons and electrons was too rapid for them to be in close enough proximity for the Coulomb force to dominate.

**Q46.22** The Universe is vast and could on its own terms get along very well without us. But as the cosmos is immense, life appears to be immensely scarce, and therefore precious. We must do our work, growing corn to feed the hungry while preserving our planet for future generations. One person has singular abilities and opportunities for effort, faithfulness, generosity, honor, curiosity, understanding, and wonder. His or her place is to use those abilities and opportunities, unique in all the Universe.

## SOLUTIONS TO PROBLEMS

### Section 46.1 The Fundamental Forces in Nature

### Section 46.2 Positrons and Other Antiparticles

**P46.1** Assuming that the proton and antiproton are left nearly at rest after they are produced, the energy *E* of the photon must be

$$E=2E_0=2(938.3\text{ MeV})=1\,876.6\text{ MeV}=3.00\times10^{-10}\text{ J}.$$

Thus, $E=hf=3.00\times10^{-10}\text{ J}$

$$f=\frac{3.00\times10^{-10}\text{ J}}{6.626\times10^{-34}\text{ J}\cdot\text{s}}=\boxed{4.53\times10^{23}\text{ Hz}}$$

$$\lambda=\frac{c}{f}=\frac{3.00\times10^{8}\text{ m/s}}{4.53\times10^{23}\text{ Hz}}=\boxed{6.62\times10^{-16}\text{ m}}.$$

**P46.2** The minimum energy is released, and hence the minimum frequency photons are produced, when the proton and antiproton are at rest when they annihilate.

That is, $E = E_0$ and $K = 0$. To conserve momentum, each photon must carry away one-half the energy.

Thus, $$E_{\min} = \frac{2E_0}{2} = E_0 = 938.3 \text{ MeV} = hf_{\min}.$$

Thus, $$f_{\min} = \frac{(938.3 \text{ MeV})\left(1.60 \times 10^{-13} \text{ J/MeV}\right)}{\left(6.626 \times 10^{-34} \text{ J} \cdot \text{s}\right)} = \boxed{2.27 \times 10^{23} \text{ Hz}}$$

$$\lambda = \frac{c}{f_{\min}} = \frac{3.00 \times 10^{8} \text{ m/s}}{2.27 \times 10^{23} \text{ Hz}} = \boxed{1.32 \times 10^{-15} \text{ m}}.$$

**P46.3** In $\gamma \to \text{p}^+ + \text{p}^-$,

we start with energy 2.09 GeV

we end with energy 938.3 MeV + 938.3 MeV + 95.0 MeV + $K_2$

where $K_2$ is the kinetic energy of the second proton.

Conservation of energy for the creation process gives $\boxed{K_2 = 118 \text{ MeV}}$.

## Section 46.3 **Mesons and the Beginning of Particle Physics**

**P46.4** The reaction is $\mu^+ + \text{e}^- \to \nu + \nu$

muon-lepton number before reaction: $(-1) + (0) = -1$

electron-lepton number before reaction: $(0) + (1) = 1$.

Therefore, after the reaction, the muon-lepton number must be –1. Thus, one of the neutrinos must be the anti-neutrino associated with muons, and one of the neutrinos must be the neutrino associated with electrons:

$\boxed{\bar{\nu}_\mu}$ and $\boxed{\nu_\text{e}}$.

Then $\mu^+ + \text{e}^- \to \bar{\nu}_\mu + \nu_\text{e}$.

**P46.5** The creation of a virtual $Z^0$ boson is an energy fluctuation $\Delta E = 93 \times 10^9$ eV. It can last no longer than $\Delta t = \frac{\hbar}{2\Delta E}$ and move no farther than

$$c(\Delta t) = \frac{hc}{4\pi \Delta E} = \frac{\left(6.626 \times 10^{-34} \text{ J} \cdot \text{s}\right)\left(3.00 \times 10^{8} \text{ m/s}\right)}{4\pi\left(93 \times 10^{9} \text{ eV}\right)}\left(\frac{1 \text{ eV}}{1.60 \times 10^{-19} \text{ J}}\right) = 1.06 \times 10^{-18} \text{ m} = \boxed{\sim 10^{-18} \text{ m}}.$$

**P46.6** A proton has rest energy 938.3 MeV. The time interval during which a virtual proton could exist is at most $\Delta t$ in $\Delta E \Delta t = \frac{\hbar}{2}$. The distance it could move is at most

$$c\Delta t = \frac{\hbar c}{2\Delta E} = \frac{\left(1.055\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3\times10^{8}\text{ m/s}\right)}{2(938.3)\left(1.6\times10^{-13}\text{ J}\right)}\boxed{\sim 10^{-16}\text{ m}}.$$

According to Yukawa's line of reasoning, this distance is the range of a force that could be associated with the exchange of virtual protons between high-energy particles.

**P46.7** By Table 46.2, $M_{\pi^0} = 135\text{ MeV}/c^2$.

Therefore, $E_\gamma = \boxed{67.5\text{ MeV}}$ for each photon

$$p = \frac{E_\gamma}{c} = \boxed{67.5\text{ MeV}/c}$$

and $f = \frac{E_\gamma}{h} = \boxed{1.63\times10^{22}\text{ Hz}}$.

**P46.8** The time interval for a particle traveling with the speed of light to travel a distance of $3\times10^{-15}$ m is

$$\Delta t = \frac{d}{v} = \frac{3\times10^{-15}\text{ m}}{3\times10^{8}\text{ m/s}} = \boxed{\sim 10^{-23}\text{ s}}.$$

**P46.9** (a) $\Delta E = \left(m_n - m_p - m_e\right)c^2$

From Table A-3, $\Delta E = (1.008\,665 - 1.007\,825)(931.5) = \boxed{0.782\text{ MeV}}$.

(b) Assuming the neutron at rest, momentum conservation for the decay process implies $p_p = p_e$. Relativistic energy for the system is conserved

$$\sqrt{\left(m_p c^2\right)^2 + p_p^2 c^2} + \sqrt{\left(m_e c^2\right)^2 + p_e^2 c^2} = m_n c^2.$$

Since $p_p = p_e$, $\sqrt{(938.3)^2 + (pc)^2} + \sqrt{(0.511)^2 + (pc)^2} = 939.6\text{ MeV}$.

Solving the algebra, $pc = 1.19\text{ MeV}$.

If $p_e c = \gamma m_e v_e c = 1.19\text{ MeV}$, then $\frac{\gamma v_e}{c} = \frac{1.19\text{ MeV}}{0.511\text{ MeV}} = \frac{x}{\sqrt{1-x^2}} = 2.33$ where $x = \frac{v_e}{c}$.

Solving, $x^2 = \left(1-x^2\right)5.43$ and $x = \frac{v_e}{c} = 0.919$

$$\boxed{v_e = 0.919c}.$$

Then $m_p v_p = \gamma_e m_e v_e$: $v_p = \frac{\gamma_e m_e v_e c}{m_p c} = \frac{(1.19\text{ MeV})\left(1.60\times10^{-13}\text{ J/MeV}\right)}{\left(1.67\times10^{-27}\right)\left(3.00\times10^{8}\text{ m/s}\right)}$

$$v_p = 3.80\times10^{5}\text{ m/s} = \boxed{380\text{ km/s}}.$$

(c) $\boxed{\text{The electron is relativistic, the proton is not.}}$

## Section 46.4 **Classification of Particles**

**P46.10** In $? + \mathrm{p}^+ \to \mathrm{n} + \mu^+$, charge conservation requires the unknown particle to be neutral. Baryon number conservation requires baryon number = 0. The muon-lepton number of ? must be –1.

So the unknown particle must be $\boxed{\overline{\nu}_\mu}$.

**P46.11** $\Omega^+ \to \overline{\Lambda}^0 + \mathrm{K}^+$ $\qquad$ $\overline{\mathrm{K}}{}^0_\mathrm{S} \to \pi^+ + \pi^-$ $\qquad$ (or $\pi^0 + \pi^0$)

$\overline{\Lambda}^0 \to \overline{\mathrm{p}} + \pi^+$ $\qquad$ $\overline{\mathrm{n}} \to \overline{\mathrm{p}} + \mathrm{e}^+ + \nu_\mathrm{e}$

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## Section 46.5 **Conservation Laws**

**P46.12** (a) $\mathrm{p} + \overline{\mathrm{p}} \to \mu^+ + \mathrm{e}^-$ $\quad \boxed{L_e}$ $\quad 0 + 0 \to 0 + 1$

and $\quad \boxed{L_\mu}$ $\quad 0 + 0 \to -1 + 0$

(b) $\pi^- + \mathrm{p} \to \mathrm{p} + \pi^+$ $\quad \boxed{\text{charge}}$ $\quad -1 + 1 \to +1 + 1$

(c) $\mathrm{p} + \mathrm{p} \to \mathrm{p} + \pi^+$ $\quad \boxed{\text{baryon number}}$: $\quad 1 + 1 \to 1 + 0$

(d) $\mathrm{p} + \mathrm{p} \to \mathrm{p} + \mathrm{p} + \mathrm{n}$ $\quad \boxed{\text{baryon number}}$: $\quad 1 + 1 \to 1 + 1 + 1$

(e) $\gamma + \mathrm{p} \to \mathrm{n} + \pi^0$ $\quad \boxed{\text{charge}}$ $\quad 0 + 1 \to 0 + 0$

**P46.13** (a) Baryon number and charge are conserved, with values of $0 + 1 = 0 + 1$ and $1 + 1 = 1 + 1$ in both reactions.

(b) $\boxed{\text{Strangeness is } \textit{not} \text{ conserved}}$ in the second reaction.

**P46.14** Baryon number conservation $\boxed{\text{allows the first and forbids the second}}$.

**P46.15** (a) $\pi^- \to \mu^- + \boxed{\overline{\nu}_\mu}$ $\qquad L_\mu$: $\quad 0 \to 1 - 1$

(b) $\mathrm{K}^+ \to \mu^+ + \boxed{\nu_\mu}$ $\qquad L_\mu$: $\quad 0 \to -1 + 1$

(c) $\boxed{\overline{\nu}_\mathrm{e}} + \mathrm{p}^+ \to \mathrm{n} + \mathrm{e}^+$ $\qquad L_e$: $\quad -1 + 0 \to 0 - 1$

(d) $\boxed{\nu_\mathrm{e}} + \mathrm{n} \to \mathrm{p}^+ + \mathrm{e}^-$ $\qquad L_e$: $\quad 1 + 0 \to 0 + 1$

(e) $\boxed{\nu_\mu} + \mathrm{n} \to \mathrm{p}^+ + \mu^-$ $\qquad L_\mu$: $\quad 1 + 0 \to 0 + 1$

(f) $\mu^- \to \mathrm{e}^- + \boxed{\overline{\nu}_\mathrm{e}} + \boxed{\nu_\mu}$ $\qquad L_\mu$: $\quad 1 \to 0 + 0 + 1$ $\quad$ and $\quad L_e$: $\quad 0 \to 1 - 1 + 0$

**P46.16** Momentum conservation for the decay requires the pions to have equal speeds.

The total energy of each is $\dfrac{497.7\text{ MeV}}{2}$

so $E^2 = p^2c^2 + \left(mc^2\right)^2$ gives

$$(248.8\text{ MeV})^2 = (pc)^2 + (139.6\text{ MeV})^2.$$

Solving, $pc = 206\text{ MeV} = \gamma\, mvc = \dfrac{mc^2}{\sqrt{1-(v/c)^2}}\left(\dfrac{v}{c}\right)$

$$\frac{pc}{mc^2} = \frac{206\text{ MeV}}{139.6\text{ MeV}} = \frac{1}{\sqrt{1-(v/c)^2}}\left(\frac{v}{c}\right) = 1.48$$

$$\frac{v}{c} = 1.48\sqrt{1-\left(\frac{v}{c}\right)^2}$$

and $\left(\dfrac{v}{c}\right)^2 = 2.18\left[1-\left(\dfrac{v}{c}\right)^2\right] = 2.18 - 2.18\left(\dfrac{v}{c}\right)^2$

$$3.18\left(\frac{v}{c}\right)^2 = 2.18$$

so $\dfrac{v}{c} = \sqrt{\dfrac{2.18}{3.18}} = 0.828$

and $\boxed{v = 0.828c}$.

**P46.17** (a) $p^+ \to \pi^+ + \pi^0$ $\boxed{\text{Baryon number}}$: $1 \to 0+0$

(b) $p^+ + p^+ \to p^+ + p^+ + \pi^0$ This reaction $\boxed{\text{can occur}}$.

(c) $p^+ + p^+ \to p^+ + \pi^+$ $\boxed{\text{Baryon number}}$ is violated: $1+1 \to 1+0$

(d) $\pi^+ \to \mu^+ + \nu_\mu$ This reaction $\boxed{\text{can occur}}$.

(e) $n^0 \to p^+ + e^- + \overline{\nu}_e$ This reaction $\boxed{\text{can occur}}$.

(f) $\pi^+ \to \mu^+ + n$ Violates $\boxed{\text{baryon number}}$: $0 \to 0+1$

Violates $\boxed{\text{muon-lepton number}}$: $0 \to -1+0$

**P46.18** (a) $p \to e^+ + \gamma$

Baryon number: $+1 \to 0 + 0$

$\Delta B \neq 0$, so baryon number conservation is violated.

(b) From conservation of momentum for the decay: $p_e = p_\gamma$.

Then, for the positron, $E_e^2 = (p_e c)^2 + E_{0,\,e}^2$

becomes $E_e^2 = (p_\gamma c)^2 + E_{0,\,e}^2 = E_\gamma^2 + E_{0,\,e}^2$.

From conservation of energy for the system: $E_{0,\,p} = E_e + E_\gamma$

or $E_e = E_{0,\,p} - E_\gamma$

so $E_e^2 = E_{0,\,p}^2 - 2E_{0,\,p}E_\gamma + E_\gamma^2$.

Equating this to the result from above gives $E_\gamma^2 + E_{0,\,e}^2 = E_{0,\,p}^2 - 2E_{0,\,p}E_\gamma + E_\gamma^2$

or $$E_\gamma = \frac{E_{0,\,p}^2 - E_{0,\,e}^2}{2E_{0,\,p}} = \frac{(938.3\text{ MeV})^2 - (0.511\text{ MeV})^2}{2(938.3\text{ MeV})} = \boxed{469\text{ MeV}}.$$

Thus, $$E_e = E_{0,\,p} - E_\gamma = 938.3\text{ MeV} - 469\text{ MeV} = \boxed{469\text{ MeV}}.$$

Also, $$p_\gamma = \frac{E_\gamma}{c} = \boxed{\frac{469\text{ MeV}}{c}}$$

and $$p_e = p_\gamma = \boxed{\frac{469\text{ MeV}}{c}}.$$

(c) The total energy of the positron is $E_e = 469\text{ MeV}$.

But, $$E_e = \gamma E_{0,\,e} = \frac{E_{0,\,e}}{\sqrt{1-(v/c)^2}}$$

so $$\sqrt{1-\left(\frac{v}{c}\right)^2} = \frac{E_{0,\,e}}{E_e} = \frac{0.511\text{ MeV}}{469\text{ MeV}} = 1.09\times10^{-3}$$

which yields: $\boxed{v = 0.999\,999\,4c}$.

**P46.19** The relevant conservation laws are: $\Delta L_e = 0$

$\Delta L_\mu = 0$

and $\Delta L_\tau = 0$.

(a) $\pi^+ \to \pi^0 + e^+ + ?$ $L_e$: $0 \to 0 - 1 + L_e$ implies $L_e = 1$ and we have a $\boxed{\nu_e}$

(b) $? + p \to \mu^- + p + \pi^+$ $L_\mu$: $L_\mu + 0 \to +1 + 0 + 0$ implies $L_\mu = 1$ and we have a $\boxed{\nu_\mu}$

*continued on next page*

(c) $\Lambda^0 \to \rho + \mu^- + ?$ $L_\mu$: $0 \to 0 + 1 + L_\mu$ implies $L_\mu = -1$ and we have a $\boxed{\bar{\nu}_\mu}$

(d) $\tau^+ \to \mu^+ + ? + ?$ $L_\mu$: $0 \to -1 + L_\mu$ implies $L_\mu = 1$ and we have a $\boxed{\nu_\mu}$

$L_\tau$: $-1 \to 0 + L_\tau$ implies $L_\tau = -1$ and we have a $\boxed{\bar{\nu}_\tau}$

Conclusion for (d): $L_\mu = 1$ for one particle, and $L_\tau = -1$ for the other particle.

We have $\boxed{\nu_\mu}$

and $\boxed{\bar{\nu}_\tau}$.

## Section 46.6 **Strange Particles and Strangeness**

**P46.20** The $\rho^0 \to \pi^+ + \pi^-$ decay must occur via the strong interaction.

The $K_S^0 \to \pi^+ + \pi^-$ decay must occur via the weak interaction.

**P46.21** (a) $\Lambda^0 \to p + \pi^-$ Strangeness: $-1 \to 0 + 0$ (strangeness is $\boxed{\text{not conserved}}$)

(b) $\pi^- + p \to \Lambda^0 + K^0$ Strangeness: $0 + 0 \to -1 + 1$ ($0 = 0$ and strangeness is $\boxed{\text{conserved}}$)

(c) $\bar{p} + p \to \bar{\Lambda}^0 + \Lambda^0$ Strangeness: $0 + 0 \to +1 - 1$ ($0 = 0$ and strangeness is $\boxed{\text{conserved}}$)

(d) $\pi^- + p \to \pi^- + \Sigma^+$ Strangeness: $0 + 0 \to 0 - 1$ ($0 \neq -1$: strangeness is $\boxed{\text{not conserved}}$)

(e) $\Xi^- \to \Lambda^0 + \pi^-$ Strangeness: $-2 \to -1 + 0$ ($-2 \neq -1$ so strangeness is $\boxed{\text{not conserved}}$)

(f) $\Xi^0 \to p + \pi^-$ Strangeness: $-2 \to 0 + 0$ ($-2 \neq 0$ so strangeness is $\boxed{\text{not conserved}}$)

**P46.22** (a) $\mu^- \to e^- + \gamma$ $L_e$: $0 \to 1 + 0$,

and $L_\mu$: $1 \to 0$

(b) $n \to p + e^- + \nu_e$ $L_e$: $0 \to 0 + 1 + 1$

(c) $\Lambda^0 \to p + \pi^0$ Strangeness: $-1 \to 0 + 0$,

and charge: $0 \to +1 + 0$

(d) $p \to e^+ + \pi^0$ Baryon number: $+1 \to 0 + 0$

(e) $\Xi^0 \to n + \pi^0$ Strangeness: $-2 \to 0 + 0$

**P46.23** (a) $\pi^- + \text{p} \rightarrow 2\eta$ violates conservation of baryon number as $0+1\rightarrow 0$, $\boxed{\text{not allowed}}$.

(b) $\text{K}^- + \text{n} \rightarrow \Lambda^0 + \pi^-$

Baryon number, $0+1\rightarrow 1+0$

Charge, $-1+0\rightarrow 0-1$

Strangeness, $-1+0\rightarrow -1+0$

Lepton number, $0\rightarrow 0$

The interaction may occur via the $\boxed{\text{strong interaction}}$ since all are conserved.

(c) $\text{K}^- \rightarrow \pi^- + \pi^0$

Strangeness, $-1\rightarrow 0+0$

Baryon number, $0\rightarrow 0$

Lepton number, $0\rightarrow 0$

Charge, $-1\rightarrow -1+0$

Strangeness is violated by one unit, but everything else is conserved. Thus, the reaction can occur via the $\boxed{\text{weak interaction}}$, but not the strong or electromagnetic interaction.

(d) $\Omega^- \rightarrow \Xi^- + \pi^0$

Baryon number, $1\rightarrow 1+0$

Lepton number, $0\rightarrow 0$

Charge, $-1\rightarrow -1+0$

Strangeness, $-3\rightarrow -2+0$

May occur by $\boxed{\text{weak interaction}}$, but not by strong or electromagnetic.

(e) $\eta \rightarrow 2\gamma$

Baryon number, $0\rightarrow 0$

Lepton number, $0\rightarrow 0$

Charge, $0\rightarrow 0$

Strangeness, $0\rightarrow 0$

No conservation laws are violated, but photons are the mediators of the electromagnetic interaction. Also, the lifetime of the $\eta$ is consistent with the $\boxed{\text{electromagnetic interaction}}$.

**P46.24** (a) $\Xi^- \to \Lambda^0 + \mu^- + \nu_\mu$

| | | | |
|---|---|---|---|
| Baryon number: | $+1 \to +1+0+0$ | Charge: | $-1 \to 0-1+0$ |
| $L_e$: | $0 \to 0+0+0$ | $L_\mu$: | $0 \to 0+1+1$ |
| $L_\tau$: | $0 \to 0+0+0$ | Strangeness: | $-2 \to -1+0+0$ |

Conserved quantities are: $\boxed{B\text{, charge, } L_e\text{, and } L_\tau}$

(b) $K_S^0 \to 2\pi^0$

| | | | |
|---|---|---|---|
| Baryon number: | $0 \to 0$ | Charge: | $0 \to 0$ |
| $L_e$: | $0 \to 0$ | $L_\mu$: | $0 \to 0$ |
| $L_\tau$: | $0 \to 0$ | Strangeness: | $+1 \to 0$ |

Conserved quantities are: $\boxed{B\text{, charge, } L_e\text{, } L_\mu\text{, and } L_\tau}$

(c) $K^- + p \to \Sigma^0 + n$

| | | | |
|---|---|---|---|
| Baryon number: | $0+1 \to 1+1$ | Charge: | $-1+1 \to 0+0$ |
| $L_e$: | $0+0 \to 0+0$ | $L_\mu$: | $0+0 \to 0+0$ |
| $L_\tau$: | $0+0 \to 0+0$ | Strangeness: | $-1+0 \to -1+0$ |

Conserved quantities are: $\boxed{S\text{, charge, } L_e\text{, } L_\mu\text{, and } L_\tau}$

(d) $\Sigma^0 + \Lambda^0 + \gamma$

| | | | |
|---|---|---|---|
| Baryon number: | $+1 \to 1+0$ | Charge: | $0 \to 0$ |
| $L_e$: | $0 \to 0+0$ | $L_\mu$: | $0 \to 0+0$ |
| $L_\tau$: | $0 \to 0+0$ | Strangeness: | $-1 \to -1+0$ |

Conserved quantities are: $\boxed{B\text{, } S\text{, charge, } L_e\text{, } L_\mu\text{, and } L_\tau}$

(e) $e^+ + e^- \to \mu^+ + \mu^-$

| | | | |
|---|---|---|---|
| Baryon number: | $0+0 \to 0+0$ | Charge: | $+1-1 \to +1-1$ |
| $L_e$: | $-1+1 \to 0+0$ | $L_\mu$: | $0+0 \to +1-1$ |
| $L_\tau$: | $0+0 \to 0+0$ | Strangeness: | $0+0 \to 0+0$ |

Conserved quantities are: $\boxed{B\text{, } S\text{, charge, } L_e\text{, } L_\mu\text{, and } L_\tau}$

(f) $\overline{p} + n \to \overline{\Lambda}^0 + \Sigma^-$

| | | | |
|---|---|---|---|
| Baryon number: | $-1+1 \to -1+1$ | Charge: | $-1+0 \to 0-1$ |
| $L_e$: | $0+0 \to 0+0$ | $L_\mu$: | $0+0 \to 0+0$ |
| $L_\tau$: | $0+0 \to 0+0$ | Strangeness: | $0+0 \to +1-1$ |

Conserved quantities are: $\boxed{B\text{, } S\text{, charge, } L_e\text{, } L_\mu\text{, and } L_\tau}$

**P46.25** (a) $\mathrm{K}^+ + \mathrm{p} \to \underline{?} + \mathrm{p}$

The strong interaction conserves everything.

Baryon number, $0+1 \to B+1$ so $B=0$

Charge, $+1+1 \to Q+1$ so $Q=+1$

Lepton numbers, $0+0 \to L+0$ so $L_e = L_\mu = L_\tau = 0$

Strangeness, $+1+0 \to S+0$ so $S=1$

The conclusion is that the particle must be positively charged, a non-baryon, with strangeness of +1. Of particles in Table 46.2, it can only be the $\boxed{\mathrm{K}^+}$. Thus, this is an elastic scattering process.

The weak interaction conserves all but strangeness, and $\Delta S = \pm 1$.

(b) $\Omega^- \to \underline{?} + \pi^-$

Baryon number, $+1 \to B+0$ so $B=1$

Charge, $-1 \to Q-1$ so $Q=0$

Lepton numbers, $0 \to L+0$ so $L_e = L_\mu = L_\tau = 0$

Strangeness, $-3 \to S+0$ so $\Delta S = 1$: $S=-2$

The particle must be a neutral baryon with strangeness of –2. Thus, it is the $\boxed{\Xi^0}$.

(c) $\mathrm{K}^+ \to \underline{?} + \mu^+ + \nu_\mu$

Baryon number, $0 \to B+0+0$ so $B=0$

Charge, $+1 \to Q+1+0$ so $Q=0$

Lepton numbers, $L_e$, $0 \to L_e+0+0$ so $L_e=0$

$L_\mu$, $0 \to L_\mu-1+1$ so $L_\mu=0$

$L_\tau$, $0 \to L_\tau+0+0$ so $L_\tau=0$

Strangeness, $1 \to S+0+0$ so $\Delta S = \pm 1$

(for weak interaction): $S=0$

The particle must be a neutral meson with strangeness $=0 \Rightarrow \boxed{\pi^0}$.

## Section 46.7 **Making Elementary Particles and Measuring Their Properties**

***P46.26** (a) $$p_{\Sigma^+}=eBr_{\Sigma^+}=\frac{\left(1.602\,177\times10^{-19}\text{ C}\right)(1.15\text{ T})(1.99\text{ m})}{5.344\,288\times10^{-22}\,(\text{kg}\cdot\text{m/s})/(\text{MeV}/c)}=\boxed{\frac{686\text{ MeV}}{c}}$$

$$p_{\pi^+}=eBr_{\pi^+}=\frac{\left(1.602\,177\times10^{-19}\text{ C}\right)(1.15\text{ T})(0.580\text{ m})}{5.344\,288\times10^{-22}\,(\text{kg}\cdot\text{m/s})/(\text{MeV}/c)}=\boxed{\frac{200\text{ MeV}}{c}}$$

(b) Let $\varphi$ be the angle made by the neutron's path with the path of the $\Sigma^+$ at the moment of decay. By conservation of momentum:

$$p_n\cos\varphi+(199.961\,581\text{ MeV}/c)\cos64.5°=686.075\,081\text{ MeV}/c$$

$$\therefore p_n\cos\varphi=599.989\,401\text{ MeV}/c \qquad (1)$$

$$p_n\sin\varphi=(199.961\,581\text{ MeV}/c)\sin64.5°=180.482\,380\text{ MeV}/c \qquad (2)$$

From (1) and (2): $$p_n=\sqrt{(599.989\,401\text{ MeV}/c)^2+(180.482\,380\text{ MeV}/c)^2}=\boxed{627\text{ MeV}/c}$$

(c) $$E_{\pi^+}=\sqrt{\left(p_{\pi^+}c\right)^2+\left(m_{\pi^+}c^2\right)^2}=\sqrt{(199.961\,581\text{ MeV})^2+(139.6\text{ MeV})^2}=\boxed{244\text{ MeV}}$$

$$E_n=\sqrt{(p_nc)^2+\left(m_nc^2\right)^2}=\sqrt{(626.547\,022\text{ MeV})^2+(939.6\text{ MeV})^2}=\boxed{1\,130\text{ MeV}}$$

$$E_{\Sigma^+}=E_{\pi^+}+E_n=243.870\,445\text{ MeV}+1\,129.340\,219\text{ MeV}=\boxed{1\,370\text{ MeV}}$$

(d) $$m_{\Sigma^+}c^2=\sqrt{E_{\Sigma^+}^2-\left(p_{\Sigma^+}c\right)^2}=\sqrt{(1\,373.210\,664\text{ MeV})^2-(686.075\,081\text{ MeV})^2}=1\,190\text{ MeV}$$

$$\therefore m_{\Sigma^+}=\boxed{1\,190\text{ MeV}/c^2}$$

$$E_{\Sigma^+}=\gamma m_{\Sigma^+}c^2,\text{ where }\gamma=\left(1-\frac{v^2}{c^2}\right)^{-1/2}=\frac{1\,373.210\,664\text{ MeV}}{1\,189.541\,303\text{ MeV}}=1.154\,4$$

Solving for $v$, $v=\boxed{0.500c}$.

**P46.27** Time-dilated lifetime:

$$T=\gamma T_0=\frac{0.900\times10^{-10}\text{ s}}{\sqrt{1-v^2/c^2}}=\frac{0.900\times10^{-10}\text{ s}}{\sqrt{1-(0.960)^2}}=3.214\times10^{-10}\text{ s}$$

$$\text{distance}=0.960\left(3.00\times10^8\text{ m/s}\right)\left(3.214\times10^{-10}\text{ s}\right)=\boxed{9.26\text{ cm}}.$$

***P46.28** (a) Let $E_{\min}$ be the minimum total energy of the bombarding particle that is needed to induce the reaction. At this energy the product particles all move with the same velocity. The product particles are then equivalent to a single particle having mass equal to the total mass of the product particles, moving with the same velocity as each product particle. By conservation of energy:

$$E_{\min} + m_2c^2 = \sqrt{\left(m_3c^2\right)^2 + \left(p_3c\right)^2}\,. \qquad (1)$$

By conservation of momentum: $p_3 = p_1$

$$\therefore \left(p_3c\right)^2 = \left(p_1c\right)^2 = E_{\min}^2 - \left(m_1c^2\right)^2. \qquad (2)$$

Substitute (2) in (1): $E_{\min} + m_2c^2 = \sqrt{\left(m_3c^2\right)^2 + E_{\min}^2 - \left(m_1c^2\right)^2}\,.$

Square both sides:

$$E_{\min}^2 + 2E_{\min}m_2c^2 + \left(m_2c^2\right)^2 = \left(m_3c^2\right)^2 + E_{\min}^2 - \left(m_1c^2\right)^2$$

$$\therefore E_{\min} = \frac{\left(m_3^2 - m_1^2 - m_2^2\right)c^2}{2m_2}$$

$$\therefore K_{\min} = E_{\min} - m_1c^2 = \frac{\left(m_3^2 - m_1^2 - m_2^2 - 2m_1m_2\right)c^2}{2m_2} = \frac{\left[m_3^2 - \left(m_1 + m_2\right)^2\right]c^2}{2m_2}$$

Refer to Table 46.2 for the particle masses.

(b) $$K_{\min} = \frac{\left[4(938.3)\right]^2 \text{ MeV}^2/c^2 - \left[2(938.3)\right]^2 \text{ MeV}^2/c^2}{2\left(938.3 \text{ MeV}/c^2\right)} = \boxed{5.63 \text{ GeV}}$$

(c) $$K_{\min} = \frac{(497.7 + 1\,115.6)^2 \text{ MeV}^2/c^2 - (139.6 + 938.3)^2 \text{ MeV}^2/c^2}{2(938.3) \text{ MeV}/c^2} = \boxed{768 \text{ MeV}}$$

(d) $$K_{\min} = \frac{\left[2(938.3) + 135\right]^2 \text{ MeV}^2/c^2 - \left[2(938.3)\right]^2 \text{ MeV}^2/c^2}{2(938.3) \text{ MeV}/c^2} = \boxed{280 \text{ MeV}}$$

(e) $$K_{\min} = \frac{\left[\left(91.2 \times 10^3\right)^2 - \left[(938.3 + 938.3)^2\right] \text{ MeV}^2/c^2\right]}{2(938.3) \text{ MeV}/c^2} = \boxed{4.43 \text{ TeV}}$$

## Section 46.8 Finding Patterns in the Particles

## Section 46.9 Quarks

## Section 46.10 Multicolored Quarks

## Section 46.11 The Standard Model

**P46.29** (a) The number of protons

$$N_p = 1\,000\text{ g}\left(\frac{6.02\times10^{23}\text{ molecules}}{18.0\text{ g}}\right)\left(\frac{10\text{ protons}}{\text{molecule}}\right) = 3.34\times10^{26}\text{ protons}$$

and there are $$N_n = 1\,000\text{ g}\left(\frac{6.02\times10^{23}\text{ molecules}}{18.0\text{ g}}\right)\left(\frac{8\text{ neutrons}}{\text{molecule}}\right) = 2.68\times10^{26}\text{ neutrons}.$$

So there are for electric neutrality $\boxed{3.34\times10^{26}\text{ electrons}}$.

The up quarks have number $2\left(3.34\times10^{26}\right)+2.68\times10^{26} = \boxed{9.36\times10^{26}\text{ up quarks}}$

and there are $2\left(2.68\times10^{26}\right)+3.34\times10^{26} = \boxed{8.70\times10^{26}\text{ down quarks}}$.

(b) Model yourself as 65 kg of water. Then you contain:

$$65\left(3.34\times10^{26}\right)\boxed{\sim10^{28}\text{ electrons}}$$

$$65\left(9.36\times10^{26}\right)\boxed{\sim10^{29}\text{ up quarks}}$$

$$65\left(8.70\times10^{26}\right)\boxed{\sim10^{29}\text{ down quarks}}.$$

Only these fundamental particles form your body. You have no strangeness, charm, topness or bottomness.

**P46.30** (a)

| | proton | u | u | d | total |
|---|---|---|---|---|---|
| strangeness | 0 | 0 | 0 | 0 | 0 |
| baryon number | 1 | 1/3 | 1/3 | 1/3 | 1 |
| charge | *e* | 2*e*/3 | 2*e*/3 | –*e*/3 | *e* |

(b)

| | neutron | u | d | d | total |
|---|---|---|---|---|---|
| strangeness | 0 | 0 | 0 | 0 | 0 |
| baryon number | 1 | 1/3 | 1/3 | 1/3 | 1 |
| charge | 0 | 2*e*/3 | –*e*/3 | –*e*/3 | 0 |

**P46.31** Quark composition of proton = uud and of neutron = udd.
Thus, if we neglect binding energies, we may write

$$m_\text{p} = 2m_\text{u} + m_\text{d} \tag{1}$$

and $$m_\text{n} = m_\text{u} + 2m_\text{d}. \tag{2}$$

Solving simultaneously,

we find $$m_\text{u} = \frac{1}{3}\left(2m_\text{p} - m_\text{n}\right) = \frac{1}{3}\left[2\left(938\text{ MeV}/c^2\right) - 939.6\text{ meV}/c^2\right] = \boxed{312\text{ MeV}/c^2}$$

and from either (1) or (2), $m_\text{d} = \boxed{314\text{ MeV}/c^2}$.

**P46.32** (a)

| | $K^0$ | d | $\bar{s}$ | total |
|---|---|---|---|---|
| strangeness | 1 | 0 | 1 | 1 |
| baryon number | 0 | 1/3 | –1/3 | 0 |
| charge | 0 | $-e/3$ | $e/3$ | 0 |

(b)

| | $\Lambda^0$ | u | d | s | total |
|---|---|---|---|---|---|
| strangeness | –1 | 0 | 0 | –1 | –1 |
| baryon number | 1 | 1/3 | 1/3 | 1/3 | 1 |
| charge | 0 | $2e/3$ | $-e/3$ | $-e/3$ | 0 |

**P46.33** (a) $\pi^- + p \to K^0 + \Lambda^0$

In terms of constituent quarks: $\boxed{\bar{u}d + uud \to d\bar{s} + uds}$

up quarks: $-1+2 \to 0+1$, or $1 \to 1$

down quarks: $1+1 \to 1+1$, or $2 \to 2$

strange quarks: $0+0 \to -1+1$, or $0 \to 0$

(b) $\pi^+ + p \to K^+ + \Sigma^+$ $\boxed{\bar{d}u + uud \to u\bar{s} + uus}$

up quarks: $1+2 \to 1+2$, or $3 \to 3$

down quarks: $-1+1 \to 0+0$, or $0 \to 0$

strange quarks: $0+0 \to -1+1$, or $0 \to 0$

(c) $K^- + p \to K^+ + K^0 + \Omega^-$ $\boxed{\bar{u}s + uud \to u\bar{s} + d\bar{s} + sss}$

up quarks: $-1+2 \to 1+0+0$, or $1 \to 1$

down quarks: $0+1 \to 0+1+0$, or $1 \to 1$

strange quarks: $1+0 \to -1-1+3$, or $1 \to 1$

(d) $p + p \to K^0 + p + \pi^+ + \underline{?}$ $uud + uud \to d\bar{s} + uud + u\bar{d} + \underline{?}$

The quark combination of ? must be such as to balance the last equation for up, down, and strange quarks.

up quarks: $2+2 = 0+2+1+?$ (has 1 u quark)

down quarks: $1+1 = 1+1-1+?$ (has 1 d quark)

strange quarks: $0+0 = -1+0+0+?$ (has 1 s quark)

quark composition $=$ uds $=$ $\boxed{\Lambda^0 \text{ or } \Sigma^0}$

**P46.34** In the first reaction, $\pi^- + p \to K^0 + \Lambda^0$, the quarks in the particles are: $\bar{u}d + uud \to d\bar{s} + uds$. There is a net of 1 up quark both before and after the reaction, a net of 2 down quarks both before and after, and a net of zero strange quarks both before and after. Thus, the reaction conserves the net number of each type of quark.

In the second reaction, $\pi^- + p \to K^0 + n$, the quarks in the particles are: $\bar{u}d + uud \to d\bar{s} + uds$. In this case, there is a net of 1 up and 2 down quarks before the reaction but a net of 1 up, 3 down, and 1 anti-strange quark after the reaction. Thus, the reaction does not conserve the net number of each type of quark.

**P46.35** $\Sigma^0 + \text{p} \to \Sigma^+ + \gamma + X$

$\text{dds} + \text{uud} \to \text{uds} + 0 + ?$

The left side has a net 3d, 2u and 1s. The right-hand side has 1d, 1u, and 1s leaving 2d and 1u missing.

$\boxed{\text{The unknown particle is a neutron, udd.}}$

Baryon and strangeness numbers are conserved.

**P46.36** Compare the given quark states to the entries in Tables 46.4 and 46.5:

(a) $\text{suu} = \boxed{\Sigma^+}$

(b) $\bar{\text{u}}\text{d} = \boxed{\pi^-}$

(c) $\bar{\text{s}}\text{d} = \boxed{\text{K}^0}$

(d) $\text{ssd} = \boxed{\Xi^-}$

**P46.37** (a) $\bar{\text{u}}\bar{\text{u}}\bar{\text{d}}$: charge $= \left(-\frac{2}{3}e\right) + \left(-\frac{2}{3}e\right) + \left(\frac{1}{3}e\right) = \boxed{-e}$. This is the $\boxed{\text{antiproton}}$.

(b) $\bar{\text{u}}\bar{\text{d}}\bar{\text{d}}$: charge $= \left(-\frac{2}{3}e\right) + \left(\frac{1}{3}e\right) + \left(\frac{1}{3}e\right) = \boxed{0}$. This is the $\boxed{\text{antineutron}}$.

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## Section 46.12 **The Cosmic Connection**

**P46.38** Section 39.4 says $f_{\text{observer}} = f_{\text{source}}\sqrt{\frac{1 + v_a/c}{1 - v_a/c}}$.

The velocity of approach, $v_a$, is the negative of the velocity of mutual recession: $v_a = -v$.

Then, $\frac{c}{\lambda'} = \frac{c}{\lambda}\sqrt{\frac{1 - v/c}{1 + v/c}}$ and $\boxed{\lambda' = \lambda\sqrt{\frac{1 + v/c}{1 - v/c}}}$.

**P46.39** (a) $\lambda' = \lambda\sqrt{\frac{1 + v/c}{1 - v/c}}$ $\quad 510 \text{ nm} = 434 \text{ nm}\sqrt{\frac{1 + v/c}{1 - v/c}}$

$$1.18^2 = \frac{1 + v/c}{1 - v/c} = 1.381$$

$$1 + \frac{v}{c} = 1.381 - 1.381\frac{v}{c} \qquad 2.38\frac{v}{c} = 0.381$$

$$\frac{v}{c} = 0.160 \quad \text{or} \quad v = \boxed{0.160c} = 4.80 \times 10^7 \text{ m/s}$$

(b) $v = HR$: $\quad R = \frac{v}{H} = \frac{4.80 \times 10^7 \text{ m/s}}{17 \times 10^{-3} \text{ m/s} \cdot \text{ly}} = \boxed{2.82 \times 10^9 \text{ ly}}$

**P46.40** (a) $\lambda_n' = \lambda_n\sqrt{\frac{1+v/c}{1-v/c}} = (Z+1)\,\lambda_n$ $\qquad \frac{1+v/c}{1-v/c} = (Z+1)^2$

$$1+\frac{v}{c} = (Z+1)^2 - \left(\frac{v}{c}\right)(Z+1)^2 \qquad \left(\frac{v}{c}\right)\left(Z^2+2Z+2\right) = Z^2+2Z$$

$$v = \boxed{c\left(\frac{Z^2+2Z}{Z^2+2Z+2}\right)}$$

(b) $R = \frac{v}{H} = \boxed{\frac{c}{H}\left(\frac{Z^2+2Z}{Z^2+2Z+2}\right)}$

**P46.41** $v = HR$ $\qquad H = \frac{\left(1.7\times10^{-2}\text{ m/s}\right)}{\text{ly}}$

(a) $v\left(2.00\times10^{6}\text{ ly}\right) = 3.4\times10^{4}\text{ m/s}$ $\qquad \lambda' = \lambda\sqrt{\frac{1+v/c}{1-v/c}} = 590(1.000\,113\,3) = \boxed{590.07\text{ nm}}$

(b) $v\left(2.00\times10^{8}\text{ ly}\right) = 3.4\times10^{6}\text{ m/s}$ $\qquad \lambda' = 590\sqrt{\frac{1+0.011\,33}{1-0.011\,33}} = \boxed{597\text{ nm}}$

(c) $v\left(2.00\times10^{9}\text{ ly}\right) = 3.4\times10^{7}\text{ m/s}$ $\qquad \lambda' = 590\sqrt{\frac{1+0.113\,3}{1-0.113\,3}} = \boxed{661\text{ nm}}$

**P46.42** (a) Wien's law: $\lambda_{\max}T = 2.898\times10^{-3}\text{ m}\cdot\text{K}$.

Thus, $\lambda_{\max} = \frac{2.898\times10^{-3}\text{ m}\cdot\text{K}}{T} = \frac{2.898\times10^{-3}\text{ m}\cdot\text{K}}{2.73\text{ K}} = 1.06\times10^{-3}\text{ m} = \boxed{1.06\text{ mm}}$.

(b) This is a $\boxed{\text{microwave}}$.

***P46.43** We suppose that the fireball of the Big Bang is a black body.

$$I = e\sigma T^4 = (1)\left(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4\right)(2.73\text{ K})^4 = \boxed{3.15\times10^{-6}\text{ W/m}^2}$$

As a bonus, we can find the current power of direct radiation from the Big Bang in the section of the universe observable to us. If it is fifteen billion years old, the fireball is a perfect sphere of radius fifteen billion light years, centered at the point halfway between your eyes:

$$\mathcal{P} = IA = I(4\pi r^2) = \left(3.15\times10^{-6}\text{ W/m}^2\right)(4\pi)\left(15\times10^{9}\text{ ly}\right)^2\left(\frac{3\times10^{8}\text{ m/s}}{1\text{ ly/yr}}\right)^2\left(3.156\times10^{7}\text{ s/yr}\right)^2$$

$$\mathcal{P} = 7.98\times10^{47}\text{ W}.$$

**P46.44** The density of the Universe is

$$\rho = 1.20\rho_c = 1.20\left(\frac{3H^2}{8\pi G}\right).$$

Consider a remote galaxy at distance $r$. The mass interior to the sphere below it is

$$M = \rho\left(\frac{4}{3}\pi r^3\right) = 1.20\left(\frac{3H^2}{8\pi G}\right)\left(\frac{4}{3}\pi r^3\right) = \frac{0.600 H^2 r^3}{G}$$

both now and in the future when it has slowed to rest from its current speed $v = Hr$. The energy of this galaxy-sphere system is constant as the galaxy moves to apogee distance $R$:

$$\frac{1}{2}mv^2 - \frac{GmM}{r} = 0 - \frac{GmM}{R} \qquad \text{so} \qquad \frac{1}{2}mH^2r^2 - \frac{Gm}{r}\left(\frac{0.600H^2r^3}{G}\right) = 0 - \frac{Gm}{R}\left(\frac{0.600H^2r^3}{G}\right)$$

$$-0.100 = -0.600\frac{r}{R} \qquad \text{so} \qquad R = 6.00r.$$

The Universe will expand by a factor of $\boxed{6.00}$ from its current dimensions.

**P46.45** (a) $k_B T \approx 2m_p c^2$

$$\text{so} \quad T \approx \frac{2m_p c^2}{k_B} = \frac{2(938.3 \text{ MeV})}{\left(1.38 \times 10^{-23} \text{ J/K}\right)}\left(\frac{1.60 \times 10^{-13} \text{ J}}{1 \text{ MeV}}\right)\boxed{\sim 10^{13} \text{ K}}$$

(b) $k_B T \approx 2m_e c^2$

$$\text{so} \quad T \approx \frac{2m_e c^2}{k_B} = \frac{2(0.511 \text{ MeV})}{\left(1.38 \times 10^{-23} \text{ J/K}\right)}\left(\frac{1.60 \times 10^{-13} \text{ J}}{1 \text{ MeV}}\right)\boxed{\sim 10^{10} \text{ K}}$$

***P46.46** (a) The Hubble constant is defined in $v = HR$. The distance $R$ between any two far-separated objects opens at constant speed according to $R = vt$. Then the time $t$ since the Big Bang is found from

$$v = Hvt \qquad 1 = Ht \qquad t = \frac{1}{H}.$$

(b) $$\frac{1}{H} = \frac{1}{17 \times 10^{-3} \text{ m/s} \cdot \text{ly}}\left(\frac{3 \times 10^8 \text{ m/s}}{1 \text{ ly/yr}}\right) = \boxed{1.76 \times 10^{10} \text{ yr}} = 17.6 \text{ billion years}$$

***P46.47** (a) Consider a sphere around us of radius $R$ large compared to the size of galaxy clusters. If the matter $M$ inside the sphere has the critical density, then a galaxy of mass $m$ at the surface of the sphere is moving just at escape speed $v$ according to

$$K+U_g=0 \qquad \frac{1}{2}mv^2-\frac{GMm}{R}=0.$$

The energy of the galaxy-sphere system is conserved, so this equation is true throughout the history of the Universe after the Big Bang, where $v=\frac{dR}{dt}$. Then

$$\left(\frac{dR}{dt}\right)^2=\frac{2GM}{R} \qquad \frac{dR}{dt}=R^{-1/2}\sqrt{2GM} \qquad \int_0^R\sqrt{R}\,dR=\sqrt{2GM}\int_0^T dt$$

$$\left.\frac{R^{3/2}}{3/2}\right|_0^R=\sqrt{2GM}\,t\Big|_0^T \qquad \frac{2}{3}R^{3/2}=\sqrt{2GM}\,T$$

$$T=\frac{2}{3}\frac{R^{3/2}}{\sqrt{2GM}}=\frac{2}{3}\frac{R}{\sqrt{2GM/R}}.$$

From above, $\sqrt{\frac{2GM}{R}}=v$

so $T=\frac{2}{3}\frac{R}{v}$.

Now Hubble's law says $v=HR$.

So $T=\frac{2}{3}\frac{R}{HR}=\frac{2}{3H}$.

(b) $$T=\frac{2}{3\left(17\times10^{-3}\text{ m/s}\cdot\text{ly}\right)}\left(\frac{3\times10^{8}\text{ m/s}}{1\text{ ly/yr}}\right)=\boxed{1.18\times10^{10}\text{ yr}}=11.8\text{ billion years}$$

***P46.48** In our frame of reference, Hubble's law is exemplified by $\mathbf{v}_1=H\mathbf{R}_1$ and $\mathbf{v}_2=H\mathbf{R}_2$. From these we may form the equations $-\mathbf{v}_1=-H\mathbf{R}_1$ and $\mathbf{v}_2-\mathbf{v}_1=H(\mathbf{R}_2-\mathbf{R}_1)$. These equations express Hubble's law as seen by the observer in the first galaxy cluster, as she looks at us to find $-\mathbf{v}_1=H(-\mathbf{R}_1)$ and as she looks at cluster two to find $\mathbf{v}_2-\mathbf{v}_1=H(\mathbf{R}_2-\mathbf{R}_1)$.

## Section 46.13 **Problems and Perspectives**

**P46.49** (a) $$L=\sqrt{\frac{\hbar G}{c^3}}=\sqrt{\frac{\left(1.055\times10^{-34}\text{ J}\cdot\text{s}\right)\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)}{\left(3.00\times10^{8}\text{ m/s}\right)^3}}=\boxed{1.61\times10^{-35}\text{ m}}$$

(b) This time is given as $T=\frac{L}{c}=\frac{1.61\times10^{-35}\text{ m}}{3.00\times10^{8}\text{ m/s}}=\boxed{5.38\times10^{-44}\text{ s}}$,

which is approximately equal to the ultra-hot epoch.

(c) Yes.

### Additional Problems

**P46.50** We find the number $N$ of neutrinos:

$$10^{46}\ \text{J} = N(6\ \text{MeV}) = N\left(6\times 1.60\times 10^{-13}\ \text{J}\right)$$

$$N = 1.0\times 10^{58}\ \text{neutrinos}$$

The intensity at our location is

$$\frac{N}{A} = \frac{N}{4\pi r^2} = \frac{1.0\times 10^{58}}{4\pi\left(1.7\times 10^5\ \text{ly}\right)^2}\left(\frac{1\ \text{ly}}{\left(3.00\times 10^8\ \text{m/s}\right)\left(3.16\times 10^7\ \text{s}\right)}\right)^2 = 3.1\times 10^{14}\ \text{m}^{-2}.$$

The number passing through a body presenting $5\,000\ \text{cm}^2 = 0.50\ \text{m}^2$

is then $$\left(3.1\times 10^{14}\ \frac{1}{\text{m}^2}\right)\left(0.50\ \text{m}^2\right) = 1.5\times 10^{14}$$

or $$\boxed{\sim 10^{14}}.$$

***P46.51** A photon travels the distance from the Large Magellanic Cloud to us in 170 000 years. The hypothetical massive neutrino travels the same distance in 170 000 years plus 10 seconds:

$$c(170\,000\ \text{yr}) = v(170\,000\ \text{yr} + 10\ \text{s})$$

$$\frac{v}{c} = \frac{170\,000\ \text{yr}}{170\,000\ \text{yr} + 10\ \text{s}} = \frac{1}{1+\left\{10\ \text{s}\Big/\left[\left(1.7\times 10^5\ \text{yr}\right)\left(3.156\times 10^7\ \text{s/yr}\right)\right]\right\}} = \frac{1}{1+1.86\times 10^{-12}}$$

For the neutrino we want to evaluate $mc^2$ in $E = \gamma mc^2$:

$$mc^2 = \frac{E}{\gamma} = E\sqrt{1-\frac{v^2}{c^2}} = 10\ \text{MeV}\sqrt{1-\frac{1}{\left(1+1.86\times 10^{-12}\right)^2}} = 10\ \text{MeV}\sqrt{\frac{\left(1+1.86\times 10^{-12}\right)^2-1}{\left(1+1.86\times 10^{-12}\right)^2}}$$

$$mc^2 \approx 10\ \text{MeV}\sqrt{\frac{2\left(1.86\times 10^{-12}\right)}{1}} = 10\ \text{MeV}\left(1.93\times 10^{-6}\right) = 19\ \text{eV}.$$

Then the upper limit on the mass is

$$m = \boxed{\frac{19\ \text{eV}}{c^2}}$$

$$m = \frac{19\ \text{eV}}{c^2}\left(\frac{\text{u}}{931.5\times 10^6\ \text{eV}/c^2}\right) = 2.1\times 10^{-8}\ \text{u}.$$

**P46.52** (a) $\pi^- + \text{p} \rightarrow \Sigma^+ + \pi^0$ is forbidden by $\boxed{\text{charge conservation}}$.

(b) $\mu^- \rightarrow \pi^- + \nu_e$ is forbidden by $\boxed{\text{energy conservation}}$.

(c) $\text{p} \rightarrow \pi^+ + \pi^+ + \pi^-$ is forbidden by $\boxed{\text{baryon number conservation}}$.

**P46.53** The total energy in neutrinos emitted per second by the Sun is:

$$(0.4)\left[4\pi\left(1.5\times10^{11}\right)^2\right]\text{W} = 1.1\times10^{23}\text{ W}.$$

Over $10^9$ years, the Sun emits $3.6\times10^{39}$ J in neutrinos. This represents an annihilated mass

$$mc^2 = 3.6\times10^{39}\text{ J}$$

$$m = 4.0\times10^{22}\text{ kg}.$$

About $\boxed{\text{1 part in 50 000 000}}$ of the Sun's mass, over $10^9$ years, has been lost to neutrinos.

**P46.54** $\text{p}+\text{p}\rightarrow\text{p}+\pi^+ + X$

We suppose the protons each have 70.4 MeV of kinetic energy. From conservation of momentum for the collision, particle $X$ has zero momentum and thus zero kinetic energy. Conservation of system energy then requires

$$M_\text{p}c^2 + M_\pi c^2 + M_X c^2 = \left(M_\text{p}c^2 + K_\text{p}\right) + \left(M_\text{p}c^2 + K_\text{p}\right)$$

$$M_X c^2 = M_\text{p}c^2 + 2K_\text{p} - M_\pi c^2 = 938.3\text{ MeV} + 2(70.4\text{ MeV}) - 139.6\text{ MeV} = 939.5\text{ MeV}$$

$X$ must be a neutral baryon of rest energy 939.5 MeV. Thus $X$ is a $\boxed{\text{neutron}}$.

***P46.55** (a) If $2N$ particles are annihilated, the energy released is $2Nmc^2$. The resulting photon momentum is $p = \dfrac{E}{c} = \dfrac{2Nmc^2}{c} = 2Nmc$. Since the momentum of the system is conserved, the rocket will have momentum $2Nmc$ directed opposite the photon momentum.

$$\boxed{p = 2Nmc}$$

(b) Consider a particle that is annihilated and gives up its rest energy $mc^2$ to another particle which also has initial rest energy $mc^2$ (but no momentum initially).

$$E^2 = p^2c^2 + \left(mc^2\right)^2$$

Thus $\left(2mc^2\right)^2 = p^2c^2 + \left(mc^2\right)^2$.

Where $p$ is the momentum the second particle acquires as a result of the annihilation of the first particle. Thus $4\left(mc^2\right)^2 = p^2c^2 + \left(mc^2\right)^2$, $p^2 = 3\left(mc^2\right)^2$. So $p = \sqrt{3}mc$.

This process is repeated $N$ times (annihilate $\dfrac{N}{2}$ protons and $\dfrac{N}{2}$ antiprotons). Thus the total momentum acquired by the ejected particles is $\sqrt{3}Nmc$, and this momentum is imparted to the rocket.

$$\boxed{p = \sqrt{3}Nmc}$$

(c) Method (a) produces greater speed since $2Nmc > \sqrt{3}Nmc$.

**P46.56** (a) $\Delta E \Delta t \approx \hbar$, and $\Delta t = \frac{r}{c} = \frac{1.4 \times 10^{-15}\ \text{m}}{3 \times 10^{8}\ \text{m/s}} = 4.7 \times 10^{-24}\ \text{s}$

$$\Delta E \approx \frac{\hbar}{\Delta t} = \frac{1.055 \times 10^{-34}\ \text{J} \cdot \text{s}}{4.7 \times 10^{-24}\ \text{s}} = \left(2.3 \times 10^{-11}\ \text{J}\right)\left(\frac{1\ \text{MeV}}{1.60 \times 10^{-13}\ \text{J}}\right) = 1.4 \times 10^{2}\ \text{MeV}$$

$$m = \frac{\Delta E}{c^2} \approx 1.4 \times 10^{2}\ \text{MeV}/c^2\ \boxed{\sim 10^{2}\ \text{MeV}/c^2}$$

(b) From Table 46.2, $m_\pi c^2 = 139.6\ \text{MeV}\ \boxed{\text{a pi-meson}}$.

**P46.57** $m_\Lambda c^2 = 1115.6\ \text{MeV}$ $\Lambda^0 \to \text{p} + \pi^-$

$m_p c^2 = 938.3\ \text{MeV}$ $m_\pi c^2 = 139.6\ \text{MeV}$

The difference between starting rest energy and final rest energy is the kinetic energy of the products.

$$K_p + K_\pi = 37.7\ \text{MeV} \quad \text{and} \quad p_p = p_\pi = p$$

Applying conservation of relativistic energy to the decay process, we have

$$\left[\sqrt{(938.3)^2 + p^2c^2} - 938.3\right] + \left[\sqrt{(139.6)^2 + p^2c^2} - 139.6\right] = 37.7\ \text{MeV}.$$

Solving the algebra yields

$$p_\pi c = p_p c = 100.4\ \text{MeV}.$$

Then, $K_p = \sqrt{\left(m_p c^2\right)^2 + (100.4)^2} - m_p c^2 = \boxed{5.35\ \text{MeV}}$

$$K_\pi = \sqrt{(139.6)^2 + (100.4)^2} - 139.6 = \boxed{32.3\ \text{MeV}}.$$

**P46.58** By relativistic energy conservation in the reaction, $E_\gamma + m_e c^2 = \frac{3m_e c^2}{\sqrt{1 - v^2/c^2}}$. (1)

By relativistic momentum conservation for the system, $\frac{E_\gamma}{c} = \frac{3m_e v}{\sqrt{1 - v^2/c^2}}$. (2)

Dividing (2) by (1), $X = \frac{E_\gamma}{E_\gamma + m_e c^2} = \frac{v}{c}$.

Subtracting (2) from (1), $m_e c^2 = \frac{3m_e c^2}{\sqrt{1 - X^2}} - \frac{3m_e c^2 X}{\sqrt{1 - X^2}}$.

Solving, $1 = \frac{3 - 3X}{\sqrt{1 - X^2}}$ and $X = \frac{4}{5}$ so $E_\gamma = 4m_e c^2 = \boxed{2.04\ \text{MeV}}$.

**P46.59** Momentum of proton is $qBr = \left(1.60\times10^{-19}\text{ C}\right)(0.250\text{ kg/C}\cdot\text{s})(1.33\text{ m})$

$p_p = 5.32\times10^{-20}\text{ kg}\cdot m/s$ $\quad cp_p = 1.60\times10^{-11}\text{ kg}\cdot\text{m}^2/\text{s}^2 = 1.60\times10^{-11}\text{ J} = 99.8\text{ MeV}$.

Therefore, $p_p = 99.8\text{ MeV}/c$.

The total energy of the proton is $E_p = \sqrt{E_0^2 + (cp)^2} = \sqrt{(983.3)^2 + (99.8)^2} = 944\text{ MeV}$.

For pion, the momentum $qBr$ is the same (as it must be from conservation of momentum in a 2-particle decay).

$p_\pi = 99.8\text{ MeV}/c$ $\quad E_{0\pi} = 139.6\text{ MeV}$

$$E_\pi = \sqrt{E_0^2 + (cp)^2} = \sqrt{(139.6)^2 + (99.8)^2} = 172\text{ MeV}$$

Thus, $E_{\text{total after}} = E_{\text{total before}} = \text{Rest energy}$.

Rest Energy of unknown particle $= 944\text{ MeV} + 172\text{ MeV} = 1\,116\text{ MeV}$ (This is a $\Lambda^0$ particle!)

Mass $= \boxed{1\,116\text{ MeV}/c^2}$.

**P46.60** $\Sigma^0 \to \Lambda^0 + \gamma$

From Table 46.2, $m_\Sigma = 1\,192.5\text{ MeV}/c^2$ and $m_\Lambda = 1115.6\text{ MeV}/c^2$.

Conservation of energy in the decay requires

$E_{0,\Sigma} = \left(E_{o,\Lambda} + K_\Lambda\right) + E_\gamma$ or $1\,192.5\text{ MeV} = \left(1\,115.6\text{ MeV} + \dfrac{{p_\Lambda}^2}{2m_\Lambda}\right) + E_\gamma$.

System momentum conservation gives $|p_\Lambda| = |p_\gamma|$, so the last result may be written as

$$1\,192.5\text{ MeV} = \left(1\,115.6\text{ MeV} + \frac{{p_\gamma}^2}{2m_\Lambda}\right) + E_\gamma$$

or $$1\,192.5\text{ MeV} = \left(1\,115.6\text{ MeV} + \frac{{p_\gamma}^2c^2}{2m_\Lambda c^2}\right) + E_\gamma.$$

Recognizing that $m_\Lambda c^2 = 1\,115.6\text{ MeV}$ and $p_\gamma c = E_\gamma$

we now have $$1\,192.5\text{ MeV} = 1\,115.6\text{ MeV} + \frac{{E_\gamma}^2}{2(1\,115.6\text{ MeV})} + E_\gamma.$$

Solving this quadratic equation, $E_\gamma = \boxed{74.4\text{ MeV}}$.

**P46.61** $\text{p} + \text{p} \to \text{p} + \text{n} + \pi^+$

The total momentum is zero before the reaction. Thus, all three particles present after the reaction may be at rest and still conserve system momentum. This will be the case when the incident protons have minimum kinetic energy. Under these conditions, conservation of energy for the reaction gives

$$2\left(m_pc^2 + K_p\right) = m_pc^2 + m_nc^2 + m_\pi c^2$$

so the kinetic energy of each of the incident protons is

$$K_p = \frac{m_nc^2 + m_\pi c^2 - m_pc^2}{2} = \frac{(939.6 + 139.6 - 938.3)\text{ MeV}}{2} = \boxed{70.4\text{ MeV}}.$$

**P46.62** $\pi^- \to \mu^- + \overline{\nu}_\mu$: From the conservation laws for the decay,

$$m_\pi c^2 = 139.6 \text{ MeV} = E_\mu + E_\nu \qquad [1]$$

and $p_\mu = p_\nu$, $E_\nu = p_\nu c$ : $E_\mu^2 = (p_\mu c)^2 + (105.7 \text{ MeV})^2 = (p_\nu c)^2 + (105.7 \text{ MeV})^2$

or $E_\mu^2 - E_\nu^2 = (105.7 \text{ MeV})^2$. [2]

Since $E_\mu + E_\nu = 139.6 \text{ MeV}$ [1]

and $(E_\mu + E_\nu)(E_\mu - E_\nu) = (105.7 \text{ MeV})^2$ [2]

then $E_\mu - E_\nu = \dfrac{(105.7 \text{ MeV})^2}{139.6 \text{ MeV}} = 80.0$. [3]

Subtracting [3] from [1], $2E_\nu = 59.6 \text{ MeV}$ and $\boxed{E_\nu = 29.8 \text{ MeV}}$.

**P46.63** The expression $e^{-E/k_BT}dE$ gives the fraction of the photons that have energy between $E$ and $E + dE$. The fraction that have energy between $E$ and infinity is

$$\frac{\int_E^\infty e^{-E/k_BT}dE}{\int_0^\infty e^{-E/k_BT}dE} = \frac{\int_E^\infty e^{-E/k_BT}(-dE/k_BT)}{\int_0^\infty e^{-E/k_BT}(-dE/k_BT)} = \frac{e^{-E/k_BT}\Big|_E^\infty}{e^{-E/k_BT}\Big|_0^\infty} = e^{-E/k_BT}.$$

We require $T$ when this fraction has a value of 0.0100 (i.e., 1.00%)

and $E = 1.00 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$.

Thus, $0.010\,0 = e^{-(1.60\times10^{-19} \text{ J})/(1.38\times10^{-23} \text{ J/K})T}$

or $\ln(0.010\,0) = -\dfrac{1.60\times10^{-19} \text{ J}}{(1.38\times10^{-23} \text{ J/K})T} = -\dfrac{1.16\times10^4 \text{ K}}{T}$ giving $T = \boxed{2.52\times10^3 \text{ K}}$.

**P46.64** (a) This diagram represents the annihilation of an electron and an antielectron. From charge and lepton-number conservation at either vertex, the exchanged particle must be an electron, $\boxed{e^-}$.

(b) This is the tough one. A neutrino collides with a neutron, changing it into a proton with release of a muon. This is a weak interaction. The exchanged particle has charge $+e$ and is a $\boxed{W^+}$.

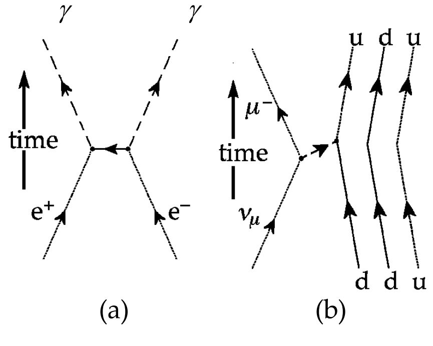

**FIG. P46.64**

**P46.65** (a) The mediator of this weak interaction is a $\boxed{\mathrm{Z}^0 \text{ boson}}$.

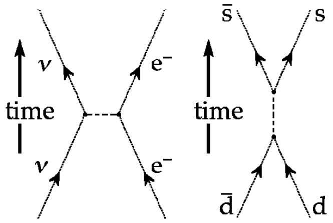

**FIG. P46.65**

(b) The Feynman diagram shows a down quark and its antiparticle annihilating each other. They can produce a particle carrying energy, momentum, and angular momentum, but zero charge, zero baryon number, and, it may be, no color charge. In this case the product particle is a $\boxed{\text{photon}}$.

For conservation of both energy and momentum in the collision we would expect two photons; but momentum need not be strictly conserved, according to the uncertainty principle, if the photon travels a sufficiently short distance before producing another matter-antimatter pair of particles, as shown in Figure P46.65. Depending on the color charges of the d and $\bar{\mathrm{d}}$ quarks, the ephemeral particle could also be a $\boxed{\text{gluon}}$, as suggested in the discussion of Figure 46.14(b).

***P46.66** (a) At threshold, we consider a photon and a proton colliding head-on to produce a proton and a pion at rest, according to $\mathrm{p}+\gamma \to \mathrm{p}+\pi^0$. Energy conservation gives

$$\frac{m_\mathrm{p}c^2}{\sqrt{1-u^2/c^2}}+E_\gamma = m_\mathrm{p}c^2+m_\pi c^2.$$

Momentum conservation gives $\dfrac{m_\mathrm{p}u}{\sqrt{1-u^2/c^2}}-\dfrac{E_\gamma}{c}=0$.

Combining the equations, we have

$$\frac{m_\mathrm{p}c^2}{\sqrt{1-u^2/c^2}}+\frac{m_\mathrm{p}c^2}{\sqrt{1-u^2/c^2}}\frac{u}{c}=m_\mathrm{p}c^2+m_\pi c^2$$

$$\frac{938.3\text{ MeV}(1+u/c)}{\sqrt{(1-u/c)(1+u/c)}}=938.3\text{ MeV}+135.0\text{ MeV}$$

so $\dfrac{u}{c}=0.134$

and $E_\gamma=\boxed{127\text{ MeV}}$.

(b) $\lambda_{\max}T=2.898\text{ mm}\cdot\text{K}$

$$\lambda_{\max}=\frac{2.898\text{ mm}\cdot\text{K}}{2.73\text{ K}}=\boxed{1.06\text{ mm}}$$

(c) $E_\gamma=hf=\dfrac{hc}{\lambda}=\dfrac{1\,240\text{ eV}\cdot 10^{-9}\text{ m}}{1.06\times10^{-3}\text{ m}}=\boxed{1.17\times10^{-3}\text{ eV}}$

*continued on next page*

(d) In the primed reference frame, the proton is moving to the right at $\frac{u'}{c} = 0.134$ and the photon is moving to the left with $hf' = 1.27 \times 10^{8}$ eV. In the unprimed frame, $hf = 1.17 \times 10^{-3}$ eV. Using the Doppler effect equation from Section 39.4, we have for the speed of the primed frame

$$1.27 \times 10^{8} = \sqrt{\frac{1 + v/c}{1 - v/c}}\, 1.17 \times 10^{-3}$$

$$\frac{v}{c} = 1 - 1.71 \times 10^{-22}$$

Then the speed of the proton is given by

$$\frac{u}{c} = \frac{u'/c + v/c}{1 + u'v/c^{2}} = \frac{0.134 + 1 - 1.71 \times 10^{-22}}{1 + 0.134\left(1 - 1.71 \times 10^{-22}\right)} = 1 - 1.30 \times 10^{-22}.$$

And the energy of the proton is

$$\frac{m_{\mathrm{p}}c^{2}}{\sqrt{1 - u^{2}/c^{2}}} = \frac{938.3 \text{ MeV}}{\sqrt{1 - \left(1 - 1.30 \times 10^{-22}\right)^{2}}} = 6.19 \times 10^{10} \times 938.3 \times 10^{6} \text{ eV} = \boxed{5.81 \times 10^{19} \text{ eV}}.$$

## ANSWERS TO EVEN PROBLEMS

| | |
|---|---|
| **P46.2** | $2.27 \times 10^{23}$ Hz; 1.32 fm |
| **P46.4** | $\overline{\nu}_{\mu}$ and $\nu_{\mathrm{e}}$ |
| **P46.6** | $\sim 10^{-16}$ m |
| **P46.8** | $\sim 10^{-23}$ s |
| **P46.10** | $\overline{\nu}_{\mu}$ |
| **P46.12** | (a) electron lepton number and muon lepton number; (b) charge;<br>(c) baryon number; (d) baryon number;<br>(e) charge |
| **P46.14** | the second violates conservation of baryon number |
| **P46.16** | $0.828c$ |
| **P46.18** | (a) see the solution;<br>(b) 469 MeV; $\frac{469 \text{ MeV}}{c}$ for both;<br>(c) $0.999\,999\,4c$ |
| **P46.20** | see the solution |
| **P46.22** | (a) electron lepton number and muon lepton number;<br>(b) electron lepton number;<br>(c) strangeness and charge;<br>(d) baryon number; (e) strangeness |
| **P46.24** | see the solution |
| **P46.26** | (a) $\frac{686 \text{ MeV}}{c}$ and $\frac{200 \text{ MeV}}{c}$;<br>(b) 627 MeV/$c$;<br>(c) 244 MeV, 1 130 MeV, 1 370 MeV;<br>(d) 1 190 MeV/$c^{2}$, $0.500c$ |
| **P46.28** | (a) see the solution; (b) 5.63 GeV;<br>(c) 768 MeV; (d) 280 MeV; (e) 4.43 TeV |
| **P46.30** | see the solution |
| **P46.32** | see the solution |
| **P46.34** | see the solution |
| **P46.36** | (a) $\Sigma^{+}$; (b) $\pi^{-}$; (c) $\mathrm{K}^{0}$; (d) $\Xi^{-}$ |
| **P46.38** | see the solution |

| | |
|---|---|
| **P46.40** | (a) $v = c\left(\dfrac{Z^2+2Z}{Z^2+2Z+2}\right)$; (b) $\dfrac{c}{H}\left(\dfrac{Z^2+2Z}{Z^2+2Z+2}\right)$ |
| **P46.42** | (a) 1.06 mm; (b) microwave |
| **P46.44** | 6.00 |
| **P46.46** | (a) see the solution; (b) 17.6 Gyr |
| **P46.48** | see the solution |
| **P46.50** | $\sim 10^{14}$ |
| **P46.52** | (a) charge; (b) energy; (c) baryon number |
| **P46.54** | neutron |
| **P46.56** | (a) $\sim 10^2\ \text{MeV}/c^2$; (b) a pi-meson |
| **P46.58** | 2.04 MeV |
| **P46.60** | 74.4 MeV |
| **P46.62** | 29.8 MeV |
| **P46.64** | (a) electron-position annihilation; $e^-$;<br>(b) a neutrino collides with a neutron, producing a proton and a muon; $W^+$ |
| **P46.66** | (a) 127 MeV; (b) 1.06 mm; (c) 1.17 meV;<br>(d) $5.81 \times 10^{19}$ eV |